

Neutrino-jet correlations in charged-current SIDIS

Weihua Yang¹, Jing Zhao^{2,*} and Zhe Zhang^{3,†}

¹*College of Nuclear Equipment and Nuclear Engineering, Yantai University,
Yantai, Shandong 264005, China*

²*Key Laboratory of Particle Physics and Particle Irradiation (MOE),
Institute of Frontier and Interdisciplinary Science, Shandong University,
Qingdao, Shandong 266237, China*

³*Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics,
Chinese Academy of Sciences, Huizhou 516000, China*



(Received 14 May 2025; accepted 26 July 2025; published 20 August 2025)

Charged-current deep inelastic scattering plays a significant role in determining parton distribution functions with flavor separation. In this work, we present a systematic calculation of the charged-current semi-inclusive deep inelastic scattering (SIDIS) in the eN collinear frame up to twist-3 level at leading order. Semi-inclusive refers to the process in which a jet is detected in addition to the scattered neutrino. We focus on neutrino-jet correlations in our calculation. We first present the differential cross section in terms of structure functions, followed by the differential cross section expressed in term of transverse momentum dependent parton distribution functions. We derive the complete set of azimuthal asymmetries and intrinsic asymmetries. We also introduce an observable A^C , defined as the ratio of the difference from the sum of differential cross sections for electron and positron semi-inclusive deep inelastic scattering. We notice that A^C provides a sensitive probe for valence and sea quark distribution functions.

DOI: [10.1103/z8tf-mzrw](https://doi.org/10.1103/z8tf-mzrw)

I. INTRODUCTION

The lepton-nucleon deep inelastic scattering (DIS) has achieved great success in our understanding of the partonic structure of the nucleon. It will still play an important role in the future Electron-Ion Collider (EIC) [1–3] experiment to explore the spin and three-dimensional structure of the nucleon over wide kinematic regions. Thanks to the factorization theorem [4], the cross section of DIS can be factorized into two parts: one is the hard part that can be calculated with perturbative theory, and the other is the soft part that involves parton distribution functions (PDFs). The one-dimensional or collinear PDFs are studied in the inclusive DIS, where only the scattered lepton is detected with high precision. However, inclusive DIS cannot access three-dimensional or transverse momentum dependent PDFs (TMDs), since no transverse momentum scale is introduced. To solve this problem, one needs to consider

the semi-inclusive DIS (SIDIS), where a hadron or a jet is also detected in addition to the scattered lepton.

For the hadron production in SIDIS [5,6], fragmentation functions (FFs) which describe the formation of hadrons from a partonic initial state are involved. The cross section will be given in terms of convolutions of PDFs and FFs. Fragmentation functions are nonperturbative quantities and can only be determined by experimental measurements. Comparing to hadron production SIDIS, the jet production one does have a simpler form of theoretical formulation and does not introduce extra uncertainties from FFs. This is helpful to improve the measurement accuracy. Furthermore, the jet can be a direct probe of analyzing properties of the partonic structure of nucleon. For example, the transverse momentum of the jet is equal to that of the quark in the VN collinear frame [7–10]. Here V denotes the intermediate exchange boson, whose transverse momentum is defined as zero. For these reasons, jet production SIDIS for TMD studies has attracted a lot of attentions. Nevertheless, the jet production SIDIS process cannot cover the low energy kinematic regions and access the chiral-odd TMDs due to the conservation of the helicity. The hadron production SIDIS process therefore has to be considered to study chiral-odd TMDs. We note here that proposals for exploring the chiral-odd TMDs via jet fragmentation functions have been studied recently in Refs. [11–14].

*Contact author: zhaojingzj@sdu.edu.cn

†Contact author: zhangzhe@impcas.ac.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

In this paper, we calculate the charged-current jet production SIDIS process in the eN collinear frame. Charged-current interactions can be used as a clean probe to separate quark flavors, which cannot be achieved in the neutral-current DIS alone. Furthermore, charged-current interactions with the high statistics samples can offer precision measurements of electroweak parameters, e.g., a weak mixing angle [15], which is used to determine the running of $\sin^2 \theta_W$ as a function of Q^2 . In our analysis, the jet is simplified as a quark. Considering the conservation of momentum, the jet and the scattered electron are produced back-to-back in the plane perpendicular to the beam direction. However, the intrinsic transverse momentum of the quark or higher-order gluon radiations would break the balance and induce measurable effects. We notice that these effects induced by the intrinsic transverse momentum of the quark provide a favorable palace to study twist-3 TMDs. To this end, we present systematic calculations of twist-3 measurable quantities for the neutrino-jet correlations ($j = l' + k'$) in the eN collinear frame. Our method is consistent with that of Refs. [16–20], which focused on higher twist calculations at leading order. On the other hand, calculations can also be done from a higher-order perspective [21–28]. In this paper, we calculate both the electron and positron scattering off a nucleon (nucleus) and write theoretical expressions in a general form. We first present the differential cross section in terms of structure functions, followed by the differential cross section given in terms of TMDs at twist-3 level. We further calculate the complete azimuthal asymmetries and intrinsic asymmetries. We also introduce an observable A^C , defined as the ratio of the difference from the sum of differential cross sections for electron and positron SIDIS processes. We notice that A^C is sensitive to quark distribution functions and can be used to study the violation of strange-antistrange symmetry. We note here that our consideration is different from that in Ref. [27], in which both neutrino-jet correlations and hadron-in-jet spin asymmetries were calculated at leading twist. Specially, we focus on the neutrino-jet correlations and perform the calculation at the twist-3 level.

To be explicit, we organize this paper as follows. In Sec. II, we present the formalism and conventions used in this paper and give a brief introduction of the eN collinear frame. In Sec. III, we present a systematic calculation of the differential cross section of the jet production SIDIS in terms of structure functions. The systematic calculations in the parton model up to twist-3 level are shown in Sec. IV. Measurable quantities and numerical estimates are given in Sec. V. A brief summary is given in Sec. VI.

II. THE FORMALISM

In this paper, we consider charged-current semi-inclusive deep inelastic scattering, in which both a jet and the scattered lepton (neutrino or antineutrino) are detected in

the final state. The electron scattering process is labeled as follows:

$$e^-(l) + N(p, S) \rightarrow \nu_e(l') + jet(k') + X, \quad (2.1)$$

where e^- denotes an electron with momentum l , N represents a nucleon with momentum p , and the jet corresponds to a quark with momentum k' , which is observed as a jet of hadrons in experiments. We require that the jet is in the current fragmentation region. For the positron scattering process, e^- is replaced by e^+ , and the final-state neutrino ν_e is replaced by the antineutrino $\bar{\nu}_e$. To be explicit, we list the standard variables for the jet production SIDIS,

$$s = (p + l)^2, \quad x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot l}, \quad (2.2)$$

where $Q^2 = -q^2 = -(l - l')^2$.

Regardless of whether the process involves electron or positron scattering, the differential cross section can always be expressed as the contraction of the leptonic tensor and the hadronic tensor,

$$d\sigma = \frac{\alpha_{em}^2}{2sQ^4} A_W L_{\mu\nu}(l, l') W^{\mu\nu}(q, p, S, k') \frac{d^3 l' d^3 k'}{(2\pi)^3 E_{l'} E_{k'}}, \quad (2.3)$$

where α_{em} is the fine structure constant. The A_W factor is calculated as

$$A_W = \frac{Q^4}{[(Q^2 + M_W^2)^2 + \Gamma_W^2 M_W^2] 16 \sin^4 \theta_W}, \quad (2.4)$$

where M_W and Γ_W indicate the mass and width of the W boson, respectively, and the θ_W denotes the weak mixing angle. The leptonic tensor is given by

$$L_{\mu\nu}(l, l') = 2(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu} l \cdot l') + 2i\lambda_n \epsilon_{\mu\nu l l'}, \quad (2.5)$$

where λ_n is introduced for convenience. For neutrino production and antineutrino production, λ_n is -1 and 1 , respectively. The hadronic tensor is given by

$$W^{\mu\nu}(q, p, k') = \sum_X (2\pi)^3 \delta^4(p + q - k' - p_X) \times \langle N, S | J^\mu(0) | k'; X \rangle \langle k'; X | J^\nu(0) | N, S \rangle. \quad (2.6)$$

The weak current is defined as $J^\mu(0) = \bar{\psi}(0) \Gamma^\mu \psi(0)$, where $\Gamma^\mu = \gamma^\mu (c_V^q - c_A^q \gamma^5)$ is the interaction vertex, with the weak couplings c_V^q and c_A^q . For charged-current weak interaction, both couplings take the value $c_V^q = c_A^q = 1$. We keep these symbols in the following context for convenience. It is also convenient to consider the k'_T -dependent cross section, i.e.,

$$d\sigma = \frac{\alpha_{\text{em}}^2}{sQ^4} A_W L_{\mu\nu}(l, \lambda_n, l') W^{\mu\nu}(q, p, S, k'_T) \frac{d^3 l' d^2 k'_T}{E_{l'}}, \quad (2.7)$$

where the k'_z -integrated hadronic tensor is given by

$$W^{\mu\nu}(q, p, S, k'_T) = \int \frac{dk'_z}{(2\pi)^3 2E_{k'}} W^{\mu\nu}(q, p, S, k'). \quad (2.8)$$

Following the convention of investigating the neutrino-jet correlations in the eN collinear frame, we define the sum of the momenta of the scattered neutrino (antineutrino) and the jet as $j = l' + k'$. In this collinear frame, we have

$$\vec{j}_T = \vec{l}'_T + \vec{k}'_T = \vec{l}'_T + \vec{k}_T + \vec{q}_T = \vec{k}_T \quad (2.9)$$

if higher-order gluon radiations are neglected. In other words, the transverse momentum j_T equals the intrinsic transverse momentum k_T of a quark in the nucleon (nucleus). Therefore, the cross section can be rewritten as

$$\frac{d\sigma}{d\eta d^2 l'_T d^2 j_T} = \frac{\alpha_{\text{em}}^2}{sQ^4} A_W L_{\mu\nu}(l, \lambda_n, l') W^{\mu\nu}(q, p, S, j_T), \quad (2.10)$$

where η is the rapidity of the scattered lepton. Here we have used the relation $d\eta = dl'_z/E_{l'}$.

As illustrated in Fig. 1, the eN collinear frame is defined such that the target moves along the $+z$ direction, while the incoming lepton travels along the $-z$ direction. The scattered (anti)neutrino lies in the xoz plane, which is known as the lepton plane. In this frame, we use the light-cone unit vectors $\bar{t}^\mu = (1, 0, \vec{0}_T)$ and $t^\mu = (0, 1, \vec{0}_T)$ to decompose the relevant momenta, and they satisfy $\bar{t}^2 = t^2 = 0$ and $\bar{t} \cdot t = 1$. Therefore, we have

$$p^\mu = p^+ \bar{t}^\mu + p^- t^\mu, \quad (2.11)$$

$$l^\mu = l^+ \bar{t}^\mu + l^- t^\mu, \quad (2.12)$$

where the plus (minus) components are defined as $p^\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^3)$, and similarly for $l^\pm$. Up to $\mathcal{O}(1/Q^2)$, the momenta can be approximated as $p^\mu \approx p^+ \bar{t}^\mu$ and $l^\mu \approx l^- t^\mu$. Correspondingly, the light-cone vector in this

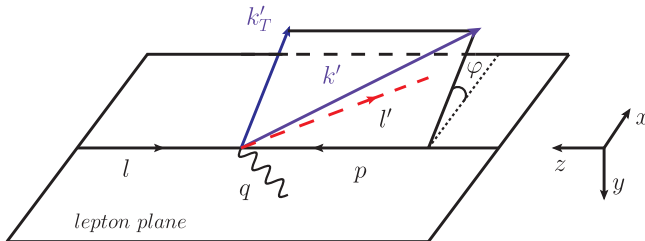


FIG. 1. The SIDIS process of the jet productions in the eN collinear frame.

frame can be defined as $\bar{t}^\mu = p^\mu/p^+$ and $t^\mu = l^\mu/l^-$. Therefore, the relevant momenta can be parametrized as

$$p^\mu = (p^+, 0, \vec{0}_T), \quad (2.13)$$

$$l^\mu = \left(0, \frac{Q^2}{2xy p^+}, \vec{0}_T\right), \quad (2.14)$$

$$l'^\mu = \left(xy p^+, \frac{(1-y)Q^2}{2xy p^+}, Q\sqrt{1-y}, 0\right), \quad (2.15)$$

$$q^\mu = \left(-xy p^+, \frac{Q^2}{2x p^+}, -Q\sqrt{1-y}, 0\right). \quad (2.16)$$

Since the transverse momentum j_T equals the intrinsic transverse momentum k_T of a quark in the nucleon in the eN collinear frame, we parametrize them as

$$j_T^\mu = k_T^\mu = |k_T|(0, 0, \cos \varphi, \sin \varphi), \quad (2.17)$$

and we do not distinguish them in the following context. The polarization vector of the nucleon is described by the helicity λ_N and the transverse polarization vector S_T^μ , where S_T^μ can be decomposed as

$$S_T^\mu = |S_T|(0, 0, \cos \varphi_S, \sin \varphi_S). \quad (2.18)$$

Here we note that the transverse component is defined with respect to the z direction determined by momenta of the incoming lepton and the target nucleon.

Another coordinate frame commonly used in SIDIS is the boson-nucleon (VN) collinear frame, illustrated in Fig. 2, where the nucleon travels along the $+z$ direction while the intermediate exchange boson moves along the $-z$ direction. In this frame, we decompose the relevant momenta using the unit vectors $(\bar{n}, n)$, which are distinguished from $(\bar{t}, t)$ defined in the eN collinear frame. We also have

$$p^\mu = \tilde{p}^+ \bar{n}^\mu + \tilde{p}^- n^\mu, \quad (2.19)$$

$$q^\mu = q^+ \bar{n}^\mu + q^- n^\mu, \quad (2.20)$$

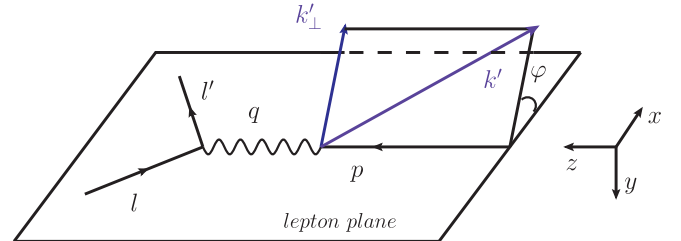


FIG. 2. The SIDIS process of the jet productions in the VN collinear frame.

where the plus component of p^μ is written as $\tilde{p}$ to distinguish that in the eN collinear frame. Up to $\mathcal{O}(1/Q^2)$, only the large plus component of p^μ survives. The light-cone vectors can be defined as $\tilde{n}^\mu = p^\mu/\tilde{p}^+$ and $n^\mu = (q^\mu + xp^\mu)/q^-$. Under these conditions, we can parametrize momenta of the involved particles as

$$\begin{aligned} p^\mu &= (\tilde{p}^+, 0, \vec{0}_\perp), \\ q^\mu &= \left(-x\tilde{p}^+, \frac{Q^2}{2x\tilde{p}^+}, \vec{0}_\perp\right), \\ l^\mu &= \left(\frac{1-y}{y}x\tilde{p}^+, \frac{Q^2}{2xy\tilde{p}^+}, \frac{Q\sqrt{1-y}}{y}, 0\right), \\ l'^\mu &= \left(\frac{1}{y}x\tilde{p}^+, \frac{(1-y)Q^2}{2xy\tilde{p}^+}, \frac{Q\sqrt{1-y}}{y}, 0\right). \end{aligned} \quad (2.21)$$

The relationships between $(\tilde{t}, t)$ and $(\tilde{n}, n)$ can be expressed as

$$\tilde{t}^\mu = \frac{(2-y)}{y} \frac{\tilde{p}^+}{p^+} \tilde{n}^\mu + \frac{y l_T^\mu + q_T^\mu}{xy p^+}, \quad (2.22)$$

$$t^\mu = \frac{2(1-y)x^2\tilde{p}^+ p^+}{yQ^2} \tilde{n}^\mu + \frac{p^+}{\tilde{p}^+} n^\mu + \frac{2xy p^+}{Q^2} l_T^\mu. \quad (2.23)$$

According to our conventions, the transverse components of l^μ and q^μ can be written as

$$l_T = l_x = \frac{Q\sqrt{1-y}}{y}, \quad (2.24)$$

$$q_T = q_x = -Q\sqrt{1-y}, \quad (2.25)$$

and they lie in the lepton plane or the x - z plane. Under this circumstance, the second term in Eq. (2.22) vanishes and

$$p^+ \tilde{t}^\mu = \frac{(2-y)}{y} \tilde{p}^+ \tilde{n}^\mu. \quad (2.26)$$

That is to say, $\tilde{p}^+$ and p^+ do not have to be equal.

III. THE CROSS SECTION IN TERMS OF STRUCTURE FUNCTIONS

In this section, we perform a general kinematic analysis for this charged-current SIDIS process. We first decompose the hadronic tensor under the limit of kinematic constraints, and then express the differential cross section in terms of structure functions.

Since the hadronic tensor contains the unperturbative information in the nucleon, it cannot be calculated by perturbative theory. It is well known that the hadronic tensor must satisfy the conditions required by the Hermiticity, gauge invariance, and parity invariance,

$$W^{*\mu\nu}(q, p, S, j_T) = W^{\mu\nu}(q, p, S, j_T), \quad (3.1)$$

$$q_\mu W^{\mu\nu}(q, p, S, j_T) = q_\nu W^{\mu\nu}(q, p, S, j_T) = 0, \quad (3.2)$$

$$W^{\mu\nu}(q, p, S, j_T) = W_{\mu\nu}(q^{\mathcal{P}}, p^{\mathcal{P}}, -S^{\mathcal{P}}, j_T^{\mathcal{P}}), \quad (3.3)$$

where the superscript $\mathcal{P}$ indicates a sign flip of all space components, such as $A_\mu^{\mathcal{P}} = A^\mu$. We note here that the parity invariance has no constraint on the hadronic tensor because weak interaction is considered in this process. With the constraints of Hermiticity and gauge invariance, we decompose the hadronic tensor in terms of basic Lorentz tensors (BLTs) multiplied by scalar functions. One can divide the hadronic tensor into a symmetric part and an antisymmetric part, i.e., $W^{\mu\nu} = W^{S\mu\nu} + iW^{A\mu\nu}$. The more explicit expression is given by

$$W^{S\mu\nu} = \sum_{\sigma,i} W_{\sigma i}^S h_{\sigma i}^{S\mu\nu} + \sum_{\sigma,i} \tilde{W}_{\sigma i}^S \tilde{h}_{\sigma i}^{S\mu\nu}, \quad (3.4)$$

$$W^{A\mu\nu} = \sum_{\sigma,i} W_{\sigma i}^A h_{\sigma i}^{A\mu\nu} + \sum_{\sigma,i} \tilde{W}_{\sigma i}^A \tilde{h}_{\sigma i}^{A\mu\nu}, \quad (3.5)$$

where $h_{\sigma i}^{\mu\nu}$'s and $\tilde{h}_{\sigma i}^{\mu\nu}$'s represent the parity conserving and flipping BLTs, respectively. These coefficients $W_{\sigma i}$'s are scalar functions. We use the subscript σ to specify the spin states of the target particle.

The BLTs can be constructed from the kinematic variables involved in the process. For the unpolarized case, we obtain nine independent BLTs as

$$h_{U_i}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, p_q^\mu p_q^\nu, j_{Tq}^\mu j_{Tq}^\nu, p_q^{\{\mu} j_{Tq}^{\nu\}} \right\}, \quad (3.6)$$

$$\tilde{h}_{U_i}^{S\mu\nu} = \{ \epsilon^{\{\mu q p j_T\}} p_q^{\nu\}}, \epsilon^{\{\mu q p j_T\}} j_{Tq}^{\nu\} \}, \quad (3.7)$$

$$h_U^{A\mu\nu} = \{ p_q^{[\mu} j_{Tq}^{\nu]} \}, \quad (3.8)$$

$$\tilde{h}_{U_i}^{A\mu\nu} = \{ \epsilon^{\mu\nu q p}, \epsilon^{\mu\nu q j_T} \}, \quad (3.9)$$

where $p_q^\mu \equiv p^\mu - q^\mu(p \cdot q)/q^2$, which satisfies $p_q \cdot q = 0$. The subscript U indicates the unpolarized part. We also use the symmetrization and antisymmetrization conventions, i.e., $A^{\{\mu} B^{\mu\}} \equiv A^\mu B^\mu + A^\nu B^\nu$ and $A^{[\mu} B^{\mu]} \equiv A^\mu B^\mu - A^\nu B^\nu$.

As shown in Refs. [29,30], the polarization dependent BLTs can be constructed from multiplying the unpolarized BLTs given in Eqs. (3.6)–(3.9) by the polarization dependent scalars or pseudoscalars. Therefore, we obtain the vector polarized BLTs as follows:

$$h_{V_i}^{S\mu\nu} = \{ [(q \cdot S), (j_T \cdot S)] \tilde{h}_{U_i}^{S\mu\nu}, \epsilon^{Sq p j} h_{U_j}^{S\mu\nu} \}, \quad (3.10)$$

$$\tilde{h}_{V_i}^{S\mu\nu} = \{ [(q \cdot S), (j_T \cdot S)] h_{U_i}^{S\mu\nu}, \epsilon^{Sq p j} \tilde{h}_{U_j}^{S\mu\nu} \}, \quad (3.11)$$

$$h_{V_i}^{A\mu\nu} = \{[(q \cdot S), (j_T \cdot S)]\tilde{h}_{U_i}^{A\mu\nu}, \varepsilon^{Sqpj}h_{U_i}^{A\mu\nu}\}, \quad (3.12)$$

$$\tilde{h}_{V_i}^{A\mu\nu} = \{[(q \cdot S), (j_T \cdot S)]h_{U_i}^{A\mu\nu}, \varepsilon^{Sqpj}\tilde{h}_{U_i}^{A\mu\nu}\}, \quad (3.13)$$

where the subscript V represents the vector polarized dependence.

Substituting the BLTs shown in Eqs. (3.6)–(3.13) into Eqs. (3.4) and (3.5), one can obtain the complete decomposition of the hadronic tensor. After contracting it with the leptonic tensor, we can express the differential cross section in a general form. Neglecting the complicated calculations,

we here show only the final result. According to the polarization states of the nucleon, the differential cross section can be divided into three parts,

$$\frac{d\sigma}{d\eta d^2l_T d^2j_T} = \frac{\alpha_{\text{em}}^2}{Q^4} xy A_W [\mathcal{F}_U + \lambda_N \mathcal{F}_L + |S_T| \mathcal{F}_T], \quad (3.14)$$

where the subscripts indicate the polarization states of the target particle. The explicit form of each part can be expressed in terms of the corresponding structure functions, which are given by

$$\mathcal{F}_U = F_U + \sin \varphi F_U^{\sin \varphi} + \sin 2\varphi F_U^{\sin 2\varphi} + \cos \varphi F_U^{\cos \varphi}, \quad (3.15)$$

$$\mathcal{F}_L = F_L + \sin \varphi F_L^{\sin \varphi} + \sin 2\varphi F_L^{\sin 2\varphi} + \cos \varphi F_L^{\cos \varphi} + \cos 2\varphi F_L^{\cos 2\varphi} + \cos 3\varphi F_L^{\cos 3\varphi}, \quad (3.16)$$

$$\begin{aligned} \mathcal{F}_T = & \sin \varphi_S F_T^{\sin \varphi_S} + \sin(\varphi + \varphi_S) F_T^{\sin(\varphi + \varphi_S)} + \sin(2\varphi + \varphi_S) F_T^{\sin(2\varphi + \varphi_S)} + \sin(\varphi - \varphi_S) F_T^{\sin(\varphi - \varphi_S)} \\ & + \sin(2\varphi - \varphi_S) F_T^{\sin(2\varphi - \varphi_S)} + \sin(3\varphi - \varphi_S) F_T^{\sin(3\varphi - \varphi_S)} + \cos \varphi_S F_T^{\cos \varphi_S} + \cos(\varphi + \varphi_S) F_T^{\cos(\varphi + \varphi_S)} \\ & + \cos(\varphi - \varphi_S) F_T^{\cos(\varphi - \varphi_S)} + \cos(2\varphi - \varphi_S) F_T^{\cos(2\varphi - \varphi_S)} + \cos(3\varphi - \varphi_S) F_T^{\cos(3\varphi - \varphi_S)}, \end{aligned} \quad (3.17)$$

where each term corresponds to a combination of nucleon polarization state and azimuthal modulation.

IV. THE CALCULATION IN THE PARTON MODEL

A. The leading-twist hadronic tensor

In the parton model, we calculate the hadronic tensor up to the twist-3 level. To show a clear calculation, it is convenient to divide the hadronic tensor into a leading-twist part and a twist-3 part. We first begin with the leading-twist contribution.

At the leading twist, the hadronic tensor can be obtained by using Eq. (2.6). According to the factorization, it can be written as

$$W^{\mu\nu} = \frac{2E_{k'}}{2p \cdot q} \text{Tr}[\hat{\Phi}^{(0)}(x, k_T) \hat{H}^{\mu\nu}(q, k)] (2\pi)^3 \delta(q_z + k_z - k'_z). \quad (4.1)$$

Integrating over k'_z [see Eq. (2.8)], we obtain the $k'_T(j_T)$ -dependent hadronic tensor

$$W^{\mu\nu} = \frac{1}{2p \cdot q} \text{Tr}[\hat{\Phi}^{(0)}(x, k_T) \hat{H}^{\mu\nu}(q, k)], \quad (4.2)$$

where the quark-quark correlator is defined as

$$\begin{aligned} \hat{\Phi}^{(0)}(x, k_T) = & \int \frac{p^+ d\xi^- d^2\vec{\xi}_T}{(2\pi)^3} e^{ixp^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \\ & \times \langle N, S | \bar{\psi}(0) \mathcal{L}(0, y) \psi(\xi) | N, S \rangle. \end{aligned} \quad (4.3)$$

The gauge link has been inserted into the quark-quark correlator to keep the gauge invariance. We decompose the correlators as

$$\hat{\Phi}^{(0)} = \frac{1}{2} [\gamma^\alpha \Phi_\alpha^{(0)} + \gamma^\alpha \gamma_5 \tilde{\Phi}_\alpha^{(0)}]. \quad (4.4)$$

In the jet-production SIDIS process, we only need to consider the γ^α and the $\gamma^\alpha \gamma_5$ terms (chiral-even terms) in the parametrization of these correlators, because there are no helicity flips. The TMDs are obtained by decomposing the correlator, which are given by

$$\begin{aligned} \Phi_\alpha^{(0)} = & p^+ \bar{t}_\alpha \left(f_1 - \frac{k_T \cdot \tilde{S}_T}{M} f_{1T}^\perp \right) + k_{T\alpha} f^\perp \\ & - M \tilde{S}_{T\alpha} f_T - \lambda_N \tilde{k}_{T\alpha} f_L^\perp - \frac{k_{T(\alpha} k_{T\beta)}}{M} \tilde{S}_T^\beta f_T^\perp, \end{aligned} \quad (4.5)$$

$$\begin{aligned} \tilde{\Phi}_\alpha^{(0)} = & p^+ \bar{t}_\alpha \left(-\lambda_N g_{1L} + \frac{k_T \cdot S_T}{M} g_{1T}^\perp \right) - \tilde{k}_{T\alpha} g^\perp \\ & - M S_{T\alpha} g_T - \lambda_N k_{T\alpha} g_L^\perp + \frac{k_{T(\alpha} k_{T\beta)}}{M} S_T^\beta g_T^\perp. \end{aligned} \quad (4.6)$$

We have introduced the notation $\tilde{A}^\alpha = \varepsilon_T^{\alpha A} = \varepsilon_T^{\alpha\beta} A_{T\beta}$, where A can be either k_T or S_T . The symmetric traceless tensor is defined as $k_{T(\alpha} k_{T\beta)} = k_{T\alpha} k_{T\beta} - \frac{1}{2} g_{T\alpha\beta} k_T^2$.

The hard part in Eq. (4.2) is abbreviated as

$$\hat{H}^{\mu\nu}(q, k) = \Gamma^\mu(\not{q} + k)\Gamma^\nu. \quad (4.7)$$

In the eN collinear frame, we have the following relationships:

$$k^- \ll k_T \ll q_T \sim q^-, \quad (4.8)$$

$$q^+ + k^+ = (1 - y)x p^+. \quad (4.9)$$

Neglecting the small components of k , we obtain

$$(\not{q} + \not{k}) = (1 - y)x p^+ \not{\bar{t}} + q^- \not{t} + \not{q}_T. \quad (4.10)$$

We notice that the transverse and the plus components also contribute in the eN collinear frame in addition to the minus component. This is important to the requirement of the current conservation of the hadronic tensor.

When the dust settles, the hadronic tensor at the leading twist can be expressed as

$$W_{t2}^{\mu\nu} = -(c_1^q \tilde{g}_T^{\mu\nu} + i c_3^q \tilde{\varepsilon}_T^{\mu\nu}) \left(f_1 - \frac{k_T \cdot \tilde{S}_T}{M} f_{1T}^\perp \right) - (c_3^q \tilde{g}_T^{\mu\nu} + i c_1^q \tilde{\varepsilon}_T^{\mu\nu}) \left(-\lambda_N g_{1L} + \frac{k_T \cdot S_T}{M} g_{1T}^\perp \right), \quad (4.11)$$

where the subscript $t2$ denotes the leading twist. The $\tilde{g}_T^{\mu\nu}$ and $\tilde{\varepsilon}_T^{\mu\nu}$ are dimensionless tensors,

$$\tilde{g}_T^{\mu\nu} = g_T^{\mu\nu} - \frac{\tilde{q}_T^2}{(q^-)^2} \bar{t}^\mu \bar{t}^\nu - \frac{1}{q^-} q_T^{\{\mu} \bar{t}^{\nu\}}, \quad (4.12)$$

$$\tilde{\varepsilon}_T^{\mu\nu} = \varepsilon_T^{\mu\nu} + \frac{1}{q^-} \varepsilon^{\mu\nu} \bar{t} q_T. \quad (4.13)$$

It is easily to check that $q_\mu \tilde{g}_T^{\mu\nu} = q_\nu \tilde{g}_T^{\mu\nu} = 0$ and $q_\mu \tilde{\varepsilon}_T^{\mu\nu} = q_\nu \tilde{\varepsilon}_T^{\mu\nu} = 0$ by using the definitions

$$g_T^{\mu\nu} = g^{\mu\nu} - \bar{t}^\mu \bar{t}^\nu - \bar{t}^\nu \bar{t}^\mu, \quad (4.14)$$

$$\varepsilon_T^{\mu\nu} = \varepsilon^{\alpha\beta\mu\nu} \bar{t}_\alpha \bar{t}_\beta. \quad (4.15)$$

$$W_{t3,q}^{\mu\nu} = [c_1^q (k_T^{\{\mu} \bar{t}^{\nu\}} q^- + k_T^{\{\mu} \bar{t}^{\nu\}} (1 - y)x p^+ + k_T^{\{\mu} q_T^{\nu\}} - g^{\mu\nu} k_T \cdot q_T) + i c_3^q (\tilde{k}_T^{\{\mu} \bar{t}^{\nu\}} q^- - \tilde{k}_T^{\{\mu} \bar{t}^{\nu\}} (1 - y)x p^+ - \tilde{t}^{\{\mu} \bar{t}^{\nu\}} \varepsilon_T^{kq})] \frac{f^\perp}{p \cdot q} + [-c_3^q (\tilde{k}_T^{\{\mu} \bar{t}^{\nu\}} q^- + \tilde{k}_T^{\{\mu} \bar{t}^{\nu\}} (1 - y)x p^+ + \tilde{k}_T^{\{\mu} q_T^{\nu\}} - g^{\mu\nu} \varepsilon_T^{kq}) + i c_1^q (k_T^{\{\mu} \bar{t}^{\nu\}} q^- - k_T^{\{\mu} \bar{t}^{\nu\}} (1 - y)x p^+ - \tilde{t}^{\{\mu} \bar{t}^{\nu\}} k_T \cdot q_T)] \frac{g^\perp}{p \cdot q}, \quad (4.20)$$

where the subscript q denotes the hadronic tensor from the quark-quark correlator. Because of the incompleteness of the twist-3 hadronic tensor, this expression does not satisfy the current conservation, i.e., $q_\mu W_{t3,q}^{\mu\nu} \neq 0$.

These equations imply that the hadronic tensor satisfies the current conservation.

B. The twist-3 hadronic tensor

The twist-3 contributions to the hadronic tensor arise from two sources. One is the quark-quark correlator given in Eq. (4.3), and the other is the quark-gluon-quark correlator whose operator definition is given by

$$\hat{\Phi}_\rho^{(1)}(x, k_T) = \int \frac{p^+ dy^- d^2 y_T}{(2\pi)^3} e^{ix p^+ y^- - i \vec{k}_T \cdot \vec{y}_T} \times \langle p, S | \bar{\psi}(0) D_{T\rho}(0) \mathcal{L}(0, y) \psi(y) | p, S \rangle, \quad (4.16)$$

where $D_\rho(y) = -i\partial_\rho + gA_\rho(y)$ is the covariant derivative.

We decompose $\hat{\Phi}_\rho^{(1)}$ as

$$\hat{\Phi}_\rho^{(1)} = \frac{1}{2} [\gamma^\alpha \Phi_{\rho\alpha}^{(1)} + \gamma^\alpha \gamma_5 \tilde{\Phi}_{\rho\alpha}^{(1)}]. \quad (4.17)$$

Similar to Eqs. (4.5) and (4.6), the coefficient functions are decomposed as

$$\Phi_{\rho\alpha}^{(1)} = p^+ \bar{t}_\alpha \left[k_{T\rho} f_d^\perp - M \tilde{S}_{T\rho} f_{dT} - \lambda_N \tilde{k}_{T\rho} f_{dL}^\perp - \frac{k_{T\langle\rho} k_{T\beta\rangle}}{M} \tilde{S}_T^\beta f_{dT}^\perp \right], \quad (4.18)$$

$$\tilde{\Phi}_{\rho\alpha}^{(1)} = i p^+ \bar{t}_\alpha \left[\tilde{k}_{T\rho} g_d^\perp + M S_{T\rho} g_{dT} + \lambda_N k_{T\rho} g_{dL}^\perp - \frac{k_{T\langle\rho} k_{T\beta\rangle}}{M} S_T^\beta g_{dT}^\perp \right], \quad (4.19)$$

where the subscript d is used to denote twist-3 TMDs defined via the quark-gluon-quark correlator.

Since the complete calculation of the twist-3 hadronic tensor is complicated, we only present the main steps by taking the $f^\perp$ and $g^\perp$ terms for example.

First of all, we calculate the contributions from the quark-quark correlator. Inserting the twist-3 TMDs (4.5) and (4.6) and the hard part (4.7) into Eq. (4.2), we obtain

Next we calculate the contributions from the quark-gluon-quark correlator. There are two parts, $W_{t3,L}^{\mu\nu}$ and $W_{t3,R}^{\mu\nu}$, which satisfy $W_{t3,L}^{\mu\nu} = (W_{t3,R}^{\nu\mu})^*$. The indices L and R denote the left cut and the right cut, respectively [31].

According to the operator definition of the hadronic tensor, we have

$$W_{i3,L}^{\mu\nu} = \frac{1}{2p \cdot q} \text{Tr}[\hat{\Phi}_\rho^{(1)}(x, k_T) \hat{H}^{\mu\nu,\rho}(q, k_1, k_2)], \quad (4.21)$$

where $\hat{H}^{\mu\nu,\rho}$ is the hard scattering amplitude,

$$\hat{H}^{\mu\nu,\rho} = \Gamma^{\mu,q} \frac{k_2 + \not{q}}{(k_2 + q)^2} \gamma^\rho (k_1 + \not{q}) \Gamma^{\nu,q}. \quad (4.22)$$

Substituting Eqs. (4.18), (4.19), and (4.22) into (4.21) gives

$$\begin{aligned} W_{i3,L}^{\mu\nu} = & \left[c_1^q \left(\tilde{t}^\mu \tilde{t}^\nu \frac{k_T \cdot q_T}{q^-} - k_T^\nu \tilde{t}^\mu \right) \right. \\ & - i c_3^q \left(\tilde{t}^\mu \tilde{t}^\nu \frac{\varepsilon_T^{qk}}{q^-} - \tilde{k}_T^\nu \tilde{t}^\mu \right) \left. \right] \frac{p^+}{p \cdot q} f_d^\perp \\ & - \left[c_1^q \left(\tilde{t}^\mu \tilde{t}^\nu \frac{k_T \cdot q_T}{q^-} - k_T^\nu \tilde{t}^\mu \right) \right. \\ & \left. - i c_3^q \left(\tilde{t}^\mu \tilde{t}^\nu \frac{\varepsilon_T^{qk}}{q^-} - \tilde{k}_T^\nu \tilde{t}^\mu \right) \right] \frac{p^+}{p \cdot q} g_d^\perp. \end{aligned} \quad (4.23)$$

We note that twist-3 TMDs marked with subscript d can be related to those without the subscript through the following relation [19]:

$$f_{dS}^K - g_{dS}^K = -x(f_S^K - i g_S^K), \quad (4.24)$$

where the superscript K can be $\perp$ and the subscript S can be L and T . Using this relation, Eq. (4.23) can be rewritten as

$$\begin{aligned} W_{i3,L}^{\mu\nu} = & - \left[c_1^q \left(\tilde{t}^\mu \tilde{t}^\nu \frac{k_T \cdot q_T}{q^-} - k_T^\nu \tilde{t}^\mu \right) \right. \\ & \left. - i c_3^q \left(\tilde{k}_T^\nu \tilde{t}^\mu - \tilde{t}^\mu \tilde{t}^\nu \frac{\varepsilon_T^{qk}}{q^-} \right) \right] \frac{x p^+}{p \cdot q} f^\perp \\ & + \left[i c_1^q \left(\tilde{t}^\mu \tilde{t}^\nu \frac{k_T \cdot q_T}{q^-} - k_T^\nu \tilde{t}^\mu \right) \right. \\ & \left. + c_3^q \left(\tilde{k}_T^\nu \tilde{t}^\mu - \tilde{t}^\mu \tilde{t}^\nu \frac{\varepsilon_T^{qk}}{q^-} \right) \right] \frac{x p^+}{p \cdot q} g^\perp. \end{aligned} \quad (4.25)$$

Note that the complete twist-3 hadronic tensor from the quark-gluon-quark correlator is given by the sum of $W_{i3,L}^{\mu\nu}$ and $W_{i3,R}^{\mu\nu}$,

$$\begin{aligned} W_{i3,L}^{\mu\nu} + W_{i3,R}^{\mu\nu} = & - \left[c_1^q \left(\tilde{t}^\mu \tilde{t}^\nu \frac{2x p^+}{q^-} k_T \cdot q_T - x p^+ k_T^{\{\mu} \tilde{t}^{\nu\}} \right) - i c_3^q x p^+ \tilde{k}_T^{[\mu} \tilde{t}^{\nu]} \right] \frac{f^\perp}{p \cdot q} \\ & + \left[c_3^q \left(\tilde{t}^\mu \tilde{t}^\nu \frac{2x p^+}{q^-} \varepsilon_T^{kq} - x p^+ \tilde{k}_T^{\{\mu} \tilde{t}^{\nu\}} \right) + i c_1^q x p^+ k_T^{[\mu} \tilde{t}^{\nu]} \right] \frac{g^\perp}{p \cdot q}. \end{aligned} \quad (4.26)$$

To obtain the complete twist-3 hadronic tensor, which satisfies the current conservation, one is supposed to sum Eqs. (4.20) and (4.26) together. Finally, we have

$$W_{i3}^{\mu\nu} = \frac{f^\perp}{p \cdot q} h_1^{\mu\nu} + \frac{g^\perp}{p \cdot q} h_3^{\mu\nu} + \dots, \quad (4.27)$$

where the tensors $h_1^{\mu\nu}$ and $h_3^{\mu\nu}$ are given by

$$\begin{aligned} h_1^{\mu\nu} = & + c_1^q \left[k_T^{\{\mu} t^{\nu\}} q^- + k_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + k_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2x p^+}{q^-} \right) k_T \cdot q_T \right] \\ & + i c_3^q [\tilde{k}_T^{[\mu} t^{\nu]} q^- - \tilde{k}_T^{[\mu} \tilde{t}^{\nu]} q^+ - \tilde{t}^{[\mu} t^{\nu]} \varepsilon_T^{qk}], \end{aligned} \quad (4.28)$$

$$\begin{aligned} h_3^{\mu\nu} = & - c_3^q \left[\tilde{k}_T^{\{\mu} t^{\nu\}} q^- + \tilde{k}_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + \tilde{k}_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2x p^+}{q^-} \right) \varepsilon_T^{qk} \right] \\ & + i c_1^q [k_T^{[\mu} t^{\nu]} q^- - k_T^{[\mu} \tilde{t}^{\nu]} q^+ - \tilde{t}^{[\mu} t^{\nu]} k_T \cdot q_T]. \end{aligned} \quad (4.29)$$

Here we show only the $f^\perp$ and $g^\perp$ terms. By performing the steps presented above, the twist-3 hadronic tensor including the previously omitted terms can be written as

$$W_{i3}^{\mu\nu} = \frac{f^\perp}{p \cdot q} h_1^{\mu\nu} + \frac{\lambda_N f_L^\perp}{p \cdot q} h_2^{\mu\nu} + \frac{g^\perp}{p \cdot q} h_3^{\mu\nu} + \frac{\lambda g_L^\perp}{p \cdot q} h_4^{\mu\nu} + \frac{M f_T}{p \cdot q} h_5^{\mu\nu} + \frac{f_T^\perp}{p \cdot q} h_6^{\mu\nu} + \frac{M g_T}{p \cdot q} h_7^{\mu\nu} + \frac{g_T^\perp}{p \cdot q} h_8^{\mu\nu}, \quad (4.30)$$

where $h_{2,4-8}^{\mu\nu}$ are defined as

$$h_2^{\mu\nu} = -c_1^q \left[\tilde{k}_T^{\{\mu} t^{\nu\}} q^- + \tilde{k}_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + \tilde{k}_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2xp^+}{q^-} \right) \varepsilon_T^{qk} \right] + ic_3^q [k_T^{\{\mu} t^{\nu\}} q^- - k_T^{\{\mu} \tilde{t}^{\nu\}} q^+ - \tilde{t}^{\{\mu} t^{\nu\}} k_T \cdot q_T], \quad (4.31)$$

$$h_4^{\mu\nu} = -c_3^q \left[k_T^{\{\mu} t^{\nu\}} q^- + k_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + k_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2xp^+}{q^-} \right) k_T \cdot q_T \right] - ic_1^q [\tilde{k}_T^{\{\mu} t^{\nu\}} q^- - \tilde{k}_T^{\{\mu} \tilde{t}^{\nu\}} q^+ - \tilde{t}^{\{\mu} t^{\nu\}} \varepsilon_T^{qk}], \quad (4.32)$$

$$h_5^{\mu\nu} = + \left\{ -c_1^q \left[\tilde{S}_T^{\{\mu} t^{\nu\}} q^- + \tilde{S}_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + \tilde{S}_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2xp^+}{q^-} \right) \varepsilon_T^{qS} \right] + ic_3^q [S_T^{\{\mu} t^{\nu\}} q^- - S_T^{\{\mu} \tilde{t}^{\nu\}} q^+ - \tilde{t}^{\{\mu} t^{\nu\}} S_T \cdot q_T] \right\}, \quad (4.33)$$

$$h_6^{\mu\nu} = - \left\{ +c_1^q \left[k_T^{\{\mu} t^{\nu\}} q^- + k_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + k_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2xp^+}{q^-} \right) k_T \cdot q_T \right] + ic_3^q [\tilde{k}_T^{\{\mu} t^{\nu\}} q^- - \tilde{k}_T^{\{\mu} \tilde{t}^{\nu\}} q^+ - \tilde{t}^{\{\mu} t^{\nu\}} \varepsilon_T^{qk}] \right\} \frac{\varepsilon_T^{kS}}{M} \quad (4.34)$$

$$- \left\{ -c_1^q \left[\tilde{S}_T^{\{\mu} t^{\nu\}} q^- + \tilde{S}_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + \tilde{S}_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2xp^+}{q^-} \right) \varepsilon_T^{qS} \right] + ic_3^q [S_T^{\{\mu} t^{\nu\}} q^- - S_T^{\{\mu} \tilde{t}^{\nu\}} q^+ - \tilde{t}^{\{\mu} t^{\nu\}} S_T \cdot q_T] \right\} \frac{k_T^2}{2M}, \quad (4.35)$$

$$h_7^{\mu\nu} = + \left\{ -c_3^q \left[S_T^{\{\mu} t^{\nu\}} q^- + S_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + S_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2xp^+}{q^-} \right) S_T \cdot q_T \right] - ic_1^q [\tilde{S}_T^{\{\mu} t^{\nu\}} q^- - \tilde{S}_T^{\{\mu} \tilde{t}^{\nu\}} q^+ - \tilde{t}^{\{\mu} t^{\nu\}} \varepsilon_T^{qS}] \right\}, \quad (4.36)$$

$$h_8^{\mu\nu} = + \left\{ +c_3^q \left[k_T^{\{\mu} t^{\nu\}} q^- + k_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + k_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2xp^+}{q^-} \right) k_T \cdot q_T \right] + ic_1^q [\tilde{k}_T^{\{\mu} t^{\nu\}} q^- - \tilde{k}_T^{\{\mu} \tilde{t}^{\nu\}} q^+ - \tilde{t}^{\{\mu} t^{\nu\}} \varepsilon_T^{qk}] \right\} \frac{k_T \cdot S_T}{M} \quad (4.37)$$

$$- \left\{ +c_3^q \left[S_T^{\{\mu} t^{\nu\}} q^- + S_T^{\{\mu} \tilde{t}^{\nu\}} (2-y) x p^+ + S_T^{\{\mu} q_T^{\nu\}} - \left(g^{\mu\nu} + \tilde{t}^\mu \tilde{t}^\nu \frac{2xp^+}{q^-} \right) S_T \cdot q_T \right] + ic_1^q [\tilde{S}_T^{\{\mu} t^{\nu\}} q^- - \tilde{S}_T^{\{\mu} \tilde{t}^{\nu\}} q^+ - \tilde{t}^{\{\mu} t^{\nu\}} \varepsilon_T^{qS}] \right\} \frac{k_T^2}{2M}. \quad (4.38)$$

We notice that the complete twist-3 hadronic tensor in Eq. (4.30) satisfies the current conservation, $q_\mu \tilde{W}_{i3}^{\mu\nu} = q_\nu \tilde{W}_{i3}^{\mu\nu} = 0$. Although the h -tensors appear somewhat complicated, they share similar structures and will lead to a simple expression for the differential cross section.

C. The cross section in the parton model

Contracting the hadronic tensor obtained in Eq. (4.30) with the leptonic tensor, the differential cross section can be

expressed in terms of TMDs in the parton model. At the leading-twist approximation, the contractions are given by

$$L_{\mu\nu} (c_1^q \tilde{g}_T^{\mu\nu} + ic_3^q \tilde{\varepsilon}_T^{\mu\nu}) = -\frac{2Q^2}{y^2} T_0^q(y), \quad (4.39)$$

$$L_{\mu\nu} (c_3^q \tilde{g}_T^{\mu\nu} + ic_1^q \tilde{\varepsilon}_T^{\mu\nu}) = -\frac{2Q^2}{y^2} T_1^q(y), \quad (4.40)$$

where T -functions are defined as

$$\begin{aligned} T_0^q(y) &= c_1^q A(y) - \lambda_n c_3^q C(y), \\ T_1^q(y) &= c_3^q A(y) - \lambda_n c_1^q C(y). \end{aligned} \quad (4.41)$$

Here $A(y) = y^2 - 2y + 2$ and $C(y) = y(2 - y)$. Since $c_1^q = c_3^q = 2$ in the charged-current interaction, it follows that $T_0^q(y) = T_1^q(y)$. For simplicity, we define $T^q(y) \equiv T_0^q(y) = T_1^q(y)$ in the following. Therefore, the leading-twist cross section for the jet-production SIDIS of the electron in the eN collinear frame can be expressed as

$$\begin{aligned} d\tilde{\sigma}_{l2} &= \frac{\alpha_{\text{em}}^2}{yQ^4} A_W T^q(y) 2x \{f_1 - \lambda_N g_{1L} \\ &\quad + |S_T| k_{TM} [\sin(\varphi - \varphi_S) f_{1T}^\perp - \cos(\varphi - \varphi_S) g_{1T}^\perp]\}, \end{aligned} \quad (4.42)$$

where $d\tilde{\sigma}_{l2} = d\sigma_{l2}/d\eta d^2 l_T d^2 j_T$ and $k_{TM} = |k_T|/M$. The subscript $l2$ denotes the leading-twist contribution.

Similarly, we can calculate the differential cross section at twist-3. For conciseness, we only show contractions of the leptonic tensor with $h_1^{\mu\nu}$ and $h_3^{\mu\nu}$,

$$L_{\mu\nu} \cdot h_1^{\mu\nu} = -\frac{2Q^3}{y^2} |k_T| T_2^q(y) \cos \varphi, \quad (4.43)$$

$$L_{\mu\nu} \cdot h_3^{\mu\nu} = -\frac{2Q^3}{y^2} |k_T| T_3^q(y) \sin \varphi. \quad (4.44)$$

Other contractions have the same forms. Here the T -functions are defined as

$$\begin{aligned} T_2^q(y) &= c_1^q B(y) - \lambda_n c_3^q D(y), \\ T_3^q(y) &= c_3^q B(y) - \lambda_n c_1^q D(y), \end{aligned} \quad (4.45)$$

with $B(y) = (2 - y^2)\sqrt{1 - y}$ and $D(y) = y^2\sqrt{1 - y}$. After simple calculations, we write down the differential cross section at twist-3,

$$\begin{aligned} d\tilde{\sigma}_{l3} &= -\frac{\alpha_{\text{em}}^2}{yQ^4} A_W 4x^2 \kappa_M \tilde{T}^q(y) \left\{ k_{TM} [\cos \varphi f^\perp + \sin \varphi g^\perp] \right. \\ &\quad + \lambda_N k_{TM} [\sin \varphi f_L^\perp - \cos \varphi g_L^\perp] \\ &\quad + |S_T| \left[\sin \varphi_S f_T + \sin(2\varphi - \varphi_S) \frac{k_{TM}^2}{2} f_T^\perp \right. \\ &\quad \left. \left. + \cos \varphi_S g_T - \cos(2\varphi - \varphi_S) \frac{k_{TM}^2}{2} g_T^\perp \right] \right\}, \end{aligned} \quad (4.46)$$

where $d\tilde{\sigma}_{l3} = d\sigma_{l3}/d\eta d^2 l_T d^2 j_T$. $\kappa_M = M/Q$ is the twist suppression factor. $\tilde{T}^q(y)$ is defined as $\tilde{T}^q(y) \equiv T_2^q(y) = T_3^q(y)$ because of $c_1^q = c_3^q = 2$. We note that in the electron scattering process, U -type quarks ($u, \bar{d}, \bar{s}, \dots$) contribute to the cross section while in the positron scattering process

and D -type quarks ($d, s, \bar{u}, \dots$) contribute to the cross section.

V. MEASURABLE QUANTITIES

A. The structure functions

Comparing the cross section in terms of structure functions in Eq. (3.14) and the parton model results in Eqs. (4.42) and (4.46), one can obtain the results of the structure functions in the parton model. In the following, we take the SIDIS of the electron as an example to present the results. There are four nonzero structure functions at the leading twist:

$$F_U = 2 \frac{T^q(y)}{y^2} f_1, \quad (5.1)$$

$$F_L = -2 \frac{T^q(y)}{y^2} g_{1L}, \quad (5.2)$$

$$F_T^{\sin(\varphi - \varphi_S)} = 2 \frac{T^q(y)}{y^2} k_{TM} f_{1T}^\perp, \quad (5.3)$$

$$F_T^{\cos(\varphi - \varphi_S)} = -2 \frac{T^q(y)}{y^2} k_{TM} g_{1T}^\perp. \quad (5.4)$$

At twist-3, we obtain eight nonzero structure functions,

$$F_U^{\cos \varphi} = -4 \frac{\tilde{T}^q(y)}{y^2} x \kappa_M k_{TM} f^\perp, \quad (5.5)$$

$$F_U^{\sin \varphi} = -4 \frac{\tilde{T}^q(y)}{y^2} x \kappa_M k_{TM} g^\perp, \quad (5.6)$$

$$F_L^{\sin \varphi} = -4 \frac{\tilde{T}^q(y)}{y^2} x \kappa_M k_{TM} f_L^\perp, \quad (5.7)$$

$$F_L^{\cos \varphi} = 4 \frac{\tilde{T}^q(y)}{y^2} x \kappa_M k_{TM} g_L^\perp, \quad (5.8)$$

$$F_T^{\sin \varphi_S} = -4 \frac{\tilde{T}^q(y)}{y^2} x \kappa_M f_T, \quad (5.9)$$

$$F_T^{\sin(2\varphi - \varphi_S)} = -4 \frac{\tilde{T}^q(y)}{y^2} x \kappa_M \frac{k_{TM}^2}{2} f_T^\perp, \quad (5.10)$$

$$F_T^{\cos \varphi_S} = -4 \frac{\tilde{T}^q(y)}{y^2} x \kappa_M g_T, \quad (5.11)$$

$$F_T^{\cos(2\varphi - \varphi_S)} = 4 \frac{\tilde{T}^q(y)}{y^2} x \kappa_M \frac{k_{TM}^2}{2} g_T^\perp. \quad (5.12)$$

According to the results above, we find that results of the structure functions including only leading-twist functions

exhibit the dependence on the even number of φ and φ_S , while the structure functions at twist-3 correspond to the dependence of the odd number of φ and φ_S . We can in principle utilize these nonvanishing structure functions to study the coupling of the weak interactions.

B. The azimuthal asymmetries

For the jet-production SIDIS, FFs are not involved. Therefore, (TMD) PDFs are the only unknown quantities related to azimuthal asymmetries. From the cross sections shown before we notice that azimuthal asymmetries refer to azimuthal asymmetries of k_T , or azimuthal asymmetries of j_T , since $\vec{j}_T = \vec{k}_T$ in the eN collinear frame. One therefore can measure j_T to determine k_T . We here use the Trento convention to define the azimuthal asymmetry. For example, we define

$$\langle \sin \varphi \rangle_U = \frac{\int d\tilde{\sigma} \sin \varphi d\varphi}{\int d\tilde{\sigma} d\varphi}, \quad (5.13)$$

for the unpolarized or longitudinally polarized target case, and

$$\langle \sin(\varphi - \varphi_S) \rangle_T = \frac{\int d\tilde{\sigma} \sin(\varphi - \varphi_S) d\varphi d\varphi_S}{\int d\tilde{\sigma} d\varphi d\varphi_S}, \quad (5.14)$$

for the transversely polarized target case.

At the leading twist, there are two polarization dependent azimuthal asymmetries that are given by

$$\langle \sin(\varphi - \varphi_S) \rangle_T = \frac{k_{TM} f_{1T}^\perp}{2 f_1}, \quad (5.15)$$

$$\langle \cos(\varphi - \varphi_S) \rangle_T = -\frac{k_{TM} g_{1T}^\perp}{2 f_1}. \quad (5.16)$$

$\langle \sin(\varphi - \varphi_S) \rangle_T$ is the famous Sivers asymmetry or single transverse spin asymmetry. In the jet-production charged-current SIDIS, it is just a ratio of Sivers function $f_{1T}^\perp$ [32,33] and f_1 . In addition, we also have eight twist-3 azimuthal asymmetries. They are given by

$$\langle \cos \varphi \rangle_U = -x\kappa_M k_{TM} \frac{\tilde{T}^q(y) f_1^\perp}{T^q(y) f_1}, \quad (5.17)$$

$$\langle \sin \varphi \rangle_U = -x\kappa_M k_{TM} \frac{\tilde{T}^q(y) g_1^\perp}{T^q(y) f_1}, \quad (5.18)$$

$$\langle \cos \varphi \rangle_L = x\kappa_M k_{TM} \frac{\tilde{T}^q(y) g_L^\perp}{T^q(y) f_1}, \quad (5.19)$$

$$\langle \sin \varphi \rangle_L = -x\kappa_M k_{TM} \frac{\tilde{T}^q(y) f_L^\perp}{T^q(y) f_1}, \quad (5.20)$$

$$\langle \cos \varphi_S \rangle_T = -x\kappa_M \frac{\tilde{T}^q(y) g_T}{T^q(y) f_1}, \quad (5.21)$$

$$\langle \sin \varphi_S \rangle_T = -x\kappa_M \frac{\tilde{T}^q(y) f_T}{T^q(y) f_1}, \quad (5.22)$$

$$\langle \cos(2\varphi - \varphi_S) \rangle_T = x\kappa_M \frac{k_{TM}^2}{2} \frac{\tilde{T}^q(y) g_T^\perp}{T^q(y) f_1}, \quad (5.23)$$

$$\langle \sin(2\varphi - \varphi_S) \rangle_T = -x\kappa_M \frac{k_{TM}^2}{2} \frac{\tilde{T}^q(y) f_T^\perp}{T^q(y) f_1}. \quad (5.24)$$

We note that the azimuthal asymmetries discussed above are defined for both the electron scattering process and the positron scattering process. For the former, U -type quarks are involved in that process and D -type quarks are involved for the latter.

C. The intrinsic asymmetries

In addition to azimuthal asymmetries, we also define intrinsic asymmetries in the eN collinear frame to explore the imbalance of the transverse momentum of the incident quark in a nucleon. Note that the transverse momentum of the incident quark (jet), which lies in the x - y plane, can be decomposed as

$$k_T^x = k_T \cos \varphi, \quad (5.25)$$

$$k_T^y = k_T \sin \varphi. \quad (5.26)$$

Therefore, we can define $k_T^x(-x) - k_T^x(+x)$ to quantify the difference of the transverse momentum between the negative x and positive x directions. The difference in the y direction is defined similarly. To be explicit, we present the general definitions of the intrinsic asymmetries,

$$A^x = \frac{\int_{-\pi/2}^{\pi/2} d\varphi d\tilde{\sigma} - \int_{\pi/2}^{3\pi/2} d\varphi d\tilde{\sigma}}{\int_{-\pi/2}^{\pi/2} d\varphi d\tilde{\sigma}_U + \int_{\pi/2}^{3\pi/2} d\varphi d\tilde{\sigma}_U}, \quad (5.27)$$

$$A^y = \frac{\int_0^\pi d\varphi d\tilde{\sigma} - \int_\pi^{2\pi} d\varphi d\tilde{\sigma}}{\int_0^\pi d\varphi d\tilde{\sigma}_U + \int_\pi^{2\pi} d\varphi d\tilde{\sigma}_U}. \quad (5.28)$$

The A^x and A^y lead to asymmetries in the x direction and y direction, respectively.

According to the definitions, four kinds of asymmetries are obtained. They are

$$A_U^x = -\frac{4x\kappa_M k_{TM}}{\pi} \frac{\tilde{T}^q(y) f_1^\perp}{T^q(y) f_1}, \quad (5.29)$$

$$A_U^y = -\frac{4x\kappa_M k_{TM}}{\pi} \frac{\tilde{T}^q(y) g_1^\perp}{T^q(y) f_1}, \quad (5.30)$$

$$A_L^x = \frac{4x\kappa_M k_{TM}}{\pi} \frac{\tilde{T}^q(y) g_L^\perp}{T^q(y) f_1}, \quad (5.31)$$

$$A_L^y = -\frac{4x\kappa_M k_{TM}}{\pi} \frac{\tilde{T}^q(y) f_L^\perp}{T^q(y) f_1}. \quad (5.32)$$

We find that the intrinsic asymmetries are all twist-3 measurable quantities.

D. The charge asymmetry

From Eq. (4.42), we can write down the differential cross sections for the electron inelastic scattering and the positron inelastic scattering, respectively. They are

$$\begin{aligned} d\tilde{\sigma}(e^-) = & \frac{\alpha_{\text{em}}^2}{yQ^4} A_W 4x \{ [A(y) + C(y)] f_1^u \\ & + [A(y) - C(y)] (f_1^{\bar{d}} + f_1^{\bar{s}}) \}, \end{aligned} \quad (5.33)$$

$$\begin{aligned} d\tilde{\sigma}(e^+) = & \frac{\alpha_{\text{em}}^2}{yQ^4} A_W 4x \{ [A(y) - C(y)] f_1^{\bar{u}} \\ & + [A(y) + C(y)] (f_1^d + f_1^s) \}, \end{aligned} \quad (5.34)$$

where $d\tilde{\sigma} = d\sigma_{I2}/d\eta d^2 l_T^2 d^2 j_T$, superscripts denote quark flavors, and only light flavors are considered here. According to Eqs. (5.33) and (5.34), we introduce the charge asymmetry, which is defined as the ratio of the difference from the sum of the differential cross sections of the electron inelastic scattering and the positron inelastic scattering,

$$A^C = \frac{d\tilde{\sigma}(e^-) - d\tilde{\sigma}(e^+)}{d\tilde{\sigma}(e^-) + d\tilde{\sigma}(e^+)}. \quad (5.35)$$

After a simple calculation, we obtain

$$A^C = \frac{[A(y) + C(y)]F_{N1} + [A(y) - C(y)]F_{N2}}{[A(y) + C(y)]F_{D1} + [A(y) - C(y)]F_{D2}}, \quad (5.36)$$

where

$$F_{N1} = f_1^u - f_1^{\bar{u}}, \quad (5.37)$$

$$F_{N2} = f_1^{\bar{d}} + f_1^{\bar{s}} - f_1^d - f_1^s, \quad (5.38)$$

$$F_{D1} = f_1^u + f_1^{\bar{u}}, \quad (5.39)$$

$$F_{D2} = f_1^{\bar{d}} + f_1^{\bar{s}} + f_1^d + f_1^s. \quad (5.40)$$

If we define $\delta f^q = f^q - f^{\bar{q}}$ with $q = u, d, s$, the numerator in Eq. (5.36) can be rewritten as

$$\mathcal{N} = [A(y) + C(y)]\delta f^u - [A(y) - C(y)](\delta f^d + \delta f^s). \quad (5.41)$$

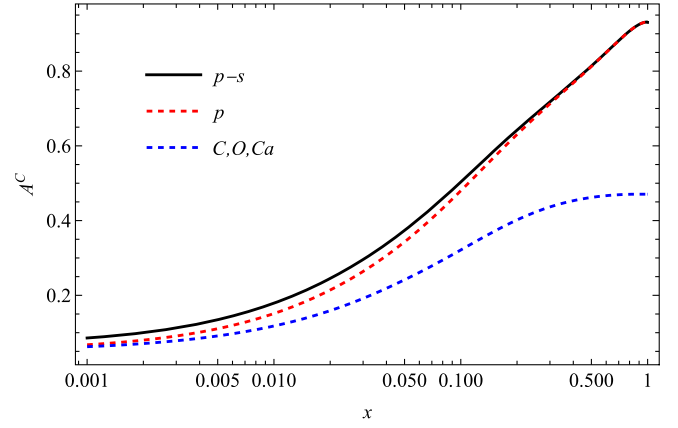


FIG. 3. Numerical estimations of ratio A^C with respect to x . Fraction y and k_T are taken as $y = 0.4$ and $k_T = 0.5$ GeV. $p-s$ denotes numerical estimates without (with) considering strange and antistrange quarks. C, O, Ca denote estimates for the scattering of nuclei, carbon, oxygen, and calcium.

We notice that A^C not only provides a sensitive probe for valence quark distribution functions but also reveals violations of strange-antistrange symmetry.

To have an intuitive impression of the charge asymmetry defined above, we present numerical estimates in Figs. 3 and 4. Our estimations are based on the Gaussian ansatz for $f_1(x, k_T)$, i.e.,

$$f_1(x, k_T) = \frac{1}{\pi\Delta^2} f_1(x) e^{-\vec{k}_T^2/\Delta^2}, \quad (5.42)$$

where $f_1(x)$ is taken from CTEQ18 [34] for proton and from EPPS21 [34,35] for carbon, oxygen, and calcium. The average squared transverse momenta are taken as $\Delta_u^2 = \Delta_d^2 = 0.34$ GeV², $\Delta_u^2 = \Delta_d^2 = 0.63$ GeV², and

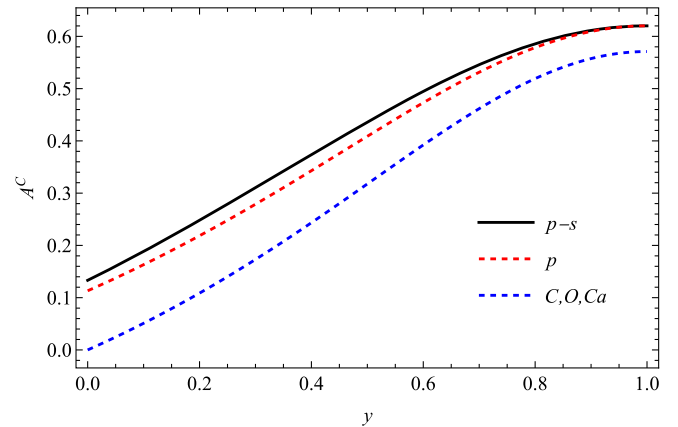


FIG. 4. Numerical estimations of ratio A^C with respect to y . Fraction x and k_T are taken as $x = 0.05$ and $k_T = 0.5$ GeV. $p-s$ denotes numerical estimates without (with) considering strange and antistrange quarks. C, O, Ca denote estimates for the scattering of nuclei, carbon, oxygen, and calcium.

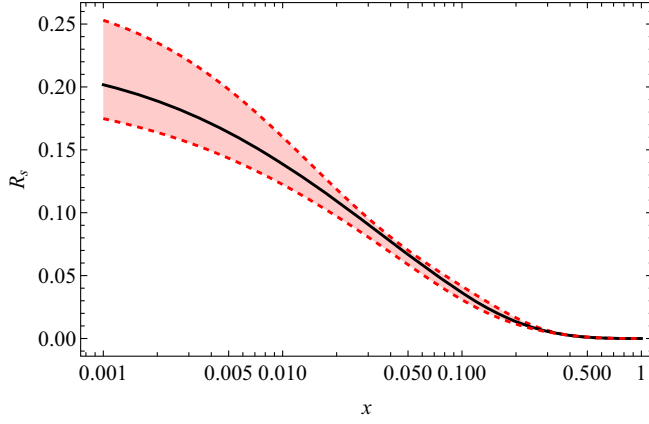


FIG. 5. Numerical estimations of ratio R_s with respect to x . Fraction y and k_T are taken as $y = 0.4$ and $k_T = 0.5$ GeV. The band denotes uncertainties caused by PDFs.

$\Delta_s^2 = \Delta_{\bar{s}}^2 = 0.22 \text{ GeV}^2$ [36–41] for numerical estimates. In Figs. 3 and 4, black solid lines show numerical estimates without considering strange and antistrange quarks while red dashed lines show estimates with considering strange and antistrange quarks. To see the difference clearly, we introduce the ratio

$$R_s = \frac{A^C(s, \bar{s} = 0) - A^C(s, \bar{s} \neq 0)}{A^C(s, \bar{s} \neq 0)} \quad (5.43)$$

and show it in Fig. 5. We notice that strange and antistrange quarks have significant influence on A^C , especially at the sea quark region. The band denotes uncertainties caused by PDFs.

Blue lines in Figs. 3 and 4 show estimates of A^C for inelastic scattering processes of nuclei, carbon, oxygen, and calcium. They are isoscalar nuclei with $N = Z$, where the proton number equals the neutron number. From these two figures, we see the difference between $A_{N \neq Z}^C$ and $A_{N=Z}^C$ becomes smaller and smaller as x becomes smaller. We also note here that the charge asymmetry defined in this part is independent of the type of target nucleus if it has the same number of neutrons and protons.

VI. SUMMARY

In this paper, we present a systematic calculation of the charged-current jet-production SIDIS process in the eN collinear frame. The eN collinear frame is defined such that the target travels along the $+z$ direction, while the incoming lepton travels along the $-z$ direction. The scattered (anti)neutrino lies in the x - z plane, which is known as the lepton plane. The differential cross section is first expressed in terms of structure functions and then expressed in terms of TMDs at the tree level twist-3. Since the W -boson gains the transverse momentum component q_T in the eN collinear frame, the gauge invariance of the hadronic tensor becomes difficult to verify directly. To achieve this goal, we present a systematic calculation of how to obtain the gauge invariant hadronic tensor at the twist-3 level. By comparing these two forms, we obtain a set of relationships between structure functions and the TMDs, shown in Eqs. (5.1)–(5.12). We also calculate azimuthal asymmetries and intrinsic asymmetries. Two leading-twist and eight twist-3 azimuthal asymmetries are obtained. However, intrinsic asymmetries are all twist-3 quantities. They provide more measurable quantities for extracting TMDs. We also introduce the charge asymmetry A^C , which is defined as the ratio of the difference from the sum of the differential cross sections of the electron semi-inclusive deep inelastic scattering and the positron semi-inclusive deep inelastic scattering. We find that A^C can be used to determine the valence quark distribution functions and the violation of strange-antistrange symmetry. Numerical values show that contributions from strange and antistrange quarks become significant as fraction x decreases.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grants No. 12405103 and No. 12447132) and the Youth Innovation Technology Project of Higher School in Shandong Province (2023KJ146).

DATA AVAILABILITY

The data that support the findings of this article are openly available [34,35].

-
- [1] A. Accardi *et al.*, *Eur. Phys. J. A* **52**, 268 (2016).
 [2] R. Abdul Khalek, A. Accardi, J. Adam, D. Adamiak, W. Akers, M. Albaladejo, A. Al-bataineh, M. G. Alexeev, F. Ameli, P. Antonioli *et al.*, *Nucl. Phys. A* **1026**, 122447 (2022).

- [3] D. P. Anderle, V. Bertone, X. Cao, L. Chang, N. Chang, G. Chen, X. Chen, Z. Chen, Z. Cui, L. Dai *et al.*, *Front. Phys. (Beijing)* **16**, 64701 (2021).
 [4] See e. g., J. C. Collins, D. E. Soper, and G. F. Sterman, *Adv. Ser. Dir. High Energy Phys.* **5**, 1 (1989); *Perturbative*

- Quantum Chromodynamics*, edited by A. H. Mueller (World Scientific, Singapore, 1989).
- [5] P. J. Mulders and R. D. Tangerman, *Nucl. Phys.* **B461**, 197 (1996); **B484**, 538(E) (1997).
 - [6] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders, and M. Schlegel, *J. High Energy Phys.* **02** (2007) 093.
 - [7] W. Yang and X. Yang, *Phys. Rev. D* **106**, 093003 (2022).
 - [8] W. Yang and X. Yang, *Nucl. Phys.* **B990**, 116181 (2023).
 - [9] W. Yang, *Phys. Rev. D* **108**, 056022 (2023).
 - [10] C. Wu, X. Yang, C. Li, and W. Yang, *Nucl. Phys.* **B997**, 116390 (2023).
 - [11] A. Accardi and A. Bacchetta, *Phys. Lett. B* **773**, 632 (2017).
 - [12] A. Accardi and A. Signori, *Phys. Lett. B* **798**, 134993 (2019).
 - [13] A. Accardi and A. Signori, *Eur. Phys. J. C* **80**, 825 (2020).
 - [14] Z. B. Kang, K. Lee, and F. Zhao, *Phys. Lett. B* **809**, 135756 (2020).
 - [15] D. Boer, M. Diehl, R. Milner, R. Venugopalan, W. Vogelsang, D. Kaplan, H. Montgomery, S. Vigdor, A. Accardi, E. C. Aschenauer *et al.*, [arXiv:1108.1713](https://arxiv.org/abs/1108.1713).
 - [16] Y.-k. Song, J.-h. Gao, Z.-t. Liang, and X.-N. Wang, *Phys. Rev. D* **83**, 054010 (2011).
 - [17] Y.-k. Song, J.-h. Gao, Z.-t. Liang, and X.-N. Wang, *Phys. Rev. D* **89**, 014005 (2014).
 - [18] S.-y. Wei, Y.-k. Song, K.-b. Chen, and Z.-t. Liang, *Phys. Rev. D* **95**, 074017 (2017).
 - [19] K. B. Chen and W. H. Yang, *Phys. Rev. D* **101**, 096017 (2020).
 - [20] W. Yang, *Phys. Rev. D* **103**, 016011 (2021).
 - [21] D. Gutierrez-Reyes, I. Scimemi, W. J. Waalewijn, and L. Zoppi, *Phys. Rev. Lett.* **121**, 162001 (2018).
 - [22] D. Gutierrez-Reyes, I. Scimemi, W. J. Waalewijn, and L. Zoppi, *J. High Energy Phys.* **10** (2019) 031.
 - [23] Z. B. Kang, X. Liu, S. Mantry, and D. Y. Shao, *Phys. Rev. Lett.* **125**, 242003 (2020).
 - [24] M. Arratia, Y. Makris, D. Neill, F. Ringer, and N. Sato, *Phys. Rev. D* **104**, 034005 (2021).
 - [25] X. Liu, F. Ringer, W. Vogelsang, and F. Yuan, *Phys. Rev. D* **102**, 094022 (2020).
 - [26] X. Liu, F. Ringer, W. Vogelsang, and F. Yuan, *Phys. Rev. Lett.* **122**, 192003 (2019).
 - [27] M. Arratia, Z. B. Kang, S. J. Paul, A. Prokudin, F. Ringer, and F. Zhao, *Phys. Rev. D* **107**, 094036 (2023).
 - [28] M. Arratia, Y. Song, F. Ringer, and B. V. Jacak, *Phys. Rev. C* **101**, 065204 (2020).
 - [29] K.-b. Chen, W.-h. Yang, S.-y. Wei, and Z.-t. Liang, *Phys. Rev. D* **94**, 034003 (2016).
 - [30] J. Zhao, Z. Zhang, Z.-t. Liang, T. Liu, and Y.-j. Zhou, *Phys. Rev. D* **109**, 074017 (2024).
 - [31] Z.-t. Liang and X. N. Wang, *Phys. Rev. D* **75**, 094002 (2007).
 - [32] D. W. Sivers, *Phys. Rev. D* **41**, 83 (1990).
 - [33] D. W. Sivers, *Phys. Rev. D* **43**, 261 (1991).
 - [34] T. J. Hou, J. Gao, T. J. Hobbs, K. Xie, S. Dulat, M. Guzzi, J. Huston, P. Nadolsky, J. Pumplin, C. Schmidt *et al.*, *Phys. Rev. D* **103**, 014013 (2021).
 - [35] K. J. Eskola, P. Paakkinen, H. Paukkunen, and C. A. Salgado, *Eur. Phys. J. C* **82**, 413 (2022).
 - [36] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, and A. Prokudin, *Phys. Rev. D* **71**, 074006 (2005).
 - [37] A. Signori, A. Bacchetta, M. Radici, and G. Schnell, *J. High Energy Phys.* **11** (2013) 194.
 - [38] M. Anselmino, M. Boglione, J. O. Gonzalez Hernandez, S. Melis, and A. Prokudin, *J. High Energy Phys.* **04** (2014) 005.
 - [39] J. Cammarota, L. Gamberg, Z.-B. Kang, J. A. Miller, D. Pitonyak, A. Prokudin, T. C. Rogers, and N. Sato (Jefferson Lab Angular Momentum Collaboration), *Phys. Rev. D* **102**, 054002 (2020).
 - [40] A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, M. Cerutti, F. Piacenza, M. Radici, and A. Signori (MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration), *J. High Energy Phys.* **10** (2022) 127.
 - [41] A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, M. Cerutti, F. Delcarro, M. Radici, L. Rossi, and A. Signori (MAP Collaboration), *J. High Energy Phys.* **08** (2024) 232.