

QCD sum rule study of excited light meson operators

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We apply the QCD sum rule method to systematically study excited light meson operators and calculate their decay constants. These operators are constructed by explicitly adding one covariant derivative to the quark-antiquark pair. In total, twelve such operators are constructed, among which ten are subjected to detailed numerical analyses. The considered quark contents include $\bar{q}q$, $\bar{q}s$, and $\bar{s}s$ ($q = u/d$), allowing the formation of various $SU(3)$ flavor nonets. For instance, our results support the interpretation that the $a_2(1320)$, $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$ states constitute a flavor nonet with quantum numbers $J^{P(C)} = 2^{+(+)}$. In addition, we predict several excited meson states, whose masses and decay constants are determined using the QCD sum rule method.

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I. INTRODUCTION

Within the conventional quark model, mesons are understood as bound states of a quark and an antiquark, while baryons are described as composite systems of three quarks [1,2]. Over the past several decades, numerous light mesons have been observed experimentally [3–32], prompting extensive theoretical investigations. These studies have employed a wide range of approaches, including the quark model [11,33], the MIT bag model [34–38], the flux-tube model [39–41], the AdS/QCD model [42–45], lattice QCD [46–64], and QCD sum rules [65–86], among others.

When applying the QCD sum rule method to investigate meson states, one requires suitable meson operators, each composed of a quark field and an antiquark field combined with an appropriate Lorentz structure Γ :

$$J = \bar{q}_a \Gamma q_a, \quad (1)$$

where a denotes the color index. Without the inclusion of covariant derivatives, there exist five independent local operators:

$$\begin{aligned} \bar{q}_a \gamma_5 q_a, & \quad \bar{q}_a q_a, \\ \bar{q}_a \gamma_\mu q_a, & \quad \bar{q}_a \gamma_\mu \gamma_5 q_a, \\ \bar{q}_a \sigma_{\mu\nu} q_a. & \end{aligned} \quad (2)$$

Additionally, the operator

$$\bar{q}_a \sigma_{\mu\nu} \gamma_5 q_a \quad (3)$$

is not independent, as it can be related to $\bar{q}_a \sigma_{\mu\nu} q_a$ via the identity $\sigma_{\mu\nu} \gamma_5 = i/2 \times \epsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta}$. These “ground-state” meson operators have been extensively studied in the literature. In contrast, “excited” meson operators involving covariant derivatives have received relatively little attention and merit further exploration.

In this paper, we shall systematically investigate excited light meson operators using the QCD sum rule method. Each operator is constructed from a quark field, an antiquark field, and a covariant derivative, combined with an appropriate Lorentz structure Γ . Since the excited states are expected to correspond to internal orbital angular momentum excitations rather than collective excitations of the meson, we have chosen to place the covariant derivatives between the quark and antiquark fields in our operator construction:

$$J = \bar{q}_a \overleftrightarrow{D}_\alpha \Gamma q_a, \quad (4)$$

where the covariant derivative is defined as

$$[X \overleftrightarrow{D}_\alpha Y] = X(D_\alpha Y) - (D_\alpha X)Y, \quad (5)$$

with

$$D_\alpha q_a = \partial_\alpha q_a + i g_s A_\alpha q_a = \partial_\alpha q_a + i g_s A_\alpha^{\lambda_{ab}} \frac{\lambda_{ab}^n}{2} q_b. \quad (6)$$

These operators can couple to meson states through

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$$\langle 0|J|X\rangle = f_X, \quad (7)$$

where f_X denotes the decay constant. In total, we shall construct twelve excited light meson operators and carry out detailed analyses for ten of them. By specifying the quark content as $\bar{q}q$, $\bar{q}s$, or $\bar{s}s$ ($q = u/d$), we shall apply the QCD sum rule method to compute their masses and decay constants.

This paper is organized as follows: In Sec. II, we present a systematic construction of excited light meson operators. Building on this foundation, we perform QCD sum rule analyses using a subset of these operators in Sec. III. Finally, in Sec. IV we summarize our findings and provide a detailed discussion of the results.

II. PHENOMENOLOGICAL ANALYSES

In this section, we systematically construct excited light meson operators by combining one quark field, one antiquark field, one covariant derivative, and appropriate Lorentz structures, as shown in Eq. (4). For example, there are several ways to combine the covariant derivative $\vec{D}_\alpha$ with the tensor operator $\bar{q}_a \sigma_{\mu\nu} q_a$:

- (1) One approach is to leave all Lorentz indices uncontracted:

$$J_{\mu,\alpha\beta} = \bar{q}_a \vec{D}_\mu \sigma_{\alpha\beta} q_a. \quad (8)$$

This operator generally contains spin-1 and spin-2 components and may also include a spin-0 component. It does not contain a spin-3 component, as the indices α and β are antisymmetric. The spin-2 component can be projected using

$$J_{\mu,\alpha\beta}^{2-+} = \mathcal{S}'[\bar{q}_a \vec{D}_\mu \sigma_{\alpha\beta} q_a], \quad (9)$$

where $\mathcal{S}'[\dots]$ denotes symmetrization and trace subtraction over the set $\{\mu, \alpha\}$, implemented via the projection operator

$$\Gamma_{\alpha'\mu'\nu';\alpha\mu\nu} = g_{\alpha'\alpha}g_{\mu'\mu}g_{\nu'\nu} + g_{\mu'\alpha}g_{\alpha'\mu}g_{\nu'\nu} - \frac{1}{2}g_{\alpha'\mu'}g_{\alpha\mu}g_{\nu'\nu}. \quad (10)$$

- (2) The spin-1 component of Eq. (8) can be more conveniently obtained by index contraction:

$$J_\alpha^{1-+} = \bar{q}_a \vec{D}_\mu \sigma_{\alpha\beta} q_a \times g^{\mu\beta}. \quad (11)$$

- (3) Equation (8) may also contain a spin-0 component, though we currently do not know how to isolate it in a relativistically covariant form.

As summarized in Fig. 1, a total of twelve such operators are constructed and analyzed in this work, denoted as $J_{\dots}^{JPC}$,

where JPC represents the quantum numbers of each operator: J denotes the total angular momentum, P the parity, and C the charge-conjugation parity. The operators are defined as follows:

$$J_\mu^{1+-} = \bar{q}_a \vec{D}_\mu \gamma_5 q_a, \quad (12)$$

$$J_\mu^{1--} = \bar{q}_a \vec{D}_\mu q_a, \quad (13)$$

$$J^{0++} = \bar{q}_a \vec{D}_\mu \gamma_\nu q_a \times g^{\mu\nu}, \quad (14)$$

$$J_{\mu\nu}^{1++} = \mathcal{A}[\bar{q}_a \vec{D}_\mu \gamma_\nu q_a], \quad (15)$$

$$J_{\mu\nu}^{2++} = \mathcal{S}[\bar{q}_a \vec{D}_\mu \gamma_\nu q_a], \quad (16)$$

$$J^{0--} = \bar{q}_a \vec{D}_\mu \gamma_\nu \gamma_5 q_a \times g^{\mu\nu}, \quad (17)$$

$$J_{\mu\nu}^{1--} = \mathcal{A}[\bar{q}_a \vec{D}_\mu \gamma_\nu \gamma_5 q_a], \quad (18)$$

$$J_{\mu\nu}^{2--} = \mathcal{S}[\bar{q}_a \vec{D}_\mu \gamma_\nu \gamma_5 q_a], \quad (19)$$

$$J_\alpha^{1-+} = \bar{q}_a \vec{D}_\mu \sigma_{\alpha\beta} q_a \times g^{\mu\beta}, \quad (20)$$

$$J_{\mu,\alpha\beta}^{2-+} = \mathcal{S}'[\bar{q}_a \vec{D}_\mu \sigma_{\alpha\beta} q_a], \quad (21)$$

$$J_\alpha^{1++} = \bar{q}_a \vec{D}_\mu \sigma_{\alpha\beta} \gamma_5 q_a \times g^{\mu\beta}, \quad (22)$$

$$J_{\mu,\alpha\beta}^{2++} = \mathcal{S}'[\bar{q}_a \vec{D}_\mu \sigma_{\alpha\beta} \gamma_5 q_a]. \quad (23)$$

The derivations of their quantum numbers will be presented in detail in Sec. IV. In the above expressions, the symbol $\mathcal{A}[\dots]$ denotes antisymmetrization over the index set $\{\mu\nu\}$, which is implemented by applying the projection operator

$$\Gamma_{\mu'\nu';\mu\nu} = g_{\mu'\mu}g_{\nu'\nu} - g_{\nu'\mu}g_{\mu'\nu}. \quad (24)$$

The symbol $\mathcal{S}[\dots]$ represents symmetrization and trace subtraction over $\{\mu\nu\}$, achieved using the projection operator

$$\Gamma_{\mu'\nu';\mu\nu} = g_{\mu'\mu}g_{\nu'\nu} + g_{\nu'\mu}g_{\mu'\nu} - \frac{1}{2}g_{\mu'\nu'}g_{\mu\nu}. \quad (25)$$

Due to the substantial computational complexity associated with the operators $J_{\mu\alpha\beta}^{2-+}$ and $J_{\mu\alpha\beta}^{2++}$, we have not carried out a QCD sum rule analysis for these specific cases. Instead, we refer the reader to Ref. [87] for a detailed discussion. In addition, the operator given in Eq. (14) can

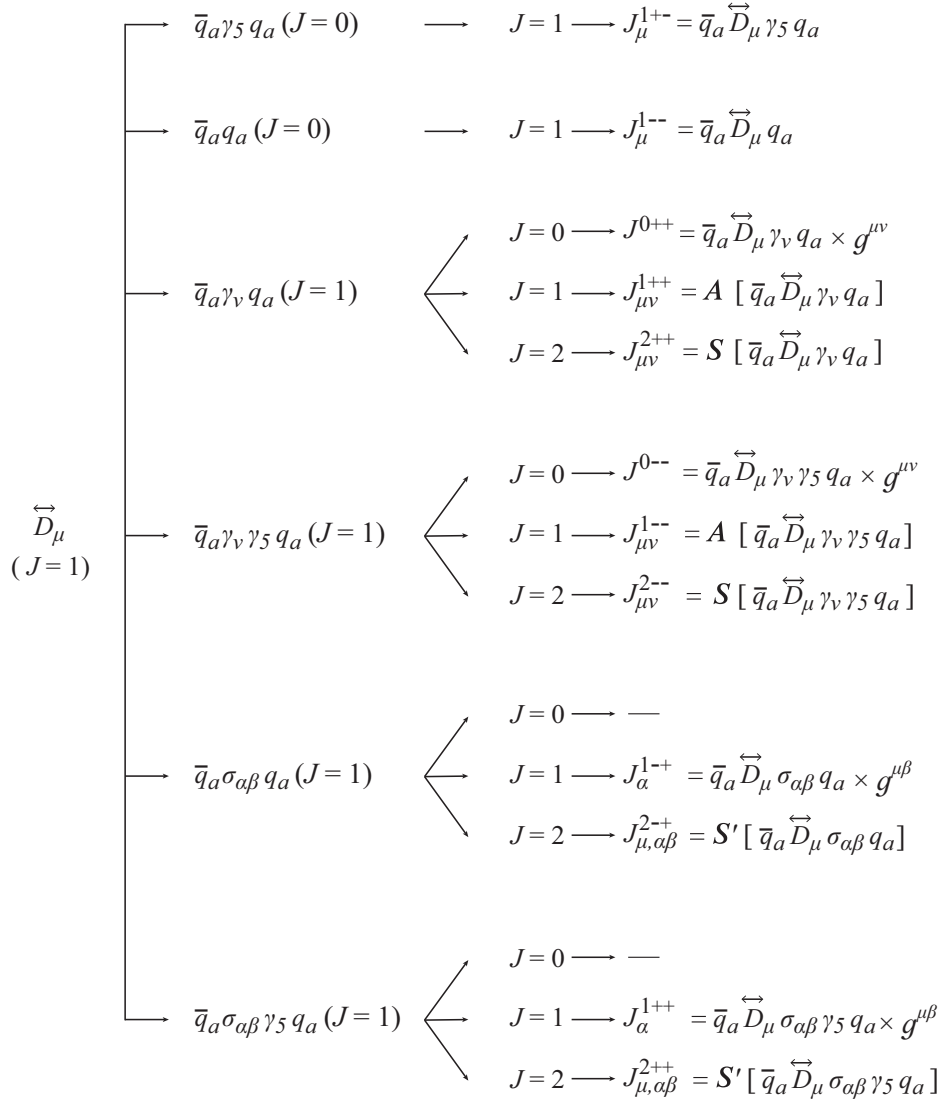


FIG. 1. Categorization of excited light meson operators: Each operator is constructed from a quark field, an antiquark field, and a covariant derivative, combined with an appropriate Lorentz structure Γ . The symbol $\mathcal{A}[\dots]$ denotes antisymmetrization over the index set $\{\mu\nu\}$, $\mathcal{S}[\dots]$ represents symmetrization and trace subtraction over $\{\mu\nu\}$ and $\mathcal{S}'[\dots]$ indicates symmetrization and trace subtraction over the set $\{\mu\alpha\}$.

be transformed through the equations of motion for the quark fields as

$$\bar{q}_a \overleftrightarrow{D}_\mu \gamma_\nu q_a \times g^{\mu\nu} = -2im_q \bar{q}_a q_a. \quad (26)$$

Similarly, the operator given in Eq. (17) can be transformed as

$$\bar{q}_a \overleftrightarrow{D}_\mu \gamma_\nu \gamma_5 q_a \times g^{\mu\nu} = 0. \quad (27)$$

Therefore, we do not use them to perform QCD sum rule analysis.

Since each operator can, in general, couple to multiple states, we analyze the Lorentz structures of the excited light meson operators listed above individually in the following:

- (1) The operators J^{0++} and J^{0--} contain only the $J^{PC} = 0^{++}$ and 0^{--} components, respectively. Accordingly, they couple exclusively to the corresponding spin-0 meson states via

$$\langle 0 | J^{0++} | X_{0++} \rangle = f_{0++}, \quad (28)$$

$$\langle 0 | J^{0--} | X_{0--} \rangle = f_{0--}, \quad (29)$$

where f_{0++} and f_{0--} denote the decay constants.

- (2) The operators $J_{\mu\nu}^{2++}$ and $J_{\mu\nu}^{2--}$ contain only the $J^{PC} = 2^{++}$ and 2^{--} components, respectively. Accordingly,

they couple exclusively to the corresponding spin-2 meson states via

$$\langle 0 | J_{\mu\nu}^{2++} | X_{2++} \rangle = \epsilon_{\mu\nu} f_{2++}, \quad (30)$$

$$\langle 0 | J_{\mu\nu}^{2--} | X_{2--} \rangle = \epsilon_{\mu\nu} f_{2--}, \quad (31)$$

where f_{2++} and f_{2--} are the decay constants, and $\epsilon_{\mu\nu}$ is the polarization tensor with two symmetric Lorentz indices $\mu\nu$.

- (3) The operator $J_{\mu\nu}^{1--}$, with two antisymmetric Lorentz indices $\{\mu\nu\}$, contains both $J^{PC} = 1^{--}$ and 1^{+-} components. Therefore, it can couple to meson states with these quantum numbers through

$$\langle 0 | J_{\mu\nu}^{1--} | X_{1--} \rangle = i f_{1--} \epsilon_{\mu\nu\alpha\beta} \epsilon^\alpha q^\beta, \quad (32)$$

$$\langle 0 | J_{\mu\nu}^{1--} | X_{1+-} \rangle = i f_{1+-} (q_\mu \epsilon_\nu - q_\nu \epsilon_\mu), \quad (33)$$

where f_{1--} and f_{1+-} are the decay constants, q_μ is the four-momentum, ϵ_μ is the polarization vector, and $\epsilon_{\mu\nu\alpha\beta}$ denotes the Levi-Civita symbol in four dimensions, which is the fully antisymmetric tensor. Since the Lorentz structures in Eqs. (32) and (33) are totally different, we can clearly separate the contributions of the two states X_{1--} and X_{1+-} at the hadron level. Specifically, we can isolate X_{1--} by considering the two-point correlation function

$$\begin{aligned} & \langle 0 | J_{\mu\nu}^{1--} | X_{1--} \rangle \langle X_{1--} | J_{\mu'\nu'}^{1--\dagger} | 0 \rangle \\ &= f_{1--}^2 \epsilon_{\mu\nu\alpha\beta} \epsilon^\alpha q^\beta \epsilon_{\mu'\nu'\alpha'\beta'} \epsilon^{*\alpha'} q^{\beta'} \\ &= -f_{1--}^2 [q^2 (g_{\mu\mu'} g_{\nu\nu'} - g_{\mu\nu} g_{\nu\mu'}) - g_{\mu\nu} q_\nu q_{\mu'} \\ &\quad - g_{\nu\mu'} q_\mu q_{\mu'} + g_{\mu\mu'} q_\nu q_{\nu'} + g_{\nu\nu'} q_\mu q_{\mu'}], \end{aligned} \quad (34)$$

and we can isolate X_{1+-} through

$$\begin{aligned} & \langle 0 | J_{\mu\nu}^{1--} | X_{1+-} \rangle \langle X_{1+-} | J_{\mu'\nu'}^{1--\dagger} | 0 \rangle \\ &= f_{1+-}^2 (q_\mu \epsilon_\nu - q_\nu \epsilon_\mu) (q_{\mu'} \epsilon_{\nu'}^* - q_{\nu'} \epsilon_{\mu'}^*) \\ &= f_{1+-}^2 (g_{\mu\mu'} q_\nu q_{\nu'} + g_{\nu\nu'} q_\mu q_{\mu'} \\ &\quad - g_{\mu\nu} q_\nu q_{\mu'} - g_{\nu\mu'} q_\mu q_{\nu'}). \end{aligned} \quad (35)$$

Similarly, we can investigate the operator $J_{\mu\nu}^{1++}$, which can couple to both $J^{PC} = 1^{++}$ and $J^{PC} = 1^{-+}$ meson states.

- (4) The operator $J_{\mu\nu}^{1-+}$ contains both $J^{PC} = 1^{-+}$ and 0^{++} components, and therefore it can couple to meson states with these quantum numbers through

$$\langle 0 | J_{\mu\nu}^{1-+} | X_{1-+} \rangle = \epsilon_{\mu\nu} f_{1-+}, \quad (36)$$

$$\langle 0 | J_{\mu\nu}^{1-+} | X_{0++} \rangle = q_\mu f_{0++}, \quad (37)$$

where f_{1-+} and f_{0++} are the decay constants. These two states, X_{1-+} and X_{0++} , can be clearly separated at the hadron level by analyzing the two-point correlation function

$$\begin{aligned} \Pi_{\mu\mu'}^{1-+}(q^2) &\equiv i \int d^4x e^{iqx} \langle 0 | \mathbf{T} [J_{\mu}^{1-+}(x) J_{\mu'}^{1-+\dagger}(0)] | 0 \rangle \\ &= (g_{\mu\mu'} - q_\mu q_{\mu'} / q^2) \Pi_{1-+}(q^2) \\ &\quad + (q_\mu q_{\mu'} / q^2) \Pi_{0++}(q^2). \end{aligned} \quad (38)$$

Similarly, we can investigate the operators J_{μ}^{1++} , J_{μ}^{1--} , and J_{μ}^{1+-} .

III. QCD SUM RULE ANALYSIS

The QCD sum rule method represents a well-established and powerful nonperturbative framework in the study of hadron physics, as demonstrated in numerous foundational works [65,88–93]. In the present work, we employ this formalism to systematically examine the properties of the ten excited light meson operators introduced in Eqs. (12)–(20) and Eq. (22). For clarity and conciseness, we focus our analysis on the representative case of the 2^{++} tensor operator $J_{\mu\nu}^{2++}$ defined in Eq. (16), with particular emphasis on the investigation of its two-point correlation function

$$\begin{aligned} \Pi_{\mu\nu,\mu'\nu'}^{2++}(q^2) &= i \int d^4x e^{iqx} \langle 0 | \mathbf{T} [J_{\mu\nu}^{2++}(x) J_{\mu'\nu'}^{2++\dagger}(0)] | 0 \rangle \\ &= \Pi_{2++}(q^2) \times \epsilon_{\mu\nu} \epsilon_{\mu'\nu'}^* + \dots \end{aligned} \quad (39)$$

at both the hadron and quark-gluon levels, where $\dots$ denotes the other structures with different Lorentz structures, such as $q_\mu q_\nu q_{\mu'}^{\nu'}$, which are omitted as they do not affect the final numerical results. The tensor structure is given by

$$\epsilon_{\mu\nu} \epsilon_{\mu'\nu'}^* = g_{\mu\mu'} g_{\nu\nu'} + g_{\mu\nu} g_{\nu\mu'} - \frac{1}{2} g_{\mu\nu} g_{\mu'\nu'}. \quad (40)$$

At the hadron level, the dispersion relation can be derived using Cauchy's contour integration, allowing the correlation function in Eq. (39) to be expressed as

$$\Pi_{2++}(q^2) = \int_{t_{\min}}^{\infty} \frac{\rho_{\text{phen}}(s)}{s - q^2 - i\epsilon} ds, \quad (41)$$

where $t_{\min} = \{M_{2++}^2, s_0^h\}$, with M_{2++} denoting the mass of the meson X_{2++} , and s_0^h is the threshold of the lowest continuum state. The phenomenological spectral density $\rho_{\text{phen}}(s)$ is parametrized as

$$\begin{aligned}\rho_{\text{phen}}(s) &\equiv \frac{1}{\pi} \text{Im} \Pi_{2^{++}}(q^2) \\ &= f_{2^{++}}^2 \delta(s - M_{2^{++}}^2) + \rho_{\text{phen}}^{\text{higher}}(s) \theta(s - s_0^h),\end{aligned}\quad (42)$$

where $\rho_{\text{phen}}^{\text{higher}}(s)$ accounts for the contributions from higher excited states and the continuum.

At the quark-gluon level, we employ the operator product expansion (OPE) to evaluate the correlation function in Eq. (39), from which the OPE spectral density $\rho_{\text{OPE}}(s)$ is extracted. The correlation function is then expressed as

$$\Pi_{2^{++}}(q^2) = \int_{s_<}^{\infty} \frac{\rho_{\text{OPE}}(s)}{s - q^2 - i\epsilon} ds, \quad (43)$$

where $s_<$ denotes the physical threshold at the OPE level. Specifically, we take $s_< = 0$, m_s^2 , and $4m_s^2$ for the quark contents $\bar{q}q$, $\bar{q}s$, and $\bar{s}s$, respectively ($q = u, d$).

Based on the principle of quark-hadron duality—which asserts that in the deep Euclidean limit $q^2 \rightarrow -\infty$, the correlation function becomes dominated by perturbative contributions while condensate effects vanish—we obtain the relation

$$\int_{s_0^h}^{\infty} \frac{\rho_{\text{phen}}^{\text{higher}}(s)}{s(s - q^2)} ds = \int_{s_0}^{\infty} \frac{\rho_{\text{OPE}}(s)}{s(s - q^2)} ds, \quad (44)$$

where the parameter s_0 serves as a continuum threshold to be determined through phenomenological analysis. Note that s_0 and s_0^h are not necessarily equal.

To enhance the ground-state contribution and suppress those from higher states and the continuum, we perform a Borel transformation on both sides. Combining Eqs. (41) and (43), we arrive at

$$\begin{aligned}&\int_{s_<}^{\infty} \rho_{\text{OPE}}(s) e^{-s/M_B^2} ds \\ &= f_{2^{++}}^2 e^{-M_{2^{++}}^2/M_B^2} + \int_{s_0^h}^{\infty} \rho_{\text{phen}}^{\text{higher}}(s) e^{-s/M_B^2} ds.\end{aligned}\quad (45)$$

This yields the QCD sum rule relation:

$$\begin{aligned}\Pi_{2^{++}}(s_0, M_B^2) &\equiv f_{2^{++}}^2 e^{-M_{2^{++}}^2/M_B^2} \\ &= \int_{s_<}^{s_0} e^{-s/M_B^2} \rho_{\text{OPE}}(s) ds.\end{aligned}\quad (46)$$

The mass and decay constant of the meson $X_{2^{++}}$ are then extracted using

$$M_{2^{++}}^2(s_0, M_B^2) = \frac{\int_{s_<}^{s_0} e^{-s/M_B^2} s \rho_{\text{OPE}}(s) ds}{\int_{s_<}^{s_0} e^{-s/M_B^2} \rho_{\text{OPE}}(s) ds}, \quad (47)$$

$$f_{2^{++}}^2(s_0, M_B^2) = \Pi_{2^{++}}(s_0, M_B^2) \cdot e^{M_{2^{++}}^2/M_B^2}. \quad (48)$$

Specifically, the sum rule equation derived from the operator $J_{\mu\nu}^{2^{++}}$ with the quark content $\bar{q}s$ ($q = u/d$) is given by

$$\begin{aligned}\Pi_{2^{++}} &= \int_{m_s^2}^{s_0} \left[-\frac{3s^2}{20\pi^2} + \left(+\frac{m_s^2}{2\pi^2} + \frac{m_q^2}{2\pi^2} - \frac{m_q m_s}{2\pi^2} \right) s \right. \\ &\quad \left. + \frac{\langle g_s^2 GG \rangle}{18\pi^2} - \frac{m_s^2 m_q^2}{2\pi^2} \right] e^{-s/M_B^2} ds \\ &\quad + (+m_q \langle g_s \bar{s} \sigma G s \rangle + m_s \langle g_s \bar{q} \sigma G q \rangle) \\ &\quad + \frac{16\pi^2}{M_B^2} \left(+\frac{\langle \bar{q}q \rangle \langle g_s \bar{s} \sigma G s \rangle}{48} + \frac{\langle \bar{s}s \rangle \langle g_s \bar{q} \sigma G q \rangle}{48} \right).\end{aligned}\quad (49)$$

Within the QCD sum rule framework, we derive the sum rule equation for the $\bar{q}q$ quark content through systematic substitutions: $m_s \rightarrow m_q$, $\langle \bar{s}s \rangle \rightarrow \langle \bar{q}q \rangle$, and $\langle g_s \bar{s} \sigma G s \rangle \rightarrow \langle g_s \bar{q} \sigma G q \rangle$. Conversely, the corresponding expression for the $\bar{s}s$ configuration can be obtained by applying the inverse replacements. Within the QCD sum rule framework, the up and down quarks are treated as degenerate. As a result, for each operator, the extracted results for the isoscalar and isovector $\bar{q}q$ states are identical. For completeness, we provide the full set of sum rule equations obtained from the remaining operators in the Appendix.

In the calculations, we take into account the Feynman diagrams depicted in Fig. 2. The covariant derivative operator $D_\alpha = \partial_\alpha + ig_s A_\alpha$ can be naturally separated into two terms, with the gluon field from the second term illustrated explicitly in Figs. 2(c–2)–2(e–2). We consider the perturbative term, the strange quark mass m_s , the quark condensates $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$, the gluon condensate $\langle g_s^2 GG \rangle$, and the quark-gluon mixed condensates $\langle g_s \bar{q} \sigma G q \rangle$ and $\langle g_s \bar{s} \sigma G s \rangle$, as well as their combinations. Vacuum saturation is assumed for higher-dimensional condensates. Other condensates, such as $\langle g_s^3 G^3 \rangle$ and $\langle g_s \bar{s} D_\mu G^{\mu\nu} \gamma_\nu s \rangle$, are not considered in the present study. We have accounted for all diagrams proportional to $g_s^{N=0}$ and $g_s^{N=1}$, but only partially for those proportional to $g_s^{N=2}$. Notably, the $D = 6$ term $m_s \langle g_s \bar{q} \sigma G q \rangle$, as well as the $D = 8$ terms $\langle \bar{q}q \rangle \langle g_s \bar{s} \sigma G s \rangle$ and $\langle \bar{s}s \rangle \langle g_s \bar{q} \sigma G q \rangle$, play significant roles.

We employ Eq. (49) for the numerical analysis and refer to the corresponding state as $|\bar{q}s; 2^{++}\rangle$. The following values for various QCD parameters are adopted, as suggested in Refs. [3,67,94–100]:

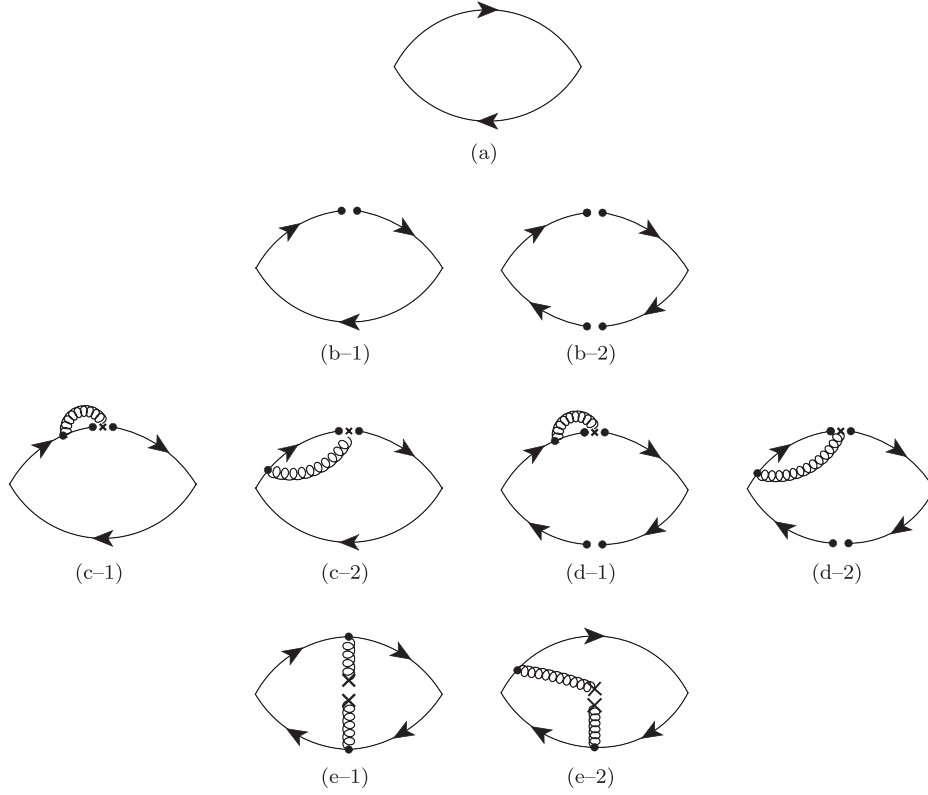


FIG. 2. Feynman diagrams for excited light meson operators. Diagrams (a) and (b-1) are proportional to $g_s^{N=0}$; diagrams (c-1) and (d-1) are proportional to $g_s^{N=1}$; and diagrams (e-1) are proportional to $g_s^{N=2}$. The gluons appearing in diagrams (c-2), (d-2), and (e-2) originate (partly) from the covariant derivative operator.

$$\begin{aligned}
 m_q &= m_u = m_d \rightarrow 0, \\
 m_s(2 \text{ GeV}) &= 93_{-5}^{+11} \text{ MeV}, \\
 \langle g_s^2 GG \rangle &= (6.35 \pm 0.35) \times 10^{-2} \text{ GeV}^4, \\
 \langle \bar{q}q \rangle &= -(0.240 \pm 0.010)^3 \text{ GeV}^3, \\
 \langle \bar{s}s \rangle &= (0.8 \pm 0.1) \times \langle \bar{q}q \rangle, \\
 \langle g_s \bar{q} \sigma G q \rangle &= -M_0^2 \times \langle \bar{q}q \rangle, \\
 \langle g_s \bar{s} \sigma G s \rangle &= -M_0^2 \times \langle \bar{s}s \rangle, \\
 M_0^2 &= (0.8 \pm 0.2) \text{ GeV}^2.
 \end{aligned} \tag{50}$$

According to Eq. (47), the mass of the state $|\bar{q}s; 2^{++}\rangle$ depends on two free parameters: the continuum threshold s_0 and the Borel mass M_B . To determine their respective working regions, we systematically analyze three key criteria: (a) the convergence of OPE, (b) the dominance of the pole contribution over the continuum, and (c) the stability of the extracted mass with respect to variations in these parameters.

First, we investigate the convergence of OPE and require the $D = 8$ and $D = 6$ terms to be less than 5% and 10%, respectively:

$$\text{CVG}_8 \equiv \left| \frac{\Pi_{2^{++}}^{D=8}(\infty, M_B^2)}{\Pi_{2^{++}}(\infty, M_B^2)} \right| < 5\%, \tag{51}$$

$$\text{CVG}_6 \equiv \left| \frac{\Pi_{2^{++}}^{D=6}(\infty, M_B^2)}{\Pi_{2^{++}}(\infty, M_B^2)} \right| < 10\%. \tag{52}$$

As illustrated in Fig. 3 (dashed curves), we determine a lower bound for the Borel mass as $M_B^2 > 0.90 \text{ GeV}^2$.

Second, we investigate the one-pole-dominance assumption and require the pole contribution to be larger than 40%:

$$\text{Pole contribution} \equiv \left| \frac{\Pi_{2^{++}}(s_0, M_B^2)}{\Pi_{2^{++}}(\infty, M_B^2)} \right| > 40\%. \tag{53}$$

As shown in Fig. 3 (solid curve), we determine an upper bound for the Borel mass, $M_B^2 < 1.14 \text{ GeV}^2$, when adopting a threshold value of $s_0 = 2.9 \text{ GeV}^2$. Based on this criterion, the valid Borel window at this threshold is identified as $0.90 < M_B^2 < 1.14 \text{ GeV}^2$. Extending the analysis to other values of s_0 , we find that viable Borel windows persist as long as the threshold satisfies $s_0 > s_0^{\min} = 2.4 \text{ GeV}^2$. In practice, s_0 is typically chosen to be 10%–20% larger than $s_0^{\min}$. In this study, we adopt

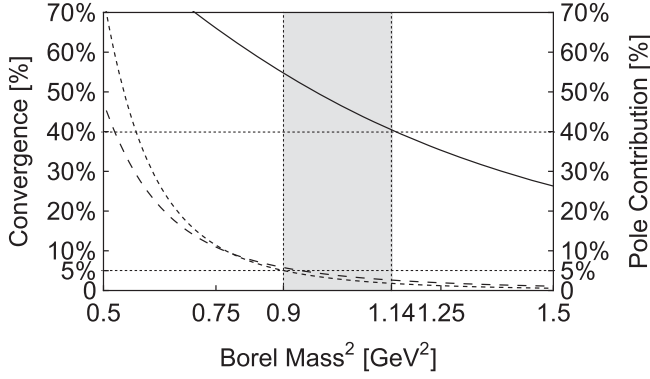


FIG. 3. CVG_8 [short dashed curve defined in Eq. (51)], CVG_6 [middle dashed curve defined in Eq. (52)], and the pole contribution [solid curve defined in Eq. (53)] as functions of the Borel mass M_B . These curves are obtained using the operator $J_{\mu\nu}^{2^{++}}$ with the quark content $\bar{q}s$ ($q = u/d$), under the threshold setting $s_0 = 2.9 \text{ GeV}^2$.

$s_0 = 2.9 \text{ GeV}^2$, within the range of $2.4 \text{ GeV}^2 < s_0 < 3.4 \text{ GeV}^2$.

Third, we examine the parameter dependence of the extracted mass, $M_{|\bar{q}s;2^{++}\rangle}$, by systematically varying both the threshold value s_0 and the Borel mass M_B . As shown in Fig. 4, our analysis reveals that the mass exhibits moderate sensitivity to s_0 near 2.9 GeV^2 , while demonstrating remarkable stability against variations in M_B across the Borel window of $0.90 \text{ GeV}^2 < M_B^2 < 1.14 \text{ GeV}^2$. Based on these observations, we establish the optimal working regions as $2.4 \text{ GeV}^2 < s_0 < 3.4 \text{ GeV}^2$ and $0.90 \text{ GeV}^2 < M_B^2 < 1.14 \text{ GeV}^2$. Within these carefully determined parameter ranges, we obtain stable predictions for both the mass and decay constant of the $|\bar{q}s;2^{++}\rangle$ state as

$$M_{|\bar{q}s;2^{++}\rangle} = 1.44^{+0.08}_{-0.09} \text{ GeV}, \quad (54)$$

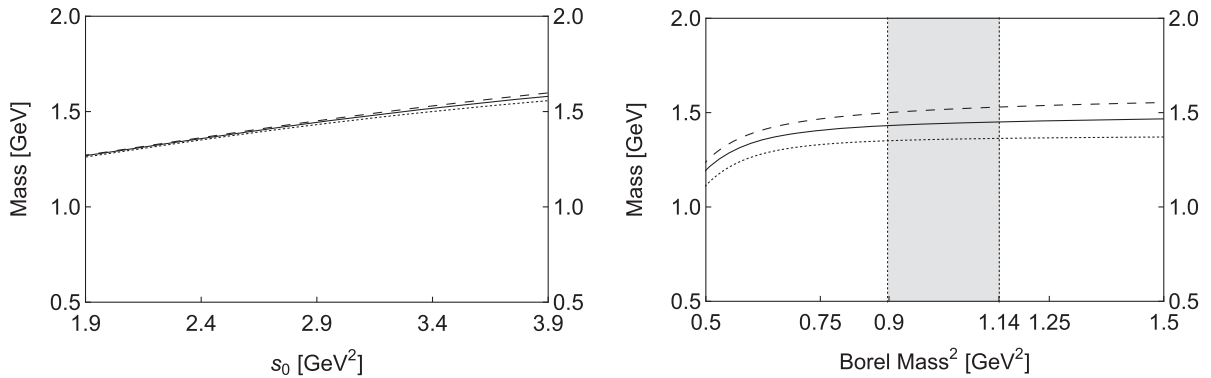


FIG. 4. Mass of the state $|\bar{q}s;2^{++}\rangle$, extracted from the operator $J_{\mu\nu}^{2^{++}}$ with the quark content $\bar{q}s$ ($q = u/d$), as a function of the threshold value s_0 (left) and the Borel mass M_B (right). In the left panel, the short-dashed/solid/long-dashed curves are plotted by setting $M_B^2 = 0.90/1.02/1.14 \text{ GeV}^2$, respectively. In the right panel, the short-dashed/solid/long-dashed curves are plotted by setting $s_0 = 2.4/2.9/3.4 \text{ GeV}^2$, respectively.

$$f_{|\bar{q}s;2^{++}\rangle} = 0.313^{+0.078}_{-0.079} \text{ GeV}^3. \quad (55)$$

Its central value corresponds to $s_0 = 2.9 \text{ GeV}^2$ and $M_B^2 = 1.02 \text{ GeV}^2$, and its uncertainty comes from the threshold value s_0 , the Borel mass M_B , and various QCD parameters listed in Eqs. (50).

We systematically apply the same methodology to analyze the remaining operators given in Eqs. (12)–(20) and Eq. (22). The comprehensive results of these analyses are summarized in Table I. Notably, for certain cases—the spin-0 states coupled by $J^{0^{++}}$ and $J^{0^{--}}$, and the spin-0 states coupled by $J_{\mu}^{1^{--}}$ and $J_{\mu}^{1^{+-}}$, as well as the spin-1 states coupled by $J_{\alpha}^{1^{+-}}$ and $J_{\alpha}^{1^{++}}$ —the derived QCD sum rule expressions lack a perturbative term. The absence of this leading-order contribution compromises the reliability of the corresponding predictions. Therefore, we do not report results for these cases.

IV. SUMMARY AND DISCUSSIONS

In this study, we employ the QCD sum rule method to systematically investigate excited light meson operators composed of one quark field, one antiquark field, and one covariant derivative. A comprehensive set of twelve such operators is constructed, of which ten are selected for detailed QCD sum rule analyses. We compute the masses and decay constants of the corresponding meson states, considering quark contents of the forms $\bar{q}q$, $\bar{s}s$, and $\bar{q}s$ ($q = u/d$). The numerical results are summarized in Table I and are in excellent agreement with the lattice QCD calculations reported in Ref. [101], thereby substantially reinforcing the credibility of our conclusions. A detailed discussion of each case is presented below ($\mu, \nu = 0 - 3$ and $i, j = 1 - 3$):

- (1) The quark-antiquark pair $\bar{q}_a \gamma_5 q_a$ carries $L = 0$, $S = 0$, and $J = 0$. By incorporating a covariant derivative, the operator

TABLE I. QCD sum rule results extracted from the excited light meson operators defined in Eqs. (12)–(20) and Eq. (22), with the quark contents $\bar{q}q$, $\bar{q}s$, and $\bar{s}s$ ($q = u/d$). For each operator, the extracted results for the isoscalar and isovector $\bar{q}q$ states are identical. The corresponding experimental candidates, as reported in PDG [3], are listed in the last column for comparison.

Operators	Meson [J^{PC}]	Working regions				Mass (GeV)	Decay constant	Candidate [3]
		$s_0^{\min}$ [GeV ²]	M_B^2 [GeV ²]	s_0 [GeV ²]	Pole (%)			
J_μ^{1--}	$ \bar{q}q; 1^{--}\rangle$	3.8	1.70–2.05	4.6	40–52	$1.70^{+0.14}_{-0.17}$	$0.602^{+0.173}_{-0.169}$ GeV ³	$\rho(1700)/\omega(1650)$
	$ \bar{q}s; 1^{--}\rangle$	3.8	1.71–2.09	4.7	40–53	$1.71^{+0.14}_{-0.17}$	$0.617^{+0.173}_{-0.170}$ GeV ³	$K^*(1680)$
	$ \bar{s}s; 1^{--}\rangle$	5.1	2.25–2.81	6.4	40–55	$2.00^{+0.15}_{-0.19}$	$0.968^{+0.260}_{-0.257}$ GeV ³	...
	$ \bar{q}q/\bar{q}s/\bar{s}s; 0^{+(-)}\rangle$					...		
J_μ^{1+-}	$ \bar{q}q; 1^{+-}\rangle$	2.1	1.00–1.21	2.6	40–53	$1.22^{+0.10}_{-0.13}$	$0.257^{+0.063}_{-0.062}$ GeV ³	$b_1(1235)/h_1(1170)$
	$ \bar{q}s; 1^{+-}\rangle$	2.2	1.01–1.23	2.7	40–53	$1.27^{+0.10}_{-0.12}$	$0.271^{+0.065}_{-0.064}$ GeV ³	$K_1(1270)$
	$ \bar{s}s; 1^{+-}\rangle$	2.6	1.18–1.43	3.2	40–53	$1.41^{+0.10}_{-0.12}$	$0.348^{+0.086}_{-0.084}$ GeV ³	$h_1(1415)$
	$ \bar{q}q/\bar{q}s/\bar{s}s; 0^{-(-)}\rangle$					...		
$J_{\mu\nu}^{1++}$	$ \bar{q}q; 1^{++}\rangle$	2.1	0.94–1.15	2.6	40–53	$1.25^{+0.10}_{-0.13}$	$0.111^{+0.010}_{-0.010}$ GeV ²	$a_1(1260)/f_1(1285)$
	$ \bar{q}s; 1^{++}\rangle$	2.1	0.92–1.13	2.6	40–53	$1.28^{+0.09}_{-0.11}$	$0.106^{+0.011}_{-0.012}$ GeV ²	$K_1(1270)$
	$ \bar{s}s; 1^{++}\rangle$	2.5	1.08–1.33	3.1	40–53	$1.42^{+0.10}_{-0.12}$	$0.114^{+0.012}_{-0.014}$ GeV ²	$f_1(1420)$
	$ \bar{q}q; 1^{++}\rangle$	3.5	2.34–2.98	4.3	40–52	$1.65^{+0.11}_{-0.13}$	$0.328^{+0.062}_{-0.066}$ GeV ²	$\pi_1(1600)$
	$ \bar{q}s; 1^{--}\rangle$	3.9	2.65–3.35	4.8	40–52	$1.72^{+0.12}_{-0.14}$	$0.364^{+0.070}_{-0.075}$ GeV ²	$K^*(1680)$
	$ \bar{s}s; 1^{--}\rangle$	4.6	3.20–4.02	5.7	40–51	$1.85^{+0.13}_{-0.15}$	$0.431^{+0.077}_{-0.081}$ GeV ²	$\eta_1(1855)$
$J_{\mu\nu}^{2++}$	$ \bar{q}q; 2^{++}\rangle$	2.3	0.93–1.14	2.8	40–53	$1.36^{+0.11}_{-0.14}$	$0.286^{+0.091}_{-0.092}$ GeV ³	$a_2(1320)/f_2(1270)$
	$ \bar{q}s; 2^{++}\rangle$	2.4	0.90–1.14	2.9	40–54	$1.44^{+0.08}_{-0.09}$	$0.313^{+0.078}_{-0.079}$ GeV ³	$K_2^*(1430)$
	$ \bar{s}s; 2^{++}\rangle$	2.5	0.92–1.22	3.1	40–57	$1.52^{+0.08}_{-0.08}$	$0.358^{+0.096}_{-0.098}$ GeV ³	$f_2'(1525)$
$J_{\mu\nu}^{1--}$	$ \bar{q}q; 1^{--}\rangle$	3.7	1.60–1.97	4.5	40–53	$1.70^{+0.12}_{-0.15}$	$0.142^{+0.015}_{-0.017}$ GeV ²	$\rho(1700)/\omega(1650)$
	$ \bar{q}s; 1^{--}\rangle$	3.7	1.61–2.01	4.6	40–54	$1.71^{+0.12}_{-0.15}$	$0.143^{+0.014}_{-0.016}$ GeV ²	...
	$ \bar{s}s; 1^{--}\rangle$	5.0	2.16–2.74	6.3	40–55	$2.00^{+0.15}_{-0.18}$	$0.166^{+0.018}_{-0.020}$ GeV ²	...
	$ \bar{q}q; 1^{+-}\rangle$	3.2	2.14–2.76	4.0	40–52	$1.60^{+0.10}_{-0.12}$	$0.305^{+0.057}_{-0.060}$ GeV ²	$h_1(1595)$
	$ \bar{q}s; 1^{+-}\rangle$	3.5	2.38–2.98	4.3	40–51	$1.64^{+0.11}_{-0.13}$	$0.327^{+0.063}_{-0.067}$ GeV ²	...
	$ \bar{s}s; 1^{+-}\rangle$	4.5	3.10–3.94	5.6	40–52	$1.84^{+0.13}_{-0.15}$	$0.423^{+0.077}_{-0.081}$ GeV ²	...
$J_{\mu\nu}^{2--}$	$ \bar{q}q; 2^{--}\rangle$	3.4	1.40–1.76	4.2	40–54	$1.69^{+0.10}_{-0.12}$	$0.567^{+0.147}_{-0.147}$ GeV ³	...
	$ \bar{q}s; 2^{--}\rangle$	4.1	1.73–2.13	5.0	40–53	$1.81^{+0.13}_{-0.15}$	$0.728^{+0.199}_{-0.197}$ GeV ³	$K_2(1820)$
	$ \bar{s}s; 2^{--}\rangle$	4.5	1.90–2.39	5.6	40–55	$1.91^{+0.13}_{-0.16}$	$0.855^{+0.230}_{-0.228}$ GeV ³	...
J_μ^{1+-}	$ \bar{q}q; 0^{++}\rangle$	2.8	1.23–1.51	3.4	40–53	$1.45^{+0.12}_{-0.14}$	$0.310^{+0.030}_{-0.032}$ GeV ²	$a_0(1450)/f_0(1370)$
	$ \bar{q}s; 0^{++}\rangle$	2.7	1.20–1.46	3.3	40–53	$1.43^{+0.12}_{-0.15}$	$0.304^{+0.030}_{-0.032}$ GeV ²	$K_0^*(1430)$
	$ \bar{s}s; 0^{++}\rangle$	4.0	1.72–2.12	4.9	40–53	$1.78^{+0.13}_{-0.16}$	$0.355^{+0.039}_{-0.044}$ GeV ²	$f_0(1770)$
	$ \bar{q}q/\bar{q}s/\bar{s}s; 1^{+(-)}\rangle$					...		
J_μ^{1++}	$ \bar{q}q; 0^{+-}\rangle$	4.1	1.80–2.19	5.0	40–53	$1.78^{+0.13}_{-0.16}$	$0.368^{+0.037}_{-0.042}$ GeV ²	$\pi(1800)/\eta(1760)$
	$ \bar{q}s; 0^{+-}\rangle$	4.2	1.84–2.28	5.2	40–54	$1.83^{+0.13}_{-0.15}$	$0.372^{+0.037}_{-0.042}$ GeV ²	$K(1830)$
	$ \bar{s}s; 0^{+-}\rangle$	5.1	2.18–2.76	6.4	40–55	$2.03^{+0.15}_{-0.18}$	$0.405^{+0.044}_{-0.050}$ GeV ²	...
	$ \bar{q}q/\bar{q}s/\bar{s}s; 1^{+(+)}\rangle$					...		

$$J_\mu^{1+-} = \bar{q}_a \overleftrightarrow{D}_\mu \gamma_5 q_a \quad (56)$$

$$\langle 0 | J_\mu^{1+-} | X_{1+-} \rangle = \epsilon_\mu f_{1+-}. \quad (57)$$

acquires $L = 1$, $S = 0$, and $J = 1$, thereby allowing the investigation of excited $^1P_1$ meson states with quantum numbers $J^{PC} = 1^{+-}$ through the coupling

Our analysis indicates that the $b_1(1235)$, $h_1(1170)$, $K_1(1270)$, and $h_1(1415)$ states can be classified as members of an $SU(3)$ flavor nonet with quantum numbers $J^{PC} = 1^{+-}$.

- (2) The quark-antiquark pair $\bar{q}_a \sigma_{0i} q_a$ carries $L = 0$, $S = 1$, and $J = 1$. By incorporating a covariant derivative, the $\mu \rightarrow 0$ component of the operator

$$J_a^{1-+} = \bar{q}_a \overleftrightarrow{D}_\mu \sigma_{\alpha\beta} q_a \times g_{\mu\beta} \quad (58)$$

acquires $L = 1$, $S = 1$, and $J = 0$, thereby allowing the investigation of excited ${}^3P_0$ meson states with quantum numbers $J^{PC} = 0^{++}$ through the coupling

$$\langle 0 | J_a^{1-+} | X_{0++} \rangle = q_\mu f_{0++}. \quad (59)$$

Our analysis indicates that the $a_0(1450)$, $f_0(1370)$, $K_0^*(1430)$, and $f_0(1770)$ states can be classified as members of an $SU(3)$ flavor nonet with quantum numbers $J^{P(C)} = 0^{+(+)}$.

- (3) The quark-antiquark pair $\bar{q}_a \gamma_i q_a$ carries $L = 0$, $S = 1$, and $J = 1$. By incorporating a covariant derivative, the $\mu\nu \rightarrow ij$ component of the operator

$$J_{\mu\nu}^{1++} = \mathcal{A}[\bar{q}_a \overleftrightarrow{D}_\mu \gamma_\nu q_a] \quad (60)$$

acquires $L = 1$, $S = 1$, and $J = 1$, thereby allowing the investigation of excited ${}^3P_1$ meson states with quantum numbers $J^{PC} = 1^{++}$ through the coupling

$$\langle 0 | J_{\mu\nu}^{1++} | X_{1++} \rangle = i f_{1++} \epsilon_{\mu\nu\alpha\beta} \epsilon^\alpha q^\beta. \quad (61)$$

Our analysis indicates that the $a_1(1260)$, $f_1(1285)$, $K_1(1270)$, and $f_1(1420)$ states can be classified as members of an $SU(3)$ flavor nonet with quantum numbers $J^{P(C)} = 1^{+(+)}$. This conclusion is well consistent with the lattice QCD calculations reported in Ref. [101].

In addition, the $\mu\nu \rightarrow 0i$ component of the operator $J_{\mu\nu}^{1++}$ can be employed to investigate meson states with the exotic quantum numbers $J^{PC} = 1^{-+}$ via the coupling

$$\langle 0 | J_{\mu\nu}^{1++} | X_{1-+} \rangle = i f_{1-+} (q_\mu \epsilon_\nu - q_\nu \epsilon_\mu). \quad (62)$$

Given that the covariant derivative includes a gluon field, our analysis suggests that the $\pi_1(1600)$ and $\eta_1(1855)$ states can be interpreted as hybrid mesons with quantum numbers $J^{PC} = 1^{-+}$.

- (4) For the quark-antiquark pair $\bar{q}_a \gamma_i q_a$, the inclusion of a covariant derivative also allows the construction of the operator

$$J_{\mu\nu}^{2++} = \mathcal{S}[\bar{q}_a \overleftrightarrow{D}_\mu \gamma_\nu q_a], \quad (63)$$

which carries $L = 1$, $S = 1$, and $J = 2$, thereby allowing the investigation of excited ${}^3P_2$ meson states with quantum numbers $J^{PC} = 2^{++}$ through

the coupling

$$\langle 0 | J_{\mu\nu}^{2++} | X_{2++} \rangle = i f_{2++} \epsilon_{\mu\nu}. \quad (64)$$

Our analysis indicates that the $a_2(1320)$, $f_2(1270)$, $K_2^*(1430)$, and $f_2'(1525)$ states can be classified as members of an $SU(3)$ flavor nonet with quantum numbers $J^{P(C)} = 2^{+(+)}$. This conclusion is also well consistent with the lattice QCD calculations reported in Ref. [101].

- (5) The quark-antiquark pair $\bar{q}_a q_a$ carries $L = 1$, $S = 1$, and $J = 0$. By incorporating a covariant derivative, the $\mu \rightarrow i$ component of the operator

$$J_\mu^{1--} = \bar{q}_a \overleftrightarrow{D}_\mu q_a \quad (65)$$

acquires $L = 2$, $S = 1$, and $J = 1$. Similarly, the quark-antiquark pair $\bar{q}_a \gamma_i \gamma_5 q_a$ carries $L = 1$, $S = 1$, and $J = 1$. By incorporating a covariant derivative, the $\mu\nu \rightarrow ij$ component of the operators

$$J_{\mu\nu}^{1--} = \mathcal{A}[\bar{q}_a \overleftrightarrow{D}_\mu \gamma_\nu \gamma_5 q_a] \quad (66)$$

also acquires $L = 2$, $S = 1$, and $J = 1$. These two operators can thus be used to investigate excited ${}^3D_1$ meson states with quantum numbers $J^{PC} = 1^{--}$, through the couplings

$$\langle 0 | J_\mu^{1--} | X_{1--} \rangle = \epsilon_\mu f_{1--}, \quad (67)$$

$$\langle 0 | J_{\mu\nu}^{1--} | X_{1--} \rangle = i f_{1--} \epsilon_{\mu\nu\alpha\beta} \epsilon^\alpha q^\beta. \quad (68)$$

Our analysis indicates that the $\rho(1700)$, $\omega(1650)$, and $K^*(1680)$ states can be classified as members of an $SU(3)$ flavor nonet with quantum numbers $J^{P(C)} = 1^{-(-)}$. The corresponding $\bar{s}s$ state has not yet been observed experimentally, and its mass is calculated to be $2.00_{-0.19}^{+0.15}$ GeV.

In addition, the quark-antiquark pair $\bar{q}_a \gamma_0 \gamma_5 q_a$ carries $L = 0$, $S = 0$, and $J = 0$. By incorporating a covariant derivative, the $\mu\nu \rightarrow 0i$ component of the operator $J_{\mu\nu}^{1--}$ acquires $L = 1$, $S = 0$, and $J = 1$. Our results suggest that this operator can be employed to investigate the excited 2^1P_1 meson state with quantum numbers $J^{PC} = 1^{+-}$ through the coupling

$$\langle 0 | J_{\mu\nu}^{1--} | X_{1+-} \rangle = i f_{1+-} (q_\mu \epsilon_\nu - q_\nu \epsilon_\mu). \quad (69)$$

Accordingly, the $h_1(1595)$ state can be assigned as a member of an $SU(3)$ flavor nonet with quantum numbers $J^P = 1^{+-}$. The corresponding states with quark contents $\bar{q}s$ and $\bar{s}s$ have not yet been observed

experimentally, and their masses are calculated to be $1.64^{+0.11}_{-0.13}$ GeV and $1.84^{+0.13}_{-0.15}$ GeV, respectively.

- (6) For the quark-antiquark pair $\bar{q}_a \gamma_i \gamma_5 q_a$, the inclusion of a covariant derivative also allows the construction of the operator

$$J_{\mu\nu}^{2--} = S[\bar{q}_a \overleftrightarrow{D}_{\mu} \gamma_{\nu} \gamma_5 q_a], \quad (70)$$

which carries $L = 2$, $S = 1$, and $J = 2$, thereby allowing the investigation of excited ${}^3D_2$ meson states with quantum numbers $J^{PC} = 2--$ through the coupling

$$\langle 0 | J_{\mu\nu}^{2--} | X_{2--} \rangle = i f_{2--} \epsilon_{\mu\nu}. \quad (71)$$

Our analysis indicates that the $K_2(1820)$ state can be classified as a member of an $SU(3)$ flavor nonet with quantum numbers $J^P = 2^-$. The corresponding states with quark contents $\bar{q}q$ and $\bar{s}s$ have not yet been observed experimentally, and their masses are calculated to be $1.69^{+0.10}_{-0.12}$ GeV and $1.91^{+0.13}_{-0.16}$ GeV, respectively.

- (7) The quark-antiquark pair $\bar{q}_a \sigma_{0i} \gamma_5 q_a$ carries $L = 1$, $S = 0$, and $J = 1$. By incorporating a covariant derivative, the $\mu \rightarrow 0$ component of the operator

$$J_a^{1-+} = \bar{q}_a \overleftrightarrow{D}_{\mu} \sigma_{a\beta} \gamma_5 q_a \times g_{\mu\beta} \quad (72)$$

acquires $L = 0$, $S = 0$, and $J = 0$, thereby allowing the investigation of pseudoscalar meson states with quantum numbers $J^{PC} = 0^{+-}$ through the coupling

$$\langle 0 | J_a^{1-+} | X_{0-+} \rangle = q_{\mu} f_{0-+}. \quad (73)$$

Our analysis reveals an unconventional behavior: this operator does not significantly couple to the ground state or the first radial excitation (i.e., the 1^1S_0 or 2^1S_0 states), but predominantly couples to the 3^1S_0 state. Accordingly, the $\pi(1800)$, $\eta(1760)$, and $K(1830)$ can be interpreted as members of an $SU(3)$ flavor nonet with quantum numbers $J^{P(C)} = 0^{-(+)}$. The corresponding $\bar{s}s$ state has not yet been observed experimentally, and its mass is calculated to be $2.03^{+0.15}_{-0.18}$ GeV.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: SPECTRAL DENSITIES

In this appendix, we present the QCD sum rule equations derived from the operators $J_{\mu(l)}^{--}$ defined in Eqs. (13)–(20) and Eq. (22), as follows:

- (1) The operator J_{μ}^{1--} couples to both the $J^{PC} = 1^{--}$ and 0^{+-} states. The corresponding QCD sum rule equations for the quark content $\bar{q}s$ ($q = u/d$) are

$$\begin{aligned} \Pi_{1--} = & \int_{m_s^2}^{s_0} \left[+\frac{s^2}{8\pi^2} + \left(-\frac{m_s^2}{2\pi^2} - \frac{m_q^2}{2\pi^2} - \frac{m_q m_s}{4\pi^2} \right) s + \frac{\langle g_s^2 GG \rangle}{32\pi^2} + \frac{3m_s^2 m_q^2}{4\pi^2} \right] e^{-s/M^2} ds \\ & + \left(+\frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{2} + \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{2} \right) + \frac{16\pi^2}{M_B^2} \left(+\frac{\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{96} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{96} + \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{96} \right), \quad (A1) \end{aligned}$$

$$\begin{aligned} \Pi_{0+-} = & \int_{m_s^2}^{s_0} \left[-\frac{3m_q \langle \bar{q} q \rangle}{2} - \frac{3m_s \langle \bar{s} s \rangle}{2} - m_q \langle \bar{s} s \rangle - m_s \langle \bar{q} q \rangle \right] e^{-s/M^2} ds + \left(+m_s m_q^2 \langle \bar{s} s \rangle + m_q m_s^2 \langle \bar{q} q \rangle \right. \\ & \left. - \frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{2} - \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{2} \right) + \frac{16\pi^2}{M_B^2} \left(+\frac{\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{96} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{96} + \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{96} \right). \quad (A2) \end{aligned}$$

- (2) The operator J_{μ}^{1+-} couples to both the $J^{PC} = 1^{+-}$ and 0^{--} states. The corresponding QCD sum rule equations for the quark content $\bar{q}s$ ($q = u/d$) are

$$\begin{aligned} \Pi_{1+-} = & \int_{m_s^2}^{s_0} \left[+\frac{s^2}{8\pi^2} + \left(-\frac{m_s^2}{2\pi^2} - \frac{m_q^2}{2\pi^2} + \frac{m_q m_s}{4\pi^2} \right) s + \frac{\langle g_s^2 GG \rangle}{32\pi^2} + \frac{3m_s^2 m_q^2}{4\pi^2} \right] e^{-s/M^2} ds \\ & + \left(-\frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{2} - \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{2} \right) + \frac{16\pi^2}{M_B^2} \left(+\frac{\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{96} - \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{96} - \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{96} \right), \end{aligned} \quad (A3)$$

$$\begin{aligned} \Pi_{0--} = & \int_{m_s^2}^{s_0} \left[-\frac{3m_q \langle \bar{q} q \rangle}{2} - \frac{3m_s \langle \bar{s} s \rangle}{2} + m_q \langle \bar{s} s \rangle + m_s \langle \bar{q} q \rangle \right] e^{-s/M^2} ds + \left(+m_s m_q^2 \langle \bar{s} s \rangle + m_q m_s^2 \langle \bar{q} q \rangle \right. \\ & \left. + \frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{2} + \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{2} \right) + \frac{16\pi^2}{M_B^2} \left(+\frac{\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{96} - \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{96} - \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{96} \right). \end{aligned} \quad (A4)$$

- (3) The operator J_{μ}^{1++} couples to both the $J^{PC} = 1^{++}$ and 1^{-+} states. The corresponding QCD sum rule equations for the quark content $\bar{q}s$ ($q = u/d$) are

$$\begin{aligned} \Pi_{1++} = & \int_{m_s^2}^{s_0} \left[+\frac{s^2}{16\pi^2} + \left(-\frac{m_s^2}{4\pi^2} - \frac{m_q^2}{4\pi^2} + \frac{m_q m_s}{8\pi^2} \right) s + \frac{3m_s^2 m_q^2}{8\pi^2} \right] e^{-s/M^2} ds + \left(-\frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{4} \right. \\ & \left. - \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{4} \right) + \frac{16\pi^2}{M_B^2} \left(+\frac{\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{96} - \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{192} - \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{192} \right), \end{aligned} \quad (A5)$$

$$\begin{aligned} \Pi_{1-+} = & \int_{m_s^2}^{s_0} \left(+\frac{s}{8\pi^2} - \frac{15m_s^2}{32\pi^2} - \frac{15m_q^2}{32\pi^2} + \frac{3m_q m_s}{16\pi^2} \right) e^{-s/M^2} ds + \left(-\frac{3m_s^2 m_q^2}{4\pi^2} - \frac{m_q \langle \bar{q} q \rangle}{2} - \frac{m_s \langle \bar{s} s \rangle}{2} \right. \\ & \left. + \frac{m_q \langle \bar{s} s \rangle}{4} + \frac{m_s \langle \bar{q} q \rangle}{4} - \frac{\langle g_s^2 GG \rangle}{192} \right) + \frac{1}{M_B^2} \left(+\frac{3\langle \bar{s} s \rangle m_q^2 m_s}{8} + \frac{3\langle \bar{q} q \rangle m_s^2 m_q}{8} + \frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{2} \right. \\ & \left. + \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{2} \right) - \frac{8\pi^2}{M_B^4} \left(+\frac{\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{96} - \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{192} - \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{192} \right). \end{aligned} \quad (A6)$$

- (4) The operator $J_{\mu\nu}^{2++}$ only couples to the $J^{PC} = 2^{++}$ state. The corresponding QCD sum rule equation for the quark content $\bar{q}s$ ($q = u/d$) is

$$\begin{aligned} \Pi_{2++} = & \int_{m_s^2}^{s_0} \left[-\frac{3s^2}{20\pi^2} + \left(+\frac{m_s^2}{2\pi^2} + \frac{m_q^2}{2\pi^2} - \frac{m_q m_s}{2\pi^2} \right) s + \frac{\langle g_s^2 GG \rangle}{18\pi^2} - \frac{m_s^2 m_q^2}{2\pi^2} \right] e^{-s/M^2} ds \\ & + \left(+m_q \langle g_s \bar{s} \sigma G s \rangle + m_s \langle g_s \bar{q} \sigma G q \rangle \right) + \frac{16\pi^2}{M_B^2} \left(+\frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{48} + \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{48} \right). \end{aligned} \quad (A7)$$

- (5) The operator $J_{\mu\nu}^{1--}$ couples to both the $J^{PC} = 1^{--}$ and 1^{+-} states. The corresponding QCD sum rule equations for the quark content $\bar{q}s$ ($q = u/d$) are

$$\begin{aligned} \Pi_{1--} = & \int_{m_s^2}^{s_0} \left[+\frac{s^2}{16\pi^2} + \left(-\frac{m_s^2}{4\pi^2} - \frac{m_q^2}{4\pi^2} - \frac{m_q m_s}{8\pi^2} \right) s + \frac{3m_s^2 m_q^2}{8\pi^2} \right] e^{-s/M^2} ds + \left(+\frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{4} \right. \\ & \left. + \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{4} \right) + \frac{16\pi^2}{M_B^2} \left(+\frac{\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{96} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{192} + \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{192} \right), \end{aligned} \quad (A8)$$

$$\begin{aligned} \Pi_{1+-} = & \int_{m_s^2}^{s_0} \left(+\frac{s}{8\pi^2} - \frac{15m_s^2}{32\pi^2} - \frac{15m_q^2}{32\pi^2} - \frac{3m_q m_s}{16\pi^2} \right) e^{-s/M^2} ds + \left(-\frac{3m_s^2 m_q^2}{4\pi^2} - \frac{m_q \langle \bar{q} q \rangle}{2} - \frac{m_s \langle \bar{s} s \rangle}{2} \right. \\ & \left. - \frac{m_q \langle \bar{s} s \rangle}{4} - \frac{m_s \langle \bar{q} q \rangle}{4} - \frac{\langle g_s^2 GG \rangle}{192} \right) + \frac{1}{M_B^2} \left(+\frac{3\langle \bar{s} s \rangle m_q^2 m_s}{8} + \frac{3\langle \bar{q} q \rangle m_s^2 m_q}{8} - \frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{2} \right. \\ & \left. - \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{2} \right) - \frac{8\pi^2}{M_B^4} \left(+\frac{\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{96} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{192} + \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{192} \right). \end{aligned} \quad (A9)$$

- (6) The operator $J_{\mu\nu}^{2--}$ only couples to the $J^{PC} = 2^{--}$ state. The corresponding QCD sum rule equation for the quark content $\bar{q}s$ ($q = u/d$) is

$$\begin{aligned} \Pi_{2--} = & \int_{m_s^2}^{s_0} \left[-\frac{3s^2}{20\pi^2} + \left(+\frac{m_s^2}{2\pi^2} + \frac{m_q^2}{2\pi^2} + \frac{m_q m_s}{2\pi^2} \right) s + \frac{\langle g_s^2 GG \rangle}{18\pi^2} - \frac{m_s^2 m_q^2}{2\pi^2} \right] e^{-s/M^2} ds \\ & + (-m_q \langle g_s \bar{s} \sigma G s \rangle - m_s \langle g_s \bar{q} \sigma G q \rangle) + \frac{16\pi^2}{M_B^2} \left(-\frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{48} - \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{48} \right). \end{aligned} \quad (A10)$$

- (7) The operator J_α^{1-+} couples to both the $J^{PC} = 1^{-+}$ and 0^{++} states. The corresponding QCD sum rule equations for the quark content $\bar{q}s$ ($q = u/d$) are

$$\begin{aligned} \Pi_{1-+} = & \int_{m_s^2}^{s_0} \left[\left(+\frac{m_s^2}{4\pi^2} + \frac{m_q^2}{4\pi^2} - \frac{m_s m_q}{2\pi^2} \right) s - \frac{9m_s^2 m_q^2}{4\pi^2} + \frac{\langle g_s^2 GG \rangle}{32\pi^2} \right] e^{-s/M^2} ds \\ & + \left(-2m_s m_q^2 \langle \bar{s} s \rangle - 2m_q m_s^2 \langle \bar{q} q \rangle + \frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{2} + \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{2} \right) \\ & + \frac{16\pi^2}{M_B^2} \left(-\frac{3\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{32} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{32} + \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{32} \right), \end{aligned} \quad (A11)$$

$$\begin{aligned} \Pi_{0++} = & \int_{m_s^2}^{s_0} \left[-\frac{3s^2}{8\pi^2} + \left(+\frac{3m_s^2}{2\pi^2} + \frac{3m_q^2}{2\pi^2} - \frac{3m_q m_s}{4\pi^2} \right) s - \frac{9m_s^2 m_q^2}{2\pi^2} - \frac{5m_q \langle \bar{q} q \rangle}{2} - \frac{5m_s \langle \bar{s} s \rangle}{2} \right. \\ & \left. + m_q \langle \bar{s} s \rangle + m_s \langle \bar{q} q \rangle - \frac{\langle g_s^2 GG \rangle}{48\pi^2} \right] e^{-s/M^2} ds + \left(+\frac{3m_q \langle g_s \bar{s} \sigma G s \rangle}{2\pi^2} + \frac{3m_s \langle g_s \bar{q} \sigma G q \rangle}{2\pi^2} \right) \\ & + \frac{16\pi^2}{M_B^2} \left(-\frac{3\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{32} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{32} + \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{32} \right). \end{aligned} \quad (A12)$$

- (8) The operator J_α^{1++} couples to both the $J^{PC} = 1^{++}$ and 0^{-+} states. The corresponding QCD sum rule equations for the quark content $\bar{q}s$ ($q = u/d$) are

$$\begin{aligned} \Pi_{1++} = & \int_{m_s^2}^{s_0} \left[\left(-\frac{m_s^2}{4\pi^2} - \frac{m_q^2}{4\pi^2} - \frac{m_s m_q}{2\pi^2} \right) s + \frac{9m_s^2 m_q^2}{4\pi^2} - \frac{\langle g_s^2 GG \rangle}{32\pi^2} \right] e^{-s/M^2} ds \\ & + \left(+2m_s m_q^2 \langle \bar{s} s \rangle + 2m_q m_s^2 \langle \bar{q} q \rangle + \frac{m_q \langle g_s \bar{s} \sigma G s \rangle}{2} + \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{2} \right) \\ & + \frac{16\pi^2}{M_B^2} \left(+\frac{3\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{32} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{32} + \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{32} \right), \end{aligned} \quad (A13)$$

$$\begin{aligned} \Pi_{0-+} = & \int_{m_s^2}^{s_0} \left[+\frac{3s^2}{8\pi^2} + \left(-\frac{3m_s^2}{2\pi^2} - \frac{3m_q^2}{2\pi^2} - \frac{3m_q m_s}{4\pi^2} \right) s + \frac{9m_s^2 m_q^2}{2\pi^2} + \frac{5m_q \langle \bar{q} q \rangle}{2} + \frac{5m_s \langle \bar{s} s \rangle}{2} \right. \\ & \left. + m_q \langle \bar{s} s \rangle + m_s \langle \bar{q} q \rangle + \frac{\langle g_s^2 GG \rangle}{48\pi^2} \right] e^{-s/M^2} ds + \left(+\frac{3m_q \langle g_s \bar{s} \sigma G s \rangle}{2\pi^2} + \frac{3m_s \langle g_s \bar{q} \sigma G q \rangle}{2\pi^2} \right) \\ & + \frac{16\pi^2}{M_B^2} \left(+\frac{3\langle \bar{q} q \rangle m_q \langle \bar{s} s \rangle m_s}{32} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{32} + \frac{\langle \bar{s} s \rangle \langle g_s \bar{q} \sigma G q \rangle}{32} \right). \end{aligned} \quad (A14)$$

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