

# Misalignment from kicks: The impact of particle interactions on ultralight dark matter

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Oscillating ultralight scalar fields are a natural explanation for the dark matter in our Universe, as long as a mechanism, often called a misalignment mechanism, exists to explain the amplitude of the scalar oscillations. If the dark matter scalar couples to the Standard Model, then the dynamics of ordinary matter can influence the behavior of dark matter in the early Universe. In this work, we show how this changes the expected value of the scalar field and the resulting amplitude of late-time scalar oscillations and therefore the abundance of dark matter at late times. For dark matter scalars that interact quadratically with Standard Model fields, we derive estimates of the size of this effect as a function of the strength of the coupling, and for axionlike fields we show that interactions with dark sector matter can temporarily destabilize the field, leading to large field displacements.

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## I. INTRODUCTION

Ultralight scalar fields are a popular explanation for the fundamental nature of the dark matter that makes up 27% of our Universe today [1]. If such a scalar field is oscillating homogeneously in a quadratic potential, then it behaves as a pressureless perfect fluid [2–4]. As long as the scalar is not too light,  $m_\phi \gtrsim 10^{-24}$  eV [5], then the scalar is also able to cluster and form halos around visible galaxies.

One challenge for ultralight dark matter models is to explain the observed amount of dark matter. The dark matter abundance in these models is set by the mass of the scalar, which controls the frequency of oscillations and the amplitude of oscillations. So an ultralight scalar explanation for dark matter requires an answer to the question: Why is the scalar field oscillating in its potential and not settled at the minimum? Solutions to this problem are referred to as the “misalignment mechanism” [5,6] because the value of the scalar field must, at some time in the early Universe, be misaligned from the position of

the minimum of its potential at late times. The scalar field will be frozen by Hubble friction at this misaligned value until a time at which the Hubble scale drops below the mass of the scalar; after this point, the field will oscillate in its potential.

Explanations for the initial misaligned value of the scalar often invoke an additional symmetry that is broken to give rise to the misaligned initial value of the scalar. For example, the scalar could be the pseudo-Nambu-Goldstone boson that emerges when a  $U(1)$  symmetry is spontaneously broken at a high-energy scale and then explicitly broken, e.g., by temperature-dependent corrections as occurs for axion models [6–12]. In this work, we show how misalignment can be enhanced or suppressed by interactions with Standard Model fields.

We focus on dark matter scalar fields with quadratic couplings to the Standard Model. There has been recent interest in such models, in part because they avoid many of the stringent constraints on linear couplings [13,14] and in part because such interactions occur for axions [15,16], symmetric Higgs portal scalars [17,18], or other scalar models where linear terms are forbidden by some mirror symmetry [19]. Such scalars are constrained by measurements with atomic clocks, and by the MICROSCOPE satellite’s test of the equivalence principle [13,20–25], with low-energy neutrons [26] and gravitational wave detectors [27], by big bang nucleosynthesis (BBN) [28–30] and observations of white dwarf mass-radius

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relations [31]. Further tests have been proposed, including with space-based atomic clocks [32], atom interferometry [33–38], hyperfine transitions [39,40], molecular spectroscopy [41], nuclear clocks [42], and pulsar timing arrays [43].<sup>1</sup>

We assume the scalar to couple to the mass of Standard Model fermions, so that the classical dynamics of the scalar field are sourced by the trace of the matter stress-energy tensor  $T_{\mu}^{\mu}$ . Such interactions can be generated, among others, from the Higgs portal or conformal-type couplings [46,47]. When matter is relativistic, as in the early Universe, one might thus expect the scalar field to be insensitive to matter fields with  $T \approx 0$ . However, whenever a particle becomes nonrelativistic, the trace of the matter stress-energy tensor becomes nonzero around an e-fold and “kicks” the scalar field, an effect that has been studied for dynamical dark energy models [48–51], coupled scalar-tensor theories [52], and dilaton fields from string theory [53,54]. This effect has also been seen previously in numerical simulations of quadratically coupled ultralight dark matter scalars in the early Universe [28–30], where constraints were placed on the theory by requiring that any variation in fundamental constants, caused by the evolution of the scalar, not disrupt BBN. In those works, the effect of the kick from electrons becoming nonrelativistic around the time of BBN was only determined numerically. In what follows, we will show how kicks from matter particles becoming nonrelativistic enhance or suppress the initial scalar misalignment, depending on the strength and sign of the coupling, and show how the size of this effect can be estimated.

In Sec. II, we introduce the scalar model we consider to describe ultralight dark matter interacting quadratically with the Standard Model, and we determine how a matter particle becoming nonrelativistic displaces the dark matter scalar. In Sec. III, we extend this to consider the potential and couplings of a dark axion model. We conclude in Sec. IV.

## II. QUADRATIC MODEL

In this section, we will introduce the effect that couplings between matter and scalar fields have on their evolution. For this, we will assume that we have an interaction of the form (mostly plus metric)

$$\mathcal{L} \supset \frac{1}{2}(\nabla\phi)^2 - V(\phi) + i\bar{\psi}_i \not{\partial} \psi_i - m_i(1 + A(\phi))\bar{\psi}_i \psi_i, \quad (2.1)$$

where  $\phi$  is the scalar field that will behave as dark matter and  $\psi_i$  is a matter field. In general, such an interaction

<sup>1</sup>We note that interactions between the scalar and matter can both enhance and suppress the sensitivity of experiments to the amplitude and time evolution of the background dark matter scalar [44,45].

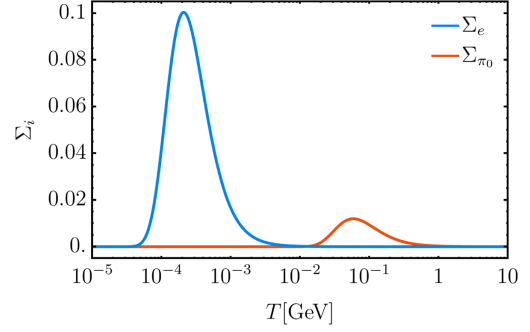


FIG. 1. Shape of the kick function  $\Sigma_i(T)$  induced by coupling a scalar field to the trace of the energy-momentum tensor of a species  $i$ . We show the fermionic case for the electron, using the + sign in Eq. (2.3), and the bosonic case for the neutral pion, using the – sign. In both cases, the kick peaks around  $T \sim m_i$ .

would be expected to couple to more than one fermion species, depending on its nature. Here, however, we focus on the effect of a single coupling, noting that the analysis can be straightforwardly generalized to multiple particles [55,56]. Assuming  $|A(\phi)| < 1$ , we can approximate  $m_i\bar{\psi}_i\psi_i \approx T_{\mu i}^{\mu}$ , with  $T_{\mu i}^{\mu}$  being the energy-momentum tensor for the fermion field. This leads to the following equation of motion for the  $\phi$  field:

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}(\phi) + A_{,\phi}(\phi)T_{\mu i}^{\mu}, \quad \approx -V_{,\phi}(\phi) - A_{,\phi}(\phi)\rho_R\Sigma_i, \quad (2.2)$$

where  $\rho_R$  is the energy density of the radiation bath, and we have assumed radiation domination to go from the first to the second line, allowing us to ignore nonrelativistic degrees of freedom.  $\Sigma_i = (\rho_i - 3P_i)/\rho_R$  reads

$$\Sigma_i(T) = \frac{15}{\pi^4} \frac{g_i}{g_*(T)} \left( \frac{m_i^2}{T^2} \right) \int_{m_i/T}^{\infty} \frac{\sqrt{u^2 - (m_i/T)^2}}{e^u \pm 1} du, \quad (2.3)$$

where the (+) sign applies to fermions and the (–) sign to bosons and  $g_*(T) = \rho_R[(\pi^2/30)T^4]^{-1}$  is the number of relativistic degrees of freedom. This expression peaks for values around  $T \sim m_i$ , as shown in Fig. 1 for the electron and neutral pion as representative fermionic and bosonic examples, respectively.  $\Sigma$  describes the influx of energy that goes from the fermion field into the  $\phi$  field when  $\psi_i$  becomes nonrelativistic, and it is usually referred to as the “kick” [51]. While the effect of this kick has been studied in various scenarios, here we are interested in its impact on the initial conditions for ultralight dark matter.

Given that linear couplings of matter fields to a scalar field  $\phi$  are very constrained due to fifth force experiments [13,14], we will impose a  $\mathbb{Z}_2$  symmetry on the model, leading to

$$V(\phi) = \frac{\mu^2}{2} \phi^2, \quad A(\phi) = \frac{\beta}{2M_{\text{Pl}}^2} \phi^2 + \dots, \quad (2.4)$$

where the ellipsis contains higher order terms in  $\beta\phi^2/M_{\text{Pl}}^2$ . Therefore, to avoid any problems regarding the effective field theory, we will always ensure  $A(\phi) < 1$ . Assuming a radiation-dominated universe ( $\rho_R/H^2 \approx 3M_{\text{Pl}}^2$ ), and thus  $\rho_\phi \ll \rho_R$ , this system will evolve under the following equation of motion:

$$\phi'' + \phi' \approx -\frac{\mu^2\phi}{H^2} - 3\beta\phi\Sigma(T_i), \quad (2.5)$$

where the prime denotes the derivative with respect to e-folds,  $N \equiv \log(a)$ . Here, we focus on the effect of a single kick-type interaction, but one would expect one such kick per coupled fermion.

At early times, the scalar field dynamics are frozen due to the Hubble friction dominating over the influence of the potential, until the Hubble rate drops below the scalar mass,  $H < m_\phi$ . In what follows, we will analyze the effect of the fermion interaction on the dynamics of the  $\phi$  field before unfreezing. The main difference to the standard evolution is that the unfreezing condition will now depend on the effective mass of the field, given by  $m_{\text{eff}} = \mu^2 + 3\beta H^2 \Sigma$ . Since we are interested in periods where  $\mu < H$  but  $m_{\text{eff}} > H$ , the equation of motion simplifies to

$$\phi'' + \phi' + 3\beta\phi\Sigma(T_i) \approx 0. \quad (2.6)$$

Note that the Planck suppression in  $A(\phi)$  is compensated by the large densities of the fields in the early Universe to make the interaction term relevant. In addition, it is important to clarify that energy is conserved, the influx of energy into the scalar sector shifts  $\phi$ , leading to a corresponding change in the mass of  $\psi_i$ .

In what follows, we provide an analytical description of the scalar field's evolution during the kick, which we will later validate with a numerical exploration of the parameter space. This provides an analytic guide to effects seen in Refs. [28,29] and extends the study to large couplings. We will focus on the effect of a fermionic kick, but the conclusion of this section can be easily extended to the bosonic case. Since Eq. (2.6) describes a simple harmonic oscillator, we will analyze the system's behavior separately in the oscillating and nonoscillating regimes:

### A. Oscillating

In this limit, for a quadratic potential, once it unfreezes the scalar field starts oscillating around the minimum of its potential, behaving as pressureless matter. One could assume that the larger the interaction strength, the more energy is being injected into the scalar field by the kick function. However, while the kick generates an instantaneous increase in the scalar potential energy of order

$3\beta\Sigma\phi_i^2$ , we also must consider that during the oscillations around the minimum the scalar energy density redshifts at a rate  $\rho_\phi \propto a^{-3}$ . This means that the field will lose a total energy of  $\rho_\phi \approx e^{-3\Delta N_*}$  because of this interaction, where  $\Delta N_*$  denotes the duration of the kick. These two effects compete with one another, and it is therefore not obvious *a priori* whether the oscillations persist long enough for the field to dissipate all of the energy injected by the kick.

To obtain the value for  $\Delta N_*$ , a first approximation is to determine the length of time that  $m_{\text{eff}} > H$  or, assuming  $m < H$ , for how long  $\sqrt{3}\beta\Sigma(N) > 1$ . Working with the kick equation from Eq. (2.3), we can approximate the growing and decaying parts of the kick function in Fig. 1, respectively, by

$$\begin{aligned} \Sigma(T_+ \gg m_i) &\approx m_i^2 T_+^{-2}, \\ \Sigma(T_- \ll m_i) &\approx m_i^{-5/2} T_-^{-5/2} e^{-m_i/T_-}. \end{aligned} \quad (2.7)$$

From this,  $\Delta N_*$  can be obtained by finding the root of  $3\beta\Sigma(T) = 1$  and then solving for  $N$ . For each part, the roots can be approximated by

$$\begin{aligned} T_+ &= m_i \sqrt{3\beta}, \\ T_- &= \frac{5}{2} m_i W_{-1} \left( \frac{2}{5} \beta^{2/5} \right) \approx m_i \frac{1}{\log(3\beta) + \frac{5}{2} \log(\log(3\beta))}, \end{aligned} \quad (2.8)$$

where  $W_{-1}(x)$  is the  $k = -1$  branch of the Lambert W function, which solves for the root of the decaying part of the kick function, for which the approximation holds as long as  $\beta \gg 1$ .  $\Delta N_*$  is related to these temperatures by

$$\Delta N_* = \log \left( \frac{T_+}{T_-} \right), \quad (2.9)$$

from which we obtain

$$\Delta N_* = \log \left( \sqrt{3\beta} \left( \log(3\beta) + \frac{5}{2} \log(\log(3\beta)) \right) \right). \quad (2.10)$$

Therefore, assuming that the kick gives all its energy to the scalar field and the field oscillates for a period of  $\Delta N_*$ , its energy density by the end of the kick will be

$$\langle \rho_\phi \rangle \propto \frac{3\phi_i^2}{2} \beta \Sigma|_{\text{max}} \exp \left( -\frac{3}{2} \Delta N_* \right), \quad (2.11)$$

where  $\Sigma|_{\text{max}}$  is the maximum value the kick function will take at its peak, which is around  $0.1\beta$ . Substituting for Eq. (2.10) and taking the term at leading order in  $\beta$ , we find

$$\langle \rho_\phi \rangle \propto \beta \exp \left( -\frac{3}{2} \log(\sqrt{\beta}) \right) = \frac{1}{\sqrt{\beta}}. \quad (2.12)$$

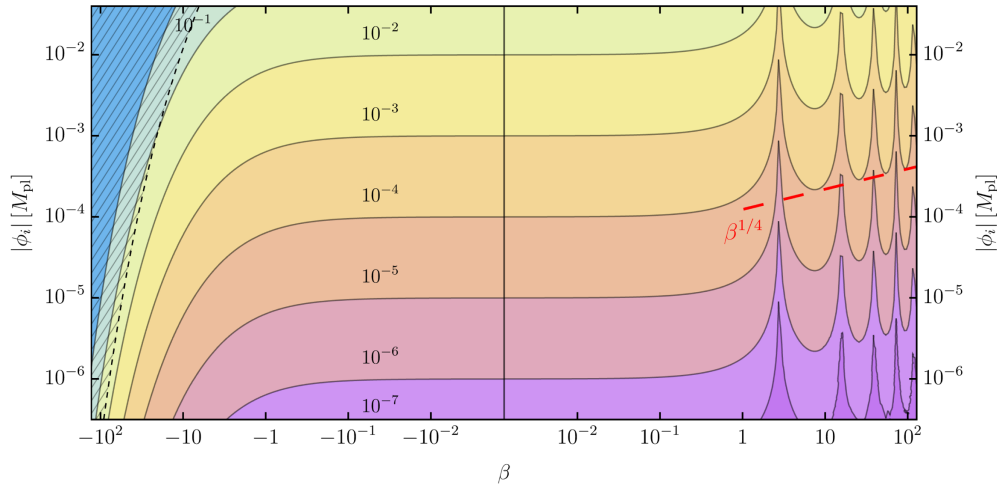


FIG. 2. Final value of the scalar fields as a function of the initial value  $|\phi_i|$  and both negative and positive kick strengths,  $\beta$ , after numerical evolution of the system. The different colored regions show the order of magnitude of the value of the field at the end of the kick ( $|\phi(N_f)|$ ). Dark blue on the left marks the region where  $\phi > M_{\text{pl}}$ , and the dashes mark the region where  $|A(\phi_f)| > 1$ . The rightmost peaks in the figure mark scenarios where, at the end of the evolution, the field is in the minimum of the quadratic potential.

Therefore, for large values of  $\beta$ , the total energy of the scalar field will always decrease due to the kick.

### B. Nonoscillating

This occurs when the interaction strength (i.e.,  $\beta\Sigma$ ) is very small or negative. If  $\Sigma(T)$  were constant, an analytical solution from Eq. (2.6) would straightforwardly follow using the ansatz  $\phi \propto e^{\lambda N}$ . In our case, however, we only assume that  $\Sigma(T)$  varies slowly, which is not always guaranteed. Nevertheless, we can average the effect of its evolution on  $\phi$  and obtain a good approximate solution,

$$\phi(N) = \phi_i \exp \left( \int_{N_i}^N \frac{-1 + \sqrt{1 - 12\beta\Sigma(\hat{N})}}{2} d\hat{N} \right), \quad (2.13)$$

which, depending on the size of  $\beta$ , we can approximate as

$$\phi_f \approx \phi_i \begin{cases} \exp(-\beta\bar{\Sigma}) & \text{for } |\beta| < 1, \\ \exp(\sqrt{-\beta\bar{\Sigma}}) & \text{for } \beta \ll -1, \end{cases} \quad (2.14)$$

where  $\bar{\Sigma}$  is the average value for  $\Sigma(N)$ .

We compare this analytical approach against numerical computations in Fig. 2, both for positive and negative interaction strengths. For large positive kicks, the field loses energy due to the extra oscillations it undergoes, with this effect peaking every time the field ends its evolution near the minimum of the potential. However, while the  $\beta$  dependence is highly oscillatory, we can still see the overall trend of  $\phi_f \propto \beta^{1/4}$  from Eq. (2.12) marked by the dashed red line. In the negative kick case, we find that, at large  $\beta$ , the field is very sensitive to the instability created by flipping the sign of the quadratic potential. In summary, the

overall dependence of the strength of quadratic couplings on the energy of a scalar field is

$$\langle \rho_f \rangle \propto \begin{cases} \beta^{-1/2} & \text{for } \beta \gg 1, \\ \exp(-\beta\bar{\Sigma}) & \text{for } |\beta| < 1, \\ \exp(\sqrt{-\beta\bar{\Sigma}}) & \text{for } \beta \ll -1. \end{cases} \quad (2.15)$$

We can see that even Planck-suppressed interactions can have a large effect on the evolution of scalar fields in the early Universe, independent of the Hubble parameter at the time. For the quadratic model presented in this section, we must remain within the regime of validity of the EFT [i.e.,  $|A(\phi)| < 1$  at all times]. This is especially challenging for large negative  $\beta$  since the instability in the potential tends to displace the field over this limit, as marked by the black dashed region in Fig. 2. We will now show how our conclusions differ for an axionlike potential.

### III. DARK QCD AXION MODEL

Axion fields are among the most compelling dark matter candidates due to their close connection to fundamental physics. Here we consider the often studied possibility that axion dark matter is part of a “dark QCD” sector [57–62]. In that case, generic interactions among the dark sector fields are expected and, because the axion is a pseudoscalar field, interactions with, for example, fermion mass terms appear first at order  $\phi^2$ , leading to similar dynamics to those discussed above. Throughout this section, QCD terminology (e.g., quarks, baryons, and hadronic parameters) refers to the dark sector unless explicitly labeled as Standard Model QCD (SM-QCD).

The bare axion potential due to the pion mass terms [63,64] is given by

$$V(a) = -\Lambda^4 \sqrt{1 - \xi \sin^2\left(\frac{a}{2f}\right)}, \quad (3.1)$$

where we take  $\xi = 4 \frac{m_u m_d}{(m_u + m_d)^2} \approx 0.86$  to be the dimensionless ratio of quark masses to mimic the SM-QCD axion behavior, and  $\Lambda^4 = \epsilon m_\pi^2 f_\pi^2$  is an energy scale related to the mass and decay constant of the neutral pion, respectively. In this way, the axion mass is given by  $m_a = \frac{\Lambda^2 \sqrt{\xi}}{2f}$ . Following [61,62], we have introduced the parameter  $\epsilon \ll 1$  that allows for finite density corrections, which we will introduce later, to overcome the vacuum potential in Eq. (3.1) without leaving the perturbative regime.

In the presence of a background density of dark baryons, the axion picks up an effective correction to the potential that can be generally expressed as

$$V_b(a) = +2\alpha \sqrt{1 - \xi \sin^2\left(\frac{a}{2f}\right)} T_\mu^{\mu(b)} + \mathcal{O}\left(\left(\frac{\alpha}{m_\pi^2 f_\pi^2} T_\mu^{\mu(b)}\right)^2\right), \quad (3.2)$$

where  $\alpha$  controls the strength of the interaction and  $T_\mu^{\mu(b)}$  is the energy density of the dark baryon species coupling to the axion field. For reference,  $\alpha$  is fixed to  $\alpha = \sigma_N/m_N$  for nucleon interactions in the SM-QCD axion, with  $\sigma_N = 59 \pm 7$  MeV being the pion-nucleon sigma term and  $m_N$  the average mass of nucleons. One can also consider corrections from lighter states such as the neutral pion itself, for which the leading order correction corresponds to taking  $\alpha = 1/2$  [64,65]. In total, the evolution of this field is dictated by the effective potential

$$V_{\text{eff}}(a) = -\Lambda^4 \left(1 - \frac{2\alpha}{\Lambda^4} T_\mu^{\mu(b)}\right) \sqrt{1 - \xi \sin^2\left(\frac{a}{2f}\right)} + \mathcal{O}\left(\left(\frac{\alpha}{m_\pi^2 f_\pi^2} T_\mu^{\mu(b)}\right)^2\right). \quad (3.3)$$

As shown in Fig. 3, large density corrections can flip the sign of the effective potential. This is the key aspect of the dark QCD axion, as when a dark baryon species becomes nonrelativistic in the thermal bath, it will displace the field to the maximum of the bare potential. For this to take place, we need to satisfy the following two conditions:

(I) Perturbative control,

$$\frac{\alpha}{m_\pi^2 f_\pi^2} T_\mu^{\mu(b)} < 1. \quad (3.4)$$

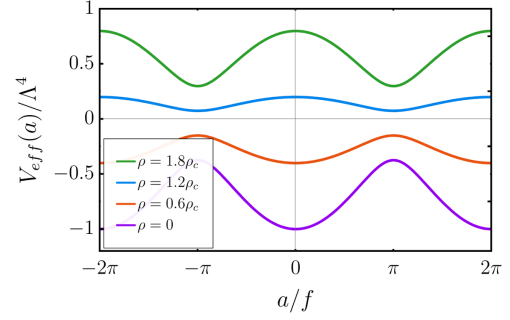


FIG. 3. Effective potential for the axion model when including interactions with dark baryons. The density corrections lead to a sign swap in the early Universe for densities larger than  $\rho_c \equiv \Lambda^4/2\alpha$ .

(II) Sign switch on effective potential,

$$\frac{2\alpha}{\Lambda^4} T_\mu^{\mu(b)} > 1. \quad (3.5)$$

For these conditions to be satisfied, we only need to impose  $\epsilon \ll 1$ , which can be achieved by having multiaxion theories with  $\mathbb{Z}_N$  exchange symmetry [65,66]. Additionally, assuming dark baryons exist as a stable species before they become nonrelativistic requires the axion scale to be larger than the energy density of baryons, meaning  $T_\mu^{\mu(b)} < m_\pi^2 f_\pi^2$ . This allows us to use the well-known formula for the kick [Eq. (2.3)] to describe the time evolution of this interaction.<sup>2</sup>

Assuming the Universe to be radiation dominated, the equation of motion will take the following form:

$$a'' + a' \approx -\frac{\sin(\frac{a}{2f}) \cos(\frac{a}{2f})}{\sqrt{1 - \xi \sin^2(\frac{a}{2f})}} \left( \frac{2m_a^2 f}{H^2} - \beta_b \pi f \Sigma^{(b)} \right), \quad (3.6)$$

where  $\beta_b = \frac{3M_{\text{Pl}}^2 \alpha \xi}{\pi f^2}$  is the interaction strength and  $\Sigma^{(b)}$  is the kick function associated with the dark baryon using Eq. (2.3), which describes the energy influx to the axion field when the baryons go from being relativistic to nonrelativistic. As in Sec. II, we will assume the bare mass of the potential to be smaller than the Hubble constant, such that the field unfreezes because of the interaction to matter. In Fig. 4, we show the field rescaling from  $a_i$  to  $a_f$  due to the pion kick, meaning  $\alpha = 1/2$ , for a dark QCD axion potential that shares the same  $m_\pi$  as in SM-QCD. For  $f > 10^{-2} M_{\text{Pl}}$ , we find a similar result to that

<sup>2</sup>If the dark baryons are formed already nonrelativistic, as is the case for nucleons in QCD without further model building, they will also provide a kicklike interaction into the dynamics of the axion field at the QCD phase transition. However, its shape and strength is difficult to estimate as it requires including non-perturbative terms, and best attempts involve complicated numerical analyses [67].

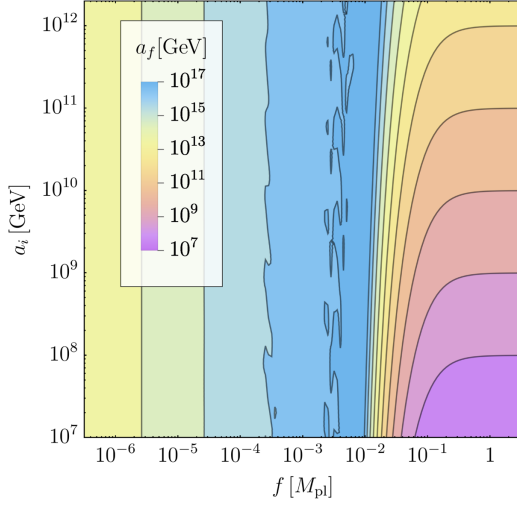


FIG. 4. Final value of the axion field at the end of the kick due to interactions with pions for different decay constants and initial values. For this figure, we fixed  $\alpha = 1/2$  as given for the pion interaction. The different colored regions indicate the order of magnitude of the value of the field at the end of the kick  $[a(N_f)]$ . For large masses, the field reaches the effective minimum placed at  $a_f = \pi f$ .

for negative  $\beta$  in the quadratic model, shown in Fig. 2. However, for smaller decay constants the interaction becomes stronger, and the field is able to reach the minimum of the effective potential, located at  $a_{\max} = \pi f$ , regardless of the starting position. Between these two regimes, the kick does not last long enough for the field to lose all of its momentum and settle at the effective minimum, leading to a nontrivial pattern in the final value of the axion field.

Although this system has multiple free parameters, we can simplify the full model by rescaling  $a \rightarrow f\pi\tilde{a}$ .<sup>3</sup> This leads to the following equation of motion:

$$\tilde{a}'' + \tilde{a}' \approx - \frac{\sin\left(\frac{\tilde{a}\pi}{2}\right) \cos\left(\frac{\tilde{a}\pi}{2}\right)}{\sqrt{1 - \xi \sin^2\left(\frac{\tilde{a}\pi}{2}\right)}} \left( \frac{2\pi m_a^2}{H^2} - \beta_b \Sigma^{(b)} \right). \quad (3.7)$$

The behavior of the generic system is illustrated in Fig. 5, which shows the mapping of initial values  $\tilde{a}_i$  to final values  $\tilde{a}_f$  for different interaction strengths  $\beta_b$ . As expected, for  $\beta_b < 10$  the dynamics closely resemble the negative- $\beta$  quadratic case. For stronger interactions, we instead observe oscillations about the new minimum at  $\tilde{a} = 1$ .

Although an analytic treatment is challenging due to the nonlinearity of both the potential and the coupling function,

<sup>3</sup>For this rescaling to be valid, we must ensure that the Universe stays radiation dominated. In general, this requires  $\tilde{a}'\pi f \ll M_{\text{Pl}}$ , which is satisfied provided  $f \ll M_{\text{Pl}}$ .

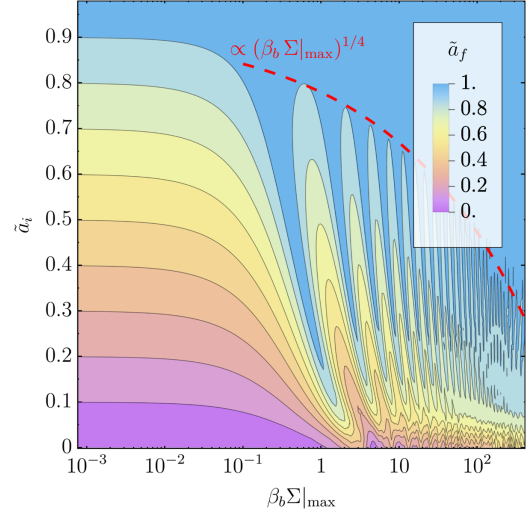


FIG. 5. Final value of the rescaled axion field,  $\tilde{a} = a\pi f$ , at the end of the kick for different coupling  $\beta_b = \frac{3M_{\text{Pl}}^2 \alpha \xi}{f^2}$  and initial conditions  $\tilde{a}_i$ , given some kick peak value  $\Sigma|_{\max}$ . The different colored regions indicate the order of magnitude of the field at the end of the kick ( $\tilde{a}_f$ ), where the same color is given for symmetric points with respect to the maximum  $\tilde{a}_f = 1$ .

we can still use the approximation from the previous section in the regime of small amplitudes of oscillation. This corresponds to initial conditions sufficiently close to the effective minimum,  $\tilde{a} = 1$ . In that limit, the scaling  $\tilde{a}_f \propto 1 - (\beta_b \Sigma|_{\max})^{1/4}$  provides a good description of the oscillation envelope, as marked in Fig. 5 by the dashed red line for  $\tilde{a}_f \approx 0.9$ . For smaller  $\tilde{a}_f$ , this approximation breaks down due to the quadratic expansion not being a good approximation for the full axion potential.

The calculations shown in this section illustrate the positive aspects of these interactions on the axion potential, as they can alleviate the fine-tuning issues associated with the misalignment mechanism. This is because, independent of their initial position, interactions controlled by scales a couple of orders of magnitude larger than the Planck scale will drive the field all the way to the top of the axion potential.

#### IV. CONCLUSIONS

In this work, we have examined how particle interactions affect the early-time dynamics of ultralight dark matter. When a particle species becomes nonrelativistic, it induces a “kicklike” effect on the scalar field, temporarily destabilizing it. We have studied this effect in the context of the misalignment mechanism, a key ingredient in ultralight dark matter models, in which the field must be displaced from its minimum so that the amplitude of its oscillations carries sufficient energy density today.

We first considered a quadratic potential for the dark matter field, with a quadratic coupling to matter. By

analytically estimating the duration of the kick, we studied the effect of this interaction for different coupling strengths,  $\beta$ . For large positive couplings, the kick induces a large effective mass on the scalar field, making it oscillate around the minimum of the potential before it would otherwise. Overall, these oscillations cause the total energy of the field to reduce by  $\rho_f \propto \beta^{-1/2}$ . On the contrary, for large negative couplings, the quadratic potential flips its sign and the total energy is increased by  $\log(\rho_f) \propto (-\beta\bar{\Sigma})^{1/2}$ , where  $\bar{\Sigma}$  is the average amplitude of the kick function as given in Eq. (2.3). We provide a numerical exploration of the parameter space for the quadratic model in Fig. 2.

We next applied the same analysis to a dark QCD axion. In the small amplitude regime, the dynamics reduce to those of a quadratic potential, but the cosinelike structure bounds the field displacement for large interactions. Because the coupling to dark baryons is negative, the kick can reverse the sign of the effective potential, placing its minimum at the top of the bare potential during the kick. While the full system cannot be easily described analytically, the quadratic analysis remains applicable for small oscillations about the effective minimum. The results for a generic potential and different coupling strengths are shown in Fig. 5. We also considered the evolution for an axion with the Standard Model's QCD parameter choices in Fig. 4. We find that in the weak-coupling limit, the behavior closely tracks the quadratic case, while for sufficiently strong couplings, the field relaxes to an approximately

constant value set by the new minimum of the effective potential.

In conclusion, particle interactions can significantly affect the early evolution of ultralight dark matter, even when the coupling strength is only a few orders of magnitude larger than gravity. The effect depends on the sign of the coupling, which may either increase or decrease the initial vacuum expectation value of the scalar field, thereby improving or worsening the misalignment needed to account for the observed dark matter abundance today. For a dark QCD axion, as the sign of the coupling is fixed, we find that particle interactions always enhance the late-time dark matter abundance.

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## DATA AVAILABILITY

The Wolfram *Mathematica* code used to generate all figures in this work is publicly available in Ref. [68].

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