

Scale-invariant open quantum systems

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We develop the complete theoretical framework for open quantum systems coupled to scale-invariant environments. Such environments, we show, are universally and uniquely described by unparticle baths [1] characterized by a single scaling dimension d_U , which is a property of the bath operator entering the coupling, not of the probe. Different probes coupled to the same bath operator $\mathcal{O}_E$ must yield the same d_U , providing a nontrivial cross-probe consistency test for a fixed coupling channel. This companion paper provides the full proof of the uniqueness theorem, the mathematical formalism of the resulting non-Markovian dynamics, and worked applications to three physical realizations omitted from the shorter letter [2]. Starting from the uniqueness theorem, we derive the complete set of non-Markovian memory kernels, the exact noise kernel, including vacuum and thermal contributions via Matsubara summation, and the fractional generalization of the Caldeira-Leggett master equation for arbitrary d_U . The unparticle dimension acts as a control parameter governing a rich phase structure, including a thermalization transition at $d_U = 3/2$, the Ohmic boundary at $d_U = 2$, and a decoherence phase transition at $d_U = 5/2$ in the thermal regime ($d_U = 2$ in the vacuum regime), beyond which quantum coherence is protected at long times. These transitions are universal for free-field, weakly interacting, and engineered power-law baths; for strongly interacting CFT baths they describe intermediate-time scaling in the window $1/\omega_{UV} \ll t \ll 1/T$. Three physical realizations are presented. For the quantum Ising model, d_U is derived from first principles from the known CFT operator dimension. For inflationary cosmology, $d_U = 2$ is obtained by matching to established results; for heavy-fermion materials, $d_U = 3/2$ is inferred empirically from two-channel transport data. Coupling to the energy operator in $(1+1)$ spacetime dimensions yields $d_U = 3/2$, providing a field-theoretic derivation of $1/f$ noise; the $(2+1)$ D case yields $d_U \approx 1.413$ from the conformal bootstrap [3]. For high-energy astrophysical neutrinos in the regime $E \gg T$, the energy- and baseline-dependent decoherence rate $\Gamma_{\text{decoh}} \propto \mathcal{B}(E, T_U) L^{5-2d_U}$ provides a direct observable imprint of the scaling dimension. A systematic comparison with the Caldeira-Leggett model, phenomenological Lindblad equations, and the non-Markovian literature establishes the precise relationship between these approaches and the unparticle framework. The regime of validity is analyzed for each physical system, including the crossover between vacuum and thermal regimes of the noise kernel. Experimental predictions and consistency tests are detailed for trapped-ion quantum simulators, neutrino telescopes, and superconducting qubits.

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I. INTRODUCTION

Our previous work [2] established that any local, Lorentz-invariant, scale-invariant quantum environment is mathematically equivalent to an unparticle bath [1], provided a streamlined proof of the uniqueness theorem,

validated the framework using multichannel transport data from the unitary Fermi gas and engineered spin-boson experiments, and illustrated three physical realizations spanning 25 orders of magnitude in energy.

The present paper provides what the Letter necessarily omitted: the complete proof with all loopholes treated explicitly, the full mathematical formalism for the resulting non-Markovian dynamics, and the detailed derivations for each physical realization. We emphasize that the setup throughout is a standard reduced-density-matrix construction: the total system $S + E$ evolves unitarily under a Hamiltonian of the form $H = H_S + H_E + gA_S\mathcal{O}_E$

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(system, environment, and interaction terms, respectively, defined precisely in Sec. III), and the open dynamics of S is obtained by tracing out the environmental degrees of freedom via the influence functional. The contribution of the paper is to establish what form this reduced dynamics must take when the bath is scale invariant, without assuming any specific microscopic model for the bath. The title refers to this reduced dynamics of a probe coupled to a scale-invariant bath, not to an intrinsically open scale-invariant many-body system.

The organization is designed so that each Section is self-contained; a reader interested only in, say, the inflationary application can proceed directly to Sec. VII after reading the framework summary in Sec. III.

A. What this article contains

Section II states and proves the uniqueness theorem in full, including explicit treatment of five classes of loophole (approximate scale invariance, nonlocal coupling, discrete scale invariance, multiple competing sectors, and quantum anomalies) and a no-go theorem in contrapositive form.

Section III develops the complete mathematical formalism: influence functional, non-Markovian master equation, exact expressions for the dissipation and noise kernels including the full vacuum-plus-thermal decomposition via Matsubara summation, the fractional Caldeira-Leggett equation, the specific heat scaling and a complete set of extraction formulas across observable channels, and the phase structure as a function of d_U .

Section IV situates the unparticle framework relative to existing approaches: the Caldeira-Leggett model (recovered as the special case $d_U = 2$ in the Markovian limit), phenomenological Lindblad master equations (incapable of describing coherence protection for $d_U > 5/2$), and the broader non-Markovian literature.

Section V presents a second many-body validation of the framework, independent of the previous Letter [2]: a two-channel consistency check in two heavy-fermion compounds at their quantum critical points yields $d_U = 3/2$ from both resistivity and specific heat, establishing that d_U discriminates between distinct universality classes.

The next three Sections present three physical realizations in full detail: Sec. VI studies critical points in the Ising model, Sec. VII shows the application of the framework to cosmic inflation, and Sec. VIII explores the effect of a scale-invariant bath on decoherence of astrophysical neutrinos. Each of these sections opens with a summary of the system, environment, coupling operator, bath correlator, and predicted observable, providing a concrete dictionary between the general formalism and the specific physical realization. The IceCube numerical analysis for the neutrino section is in preparation and will appear separately.

Section IX analyzes the regime of validity for each system, establishing when scale invariance holds and how

thermal corrections modify the framework and the regime of validity of the universal exponents.

Section X provides a complete roadmap for proposed future experimental applications in various fields.

Finally, we summarize our results and conclude in Sec. XI.

B. Notation and conventions

We employ natural units $\hbar = c = k_B = 1$ throughout the article unless otherwise stated. Spatial dimension is d (not including time), so spacetime is $(d + 1)$ -dimensional. We use the Lorentzian signature $\eta^{\mu\nu} = \text{diag}(-1, +1, +1, +1)$. The unparticle dimension d_U and CFT scaling dimension Δ are related by $d_U = \Delta - (d - 2)/2$.

II. UNIQUENESS THEOREM: COMPLETE PROOF

A. Statement

Theorem 1. (Unparticle universality). Let a quantum system S couple locally to an environment E in d spatial dimensions. Assume:

- (1) Scale invariance: E exhibits exact continuous scale invariance (no intrinsic mass or energy scales).
- (2) Lorentz invariance: The theory is relativistically invariant.
- (3) Locality: The coupling Hamiltonian between the system and the environment is $H_{\text{int}} = gA_S(x)\mathcal{O}_E(x)$, where $A_S(x)$ is the system operator, $\mathcal{O}_E(x)$ is the environment operator at the same point x , and g is a dimensionless coupling constant.
- (4) Unitarity: The full system $S + E$ evolves unitarily.

Then:

- (1) E is described by a conformal field theory.¹
- (2) The spectral density takes the unique form

$$J(\omega) = A\omega^{2\Delta-d-1}, \quad (1)$$

where Δ is the scaling dimension of $\mathcal{O}_E$ and A is a normalization constant.

- (3) This is mathematically equivalent to an unparticle bath with²

$$d_U = \Delta - \frac{d-2}{2}. \quad (2)$$

¹In $d = 2$, this follows rigorously [4]; in $d \geq 3$ it holds under the additional assumption of unitarity and absence of a virial current, supported by strong evidence in $d = 4$ [5,6].

²Note that this convention differs from Georgi's original definition [1], in which d_U is identified directly with the scaling dimension Δ ; our convention is chosen to make the spectral exponent $s = 2d_U - 3$ take the standard spin-boson form in $d = 3$ spatial dimensions. With this convention, we have $s = 1$ Ohmic, $s > 1$ super-Ohmic, and $s < 1$ sub-Ohmic.

TABLE I. Complete set of scaling exponents for a system coupled to an unparticle bath with dimension d_U . Unless otherwise stated, decoherence and noise exponents refer to the thermal regime ($t \gg \beta$); the corresponding vacuum-regime ($t \ll \beta$) exponents are shifted by -1 (see Sec. IX). All quantities are determined by a single parameter d_U ; the consistency relations in the final row are algebraic identities.

Quantity	Scaling	Exponent	Constraint
Spectral density	$J(\omega) \propto \omega^s$	$s = 2d_U - 3$	$\dots$
Dissipation kernel	$\eta(t) \propto t^{-\alpha_\eta}$	$\alpha_\eta = 2d_U - 2$	$d_U > 1$
Noise (high- T)	$\nu(t) \propto T t^{-\alpha_\nu}$	$\alpha_\nu = 2d_U - 3$	$d_U > 3/2$
Damping	$\Gamma_{\text{damp}} \propto t^{\beta_{\text{damp}}}$	$\beta_{\text{damp}} = 3 - 2d_U$	$d_U \neq 3/2$
Decoherence	$\Gamma_{\text{decoh}} \propto t^{\gamma_{\text{decoh}}}$	$\gamma_{\text{decoh}} = 5 - 2d_U$	$d_U \neq 2, 5/2$
Decoh. rate	$\dot{\Gamma}_{\text{decoh}} \propto t^{\delta_{\text{decoh}}}$	$\delta_{\text{decoh}} = 4 - 2d_U$	$\dots$
Consistency relations: $s + \gamma_{\text{decoh}} = 2$; $\alpha_\eta + \delta_{\text{decoh}} = 2$; $\alpha_\nu + \beta_{\text{damp}} = 0$.			

- (4) All dynamical exponents are uniquely determined by d_U via the relations in Table I.

B. Proof via the Källén-Lehmann spectral representation

In a previous work [2], we proved the theorem via conformal Ward identities in position space. Here, we give an independent derivation that works entirely in momentum space, using the Källén-Lehmann (KL) spectral representation. The KL representation is the most general statement about two-point functions in any Lorentz-invariant quantum field theory; scale invariance is then injected as an additional constraint that uniquely fixes its form. The two proofs are logically independent and reach the same conclusion by different routes, providing a consistency cross-check on the framework.

1. Step 1: The Källén-Lehmann representation

The KL representation answers the following question: what is the most general form of the two-point correlator of a local operator can take in a Lorentz-invariant quantum field theory, with no further assumptions? The answer, derived from Lorentz invariance and the positivity of the spectral measure alone [7,8], is a superposition of free propagators,

$$G_E(p_E^2) = \int_0^\infty d\sigma \frac{\rho(\sigma)}{p_E^2 + \sigma}, \quad (3)$$

where $G_E(p_E^2)$ is the Euclidean two-point function in momentum space evaluated at zero spatial momentum, $p_E^2 = p_0^2$ is the squared Euclidean frequency, and σ has dimensions of $(\text{mass})^2$. The function $\rho(\sigma) \geq 0$ is the *spectral weight*: it encodes the density of states at each invariant mass scale $\sqrt{\sigma}$ contributing to the correlator.

To understand Eq. (3) physically: even in a strongly interacting theory with no free particles, the two-point function decomposes as an integral over free propagators of

all possible masses, weighted by $\rho(\sigma)$. For a free massive particle of mass m , $\rho(\sigma) = \delta(\sigma - m^2)$; for a theory with a continuum of states, $\rho(\sigma)$ is a smooth function. Crucially, Eq. (3) uses no dynamical information about the theory; it is a kinematic identity. Scale invariance—the assumption we have not yet imposed—will now sharply constrain the form of $\rho(\sigma)$.

2. Step 2: Scale invariance forces a power-law spectral weight

Under a scale transformation $x \rightarrow \lambda x$, the operator $\mathcal{O}_E$ with scaling dimension Δ transforms as $\mathcal{O}_E(\lambda x) = \lambda^{-\Delta} \mathcal{O}_E(x)$, so the position-space two-point function satisfies

$$G_E(\lambda x) = \lambda^{-2\Delta} G_E(x). \quad (4)$$

Passing to momentum space via the $(d+1)$ -dimensional Fourier transform, the rescaling $x \rightarrow \lambda x$ corresponds to $p_E \rightarrow p_E/\lambda$. Tracking the Jacobian $d^{d+1}x \rightarrow \lambda^{d+1} d^{d+1}x$, Eq. (4) becomes

$$G_E\left(\frac{p_E^2}{\lambda^2}\right) = \lambda^{d+1-2\Delta} G_E(p_E^2). \quad (5)$$

From the KL representation (3), multiplying numerator and denominator by λ^2 ,

$$G_E\left(\frac{p_E^2}{\lambda^2}\right) = \lambda^2 \int_0^\infty d\sigma \frac{\rho(\sigma)}{p_E^2 + \lambda^2 \sigma}. \quad (6)$$

Substituting $\sigma \rightarrow \sigma/\lambda^2$ (so $d\sigma \rightarrow d\sigma/\lambda^2$) brings the denominator back to the standard form $p_E^2 + \sigma$,

$$G_E\left(\frac{p_E^2}{\lambda^2}\right) = \int_0^\infty d\sigma \frac{\rho(\sigma/\lambda^2)}{p_E^2 + \sigma}. \quad (7)$$

Equating with the right-hand side of Eq. (5) and using the linear independence of $1/(p_E^2 + \sigma)$ at different values of σ

(they have poles at different locations and cannot cancel each other),

$$\rho(\sigma/\lambda^2) = \lambda^{d+1-2\Delta}\rho(\sigma), \quad \forall \lambda > 0, \sigma > 0. \quad (8)$$

Replacing $\sigma \rightarrow \lambda^2\sigma$ gives the equivalent form

$$\rho(\lambda^2\sigma) = \lambda^{2\Delta-d-1}\rho(\sigma), \quad (9)$$

a homogeneity condition: rescaling the argument of ρ by λ^2 multiplies ρ by a pure power of λ .

3. Step 3: The unique solution is a power law

Equation (9) is a functional equation for ρ . Setting $\sigma_0 = 1$ and $\lambda^2 = \sigma$,

$$\rho(\sigma) = C_\rho \sigma^{(2\Delta-d-1)/2} = C_\rho \sigma^{\Delta-(d+1)/2}, \quad (10)$$

where $C_\rho = \rho(1)$ is a normalization constant. The unique solution is a pure power law. This is the key step: scale invariance leaves no freedom in the functional form of the spectral weight.

The physical interpretation is transparent. A massive theory has $\rho(\sigma)$ peaked at $\sigma \sim m^2$ and exponentially suppressed for $\sigma \gg m^2$; the mass scale m breaks scale invariance. If the theory has a continuum of states but no mass gap, $\rho(\sigma)$ can extend to $\sigma = 0$. Scale invariance demands more: $\rho(\sigma)$ must have no preferred mass scale at all, which forces it to be a power law. A power law is the unique scale-free function.

Unitarity requires $\rho(\sigma) \geq 0$, which is satisfied for $C_\rho > 0$ when $\Delta > (d+1)/2$, i.e., $d_U > 0$. The stronger condition $d_U > 1$ arises from the infrared convergence of the memory kernels and is derived separately in Sec. III.

4. Step 4: From spectral weight to bath spectral density

The bath spectral density $J(\omega)$ is related to the retarded Green's function via [Eq. (16)]

$$J(\omega) = -2 \operatorname{Im} G_R(\omega, \mathbf{k} = 0), \quad \omega > 0. \quad (11)$$

The retarded Green's function is obtained from Eq. (3) by analytic continuation $p_0^E \rightarrow -i(\omega + i\epsilon)$, giving $p_E^2 \rightarrow -(\omega + i\epsilon)^2$,

$$G_R(\omega, \mathbf{k} = 0) = \int_0^\infty d\sigma \frac{\rho(\sigma)}{\sigma - (\omega + i\epsilon)^2}. \quad (12)$$

Taking the imaginary part using $\operatorname{Im}[(\sigma - (\omega + i\epsilon)^2)^{-1}] = -\pi\delta(\sigma - \omega^2)$ for $\omega > 0$,

$$J(\omega) = 2\pi\rho(\omega^2) = 2\pi C_\rho(\omega^2)^{\Delta-(d+1)/2} = 2\pi C_\rho \omega^{2\Delta-d-1}, \quad (13)$$

which, reabsorbing the factor of $2\pi C_\rho$ into the normalization constant A , gives

$$J(\omega) = A \omega^{2\Delta-d-1}, \quad (14)$$

reproducing Eq. (1) exactly.

5. Step 5: Identification with the unparticle form

The unparticle spectral density [1] is $\rho_U(\omega) \propto \omega^{2d_U-3}$. Matching exponents with Eq. (14),

$$2d_U - 3 = 2\Delta - d - 1 \Rightarrow d_U = \Delta - \frac{d-2}{2}, \quad (15)$$

which is Eq. (2). The dynamical exponents in Table I follow by Fourier transformation of the kernels; full derivations are given in Sec. III.

The identification with the unparticle form has a transparent interpretation in the KL language. The spectral weight $\rho(\sigma) \propto \sigma^{\Delta-(d+3)/2}$ is a pure power law, meaning the bath has a continuous spectrum of states at every invariant mass scale, with no preferred mass and no particle poles. This is precisely the defining feature of an unparticle sector [1]. The KL representation makes this identification automatic: scale invariance does not merely suggest the unparticle picture, it forces it. ■

Remark. The Källén-Lehmann proof derives conclusions (2)–(4) of Theorem 1 directly from Lorentz invariance and scale invariance, without invoking the CFT identification of conclusion (1). Conclusion (1)—that continuous scale invariance implies full conformal invariance—is established independently: in $d = 2$ it follows rigorously from Polchinski's completion of Zamolodchikov's c -theorem argument [4]; in $d \geq 3$ it holds under the additional assumption of unitarity and absence of a virial current, for which strong evidence exists in $d = 4$ [5,6] but no general proof is available. The logical structure of the theorem is therefore: conclusion (1) is an input from the conformal field theory literature; conclusions (2)–(4) are derived consequences proved here.

C. Loopholes and limitations

The theorem's assumptions admit five classes of failure, each physically realizable.

- (1) *Approximate scale invariance.* Real systems are scale invariant only over a finite window $\omega_{\text{IR}} < \omega < \omega_{\text{UV}}$. Infrared cutoffs arise from finite temperature T , system size L , or mass gap m ; ultraviolet cutoffs from lattice spacing or dynamically generated scales. Outside the window, the spectral density takes the form $J(\omega) = \omega^{2d_U-3} f_{\text{IR}}(\omega/\omega_{\text{IR}}) f_{\text{UV}}(\omega/\omega_{\text{UV}})$ with cutoff functions approaching unity in the scaling regime. Experimentally, this produces crossovers in $\Gamma_{\text{decoh}}(t)$ at early ($t \sim 1/\omega_{\text{UV}}$) and late ($t \sim 1/\omega_{\text{IR}}$) times.

- (2) *Nonlocal coupling.* If the coupling involves a kernel $K(x-y)$, $H_{\text{int}} = \int dx dy K(x-y) A_S(x) \mathcal{O}_E(y)$, the spectral density acquires a momentum-dependent form factor $|\tilde{K}(\omega, \mathbf{k})|^2$ that modifies the pure power law. Physical examples include dipole couplings with spatial averaging over the probe wave function.
- (3) *Discrete scale invariance.* Systems invariant only under $x \rightarrow \lambda_0^n x$ (Efimov states, hierarchical spin models) have a spectral weight that is no longer a pure power law; instead, Eq. (9) admits oscillatory solutions, producing log-periodic modulations: $J(\omega) \propto \omega^s [1 + A \cos(b \ln \omega + \phi)]$ with $b = 2\pi / \ln \lambda_0$. The functional equation argument in Step 3 requires continuity of $\rho(\sigma)$, which fails for discrete scale invariance.
- (4) *Multiple competing sectors.* If the environment contains several unparticle sectors with dimensions $d_U^{(i)}$, the total spectral density is $J_{\text{tot}}(\omega) = \sum_i A_i \omega^{2d_U^{(i)}-3}$. In the KL language, each sector contributes an independent power-law term to $\rho(\sigma)$. Different sectors dominate at different time-scales, producing observable crossovers.
- (5) *Quantum anomalies.* Classical scale invariance may be broken by quantum effects, as in the trace anomaly of QCD where $\langle T_\mu^\mu \rangle \neq 0$. The unparticle description applies only above the anomaly-generated scale, where the classical symmetry is approximately restored.

D. Converse: A no-go theorem

Theorem 2. (No scale-invariant alternatives) If the measured exponents γ_{decoh} and s are inconsistent with any single real value of d_U (i.e., $s + \gamma_{\text{decoh}} \neq 2$), then at least one of the following holds: the environment is not scale invariant; the coupling is nonlocal; Lorentz invariance or unitarity is violated; multiple competing sectors are present.

This provides an experimental diagnostic: measuring any two exponents from Table I independently extracts two values of d_U ; inconsistency falsifies scale invariance and identifies which loophole is operative. In the KL language, the no-go theorem is particularly transparent: a sum of power laws $\sum_i A_i \sigma^{\alpha_i}$ can mimic a single power law only over a limited frequency range; over a wide enough range, the individual exponents become separately resolvable, and their inconsistency with a single d_U is a direct signal of multiple sectors.

The two proofs of the unparticle universality theorem complement each other while relying on distinct frameworks. The CFT-based proof [2] works in position space and leverages conformal symmetry: scale invariance, together with Lorentz invariance and unitarity, implies full conformal invariance, which uniquely fixes the two-point function of the primary operator via Ward identities.

Analytic continuation then produces the retarded Green's function and the spectral density, leading directly to the unparticle exponent. By contrast, the Källén-Lehmann proof above operates in momentum space and begins from the most general Lorentz-invariant two-point function expressed as an integral over spectral weights. Imposing scale invariance on the spectral weight forces a unique power-law form, which in turn yields the same spectral density and unparticle exponent without assuming full conformal invariance. The CFT proof emphasizes symmetry constraints in position space; the KL proof emphasizes kinematic constraints in momentum space. Both independently converge on the same quantitative predictions, providing a cross-validation of the theorem.

It is essential to distinguish the logical direction of this result from that of the existing literature. The spin-boson and Caldeira-Leggett literature begins by *assuming* a power-law spectral density $J(\omega) \propto \omega^s$ and studies the resulting dynamics—a rich program that produced the well-known sub-Ohmic/Ohmic/super-Ohmic classification [9]. Our theorem addresses the prior question: for which physical environments is this spectral form *forced by symmetry*, rather than chosen as a model? The answer is: exactly and only those environments that are scale invariant. The power-law form is not an assumption; it is a consequence of Steps 1–3 of the proof, which leave no freedom in the functional form of $J(\omega)$ once scale invariance is imposed. Once this is established, applying the standard open-system formalism to the *predicted* spectral density yields theoretical values for all dynamical exponents (Table I) that can be directly compared with experiment. The consistency relations listed there hold in the spin-boson literature too, but there s is a free fitting parameter; here s is a theoretical prediction fixed by d_U , so the relations become falsifiable tests rather than tautologies. The fundamental content beyond CFT scaling is therefore that d_U is a *bath operator* property, not a system property: different probes coupled to the same bath operator $\mathcal{O}_E$ must yield the same d_U , and independent measurements of s and γ_{decoh} in unrelated experiments constitute genuine cross-probe consistency tests of the scale-invariance hypothesis for a fixed coupling channel.

III. MATHEMATICAL FRAMEWORK

The bath spectral density is defined throughout via the retarded Green's function of the environmental operator $\mathcal{O}_E$ evaluated at zero spatial momentum,

$$J(\omega) \equiv -2 \text{Im} G_R(\omega, \mathbf{k} = 0), \quad \omega > 0. \quad (16)$$

The overall normalization constant absorbs the coupling strength g^2 and the CFT coefficient $C_{\mathcal{O}}$; all universal exponents depend only on d_U and are independent of the normalization.

A. Unparticle operators and correlators

Theorem 1 establishes that the environment operator $\mathcal{O}_E$ of any scale-invariant bath is mathematically equivalent to an unparticle operator; in what follows, we adopt the notation $\mathcal{O}_U$ for the unparticle operator. This operator, $\mathcal{O}_U$, is defined by the scaling dimension d_U and has spectral density

$$\rho_U(\omega) = A_{d_U} \omega^{2d_U-3} \Theta(\omega), \quad (17)$$

where the normalization constant is [1]

$$A_{d_U} = \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1)\Gamma(2d_U)}. \quad (18)$$

Fourier transforming to real time, the two-point correlator at equal spatial position decays as

$$G_U(t) \propto \frac{e^{i\phi}}{t^{2d_U-2}}, \quad \phi = \frac{\pi}{2}(2d_U - 2), \quad (19)$$

which is a pure power law as expected from scale invariance and in contrast to the exponential decay e^{-mt} of massive particle propagators.

B. Influence functional and master equation

Consider a system with Hamiltonian H_S coupled to an unparticle bath via

$$H_{\text{int}} = g A_S \mathcal{O}_U, \quad (20)$$

$$\frac{d\rho_S(t)}{dt} = -i[H_S, \rho_S(t)] - g^2 \int_0^t ds [\nu(t-s)[A_S, [A_S(s), \rho_S(t)]] - i\eta(t-s)[A_S, \{A_S(s), \rho_S(t)\}]], \quad (25)$$

where $\nu(t) \equiv K'(t)$ is the noise kernel and $\eta(t) \equiv K''(t)$ is the dissipation kernel. The convolution over all past times $\int_0^t ds$ encodes memory: the system's current state depends on its entire history.

C. Dissipation kernel

Substituting $\rho_U(\omega) \propto \omega^{2d_U-3}$ into Eq. (24) and applying the Fourier identity

$$\int_0^\infty d\omega \omega^\mu \sin(\omega t) = \Gamma(\mu + 1) \sin\left(\frac{\pi(\mu + 1)}{2}\right) t^{-(\mu+1)}, \quad (26)$$

$$\text{Re}(\mu) > -1,$$

gives

where g and A_S are defined as in Theorem 1. Tracing out the environmental degrees of freedom in the path integral gives the reduced density matrix

$$\rho_S(x_f, x'_f; t) = \int \mathcal{D}x \mathcal{D}x' e^{i(S_S[x] - S_S[x'])} \mathcal{F}[x, x'], \quad (21)$$

where the influence functional for a Gaussian environment is

$$\mathcal{F}[x, x'] = \exp\left\{-g^2 \int_0^t ds \int_0^t ds' [A_S(s) - A'_S(s)] \times K(s-s') [A_S(s') - A'_S(s')]\right\}. \quad (22)$$

The kernel $K(t) = K'(t) + iK''(t)$ decomposes into noise and dissipation parts,

$$K'(t) = \int_0^\infty d\omega \rho_U(\omega) \coth\left(\frac{\beta\omega}{2}\right) \cos(\omega t), \quad (23)$$

$$K''(t) = -\int_0^\infty d\omega \rho_U(\omega) \sin(\omega t), \quad (24)$$

where $\beta = 1/T$ is the inverse temperature of the unparticle bath.

Expanding to second order in g without invoking the Markov approximation yields the non-Markovian master equation,

$$\eta(t) = \frac{2\alpha}{\pi} \Gamma(2d_U - 2) \sin\left(\frac{\pi(2d_U - 2)}{2}\right) \frac{1}{t^{2d_U-2}}, \quad (27)$$

where $\alpha = g^2 A_{d_U}$. The power-law exponent is

$$\alpha_{\text{dissip}} = 2d_U - 2, \quad (28)$$

requiring $d_U > 1$ for infrared convergence.

For integer d_U , the sine prefactor vanishes in the strictly scale-invariant limit. This is a marginal case: any physical UV completion restores a nonvanishing kernel while preserving the long-time power-law scaling. Integer d_U should be understood as a marginal dissipation point, not a dissipationless regime.

D. Noise kernel: Exact expression

We decompose the noise kernel into vacuum (I_V) and thermal (I_T) contributions using $\coth(x) = 1 + 2/(e^{2x} - 1)$,

$$\frac{\pi}{2\alpha}\nu(t) = I_V + I_T, \quad (29)$$

where

$$I_V = \int_0^\infty d\omega \omega^{2d_U-3} \cos(\omega t), \quad (30)$$

$$I_T = \int_0^\infty d\omega \omega^{2d_U-3} \cos(\omega t) \frac{2}{e^{\beta\omega} - 1}. \quad (31)$$

I_V evaluates to

$$I_V = \Gamma(2d_U - 2) \cos\left(\frac{\pi(2d_U - 2)}{2}\right) \frac{1}{t^{2d_U-2}}. \quad (32)$$

For the second one, I_T , we can use $1/(e^x - 1) = \sum_{n=1}^\infty e^{-nx}$ and Cauchy's theorem to evaluate the resulting integrals,

$$I_T = \sum_{n=1}^\infty \text{Re} \frac{2\Gamma(2d_U - 2)}{(n\beta - it)^{2d_U-2}}. \quad (33)$$

The full noise kernel is therefore

$$\begin{aligned} \nu(t) = & \frac{2\alpha}{\pi} \Gamma(2d_U - 2) \left[\cos\left(\frac{\pi(2d_U - 2)}{2}\right) \frac{1}{t^{2d_U-2}} \right. \\ & \left. + \sum_{n=1}^\infty \text{Re} \frac{2}{(n\beta - it)^{2d_U-2}} \right]. \end{aligned} \quad (34)$$

This expression is exact and valid for all t and T . Both vacuum and thermal terms require $d_U > 1$ for regularity.

a. Limiting regimes. In the *vacuum regime* $t \ll \beta$, the thermal sum is negligible and

$$\nu(t) \propto t^{-(2d_U-2)}. \quad (35)$$

In the *thermal regime* $t \gg \beta$, the sum approximates an integral; requiring $d_U > 3/2$ for convergence and using $\Gamma(z+1) = z\Gamma(z)$ recovers the high-temperature result

$$\nu(t) \approx \frac{4\alpha T}{\pi} \Gamma(2d_U - 3) \cos\left(\frac{\pi(2d_U - 3)}{2}\right) \frac{1}{t^{2d_U-3}}, \quad (36)$$

with noise exponent $\alpha_\nu = 2d_U - 3$. The crossover between regimes occurs at $t \sim \beta$. Crucially, *both regimes are power laws*; the exponent shifts by unity between them but d_U is the same parameter throughout.

E. Damping and decoherence functionals

a. Damping function. The accumulated damping is

$$\Gamma_{\text{damp}}(t) = \int_0^t ds \eta(s) \propto t^{3-2d_U}, \quad d_U \neq 3/2, \quad (37)$$

with logarithmic growth at the marginal case $d_U = 3/2$.

b. Decoherence functional. Coherence between position eigenstates $|x\rangle$ and $|x'\rangle$ decays as $|\rho(x, x'; t)| \propto \exp[-(x - x')^2 \Gamma_{\text{decoh}}(t)/2\hbar^2]$, where

$$\Gamma_{\text{decoh}}(t) = \int_0^t ds \int_0^s ds' \nu(s - s'). \quad (38)$$

Substituting the high-temperature noise kernel $\nu(u) \propto Tu^{-(2d_U-3)}$ and performing the double integral,

$$\Gamma_{\text{decoh}}(t) \propto t^{5-2d_U}, \quad d_U \neq 2, 5/2. \quad (39)$$

Similarly, in the low-temperature regime, where $\nu(u) \propto u^{-(2d_U-2)}$,

$$\Gamma_{\text{decoh}}(t) \propto t^{4-2d_U}, \quad d_U \neq 3/2, 2. \quad (40)$$

Alternatively, via the spectral representation,

$$\Gamma_{\text{decoh}}(t) = \int_0^\infty d\omega \frac{J(\omega)}{\omega^2} \coth\left(\frac{\beta\omega}{2}\right) (1 - \cos \omega t), \quad (41)$$

which is UV finite for $J(\omega) \propto \omega^s$ with $s < 1$ and provides the correct regularization for super-Ohmic cases.

Using Eq. (41) and introducing a UV cutoff Ω_{UV} , one can compute the decoherence functional numerically. Figure 1 shows the result for this calculation in different regimes. In the sub-Ohmic case, we see that the cutoff only affects the functional at times $t \lesssim 2\pi/\Omega_{\text{UV}}$. Above this time, the functional is well described by the unparticle prediction, the power laws with different exponents in the $t \ll \beta$ and $t \gg \beta$ regimes are visible and match the prediction, and the limit $\Omega_{\text{UV}} \rightarrow \infty$ is finite and converges to this prediction. This is not the case in the super-Ohmic regime, where the cutoff does impact the decoherence functional at times far above the cutoff scale, as the high-frequency modes in the integral in Eq. (41) add a time-independent contribution to the functional, which diverges in the $\Omega_{\text{UV}} \rightarrow \infty$ limit. If $1 < s < 2$ ($2 < d_U < 5/2$), the power law from the low-frequency modes of the integral eventually overtakes this constant contribution at sufficiently long times, recovering the result from the unparticle description. But if $s > 2$ ($d_U > 5/2$), the power law contribution has a negative exponent, so it saturates and never overtakes the constant; thus, coherence survives indefinitely in this regime.

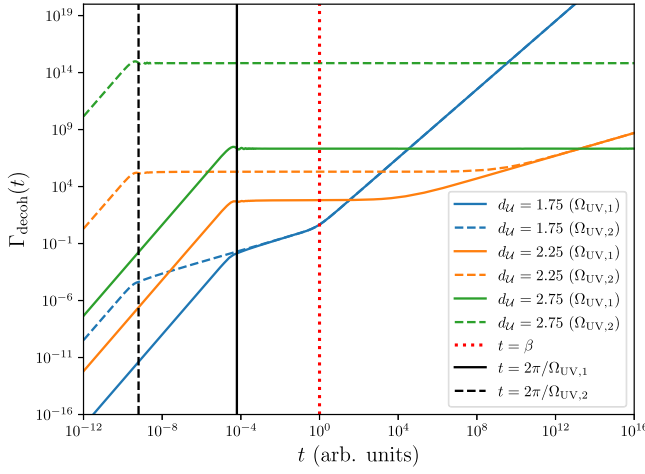


FIG. 1. Decoherence functional $\Gamma_{\text{decoh}}(t)$ for the spectral density $J(\omega) = \alpha\omega^s$, $s = 2d_U - 3$, $\alpha = 1$, with time in units of β . The plot shows the evolution of the decoherence functional for three different values of d_U in the sub-Ohmic (blue), super-Ohmic (orange) and ultra-super-Ohmic (green) regimes (See Table II), and two different UV cutoffs $\Omega_{\text{UV},1} = 10^5/\beta$ (solid), $\Omega_{\text{UV},2} = 10^{10}/\beta$ (dashed).

F. Specific heat scaling and thermodynamic observables

The internal energy of the bath follows from the spectral density $J(\omega) \propto \omega^{2d_U-3}$ and the Bose-Einstein occupation $n(\omega) = (e^{\beta\omega} - 1)^{-1}$,

$$U \propto \int_0^\infty d\omega \omega J(\omega) n(\omega) = \int_0^\infty d\omega \frac{\omega^{2d_U-2}}{e^{\beta\omega} - 1}. \quad (42)$$

Here U is the internal energy of the bath degrees of freedom; in a many-body system at a quantum critical point, this corresponds to the electronic contribution to the measured specific heat. Substituting $u = \beta\omega = \omega/T$,

$$U \propto T^{2d_U-1} \int_0^\infty du \frac{u^{2d_U-2}}{e^u - 1} = T^{2d_U-1} \Gamma(2d_U-1) \zeta(2d_U-1), \quad (43)$$

where the integral converges for $d_U > 1$. Differentiating with respect to T ,

$$C(T) = \frac{\partial U}{\partial T} \propto T^{2d_U-2}, \quad (44)$$

or equivalently

$$\frac{C(T)}{T} \propto T^{2d_U-3}. \quad (45)$$

The extraction formula is therefore

$$d_U = \frac{x_{C/T} + 3}{2}, \quad (46)$$

where $x_{C/T}$ is the power-law exponent of C/T versus T , in direct analogy with the resistivity formula $d_U = (x_\rho + 2)/2$.

a. Marginal case and the $-\ln T$ anomaly. At $d_U = 3/2$, the exponent in Eq. (45) vanishes: $C/T \propto T^0$. This is a marginal point in precisely the same sense as the thermalization transition in the phase diagram (Table II, derived in the next Subsection): the leading power-law contribution is constant, and the first nontrivial correction is logarithmic. Expanding the integrand near $d_U = 3/2$, the correction to U at order $(d_U - 3/2)$ introduces a factor of $\ln T$, yielding

$$\left. \frac{C(T)}{T} \right|_{d_U=3/2} \propto -\ln \frac{T}{T_0}, \quad (47)$$

where T_0 is a nonuniversal scale set by the UV cutoff.

b. Extraction formulas across channels. The following summary collects the d_U extraction formula for each independently measurable observable channel (see also Table I):

$$d_U = \begin{cases} \frac{x_\rho + 2}{2} & \rho(T) \propto T^{x_\rho}, \\ \frac{x_\eta + 2}{2} & \eta(T) \propto T^{x_\eta}, \\ \frac{x_{C/T} + 3}{2} & C(T)/T \propto T^{x_{C/T}}, \\ \frac{3}{2} & C(T)/T \propto -\ln T \text{ (marginal)}. \end{cases} \quad (48)$$

These formulas are used in Sec. VA to perform a two-channel consistency check in heavy-fermion compounds. All four follow from the single spectral density $J(\omega) \propto$

TABLE II. Phase diagram of open-system dynamics as a function of d_U .

d_U	Bath type	Thermalization	Decoherence	Memory
$< 3/2$	Deep sub-Ohmic	Power-law	Accelerating	Very strong
$= 3/2$	Critical sub-Ohmic	Logarithmic	$\propto t^2$	Strong
$3/2 - 2$	Sub-Ohmic	Slowing	Growing	Moderate
$= 2$	Ohmic	Slow	Linear	Marginal
$2 - 5/2$	Super-Ohmic	Fails	Decelerating	Weak
$= 5/2$	Decoherence critical	Fails	Logarithmic	Very weak
$> 5/2$	Ultra-super-Ohmic	Fails	Saturates	Negligible

ω^{2d_U-3} via the appropriate Kubo relation; they are consistency checks, not independent definitions of d_U .

G. Phase structure

The three critical dimensions marking qualitative transitions are:

a. $d_U = 3/2$ (*thermalization transition*). The damping exponent $\beta_{\text{damp}} = 3 - 2d_U$ changes sign: for $d_U < 3/2$, Γ_{damp} grows (efficient thermalization); for $d_U > 3/2$, it decays (thermalization fails at long times). At exactly $d_U = 3/2$, thermalization is logarithmic.

b. $d_U = 2$ (*Ohmic boundary*). $J(\omega) \propto \omega$; the dissipation kernel $\eta(t) \propto t^{-2}$ marks the boundary between non-Markovian ($d_U < 2$, slow kernel decay, strong memory) and quasi-Markovian ($d_U > 2$, fast decay, finite memory time). The memory time $\tau_{\text{mem}} = \int_0^\infty dt t \eta(t)$ diverges for $d_U \leq 2$ and is finite for $d_U > 2$.

c. $d_U = 5/2$ (*decoherence phase transition*). The decoherence exponent $\gamma_{\text{decoh}} = 5 - 2d_U$ changes sign. For $d_U \leq 5/2$: $\Gamma_{\text{decoh}}(t \rightarrow \infty) \rightarrow \infty$, coherence is irreversibly lost (for $d_U = 5/2$, it grows logarithmically). For $d_U > 5/2$: $\Gamma_{\text{decoh}}(t \rightarrow \infty) \rightarrow \text{constant}$, coherence is *protected* at long times (for free-field, weakly interacting, or engineered power-law baths; see Sec. IX).

This last transition—coherence protection by a super-Ohmic bath—is impossible in any Markovian (Lindblad) description. Its physical origin is the extremely short correlation time of highly super-Ohmic baths: high-frequency modes dominate, but their rapid oscillations average to zero on the system's timescale, effectively decoupling the bath at late times.

The complete phase diagram is summarized in Table II.

IV. COMPARISON WITH EXISTING FRAMEWORKS

A. Caldeira-Leggett model

The standard Caldeira-Leggett (CL) master equation for a particle of mass m in the high-temperature limit is

$$\frac{d\rho_S}{dt} = -\frac{i}{\hbar}[H_S, \rho_S] - \frac{i\gamma}{2\hbar}[x, \{p, \rho_S\}] - \frac{\gamma m T}{\hbar^2}[x, [x, \rho_S]], \quad (49)$$

where γ is the damping coefficient of the model. Here we assume an Ohmic spectral density $J(\omega) = \eta_0 \omega$, where η_0 is the Ohmic coupling constant, with high-frequency cutoff, weak coupling, and Markovian dynamics.

The unparticle master equation (25) reduces to Eq. (49) in two limits:

- (i) *Ohmic case with coarse graining*. For $d_U = 2$, the spectral density is Ohmic. Coarse-graining over times $t \gg \omega_0^{-1}$ allows approximating the

time-nonlocal integral by an effective local damping coefficient $\gamma_{\text{eff}} = \int_0^\infty ds \eta(s)$.

- (ii) *Large scaling dimension*. As $d_U \rightarrow \infty$, $\eta(t) \propto t^{-(2d_U-2)}$ decays infinitely rapidly, approaching $\gamma \delta(t-s)$, exactly reproducing the Markovian limit.

For general d_U , the unparticle master equation constitutes a fractional generalization of the CL equation,

$$\frac{d\rho_S(t)}{dt} = -\frac{i}{\hbar}[H_S, \rho_S(t)] - \frac{C_\alpha}{\hbar^2} \int_0^t \frac{ds}{(t-s)^{2d_U-2}} [x, [x(s), \rho_S(t)]] + \dots, \quad (50)$$

featuring a Riemann-Liouville fractional integral of order $\alpha_{\text{dissip}} = 2d_U - 2$. The Caldeira-Leggett model is thus a special case of our formalism, not a competitor.

B. Lindblad master equations

Lindblad master equations treat decoherence rates as free parameters and predict exponential decay of coherence, $|\rho_{\alpha\beta}(t)| \propto e^{-\Gamma_{\alpha\beta} t}$, corresponding to a constant decoherence rate. Within the unparticle framework, the Lindblad form is recovered only when $\Gamma_{\text{decoh}}(t) \propto t$, i.e., $d_U = 2$ (Ohmic).

For all other values of d_U , the decoherence functional is a power law t^{5-2d_U} , giving stretched-exponential rather than purely exponential decay. Most importantly, for $d_U > 5/2$ the decoherence functional *saturates* at long times—a regime entirely inaccessible to Lindblad dynamics and representing a genuine qualitative difference.

C. Non-Markovian approaches

The broader non-Markovian literature [10,11] encompasses many techniques including Nakajima-Zwanzig and time-convolutionless master equations [12], hierarchy of equations of motion [10], and reaction coordinate methods [13]. The unparticle framework is complementary: it does not provide a numerical solution method but identifies the universality class. Systems with power-law spectral densities previously studied as the spin-boson model [9] are equivalent to the unparticle framework with $d_U = (s+3)/2$ in the limit where the UV cutoff can be taken to infinity, or in situations where it is large compared to all relevant physical scales. In practice, the spin-boson model with a finite UV cutoff is the more general case; the pure unparticle description corresponds to the special limit of exact scale invariance over the full frequency range. The coherence protection for $s > 2$ (i.e., $d_U > 5/2$) found in Ref. [14] is reproduced here as a consequence of conformal symmetry rather than a model-specific result.

The relationship to the spin-boson literature deserves explicit statement. Leggett *et al.* [9] assume $J(\omega) \propto \omega^s$ and classify the resulting dynamics; the coherence protection for $s > 2$ found there is reproduced here as a consequence of conformal symmetry. The unparticle framework is not a

restatement of that classification: it is the symmetry principle that forces the spectral form, so that the classification applies universally to any scale-invariant bath. Prior work on decoherence in specific holographic and gravitational settings [15,16] and on the emergence of anomalous dimensions and power-law spectral functions from infrared dressing [17] identifies important physical phenomena within particular models; the present theorem provides the model-independent foundation within which those results are organized and compared.

D. SYK models and quantum chaos

A natural physical realization of the unparticle framework arises in Sachdev-Ye-Kitaev (SYK) models [18,19]. At low energies, the q -body SYK Hamiltonian develops emergent conformal symmetry with fermion scaling dimension $\Delta = 1/q$ [20], producing a bath spectral density of the power-law form dictated by the present framework, with

$$d_U = \frac{1}{q} + 1, \quad (51)$$

in $(0+1)$ dimensions. Simultaneously, the energy-level statistics of the SYK Hamiltonian fall in the GUE (Gaussian Unitary Ensemble, the random matrix universality class of Hermitian matrices with broken time-reversal symmetry) universality class of random matrix theory, reflecting its maximally chaotic nature and saturation of the Maldacena–Shenker–Stanford bound [21]. An unparticle bath realized by an SYK environment therefore admits a twofold description: its two-point function, which drives decoherence of the open system, is fixed by conformal symmetry and parametrized by d_U , while its internal spectral statistics are governed by random matrix universality. Whether the decoherence phase transition at $d_U = 5/2$ predicted by the present framework corresponds to a transition in spectral statistics—such as a crossover from chaotic to integrable behavior—in systems where d_U can be tuned through this value remains a concrete open problem.

V. EXPERIMENTAL VALIDATION

Our previous work [2] provided the primary empirical validation of the framework: multichannel transport data from the unitary Fermi gas yield $d_U = 7/4$ consistently from shear viscosity and thermal conductivity, two genuinely independent Green’s functions, and the engineered spin-boson experiments of Sun *et al.* directly test the consistency relation $s + \gamma_{\text{decoh}} = 2$. Here, we present a second many-body validation at a different scaling dimension, establishing that d_U has nontrivial resolution across universality classes.

A. Heavy-fermion metals at quantum criticality

Heavy-fermion compounds tuned to a magnetic quantum critical point provide a second many-body validation of the unparticle framework, at a scaling dimension distinct from

the unitary Fermi gas ($d_U = 7/4$). Unlike the Ising model realization of Sec. VI, where d_U is derived from first principles from the known CFT operator dimension, the result here is obtained empirically: two independent transport channels are fit within the same scale-invariant window, and both yield $d_U = 3/2$. This value coincides with the analytical prediction for the $(1+1)$ -dimensional Ising model coupled to the energy operator (Sec. VI), though the microscopic identification of the relevant CFT for heavy-fermion quantum criticality lies beyond the scope of the present work. The critical sub-Ohmic point $d_U = 3/2$ predicts linear-in- T resistivity $\rho \propto T$ and logarithmic specific heat $C/T \propto -\ln T$ as the two leading observables in the scale-invariant window [Eq. (47) and Table I].

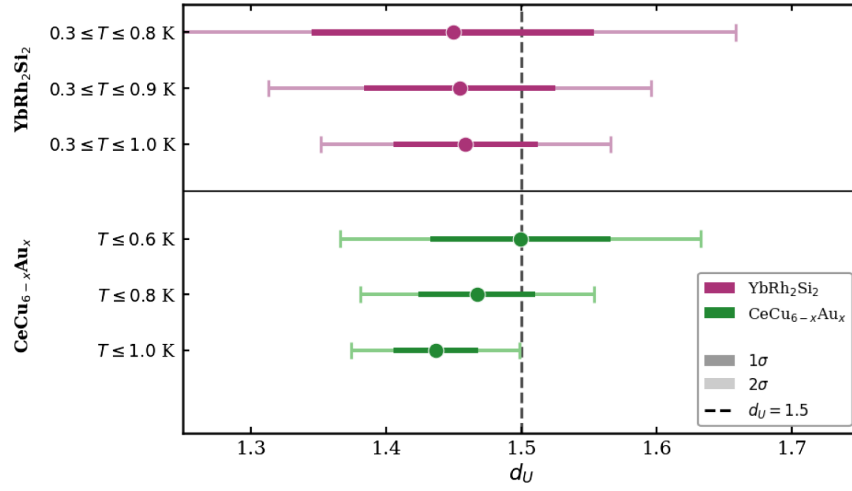
a. Two-channel consistency check. We test these predictions against published transport and thermodynamic data for two materials at their respective quantum critical points: YbRh₂Si₂ [22] and CeCu_{5.9}Au_{0.1} [23]. For each material we fit the resistivity to $\rho = \rho_0 + AT^x$ and the specific heat coefficient to $C/T = a - b \ln T$ within the same scale-invariant temperature window, extracting $d_U = (x+2)/2$ from the resistivity and verifying that the logarithmic form holds in the same window. Results are shown in Fig. 2.

For YbRh₂Si₂, the scale-invariant regime is $0.3 \leq T \leq 1.0$ K; a lower cut is imposed because this material is close to but not exactly at the QCP, so ordered-phase behavior appears below ~ 0.3 K. The resistivity fit yields $x = 0.92 \pm 0.11$, giving $d_U = 1.46 \pm 0.06$, consistent with $3/2$ at the 1σ level. The specific heat is logarithmic in the same window ($a = 0.519 \pm 0.005$, $b = 0.189 \pm 0.008$ J mol⁻¹ K⁻²), in agreement with the marginal prediction of Eq. (47).

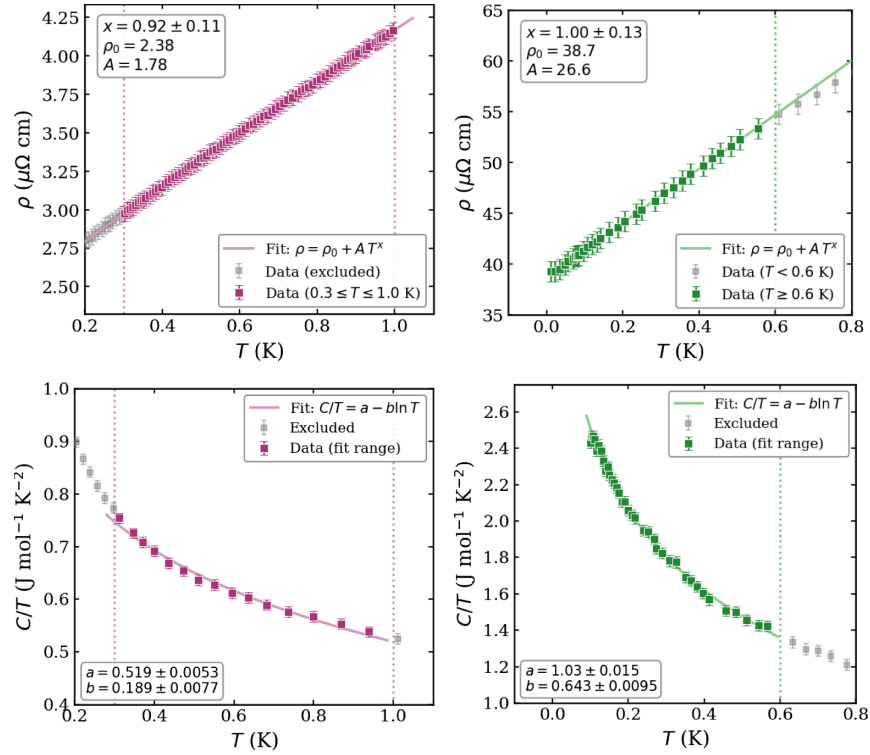
For CeCu_{5.9}Au_{0.1}, tuned to the QCP at $T = 0$ with no lower cut required, the scale-invariant window is $T \leq 0.6$ K, above which Fermi-liquid corrections become visible. The resistivity yields $x = 1.00 \pm 0.13$, giving $d_U = 1.50 \pm 0.07$, sitting exactly on the predicted value. The specific heat is again logarithmic ($a = 1.03 \pm 0.015$, $b = 0.643 \pm 0.010$ J mol⁻¹ K⁻²) in the same window.

In both materials, the resistivity and specific heat channels are fit independently within the same temperature window and yield mutually consistent results, constituting a two-channel test of $d_U = 3/2$ in each system. The high- T breakdown of the power-law scaling, clearly visible in Fig. 2, is the expected crossover out of the scale-invariant window rather than a failure of the framework: at temperatures where Fermi-liquid corrections or thermal fluctuations above the QCP regime become relevant, $J(\omega)$ acquires corrections to the pure power law, consistent with the loophole analysis of Sec. II C.

Together with the unitary Fermi gas ($d_U = 7/4$) from our earlier work, these results demonstrate that d_U has nontrivial resolution across universality classes: two distinct



(a) Extracted d_U from resistivity fits $\rho = \rho_0 + AT^x$ with $d_U = (x + 2)/2$, under progressively lower upper- T cuts. Both materials converge to $d_U = 3/2$ (dashed line) at low T , where competing effects are suppressed. Deviations at higher cuts reflect crossover out of the scale-invariant window (Sec. II C). Error bars denote 1σ (inner) and 2σ (outer).



(b) Two-channel test for YbRh₂Si₂ [22] (purple) and CeCu_{5.9}Au_{0.1} [23] (green). Top: resistivity fits within the scaling window (colored; excluded data grey). For YbRh₂Si₂, a lower cut $T = 0.3$ K removes ordered-phase contamination; for CeCu_{5.9}Au_{0.1}, tuned to the QCP, only an upper cut (0.6 K) is applied. Bottom: $C/T = a - b \ln T$ fits in the same window, consistent with $d_U = 3/2$. Agreement between channels constitutes the consistency test.

FIG. 2. Heavy-fermion validation at $d_U = 3/2$. Resistivity and specific heat independently support the same scaling in two heavy-fermion systems at their quantum critical points, providing a two-channel check distinct from the unitary Fermi gas ($d_U = 7/4$) [2].

values are measured in three physically unrelated many-body systems, with no shared fitting parameters.

VI. QUANTUM CRITICAL POINTS: THE 2D ISING MODEL

A. Critical theory

The quantum Ising Hamiltonian

$$H_{\text{Ising}} = -J \sum_{\langle ij \rangle} \sigma_i^x \sigma_j^x - h \sum_i \sigma_i^z \quad (52)$$

exhibits a continuous quantum phase transition at $h = h_c$. At criticality the low-energy effective theory is the two-dimensional Ising CFT with central charge $c = 1/2$. This is the minimal unitary CFT and is exactly solvable. The primary operator spectrum contains three operators: the identity $\mathbb{1}$ ($\Delta = 0$), the spin (order parameter) σ ($\Delta = 1/8$), and the energy density ε ($\Delta = 1$).

We consider two spacetime dimensionalities: the $(1+1)$ -dimensional quantum Ising chain ($d_{\text{space}} = 1$) and the $(2+1)$ -dimensional quantum Ising model ($d_{\text{space}} = 2$). These are *different* conformal field theories. The $(1+1)$ -dimensional chain flows to the 2D Ising CFT, a rational CFT with central charge $c = 1/2$ and exactly known operator dimensions [24],

$$\Delta_\sigma = \frac{1}{8}, \quad \Delta_\varepsilon = 1. \quad (53)$$

The $(2+1)$ -dimensional model flows instead to the 3D Ising CFT, a nonrational interacting fixed point with no closed-form description. Its scaling dimensions have been determined to high precision by the conformal bootstrap [3],

$$\Delta_\sigma \approx 0.5181, \quad \Delta_\varepsilon \approx 1.4126. \quad (54)$$

The two CFTs share the qualitative $\mathbb{Z}_2$ -symmetry-breaking structure of the Ising universality class, but they are distinct theories with different operator dimensions, which propagate directly to the unparticle predictions via Eq. (2). For the $(2+1)$ D model coupled to the energy operator, $\Delta_\varepsilon \approx 1.4126$ and $d = 2$ give $d_{\mathcal{U}} \approx 1.413$, with spectral exponent $s \approx -0.17$ —close to but distinct from the $1/f$ spectrum, and with decoherence exponent $\gamma_{\text{decoh}} \approx 2.17$.

B. Setup: System, bath, and coupling

- (i) *System*: a probe spin (external qubit or localized impurity) with Hilbert space $\mathcal{H}_S = \mathbb{C}^2$.
- (ii) *Environment*: the critical quantum Ising chain (or planar model) at its quantum critical point, described by the $(1+1)$ D (or $(2+1)$ D) Ising CFT.

- (iii) *Coupling operator*: $H_{\text{int}} = g \sigma_{\text{probe}}^z \mathcal{O}(\mathbf{r}_0)$, where $\mathcal{O}$ is either the energy density ε or spin operator σ of the Ising CFT, evaluated at the probe position $\mathbf{r}_0$.
- (iv) *Bath correlator defining $J(\omega)$* : $G_R(\omega, \mathbf{k} = 0) \propto (-i\omega)^{2\Delta-d}$, with Δ the scaling dimension of $\mathcal{O}$ and d the spatial dimension; the spectral density is $J(\omega) \propto \omega^{2\Delta-d-1}$ [Eq. (56)].
- (v) *Predicted observable*: the off-diagonal density matrix element of the probe decays as $|\rho_{\uparrow\downarrow}(t)| = \exp(-Ct^{\gamma_{\text{decoh}}})$ with $\gamma_{\text{decoh}} = 5 - 2d_{\mathcal{U}}$, measurable via Ramsey interferometry. For the $(1+1)$ D energy-operator coupling: $\gamma_{\text{decoh}} = 2$ exactly; for the $(2+1)$ D case: $\gamma_{\text{decoh}} \approx 2.174$.

At criticality the Ising spins exhibit scale-invariant fluctuations with no characteristic length or time scale, forming an unparticle bath for the probe.

C. Spectral density from CFT correlators

The Euclidean two-point function is fixed by conformal symmetry,

$$\langle \mathcal{O}(\mathbf{x}, \tau) \mathcal{O}(0, 0) \rangle_E = \frac{C_{\mathcal{O}}}{(|\mathbf{x}|^2 + \tau^2)^\Delta}. \quad (55)$$

After analytic continuation and Fourier transformation at $\mathbf{k} = 0$, the spectral density is [from Eq. (1)],

$$J(\omega) \propto \omega^{2\Delta-d_{\text{space}}-1}, \quad \omega > 0. \quad (56)$$

D. Results for all cases

We apply Eq. (56) to each operator and spacetime dimension, then use Eq. (2) to extract $d_{\mathcal{U}}$ and Table I for all predictions. Results are collected in Table III.

- a. $(1+1)$ D, coupling to ε ($\Delta = 1$, $d = 1$).

$$J(\omega) \propto \omega^{2 \cdot 1 - 1 - 1} = \omega^0, \quad d_{\mathcal{U}} = \frac{3}{2}. \quad (57)$$

Flat spectral density; sub-Ohmic. Predicted decoherence: $\Gamma_{\text{decoh}}(t) \propto t^2$ (quadratic growth). This is the marginal thermalization case.

- b. $(2+1)$ D, coupling to ε ($\Delta \approx 1.4126$, $d = 2$). Using the conformal bootstrap value [3] $\Delta_\varepsilon \approx 1.4126$,

$$J(\omega) \propto \omega^{2 \times 1.4126 - 2 - 1} \approx \omega^{-0.174}, \quad d_{\mathcal{U}} \approx 1.413. \quad (58)$$

The spectral density is close to but distinct from the $1/f$ spectrum ($s = -1$); the spectral exponent $s \approx -0.174$ lies between -1 and 0 , placing this case in the deep sub-Ohmic regime but without the marginal IR behavior of the exact $1/f$ case. The predicted decoherence exponent is $\gamma_{\text{decoh}} \approx 5 - 2(1.413) = 2.174$ (slightly superquadratic growth). As with the $(1+1)$ D spin-operator cases, the mildly divergent spectral density at $\omega \rightarrow 0$ requires IR regularization at

TABLE III. Unparticle dimensions and scaling exponents for the quantum Ising model at criticality. The $(1+1)$ D values are exact (2D Ising CFT, $c = 1/2$); the $(2+1)$ D values use conformal bootstrap results [3]. Cases with $d_U < 1$ require IR regularization. The $(2+1)$ D energy-operator case ($d_U \approx 1.413$, $s \approx -0.17$) has a mildly divergent spectral density as $\omega \rightarrow 0$ and likewise requires IR regularization at strictly zero temperature and infinite system size; for any finite T or system size the spectral density is cut off below ω_{IR} and all integrals are finite.

System	Op.	Δ	d_U	s	γ_{decoh}	Regime
$(1+1)$ D	σ	1/8	5/8	-7/4	15/4	IR divergent
$(1+1)$ D	ε	1	3/2	0	2	Sub-Ohmic
$(2+1)$ D	σ	0.5181	0.518	-1.964	3.964	IR divergent
$(2+1)$ D	ε	1.4126	1.413	-0.174	2.174	Sub-Ohmic

strictly zero temperature; for any finite T or system size, integrals are finite.

c. Cases with $d_U < 1$ (spin operator σ). For coupling to σ , both $(1+1)$ D ($d_U = 5/8$) and $(2+1)$ D ($d_U = 1/8$) give $d_U < 1$. The spectral density diverges as $\omega \rightarrow 0$ with exponent stronger than ω^{-1} , making the Fourier integrals infrared divergent without a physical cutoff. This is not a failure of the framework but a physical signal: the spin operator σ is the order-parameter field at criticality, whose fluctuations are too strongly correlated to serve as a well-defined dissipative bath at exactly zero temperature and infinite system size. In any real system, finite temperature or system size regularizes the IR, and the unparticle description applies for $\omega > \omega_{\text{IR}}$.

E. Experimental predictions

a. Decoherence decay. Prepare the probe in $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ and measure $|\rho_{\uparrow\downarrow}(t)|$ via Ramsey interferometry. Fit to $|\rho_{\uparrow\downarrow}(t)| = \exp(-Ct^{\gamma_{\text{decoh}}})$ and extract γ_{decoh} . Predicted values: $\gamma_{\text{decoh}} = 2$ for the $(1+1)$ D chain coupled to ε ; $\gamma_{\text{decoh}} \approx 2.174$ for the $(2+1)$ D model (from $\Delta_\varepsilon \approx 1.4126$, Ref. [3]).

b. Temperature scaling. In the high- T regime, the prefactor $C \propto g^2 T$ [from Eq. (36)]. Varying temperature tests this linear scaling independently of the exponent measurement.

c. Consistency check. Independently measuring the noise spectrum exponent s and the decoherence exponent γ_{decoh} allows verification of $s + \gamma_{\text{decoh}} = 2$. For the $(1+1)$ D energy-operator coupling: $s = 0$, $\gamma_{\text{decoh}} = 2$, so $s + \gamma_{\text{decoh}} = 2$ exactly.

d. Deviation from criticality. Tuning h away from h_c introduces a finite correlation length ξ , setting $\omega_{\text{IR}} \sim v/\xi$. The power-law decoherence crosses over to exponential decay at $t \sim \xi/v$. Observing this crossover and extracting the crossover scale provides an independent determination of ξ .

e. Experimental platforms. The $(1+1)$ D chain is accessible in trapped-ion simulators [25] and Rydberg atom arrays [26]. For typical parameters ($v \sim 10^3$ m/s, $g \sim 0.1$, $T \sim 100$ μ K), the decoherence time is of order tens of milliseconds, well within the reach of current experiments. The $(2+1)$ D model is accessible through layered magnetic

materials or nitrogen-vacancy centers near quantum critical materials.

VII. INFLATIONARY COSMOLOGY

A. Background and setup

Inflation provides a natural setting in which to apply the framework developed above, in a context where scale-invariant correlators arise from geometry rather than from an interacting fixed point. In the de Sitter phase of slow-roll inflation, the scale factor grows approximately exponentially, $a(t) \simeq e^{Ht}$, and the de Sitter isometry group $SO(4, 1)$ contains dilatations, enforcing approximate dilation invariance of correlation functions of light fields in the Bunch-Davies vacuum [27].

The uniqueness theorem of Sec. II is proved under flat-space conformal invariance and does not strictly apply in curved spacetime; the de Sitter isometry group $SO(4, 1)$ differs from the flat-space conformal group $SO(d+1, 1)$, and the relation (2) must be rederived in this geometry. Nevertheless, the non-Markovian formalism can be applied in the same spirit, and we show below that doing so reproduces established results on the quantum-to-classical transition during inflation [28,29], identifying inflation as a particular realization of the framework's phase structure with $d_U = 2$ defined by matching to those results.

We model the inflaton ϕ (the system) as coupled to an environment consisting of light scalar fields χ and/or gravitons, which acquire scale-invariant correlators from the de Sitter background. The interaction takes the form $\mathcal{L}_{\text{int}} = -(g/2)\phi^2\chi$.

B. De Sitter correlators and scaling dimensions

For a scalar field of mass m in $(3+1)$ -dimensional de Sitter space, the two-point function in the Bunch-Davies vacuum is

$$\langle \chi(x)\chi(0) \rangle_{\text{ds}} = \frac{C_\Delta}{(-x^2 + i\epsilon)^\Delta}, \quad (59)$$

with scaling dimension

$$\Delta = \frac{3}{2} + i\nu, \quad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}. \quad (60)$$

In particular, for a *massless* scalar ($m = 0$),

$$\Delta_{\text{massless}} = \frac{3}{2} + i\frac{3}{2}. \quad (61)$$

The real part, which governs the spectral density, is $\text{Re}[\Delta] = 3/2$. For a massless *graviton*, the transverse-traceless modes satisfy the same equation of motion as a massless minimally coupled scalar,³ so $\text{Re}[\Delta_h] = 3/2$ likewise.

C. Spectral density and unparticle dimension

The spectral density relevant to inflaton decoherence cannot be read off from the flat-space relation Eq. (2), because the de Sitter isometry group differs from the flat-space conformal group and the resulting two-point function of bath operators carries different geometric weight. A direct derivation of $J(\omega)$ for the Bunch-Davies bath of light scalars or gravitons along a comoving worldline lies beyond the scope of this article; we therefore proceed by matching the established literature on inflationary decoherence [28,29], which finds linear growth of the decoherence functional in proper time (or in e-folds $N = Ht$),

$$\Gamma_{\text{decoh}}(t) \propto Ht. \quad (62)$$

Comparing to the framework's prediction $\Gamma_{\text{decoh}} \propto t^{5-2d_U}$ in the thermal regime—appropriate here because the de Sitter Gibbons-Hawking temperature $T_{\text{GH}} = H/2\pi$ gives $\beta_{\text{GH}} \sim H^{-1}$, much shorter than the inflationary decoherence timescale of ~ 50 – 60 e-folds, so the cumulative decoherence is firmly in the regime $t \gg \beta_{\text{GH}}$ —fixes

$$d_U = 2 \quad (\text{Ohmic}). \quad (63)$$

We thus identify inflation as the Ohmic boundary ($d_U = 2$) of the framework's phase structure, where the framework's spectral density would take the form $J(\omega) \propto \omega$ if its assumptions applied directly. This identification is empirical within the family of inflationary decoherence calculations, not a derivation from the universality theorem; the latter would require extending the theorem to de Sitter isometries, which we do not pursue here. The resulting mapping between inflationary bath types and unparticle framework quantities is summarized in Table IV.

D. Setup: System, bath, and coupling

- (i) *System*: the inflaton field ϕ , treated as a probe of the inflationary environment.

³The transverse-traceless (TT) modes satisfy $\square h_{ij}^{\text{TT}} = 0$, identical to the massless scalar equation, giving $m_{\text{eff}}^2 = 0$ and hence $\nu = 3/2$ in Eq. (60).

TABLE IV. Mapping between inflationary bath types and unparticle framework quantities, obtained by matching to established inflationary decoherence calculations [28,29]. The d_U values are not derived from the flat-space universality theorem but inferred from the decoherence exponent γ_{decoh} via $d_U = (5 - \gamma_{\text{decoh}})/2$.

Bath	d_U	s	γ_{decoh}
Massless scalar ($m = 0$)	2	1	1
Graviton	2	1	1
Massive scalar ($m \sim H$)	> 2	> 1	< 1

- (ii) *Environment*: light scalar fields χ and/or graviton transverse-traceless modes, acquiring scale-invariant correlators from the de Sitter background in the Bunch-Davies vacuum.
- (iii) *Coupling operator*: $\mathcal{L}_{\text{int}} = -(g/2)\phi^2\chi$ for the scalar case; analogous gravitational coupling for the tensor modes.
- (iv) *Bath correlator defining $J(\omega)$* : the de Sitter two-point function with $\text{Re}[\Delta] = 3/2$ for both massless scalars and gravitons (Sec. VII). The flat-space relation (2) does not apply directly; d_U is determined by matching to established results [28,29].
- (v) *Predicted observable*: linear decoherence growth $\Gamma_{\text{decoh}} \propto Ht$, corresponding to $d_U = 2$ (Ohmic), consistent with the quantum-to-classical transition timescale for CMB modes exiting the horizon.

E. Why de Sitter is special

A key distinction from condensed matter applications is that de Sitter isometries enforce approximate dilation invariance even in the thermal (Bunch-Davies) state. The Gibbons-Hawking temperature $T_{\text{GH}} = H/2\pi$ gives $\beta_{\text{GH}} \sim H^{-1}$, identical to the fundamental dynamical timescale. Unlike condensed matter systems where thermal effects at $t \gg \beta$ break scale invariance, the de Sitter-invariant correlator maintains its power-law form at all timescales by symmetry, and the two-point function does not develop an exponential Boltzmann factor. Crucially, $\beta_{\text{GH}} \sim H^{-1}$ is much shorter than the inflationary decoherence timescale of ~ 50 – 60 e-folds, placing the cumulative decoherence firmly in the thermal regime $t \gg \beta_{\text{GH}}$, which justifies the use of the thermal-regime exponent $\gamma_{\text{decoh}} = 5 - 2d_U$ in matching to Eq. (62).

F. Predictions

a. Linear decoherence growth. The matching result $d_U = 2$ reproduces linear decoherence growth $\Gamma_{\text{decoh}}(t) \propto Ht$ [Eq. (62)], in agreement with the results of Kiefer and Polarski [28] on the quantum-to-classical transition during inflation and with the systematic open-EFT treatment of Colas *et al.* [29], from which the matching is performed.

b. Dissipation kernel. For $d_U = 2$: $\eta(t) \propto t^{-2}$, decaying slowly enough to produce non-negligible memory effects over times $t \lesssim H^{-1}$, but fast enough that $\int_0^\infty dt \eta(t)$ converges—consistent with the marginal Ohmic case.

c. Decoherence timescale. $\Gamma_{\text{decoh}} \sim 1$ when $t_{\text{decoh}} \sim (g^2 H)^{-1}$, or $N_{\text{decoh}} \sim g^{-2}$ e-folds. For observable CMB modes, which exit the horizon ~ 50 – 60 e-folds before the end of inflation, even weak couplings $g \sim 0.1$ ensure complete decoherence.

d. Massive bath and deviations from $d_U = 2$. For a massive bath with $m \sim H$, the decoherence functional is known to grow sublinearly in time [29], implying $\gamma_{\text{decoh}} < 1$ and hence $d_U > 2$ within the framework's phase structure. This would delay the quantum-to-classical transition, which could in turn affect the primordial power spectrum, although a detailed characterization of this effect lies beyond the scope of this work.

VIII. HIGH-ENERGY ASTROPHYSICAL NEUTRINOS

A. Neutrino oscillations and decoherence

Standard neutrino oscillations are described by the PMNS mixing matrix. For two-flavor mixing with angle θ , the vacuum survival probability is

$$P(\nu_e \rightarrow \nu_e; L) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \quad (64)$$

where Δm^2 is the difference in squared neutrino masses, E is the neutrino energy, and L the distance traveled from production to detection. Environmental coupling modifies this to

$$P(\nu_e \rightarrow \nu_e; L) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right) e^{-2\Gamma_{\text{decoh}}(L)}, \quad (65)$$

where $\Gamma_{\text{decoh}}(L)$ is the accumulated decoherence over baseline L .

B. The unparticle bath and the choice of regime

We first clarify what bath we have in mind. The scale-invariant environment relevant to neutrino propagation is not a Standard Model background. The CMB ($T \approx 2.725\text{K} \approx 0.235\text{ meV}$ [30]) and the cosmic neutrino background (predicted $T \approx 1.95\text{K} \approx 0.168\text{ meV}$ [31]) are SM thermal relics whose interactions with high-energy neutrinos are described by ordinary electroweak cross sections, not by a CFT bath; their effects are independent of the unparticle analysis. The bath in our framework is a hypothesized beyond-Standard-Model scale-invariant sector—a hidden CFT, an unparticle sector, or a gravitational sector if quantum gravity contributes scale-invariant

fluctuations—to which propagating neutrinos couple via a higher-dimension operator

$$\mathcal{L}_{\text{int}} \sim \frac{g}{M_*^n} \bar{\nu} \nu \mathcal{O}_U, \quad (66)$$

where n is fixed by requiring the operator to have dimension four; the specific value of n depends on the scaling dimension d_U of $\mathcal{O}_U$ and on the UV completion. The simplest case $n = 1$ (dimension-5 operator) is worked out explicitly in Sec. VIII F. The effective temperature T_U of this hidden sector is set by its own cosmological or local history and is, *a priori*, independent of any SM background temperature.

The system-bath-coupling structure for this application is as follows:

- (i) *System*: a propagating neutrino mass eigenstate, with the two-dimensional flavor subspace as the relevant Hilbert space.
- (ii) *Environment*: a hypothesized hidden scale-invariant sector (hidden CFT, unparticle sector, or gravitational) with scaling dimension d_U and effective temperature T_U .
- (iii) *Coupling operator*: $\mathcal{L}_{\text{int}} \sim (g/M_*^n) \bar{\nu} \nu \mathcal{O}_U$, where n is fixed by requiring the operator to have dimension four.
- (iv) *Bath correlator defining $J(\omega)$* : $G_R(\omega, \mathbf{k} = 0) \propto (-i\omega)^{2d_U-1}$, giving $J(\omega) \propto \omega^{2d_U-3}$.
- (v) *Predicted observable*: $\Gamma_{\text{decoh}} \propto \mathcal{B}(E, T_U) L^{5-2d_U}$ in the thermal regime, measurable at IceCube and next-generation neutrino telescopes via the baseline and energy dependence of the flavor ratio and survival probability [Eq. (65)].

a. The correct regime criterion. The vacuum-versus-thermal distinction is governed by the comparison of the neutrino propagation time t_{prop} to the thermal correlation time of the bath $\beta_U = 1/T_U$, as established in Sec. IX and Table VI. The relevant question is not whether $E \gg T_U$ but whether $t_{\text{prop}} \gg \beta_U$ or $t_{\text{prop}} \ll \beta_U$.

For IceCube astrophysical neutrinos, $t_{\text{prop}} \sim 10^{17}\text{ s}$, corresponding to a frequency scale $t_{\text{prop}}^{-1} \sim 10^{-32}\text{ eV}$. The thermal regime therefore applies whenever $T_U \gg 10^{-32}\text{ eV}$ —a condition satisfied by any hidden sector that has ever been in thermal equilibrium, regardless of how cold it is today. The vacuum regime, by contrast, requires $T_U \ll 10^{-32}\text{ eV}$, which is an extraordinarily cold hidden sector with no natural cosmological motivation. The thermal regime is therefore the generic expectation for astrophysical neutrinos propagating over cosmological baselines; the vacuum regime is the special case.

This is the opposite of the naïve expectation based on comparing $E \sim \text{TeV}$ to T_U : the neutrino energy is irrelevant to the regime criterion because it sets the frequency of the bath modes being sampled, not the timescale over which the bath correlation function is integrated. A high-energy neutrino propagating for 10^{17} s accumulates decoherence

from many thermal bath fluctuations even if each individual interaction samples modes at $\omega \sim E \gg T_U$; what matters is how many correlation times β_U fit within t_{prop} .

b. Thermal regime ($t_{\text{prop}} \gg \beta_U$). This is the generic case for IceCube baselines. The decoherence exponent is $\gamma_{\text{decoh}} = 5 - 2d_U$, with coherence-protection threshold at $d_U = 5/2$. The decoherence functional acquires a prefactor $\propto T_U$. The constraint from $\Delta N_{\text{eff}} \lesssim 0.3$ [32,33] bounds T_U from above for a hidden sector that thermalized before BBN, but imposes no lower bound; T_U can be arbitrarily small and the thermal regime still applies provided $T_U \gg t_{\text{prop}}^{-1}$.

c. Vacuum regime ($t_{\text{prop}} \ll \beta_U$). This applies only for an extremely cold hidden sector with $T_U \ll 10^{-32}$ eV. The decoherence exponent shifts to $\gamma_{\text{decoh}} = 4 - 2d_U$, with coherence-protection threshold at $d_U = 2$. There is no T_U prefactor; the decoherence is driven purely by vacuum fluctuations of the bath. For point sources at known distances, the vacuum regime could in principle apply to short-baseline experiments where $t_{\text{prop}} \sim \beta_U$, but for cosmological baselines it requires implausibly cold new physics.

Since T_U is unknown, both regimes are physically open and we develop predictions for each. The two regimes predict γ_{decoh} values differing by unity for the same d_U , so a sufficiently precise measurement of the baseline exponent can identify the operative regime independently of d_U itself. The unparticle signal is isolated from SM backgrounds not by its magnitude but by its distinctive baseline dependence $\propto L^{\gamma_{\text{decoh}}}$ and energy scaling through $\mathcal{A}(E)$, neither of which has an SM analog.

C. Coupling and decoherence rate

For a neutrino system coupled to a scale-invariant sector with dimension d_U via a dephasing operator $A_\nu = \sigma_z$ in the flavor basis, the master equation is Eq. (25). The off-diagonal density matrix element evolves as

$$\rho_{e\mu}(L) = \rho_{e\mu}(0) e^{-i\Delta m^2 L/4E} e^{-\Gamma_{\text{decoh}}(L,E)}. \quad (67)$$

As established in Sec. VIII, the thermal regime ($t_{\text{prop}} \gg \beta_U$) is the generic expectation for cosmological baselines. We therefore lead with the thermal-regime result and treat the vacuum regime as the special case.

a. Thermal regime ($t_{\text{prop}} \gg \beta_U$). The noise kernel is $\nu(t) \propto T_U t^{-(2d_U-3)}$ and the decoherence functional is

$$\Gamma_{\text{decoh}}(L, E) = \mathcal{B}(E, T_U) L^{5-2d_U}, \quad (68)$$

where the prefactor $\mathcal{B}$ carries an explicit factor of T_U and has mass dimension $[\mathcal{B}] = [\text{energy}]^{5-2d_U}$. The baseline exponent $5 - 2d_U$ is universal, independent of the UV completion; the coherence-protection threshold sits

at $d_U = 5/2$. The corresponding rate per unit baseline length,

$$\frac{d\Gamma_{\text{decoh}}}{dL} = (5 - 2d_U) \mathcal{B}(E, T_U) L^{4-2d_U}, \quad (69)$$

is explicitly L dependent—a non-Markovian feature inaccessible to a Lindblad treatment.

b. Vacuum regime ($t_{\text{prop}} \ll \beta_U$). For an extremely cold hidden sector, the noise kernel is $\nu(t) \propto t^{-(2d_U-2)}$ and

$$\Gamma_{\text{decoh}}(L, E) = \mathcal{A}(E) L^{4-2d_U}, \quad (70)$$

with no T_U prefactor. The baseline exponent shifts by -1 relative to the thermal regime, and the coherence-protection threshold shifts to $d_U = 2$.

c. General parametrization of the prefactor. In both regimes the prefactor has mass dimension fixed by requiring Γ_{decoh} to be dimensionless. We parametrize

$$\mathcal{A}(E) \sim \frac{g^2}{M_*^m} E^n, \quad n - m = 4 - 2d_U, \quad (71)$$

$$\mathcal{B}(E, T_U) \sim \frac{g^2 T_U}{M_*^m} E^{n'}, \quad n' - m = 4 - 2d_U, \quad (72)$$

with g dimensionless and M_* the scale of the underlying physics. The dimensional constraints leave one combination of (n, m) or (n', m) free, to be fixed by the UV completion or extracted from data alongside d_U .

d. Concrete example: Dimension-5 operator. For the dimension-5 operator $\mathcal{L}_{\text{int}} \sim (g/M_*) \bar{\nu} \nu \mathcal{O}_U$, computing the prefactors from the two-point function gives,

$$\mathcal{A}(E) \sim \frac{g^2}{M_*^2} E^{6-2d_U}, \quad (73)$$

$$\mathcal{B}(E, T_U) \sim \frac{g^2 T_U}{M_*^2} E^{6-2d_U}. \quad (74)$$

The combined experimental signatures are therefore

$$\Gamma_{\text{decoh}} \sim \begin{cases} \frac{g^2 T_U}{M_*^2} E^{6-2d_U} L^{5-2d_U} & (\text{thermal}), \\ \frac{g^2}{M_*^2} E^{6-2d_U} L^{4-2d_U} & (\text{vacuum}). \end{cases} \quad (75)$$

In the thermal regime, the additional factor of T_U/E relative to the vacuum case reflects the enhanced bath occupation at finite temperature. Both regimes produce independently measurable baseline and energy exponents, providing two handles on d_U in each case.

D. Comparison with standard Lindblad treatment

The standard phenomenological treatment employs a Lindblad master equation with constant rate Γ_0 , giving

TABLE V. Comparison of Lindblad and unparticle bath approaches to neutrino decoherence. Unparticle exponents are given for the thermal regime ($t_{\text{prop}} \gg \beta_U$), which is the generic expectation for cosmological baselines; vacuum-regime values are given in parentheses.

Property	Lindblad	Unparticle bath
Coherence decay	$e^{-\Gamma_0 L}$	$\exp(-\mathcal{B} L^{\gamma_{\text{decoh}}})$
Exponent γ_{decoh}	1 (fixed)	$5 - 2d_U$ ($4 - 2d_U$ in vacuum)
Rate	Constant	$\propto L^{\gamma_{\text{decoh}} - 1}$
Energy scaling	Ad hoc	via $\mathcal{B}(E, T_U)$, UV-dependent
Long-baseline	Always $\rightarrow 0$	Protected if $d_U > 5/2$ (> 2 in vacuum)
Memory	None (Markovian)	Power-law kernel

exponential suppression $e^{-\Gamma_0 L}$. Table V summarizes the qualitative differences.

E. Five dynamical regimes

The five regimes as a function of d_U give qualitatively distinct oscillation patterns in Eq. (65). We quote the thermal-regime exponent $\gamma_{\text{decoh}} = 5 - 2d_U$, which is the generic expectation for cosmological baselines; the vacuum-regime exponent $\gamma_{\text{decoh}} = 4 - 2d_U$ is obtained by shifting all thresholds down by $1/2$ in d_U , as noted in parentheses.

a. $d_U < 3/2$ (*deep sub-Ohmic*). $\gamma_{\text{decoh}} > 2$: decoherence accelerates faster than quadratic, coherence lost rapidly.

b. $d_U = 3/2$ (*critical sub-Ohmic*). $\gamma_{\text{decoh}} = 2$: quadratic decoherence growth, $\Gamma_{\text{decoh}} \propto L^2$.

c. $3/2 < d_U < 5/2$ (*sub-to super-Ohmic*). $0 < \gamma_{\text{decoh}} < 2$: decoherence grows as a power law with exponent between zero and two; coherence is lost at long baselines, more slowly for larger d_U .

d. $d_U = 5/2$ (*decoherence critical, thermal*); $d_U = 2$ (*decoherence critical, vacuum*). $\gamma_{\text{decoh}} = 0$: marginal case; the power-law formula gives L^0 and the actual growth is logarithmic, $\Gamma \sim \mathcal{B} \ln(L/L_0)$. Coherence survives as a power law: $|\rho_{e\mu}(L)| \propto L^{-C}$.

e. $d_U > 5/2$ (*super-Ohmic, thermal*); $d_U > 2$ (*super-Ohmic, vacuum*). $\gamma_{\text{decoh}} < 0$: coherence is *protected* at long distances. Oscillations survive at arbitrarily large baselines, a signature impossible in any Lindblad treatment.

F. Experimental strategy

The primary observable is the joint baseline and energy dependence of the oscillation suppression. Using the dimension-5 operator result in the thermal regime [Eq. (75), upper case] as the default parametrization, the survival probability takes the form

$$P(\nu_e \rightarrow \nu_e; L) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \times \exp\left[-C \frac{g^2 T_U}{M_*^2} E^{6-2d_U} L^{5-2d_U}\right], \quad (76)$$

with $Cg^2 T_U/M_*^2$ and d_U as free parameters. For the vacuum regime [Eq. (75), lower case], the E exponent remains the same, but the L exponent shifts to L^{4-2d_U} , and there is no T_U prefactor. In both cases the baseline exponent γ_{decoh} is the primary observable; the energy exponent and the presence or absence of a T_U prefactor provide additional handles.

Test 1: Baseline exponent γ_{decoh} . Lindblad predicts $\gamma_{\text{decoh}} = 1$. In the thermal regime, the unparticle framework predicts $\gamma_{\text{decoh}} = 5 - 2d_U$; measuring $\gamma_{\text{decoh}} \neq 1$ with statistical significance rules out the standard Markovian description. The coherence-protection threshold at $d_U = 5/2$ ($\gamma_{\text{decoh}} = 0$) is a qualitative target: survival of oscillations at large baselines with no baseline suppression would be a striking signature.

Test 2: Energy dependence. At fixed L , the suppression scales with energy through $\mathcal{B}(E, T_U)$. For the dimension-5 operator in the thermal regime this gives E^{6-2d_U} ; the energy slope constrains d_U independently of the baseline measurement. IceCube's wide energy range (TeV to PeV) provides a lever arm of three orders of magnitude for this test. A dedicated analysis is in preparation.

Test 3: Regime identification. The thermal and vacuum regimes predict baseline exponents $\gamma_{\text{decoh}} = 5 - 2d_U$ and $4 - 2d_U$ respectively, differing by unity for the same d_U . However, they predict the same energy exponent: E^{6-2d_U} . Jointly fitting both the baseline and energy dependence therefore determines d_U and identifies the operative regime simultaneously, without prior knowledge of T_U . A measurement preferring $\gamma_{\text{decoh}} = 5 - 2d_U$ over $4 - 2d_U$ would confirm the thermal regime and provide an indirect constraint on T_U through the overall amplitude.

IX. REGIME OF VALIDITY

The unparticle framework applies wherever scale invariance holds. Here we establish the relevant windows for each physical system. For free-field, weakly interacting, and engineered power-law baths, thermal corrections modify amplitudes but not universal exponents; for strongly interacting CFT baths the power-law predictions hold as intermediate-time scaling in the window $1/\omega_{\text{UV}} \ll t \ll 1/T$.

TABLE VI. Regime of validity of the unparticle description for each physical system.

System	T	β	t_{dyn}	Regime
Cond. mat. (warm)	100 K	10^{-13} s	$\gg \beta$	Thermal
Cond. mat. (cold)	10 mK	10^{-9} s	$\sim \beta$	Crossover
Inflation	$H/2\pi$	H^{-1}	H^{-1}	de Sitter (exact)
High- E neutrinos	$T_U \gg t_{\text{prop}}^{-1}$	β_U	10^{17} s	Thermal (generic)

A. The two regimes and their exponents

From Eq. (34), the noise kernel has two power-law regimes,

$$\nu(t) \propto \begin{cases} t^{-(2d_U-2)} & t \ll \beta \text{ (vacuum)}, \\ T \cdot t^{-(2d_U-3)} & t \gg \beta \text{ (thermal)}, \end{cases} \quad (77)$$

with crossover at $t \sim \beta = 1/T$. The decoherence exponent accordingly shifts: $\gamma_{\text{decoh}} = 4 - 2d_U$ in the vacuum regime and $5 - 2d_U$ in the thermal regime. In both cases d_U is the same parameter; only the exponent and prefactor change. Scale invariance is broken in amplitude at the crossover $t \sim \beta$, but the power-law structure is preserved on both sides.

B. Condensed matter

At $T \sim 100$ K, $\beta \sim 10^{-13}$ s. Standard decoherence experiments operate at nanosecond to microsecond timescales, deep in the thermal regime. The framework applies with the thermal-regime exponent $\gamma_{\text{decoh}} = 5 - 2d_U$ and prefactor $\propto T$.

Crossover test. At millikelvin temperatures, observe the transition from $\gamma_{\text{decoh}} = 4 - 2d_U$ (vacuum regime, short times) to $\gamma_{\text{decoh}} = 5 - 2d_U$ (thermal regime, long times) at $t \sim \beta$ (Sec. IX). For the (1 + 1)D energy-operator coupling, $d_U = 3/2$, this predicts a crossover from $\gamma_{\text{decoh}} = 1$ to $\gamma_{\text{decoh}} = 2$ at $t \sim \beta$.

C. Inflation

As discussed in Sec. VII, de Sitter isometries enforce power-law correlators at all timescales. The Gibbons-Hawking temperature $T_{\text{GH}} = H/2\pi$ gives $\beta_{\text{GH}} \sim H^{-1}$, identical to the dynamical timescale. There is no separation between vacuum and thermal regimes; instead, de Sitter symmetry guarantees that the correlator maintains its power-law form throughout. Inflation is thus the application where the framework's predictions most cleanly align with established results, although the matching to those results, rather than a direct derivation from the universality theorem, is what fixes $d_U = 2$ in this case.

D. High-energy neutrinos

As established in Sec. VIII, the thermal regime is the generic expectation for IceCube baselines, since

$t_{\text{prop}} \sim 10^{17}$ s $\gg \beta_U$ for any cosmologically motivated hidden sector. The thermal-regime exponent $\gamma_{\text{decoh}} = 5 - 2d_U$ applies, with the vacuum regime ($\gamma_{\text{decoh}} = 4 - 2d_U$) reserved for the special case of an extremely cold hidden sector with $T_U \ll 10^{-32}$ eV.

E. Regime of validity for interacting CFT baths

The framework uses the vacuum spectral function $J(\omega) \propto \omega^{2d_U-3}$ as a temperature-independent model input, following the spin-boson tradition [9]. This is exact for free-field baths and a good approximation for weakly interacting ones, where thermal corrections to the retarded Green's function at $\omega \lesssim T$ are suppressed. For strongly interacting CFTs, the exact thermal spectral function $J_T(\omega)$ differs from the vacuum form at low frequencies: in (1 + 1) D, for example, $J_T(\omega) \propto \omega T^{2\Delta-2}$ at $\omega \ll T$, giving a noise spectrum $S(\omega) \rightarrow 2T^{2\Delta-1}$ as $\omega \rightarrow 0$ rather than the power-law divergence of the vacuum approximation. The Fourier transform of this analytic noise spectrum decays exponentially in real time, giving Markovian decoherence $\Gamma_{\text{decoh}} \propto t$ at $t \gg 1/T$, independently of d_U . The power-law predictions of Table I are therefore valid in the window $1/\omega_{\text{UV}} \ll t \ll 1/T$ for strongly interacting CFT baths; the Markovian limit is recovered at $t \gg 1/T$. For the (1 + 1)D Ising model application specifically, trapped-ion experiments probing times $t \gg 1/T$ will observe Markovian behavior; the predicted exponent $\gamma_{\text{decoh}} = 2$ is accessible in the intermediate window. For the inflation and neutrino applications, where the bath is either free-field-like or weakly coupled, this limitation does not apply.

X. EXPERIMENTAL ROADMAP

A. Trapped-ion quantum simulators

Trapped-ion systems can simulate the quantum Ising model with tunable parameters and provide a probe spin weakly coupled to the critical chain.

Protocol. (1) Tune the transverse field to $h = h_c$ by monitoring the vanishing of the energy gap. (2) Prepare the probe in $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$. (3) Measure $|\rho_{\uparrow\downarrow}(t)|$ at a series of times using Ramsey interferometry or spin-echo sequences. (4) Extract the decoherence exponent γ_{decoh} by fitting $\ln(-\ln|\rho_{\uparrow\downarrow}|)$ vs $\ln t$.

Primary prediction. $\gamma_{\text{decoh}} = 2$ for (1 + 1)D coupling to ε .

Consistency test. Independently measure the noise spectrum $S(\omega)$ and extract s . Verify $s + \gamma_{\text{decoh}} = 2$.

Crossover test. At millikelvin temperatures, observe the transition from $\gamma_{\text{decoh}} = 4 - 2d_U$ (vacuum regime, short times) to $\gamma_{\text{decoh}} = 5 - 2d_U$ (thermal regime, long times) at $t \sim \beta$.

Direct test of the coherence-protection transition. The decoherence phase transition at $d_U = 5/2$ (equivalently $s = 2$) can be tested directly by *preparing* the spectral exponent s *a priori*, rather than fitting it. Extending the engineered spin-boson platform of Sun *et al.* by tuning the mode-coupling profile to target $s = 2.0$, $s = 2.5$, and $s = 3.0$, one can measure the Ramsey coherence envelope $|\rho_{\uparrow\downarrow}(t)|$ at each value and test whether it decays or saturates at long times. Below the transition ($s < 2$, $d_U < 5/2$), the coherence envelope should decay to zero; above it ($s > 2$, $d_U > 5/2$), it should saturate to a finite value $|\rho_{\uparrow\downarrow}(\infty)| > 0$ [Eq. (79)]. This experiment is qualitatively different from the exponent-fitting tests described above: the spectral exponent is a controlled input rather than a fit parameter, so a saturation observation would be direct evidence for the coherence-protected phase predicted by the universality theorem, inaccessible to any Markovian description.

B. Neutrino telescopes

A dedicated analysis of IceCube data within the unparticle framework is in preparation and will be described in a future publication. The primary observables are:

Baseline and energy dependence. Fit the joint baseline and energy dependence of the oscillation suppression in the parametrization of Eq. (75). In the thermal regime (generic for cosmological baselines), the signature is $\Gamma_{\text{decoh}} \propto T_U E^{6-2d_U} L^{5-2d_U}$; in the vacuum regime it is $\Gamma_{\text{decoh}} \propto E^{6-2d_U} L^{4-2d_U}$. The baseline exponent γ_{decoh} is the primary handle on d_U ; the energy exponent and the overall amplitude carry additional information, including T_U and g^2/M_*^2 .

Regime identification. Since the thermal and vacuum regimes predict baseline exponents differing by unity but the same energy exponent, E^{6-2d_U} , a joint fit to baseline and energy dependence identifies the operative regime alongside d_U , without prior knowledge of T_U .

Coherence protection signature. For $d_U > 5/2$ (thermal regime) or $d_U > 2$ (vacuum regime), the oscillation pattern survives at arbitrarily large baselines. A nonobservation of complete coherence loss at Gpc baselines would be a qualitative signature of coherence protection.

Future experiments IceCube-Gen2 and KM3NeT will extend the accessible energy range and source statistics, improving sensitivity to d_U by roughly an order of magnitude.

C. Superconducting qubits and engineered coherence protection

a. 1/f noise in superconducting qubits. The dominant decoherence source in superconducting

qubits—transmons, flux qubits, and charge qubits—is $1/f$ charge and flux noise [34,35], characterized by a noise power spectrum $S(\omega) \propto 1/\omega$. In the high-temperature limit, $S(\omega)$ and the bath spectral density $J(\omega)$ are related by $S(\omega) \propto TJ(\omega)/\omega$ [36], so $S(\omega) \propto \omega^{-1}$ corresponds to $J(\omega) \propto \omega^0$, i.e., $s = 0$ and $d_U = 3/2$. This is precisely the unparticle dimension derived for the $(1+1)$ -dimensional quantum Ising chain coupled to the energy operator (Sec. VI), providing a field-theoretic interpretation of $1/f$ noise in superconducting qubits as the signature of a sub-Ohmic bath at the critical sub-Ohmic point.

From Table I, $d_U = 3/2$ gives a decoherence exponent

$$\gamma_{\text{decoh}} = 5 - 2d_U = 2, \quad (78)$$

predicting quadratic growth $\Gamma_{\text{decoh}}(t) \propto g^2 T t^2$. This is consistent with existing measurements of qubit decoherence under $1/f$ noise [37], and stands in contrast to the linear growth ($\gamma_{\text{decoh}} = 1$) assumed in standard Lindblad treatments. The consistency relation $s + \gamma_{\text{decoh}} = 2$ with $s = 0$ fixes $\gamma_{\text{decoh}} = 2$ as a falsifiable prediction, extractable by fitting the Ramsey coherence envelope $|\rho_{\uparrow\downarrow}(t)|$ to a stretched exponential $\exp(-Ct^{\gamma_{\text{decoh}}})$ —a minimal modification of the standard T_2 measurement.

b. Bath-engineered coherence protection. The decoherence phase transition at $d_U = 5/2$ (Sec. III) suggests a qualitative alternative to active quantum error correction: engineering the bath spectral density to suppress decoherence passively. For $d_U > 5/2$, the decoherence functional acquires a negative exponent and *saturates* at long times, so that

$$\lim_{t \rightarrow \infty} |\rho_{\uparrow\downarrow}(t)| = |\rho_{\uparrow\downarrow}(0)| e^{-\Gamma_{\text{decoh}}(\infty)}, \quad \Gamma_{\text{decoh}}(\infty) < \infty. \quad (79)$$

Coherence is protected by the rapid phase averaging of high-frequency bath modes, which decouple from the system at late times—a mechanism absent from any Markovian description and equivalent to the result of Leggett *et al.* [9] in the spin-boson model, here derived from conformal symmetry. The tunable parameter is the spectral exponent s of the electromagnetic or phononic environment seen by the qubit: engineering $s > 2$ through structured cavities, bandgap materials, or coupled resonator arrays places the qubit in the coherence-protected phase. This engineering approach produces a bath of the power-law type by construction, placing it firmly in the regime where the coherence-protection transition is a genuine long-time property. The saturation (79) implies that idle qubit storage becomes asymptotically immune to bath-induced decoherence, relaxing the requirements on quantum memory in architectures where idle time between gate operations is a significant error source.

XI. SUMMARY AND CONCLUSIONS

We have developed the complete theoretical framework for open quantum systems coupled to scale-invariant environments, establishing the following results:

The uniqueness theorem (Sec. II) proves that under locality, Lorentz invariance, unitarity, and continuous scale invariance, the environment is necessarily equivalent to an unparticle bath. Five classes of loophole are identified and treated. The contrapositive provides a falsifiability criterion: inconsistent exponents signal a breakdown of scale invariance.

The complete mathematical framework (Sec. III) provides exact expressions for all memory kernels including the full vacuum-plus-thermal decomposition via Matsubara summation, the fractional Caldeira-Leggett equation valid for arbitrary d_U , and the phase diagram with three critical dimensions ($3/2$, 2 , $5/2$).

The Caldeira-Leggett model (Sec. IV) is recovered as a special case at $d_U = 2$ in the Markovian limit. Lindblad master equations are incapable of describing the decoherence phase transition at $d_U = 5/2$.

Experimental validation (Sec. V) demonstrates that d_U has nontrivial resolution across universality classes. Two-channel fits to resistivity and specific heat data for YbRh_2Si_2 [22] and $\text{CeCu}_{5.9}\text{Au}_{0.1}$ [23] at their respective quantum critical points yield $d_U = 3/2$ independently from both observables, consistent with the marginal prediction $C/T \propto -\ln T$ of Eq. (47). Together with the unitary Fermi gas result ($d_U = 7/4$) from [2], two distinct values of d_U are now measured across three physically unrelated many-body systems with no shared fitting parameters.

We note that the heavy-fermion validation is empirical rather than a direct application of Theorem 1: the theorem derives d_U from a known CFT operator dimension, whereas here $d_U = 3/2$ is inferred from transport data. The consistency of two independent channels at the same d_U value is evidence for the bath-operator interpretation of d_U —namely, that resistivity and specific heat probe the same underlying bath—but does not constitute a proof that the bath is a CFT in the sense of Theorem 1. Identifying the relevant CFT at the heavy-fermion quantum critical point remains an open problem.

Quantum Ising criticality (Sec. VI) yields $d_U = 3/2$ for the $(1+1)\text{D}$ chain coupled to the energy operator, providing a field-theoretic derivation of $1/f$ noise. For the $(2+1)\text{D}$ model, the conformal bootstrap value [3] $\Delta_\epsilon \approx 1.4126$ gives $d_U \approx 1.413$ and $\gamma_{\text{decoh}} \approx 2.17$, close to but distinct from the $1/f$ case.

Inflationary cosmology (Sec. VII) gives $d_U = 2$ for massless scalar and graviton baths by matching to

established results on inflationary decoherence, predicting linear decoherence growth $\Gamma \propto Ht$ in agreement with the quantum-to-classical transition literature.

High-energy neutrinos (Sec. VIII) generically access the thermal regime of the unparticle bath owing to their cosmological propagation times, and exhibit energy- and baseline-dependent decoherence $\Gamma_{\text{decoh}} \propto \mathcal{B}(E, T_U)L^{5-2d_U}$ (thermal regime), qualitatively distinct from Lindblad predictions. A dedicated IceCube analysis is in preparation.

Regime of validity (Sec. IX) establishes that the phase diagram and universal exponents hold for free-field, weakly interacting, and engineered power-law baths, and as intermediate-time scaling behavior for strongly interacting CFT baths in the window $1/\omega_{\text{UV}} \ll t \ll 1/T$. Inflation and high-energy neutrinos are identified as the cleanest applications, unaffected by this limitation.

Experimental roadmap (Sec. X) concretely shows how the framework can be applied in the future in various fields.

The primary experimental handles are the consistency relations $s + \gamma_{\text{decoh}} = 2$, $\alpha_\eta + \delta_{\text{decoh}} = 2$, and $\alpha_\nu + \beta_{\text{damp}} = 0$. These algebraic identities, not fitting parameters, provide multiobservable tests of the scale-invariance hypothesis across all platforms.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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