

Search for neutral triple gauge couplings with ZZ production at future electron-positron colliders

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This study investigates neutral triple gauge couplings (nTGCs) through ZZ production at future electron-positron colliders. The impact of beam polarization on the cross section is analyzed. We compare the signals and backgrounds for five different ZZ decay channels and present our event selection strategies for future e^+e^- colliders. The expected coefficient constraints for all decay channels are provided, and final expected constraints are derived by combining results from the different decay patterns. Our analysis indicates that future electron-positron colliders will have significantly enhanced detection capabilities for nTGCs compared to the current LHC experiments, with expected improvements in constraints by 1 to 2 orders of magnitude.

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I. INTRODUCTION

In the absence of clear signals of new physics (NP) beyond the standard model (BSM) from the Large Hadron Collider (LHC) experiments so far, the indirect detection of signs of NP through precise measurements of standard model (SM) processes has gained more attention. When the mass of a new particle is significantly larger than the electroweak symmetry breaking scale, the standard model effective field theory (SMEFT) provides a systematic parametrization of NP effects [1–4], with the advantage of not relying on any particular model. The Lagrangian of SMEFT consists of operators of dimension less than or equal to four, representing the SM itself, as well as

higher-dimensional operators constructed from SM fields. The Lagrangian of SMEFT then takes the form

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{d=5}^{\infty} \sum_j \frac{C_j^{(d)}}{\Lambda^{d-4}} \mathcal{O}_j^{(d)},$$

where $\mathcal{O}^{(d)}$ is an operator of dimension d invariant under the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group of the SM, Λ represents the mass scale of new particles, with the power of Λ in each term determined by dimensional analysis, and $C_j^{(d)}$ is the Wilson coefficient of the operator. For any NP model that satisfies the above assumptions, integrating out the new fields will produce a Lagrangian with this structure.

Depending on the gauge symmetry, triple gauge couplings (TGCs) and quartic gauge couplings (QGCs) exist in the SM. Measurements of these couplings not only test the nature of spontaneous electroweak symmetry breaking (EWSB) but also provide a unique window for searching for NP. Neutral triple gauge couplings (nTGCs) [5–8], gluonic quadratic gauge coupling (gQGC) [9], and anomalous quartic gauge coupling (aQGC), which is not related to

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anomalous TGC (aTGC) at dimension-6, all arise first in gauge-invariant dimension-8 operators. Any sign of their detection would provide direct *prima facie* evidence for NP. Additionally, it has been shown that dimension-8 operators play a crucial role from the convex geometry perspective within the SMEFT space [10]. Many researchers have emphasized the importance of dimension-8 operators [11–18].

In general, the contribution of dimension-6 operators to NP is larger than that of dimension-8 or higher operators. Consequently, there are fewer experimental probes and constraint studies on dimension-8 SMEFT interactions. However, in some cases, dimension-8 operators can make leading contributions to NP processes. Examples include dimension-8 operators contributing to light-by-light scattering [19], vector boson scattering [20–28], di-boson productions [9,29–38], and tri-boson productions [39–42]. Many experimental measurements have been made at the LHC [43–57].

At dimension-8, there are four CP -conserving operators contributing to nTGCs, they are [5,7,30]

$$\begin{aligned}\mathcal{L}_{\text{nTGC}} = & \frac{c_{\tilde{B}W}}{\Lambda^4} \mathcal{O}_{\tilde{B}W} + \frac{c_{B\tilde{W}}}{\Lambda^4} \mathcal{O}_{B\tilde{W}} \\ & + \frac{c_{W\tilde{W}}}{\Lambda^4} \mathcal{O}_{W\tilde{W}} + \frac{c_{\tilde{B}\tilde{B}}}{\Lambda^4} \mathcal{O}_{\tilde{B}\tilde{B}},\end{aligned}\quad (1)$$

with

$$\begin{aligned}\mathcal{O}_{\tilde{B}W} &= iH^+ \tilde{B}_{\mu\nu} W^{\mu\rho} \{D_\rho, D^\nu\} H + \text{H.c.}, \\ \mathcal{O}_{B\tilde{W}} &= iH^+ B_{\mu\nu} \tilde{W}^{\mu\rho} \{D_\rho, D^\nu\} H + \text{H.c.}, \\ \mathcal{O}_{W\tilde{W}} &= iH^+ \tilde{W}_{\mu\nu} W^{\mu\rho} \{D_\rho, D^\nu\} H + \text{H.c.}, \\ \mathcal{O}_{\tilde{B}\tilde{B}} &= iH^+ \tilde{B}_{\mu\nu} \tilde{B}^{\mu\rho} \{D_\rho, D^\nu\} H + \text{H.c.},\end{aligned}\quad (2)$$

where $\tilde{B}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta} B^{\alpha\beta}$, $\tilde{W}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta} W^{\alpha\beta}$, and $W_{\mu\nu} \equiv W_{\mu\nu}^a \sigma^a / 2$, where σ^a are Pauli matrices. For simplicity, we define $f_i \equiv (c_i)/\Lambda^4$ with the corresponding dimensionless coupling coefficient c_i and the cutoff scale Λ . Recent studies [32,33] have presented a comprehensive basis of dimension-8 CP -conserving and CP -violating operators contributing to nTGCs. These operators are categorized as $F^2 \phi^2 D^2$ and $F^3 D^2$, with the $F^2 H^2 D^2$ corresponding to the operator presented in Eq. (2). The $F^2 \phi^2 D^2$ and $F^3 D^2$ terms have a different structure and are not covered by the calculations in this work. The vertices are listed in the Appendix.

While dimension-6 operators do not contribute to the nTGCs at the tree level, they can arise at the one-loop level. A comparison of the contributions to the cross section shows that the one-loop dimension-6 operators contribute at the order of $\mathcal{O}(\alpha_{\text{EM}} s / 4\pi\Lambda^2)$, whereas the tree level dimension-8 operators contribute at the order of $\mathcal{O}(s v^2 / \Lambda^4)$. Consequently, within the limit of $\Lambda < \sqrt{4\pi/\alpha_{\text{EM}}} v \sim 10$ TeV, the contribution from the tree level dimension-8 nTGCs to the production cross section

significantly surpasses that of the one-loop dimension-6 operators.

On the theoretical side, anomalous tensorial structures can be generated within a renormalizable theory at higher-order loop levels. Examples include the fermion triangle diagram in the SM, as well as studies in BSM models such as the minimal supersymmetric standard model [58–60] and the little Higgs model [61]. Furthermore, a noncommutative extension of the SM [62,63] can also provide anomalous coupling structures in the neutral sector, potentially leading to a trilinear coupling of the form $\gamma\gamma\gamma$ [62]. Recent work has demonstrated the emergence of dimension-8 nTGC operators in scenarios involving extensions of the SM with heavy fermions [64,65]. Additionally, deviations in the measured Z -pair cross section from SM predictions can manifest as effective couplings of the form ZZZ and $ZZ\gamma$, as explored in the context of two-Higgs-doublet models [66,67] and low-scale gravity theories [68]. Some work has shown that nTGCs can be faked by NP via the process $pp \rightarrow ZZ \rightarrow 4\ell$ [69]. A recent study identified the dimension-8 nTGC operators contributing to the pure triple Z boson coupling on a comprehensive basis via ZZ production at high-energy e^+e^- colliders [70].

Future e^+e^- colliders have great physics potential and provide one of the rare direct windows to NP at dimension-8. Previous studies have thoroughly studied probing nTGCs by measuring $Z\gamma$ production at future e^+e^- colliders [30,31,34–37,71–73], pp colliders [32,33,74], and ep colliders [75]. In this work, we study how nTGCs can be probed by measuring the $e^+e^- \rightarrow ZZ$ process with decays at different versions of circular and linear e^+e^- colliders, including the circular electron-positron collider (CEPC) [76,77], future circular collider-electron-positron (FCC-ee) [78], international linear collider (ILC) [79], and compact linear collider (CLIC) [80]. At large energies, the validity of the dimension-8 operators is usually a noteworthy issue and can be examined by using the partial wave unitarity bounds. However, based on the results of Ref. [35], the amplitude of $e^+e^- \rightarrow VV$ is suppressed by a s -channel propagator. Consequently, the constraints from the partial wave unitarity bounds are moderate and can be safely neglected at typical high luminosities of future lepton colliders.

We note that additional dimension-8 fermionic operators exist that contribute to the $e^+e^- \rightarrow ZZ$ process through contact interactions $ZZf\bar{f}$. For this study of nTGCs at future colliders, we focus on the dimension-8 operator [Eq. (2)], as it directly contributes to the nTGCs, unlike the fermionic contact operator. In addition, within the SMEFT formulation for the current study, we follow the common approach of analyzing one operator at a time, and neglecting possible correlations between different dimension-8 operators, which might arise in specific ultraviolet models [30].

The plan of this paper is as follows. In Sec. II, we discuss the beam polarization effects on the cross section of ZZ

production at e^+e^- colliders. In Sec. III, we compare the signals and backgrounds of five ZZ decay channels and present our event selection strategy for e^+e^- colliders with different energies and luminosities. The expected constraints on the coefficients with combined results are also presented in Sec. III. Finally, we summarize our conclusions in Sec. IV. We compare the results obtained with experimental measurements of nTGCs from the LHC [81]. Depending on the capabilities of different future lepton colliders to detect nTGCs, their sensitivity could exceed that of the current LHC. The $e^+e^- \rightarrow ZZ$ process will thus provide a unique and promising channel for the detection of NP at future lepton colliders.

II. ZZ PRODUCTION AT e^+e^- COLLIDERS

At future electron-positron colliders, the di-boson pair production can be an interesting topic and ideal motivation [82–84]. In this section, we discuss the cross section of the $e^+e^- \rightarrow ZZ$ process induced by nTGCs and the contribution of the SM to this process. We then analyze the influence of initial electron polarization on the cross section.

The Feynman diagrams of the SM and nTGC contributions to ZZ production at e^+e^- colliders are shown in Fig. 1. In the SM, the $e^+e^- \rightarrow ZZ$ process is primarily governed by the electron-mediated t - and u -channel exchange diagrams at tree level. The BSM contribution due to dimension-8 operators manifests in the s -channel e^+e^- scattering, involving the exchange of either a photon or Z boson. Anomalous ZZZ and $ZZ\gamma$ couplings are allowed by electromagnetic gauge invariance and Lorentz invariance for on-shell Z or γ bosons. In this context, only one independent operator exists, as $\mathcal{O}_{B\bar{W}}$ is equivalent to $\mathcal{O}_{\bar{B}W}$, and $\mathcal{O}_{\bar{W}W}$ and $\mathcal{O}_{\bar{B}B}$ do not contribute [7]. Specifically, while $\mathcal{O}_{B\bar{W}}$ generates a $ZZ\gamma^*$ vertex it does not

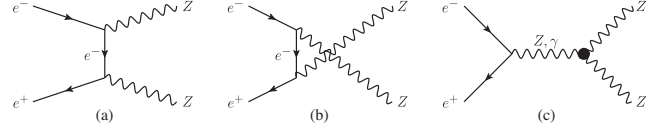


FIG. 1. Feynman diagrams of the $e^+e^- \rightarrow ZZ$ production. Diagrams (a) and (b) are the SM contributions. Diagram (c) corresponds to contributions from the nTGCs.

contribute to the ZZZ vertex [65]. Consequently, the $\mathcal{O}_{B\bar{W}}$ operator does not contribute to the ZZZ vertex. Therefore, this study of nTGCs focuses on the $\mathcal{O}_{B\bar{W}}$ operator. It should be emphasized that the $\mathcal{O}_{B\bar{W}}$ and $\mathcal{O}_{\bar{W}B}$ can be distinguished in other multiboson channels, and the $\mathcal{O}_{B\bar{B}}$ and $\mathcal{O}_{W\bar{W}}$ can also be accessed by other multiboson channels.

The $ZZ\gamma$ coupling is absent at tree level and is highly suppressed when allowed by internal particle loops in the SM, thus forbidding the s -channel production of ZZ . Therefore, any deviation from the tree level SM predictions will indicate the presence of BSM physics. Dimension-6 operators could contribute to ZZ production at tree level through $\gamma f\bar{f}$ and $Z f\bar{f}$ couplings. However, LEP2 and LHC data indicate that deviations from SM predictions are significantly constrained, with $\gamma f\bar{f}$ coupling showing deviations $\ll 0.1\%$ and $Z f\bar{f}$ coupling deviations $\leq 0.1\%$ [85]. The contributions of dimension-8 increase as s^4 , faster than dimension-6. Therefore, in this nTGC analysis for ZZ production at future colliders, we have ignored the effect of dimension-6 operators.

The scattering process at the positron collider can be enhanced or reduced by the initial beam polarization. The cross section for polarized $e^+e^- \rightarrow ZZ$ scattering can be obtained as

$$\begin{aligned}
 \sigma_{\text{pol}}^{\text{SM}}(ZZ) &= -\frac{e^4(16s_W^8(P_{e^+} + 1)(P_{e^-} - 1) + (1 - 2c_W^2)^4(P_{e^+} - 1)(P_{e^-} + 1))}{128\pi c_W^4 s^2 s_W^4 (2M_Z^2 - s)} \\
 &\quad \times \left(\sqrt{s}(s - 2M_Z^2) \sqrt{s - 4M_Z^2} - (4M_Z^4 + s^2) \tanh^{-1} \left(\frac{\sqrt{s} \sqrt{s - 4M_Z^2}}{s - 2M_Z^2} \right) \right), \\
 \sigma_{\text{pol}}^{\text{int}}(ZZ) &= \frac{f_{B\bar{W}} e^2 M_Z^2 (4(c_W^2 - 1)(P_{e^+} + 1)(P_{e^-} - 1) s_W^2 + (1 - 2c_W^2)^2 (P_{e^+} - 1)(P_{e^-} + 1))}{32\pi c_W s^2 s_W} \\
 &\quad \times \left(\sqrt{s}(2M_Z^2 + s) \sqrt{s - 4M_Z^2} - 4M_Z^2(s - M_Z^2) \tanh^{-1} \left(\frac{\sqrt{s} \sqrt{s - 4M_Z^2}}{s - 2M_Z^2} \right) \right), \\
 \sigma_{\text{pol}}^{\text{nTGC}}(ZZ) &= -\frac{f_{B\bar{W}}^2 c_W^2 M_Z^2 s_W^2 (s - 4M_Z^2)^{\frac{5}{2}} (P_{e^-} P_{e^+} - 1)}{24\pi \sqrt{s}},
 \end{aligned} \tag{3}$$

where P_{e^-} and P_{e^+} denote the polarization parameters of the electron and positron, respectively, ranging from -1 to $+1$, with $+1$ (-1) for the left (right)-handed helicity.

Numerical results are evaluated for two energy points and the following helicity fraction of the electron (P_{e^-}) and positron (P_{e^+}) beam polarization:

TABLE I. Comparison of the hard bremsstrahlung cross section (fb) of the SM and nTGC contributions for 100% polarized $e^-e^+ \rightarrow ZZ$ scattering at the characteristic c.m. energies of CEPC (FCC-ee), ILC, and CLIC.

P_{e^-}, P_{e^+}		0, 0	-1, -1	-1, 1	1, -1	1, 1
$\sqrt{s} = 250$ GeV	nTGCs	4.50×10^{-3}	0	8.99×10^{-3}	8.99×10^{-3}	0
	SM	1050	0	2637	1564	0
$\sqrt{s} = 360$ GeV	nTGCs	0.06	0	0.12	0.12	0
	SM	627	0	1574	933	0
$\sqrt{s} = 500$ GeV	nTGCs	0.336	0	0.673	0.673	0
	SM	397	0	996	591	0
$\sqrt{s} = 1$ TeV	nTGCs	7.07	0	14.1	14.1	0
	SM	145	0	363	215	0
$\sqrt{s} = 3$ TeV	nTGCs	617	0	1234	1234	0
	SM	24.9	0	62.5	37.1	0

$$(P_{e^-}, P_{e^+}) = (0, 0), (-1, -1), (-1, +1), (+1, -1), (+1, +1). \quad (4)$$

It is convenient to make a comparison of the cross sections induced by the SM couplings and nTGCs without the interference contribution with different values of polarization in Table I. The results are calculated with $f_{BW} = 1(\text{TeV}^{-4})$ as the characteristic parameter. The e^+e^- colliders will be constructed in several center-of-mass (c.m.) energy stages, 250 GeV, 360 GeV, 500 GeV, 1 TeV, and 3 TeV [76–80]. For each energy, the cross section of contributions of nTGCs $\sigma_{--,++}$ is 0, and σ_{+-} or σ_{-+} are about 2 times larger than the unpolarized cross section. For the SM contributions, σ_{+-} is about 1.5 times larger than the unpolarized cross section.

The effects of beam polarization on both SM and BSM contributions can be systematically analyzed. Beam polarization modifies the relative contributions of the SM, interference, and NP terms to the total cross section. The ratios are defined by

$$\begin{aligned} R_{\text{SM}} &= \frac{\sigma_{\text{SM}}}{\sigma_{\text{SM}} + \sigma_{\text{int}} + \sigma_{\text{nTGC}}}, \\ R_{\text{int}} &= \frac{\sigma_{\text{int}}}{\sigma_{\text{SM}} + \sigma_{\text{int}} + \sigma_{\text{nTGC}}}, \\ R_{\text{nTGC}} &= \frac{\sigma_{\text{nTGC}}}{\sigma_{\text{SM}} + \sigma_{\text{int}} + \sigma_{\text{nTGC}}}. \end{aligned} \quad (5)$$

The contribution of NP to the total cross section increases significantly with rising energy, while the SM contribution decreases. The polarization effects of R_{SM} and R_{nTGC} have mild dependence on the collision energy, as shown in Fig. 2. The sign of R_{int} , an interference term, is determined by the initial beam polarization. The magnitudes of these ratios exhibit significant energy dependence, and the interference effect is most pronounced at 1.5 TeV. Among the polarization effects of settings, polarization $e_L^- e_R^+$ demonstrates the most effective enhancement of the

signal and suppression of the SM background, with a positive interference term. It is more advantageous to measure nTGC signals with a left-hand polarized electron beam and right-hand polarized positron.

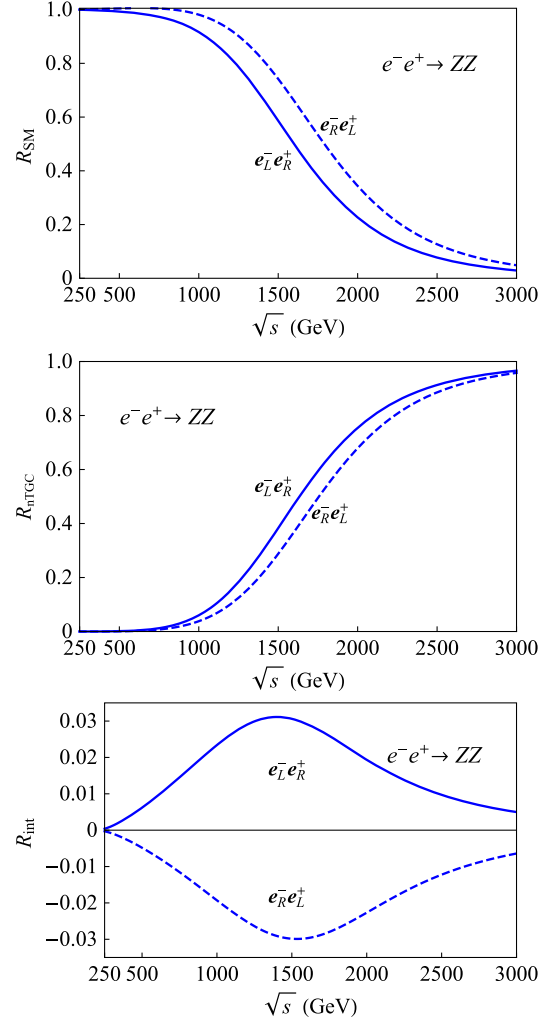


FIG. 2. Ratios R_{SM} , R_{nTGC} , and R_{int} with different electron polarizations as a function of energy.

III. PROBING NTGCS VIA ZZ PRODUCTION AT FUTURE e^+e^- COLLIDERS

ZZ production processes can be further divided into five decay modes in the final states. In this analysis, we study the following subprocesses:

$$e^+e^- \rightarrow Z_1(\ell^+\ell^-)Z_2(\ell^+\ell^-), \quad (6)$$

$$e^+e^- \rightarrow Z_1(jj)Z_2(jj), \quad (7)$$

$$e^+e^- \rightarrow Z_1(\ell^+\ell^-)Z_2(jj), \quad (8)$$

$$e^+e^- \rightarrow Z_1(\ell^+\ell^-)Z_2(\nu\bar{\nu}), \quad (9)$$

$$e^+e^- \rightarrow Z_1(jj)Z_2(\nu\bar{\nu}), \quad (10)$$

where ℓ is either e or μ . All processes are generated using the MadGraph5_aMC@NLO toolkit [86,87] with the large hadron collider olympic output format, while hadronization is performed by PYTHIA8[88]. The simulation relies on the tree level calculation with the full matrix elements.

A fast detector simulation is performed using Delphes [89], with detector configuration cards corresponding to the CEPC, FCC-ee, ILC, and CLIC experiments. These cards model the expected performance of detectors at future electron-positron colliders. Taking the detector simulation of the CEPC as an example, the concept design report detector [90] adopts a particle flow algorithm-oriented design. The detector features a large-volume gaseous detector to provide high-efficiency tracking and promising performance on particle identification. The core component of the detector is a high-granularity calorimeter that emphasizes the separation performance of final-state particles. The CEPC detector achieves a jet energy resolution ranging from 3% to 6% [91], depending on the polar angle and energy of the objective jets. Alternatively, its performance can be characterized by a boson mass resolution of approximately 3.8%. It also demonstrates excellent efficiency in reconstructing photons and leptons. A comprehensive description of the detector design and its performance can be found in Refs. [90,92].

Finally, the events are reconstructed with the MLAnalysis [93] package. The future e^+e^- collider projects CEPC, FCC, ILC, and CLIC, are associated with c.m. energy values, $\sqrt{s} = 250$ GeV, 360 GeV, 500 GeV, 1 TeV, and 3 TeV. For this work, a configuration of detector simulation and luminosity is assigned to each energy stage as follows:

- (i) 250 GeV: CEPC with $\mathcal{L} = 5 \text{ ab}^{-1}$
- (ii) 360 GeV: FCC-ee with $\mathcal{L} = 1 \text{ ab}^{-1}$
- (iii) 500 GeV: ILC with $\mathcal{L} = 500 \text{ fb}^{-1}$
- (iv) 1 TeV: ILC with $\mathcal{L} = 1 \text{ ab}^{-1}$
- (v) 3 TeV: CLIC with $\mathcal{L} = 5 \text{ ab}^{-1}$

It should be emphasized that the 250 GeV and 360 GeV benchmarks are not attached to particular collider setups,

since that energy range is accounted for by all Higgs factory proposals. CEPC, FCC-ee, and the initial stages of both ILC and CLIC are expected to deliver comparable integrated luminosities at $\sqrt{s} = 250$ GeV and 360 GeV. Thus, all events simulated at future e^+e^- colliders will use an electron-positron beam longitudinal polarization of $(+0.8, -0.8)$, serving as a reference point for future studies on e^+e^- collider polarization. In addition, it is worth noting that although achieving beam polarization at high-energy circular colliders remains technically challenging, it constitutes a significant topic of ongoing research. With experimental efforts, polarization has already been successfully demonstrated at the Z-pole (91 GeV) [94–96].

In the following signal background analysis, we will propose a set of cuts based on the kinematic performance of the contributions from SM and nTGC squared terms. However, when fitting the signal cross section with our event selection strategy, the contribution of nTGC and SM interference is included in the signal and total contribution. Consequently, the effects of these interference terms are incorporated into the constraints on operator coefficients.

A. Selection of $e^+e^- \rightarrow 2\ell 2\ell'$

Z-pairs decaying to final states with four charged leptons ($\ell = e, \mu$) produce low-multiplicity events with a clear topological signature that is exploited to maximize the selection efficiency. In addition to ZZ production, nTGCs can contribute to the $e^+e^- \rightarrow 2\ell 2\ell'$ process via di-boson production ($\gamma\gamma, Z\gamma$) and vector boson fusion (VBF) processes, as shown in Fig. 3. Therefore, we need to compare the contribution of nTGCs in both the annihilation process and the VBF process. As shown in Fig. 4, di-boson production dominates in the $e^+e^- \rightarrow 2\ell 2\ell'$ process, where the cross section is approximately 10 times larger than that of the VBF contribution. This comparison does not take into account the contribution of interference effects between nTGCs and the SM. The annihilation process is more sensitive to the dimension-8 operators, which suggests that, in signal and background analysis, the kinematic distributions of the nTGC signal will primarily reflect the characteristics of the annihilation process.

The corresponding Feynman diagrams of the SM backgrounds are shown in Fig. 5. The major contributions to the

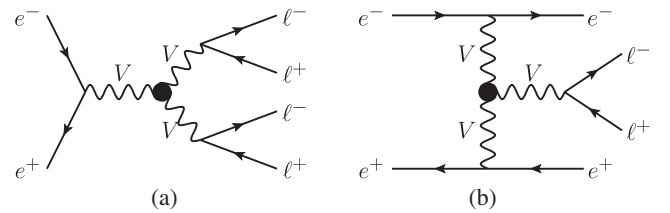


FIG. 3. The Feynman diagrams of the nTGCs contributions for the process $e^+e^- \rightarrow 2\ell 2\ell'$, where VVV stands for ZZZ, ZZ γ , or Z $\gamma\gamma$ couplings. The di-boson diagram is shown in (a), while the VBF diagram is shown in (b).

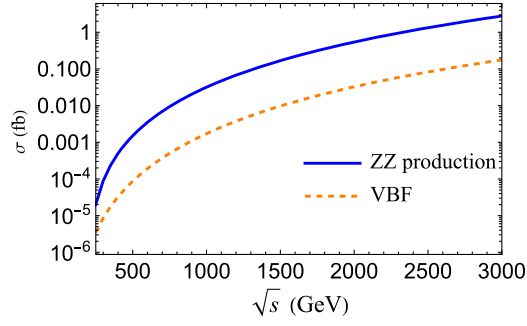


FIG. 4. A comparison of σ_{diboson} and σ_{VBF} for $e^+e^- \rightarrow 2\ell 2\ell'$ as a function of $\sqrt{s}$ with $f_{\bar{B}W} = 1(\text{TeV}^{-4})$.

background are due to Z production in association with two isolated leptons, as shown in Figs. 5(b)–5(d) and . The four-lepton final state from the di-boson production in the SM comes from the t -channel diagrams, as in Fig. 5(a).

Variables used in this analysis include the transverse momentum $p_T = \sqrt{p_x^2 + p_y^2}$ and the pseudorapidity defined in terms of the polar angle θ as $\eta = -\ln[\tan(\theta/2)]$. The basic cuts are set as same as the default settings of MadGraph5_aMC@NLO, except for $\Delta R_{\ell\ell}$, which is defined as $\sqrt{\Delta\eta_{\ell\ell}^2 + \Delta\phi_{\ell\ell}^2}$, where $\Delta\eta_{\ell\ell}$ and $\Delta\phi_{\ell\ell}$ are the differences between pseudorapidities and azimuth angles of the leptons, respectively. In the basic cuts, we use $\Delta R_{\ell\ell} > 0.2$ [97] as configurations with collinear leptons excluded. In the Monte Carlo (MC) simulation, we use the coefficients in the ranges listed in Table II for the representative

e^+e^- collider energies $\sqrt{s} = 250, 360, 500, 1000$, and 3000 GeV.

To study the kinematic features of the signal and background, we adopt the conventional approach of considering one operator at a time, as is widely used in analyses within the SMEFT framework. The largest coefficients in Table II

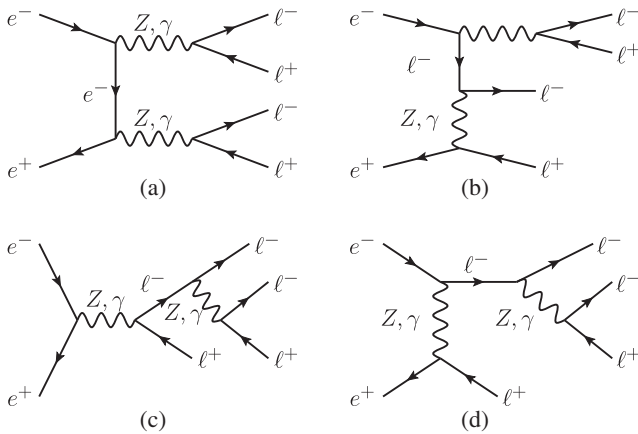


FIG. 5. Typical Feynman diagrams of the SM backgrounds for the process $e^+e^- \rightarrow 2\ell 2\ell'$.

TABLE II. The ranges of coefficients $f_{\bar{B}W}(\text{TeV}^{-4})$ used for each energy point in the MC simulation.

$\sqrt{s}$	250 GeV	360 GeV	500 GeV	1 TeV	3 TeV
$f_{\bar{B}W}$	$[-100, 100]$	$[-100, 100]$	$[-10, 10]$	$[-2.0, 2.0]$	$[-0.15, 0.15]$

are adopted. We require at least four leptons in the 4ℓ signal mode. All the numerical results are presented after lepton number cut $N_\ell \geq 4$ (denoted as N_ℓ cut).

For the SM diagram in Fig. 5(a), we expect that the distribution of SM background events is dominantly around small t and thus the transverse momentum of one of the vectors is small. Denoting p_T^ℓ as the transverse momentum of the hardest lepton and E_{cm} as the c.m. energy, the normalized distributions of p_T^ℓ/E_{cm} for the signal of $\mathcal{O}_{\bar{B}W}$ and the background are displayed in Fig. 6. The leptons from the SM background are softer than those from the nTGC signal. In addition, we also checked the η^ℓ distribution of the final-state leptons. At low energies, the scattering angle of low-energy particles is smaller, and the direction of motion of the final-state leptons is more forward. At high energies, the scattering of high-energy particles will cause the final-state leptons to be more widely distributed in different directions. The distribution of η^ℓ of the signal will approach the SM background with the increase of the collision energy. Therefore, we did not use η^ℓ as the cut in the signal background analysis for the TeV scale.

The Lorentz structure of the new interactions, which includes the epsilon tensor, could be a distinguishing

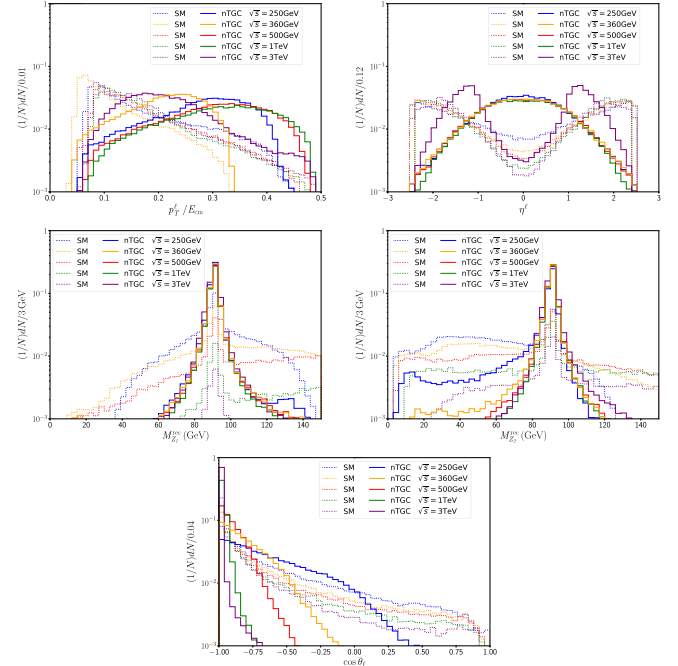


FIG. 6. The normalized distributions of p_T^ℓ/E_{cm} , η^ℓ , $M_{Z_1}^{\text{rec}}$, $M_{Z_2}^{\text{rec}}$, and $\cos\theta_\ell$ for $e^+e^- \rightarrow 2\ell 2\ell'$.

characteristic of neutral tri-boson interactions. Some recent studies have proven that angular distribution analyses are highly effective in elucidating unusual interaction patterns [70,75]. In this channel, we specifically investigate the angular distribution between two Z bosons. At high energies, the direction of the lepton is approximately consistent with that of the highly boosted Z , so the angle between leptons of the same sign is approximately the angle between two parent particles. The angle between the same sign leptons $\cos\theta_\ell$ is used at high energy. The dominant contribution of nTGCs is through the s -channel of the di-boson diagrams. Therefore, the angular distributions of di-boson production induced by nTGCs and the SM should be different.

Since the main contribution to the signal arises from di-boson production, the SM background from mono- Z or γ production can be suppressed by requiring the event to contain two reconstructed Z bosons. In the process of reconstructing the two Z bosons decaying into leptons, the four leptons with the highest energies are grouped into two pairs based on minimizing the angular separation between any two leptons. The invariant mass of each pair is then reconstructed, and the differences between the reconstructed masses and the Z boson mass are evaluated. Specifically, the invariant mass corresponding to the smallest mass difference with the Z boson $|M_{\ell\ell} - M_Z|$, denoted as $M_{Z_1}^{\text{rec}}$, while the other invariant mass $M_{Z_2}^{\text{rec}}$ corresponds to the second pair of leptons. The distributions of $M_{Z_1}^{\text{rec}}$ and $M_{Z_2}^{\text{rec}}$ shown in Fig. 6 exhibit peaks around the mass of the Z boson, which is consistent with the signal. Most background events reconstruct $M_{Z_1}^{\text{rec}}$, but do not work well for reconstructing $M_{Z_2}^{\text{rec}}$. The joint requirement of the invariant mass of the reconstructed $Z_{1,2}$ is required to satisfy $80 < M_{\ell\ell} < 100$ GeV, which can effectively suppress the background events from Figs. 5(b)–5(d). However, other kinematic observable measurements are needed to further optimize the event screening strategy to

eliminate the background from Fig. 5(a). Table III shows the cross sections with event selection strategy and cut efficiencies obtained in all energy cases.

The total cross sections after the event selection strategy (denoted by $\sigma_{\text{nTGC}}^{\text{ac}}$) are obtained by scanning various values of the coefficient of $\mathcal{O}_{\bar{B}W}$, and the results are shown in Fig. 7. In Figs. 7(a)–7(d), the fitted cross section's distribution of $f_{\bar{B}W}$ exhibits a noticeable asymmetry between the positive and negative regions, which can be attributed to the nonzero interference terms. Furthermore, this asymmetry is observed to decrease with increasing energy. This phenomenon occurs because the contributions from dimension-8 operators grow rapidly with energy, leading to a reduced overlap between NP and the SM in phase space, which consequently reduces the interference effects. At an energy of 3 TeV, as shown in Fig. 7(e), no significant asymmetry is observed, indicating that the impact of the interference term becomes negligible at this energy.

Taking into account the impact of interference terms, the cross sections $\sigma_{\text{nTGC}}^{\text{ac}}$ can be fitted as bilinear functions of $f_{\bar{B}W}$ (TeV^{-4}), as

$$\sigma_{\text{nTGC}}^{\text{ac}} = \epsilon_{\text{SM}}\sigma_{\text{SM}} + f_{\bar{B}W}^2\sigma_{\text{NP}}^{\text{ac}} + f_{\bar{B}W}\hat{\sigma}_{\text{int}}, \quad (11)$$

where ϵ_{SM} and ϵ_{NP} are the cut efficiencies of SM and nTGC events, respectively, which are listed in Table III. $\sigma_{\text{NP}}^{\text{ac}}$ is the NP contribution after cuts to be fitted, and $\hat{\sigma}_{\text{int}}$ is the interference parameter to be fitted. The numerical results of efficiencies are given in Table III. The fitted results of $\hat{\sigma}_{\text{int}}$ can then be obtained and are given in Table IV.

The sensitivity of future e^-e^+ colliders to the nTGC operators and the expected constraints on the coefficients can be obtained with respect to the significance, defined as [98,99]

$$\mathcal{S}_{\text{stat}} = \sqrt{2[(N_{\text{bg}} + N_s) \ln(1 + N_s/N_{\text{bg}}) - N_s]}, \quad (12)$$

TABLE III. The event selection strategy and cross sections (fb) after cuts for each energy point. The results of nTGCs are obtained using the upper bounds of the coefficients in Table II.

	$\sqrt{s} = 250$ GeV		$\sqrt{s} = 360$ GeV		$\sqrt{s} = 500$ GeV		$\sqrt{s} = 1$ TeV		$\sqrt{s} = 3$ TeV	
	SM	NP	SM	NP	SM	NP	SM	NP	SM	NP
Basic Cuts	22.83	0.254	16.50	3.015	11.64	0.161	5.301	0.109	1.336	0.022
$p_T^\ell/E_{\text{cm}} > 0.15$	11.01	0.228	7.387	2.719						
$p_T^\ell/E_{\text{cm}} > 0.2$					3.584	0.133	1.555	0.092		
$ \eta^\ell < 1.5$	9.244	0.225	5.712	2.652	3.519	0.133	1.502	0.092		
$\cos\theta_\ell < 0.4$	8.882	0.225								
$\cos\theta_\ell < -0.2$			4.909	2.620						
$\cos\theta_\ell < -0.6$					2.534	0.130	1.102	0.092	1.060	0.022
$ M_{Z_1, Z_2}^{\text{rec}} - M_Z < 10$ GeV	2.460	0.157	0.976	2.074	0.361	0.104	0.079	0.073	0.014	0.016
Efficiency ϵ	6.8%	40%	3.5%	43%	1.8%	40%	0.8%	40%	0.6%	53%

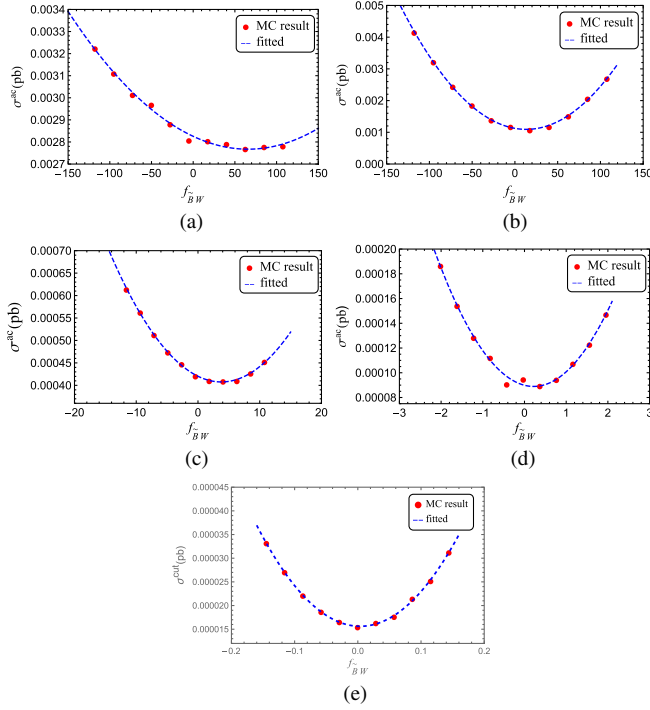


FIG. 7. Cross sections as a function of f_{BW} (TeV^{-4}) for $e^+e^- \rightarrow 2\ell 2\ell'$. (a) 250 GeV. (b) 360 GeV. (c) 500 GeV. (d) 1 TeV. (e) 3 TeV.

where the number of signal events $N_s = N_{\text{nTGC}} - N_{\text{SM}}$, and the number of background events $N_{\text{bg}} = N_{\text{SM}}$. N_{nTGC} is obtained by setting the certain high-dimensional coefficient f_{BW} in SMEFT. N_{SM} denotes the SM prediction with the high-dimensional coefficient vanishing. The likelihood function for N_s is defined as

$$L(N_s) = \frac{(N_{\text{bg}} + N_s)^n}{n!} e^{-(N_{\text{bg}} + N_s)}, \quad (13)$$

TABLE IV. The fitted values of $\sigma_{\text{NP}}^{\text{ac}}$ ($\text{fb} \times \text{TeV}^8$) and $\hat{\sigma}_{\text{int}}$ ($\text{fb} \times \text{TeV}^4$) for each energy point, where $\sigma_{\text{NP}}^{\text{ac}}$ and $\hat{\sigma}_{\text{int}}$ are defined in Eq. (11).

$\sqrt{s}$	250 GeV	360 GeV	500 GeV	1 TeV	3 TeV
$\hat{\sigma}_{\text{int}}$	-0.0018	-0.0048	-0.0067	-0.0085	-0.0062
$\sigma_{\text{NP}}^{\text{ac}}$	0.00001	0.0002	0.0008	0.0194	0.7942

TABLE V. The expected constraints on f_{BW} (TeV^{-4}) for $e^+e^- \rightarrow 2\ell 2\ell'$ at each energy point of e^+e^- colliders with corresponding design luminosities.

S_{stat}	$\sqrt{s}$ (GeV)				
	250	360	500	1000	3000
2	[-23.1, 155.0]	[-10.4, 36.9]	[-5.4, 13.0]	[-0.84, 1.28]	[-0.088, 0.096]
3	[-32.8, 164.6]	[-14.3, 40.8]	[-7.3, 14.9]	[-1.11, 1.55]	[-0.114, 0.121]
5	[-49.9, 181.8]	[-21.1, 47.5]	[-10.7, 18.2]	[-1.57, 2.01]	[-0.160, 0.169]

and one can find the maximum by setting $\partial \ln L / \partial N_s = 0$. The approximate significance is then obtained by taking the square root of the likelihood ratio statistic $-2 \ln(L(0)/L(N_s))$ after letting $n = N_{\text{bg}} + N_s$ [98].

As a result, we can obtain the projected sensitivity on f_{BW} by taking 2σ , 3σ , or 5σ significance. The results are shown in Table V.

B. Selection of $e^+e^- \rightarrow 4j$

The hadronic selection of ZZ production is very important, which represents 49% of the ZZ decay topologies. The signal is characterized by four or more jets in the final state. The contribution to the signal is not only the pair production of the Z boson, but also the W boson. The $e^+e^- \rightarrow 4j$ channel has only one topology induced by five electroweak vertices, ZZZ , $Z\gamma\gamma$, $ZZ\gamma$, WWZ , and $WW\gamma$. At tree level, the Feynman diagram is shown in Fig. 8. The background includes reducible contributions from QCD processes and di-boson productions. Figure 9 shows the typical Feynman diagrams.

The event selection strategy is optimized separately for each energy point of future e^+e^- colliders. The basic cuts are the requirements of the isolated jet. A four-jet event sample is formed by requiring the jet resolution parameter, the transverse momentum $p_T^j > 20$ GeV, and pseudorapidity $\eta_j < 5.0$. Considering that, in high-energy scenarios, collinear jets may be misidentified as a single fat jet, it is required that both signal and background events contain at least two jets, i.e., $N_j \geq 2$. In signal events, there are expected to be two back-to-back vector boson candidates decaying into two jets, denoted as V_1 and V_2 . The reconstruction method for these two bosons involves selecting the two jets with the largest angular separation from the highest-energy jets in the final state. If there are remaining jets, they are then paired based on minimizing the angular separation with these two selected jets. Afterward, the invariant mass of each pair of jets is reconstructed. The mass difference is then evaluated as $\Delta_M = |M_{jj_1} - M_{V_1}| + |M_{jj_2} - M_{V_2}|$, where M_{jj_1} and M_{jj_2} are the invariant masses of the two jet pairs. This calculation is performed twice, once for M_{V_1} and M_{V_2} set to the mass of the W boson, and once for the Z boson mass. The pairing configuration is then selected based on which pairing minimizes Δ_M when V_1 and V_2 are assigned the

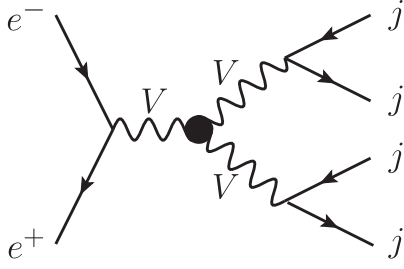


FIG. 8. The signal Feynman diagrams for the process $e^+e^- \rightarrow 4j$, where VVV stands for ZZZ, ZZ γ , Z $\gamma\gamma$, ZWW, or γWW couplings.

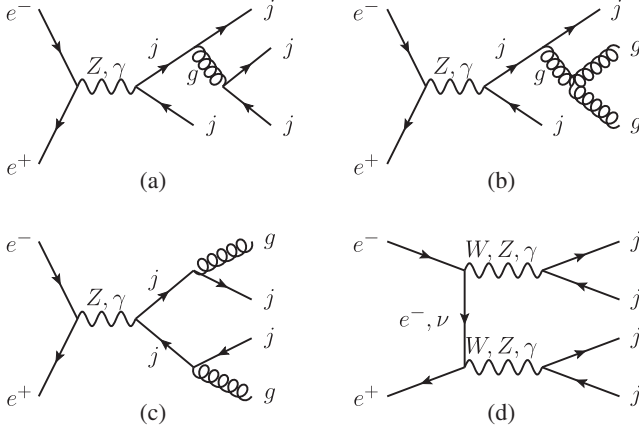


FIG. 9. The background Feynman diagrams for the process $e^+e^- \rightarrow 4j$.

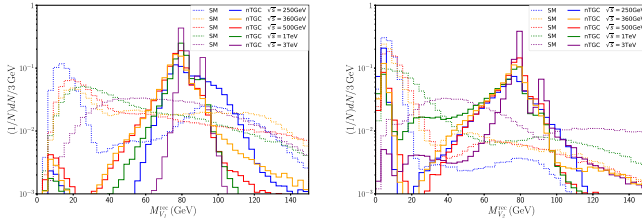


FIG. 10. The normalized distributions of $M_{V_1}^{\text{rec}}$ and $M_{V_2}^{\text{rec}}$ for $e^+e^- \rightarrow 4j$.

W or Z boson masses. The mass window of M_{V_1} and M_{V_2} is used as the improved cuts to discriminate the QCD background events against the signal events of hadronic vector boson pair events. The invariant mass distribution of the

signal exhibits distinct peaks within the mass range of the W and Z bosons, as shown in Fig. 10. The distributions of M_{V_1} and M_{V_2} display small peaks around 15 GeV and 10 GeV, respectively, which could be attributed to $Z\gamma^*$ and $\gamma^*\gamma^*$ production. Considering that $Z\gamma$ production constitutes a minor portion of the signal and that background events dominate the low-energy region for M_{V_1} and M_{V_2} , we require that both M_{V_1} and M_{V_2} of the signal event satisfy the invariant mass range of the massive vector boson. We apply a mass window cut of $60 < M_{V_1, V_2} < 110$ GeV to ensure that the selected events consist of four jets from W and Z boson pair production induced by nTGCs. The invariant mass selection aims for an inclusive selection of fully hadronic vector boson pair decays.

The specific values of the event selection strategy and the respective cut efficiencies are presented in Table VI. With the cut flow applied, the total cross sections are obtained by scanning the values of coefficients with $\mathcal{O}_{\tilde{B}W}$, and the fitting functions are shown in Fig. 11. The fitting coefficients of interference terms are given in Table VII.

The sensitivities for the $e^+e^- \rightarrow 4j$ channel at future e^+e^- colliders and the expected constraints on the coefficients are as obtained with Eq. (12). The integrated luminosity $\mathcal{L}$ is based on the design values of the CEPC, ILC, and CLIC at four energies. The projected sensitivity on $f_{\tilde{B}W}$ is found by taking 2σ , 3σ , or 5σ significance levels. The results are presented in Table VIII. While the hadronic modes are considerably more challenging, the $4j$ signal channel at the e^+e^- colliders exhibits a high degree of sensitivity to the $\mathcal{O}_{\tilde{B}W}$ operators, which are even more pronounced than the four charged leptons mode.

C. Selection of $e^+e^- \rightarrow 2\ell 2j$

The signal of $2\ell 2j$ induced by nTGCs is composed of both VBF and di-boson production topologies with the decay of massive electroweak gauge bosons. The signal from ZZ production is the main component of the nTGC signal, which has a branching ratio in ZZ production of 4.7% per lepton flavor. The kinematic characteristics of ZZ decay products will provide the main feature of the nTGC signal. The SM background can be divided into three categories according to whether the final-state leptons and jets can reconstruct Z . The typical Feynman diagrams of signal and background are shown in Figs. 12 and 13. The on-shell ZZ event can be selected by applying simultaneous

TABLE VI. The cross sections σ_{SM} and σ_{nTGC} of $e^+e^- \rightarrow 4j$ (in unit of fb) and cut efficiencies after the event selection strategy.

	$\sqrt{s} = 250$ GeV		$\sqrt{s} = 360$ GeV		$\sqrt{s} = 500$ GeV		$\sqrt{s} = 1$ TeV		$\sqrt{s} = 3$ TeV	
	SM	NP	SM	NP	SM	NP	SM	NP	SM	NP
Basic Cuts	2898	2.216	251.5	23.28	2103	1.684	994.2	1.655	208.1	0.988
$M_{V_1, V_2}^{\text{rec}} \in (60, 110)$ GeV	103.5	1.156	14.50	14.05	78.73	1.127	42.67	0.925	52.67	0.911
Efficiency ϵ	3.6%	52%	1.6%	60%	3.7%	67%	4.3%	55%	25%	92%

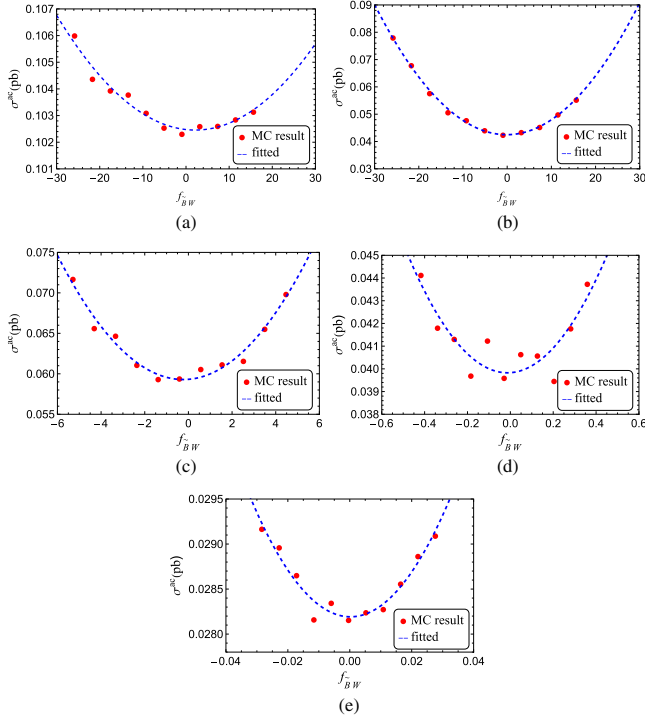


FIG. 11. Cross sections as functions of f_{BW} (TeV^{-4}) for $e^+e^- \rightarrow 4j$. (a) 250 GeV. (b) 360 GeV. (c) 500 GeV. (d) 1 TeV. (e) 3 TeV.

cuts on the masses of the $\ell^+\ell^-$ pair, on the remaining hadronic system, and on their sum.

As in the previous analysis, the signal events are generated by assuming one operator at a time, using the largest coefficients from Table II. In the following studies, we require at least two leptons and two jets. All numerical results are presented after applying the particle number

TABLE VII. The fitted values of $\sigma_{\text{NP}}^{\text{ac}}$ ($\text{fb} \times \text{TeV}^8$) and $\hat{\sigma}_{\text{int}}$ ($\text{fb} \times \text{TeV}^4$) for $e^+e^- \rightarrow 4j$ at each energy point, where $\sigma_{\text{NP}}^{\text{ac}}$ and $\hat{\sigma}_{\text{int}}$ are defined in Eq. (11).

$\sqrt{s}$	250 GeV	360 GeV	500 GeV	1 TeV	3 TeV
$\hat{\sigma}_{\text{int}}$	-0.0178	-0.0112	-0.2134	-0.1332	-0.5256
$\sigma_{\text{NP}}^{\text{ac}}$	0.0042	0.0533	0.4614	2.0408	1268.4

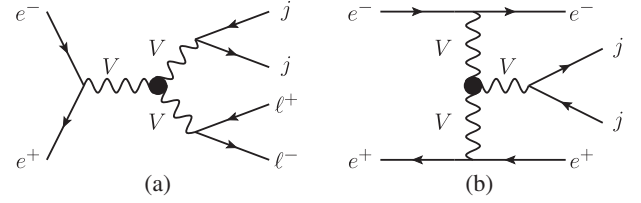


FIG. 12. Typical Feynman diagrams which contribute to the process $e^+e^- \rightarrow 2\ell 2j$ for signal events, where VVV stands for ZZZ, ZZ γ , or Z $\gamma\gamma$ couplings. The di-boson diagram is shown in (a), while the VBF diagram is shown in (b).

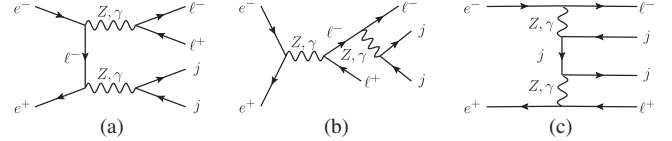


FIG. 13. Typical Feynman diagrams, which contribute to the process $e^+e^- \rightarrow 2\ell 2j$ for background events.

cuts: $N_{\ell^+} \geq 1$, $N_{\ell^-} \geq 1$, and $N_j \geq 2$, and the two hardest leptons will be a lepton and an antilepton of the same flavor. This requirement is denoted as the $N_{\ell,j}$ cut.

Another important factor is $\Delta R_{\ell\ell}$ and ΔR_{jj} . When the Z boson is energetic, the leptons tend to be collinear, resulting in a small $\Delta R_{\ell\ell}$ and a small ΔR_{jj} for the signal events. However, since $\Delta R_{\ell\ell}$ is related to lepton isolation, we impose a basic cut of $\Delta R_{\ell\ell} > 0.2$, which can be achieved in experiments [45].

To remove the backgrounds without a Z resonance, we require the invariant mass of the leptons and jets, denoted as $M_{\ell\ell}$ and M_{jj} , to be close to M_Z . For events in the $\ell^+\ell^-jj$ final states, the Z direction was taken to be the direction of the reconstructed di-jet or $\ell^+\ell^-$ pair, respectively. The normalized distributions of $M_{\ell\ell}$ and M_{jj} are shown in Fig. 14. In the distributions of the signal, the peaks at M_Z are much sharper than those in the distributions of the SM backgrounds. We cut off the events with the mass window cut $70 < M_{\ell\ell} < 110$ GeV. It should be noted that in the case of $\sqrt{s} = 250$ and 360 GeV, $M_{\ell\ell}$ has a resonance peak mainly from the $Z\gamma^*$ production processes at about 10 GeV. For $\sqrt{s} = 250$ and 360 GeV, the mass window cut increases the range by an additional $5 < M_{\ell\ell} < 20$ GeV.

TABLE VIII. The expected constraints on f_{BW} (TeV^{-4}) for $e^+e^- \rightarrow 4j$.

S_{stat}	$\sqrt{s}$ (GeV)				
	250	360	500	1000	3000
2	[-6.4, 10.7]	[-2.7, 2.9]	[-1.0, 1.5]	[-0.15, 0.22]	[-0.011, 0.011]
3	[-8.2, 12.5]	[-3.3, 3.5]	[-1.3, 1.8]	[-0.20, 0.26]	[-0.013, 0.014]
5	[-11.2, 15.4]	[-4.3, 4.4]	[-1.7, 2.2]	[-0.27, 0.33]	[-0.017, 0.017]

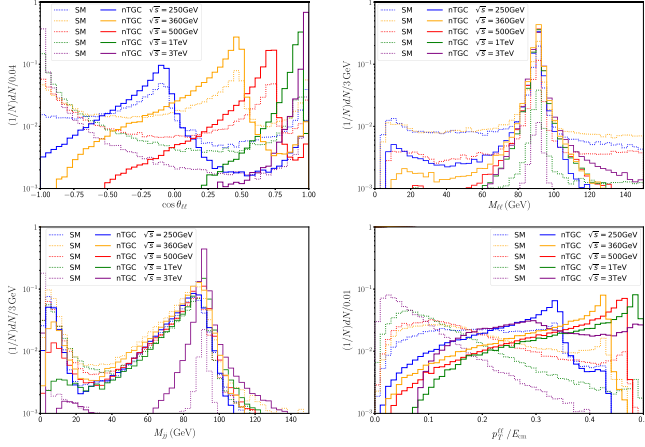


FIG. 14. The normalized distributions of $\cos \theta$, $M_{\ell\ell}$, M_{jj} , and $p_T^{\ell\ell}/E_{\text{cm}}$ for $e^+e^- \rightarrow 2\ell 2j$.

The M_{jj} distributions for the nTGC signal and the SM background show similarity at low energies. However, these distributions become more distinguishable at higher energies. Consequently, we included M_{jj} as a selection criterion only in the 3 TeV analysis.

After applying the $M_{\ell\ell,jj}$ cut, the SM backgrounds in Figs. 13(b) and 13(c) can be excluded, leaving the events from Fig. 13(a). The dominant contribution of nTGCs is through the s -channel of the di-boson diagram, Fig. 12(a). In the signal event, leptons and jets come from the decay of bosons emitted in the s -channel diagram. We expect the leptons and jets in the signal to be more energetic than those in the background, and anticipate the same for their parent

particle, the reconstructed Z boson. Thus, $p_T^{\ell\ell}/E_{\text{cm}}$ will work for this channel. At high energies, the direction of the lepton or jet is approximately consistent with that of the highly boosted Z . Therefore, in the c.m. frame, the angle $\theta_{\ell\ell}$ between the two leptons in the signal is different from those in the background, as shown in Fig. 14. We cut off the events with a large $\cos \theta_{\ell\ell}$ at the high-energy scale. This effect is not obvious for lower c.m. energy, but the signal selection efficiency can still be slightly improved by a cut of $-0.6 < \cos \theta < 0.2$. The specific values of the event selection strategy and the respective cut efficiencies are presented in Table IX.

With the event selection strategy of $e^+e^- \rightarrow 2\ell 2j$ applied, the total cross sections are obtained by scanning the values of coefficients with $\mathcal{O}_{\bar{B}W}$, and the fitting functions are shown in Fig. 15. The fitting coefficients of interference terms are given in Table X.

The sensitivities for the $e^+e^- \rightarrow 2\ell 2j$ channel and the expected constraints on the coefficients are obtained with Eq. (12). We obtain the projected sensitivity on $f_{\bar{B}W}$ by taking 2σ , 3σ , or 5σ significance. The results are shown in Table XI.

The lepton pairs in the $2\ell 2j$ final states have a distinctive signature, making possible selections with high efficiencies and low background contaminations. In comparison to the previous section, the $2\ell 2j$ has the most sensitive detection capability for the $\mathcal{O}_{\bar{B}W}$ operator in the high-energy region.

D. Selection of $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$

The Feynman diagrams of the contribution induced by nTGCs to the process $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$ are given in Fig. 16.

TABLE IX. The event selection strategy and cross sections of $e^+e^- \rightarrow 2\ell 2j$ (in unit of fb) after cuts.

	$\sqrt{s} = 250 \text{ GeV}$		$\sqrt{s} = 360 \text{ GeV}$			
	SM	NP	SM	NP		
Basic Cuts	86.08	0.556	63.43	7.454		
$-0.7 < \cos \theta_{\ell\ell} < 0.2$	53.66	0.482				
$-0.5 < \cos \theta_{\ell\ell} < 0.7$			38.55	7.058		
$M_{\ell\ell} \in (5, 20) \cup (70, 110) \text{ GeV}$	47.79	0.471	30.21	6.814		
$p_T^{\ell\ell}/E_{\text{cm}} > 0.1$	40.95	0.440	23.52	6.540		
Efficiency ϵ	35%	60%	27%	68%		
	$\sqrt{s} = 500 \text{ GeV}$		$\sqrt{s} = 1 \text{ TeV}$		$\sqrt{s} = 3 \text{ TeV}$	
	SM	NP	SM	NP	SM	NP
Basic Cuts	52.08	0.484	25.69	0.430	11.35	0.162
$\cos \theta_{\ell\ell} > 0$	25.79	0.460	6.724	0.426		
$\cos \theta_{\ell\ell} > 0.5$					0.898	0.157
$M_{\ell\ell} \in (70, 110) \text{ GeV}$	17.54	0.436	3.187	0.402		
$M_{\ell\ell, \text{jj}} \in (70, 110) \text{ GeV}$					0.334	0.134
$p_T^{\ell\ell}/E_{\text{cm}} > 0.3$	5.305	0.312	1.084	0.305		
$p_T^{\ell\ell}/E_{\text{cm}} > 0.1$					0.325	0.133
Efficiency ϵ	7.4%	51%	2.9%	54%	2.1%	71%

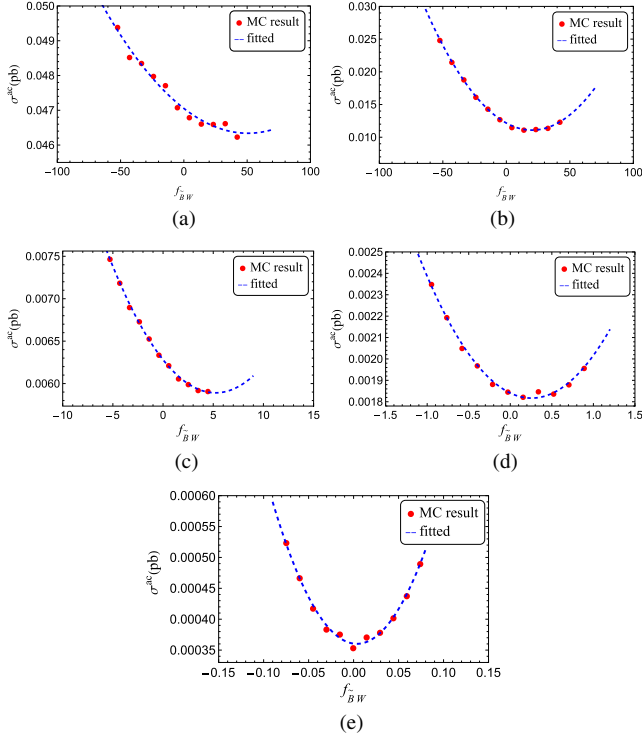


FIG. 15. Cross sections as functions of f_{BW} (TeV^{-4}) for $e^+e^- \rightarrow 2\ell 2j$. (a) 250 GeV. (b) 360 GeV. (c) 500 GeV. (d) 1 TeV. (e) 3 TeV.

The dominant signal is the di-boson contribution induced by nTGCs. The contribution from the VBF process is negligible compared to the bi-bosonic process. As shown in Fig. 17, the contribution from WW production is more significant than that from ZZ production. Therefore, in the analysis of signal and background, the $\ell^+\ell^-\nu\bar{\nu}$ signal will primarily reflect the kinematic distribution characteristics of WW and ZZ production.

The irreducible backgrounds in the $\ell^+\ell^-\nu\bar{\nu}$ channel arise from di-boson production (WW and ZZ), electroweak

TABLE X. The fitted values of $\sigma_{\text{NP}}^{\text{ac}}$ ($\text{fb} \times \text{TeV}^8$) and $\hat{\sigma}_{\text{int}}$ ($\text{fb} \times \text{TeV}^4$) for $e^+e^- \rightarrow 2\ell 2j$ at each energy point, where $\sigma_{\text{NP}}^{\text{ac}}$ and $\hat{\sigma}_{\text{int}}$ are defined in Eq. (11).

$\sqrt{s}$	250 GeV	360 GeV	500 GeV	1 TeV	3 TeV
$\hat{\sigma}_{\text{int}}$	-0.0282	-0.1064	-0.1489	-0.1848	-0.1219
$\sigma_{\text{NP}}^{\text{ac}}$	0.0003	0.0026	0.0142	0.3607	24.179

TABLE XI. The excepted constraints on f_{BW} (TeV^{-4}) for $e^+e^- \rightarrow 2\ell 2j$.

S_{stat}	$\sqrt{s}$ (GeV)				
	250	360	500	1000	3000
2	[-6.5, 107.7]	[-2.0, 42.5]	[-1.4, 11.8]	[-0.30, 0.81]	[-0.022, 0.030]
3	[-9.5, 110.7]	[-2.9, 43.4]	[-2.0, 12.4]	[-0.40, 0.92]	[-0.028, 0.036]
5	[-15.0, 116.3]	[-4.7, 45.2]	[-3.0, 13.5]	[-0.58, 1.09]	[-0.038, 0.045]

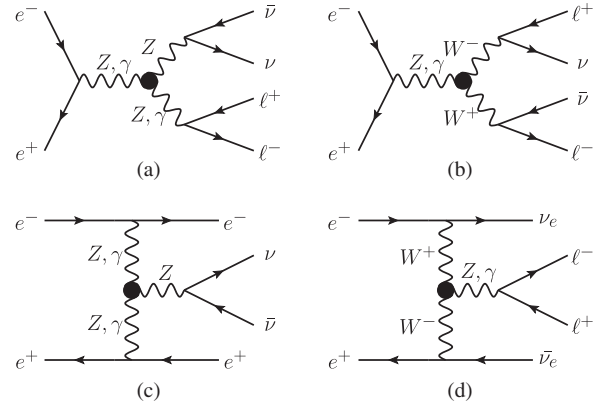


FIG. 16. Typical Feynman diagrams which contribute to the process $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$ for signal events. The di-boson diagrams are shown in (a) and (b), while the VBF diagrams are shown in (c) and (d).

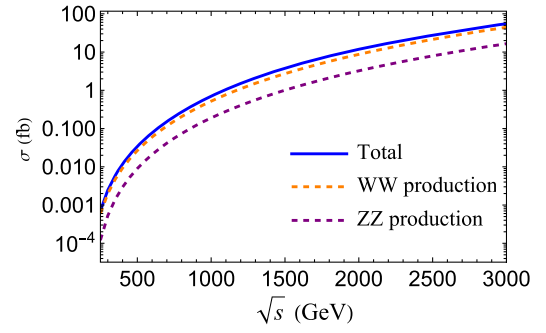


FIG. 17. The comparison of σ_{Total} , σ_{WW} , and σ_{ZZ} for $e^+e^- \rightarrow \ell\ell\nu\bar{\nu}$ as a function of c.m. energy $\sqrt{s}$ with $f_{BW} = 1(\text{TeV}^{-4})$.

VBF, and neutrino production in association with a vector boson that decays leptonically. The typical Feynman diagrams of the SM backgrounds are shown in Fig. 18. In order to suppress the SM backgrounds, the following basic cuts are used: $\Delta R_{\ell\ell} = 0.2$, $p_T^\ell = 10$ GeV, and $N_\ell \geq 2$, and the two leptons must be a lepton and an antilepton of the same flavor. After these cuts, we further employ optimized kinematical cuts according to the kinematical distributions of the signal and background.

In the event of two detectable leptons and invisible particles in the final state, the invariant mass, pseudorapidity, missing energy, and reconstructed invariant mass

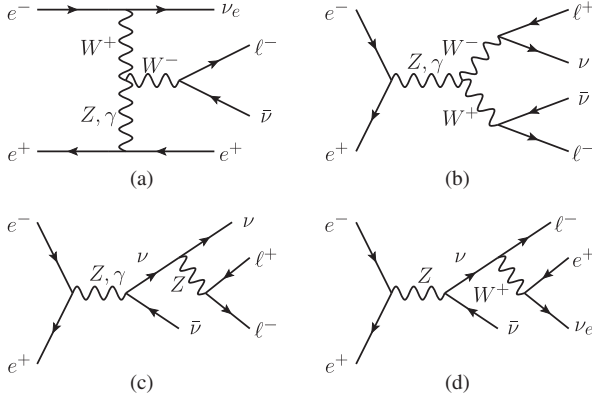


FIG. 18. Typical Feynman diagrams, which contribute to the process $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$ for background events.

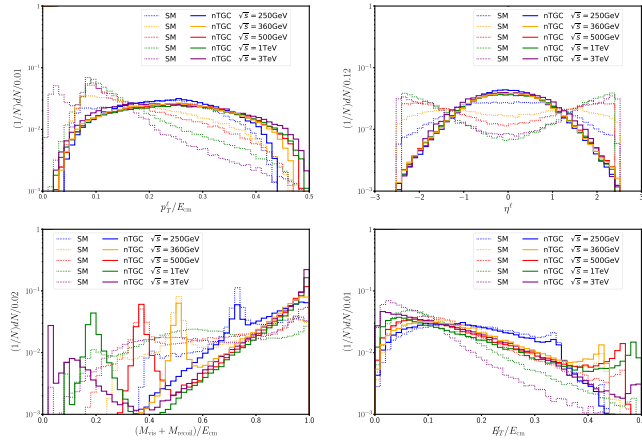


FIG. 19. The normalized distributions of p_T^ℓ/E_{cm} , η^ℓ , $(M_{\text{vis}} + M_{\text{recoil}})/\sqrt{s}$, and $E_T/\sqrt{s}$ for $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$.

can be extracted. We display the normalized distribution of these kinematic variables in Fig. 19, with the largest coefficients listed in Table II. As the majority of leptons in the background are produced via radiative decay, it can be observed that the transverse momentum of these leptons

is comparatively smaller than that of the signal. The Z boson is typically produced in the central region in the nTGC process, and the leptons of Z decay are nearly collinear with the highly boosted Z , resulting in a relatively small distribution of pseudorapidity for the associated lepton. The candidate lepton is required to have $p_T^\ell > 0.2E_{\text{cm}}$, with $|\eta^\ell| < 1.5$ for $\sqrt{s} \leq 360$ GeV and $|\eta^\ell| < 1$ for $\sqrt{s} \geq 500$ GeV.

The most significant SM background comes from WW production, where both W bosons decay leptonically. The on-shell ZZ events for this final state can be selected by applying cuts on the invariant mass of the $\ell^+\ell^-$ pair and the missing transverse energy E_T . The missing energy distributions for both the signal and background are similar. Therefore, E_T is not utilized in the event selection strategy. M_{vis} and M_{recoil} are calculated from the reconstructed Z boson and invisible recoil mass obtained through energy conservation. Then, the normalized sum of visible and recoil masses $(M_{\text{vis}} + M_{\text{recoil}})/\sqrt{s}$ can be used in the analysis. Figure 19 shows that the distribution of $(M_{\text{vis}} + M_{\text{recoil}})/\sqrt{s}$ exhibits two peaks: one near $2M_Z$, corresponding to ZZ production, and another in a higher-mass region, which is attributed to WW production. Given that the distribution of this observable varies with different collision energies, we propose two distinct sets of cut schemes for $\sqrt{s} \leq 360$ GeV and $\sqrt{s} \geq 500$ GeV to enhance selection performance. The specific values of the event selection strategy and the respective cut efficiencies are presented in Table XII.

With the event selection strategy applied for the final state $\ell^+\ell^-\nu\bar{\nu}$, the cross section fitting functions of $\mathcal{O}_{\tilde{B}W}$ are shown in Fig. 20, and the fitting coefficients of the interference terms are provided in Table XIII. The sensitivity analysis and the expected constraints on the $\mathcal{O}_{\tilde{B}W}$ operator are presented in Table XIV. Although the $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$ channel is not the most optimal, the expected constraints on $f_{\tilde{B}W}$ at the 95% C.L. can still reach approximately -4 TeV^{-4} to 8 TeV^{-4} for $\mathcal{O}_{\tilde{B}W}$ at 360 GeV. At $\sqrt{s} \geq 500$ GeV, with higher energy and

TABLE XII. The event selection strategy and cross sections of $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$ (fb) after cuts.

	$\sqrt{s} = 250$ GeV		$\sqrt{s} = 360$ GeV		$\sqrt{s} = 500$ GeV		$\sqrt{s} = 1$ TeV		$\sqrt{s} = 3$ TeV	
	SM	NP	SM	NP	SM	NP	SM	NP	SM	NP
Basic Cuts	87.72	12.03	62.04	86.36	49.49	5.686	32.00	3.794	16.33	2.230
$p_T^\ell/E_{\text{cm}} > 0.2$					19.29	3.903	9.075	2.600	2.906	1.462
$ \eta^\ell < 1.5$	68.15	10.87	38.03	77.06						
$ \eta^\ell < 1.0$					12.90	3.409	4.827	2.270	1.175	1.252
$(M_{\text{vis}} + M_{\text{recoil}}) > 120$ GeV	64.54	40.69	36.81	76.85						
$(M_{\text{vis}} + M_{\text{recoil}}) \in (150, 210) \text{ GeV} \cup (0.7E_{\text{cm}}, E_{\text{cm}})$					9.625	3.117	2.719	2.002	0.584	1.129
Efficiency ϵ	59%	72%	46%	70%	15%	43%	6.4%	41%	2.7%	44%

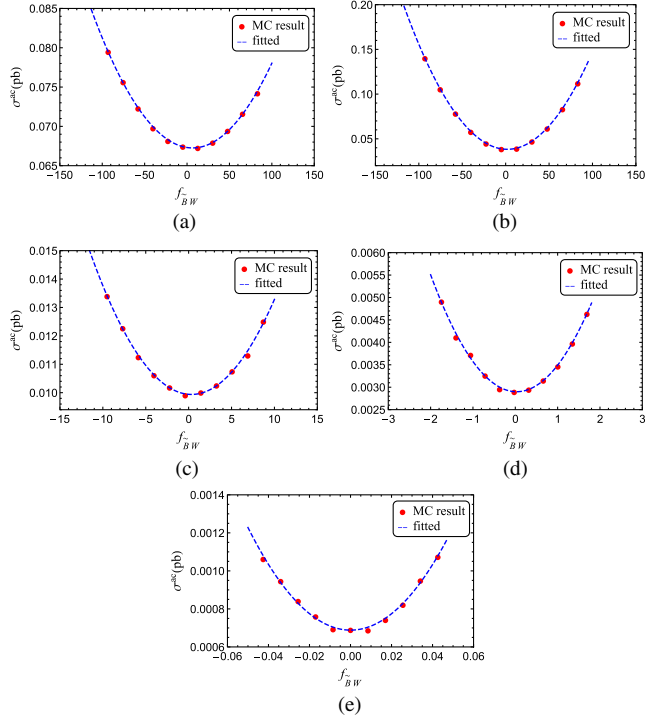


FIG. 20. Cross sections as functions of f_{BW} (TeV^{-4}) for $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$. (a) 250 GeV. (b) 360 GeV. (c) 500 GeV. (d) 1 TeV. (e) 3 TeV.

luminosity, these constraints are further improved, achieving orders of magnitude around 10^{-1} TeV^{-4} and 10^{-2} TeV^{-4} , respectively.

E. Selection of $e^+e^- \rightarrow jj\nu\bar{\nu}$

The signal $jj\nu\bar{\nu}$ from the $\mathcal{O}_{BW}$ operator originates from the diagrams depicted in Fig. 21. In this channel, di-boson production provides the dominant contribution, with the $jj\nu\bar{\nu}$ decay mode representing approximately 28% of the ZZ final states. The key signature of this decay is a large invisible energy component alongside a pair of jets with a reconstructed mass close to the Z boson mass.

The most challenging backgrounds arise from multiple sources: di-boson production, with one boson undergoing an invisible decay, di-jet events accompanied by energetic vector boson decays, single resonant $Z\nu\bar{\nu}$ production, and the VBF process, as illustrated in Fig. 22.

The $\ell^+\ell^-\nu\bar{\nu}$ channel discussed in the previous section exhibits very clean signals at e^+e^- colliders. However, hadronic decay modes hold certain advantages due to their

TABLE XIII. The fitted values of $\sigma_{\text{NP}}^{\text{ac}}$ ($\text{fb} \times \text{TeV}^8$) and δ_{int} ($\text{fb} \times \text{TeV}^4$) for $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$ at each energy point.

$\sqrt{s}$	250 GeV	360 GeV	500 GeV	1 TeV	3 TeV
δ_{int}	-0.0161	-0.0429	-0.0241	-0.0403	-0.0132
$\sigma_{\text{NP}}^{\text{ac}}$	0.0012	0.0113	0.0358	0.6316	215.60

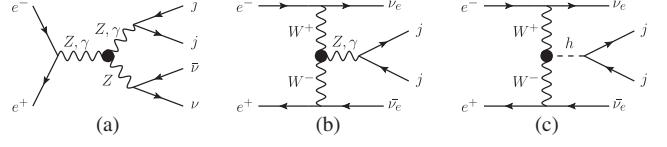


FIG. 21. Typical Feynman diagrams which contribute to the process $e^+e^- \rightarrow jj\nu\bar{\nu}$ for signal events. The di-boson diagram is shown in (a), while the VBF diagrams are shown in (b) and (c).

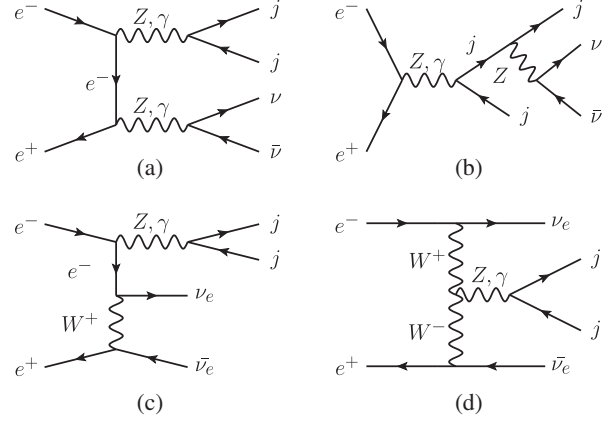


FIG. 22. The Feynman diagrams, which contribute to the process $e^+e^- \rightarrow jj\nu\bar{\nu}$ for background events.

larger branching ratios. For the nTGC signal, events were generated using the largest coefficients listed in Table II. The kinematic analysis initially selected events containing a hadronic jet with transverse momentum $p_T^j = 10 \text{ GeV}$ and pseudorapidity $|\eta^j| < 2.0$. Following these basic cuts, we required a minimum of two jets in each event, and applied a jet separation criterion of $\Delta R_{jj} = 0.2$, as previously discussed.

The properties of hadronic and invisible decays can effectively differentiate the signal from the SM background. Figure 23 presents the normalized distributions of p_T^j/E_{cm} ,

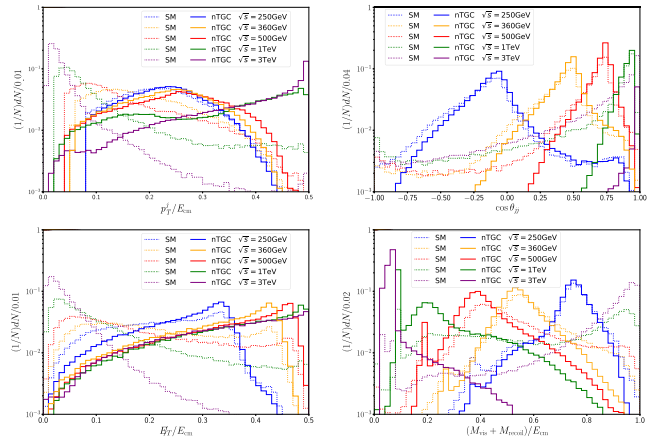


FIG. 23. The normalized distributions of p_T^j/E_{cm} , $\cos\theta_{jj}$, E_T/E_{cm} , and $(M_{\text{vis}} + M_{\text{recoil}})/\sqrt{s}$ for $e^+e^- \rightarrow jj\nu\bar{\nu}$.

$\cos \theta_{jj}$, $\vec{E}_T/E_{\text{cm}}$, and $(M_{\text{vis}} + M_{\text{recoil}})/E_{\text{cm}}$. In signal events, the Z or Higgs bosons are generally more energetic than those in background events, leading to a smaller angle between the jets and higher p_T^j values in the signal, predominantly in the high-energy region. It is worth noting that the vector bosons in the high-energy region are highly boosted. In that case, the two jets merged into one, such as the distributions of p_T^j/E_{cm} for $\sqrt{s} \geq 1$ TeV shown in Fig. 23. The transverse missing energy $\vec{E}_T$ serves to approximate the transverse energy of the Z boson in its $\nu\bar{\nu}$ decay mode. As expected, $\vec{E}_T$ in the signal is generally larger than in the background. These distinctions become increasingly pronounced with rising collision energies, particularly at TeV-scale colliders.

At several hundred GeV, where ZZ production is the main contributor to both signal and background, the peak of $(M_{\text{vis}} + M_{\text{recoil}})$ primarily appears near $2M_Z$. As the collision energy increases, the non- ZZ production contributions to the background become more significant, shifting the $(M_{\text{vis}} + M_{\text{recoil}})$ distribution to higher-mass regions. This shift improves the background rejection performance of $(M_{\text{vis}} + M_{\text{recoil}})$ at high-energy colliders.

The specific values of the event selection strategy and corresponding cut efficiencies are presented in Table XV. For the final state $jj\nu\bar{\nu}$, the cross section fitting functions for $\mathcal{O}_{\tilde{B}W}$ are shown in Fig. 24, and the fitting coefficients for the interference terms are provided in Table XVI. The longitudinal polarization contribution of vector bosons

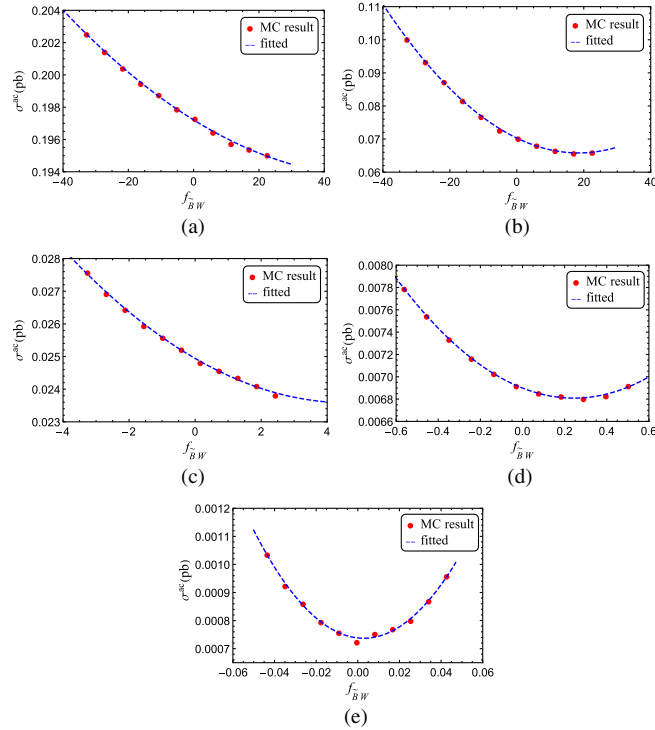


FIG. 24. Cross sections as a functions of f_{BW} (TeV^{-4}) for the process $e^+e^- \rightarrow jj\nu\bar{\nu}$ at different c.m. energies. (a) 250 GeV. (b) 360 GeV. (c) 500 GeV. (d) 1 TeV. (e) 3 TeV.

dominates the SM background, and this contribution is suppressed by $\sqrt{s}$ due to an exact cancellation with the VBF process. In contrast, the cross section of the signal induced by nTGCs increases with rising $\sqrt{s}$. The energy-dependent interference term depends on the combined effect of this enhancement and depression. Consequently, as shown in Fig. 24, the interference between the nTGC signal and the SM background for the process $e^+e^- \rightarrow jj\nu\bar{\nu}$ decreases with increasing energy. Table XVII presents the expected constraints on the nTGC operators, obtained by fitting the observed $jj\nu\bar{\nu}$ events for 2σ , 3σ , and 5σ significance levels at future e^+e^- colliders.

Summarizing the expected constraints obtained for the five channels in this section, the $4j$ signal provides the strongest constraint among the five possible nTGC signals produced by the $e^+e^- \rightarrow ZZ$ process. Potential improvements in these constraints through combined analyses are discussed in Sec. III F.

F. Constraints on the coefficients with combined results

By combining the results obtained from the five decay modes analyzed in this section, it is possible to extract more competitive constraints beyond those provided by the each decay mode alone, based on the formula

$$\mathcal{S}_{\text{stat}}^{\text{comb}} = \sqrt{\sum \mathcal{S}_{\text{stat}}^2(i)}, \quad (14)$$

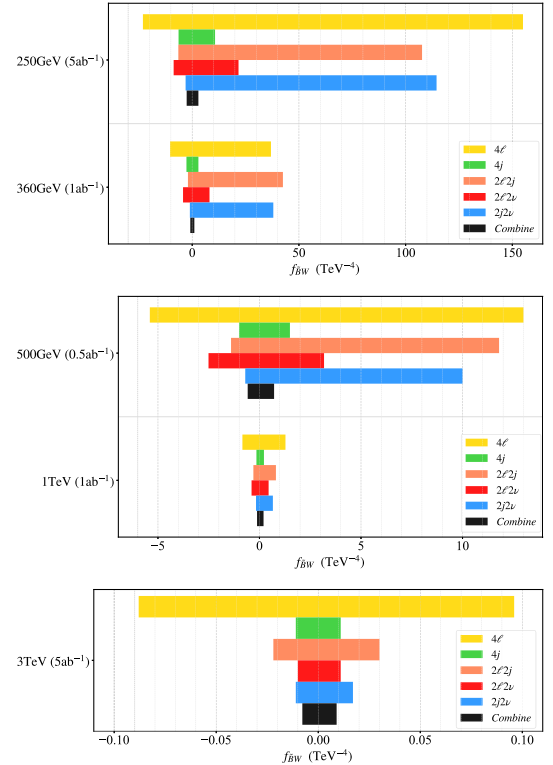


FIG. 25. Constraints on f_{BW} (TeV^{-4}) for different colliders with combined results at the 95% C.L.

TABLE XIV. The expected constraints on $f_{\tilde{B}W}$ (TeV^{-4}) for $e^+e^- \rightarrow \ell^+\ell^-\nu\bar{\nu}$.

S_{stat}	$\sqrt{s}$ (GeV)				
	250	360	500	1000	3000
2	[-8.7, 21.7]	[-4.3, 8.1]	[-2.51, 3.18]	[-0.39, 0.45]	[-0.010, 0.011]
3	[-11.5, 24.6]	[-5.6, 9.4]	[-3.15, 3.83]	[-0.48, 0.55]	[-0.013, 0.013]
5	[-16.2, 29.2]	[-7.7, 11.5]	[-4.19, 4.87]	[-0.64, 0.70]	[-0.017, 0.017]

TABLE XV. The event selection strategy and cross sections of $e^+e^- \rightarrow jj\nu\bar{\nu}$ (fb) after cuts.

	$\sqrt{s} = 250$ GeV		$\sqrt{s} = 360$ GeV	
	SM	NP	SM	NP
Basic Cuts	198.5	0.614	120.4	8.407
$E_T/E_{\text{cm}} > 0.1$	173.1	0.574		
$E_T/E_{\text{cm}} > 0.2$			62.61	6.996
$(M_{\text{vis}} + M_{\text{recoil}}) < 0.9E_{\text{cm}}$	168.6	0.567		
$(M_{\text{vis}} + M_{\text{recoil}}) < 0.8E_{\text{cm}}$			60.92	6.928
$\cos\theta_{jj} > -0.6$	162.5	0.55		
$\cos\theta_{jj} > -0.2$			59.93	6.894
Efficiency ϵ	80.3%	89.9%	49%	82%

	$\sqrt{s} = 500$ GeV		$\sqrt{s} = 1$ TeV		$\sqrt{s} = 3$ TeV	
	SM	NP	SM	NP	SM	NP
Basic Cuts	84.46	0.565	46.12	0.558	44.09	0.385
$E_T/E_{\text{cm}} > 0.3$	21.50	0.379	6.230	0.387	0.805	0.256
$(M_{\text{vis}} + M_{\text{recoil}}) < 0.7E_{\text{cm}}$	21.20	0.377				
$(M_{\text{vis}} + M_{\text{recoil}}) < 0.6E_{\text{cm}}$			5.915	0.381		
$(M_{\text{vis}} + M_{\text{recoil}}) < 0.3E_{\text{cm}}$					0.657	0.249
$\cos\theta_{jj} > 0.2$	20.80	0.375				
$\cos\theta_{jj} > 0.5$			5.787	0.379		
$\cos\theta_{jj} > 0.7$					0.648	0.249
Efficiency ϵ	24%	66%	12%	68%	1.4%	65%

TABLE XVI. The fitted values of $\sigma_{\text{NP}}^{\text{ac}}$ ($\text{fb} \times \text{TeV}^8$) and $\hat{\sigma}_{\text{int}}$ ($\text{fb} \times \text{TeV}^4$) for $e^+e^- \rightarrow jj\nu\bar{\nu}$ at each energy point.

$\sqrt{s}$	250 GeV	360 GeV	500 GeV	1 TeV	3 TeV
$\hat{\sigma}_{\text{int}}$	-0.1252	-0.4851	-0.6432	-0.7372	-0.7797
$\sigma_{\text{NP}}^{\text{ac}}$	0.0011	0.0131	0.0660	1.5071	137.88

where $S_{\text{stat}}(i)$ is the signal significance function for each decay mode of ZZ production. The numerical results of expected coefficient constraints, combining data from the five decay patterns, are presented in Table XVIII and Fig. 25. These constraints provide insight into the behavior of the coefficients for the $\mathcal{O}_{\tilde{B}W}$ operator across different $\sqrt{s}$ ranging from 250 GeV to 3 TeV. For $\sqrt{s} = 250$ GeV and 360 GeV, the $2j2\nu$ and $4j$ channels provide significant

TABLE XVII. The expected constraints on $f_{\tilde{B}W}$ (TeV^{-4}) for $e^+e^- \rightarrow jj\nu\bar{\nu}$.

S_{stat}	$\sqrt{s}$ (GeV)				
	250	360	500	1000	3000
2	[-3.1, 114.5]	[-1.1, 38.0]	[-0.7, 10.0]	[-0.17, 0.66]	[-0.011, 0.017]
3	[-4.6, 116.0]	[-1.6, 38.5]	[-1.0, 10.3]	[-0.23, 0.72]	[-0.014, 0.020]
5	[-7.5, 118.9]	[-2.6, 39.5]	[-1.7, 10.9]	[-0.34, 0.83]	[-0.019, 0.024]

TABLE XVIII. Combined constraints on $f_{\bar{B}W}$ (TeV^{-4}) for five signal channels of ZZ production.

S_{stat}	$\sqrt{s}$ (GeV)				
	250	360	500	1000	3000
2	[-2.43, 2.64]	[-0.90, 0.96]	[-0.52, 0.61]	[-0.14, 0.36]	[-0.0079, 0.0093]
3	[-3.57, 4.02]	[-1.31, 1.42]	[-0.75, 0.92]	[-0.19, 0.47]	[-0.0099, 0.0112]
5	[-5.69, 6.88]	[-2.05, 2.28]	[-1.14, 1.49]	[-0.28, 0.59]	[-0.0131, 0.0145]

TABLE XIX. Projected lower limit on Λ (GeV) with $c_{\bar{B}W} = 1$, consistent with the results in Table XVIII.

S_{stat}	$\sqrt{s}$ (GeV)				
	250	360	500	1000	3000
2	785	1010	1132	1291	3220
3	706	916	1021	1208	3074
5	617	814	905	1141	2882

constraint capabilities for the combined results. For the TeV scale, the $2\ell 2\nu$ channel plays a dominant role in determining the combined constraints. Table XIX presents the projected limit on Λ (GeV) with $c_{\bar{B}W} = 1$, which is consistent with the results shown in Table XVIII.

We further made comparisons between the LHC constraints on $\mathcal{O}_{\bar{B}W}$ at $\sqrt{s} = 13$ TeV [81] and our predictions. The observed 95% C.L. on the $f_{\bar{B}W}$ is $(-1.1, 1.1)$ via measurement of the $Z\gamma \rightarrow \nu\bar{\nu}\gamma$ production. We found that with an appropriate event selection strategy, the $e^+e^- \rightarrow ZZ$ process signal with decays at 250 GeV is slightly less sensitive to nTGCs compared to measurements at the LHC. However, as the collision energy increases to 360 GeV, the detection capability of future e^+e^- colliders for nTGCs is anticipated to surpass that of the LHC. The expected constraints on nTGCs at $\sqrt{s} = 1$ TeV with $\mathcal{L} = 1 \text{ ab}^{-1}$ can be improved by an order of magnitude over the LHC of 13 TeV, and for those at $\sqrt{s} = 3$ TeV and $\mathcal{L} = 5 \text{ ab}^{-1}$, the expected constraints at $S_{\text{stat}} = 2$ are about 2 orders of magnitude stronger than those at the $\sqrt{s} = 13$ TeV LHC at the 95% C.L. We anticipate that machine learning techniques can further enhance the sensitivity of nTGC measurements.

IV. CONCLUSIONS

Precision measurements of the self-couplings of electro-weak gauge bosons are essential for testing the EWSB mechanism and for probing potential NP. nTGCs represent a possible class of anomalous multiboson couplings within the framework of the SMEFT. Unlike the more commonly studied dimension-6 TGCs in SMEFT, nTGCs arise first from gauge-invariant dimension-8 operators. nTGCs can contribute to the process $e^+e^- \rightarrow ZZ$ through the ZZZ and $ZZ\gamma$ vertices. The potential contributions of nTGCs to this process can be investigated at future e^+e^- colliders, such as

CEPC, FCC-ee, ILC, and CLIC. In this paper, we have demonstrated that these future e^+e^- colliders possess the capability to measure nTGCs through ZZ -pair production.

We first studied the influence of initial beam polarization on the measurement of $e^+e^- \rightarrow ZZ$. Specific polarization settings can enhance the cross section at an e^+e^- collider. For instance, the cross section $\sigma_{+,+}$ of nTGCs can be twice that of the unpolarized cross section. In the case of SM contributions, the polarized cross sections σ_{-+} and σ_{+-} were approximately 2.5 and 1.5 times larger, respectively, than the unpolarized cross section. Consequently, we adopted a left-hand polarized electron beam (+80%) and right-hand polarized positron (-80%) in our simulations and analyses for future e^+e^- colliders.

Additionally, we presented a kinematic analysis of the sensitivity of measurements of five distinct final states of ZZ production induced by nTGCs at future e^+e^- colliders. To this end, we studied the kinematic features of the signal and background events using MC simulations and proposed an event selection strategy for nTGCs. The expected constraints on the coefficients of dimension-8 operators were calculated by combining results from future e^+e^- colliders with c.m. energies of 250 GeV, 360 GeV, 500 GeV, 1 TeV, and 3 TeV. As expected, higher collision energies led to greater sensitivities. The greatest sensitivity to ZZ production was generally provided by the $ZZ \rightarrow jj\nu\bar{\nu}$ decay channel, while combining results from multiple channels further enhanced the sensitivity to nTGC operators.

We quantitatively compared existing 13 TeV LHC limits on $\mathcal{O}_{\bar{B}W}$ with our results. The CEPC showed comparable sensitivity to the LHC in probing nTGCs. When the collision energy increased from 250 GeV to 360 GeV, the sensitivity of both CEPC and FCC-ee improved by roughly a factor of 3. Looking ahead, higher-energy e^+e^- colliders ($\sqrt{s} = 1\text{--}3$ TeV) are expected to surpass the LHC in sensitivity. Measurements of nTGCs at the ILC and CLIC could improve current constraints on dimension-8 operators by 1 to 2 orders of magnitude. This highlights the strong potential of these colliders to probe the SMEFT framework and complements ongoing searches for aQGCs [20–25,39–42] and gQGCs [27].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing,

and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: FEYNMAN RULES OF ANOMALOUS TRI-BOSON VERTICES

The Feynman rules of anomalous tri-boson vertices for the $e^+e^- \rightarrow ZZ$ process are given below. The Levi-Civita tensor is denoted by $\epsilon^{\mu\nu\alpha\beta}$, $c_W = \cos \theta_W$, and $s_W = \sin \theta_W$, and Hermitian conjugate fields are marked with $\dagger$,

$$V_{WW\gamma} = \frac{f_{\bar{B}W} e v^2 c_W}{4s_W^2} (\partial_\beta A_\alpha - \partial_\alpha A_\beta) [\partial_\rho W_\nu (\partial_\rho W_\mu^\dagger - \partial_\mu W_\rho^\dagger) + \partial_\nu W_\rho (\partial_\rho W_\mu^\dagger - \partial_\mu W_\rho^\dagger) + (\partial_\rho W_\mu - \partial_\mu W_\rho) (\partial_\rho W_\nu^\dagger + \partial_\nu W_\rho^\dagger)] \epsilon^{\mu\nu\alpha\beta}, \quad (\text{A1})$$

$$V_{WWZ} = -\frac{f_{\bar{B}W} e v^2}{4} [\partial_\rho W_\nu (\partial_\rho W_\mu^\dagger - \partial_\mu W_\rho^\dagger) + \partial_\nu W_\rho (\partial_\rho W_\mu^\dagger - \partial_\mu W_\rho^\dagger) + (\partial_\rho W_\mu - \partial_\mu W_\rho) (\partial_\rho W_\nu^\dagger + \partial_\nu W_\rho^\dagger)] (\partial_\beta Z_\alpha - \partial_\alpha Z_\beta) \epsilon^{\mu\nu\alpha\beta}, \quad (\text{A2})$$

$$V_{ZZZ} = -\frac{f_{\bar{B}W} e v^2}{4} (\partial_\beta Z_\alpha - \partial_\alpha Z_\beta) (\partial_\rho Z_\mu - \partial_\mu Z_\rho) (\partial_\rho Z_\nu + \partial_\nu Z_\rho) \epsilon^{\mu\nu\alpha\beta}, \quad (\text{A3})$$

$$V_{Z\gamma\gamma} = \frac{f_{\bar{B}W} e v^2}{4} (\partial_\beta A_\alpha - \partial_\alpha A_\beta) (\partial_\rho A_\mu - \partial_\mu A_\rho) (\partial_\rho Z_\nu + \partial_\nu Z_\rho) \epsilon^{\mu\nu\alpha\beta}, \quad (\text{A4})$$

$$V_{ZZ\gamma} = \frac{f_{\bar{B}W} e v^2}{4c_W s_W} [s_W^2 \partial_\mu A_\rho (\partial_\beta Z_\alpha - \partial_\alpha Z_\beta) - s_W^2 \partial_\rho A_\mu (\partial_\beta Z_\alpha - \partial_\alpha Z_\beta) + c_W^2 (\partial_\beta A_\alpha - \partial_\alpha A_\beta) (\partial_\rho Z_\mu - \partial_\mu Z_\rho)] (\partial_\rho Z_\nu + \partial_\nu Z_\rho) \epsilon^{\mu\nu\alpha\beta}. \quad (\text{A5})$$

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