

Degrees of freedom of multiway junctions in three dimensional gravity

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(Received 22 October 2025; revised 19 February 2026; accepted 28 April 2026; published 22 May 2026)

We demonstrate that n -way junctions in three-dimensional gravity correspond to coupled $n - 1$ strings each satisfying the Nambu-Goto equation in the smoothened background and with sources consisting of Monge-Ampère like terms which couple the strings. For $n \geq 3$, these $n - 1$ degrees of freedom survive the tensionless limit implying that matterlike behavior can arise out of *pure* gravity. We interpret these stringy degrees of freedom of gravitational junctions holographically in terms of wave packets, which collectively undergo perfect reflection at the multi-interface in the dual conformal field theory.

DOI: [10.1103/yy6l-1b4b](https://doi.org/10.1103/yy6l-1b4b)

I. INTRODUCTION

Geometrizing matter provides a route towards understanding its fundamental origin. String theory concretely exemplifies this by demonstrating that both matter and gravity can emerge from the vibrations of a quantized fundamental string [1–4]. Alternatively, one can ask if matterlike degrees of freedom can emerge from gravity

itself. This possibility can be probed by studying gravitational multiway junctions that glue two [5–8] or more [9,10] spacetimes. In this article, we consider general multiway gravitational junctions, both with and without tension, and explore if matterlike degrees of freedom can emerge even in the tensionless limit from *pure* gravity. We generalize the earlier result showing that the nonlinear Nambu-Goto equation for a fundamental string, with tension-induced corrections, emerges just from the junction conditions of a two-way junction gluing two three-dimensional Einstein manifolds [11]. We demonstrate that junctions gluing three or more such spacetimes have nontrivial degrees of freedom corresponding to coupled Nambu-Goto equations with non-trivial solutions even in the tensionless limit. We also discuss extensions to four-dimensional settings.

The holographic correspondence [12–14], which relates a strongly coupled quantum field theory to a classical gravitational theory in one higher dimension, implies that

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the gravitational junction gluing multiple three-dimensional locally anti-de Sitter (AdS) spacetimes can be translated to an interface between quantum wires described by strongly coupled two-dimensional field theories [5–10]. The presence of degrees of freedom of the gravitational junction imply that the interface can be viewed as a tunable quantum processor [15]. Therefore, the classical gravitational junction is of fundamental importance for a deeper understanding of how spacetime emerges from the quantum information structure of strongly interacting quantum field theories [16–19]. Furthermore, it opens new directions in the understanding of quantum information processing in many-body systems. Motivated by this, we focus on gravitational junctions gluing three-dimensional locally AdS spacetimes.

II. GRAVITATIONAL MULTIWAY JUNCTIONS

A general three-dimensional Einstein spacetime $\mathcal{M}$ is locally flat, locally AdS₃, or locally dS₃. Consider n identical copies, $\mathcal{M}_i$ of $\mathcal{M}$, each of which is divided into two parts, $\mathcal{M}_{iL}$ and $\mathcal{M}_{iR}$ along *distinct* boundaries Σ_i . As each copy $\mathcal{M}_i$ inherits the coordinate charts of $\mathcal{M}$, we can clearly distinguish between the left and right halves $\mathcal{M}_{iL}$ and $\mathcal{M}_{iR}$ in each $\mathcal{M}_i$. Discarding $\mathcal{M}_{iL}$ or $\mathcal{M}_{iR}$ for each copy i , we glue the n fragments $\mathcal{M}_{i\alpha_i}$ (with $\alpha_i = L, R$) to form a spacetime $\tilde{\mathcal{M}}$ which satisfies Einstein's equations, including the necessary junction conditions [20]. The junction in $\tilde{\mathcal{M}}$ is the codimension one hypersurface Σ , each point P of which is formed by the identification of corresponding points P_i in Σ_i . Therefore, each Σ_i should be regarded as the image of Σ in the corresponding $\mathcal{M}_i$. See Fig. 1. The *embeddings* of Σ_i in $\mathcal{M}_i$ and the *identifications* of the points P_i of Σ_i should be determined so that the junction conditions are satisfied at Σ . Our purpose is to

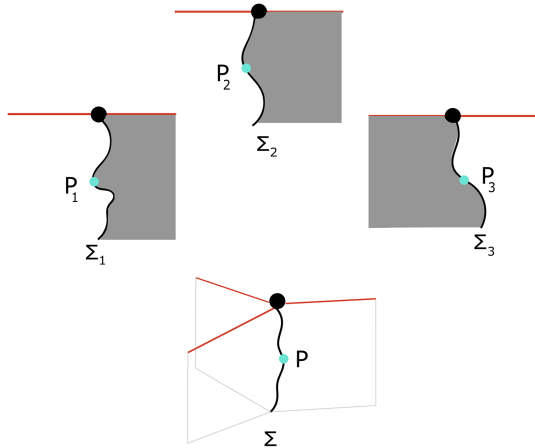


FIG. 1. A three-way junction (below) gluing asymptotically AdS spaces (on top). Red line is the asymptotic boundary. Black dot is the interface in the dual CFT. The gray regions on left/right of Σ_i ($i = 1, 2, 3$) are excised. The points P_i on Σ_i are identified with P on Σ , the gravitational junction in $\tilde{\mathcal{M}}$.

investigate the nature of general solutions. Before discussing the junction conditions explicitly, it is useful to set up our notations and identify the variables that need to be determined. Let the coordinates of $\mathcal{M}$ be t, z and x (with t , the time coordinate, and z and x the spatial coordinates), and so the fragments $\mathcal{M}_{i\alpha_i}$ have coordinates t_i, z_i and x_i . The boundaries Σ_i of $\mathcal{M}_{i\alpha_i}$ that are glued at the junction Σ assume the form,

$$\Sigma_i: x_i = f_i(t_i, z_i), \quad i = 1, 2, \dots, n. \quad (1)$$

We identify the points P_i in Σ_i with coordinates (τ_i, σ_i) with each point P in Σ with coordinates (τ, σ) . We fix the (world sheet) gauge freedom corresponding to the choice of coordinates (τ, σ) by

$$\tau(P) = \frac{1}{n} \sum_{i=1}^n t_i(P_i), \quad \sigma(P) = \frac{1}{n} \sum_{i=1}^n z_i(P_i), \quad (2)$$

with $t_i(P_i)$ and $z_i(P_i)$ being the time and longitudinal coordinates of the points P_i in Σ_i that are identified with P . We note that above and in (1), t_i, z_i and thus, x_i are functions of σ and τ , the world sheet coordinates. After gauge-fixing (2) (up to residual world sheet gauge symmetries to be discussed later), we are left with $2(n-1)$ independent variables $t_i - t_j$ and $z_i - z_j$ ($i \neq j$), which are the relative shifts of space and time as we move from $\mathcal{M}_{i\alpha_i}$ to $\mathcal{M}_{j\beta_j}$ across the junction Σ . Along with the n embedding functions f_i of Σ_i , we have in total $3n-2$ variables which need to be determined as functions of τ and σ .

The full gravitational action which gives the junction conditions is

$$S = \frac{1}{16\pi G_N} \int_{\tilde{\mathcal{M}}} d^3x \sqrt{-g} (R - 2\Lambda) + T_0 \int_{\Sigma} d\tau d\sigma \sqrt{-\gamma} + \text{GHY terms}. \quad (3)$$

Note that the bulk metric g is the *only* degree of freedom in this action, and the second line consists of Gibbons-Hawking-York (GHY) terms corresponding to the boundaries of $\mathcal{M}_{i\alpha_i}$. The action (3) assumes that the induced metrics γ_i on Σ_i are identical at each point $P(\tau, \sigma)$ on Σ , thus, defining the world sheet metric via

$$\gamma_{\mu\nu}(\tau, \sigma) := \gamma_{1,\mu\nu}(\tau, \sigma) = \dots = \gamma_{n,\mu\nu}(\tau, \sigma). \quad (4)$$

Varying the action with respect to g away from the junction, we obtain that each fragment $\mathcal{M}_{i\alpha_i}$ is an Einstein manifold. The variation with respect to g at the junction gives

$$\sum_{i=1}^n (-1)^{s(\alpha_i)} (K_{i,\mu\nu} - K_i \gamma_{i,\mu\nu}) = 8\pi G_N T_0 \gamma_{\mu\nu}, \quad (5)$$

with $s(\alpha_i) = 0$ if $\alpha_i = L$ and $s(\alpha_i) = 1$ if $\alpha_i = R$ (in each fragment, $x < 0$ and $x > 0$ denotes the left and right halves,

respectively). Above, $K_{i,\mu\nu}$ is the extrinsic curvature of Σ_i in $\mathcal{M}_{i\alpha_i}$ and $K_i = \gamma^{\mu\nu} K_{i,\mu\nu}$. The bulk diffeomorphism symmetry implies the conservation of the *total* Brown-York tensor of the junction, which is the left-hand side of (5). Therefore, the above equation has only one independent component.

We have $3n - 3$ independent equations from (4) and only one from (5). Thus, the total number of independent junction conditions is $3n - 2$, matching the total number of variables *exactly*. We will show that the general solutions are in one-to-one correspondence with appropriately coupled Nambu-Goto equations, and for $n \geq 3$, we can also obtain dynamical variables in the tensionless limit implying that matter like degrees of freedom can emerge from pure gravity.

Generalization of our results to higher dimensions is not straightforward because the number of independent junction conditions for the (codimension one) junction exceeds the number of variables, precluding emergent degrees of freedom in this case. Nevertheless, matter like degrees of freedom in the form of coupled strings can emerge also from pure gravity in higher dimensions via embedding of the multiway junction between three-dimensional spaces. See the Appendix for more details.

In order to solve the dynamics of the gravitational junctions in $3D$, it is useful to appropriately parametrize the $3n - 2$ variables. For simplicity, let us first assume that $\alpha_i = L$ for all i . We define $3n - 3$ independent relative shifts of the time (τ_{d_i}), the longitudinal coordinate (σ_{d_i}) and the transverse coordinate (x_{d_i}) across the junction as follows:

$$\begin{aligned} \tau_{d_i} &= \begin{cases} \frac{1}{n}(t_n - t_{i+1}) & \text{for } i = 1, \dots, n-2 \\ \frac{1}{n}(t_n - t_1) & \text{for } i = n-1 \end{cases}, \\ \sigma_{d_i} &= \begin{cases} \frac{1}{n}(z_n - z_{i+1}) & \text{for } i = 1, \dots, n-2 \\ \frac{1}{n}(z_n - z_1) & \text{for } i = n-1 \end{cases}, \\ x_{d_i} &= \begin{cases} \frac{1}{n}(x_n - x_{i+1}) & \text{for } i = 1, \dots, n-2 \\ \frac{1}{n}(x_n - x_1) & \text{for } i = n-1 \end{cases}. \end{aligned} \quad (6)$$

The above and the averaged transverse coordinate,

$$x_s = \frac{1}{n} \sum_i x_i, \quad (7)$$

give the requisite $3n - 2$ functions of τ and σ that need to be determined.

If a subset of the n fragments, $\mathcal{M}_{i\alpha_i}$ are $\mathcal{M}_{iR}$ instead of $\mathcal{M}_{iL}$, we simply reverse the sign of the transverse coordinate x_i in the parametrization (6) for the values of i in this subset.

For $n = 2$, the general solutions of the junction conditions in three dimensional gravity [11] are in one-to-one

correspondence with the solutions of Nambu-Goto equation in $\mathcal{M}$ up to six rigid parameters which are related to world sheet and spacetime isometries (to be described later). If we glue $\mathcal{M}_{1L}$ and $\mathcal{M}_{2R}$, then for a generic solution of the junction conditions:

(1) The hypersurface

$$\Sigma_{NG}: t = \tau, \quad z = \sigma, \quad x = x_s(\tau, \sigma)$$

in $\mathcal{M}$ (whose embedding is the average of Σ_1 and Σ_2) corresponds to a solution of the nonlinear Nambu-Goto equations for a world sheet in $\mathcal{M}$ when the tension T_0 and the rigid parameters vanish, and

(2) x_s is the *only* degree of freedom implying that x_d , τ_d and σ_d are completely determined as functions of τ , σ , the tension and the rigid parameters for any given choice of the solution of the Nambu-Goto equation in $\mathcal{M}$ corresponding to x_s .

If we glue $\mathcal{M}_{1R}$ and $\mathcal{M}_{2R}$ (or $\mathcal{M}_{1L}$ and $\mathcal{M}_{2L}$), then the roles of x_s and x_d are reversed. In what follows, we will generalize these results and the perturbative analysis in [11] for $n \geq 3$. We will focus our discussion on the n -way junction in AdS_3 with a view to its holographic interpretation.

III. PERTURBATIVE ANALYSIS

Let us first consider $n \geq 3$ fragments of locally AdS_3 spacetimes, each of which is a copy of the Bañados-Teitelboim-Zanelli (BTZ) black brane [21,22] ($\mathcal{M}$),

$$ds^2 = \frac{\ell^2}{z^2} \left(-(1 - Mz^2)dt^2 + \frac{dz^2}{1 - Mz^2} + dx^2 \right), \quad (8)$$

which is dual to the thermal state of the dual CFT (more on this later). For convenience, we set $\ell = 1$. We define $\lambda = 8\pi G_N T_0$ and consider $\lambda = \mathcal{O}(\epsilon)$. We solve the general junction conditions perturbatively in λ for the gluing of the n fragments $\mathcal{M}_{iL}$. At the zeroth order, we choose the simplest (permutation-symmetric) solution,

$$\begin{aligned} x_i &= x_0 + \mathcal{O}(\epsilon) \Rightarrow x_s = x_0 + \mathcal{O}(\epsilon), \quad x_{d_i} = \mathcal{O}(\epsilon), \\ \text{and } \tau_{d_i} &= \sigma_{d_i} = \mathcal{O}(\epsilon^2), \end{aligned} \quad (9)$$

so that we find at least one static solution to all orders in λ which has the interpretation of a *conformal* defect joining n copies of the dual CFT living on semi-infinite lines (wires).

At the first order in ϵ , the solutions for τ_{d_i} and σ_{d_i} correspond to the three world sheet Killing vectors whereas the solution for $x_s - x_0$ feature the three spacetime isometries of $\mathcal{M}$ which do not preserve the zeroth order hypersurface $x_s = x_0$. In what follows, we set these $3n$ rigid parameters to zero. We also look for solutions in which

$$\lim_{\sigma \rightarrow 0} x_i = x_0 \Rightarrow \lim_{\sigma \rightarrow 0} x_s = x_0, \quad \lim_{\sigma \rightarrow 0} x_{d_i} = 0, \quad (10)$$

corresponding to the Dirichlet boundary condition at the boundary of AdS. Such solutions have the interpretation of a n way interface in the dual CFT.

We find that

$$x_s = x_0 + \frac{\lambda}{n}\sigma + \mathcal{O}(\epsilon^3). \quad (11)$$

Remarkably, the $n-1$ variables x_{d_i} correspond to coupled Nambu-Goto equations in $\mathcal{M}$. Up to third order in ϵ , these explicitly are

$$\lambda \mathcal{N}_i = n \left((n-2) \mathcal{X}_i - \sum_{j \neq i} \mathcal{Z}_{ij} \right) + \mathcal{O}(\epsilon^3), \quad (12)$$

for $i = 1, \dots, n-1$, with primes and dots denoting ∂_σ and ∂_τ , respectively, and

$$\mathcal{N}_i = (1 - M\sigma^2)((1 - M\sigma^2)(-2x'_{d_i} + \sigma x''_{d_i}(1 - M\sigma^2)) - \sigma \ddot{x}_{d_i}), \quad (13)$$

corresponding to the linearized Nambu-Goto equation for the hypersurface,

$$\Sigma_{NG_i}: t = \tau, \quad z = \sigma, \quad x = x_{d_i}(\tau, \sigma), \quad (14)$$

in $\mathcal{M}$, while

$$\begin{aligned} \mathcal{X}_i &= x_{d_i}^{\prime 2}(1 - M\sigma^2)^2 \\ &+ \sigma x'_{d_i}(1 - M\sigma^2)((1 - M\sigma^2)^2 x''_{d_i} + \ddot{x}_{d_i}) \\ &+ \sigma^2((M\sigma \dot{x}_{d_i} + (1 - M\sigma^2)\dot{x}'_{d_i})^2 \\ &- (1 - M\sigma^2)^2 x''_{d_i} \ddot{x}_{d_i}), \end{aligned} \quad (15)$$

and

$$\begin{aligned} \mathcal{Z}_{ij} &= 2x'_{d_i}x'_{d_j}(1 - M\sigma^2)^2 \\ &+ \sigma x'_{d_i}(1 - M\sigma^2)((1 - M\sigma^2)^2 x''_{d_j} + \ddot{x}_{d_j}) \\ &+ \sigma x'_{d_j}(1 - M\sigma^2)((1 - M\sigma^2)^2 x''_{d_i} + \ddot{x}_{d_i}) \\ &+ \sigma^2(2(M\sigma \dot{x}_{d_i} + (1 - M\sigma^2)\dot{x}'_{d_i})(M\sigma \dot{x}_{d_j} + (1 - M\sigma^2)\dot{x}'_{d_j}) \\ &- (1 - M\sigma^2)^2(x''_{d_i}\ddot{x}_{d_j} + x''_{d_j}\ddot{x}_{d_i})), \end{aligned} \quad (16)$$

are Monge-Ampère like terms coupling the Nambu-Goto equations. Notice that the rhs of (12) vanishes for $n = 2$.

The physical solutions corresponding to a holographic interface of n wires should satisfy the Dirichlet boundary conditions (10). While x_s is given by (11), the solutions of x_{d_i} with such Dirichlet boundary conditions are of different nature when $\lambda > 0$ as opposed to $\lambda = 0$ for $n \geq 3$, as turning on λ is a singular perturbation of (12). (For $n = 2$, the equation for x_d trivially vanishes when $\lambda = 0$.) For illustration, consider $n = 3$. In this case, when $\lambda \neq 0$, the solutions of (12) to $\mathcal{O}(\sigma^7, \epsilon^2)$ are

$$\begin{aligned} x_{d_1} &= \mathcal{A}_1 \sigma^3 + \frac{3\mathcal{A}_1(2M\lambda + 9\mathcal{A}_1 - 18\mathcal{A}_2) + \lambda \ddot{\mathcal{A}}_1}{10\lambda} \sigma^5, \\ x_{d_2} &= \mathcal{A}_2 \sigma^3 + \frac{3\mathcal{A}_2(2M\lambda - 18\mathcal{A}_1 + 9\mathcal{A}_2) + \lambda \ddot{\mathcal{A}}_2}{10\lambda} \sigma^5, \end{aligned} \quad (17)$$

where the normalizable modes $\mathcal{A}_1(\tau)$ and $\mathcal{A}_2(\tau)$, which are both $\mathcal{O}(\epsilon)$ (like λ), determine the full solutions; and when $\lambda = 0$, the analogous solutions of (12) are

$$\begin{aligned} x_{d_1} &= \mathcal{B}_1 \sigma^2 + \frac{\mathcal{B}_1(M(\mathcal{B}_1^2 - \mathcal{B}_1\mathcal{B}_2 + \mathcal{B}_2^2) - \dot{\mathcal{B}}_2^2) + 2\mathcal{B}_2\dot{\mathcal{B}}_1\dot{\mathcal{B}}_2 + (\mathcal{B}_1 - \mathcal{B}_2)\dot{\mathcal{B}}_1^2}{4(\mathcal{B}_1^2 - \mathcal{B}_1\mathcal{B}_2 + \mathcal{B}_2^2)} \sigma^4 + \mathcal{O}(\sigma^6, \epsilon^2), \\ x_{d_2} &= \mathcal{B}_2 \sigma^2 + \frac{\mathcal{B}_2(M(\mathcal{B}_1^2 - \mathcal{B}_1\mathcal{B}_2 + \mathcal{B}_2^2) - \dot{\mathcal{B}}_1^2) + 2\mathcal{B}_1\dot{\mathcal{B}}_1\dot{\mathcal{B}}_2 + (-\mathcal{B}_1 + \mathcal{B}_2)\dot{\mathcal{B}}_2^2}{4(\mathcal{B}_1^2 - \mathcal{B}_1\mathcal{B}_2 + \mathcal{B}_2^2)} \sigma^4 + \mathcal{O}(\sigma^6, \epsilon^2), \end{aligned} \quad (18)$$

where the normalizable modes $\mathcal{B}_1(\tau)$ and $\mathcal{B}_2(\tau)$, both of which are $\mathcal{O}(\epsilon)$, determine the full solutions. Generally, when the Dirichlet boundary conditions (10) are imposed, x_{d_i} are even in σ and vanish as σ^2 near the boundary for $\lambda = 0$, while they are odd in σ and vanish as σ^3 near the boundary when $\lambda \neq 0$. For the holographic interpretation of the multiway junction, the most relevant aspect of τ_{d_i} is that

$$\lim_{\sigma \rightarrow 0} \tau_{d_i}(\sigma, \tau) = \mathbb{t}_{d_i}(\tau), \quad i = 1, \dots, n-1 \quad (19)$$

are nonvanishing indicating that the relative time differences (reparametrizations) are nontrivial at the dual interface. Furthermore, $\mathbb{t}_{d_i}(\tau)$ are determined by the normalizable modes of x_{d_i} . When $\lambda \neq 0$,

$$\dots \mathbb{t}_{d_i} - M \dot{\mathbb{t}}_{d_i} = -\frac{3}{n} \lambda \mathcal{A}_i + \mathcal{O}(\epsilon^3), \quad i = 1, \dots, n-1, \quad (20)$$

for $n \geq 2$, and when $\lambda = 0$,

$$\mathbb{t}_{d_i} - M\dot{\mathbb{t}}_{d_i} = 2\mathcal{B}_i \left((n-2)\mathcal{B}_i - 2 \sum_{j \neq i} \mathcal{B}_j \right) + \mathcal{O}(\epsilon^3), \quad (21)$$

for $n \geq 3$. In order to eliminate rigid transformations related to world sheet isometries mentioned before, we require that $\mathbb{t}_{d_i}$ decay faster than $e^{-\sqrt{M}\tau}$ at large τ . (It can be checked that $\mathcal{A}_i$ decay as $e^{-m\sqrt{M}\tau}$ with $m = 2, 3, \dots$ at large τ for x_{d_i} to be ingoing at the world sheet horizon.¹) As in the case of $n = 2$, τ_{d_i} and σ_{d_i} are also then completely determined by the normalizable modes of x_{d_i} .

Crucially, $\mathbb{t}_{d_i}$, the relative time reparametrizations at the dual interface completely encode the normalizable modes of x_{d_i} , and therefore the solution of the coupled Nambu-Goto equations for x_{d_i} which constitute the $n - 1$ degrees of freedom of the gravitational multiway junction. When $n \geq 3$, there are nontrivial degrees of freedom even when $\lambda = 0$, which can be decoded from the relative time reparametrizations $\mathbb{t}_{d_i}$ at the dual interface.

IV. HOLOGRAPHIC INTERPRETATION

We can proceed with the holographic interpretation of gravitational junctions in terms of interfaces of dual large c conformal field theories (CFT) by applying the holographic dictionary [23,24]. The simplest solution in which x_{d_i} , t_{d_i} and σ_{d_i} vanish corresponds to a static conformal interface in the dual CFT where n thermal wires with equal temperature are glued. The tension λ characterizes the defect operator at the interface, e.g., via the boundary entropy [25–27].

For the holographic interpretation of the general solutions of the gravitational junction in terms of physical operations, we note that the relative time reparametrizations $\mathbb{t}_{d_i}$ (encoding the normalizable modes of x_{d_i}) can be undone by conformal transformations acting on $n - 1$ of the n wires glued at the dual interface. Generalizing the case of $n = 2$ analyzed in [15], we can proceed by first interchanging τ with $t_n(\sigma = 0, \tau)$, the time coordinate of the n th wire at the boundary, and then $\mathbb{t}_{d_i}$ with

$$t_i = \mathbb{h}_i(t_n), \quad i = 1, \dots, n-1, \quad (22)$$

at $\sigma = 0$, relating the time of the i th wire to that of the n th wire at the interface, which can be located at $x_i = x_0 = 0$ without loss of generality (at the boundary). Let $x_i^\pm = t_i \pm x_i$ be the light-cone coordinates of the wires. The $n - 1$ conformal transformations with

$$\tilde{x}_i^\pm = \mathbb{h}_i^{-1}(x_i^\pm), \quad i = 1, \dots, n-1 \quad (23)$$

give new coordinates $\tilde{x}_i = (1/2)(\tilde{x}_i^+ - \tilde{x}_i^-)$ and $\tilde{t}_i = (1/2) \times (\tilde{x}_i^+ + \tilde{x}_i^-)$. We note from (23), that the position of the

¹We intend to study the behavior of the $\mathcal{B}_i(\tau)$ modes explicitly in the future. Note that the Eq. (12) cannot be linearized when $\lambda = 0$.

interface is preserved at $\tilde{x}_i = 0$ while $\tilde{t}_i = t_n$ at the interface for $i = 1, \dots, n-1$. Therefore, we obtain continuous coordinates and a continuous metric across the interface as a result of the conformal transformations (23). However, the energy-momentum tensor becomes discontinuous at the interface with the following nonvanishing components for $i = 1, \dots, n-1$:

$$\tilde{T}_{\pm\pm}(\tilde{x}_i^\pm) = \frac{\pi c}{12} \mathbb{h}'_i(\tilde{x}_i^\pm)^2 T^2 - \frac{c}{24\pi} \text{Sch}(\mathbb{h}_i(\tilde{x}_i^\pm), \tilde{x}_i^\pm). \quad (24)$$

In the bulk, the conformal transformations can be lifted to (improper) bulk diffeomorphisms [28] on each of the $n - 1$ fragments $\mathcal{M}_{i\alpha_i}$ for $i = 1, \dots, n-1$, and the holographic dictionary reproduces (24) with $c = \frac{3\ell}{2G_N}$ and the temperature $T = \frac{\sqrt{M}}{2\pi}$ [23,24,28]. As the induced metric of a hypersurface and its extrinsic curvature transform as scalars under bulk diffeomorphism, we obtain a gauge equivalent solution of the gravitational junction. It is to be noted that the conformal transformations (23) do not involve any rigid $\text{SL}(2, \mathbb{R})$ transformation, which preserves the thermal state as we discarded the corresponding rigid parameters in the gravitational solution [involving homogeneous solutions of (20) or (21)]. For a statement of Ward identities of the dual interface, see the Appendix.

The solution of the coupled Nambu-Goto equations given by the $n - 1$ normalizable modes of x_{d_i} determines the functions $\mathbb{h}_i$. Thus, the $n - 1$ degrees of freedom of the gravitational junction translate to state-dependent wave packets on $n - 1$ wires converging to the interface collectively and reflected *without* distortion at the interface individually, as implied by (24), when all the wires are at the same background temperature. The state-dependence is a consequence of the fact that the functions $\mathbb{h}_i$ encode the solution of the coupled Nambu-Goto equations in the BTZ black brane background which is dual to the thermal state.

V. CONCLUSIONS

We have shown that gravitational multiway junctions gluing n copies of BTZ black branes correspond to $n - 1$ coupled Nambu-Goto equations, implying emergent matter like degrees of freedom from gravity, which survive in the limit in which the tension of the junction is taken to zero for $n \geq 3$. Given that we obtain a nonsmooth spacetime manifold with emergent matterlike degrees of freedom even in the tensionless limit, our solutions are entirely novel in the context of pure gravity in three dimensions. As discussed in the Appendix, it is possible to generalize our results to higher dimensional pure gravity.

We have also shown that a n -way gravitational junction gluing n copies of BTZ black branes is holographically dual to an interface of n wires at the same temperature, each described by a large c strongly interacting two-dimensional CFT. The modes described by coupled Nambu-Goto equations correspond to possible combinations of wave

packets on the thermal wires, which can scatter off without distortion at the dual interface from past null infinity to future null infinity at each wire individually.

Extending our constructions to junctions gluing multiple *dissimilar* locally AdS_3 spacetimes is a problem of fundamental importance. Deformations of the wave packets which scatter off at the dual interface without distortion correspond to the deformations of the corresponding background geometries from BTZ black branes to general (locally AdS_3) Bañados geometries [29], and should lead to $n \rightarrow n$ scattering with tunable energy transmissions/reflections/redistributions. As in the case of the two-way junction [15,30,31], the stringy degrees of freedom can be reconstructed as universal quantum maps. Crucially, our results together with those discussed in [15,31] indicate that the multiway interfaces of CFTs [5–8,32] can be generalized by involving relative automorphisms of the Virasoro algebra, and can be realized in spin and quantum Hall systems [33–37].

Furthermore, holographic bulk spacetime reconstruction is best understood as the recovery map in a quantum error correcting code encoding bulk subregions to boundary subregions [16–19]. The bulk reconstruction of the emergent matterlike degrees of freedom of gravitational junctions from subregions of the dual dynamic multiway interface poses a fundamental challenge within the holographic framework. Primarily, we should understand how these modes affect entanglement at the dual interface following the methodology of [26,27,38–41].

ACKNOWLEDGMENTS

We especially thank Cheng Peng as discussions between him and AM motivated the present research. A. C., A. M., and M. M. acknowledge support from FONDECYT post-doctoral Grant No. 3230222, FONDECYT regular Grant No. 1240955 and “Doctorado Nacional” Grant No. 21250596 of La Agencia Nacional de Investigación y Desarrollo (ANID), Chile, respectively. A. C. also appreciates the warm hospitality extended by A. M. and Instituto de Física, Pontificia Universidad Católica de Valparaíso, Chile where majority of the work was carried out. A. M. gratefully acknowledges the hospitality of LPENS, where a substantial part of this work was carried out during his tenure as a CNRS invited professor.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX

1. Generalizations for $D > 3$

Let us first see how the count of the number of variables and number of independent junction conditions generalize

for multiway codimension one gravitational junctions in dimensions $D > 3$.

If we join n fragments of D -dimensional spacetimes with $D \geq 3$, the total number of variables in an arbitrary gluing can be counted as follows. Firstly, we have n embedding functions giving the transverse coordinate of the codimension one boundaries Σ_i [like (1)] and secondly, we have $(D-1)(n-1)$ independent relative spacetime shifts in time and the $D-2$ longitudinal coordinates across the junction [we can identify the average time and longitudinal coordinates of the images Σ_i of Σ with the $D-1$ world sheet coordinates like in (2)]. In total, there are thus

$$(D-1)(n-1) + n = Dn - D + 1$$

variables. The junction conditions involve $D(D-1) \times (n-1)/2$ conditions involving continuity of the induced metric and $D(D-1)/2 - (D-1) = (D-1)(D-2)/2$ independent conditions involving the total Brown-York tensor after accounting for the $(D-1)$ constraints implying its conservation. [The constraints originate from the symmetry of the action (3) with respect to bulk diffeomorphisms.] Therefore, the number of independent junction conditions total

$$\frac{D(D-1)(n-1)}{2} + \frac{(D-1)(D-2)}{2} = \frac{D-1}{2}(Dn-2).$$

We note that

$$\frac{D-1}{2}(Dn-2) \geq Dn - D + 1$$

for $D \geq 3$ with the inequality saturated if and only if $(D-1)/2 = 1$, i.e., $D = 3$. Therefore, we do not expect degrees of freedom to emerge at codimension one gravitational junctions in $D > 3$.

However, we can consider the embedding of the multiway junctions gluing slices of three-dimensional manifolds in $D = 3 + k$, and our results suggest that matterlike degrees of freedom in the form of vibrating strings can emerge at these codimension $k+1$ junctions. This is more easily implementable for flat spacetimes where our results for the multiway junctions gluing three-dimensional spacetimes remain valid. Consider a tessellation of $\mathbb{R}^{3,1}$ by four-dimensional hypercubes each bounded by six $2+1$ -dimensional faces as shown in Fig. 2. The $1+1$ -dimensional edges of the hypercubes are junctions where four three-dimensional spacetimes are glued. This embeds the four way junction gluing four $2+1$ -dimensional spacetimes studied in this letter in $\mathbb{R}^{3,1}$, providing a concrete setup where we can investigate whether stringlike degrees of freedom can emerge even in *pure* gravity in four dimensions. Similarly, we can proceed with polygonal tessellations of AdS_4 . We postpone the study of such

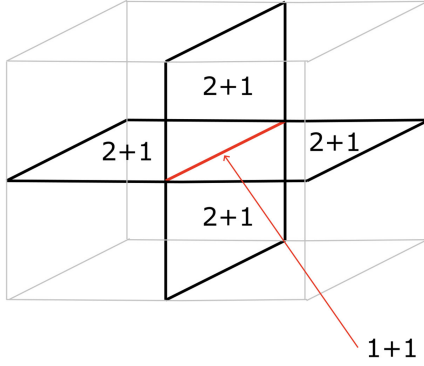


FIG. 2. A sketch of the junction in 3 + 1 dimensions. The four planes with solid black boundaries are 2 + 1 dimensional and bound 3 + 1 dimensional hypercubes (gray boundaries). Red line is a 1 + 1 dimensional four-way junction.

gravitational junctions in $D > 3$ and the analysis of their holographic interpretations for the future.

2. Ward identities at the dual multiinterfaces

Consider the n – way junction studied in this article. In the dual 1 + 1-dimensional CFT, it corresponds to n wires joined at a single point, say $x_i = 0$, with relative time reparametrizations between the wires. Let us fix the time coordinate of the n th wire t_n to be the global time. As shown in (22) in the main text, the relative time-reparametrization between the i th and n th wire can be expressed as

$$t_i = \mathbb{h}_i(t_n), \quad i = 1, \dots, n-1, \quad (\text{A1})$$

and we can undo these time reparametrizations by using different conformal transformations on each of the $i \neq n$ wires. These transformations are

$$\tilde{x}_i^\pm = \mathbb{h}_i^{-1}(x_i^\pm), \quad i = 1, \dots, n-1, \quad (\text{A2})$$

where $x_i^\pm = t_i \pm x_i$ are the light-cone coordinates for each of the wires. It is easy to see that the above transformations preserve the location of the interface at $\tilde{x}_i = 0$ and ensure that

$$\tilde{t}_i = t_n, \quad i = 1, \dots, n-1, \quad (\text{A3})$$

so that we have continuous coordinates and metric across any pair of wires.

After the conformal transformations, the nonvanishing components of the energy-momentum $T_\pm^i = \frac{\pi c}{12} T^2$ on the i th wire transform to

$$\tilde{T}_{\pm\pm}^i(\tilde{x}_i^\pm) = (\mathbb{h}'_i(\tilde{x}_i^\pm))^2 \frac{\pi c T^2}{12} - \frac{c}{24\pi} \text{Sch}(\mathbb{h}_i(\tilde{x}_i^\pm), \tilde{x}_i^\pm), \quad (\text{A4})$$

for $i \neq n$, and for $i = n$, we have $\tilde{T}_{\pm\pm}^n = T_{\pm\pm}^n$.

For concreteness, let the spatial $\tilde{x}$ coordinate of each wire extend from $-\infty$ to 0. In order to formulate the Ward identity, it is useful to consider the i th wire described by CFT_i on the left side of the multi-interface and reflect the remaining $n-1$ wires described by the combined $\overline{\text{CFT}}_1 \otimes \dots \otimes \overline{\text{CFT}}_{i-1} \otimes \overline{\text{CFT}}_{i+1} \otimes \dots \otimes \overline{\text{CFT}}_n$ theory living in the semi-infinite interval extending from 0 to ∞ and with $\overline{\text{CFT}}$ denoting the exchange of left and right movers in the CFT due to the reflection. In this continuous $\tilde{x}$ coordinates, the nonvanishing energy momentum tensor is then

$$\tilde{T}_{++}(\tilde{t}, \tilde{x}) = \Theta(-\tilde{x}) \tilde{T}_{++}^i(\tilde{t}, \tilde{x})(\tilde{t}, \tilde{x}) + \Theta(\tilde{x}) \sum_{j \neq i} \tilde{T}_{--}^j(\tilde{t}, \tilde{x}),$$

$$\tilde{T}_{--}(\tilde{t}, \tilde{x}) = \Theta(-\tilde{x}) \tilde{T}_{--}^i(\tilde{t}, \tilde{x})(\tilde{t}, \tilde{x}) + \Theta(\tilde{x}) \sum_{j \neq i} \tilde{T}_{++}^j(\tilde{t}, \tilde{x}).$$

The Ward identities are

$$\partial_{\tilde{t}} \tilde{T}^{\tilde{t}\tilde{t}}(\tilde{t}, \tilde{x}) + \partial_{\tilde{x}} \tilde{T}^{\tilde{x}\tilde{t}}(\tilde{t}, \tilde{x}) = 0, \quad (\text{A5})$$

$$\partial_{\tilde{t}} \tilde{T}^{\tilde{t}\tilde{x}}(\tilde{t}, \tilde{x}) + \partial_{\tilde{x}} \tilde{T}^{\tilde{x}\tilde{x}}(\tilde{t}, \tilde{x}) = \delta(\tilde{x}) q(\tilde{t}). \quad (\text{A6})$$

The first equation above follows as $T^{xt} \propto T_{++} - T_{--}$ and as evident from (A4), $\tilde{T}_{++}^k - \tilde{T}_{--}^k$ vanishes at $\tilde{x} = 0$ for all $k = 1, \dots, n$. More generally, the right-hand side of (A5) should be proportional to

$$\sum_i \tilde{T}_{--}^i(\tilde{t}, \tilde{x} = 0) - \sum_i \tilde{T}_{++}^i(\tilde{t}, \tilde{x} = 0), \quad (\text{A7})$$

which vanishes if the conformal boundary condition is preserved as is indeed the case generally for the two-way junction (see [15]). The source appearing in (A6),

$$q(\tilde{t}) = \sum_{j \neq i} (\tilde{T}_{++}^j + \tilde{T}_{--}^j)(\tilde{t}, \tilde{x} = 0) - (\tilde{T}_{++}^i + \tilde{T}_{--}^i)(\tilde{t}, \tilde{x} = 0), \quad (\text{A8})$$

is the expectation value of a generalized displacement operator which generates the displacement of the i th wire away from the interface.

3. Details of the perturbative expansion

The details of the derivation of the Nambu-Goto-Monge-Ampère equations (12) from the $3n-2$ junction conditions via perturbative expansion in λ are as follows. At zeroth order, permutation symmetry implies trivial solutions $x_s = x_0$ and $x_{d_i} = \tau_{d_i} = \sigma_{d_i} = 0$ of the metric continuity equations (4) and the condition for the extrinsic curvature discontinuity (5). At first order in λ , only the $\mathcal{O}(\epsilon)$ terms of τ_{d_i} and σ_{d_i} appear in the $3n-3$ metric continuity equations (4). Permutation symmetry implies τ_{d_i} and σ_{d_i} should vanish at first order. At first order, the extrinsic curvature discontinuity (5) gives

$$(1 - M\sigma^2)(\lambda - nx'_s) - n\sigma\ddot{x}_s = \mathcal{O}(\epsilon^2), \quad (\text{A9})$$

$$M\sigma\dot{x}_s + (1 - M\sigma^2)\dot{x}'_s = \mathcal{O}(\epsilon^2), \quad (\text{A10})$$

$$(1 - M\sigma^2)n\sigma x''_s - nx'_s + \lambda = \mathcal{O}(\epsilon^2), \quad (\text{A11})$$

which determines x_s at the first order as $x_s = \lambda\sigma/n + \mathcal{O}(\epsilon^2)$ given the Dirichlet boundary condition (10). This part is similar to the 2-way junction case discussed in [11].

Generally, $\mathcal{O}(\epsilon^k)$ terms of τ_{d_i} and σ_{d_i} , and $\mathcal{O}(\epsilon^{k-1})$ term of x_{d_i} are determined at the k th order in the perturbative expansion of the $3n - 3$ metric continuity conditions (4) for $k \geq 2$. The $\mathcal{O}(\epsilon^k)$ term of x_s is determined at the k th order of the expansion of the extrinsic curvature discontinuity condition (5) for $k \geq 1$. Also, the $\mathcal{O}(\epsilon^k)$ terms of x_{d_i} and x_s vanish when k is odd and even, respectively.

We generalize the procedure in [11] to solve the $3n - 3$ metric continuity conditions (4) at k th order in the perturbative expansion for $k \geq 2$. From the diagonal $\tau\tau$ and $\sigma\sigma$ components of the metric continuity conditions, we can obtain τ_{d_i} and σ_{d_i} explicitly in terms of x_{d_i} up to terms which depend only on σ or τ . Substituting these forms of τ_{d_i} and σ_{d_i} into the off diagonal components of the metric continuity conditions yield $n - 1$ equations that are schematically $\mathcal{E}_i = 0$. Then, $\partial_\sigma \partial_\tau (\mathcal{E}_i \sigma \sqrt{1 - M\sigma^2}) = 0$ give the coupled Nambu-Goto-Monge-Ampère equations (12) involving *only* the $n - 1$ variables x_{d_i} . Given that x_{d_i} satisfies these equations, we can determine τ_{d_i} and σ_{d_i} completely from $\partial_\tau \mathcal{E}_i = 0$ and $\partial_\sigma (\mathcal{E}_i \sigma \sqrt{1 - M\sigma^2}) = 0$. This procedure works at higher orders in the perturbative expansion also.

For $k \geq 2$, the extrinsic curvature discontinuity takes the same linear form as (A9) for the $\mathcal{O}(\epsilon^k)$ term of x_s only with source terms containing lower order terms of x_s and other variables. These can be readily solved as in [11].

4. A special case of Nambu-Goto-Monge-Ampère equations

The Nambu-Goto-Monge-Ampère equations (12) can be solved explicitly when the n -way junction can be reduced

to the 2-way junction. This reduction corresponds to setting $x_k = x_s$ for $k \neq i$ and $k \neq j$ (freezing the relative motions of $n - 2$ of the n hypersurfaces glued at the junction) which impose $n - 2$ linear constraints on the $n - 1$ variables x_{d_i} so that only one linear combinations of the x_{d_i} is independent. This linear combination satisfies the linearized Nambu-Goto equation (13) since the $n - 2$ constraints imply that the quadratic Monge-Ampère terms vanish identically. As for instance, setting $x_1 = x_3 = x_s$ in the case of the 4-way junction impose that $x_{d_1} = 2x_{d_2}$ and $x_{d_3} = x_{d_2}$, and the three coupled equations (12) simply imply that x_{d_2} satisfies (13).

The normalizable solutions of the linearized Nambu-Goto equation (13) in the BTZ black hole background corresponding to the quasinormal modes of the string decay as $e^{-(2+m)\sqrt{M}\tau}$ for $m = 0, 1, 2, \dots$ [11]. Explicitly, the general solution is of the form,

$$x_{d_i} = \sum_{m=0}^{\infty} b_m e^{-(2+m)\sqrt{M}\tau} P_m(\sigma), \quad (\text{A12})$$

with

$$P_m(\sigma) = \frac{3}{m^3 - m} \left(\frac{m\sigma \cosh(\text{marctanh}(\sqrt{M}\sigma))}{M} - \frac{\sinh(\text{marctanh}(\sqrt{M}\sigma))}{M^{\frac{3}{2}}} \right). \quad (\text{A13})$$

We can check that

$$P_2(\sigma) = \frac{\sigma^3}{1 - M\sigma^2}, \quad P_3(\sigma) = \frac{\sigma^3}{(1 - M\sigma^2)^{3/2}}, \quad \text{etc.} \quad (\text{A14})$$

As discussed in the main text, the normalizable solutions of the full coupled Nambu-Goto-Monge-Ampère equations (12) also decay as $e^{-(2+m)\sqrt{M}\tau}$ for $m = 0, 1, 2, \dots$ when $\lambda \neq 0$.

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