

Gravitational wave signals in a promising realization of $SO(10)$ unification

Injun Jeong^{1,2}, Jörn Kersten^{3,4}, Stefano Scopel^{1,2} and Liliana Velasco-Sevilla^{1,2}

¹*Center for Quantum Spacetime, Sogang University, 35 Baekbeom-ro, Seoul 121-742, South Korea*

²*Department of Physics, Sogang University, 35 Baekbeom-ro, Seoul 121-742, South Korea*

³*Department of Physics and Technology, University of Bergen, Postboks 7803, 5020 Bergen, Norway*

⁴*Department of Physics and IPAP, Yonsei University, 50 Yonsei-ro, Seoul 03722, South Korea*



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We investigate gravitational wave signals in a nonsupersymmetric grand unified model where the group $SO(10)$ is broken in two steps to the Standard Model gauge group. We calculate the analytical form of the one-loop effective potential responsible for the first step of symmetry breaking and show that it can lead to a first-order phase transition with gravitational wave production. We also determine the gravitational wave background produced by the primordial plasma of relativistic particles. The present experimental sensitivity is still far from the expected signals but could be in reach of novel detector concepts.

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I. INTRODUCTION

Before the advent of gravitational wave (GW) experiments, physics beyond the Standard Model (SM) could be probed at early cosmological times using photons and neutrinos. The earliest observable photons come from the cosmic microwave background (CMB), because before recombination the Universe was optically thick, while detection of cosmic neutrinos could probe the primordial nucleosynthesis era. However, times at which grand unified theories (GUTs) are unbroken remain inaccessible to cosmological information from photons and neutrinos. On the other hand, the Large Hadron Collider has not found any indication for physics beyond the SM, and hence observations from other experiments are crucial to keep probing compelling frameworks motivated by unsolved issues in the SM. As has been known for a long time [1], GUTs provide one such framework that could leave a signal in the form of a stochastic GW background.

Due to the possible observation of a stochastic GW background by pulsar timing arrays [2–5] there has been a resurgence in studying GWs from GUTs (see [6] for extensive references). The first evidence reported by the NANOGrav collaboration [7] appeared to be consistent with the expected signal from cosmic strings, considered the smoking gun of the characteristic sequence of symmetry breakings expected in GUTs. It now appears likely that what is observed by the pulsar timing arrays cannot be the signal of a Nambu-Goto string arising from a GUT [8].

Nevertheless, signals of this kind can appear below scales of order 10^{15} GeV, owing to the fact that there could be different stages of symmetry breaking in GUT models, both supersymmetric and nonsupersymmetric. In addition, specific GUT scenarios can be characterized by the appearance of a rich pattern of mutually interacting topological defects [9,10].

The current, upcoming, and projected experiments have the potential to detect GW signals in an extended frequency range. This provides an invaluable opportunity to analyze under which conditions these could come from a GUT group, to which specific scenario they correspond and at what scale they appear. In this work, we will focus on two types of signals: the GW produced by the high-temperature first-order phase transitions (FOPT) induced by the GUT symmetry breaking (Sec. III) and the stochastic background produced by the shear viscosity of the relativistic plasma in thermal equilibrium throughout the expansion history of the Universe (Sec. V).

As far as the former signal is concerned, it is not present in the SM and is considered a smoking gun for GUTs. Due to the complexity of the construction of such models, there have been no studies on FOPT in detailed GUT models, particularly at the GUT scale. It is clear that one reason for this is the high frequency required for the observation of the signals, and hence the majority of FOPT analyzed within the context of GUTs refer to phase transitions in the different chain of breakings of GUT models in the LIGO or LISA region [11–14]. However, given the panacea that GUT models are for addressing issues that in the SM remain unsolved, the study of all possible signals that GUT theories can produce merits consideration and therefore we begin the task of studying carefully transitions from the GUT scale.

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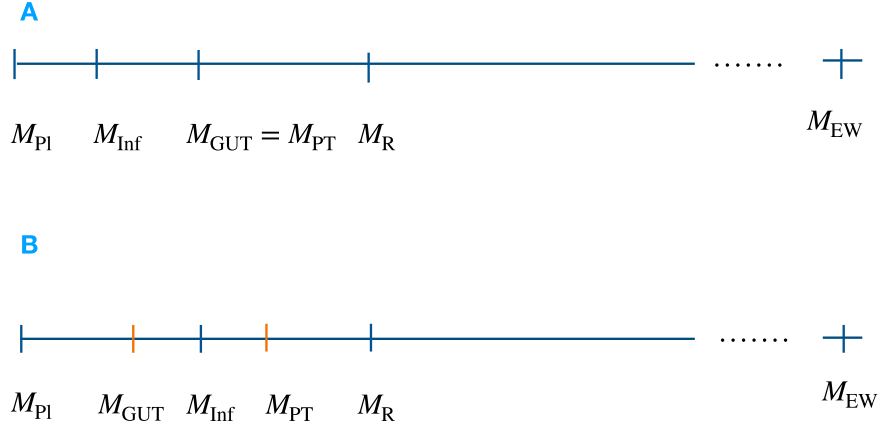


FIG. 1. Possible scenarios for the completion of the phase transition. In scenario “A,” the phase transition happens at the scale of grand unification and after the inflation scale, so its effects are visible today. Scenario “B” refers to the case that the phase transition does not complete at M_{GUT} but is completed after inflation. We briefly mention this possibility in Sec. IV. Here M_{Pl} , M_{Inf} , M_{GUT} , M_{PT} , M_{R} , and M_{EW} are, respectively, the Planck mass, the inflation scale, the FOPT scale, the GUT scale, the scale at which $SU(2)_R$ is broken, and finally the EW scale.

On the other hand, the signal from the plasma does not require physics beyond the SM and is thus also expected in the SM. Apart from the already mentioned cosmic strings and other topological defects, GW can also be produced in an incomplete phase transition [15], which could trigger observable effects in the CMB. We will briefly consider this option in Sec. IV. Before turning to these various sources of GW, we will introduce the concrete GUT scenario we consider, calculate the effective scalar potential governing the FOPT, and discuss the viable parameter space in Sec. II.

II. $SO(10)$ MODEL

A. General assumptions

In order to shed light on the possible GW signals produced by GUTs in the early Universe, we adopt the following breaking chain pattern in this work,

$$\begin{aligned}
 &SO(10) \xrightarrow[45[1]]{M_{\text{GUT}}} (SU(3)_C \times SU(2)_L \times SU(2)_R) \\
 &\quad \times U(1)_{B-L} / \mathbb{Z}_2 \\
 &\xrightarrow[126[2]]{M_{\text{R}}} (SU(3)_C \times SU(2)_L \times U(1)_Y) \xrightarrow[10]{M_{\text{EW}}} SU(3)_C \\
 &\quad \times U(1)_{\text{em}}, \tag{1}
 \end{aligned}$$

where we denote the appearance of monopoles and cosmic strings with [1] and [2], respectively [16–19]. We choose this chain because it allows gauge coupling unification without introducing supersymmetry and is in compliance with the current experimental limits on the proton decay rate. An additional appeal of this breaking chain is that, at least in its minimal version [with just the scalar content of Eq. (1), plus three generations of fermions in the spinorial representation of $SO(10)$] it has been shown to have promising

phenomenological prospects [20–24] [with the possible exception of some tension with electroweak observables that might be relaxed by extending the **10** representation at the last stage in Eq. (1) with an extended Higgs sector].

In the following, we will focus on the breaking chain (1), and the scenario “A” in Fig. 1 where the thermal evolution of the Universe after inflation starts in the $SO(10)^1$ -symmetric phase. The first step of symmetry breaking at the GUT scale M_{GUT} leads to the production of monopoles. In order to avoid overclosing the Universe, the monopole density has to be drastically reduced during the evolution below M_{GUT} . A number of mechanisms are known that achieve this goal. The first one is embedding $SO(10)$ into a larger group and breaking that group to $SO(10) \times U(1)_\psi$ before inflation. If both $SO(10)$ and $U(1)_\psi$ are broken at the same scale, the monopoles appearing in the breaking of the former can attach to the strings from the breaking of the latter if the magnetic fluxes are configured appropriately, leading to their rapid annihilation [25–30]. As a proof of principle, we give an explicit example realizing this possibility with an E_7 embedding in Appendix D. Developing more elegant solutions would be an interesting direction for GUT model building, but we do not expect significant changes of the phenomenology of the scenario that we study here. Alternatively, a period of thermal inflation could dilute the monopole density [31,32]. In this case, the GW spectrum is also diluted and deformed [33], likely rendering the GW produced by the FOPT unobservable. However, the GW produced by the thermal plasma of the SM and the intermediate gauge group (see Sec. V) will still be present. Hence, this case would require a separate analysis, which we do not pursue here. Furthermore, in some gauge-Higgs theories, by enlarging the scalar sector one may admit

¹Recall that formally we have $\text{Spin}(10)$.

embedded string solutions within the vacuum manifold. These strings can connect to or end at monopoles, so that the would-be “free” monopole is actually part of a monopole-string configuration. Because the string can absorb part of the magnetic flux or change the boundary conditions, the isolated monopole loses its topological protection and can become unstable [34,35]. The second step of symmetry breaking at M_R of the particular chain of Eq. (1) produces cosmic strings, which are topologically stable and produce GW that can be detectable [9,10]. We do not elaborate further on topological defects and their GW signatures, which would be a source of GW production in addition to the mechanism which are the main objective of our studies (i.e., the GW production by the FOPT and the thermal plasma).

It is worth pointing out that in this scenario the unification scale turns out to be close to the upper bound on the inflation scale $V_{\text{Inf}} \sim (1.6 \times 10^{16} \text{ GeV})^4$ for slow-roll inflation driven by a single scalar with canonical kinetic term [36]. An upper bound of the same order of magnitude exists also for the temperature of the plasma after reheating, in order for the latter to be in thermal equilibrium [37]. Given this tight space between inflation and the GUT scale, one can consider instead that unification takes place before inflation, as schematically depicted as option “B” in Fig. 1. In Sec. II C we describe the potential that would give rise to the FOPT and in Sec. IV we discuss realizations of the option “B” in the context of transitions that start before inflation and end after inflation [15], leaving an imprint in the CMB and with effects in a broad bandwidth.

B. Definition and matter content

We consider the minimal $SO(10)$ model containing only the gauge bosons, the three families of fermions transforming under the representation **16**, as well as scalars transforming under **10**, **45**, and **126** [22–24].² We will refer to this model as $\mathbf{G}_{3221}^M$, since the first step of symmetry breaking leads to the gauge group

$$G_{3221} := SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}. \quad (2)$$

We will investigate the potential for a first-order phase transition in this step, triggered by a vev, vacuum expectation value, v of the G_{3221} -singlet component of the **45** (see Appendix A for details).

As indicated in Eq. (1), we assume that a **126** vev is solely responsible for the later symmetry breaking to the SM gauge group, which implies that the $SU(2)_R$ triplet component of the **45** does not obtain a vev, i.e., $\omega_R = 0$ in the notation of [23,24]. We also assume that only the component of **126** obtaining the vev has a mass of $\mathcal{O}(M_R)$, while the masses of the other components are close to the

unification scale [38,39]. The masses of the **45** scalars are of $\mathcal{O}(M_{\text{GUT}})$ as well. For the multiplet **10**, a large mass splitting is required between the doublet and triplet components to avoid rapid proton decay, cf. Sec. II E 2. Below M_R we have exactly the gauge group and particle content of the SM.

C. Effective potential

In this section, we present the main results for the contributions to the effective scalar potential, including one-loop and thermal corrections. Details of the computation are given in Appendix B. We start with the tree-level $SO(10)$ -symmetric potential for the scalar **45**,

$$V_0(\phi) = -\frac{\mu^2}{2} \text{Tr} \phi^2 + \frac{a_0}{4} (\text{Tr} \phi^2)^2 + \frac{a_2}{4} \text{Tr} \phi^4 \quad (3)$$

with $\mu^2 > 0$. In terms of the classical field ϕ_c , the tree-level potential corresponding to the first step of the chain in Eq. (1), is obtained by plugging Eq. (B1) into Eq. (3), resulting in

$$V_0(\phi_c) = -\frac{1}{2} \mu^2 \varphi_c^2 + a_0 \varphi_c^4 + \frac{1}{6} a_2 \varphi_c^4. \quad (4)$$

Defining the unification scale $v \equiv M_{\text{GUT}}$ as the value of the classical field φ_c at the potential minimum,³ the tree-level minimization condition yields

$$\mu^2 = \left(4a_0 + \frac{2}{3}a_2\right)v^2. \quad (5)$$

The one-loop contribution to the effective potential at zero temperature, $V_1(\phi_c)$, contains two parts, the gauge boson contribution

$$V_1^g(\phi_c) = \frac{3}{64\pi^2} \sum_{i=g1,g2} n_i m_i^4(\phi_c) \left(\ln \frac{m_i^2(\phi_c)}{\mu_r^2} - \frac{5}{6} \right), \quad (6)$$

and the scalar contribution

$$V_1^s(\phi_c) = \frac{1}{64\pi^2} \sum_{i=s1,s2,s3,\chi} n_i m_i^4(\phi_c) \left(\ln \frac{|m_i^2(\phi_c)|}{\mu_r^2} - \frac{3}{2} \right), \quad (7)$$

where $m_i(\phi_c)$ is the field-dependent mass of the gauge boson or scalar i and n_i the number of particles with this mass. Both quantities are given in Table I. These expressions have been obtained in the $\overline{\text{MS}}$ renormalization scheme with the renormalization scale μ_r , for which we choose $\mu_r = v$ in numerical calculations. Note that the

²In [24] it has been argued that is challenging to accommodate all the necessary phenomenology for a successful model, so in Secs. II D and II E 2 we comment on possible departures from the minimum contents.

³In the notation of [23,24], this implies $v = \sqrt{3}\omega_{BL}$. Note that this vev is not at the global minimum of the tree-level potential, but this is possible once one-loop corrections are taken into account [20].

TABLE I. Field-dependent masses. The multiplicity indicates the number of particles with this mass. The gauge coupling g is the gauge coupling of the $SO(10)$ theory, defined in Eq. (A6).

Field-dependent mass squared	Particle type	Multiplicity n_i
$m_{g1}^2(\phi_c)$	$\frac{1}{6}g^2\varphi_c^2$	Gauge boson 24
$m_{g2}^2(\phi_c)$	$\frac{2}{3}g^2\varphi_c^2$	Gauge boson 6
$m_{s1}^2(\phi_c)$	$-\mu^2 + 4a_0\varphi_c^2$	Scalar 6
$m_{s2}^2(\phi_c)$	$-\mu^2 + 12a_0\varphi_c^2 + 2a_2\varphi_c^2$	Scalar 1
$m_{s3}^2(\phi_c)$	$-\mu^2 + 4a_0\varphi_c^2 + 2a_2\varphi_c^2$	Scalar 8
$m_{\chi}^2(\phi_c)$	$-\mu^2 + 4a_0\varphi_c^2 + \frac{2}{3}a_2\varphi_c^2$	NGB 30

scalar contribution contains both the physical scalars and the Nambu-Goldstone bosons (NGB). We neglect the contributions of the scalars from representations other than **45**, since they would introduce many additional parameters but are not expected to have a sizable impact on the phase transition. Note that the scalar potential that we are considering here for the first step of the symmetry breaking is independent of any physics below the unification scale, of course apart from the value of the unification scale, which is determined by the breaking chain and the matter content.

In Eq. (B6) we specify the one-loop contributions to the effective potential at finite temperature, $V_{\text{th}}(\phi_c, T)$. Then the complete one-loop effective potential is given by

$$V(\phi_c, T) = V_0(\phi_c) + V_1^g(\phi_c) + V_1^s(\phi_c) + V_{\text{th}}(\phi_c, T). \quad (8)$$

We need to find the minimum of the potential including these corrections. The condition for the minimum at zero temperature is

$$\frac{\partial V(\phi_c, 0)}{\partial \phi_c} = \frac{\partial V_0(\phi_c)}{\partial \phi_c} + \frac{\partial V_1^g(\phi_c)}{\partial \phi_c} + \frac{\partial V_1^s(\phi_c)}{\partial \phi_c} = 0, \quad (9)$$

which can be solved numerically. What we adopt here is a similar strategy to the one followed in [23] in that we adjust the value of μ^2 such that the minimum at the one-loop level is roughly equal to v . We remind the reader that v is fixed by the gauge coupling unification condition, as we will discuss in Sec. IID. Specifically, we calculate μ^2 at the tree level from Eq. (5) and use it in Eq. (9) to find the value of the scalar field in the minimum, denoted by ϕ_m . This value differs from v , so we update μ^2 iteratively using Newton's method until ϕ_m differs from v by no more than 1%. As the value of μ^2 thus obtained approaches zero, the calculation becomes numerically challenging due to convergence issues and large higher-order loop corrections. Therefore, we exclude points with $\mu^2 < 0$.

Given the size of the gauge coupling at the GUT scale (around 0.5 as we explain in Sec. IID) and the particle content of the model (30 massive gauge bosons and 15 scalar bosons), we expect that loop corrections can

become big and one-loop precision may not be sufficient. As a two-loop computation is beyond the scope of this work, we adopt the usual strategy for estimating the uncertainty caused by neglecting higher loop orders, see, e.g., [40,41], defining the quantity

$$\Delta^{\text{2-loop}} := \left\langle \frac{V(\phi_c, 0)|_{\mu_r=2v} - V(\phi_c, 0)|_{\mu_r=v/2}}{(V(\phi_c, 0)|_{\mu_r=2v} + V(\phi_c, 0)|_{\mu_r=v/2})/2} \right\rangle_{0 < \varphi_c < 1.2v} \quad (10)$$

to estimate the importance of higher loop orders, since the exact potential is independent of μ_r . The brackets in Eq. (10) indicate an average over 1000 linearly sampled values of φ_c between 0 and $1.2v$ to reduce the sensitivity to single points in field space where the denominator becomes very small due to cancellations.⁴

Before presenting the parameter space where a first-order phase transition would be possible, we discuss in the next sections constraints on the model that lead to bounds on the unification scale and on the parameter space of the coefficients in the potential.

D. Gauge coupling running

The minimal version of the model with the breaking chain of Eq. (1) contains the scalar **45**, which breaks the $SO(10)$ group, then the **126** breaks subsequently to G_{SM} and then the **10** causes the breaking at the electroweak (EW) scale. Our convention for the order of the indices is $a = 3, 2L, 2R, B - L$. For this model the component acquiring a vev in the **126** is $(1, 1, 3, 2)$ and that of **10** is $(1, 2, 2, 0) \supset (2, 1, \pm 1/2)$, where this last decomposition is under the SM group, $SU(2)_L \times SU(3)_C \times U(1)_Y$. For the matter, we have the three families of fermions accommodated in the following representations: $(3, 2, 1, -1/3)$, $(1, 2, 1, 1)$, $(\bar{3}, 1, 2, 1/3)$, and $(1, 1, 2, -1)$. However, we just take into account the running of the top mass up to the M_R scale. With this minimal matter content, the beta function coefficients at one and two loops are, respectively [10,38,42],

$$b_a^{(1)} = \begin{pmatrix} -7 \\ -3 \\ -7/3 \\ 11/2 \end{pmatrix}, \quad b_{ab}^{(2)} = \begin{pmatrix} -26 & 9/2 & 9/2 & 1/2 \\ 12 & 8 & 3 & 3/2 \\ 12 & 3 & 80/3 & 27/2 \\ 12 & 9/2 & 81/3 & 61/2 \end{pmatrix}. \quad (11)$$

⁴Such cancellations can occur if a potential barrier is present in $V(\phi_c, 0)$, since then the potential necessarily crosses zero between the minimum at the origin and the global minimum.

The matching conditions at the scale M_R are

$$\begin{aligned} g_2^{\text{SM}}(M_R) &= g_{2L}^{G_{3221}}(M_R), \\ g_1^{\text{SM}}(M_R) &= \left[\frac{3}{5} \frac{1}{g_{2R}^{G_{3221}{}^2}(M_R)} + \frac{2}{5} \frac{1}{g_{B-L}^{G_{3221}{}^2}(M_R)} \right]^{-1/2}, \\ g_3^{\text{SM}}(M_R) &= g_3^{G_{3221}}(M_R), \end{aligned} \quad (12)$$

where $g_{2L}^{G_{3221}}$, $g_{2R}^{G_{3221}}$, $g_{B-L}^{G_{3221}}$, and $g_3^{G_{3221}}$ are, respectively, the couplings of the group factors $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$, and $SU(3)_C$. We use the standard GUT normalization for the $U(1)$ gauge couplings. To unclutter the notation, from now on we suppress the superscript G_{3221} .

When running up the SM gauge couplings and matching to the G_{3221} group we need to make a guess about the relation between g_{2R} and g_{2L} but after running back and forth, in the final run from M_{GUT} to M_R , we can then obtain the final predicted value for $g_{2R}(M_R)$ and $g_{2L}(M_R)$. The beta functions take the form $dg_a/dt = g_a^3 b_a / 16\pi^2 + g_a^3 / (16\pi^2)^2 [\sum_{b=1}^4 b_{ab} g_b^2]$, for $a = 3, 2L, 2R, B-L$ and the coefficients b_a and b_{ab} as in Eq. (11). For the SM, $dg_a/dt = g_a^3 b_a / 16\pi^2 + g_a^3 / (16\pi^2)^2 [\sum_{b=1}^3 b_{ab} g_b^2 - C_a \gamma_i^2]$ with $C_1 = 17/10$, $C_2 = 3/2$, $C_3 = 2$, for $a = 1, 2, 3$ representing respectively the SM group factors $U(1)_Y$, $SU(2)_L$, $SU(3)_C$. The coefficients b_a and b_{ab} for this case are very well known and can be found in [43]. Just with this

minimal content the scales are

$$\begin{aligned} M_R^{2\text{ loop}} &= (2.9 \pm 1.0) \times 10^9 \text{ GeV}, \\ M_{\text{GUT}}^{2\text{ loop}} &= (1.6 \pm 1.0) \times 10^{16} \text{ GeV}, \end{aligned} \quad (13)$$

where the uncertainties stem mainly from threshold corrections and numerical precision. Once more constraints are taken into account [24], the scale M_{GUT} can be as low as $\mathcal{O}(10^{15})$ GeV. However, finding all phenomenology to be viable, especially in the Higgs sector at the EW scale [24], seems challenging. Adding matter content impacts the running and so the unification scale. Therefore, in what we consider here we set values of the unification scale in the range from 10^{15} GeV to 1.6×10^{16} GeV. Since it is challenging to accommodate the EW physics in the minimal G_{3221}^{M} , we may consider the addition of more scalars or fermions. For this purpose, we recall the general expression for the one-loop beta function of the gauge coupling of the group $\mathcal{G}_n$,

$$\begin{aligned} \beta_{g_n} &= \frac{g_n^3}{16\pi^2} b_n, \\ b_n &= -\frac{11}{3} I(\text{ad}^{(n)}) + \frac{2}{3} \sum_i I(F_i^{(n)}) + \frac{1}{6} \sum_j I(S_j^{(n)}), \end{aligned} \quad (14)$$

where $\text{ad}^{(n)}$, $F_i^{(n)}$, and $S_j^{(n)}$ denote the representations of $\mathcal{G}_n$ transforming the gauge bosons, chiral fermions, and real

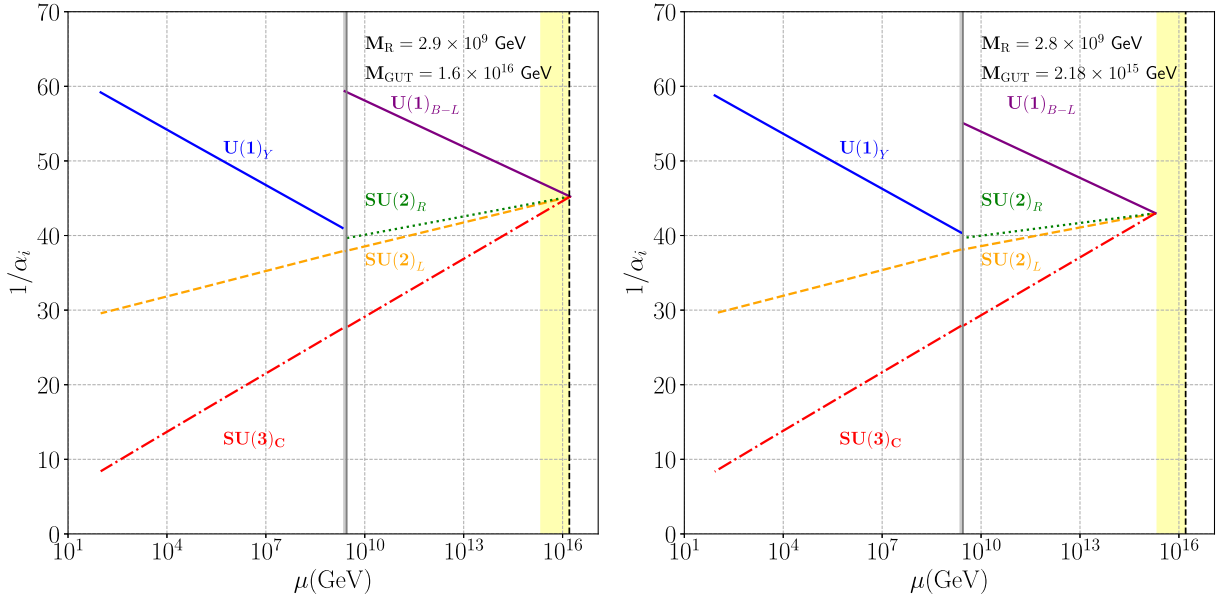


FIG. 2. Left panel represents the running of the Minimal G_{3221}^{M} model, G_{3221}^{M} (containing only $45 \oplus 126 \oplus 10$), with $M_R = (2.9 \pm 1.0) \times 10^9$ GeV and $M_{\text{GUT}} = (1.60 \pm 1.0) \times 10^{16}$ GeV. The gauge couplings of the groups of the SM are run up to M_R and those of $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ from M_R and M_{GUT} . The vertical gray band around 2.8×10^9 GeV represents the band allowed for M_R for this model and the model of the right plot. The right plot represents the running of the G_{3221}^{M} model plus a fermionic $SU(2)_R - SU(2)_L$ bidoublet dark matter candidate (that of the third panel Fig. 6 of [39]). We present this last plot as an example that the addition of matter can alter the running [see Eq. (14) and the text below] of the gauge coupling constants and reduce the unification scale.

scalars, respectively, and I is the Dynkin index of a representation. As is well known, for the SM $SU(3)_C$ factor, we have $I(\text{ad}^{(3)}) = 3$ and $\sum_i I(F_i^{(3)}) = 6$, rendering $b_3 = -7$. We can see that the addition of particles that make b_3 less negative, for example, color triplets, flattens the running of $1/\alpha_3$ and shifts M_{GUT} to higher scales and M_R to lower scales. But as some of us noted in [39], the addition of color triplets is not enough because multiplets of all the other groups are required in order to ensure that the couplings g_{2L} and g_{2R} unify at high scale. As an example, in Fig. 2 we plot the running of the gauge couplings when a dark matter candidate in the form of a self-conjugate fermion with $B - L = 0$, which is a doublet of both $SU(2)_R$ and $SU(2)_L$ (referred to as “bidoublet”) and a singlet of $SU(3)_C$ has been added to the model G_{3221}^M . For this case the GUT scale is located at 2.8×10^{15} GeV. Further examples of adding dark matter candidates can be found in [38,39]. Note that in our analysis of FOPT, we consider both the maximum scale $M_{\text{GUT}} = 1.6 \times 10^{16}$ GeV and the scale $M_{\text{GUT}} = 10^{15}$ GeV because that scale is still compatible with proton decay. One further justification for this is that we can add matter at lower energies, which will alter the scale of gauge coupling unification but not the basic characteristics of the potential of Eq. (3).

E. Constraints within the model

1. Mass spectrum

Calculating the mass spectrum arising from the GUT-scale symmetry breaking step, one encounters particles with negative mass squared, i.e., tachyons. However, it is necessary to take into account loop corrections to the masses. At the one-loop level, this entails adding all terms involving the **126** scalar to the potential in Eq. (3), determining the effective potential by extending the sums in Eqs. (6) and (7) to the full particle content, and calculating the eigenvalues of the scalar mass matrix, which is only possible numerically. This was done in [23], where it was shown that the tachyon problem can be avoided if the quartic couplings of the scalar **45** are restricted to [see Figs. 4 and 6, Sec. IV.C.1 point (iii), and Appendix A of [23]]

$$a_0 \in (0.0, 0.2), \quad a_2 \in (-0.05, -0.01). \quad (15)$$

Hence, we will only consider parameter values within these ranges in our analysis. This also ensures that the vev of the **45** is located at the global minimum of the one-loop effective potential.

2. Proton decay

Here we follow [10,38,44] and give a brief account of the way proton decay rates are calculated. The most sensitive channel for nonsupersymmetric models is the dimension-6 operator induced decay $p \rightarrow \pi^0 e^+$, which is mediated by

gauge fields.⁵ The proton decay for this channel can be estimated as [38]

$$\Gamma(p \rightarrow \pi^0 e^+) = \frac{m_p}{32\pi} \left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right)^2 [|\mathcal{A}_L(p \rightarrow \pi^0 e^+)|^2 + |\mathcal{A}_R(p \rightarrow \pi^0 e^+)|^2], \quad (16)$$

where m_p and m_{π^0} are, respectively, the proton and the neutral pion masses. The amplitudes at the weak scale are given by

$$\begin{aligned} \mathcal{A}_L(p \rightarrow \pi^0 e^+) &= C_{RL}((ud)_R u_L)(\mu = 2 \text{ GeV}) \\ &\quad \times \langle \pi^0 | (ud)_R u_R | p \rangle, \\ \mathcal{A}_R(p \rightarrow \pi^0 e^+) &= 2C_{LR}((ud)_L u_R)(\mu = 2 \text{ GeV}) \\ &\quad \times \langle \pi^0 | (ud)_R u_R | p \rangle, \end{aligned} \quad (17)$$

where the Wilson coefficients $C_{RL}((ud)_R u_L)$ and $C_{LR}((ud)_L u_R)$ correspond, respectively, to C_1 and C_2 of [38]. For numerical values we use the inputs given in Table I in [10], including the value of the hadronic matrix elements and the evolution of the Wilson coefficients from M_{GUT} given in [44]. For the minimal G_{3221} model, we obtain $(2.4 \pm 1.6) \times 10^{36}$ years, a safe value given the current experimental bound of $\tau(p \rightarrow \pi^0 e^+) > 2.34 \times 10^{34}$ years at 95% CL [45] and the projected bound of 7.8×10^{34} years at 95% CL [46].

III. APPEARANCE OF A FIRST ORDER PHASE TRANSITION

A. Preliminaries

In this section, we briefly mention only the salient concepts pertaining FOPT, which have been studied extensively (see for example [47]), in order to explain our results. As it is well known, FOPT can generate GW through nucleation, expansion, collision, and merger of bubbles in the broken phase. In vacuum, GWs originate solely from bubble collisions and can be described by the envelope approximation [48]. In a thermal plasma, however, friction slows bubble expansion, transferring most of the energy to the plasma and making scalar field contributions subdominant. Lattice simulations show that this energy drives sound waves in the plasma, leading to an acoustic phase that dominates GW production [49,50]. For stronger transitions, turbulence may emerge, sustaining GW emission until it dissipates [51–55]. To model these effects, we use GW templates from 3D simulations that fit the spectrum in terms of phase transition parameters and frequency [56–58]. Key quantities characterizing the GW are α and β , respectively,

⁵Color triplet scalars can also induce proton decay but since the dominant decays in nonsupersymmetric models are gauge interactions we do not consider them here.

the ratio of released vacuum energy to the plasma's radiation energy in the symmetric phase and a measure of the duration of the phase transition. For completeness, the formulas that we use for α and β are presented in Eqs. (C2) and (C3), while the 3D action, S_3 , describing the bubbles forming when the transition between the metastable and the true vacuum takes place is given in Eq. (C1). According to [59–61] the decay rate, $\Gamma(T)$, of a bubble is given by

$$\Gamma(T) \approx T^4 \left(\frac{S_3}{2\pi T} \right)^{\frac{3}{2}} \exp(-S_3/T). \quad (18)$$

Taking the expression above to be valid we can calculate the nucleation temperature T_n , at which the average number of bubbles nucleated per Hubble horizon is of order 1,

$$N(T_n) = \left(\frac{3\bar{M}_{\text{Pl}}}{\pi} \right)^4 \left(\frac{10}{g_*} \right)^2 \int_{T_n}^{T_c} \frac{dT}{T^5} \left(\frac{S_3}{2\pi T} \right)^{3/2} \times \exp(-S_3/T) \sim 1, \quad (19)$$

which is equivalent to computing the nucleation temperature by taking Γ/V , where V is the comoving volume, as

$$\frac{\Gamma}{V} \approx T_n^4 e^{S_3(T_n)/T_n}, \quad (20)$$

integrating to the time t_* at which the phase transition takes place and requiring that the total number of bubbles nucleated from time $t = 0$ to $t = t_*$ is of order 1.⁶ Equation (20) has no solution when

$$\frac{\Gamma}{V} < H^4, \quad (21)$$

which can be written as

$$\frac{S_3(T)}{T} > 4 \log\left(\frac{\bar{M}_{\text{Pl}}}{T}\right) - 2 \log\left(\frac{90}{\pi^2 g_*(T)}\right), \quad (22)$$

where $\bar{M}_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. In this case the transition is not completed. For finding the suitable parameter space where a FOPT may occur, we consider the one-loop corrected potential with thermal corrections in Eq. (8) and the criterion Eq. (19) to determine whether nucleation is possible. We note that for the potential that we are considering, the temperature correction always grows and has a different sign with respect to the tree-level and one-loop contributions, which need to have a minimum and become negative. In this regard, the thermal contribution is typically necessary to produce a barrier. As is well understood, a barrier can occur if there is a cubic term in the field in the one-loop thermally corrected potential (where the total contribution to the potential is $\phi^3 E$, where E has mass dimension 1 and could be temperature independent).

E is proportional to gauge coupling of the theory to the third power, g^3 , and hence the appearance of a barrier enhances with g^3 .

Above the unification scale, the considered model contains 96 fermionic (three generations in the representation **16**, two possible spin orientations) and 307 scalar degrees of freedom (45 massless gauge bosons, $10 + 45 + 126 \times 2$ real scalars), which yields $g_* = 481$ effective relativistic degrees of freedom. However, nucleation happens at temperatures below M_{GUT} , where some of the particles obtaining GUT-scale masses are nonrelativistic. To account for this, we use $g_* = 300$ in our computations.

B. Results

We start by discussing the behavior of the phase transition parameters α and β/H_* , which indicate the strength and the inverse duration, respectively, see Appendix C. In Fig. 3 we collect the results for the scales $M_{\text{GUT}} = 1.6 \times 10^{16}$ GeV and $M_{\text{GUT}} = 1.0 \times 10^{15}$ GeV. As is known, α and β/H_* are not completely independent parameters, since both of them depend on the 3D action, Eq. (C1) and they have opposite behaviors, that is, when α increases, β/H_* decreases. This behavior can be clearly seen in Fig. 3, where we plot α and β as a function of the parameter a_0 , Eq. (4), and whose range is constrained, such as to avoid a tachyonic spectrum, Eq. (15). We have also marked four representative benchmark points (BPs) and we give the critical, nucleation temperature, α and β in Table II. In addition, in Fig. 3, we have plotted the GW energy density as computed with the equations of Appendix C. Since $\Omega_{\text{GW}} h^2$ is proportional to $\alpha^2/(\beta/H_*)$, $\Omega_{\text{GW}} h^2$ follows a decreasing behavior as a_0 grows, just as in the case of $\alpha(a_0)$. For each point in a_0 , μ^2 and a_2 are computed following the algorithm described in Sec. II C.

In order to see the behavior of the observable $\Omega_{\text{GW}} h^2$ as a function of both parameters a_0 and a_2 , we present Figs. 4 and 5 for $M_{\text{GUT}} = 1.6 \times 10^{16}$ GeV and $M_{\text{GUT}} = 1.0 \times 10^{15}$ GeV, respectively. We have also marked the four BPs as well as the lines corresponding to the parameter values for which the curves in Fig. 3 have been computed. We show in solid gray the part of the allowed parameter space of a_0 and a_2 that allows for a first-order phase transition. The shading indicates the order of magnitude of the GW signal, with lighter shading corresponding to a stronger signal. Inside the yellow region indicated by “No nucleation,” there is no potential barrier and nucleation is not possible. Its exact boundary is difficult to determine because close to it the numerical algorithm eventually fails when $T_0 \rightarrow T_c$, where “ T_0 ” is the spinodal temperature, defined from the condition: $\partial^2 V(\phi, T)/\partial \phi^2 = 0$.

We see that the borders of the shaded regions are nearly parallel. Hence, the signal strength does not depend on a_0 and a_2 separately, but to a good approximation only on a linear combination. Consequently, the two-dimensional plots in Fig. 3 are sufficient to encode most of the parameter

⁶ $\int_0^{t_*} dt \Gamma/v/H^3(t) = \mathcal{O}(1)$.

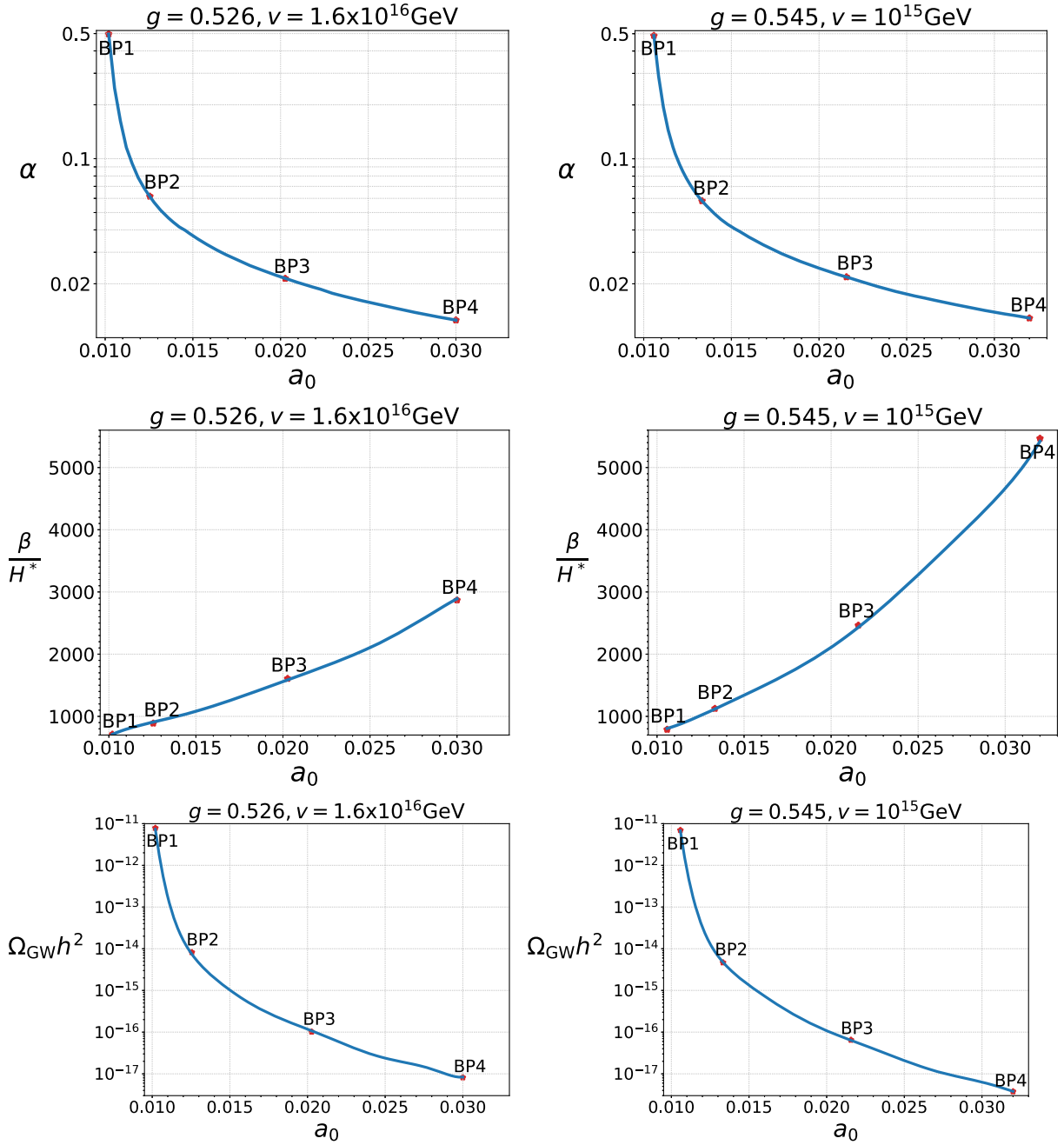


FIG. 3. Phase transition parameters ($\alpha, \beta/H_*$) and GW density as a function of a_0 for the scales $M_{\text{GUT}} = 1.6 \times 10^{16} \text{ GeV}$, left, and $M_{\text{GUT}} = 1.0 \times 10^{15} \text{ GeV}$, right. The (red) dots represent the benchmark points listed in Tables II and III.

dependence, as a parallel shift of the line connecting the benchmark points would lead to curves very similar to those in Fig. 3.

The hatched regions mark parameter space points where $\Delta^{2\text{-loop}}$ of Eq. (10) is larger than 0.5, indicating that the precision of our one-loop calculation is likely to be insufficient. As the GW signal is strongest in these regions, extending the calculation to include higher loop orders would be a highly desirable goal for future work. We see that BP2 is at the border of the region with $\Delta^{2\text{-loop}} > 0.5$

and thus corresponds to the largest signal strength for which we are confident in the accuracy of our calculation. In contrast, BP1 has the largest value of $\Delta^{2\text{-loop}}$ and should therefore be regarded with caution.

The plots also include the lines on which the quadratic term in the scalar potential vanishes at tree level [$\mu_0^2 = 0$, by which we mean that the rhs of Eq. (5) vanishes] and at the one-loop level ($\mu^2 = 0$, cf. the discussion in Sec. II C). Evidently, a strong signal corresponds to small values of μ^2 .

TABLE II. Phase transition parameters for four benchmark points with $M_{\text{GUT}} = 1.6 \times 10^{16}$ GeV.

BP	T_0/M_{GUT}	T_c/M_{GUT}	T_n/M_{GUT}	α	β/H_*	a_0	a_2
BP1	0.0603	0.119	0.0638	0.50	7.1×10^2	0.0102	-0.04
BP2	0.136	0.170	0.142	0.062	8.9×10^2	0.0125	-0.0394
BP3	0.242	0.259	0.249	0.021	1.6×10^3	0.0202	-0.0377
BP4	0.308	0.318	0.313	0.013	2.9×10^3	0.03	-0.0356

TABLE III. Phase transition parameters for four benchmark points with $M_{\text{GUT}} = 1.0 \times 10^{15}$ GeV.

BP	T_0/M_{GUT}	T_c/M_{GUT}	T_n/M_{GUT}	α	β/H_*	a_0	a_2
BP1	0.0603	0.122	0.0659	0.49	7.9×10^2	0.01058	-0.04
BP2	0.141	0.176	0.150	0.058	1.1×10^3	0.0133	-0.0391
BP3	0.244	0.261	0.252	0.022	2.5×10^3	0.0215	-0.0366
BP4	0.309	0.319	0.316	0.013	5.5×10^3	0.032	-0.0335

IV. INCOMPLETE PHASE TRANSITIONS

In Sec. III B, we have seen that for both cases of $M_{\text{GUT}} = 1.6 \times 10^{16}$ GeV and $M_{\text{GUT}} = 1.0 \times 10^{15}$ GeV, there is a parameter space in a_0 and a_2 (see the yellow region of Figs. 4 and 5) where the phase transition does not complete at the GUT scale. For these cases, we could have the scenario “B,” depicted in Fig. 1, because the criterion of Eq. (22) is

satisfied. As pointed out in [15], the phase transition can still occur at a lower scale, leaving an imprint in the CMB. For this to happen the scalar field breaking the GUT theory needs to act like a spectator value during inflation, Eq. (21) has to be satisfied, and $V_{\text{Inf}}(\phi_{\text{Inf}}) \gg V(\phi)$, where we have called ϕ_{Inf} the field triggering inflation. What is relevant for the breaking chain that we are considering is that the scalar spectrum is modified and the induced GW can be seen at

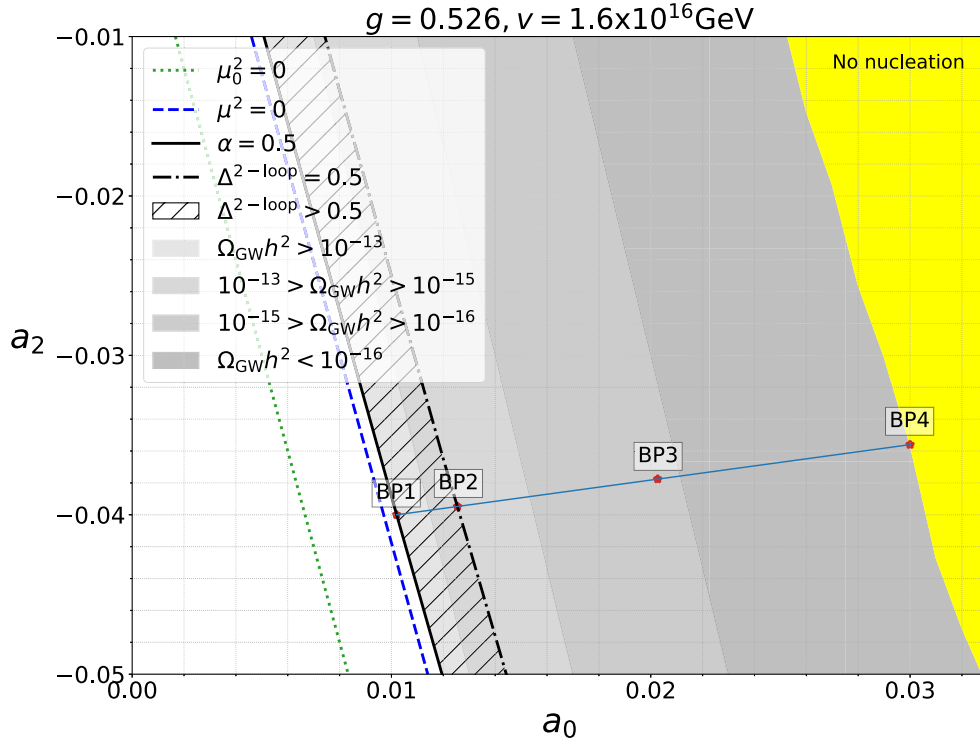
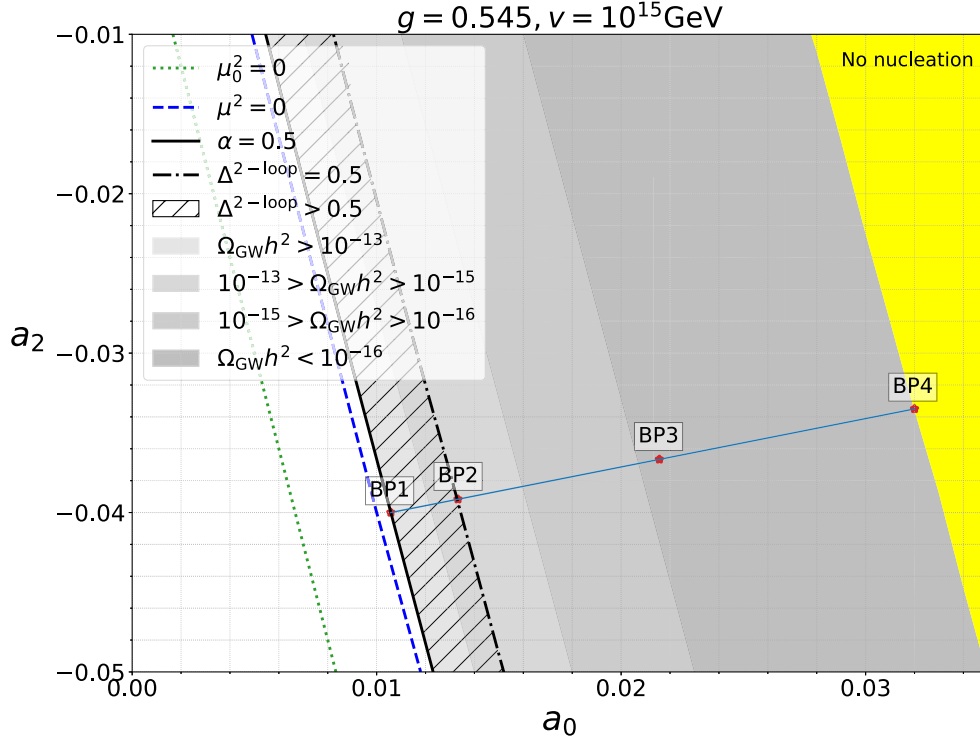


FIG. 4. GW parameter space for a_0 vs a_2 , where a_2 is restricted to the region where the spectrum does not contain tachyons. The four points marked by stars are our benchmark points. The yellow region has no FOPT or is numerically difficult to calculate. In the hatched region, $\Delta^{2\text{-loop}} > 0.5$. We denote the quadratic term in the scalar potential obtained from the tree-level condition (5) by μ_0^2 . In contrast, μ^2 is the corrected value of this parameter ensuring that the one-loop-corrected scalar potential has a minimum at v .

FIG. 5. Same as in Fig. 4 but for $M_{\text{GUT}} = 1.0 \times 10^{15}$ GeV.

lower frequencies than those of the phase transitions and the plasma. This depends on the phase transition parameters and is governed by the dimensionless parameter $\gamma_{PT} := 1/H^4\Gamma/V(\Delta V(\phi)/\dot{\phi}_{\text{Inf}})^2$, defined in [15]. In the yellow region of Figs. 4 and 5, $1/H^4\Gamma/V < 1$ and we can compute it in those regions. The other term in γ_{PT} depends on details of inflation and since we do not have a model of inflation we can give an upper bound $\gamma_{PT} \leq 10^{-3}(\Delta V(\phi)/\dot{\phi}_{\text{Inf}})^2$, since for $V(\phi) \ll V_{\text{Inf}}(\phi_{\text{Inf}})$ we expect that $V(\phi) \ll V_{\text{Inf}}(\phi_{\text{Inf}}) \ll M_{\text{Pl}}^4$ at the end of inflation when the slow-roll parameter is close to unity. For illustration, we plot in Fig. 8 the region in the plane of frequency vs $\Omega_{\text{GW}} h^2$ at which these signals could potentially be observed.

V. GRAVITATIONAL WAVES FROM THE SO(10) PLASMA

Any plasma in thermal equilibrium emits GWs [62]. The signal is determined by the shear viscosity of the plasma. It is guaranteed to be present in the SM, although the ultrahigh frequency of the GW makes their detection challenging. Scenarios of physics beyond the SM can both lower the peak frequency and enhance the signal as a result of the many additional degrees of freedom. As the shear viscosity changes during the different stages of the GUT symmetry breaking down to the SM, we obtain a different signal during each particular stage. Furthermore and more importantly, as the breaking scales M_{GUT} and M_R are fixed by gauge coupling unification, the maximum temperature

of each stage is fixed, unlike the SM, where the maximum temperature is the reheating temperature and thus a free parameter. In the following, we determine the GW signal for each breaking stage, following [62,63].

In this work, we present the results for the GW signal from the plasma during the three steps of symmetry breaking shown in Eq. (1) with the minimal content of matter. For each step, we take Eq. (26) and use the well-known expression for the GW produced by a plasma in equilibrium [62],

$$\begin{aligned} \Omega_{\text{GW}}(f, T_0) h^2 \\ = \Omega_{r_0} h^2 \frac{\lambda}{\bar{M}_{\text{Pl}}} \int_{T_{\text{EWCO}}}^{T_{\text{Max}}} dT \left(\frac{g_{*0}}{g_*(T)} \right)^{4/3} T^2 \hat{k}(f, T)^3 \frac{\eta(\hat{k}, T)}{\sqrt{\rho(T)}} \beta(T), \end{aligned} \quad (23)$$

where $\Omega_{\text{GW}}(f, T_0)$ is the fraction of energy released into GW radiation per frequency octave [1], $\lambda = 30\sqrt{3}/\pi^4$ and

$$\begin{aligned} \hat{k}(f, T) &:= \frac{k}{T} = \left[\frac{g_{*s}(T)}{g_{*s}(T_0)} \right]^{\frac{1}{3}} \frac{2\pi f}{T_0}, \\ f &= \frac{1}{2\pi} \left[\frac{g_{*s}(T_0)}{g_{*s}(T_{\text{EWCO}})} \right]^{\frac{1}{3}} \left(\frac{T_0}{T_{\text{EWCO}}} \right) k_{\text{EWCO}}. \end{aligned} \quad (24)$$

In these equations, $T_{\text{EWCO}} = 160$ GeV,

$$k_{\text{EWCO}} = k(T)(g_{*s}(T_{\text{EWCO}})/g_{*s}(T))^{1/3} T_{\text{EWCO}}/T$$

is the wave number at $T = T_{\text{EWCO}}$, T_{Max} is the maximum temperature that we identify with the reheating temperature.

The shear viscosity of the SM was computed up to the leading contribution in [62], while accounting for temperature dependence in [63]. For a general theory, we have

$$\eta(\hat{k}, T) = \begin{cases} \frac{1}{8\pi} \frac{16}{g_1^4 \ln(5T/m_{D_1})}, & \hat{k} \lesssim \alpha_1^2, \\ \eta_{\text{HTL}}(\hat{k}, T) + \eta^T(\hat{k}, T), & \hat{k} \gtrsim 3, \end{cases} \quad (25)$$

where η_{HTL} is the hard thermal logarithmic (HTL) expression [63,64]

$$\eta_{\text{HTL}}(\hat{k}, T) = \frac{1}{16\pi} \hat{k} n_B(\hat{k}) \sum_{n=1}^m d_n \hat{m}_{D_{\mathcal{G}_n}}^2 \ln \left(4 \frac{\hat{k}^2}{\hat{m}_{D_{\mathcal{G}_n}}^2} + 1 \right), \quad (26)$$

where $n_B(\hat{k}) = \frac{1}{e^{\hat{k}} - 1}$, and the sum is over the m different groups if the model contains more than one group factor, d_n is the number of generators of the given group, and the normalized Debye masses are $\hat{m}_{D_{\mathcal{G}_n}}^2 = m_{D_{\mathcal{G}_n}}^2/T^2$. The η^T part is a function that depends on the gauge couplings at definite temperature. For the SM, this has been calculated in [63]. In this work, we assume $\eta^T \rightarrow 0$ for the SO(10) model. Note that the first part in Eq. (25) corresponds to the hydrodynamic limit (low frequency) and the second term to the leading log (high frequency) [62].

In practice, since Eq. (23) takes its dominant contribution at T_{Max} and since the production of the GW takes place in the radiation era, the expression for the signal reduces to

$$\frac{\Omega_{\text{GW}}(f, T_0) h^2}{\Omega_{\gamma_0} h^2} = \lambda \left[\frac{a_{\text{Max}} T_{\text{Max}}}{a_0 T_0} \right]^4 \frac{T_{\text{EWCO}} - T_{\text{Max}}}{\bar{M}_{\text{Pl}}} \frac{T_{\text{in}}^2}{\sqrt{\rho}} \hat{k}^3 \eta \times (T_{\text{Max}}, \hat{k}), \quad (27)$$

which we use to produce our plots.

We recall that for a general gauge theory the Debye masses of a group factor $\mathcal{G}_n$ are given by [65]

$$m_{D_{\mathcal{G}_n}}^2(T) = g_n^2(T) T^2 \left[\frac{1}{3} I(\text{ad}^{(n)}) + \frac{1}{6} \sum_i I(F_i^{(n)}) + \frac{1}{6} \sum_j I(S_j^{(n)}) \right], \quad (28)$$

where the notation for representations and Dynkin indices is explained below Eq. (14). Besides, $g_n(T)$ is the temperature-dependent gauge coupling. From now on, we suppress the temperature dependence of the Debye masses and the gauge couplings in our formulas. In our model, the results for the Debye masses, calculated with the help of GroupMath [66], are as follows.

SO(10) plasma. For the stage above the unification scale, we have a single group $\mathcal{G} = \text{SO}(10)$. There are three

families of fermions in the **16** representation, that is, $I(F_i) = I(\mathbf{16}) = 2$ with $i = 1, 2, 3$. In addition, we have gauge bosons and scalars in the adjoint, $I(\text{ad}) = I(\mathbf{45}) = 8$, a real scalar multiplet with $I(\mathbf{10}) = 1$, and complex scalars in the **126**, so $I(\mathbf{126}_j) = 35$ for $j = 1, 2$.⁷ Summing up these contributions, Eq. (28) yields

$$m_{D_{\text{SO}(10)}}^2 = \frac{101}{6} g^2 T^2, \quad (29)$$

so the contribution to Eq. (26) is

$$d_{\text{SO}(10)} m_{D_{\text{SO}(10)}}^2 = 45 \times \frac{101}{6} g^2 T^2. \quad (30)$$

This value is conservative, since a complete model will include additional contributions from multiplets needed to obtain realistic Yukawa couplings and to accommodate dark matter.

SU(3)_C × SU(2)_L × SU(2)_R × U(1)_{B-L} plasma. Between the unification scale and the scale of $B-L$ breaking, the plasma contains the fermions, the gauge bosons of the intermediate gauge group G_{3221} , the doublet scalars originating from the **10**, and the triplet scalar breaking $SU(2)_R \times U(1)_{B-L}$ from the **126**. The representations decompose as (for the indices $a = 3, 2L, 2R, B-L$)

$$\begin{aligned} \mathbf{10} &= (\mathbf{1}, \mathbf{2}, \mathbf{2}, 0) \oplus (\text{heavy triplets}), \\ \mathbf{16} &= \left(\mathbf{3}, \mathbf{2}, \mathbf{1}, \frac{1}{3} \right) \oplus \left(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{2}, -\frac{1}{3} \right) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{1}, -1) \\ &\quad \oplus (\mathbf{1}, \mathbf{1}, \mathbf{2}, 1), \\ \mathbf{126} &= (\mathbf{1}, \mathbf{1}, \mathbf{3}, 2) \oplus (\text{heavy states}) \end{aligned} \quad (31)$$

in terms of G_{3221} multiplets, where the last number in parentheses indicates $B-L$. Plugging the corresponding $SU(2)$ and $SU(3)$ Dynkin indices as well as $\frac{3}{8}(B-L)^2$ (where the factor $\frac{3}{8}$ is due to the GUT normalization of g_{B-L}) into Eq. (28), we find

$$d_i m_{D_i}^2 = \begin{cases} d_1 \frac{5}{2} g_{B-L}^2 T^2, d_1 = 1, \\ d_2 2 g_{2L}^2 T^2, d_2 = 3, \\ d_2 \frac{8}{3} g_{2R}^2 T^2, d_2 = 3, \\ d_3 2 g_3^2 T^2, d_3 = 8. \end{cases} \quad (32)$$

In the case with $M_{\text{GUT}} = 10^{15}$ GeV, the dark matter fermionic bidoublet with $B-L = 0$ contributes to the $SU(2)$ group factors an additional $1/6$, resulting in $m_{D_{SU(2)_L}}^2 = 13/6 g_{2L}^2 T^2$ and $m_{D_{SU(2)_R}}^2 = 17/6 g_{2R}^2 T^2$.

⁷Above each symmetry breaking scale, we obviously need to include all the particles that later on will be the would-be Nambu-Goldstone bosons of the broken theory.

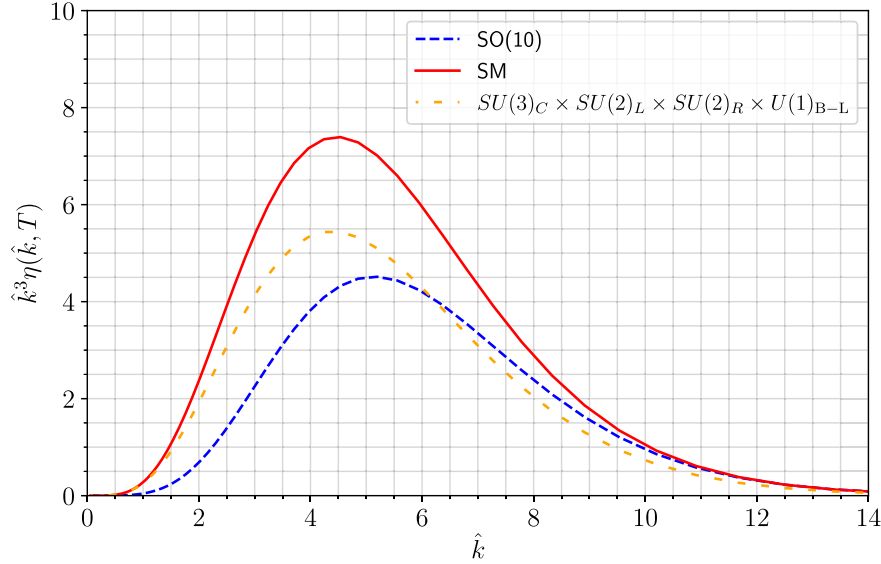


FIG. 6. Comparison of the shear viscosity, η , for the case of the SM, red solid, for the $SO(10)$ case blue long dashed, and for the group $SU(3)_C \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ orange short dashed.

SM plasma. Below the $B - L$ breaking scale, we have the gauge group and particle content of the SM, so we can use the known results [62]⁸

$$d_i m_{D_i}^2 = \begin{cases} d_1 \frac{11}{10} (g_1^{\text{SM}})^2 T^2, d_1 = 1, \\ d_2 \frac{11}{6} (g_2^{\text{SM}})^2 T^2, d_2 = 3, \\ d_3 2 (g_3^{\text{SM}})^2 T^2, d_3 = 8. \end{cases} \quad (33)$$

In Fig. 6 we show the comparison of the shear viscosity η of the $SO(10)$ group, the G_{3221} group and the SM group. We can see that the shear viscosity for G_{3221} and $SO(10)$ is suppressed with respect to the SM.

Note, from Eq. (23), that the location of the peak of the GW density, as a function of the frequency, scales as $(g_{*0}/g_*(T))^{4/3}$ and hence the more degrees of freedom the lower the frequency of the peak of the GW density. This is illustrated in Fig. 7 where we have plotted each of the contributions to the GW plasma of the breaking chain of Eq. (1). The height of the peak is determined by the maximum attainable temperature at each stage. Note that the $SO(10)$ group would be unbroken from M_{Pl} down to M_{GUT} but due to constraints on inflation and thermal equilibrium of the plasma, as mentioned in Sec. II, the plasma would enter into thermal equilibrium just close to the GUT scale and therefore we consider the temperature of the $SO(10)$ (dashed blue line) plasma at this temperature. The short-dashed (orange) line represents the contribution from G_{3221} and the solid red line, which is suppressed by more than four orders of magnitude with respect to the $SO(10)$ prediction, the SM contribution. Note that the contribution from the G_{3221} stage leads the

overall signal, although technically the maximum attainable temperature is lower than for the complete $SO(10)$ group. For comparison, we have also plotted the GW density of the plasma of the SM at the maximum temperature of $SO(10)$ so that we can see how we could distinguish the signals for each case. An advantage of the signal predicted in $SO(10)$ compared to the SM is that, even if the height of the peak is lower, its location has a lower frequency. Furthermore, in the $SO(10)$ case, the reheating temperature is fixed by the hierarchy of the scales of the breaking chain (1), while for the case of the SM, there is not a way to fix the reheating temperature.

VI. CONCLUSIONS

We have investigated expected GW signals within one of the most promising breaking chains of a nonsupersymmetric $SO(10)$ model [22–24,38], Eq. (1). We focused on two types of signal: the GWs produced by the high-temperature FOPT induced by the symmetry breaking expected in GUTs, and the stochastic background produced by the shear viscosity of the relativistic plasma in thermal equilibrium throughout the expansion history of the Universe.

As far as the FOPT is concerned, we considered a minimal particle content, including the gauge bosons, three families of fermions transforming under the representation **16**, and three scalars multiplets transforming under **10**, **45**, and **126**. Specifically, we analyzed the parameter space where the symmetry breaking is triggered by the **45**, whose tree-level potential is given in Eq. (3) and governed by two free parameters a_0 and a_2 . We studied the corresponding effective potential including the one-loop and thermal corrections, providing for the first time a compact analytic

⁸Taking into account the GUT normalization of g_1 .

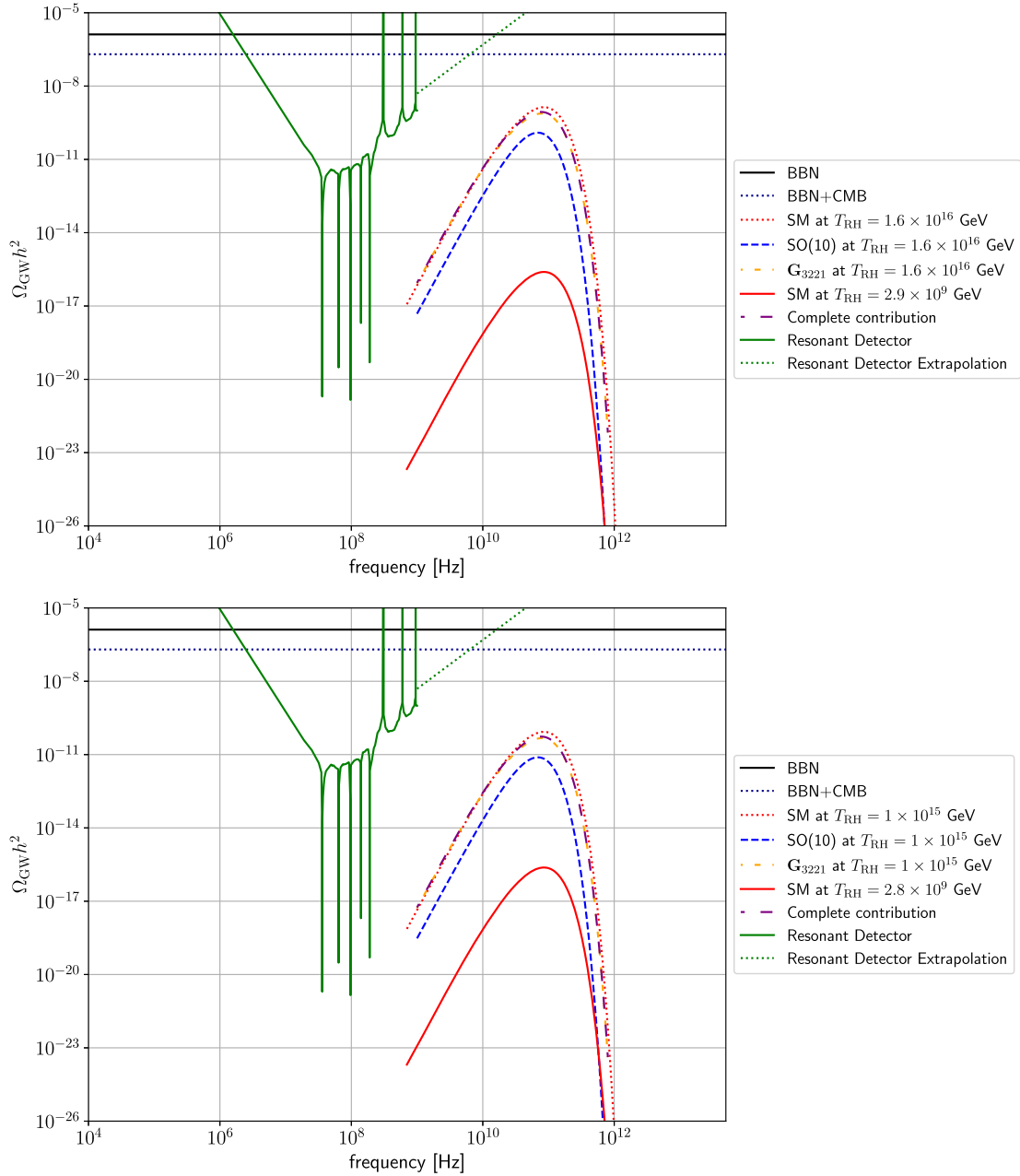


FIG. 7. Contributions to GW from plasma fluctuations, shown separately for each step of the breaking chain in Eq. (1) for $M_{\text{GUT}} = 1.6 \times 10^{16}$ GeV (top) and $M_{\text{GUT}} = 10^{15}$ GeV (bottom). For comparison, we have also plotted the SM expectation for the same reheating temperature. “Resonant Detector” refers to the proposal in [67]. In this plot we show only the leading-log part in η , Eq. (25), since it is the one that determines the location of the peak. In Fig. 8 we show also the hydrodynamic limit, relevant for disentangling the signal in the low-frequency band from other sources.

expression. We explored gauge coupling unification in this scenario, finding that it takes place between 10^{15} and 10^{16} GeV, and adopted an iterative procedure to fix the quadratic term in the potential ensuring a vev of the **45** in the correct range for each value of a_0 and a_2 . To calculate the ensuing expected signals we used the parameters of the phase transition obtained from the effective potential in the well-known expressions for the sound wave [49,50] and

turbulence [56,57] contributions to the density of GW when the barrier is mainly of thermal origin.

We found that the FOPT takes place in a significant part of the parameter space allowed by the absence of tachyons and by proton decay. The GW signal peaks at frequencies of order 10^{10} to 10^{11} Hz, depending on the unification scale, as expected for FOPT occurring at very high energy scales. This puts it beyond the sensitivity range of pulsar

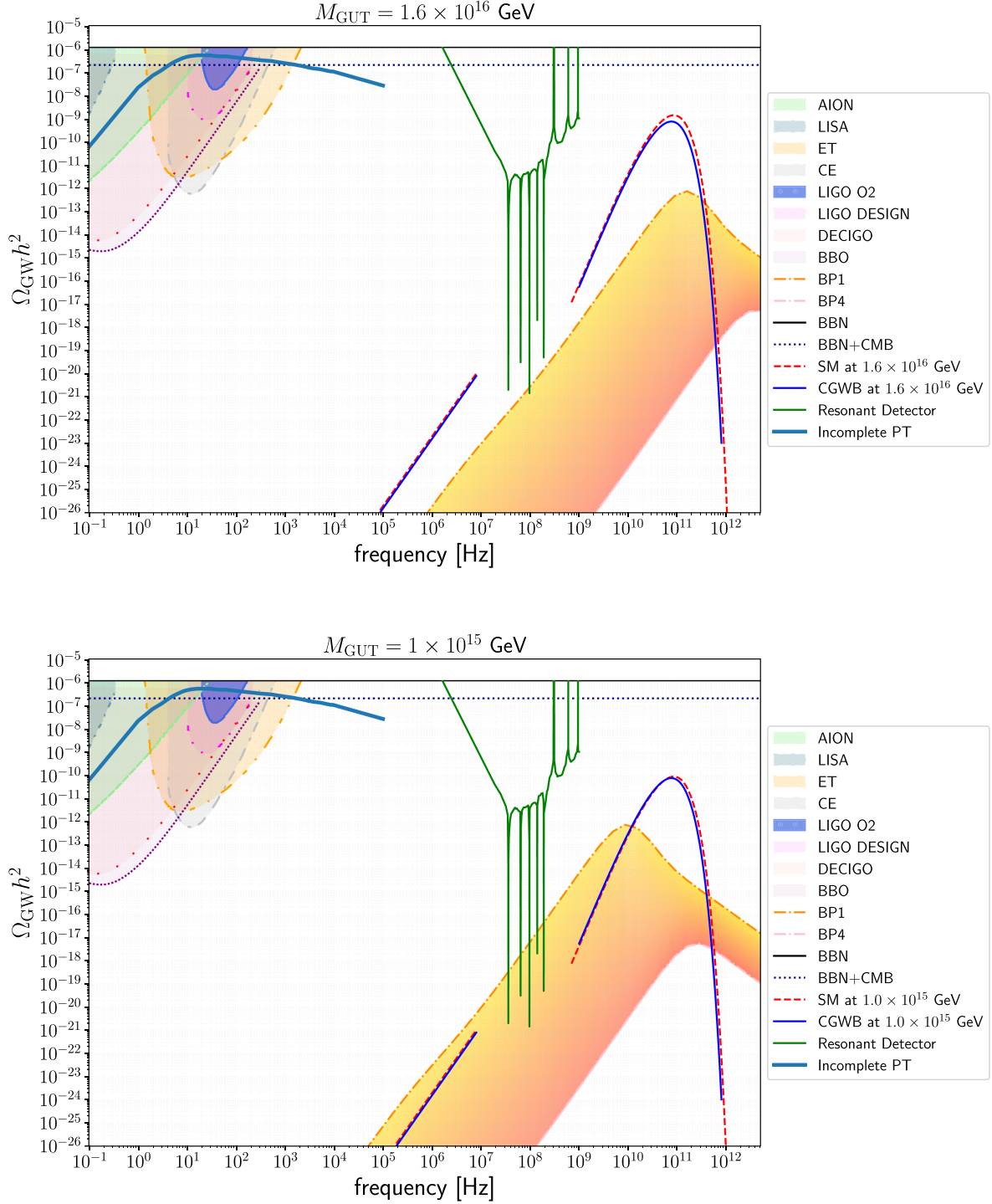


FIG. 8. GW signals from the considered $SO(10)$ model for two different values of the unification scale, $M_{\text{GUT}} = 1.6 \times 10^{16}$ GeV (top) and $M_{\text{GUT}} = 10^{15}$ GeV (bottom). The colored region shows the GW spectrum that can be obtained from the FOPT at the GUT scale by varying the scalar quartic couplings within the allowed region of the parameter space; the upper boundary corresponds to BP 1, while the lower boundary is obtained for BP 4. Solid and dashed lines show the GW produced by fluctuations in the plasma, with the $SO(10)$ prediction plotted in blue (denoted by cosmic GW background), while the signal expected in the SM with the same reheating temperature is plotted in red for comparison. We also show the sensitivity of current and planned experiments, along with the BBN and the CMB constraints.

timing arrays and most GW detectors but possibly within the range of proposed resonant detectors [67].⁹ We found the strongest signal in the region of the parameter space where the accuracy of the computation is limited by large contributions of higher loop orders. Consequently, an important direction for future work is to improve the calculation of the effective potential beyond one-loop precision. We remark that this is often also needed for models that seem to have a promising observable signal [71–73].

Fluctuations in the primordial plasma can yield a stronger GW signal than the FOPT, at frequencies of the same order, as shown by the solid and dashed lines in Fig. 8. However, the $SO(10)$ signal is somewhat suppressed compared to the SM with the same reheating temperature. Consequently, observing a signal at the SM strength would in principle disfavor an interpretation in terms of the $SO(10)$ model. The lower-frequency part of the plotted signal corresponds to the hydrodynamic limit, i.e., the first expression in Eq. (25), and the high-frequency part to the leading-log result (second expression).

In Fig. 8 we plot all GW signals that we have considered in this work, along with some current and planned experiments, in particular the resonant detector proposed in [67]. As to the possibilities of observing GW produced by the scalar power spectrum when the phase transition is not completed before inflation (see Sec. IV), the details depend on the particular inflation scenario and are therefore more model dependent. Nevertheless, in our scenario we can put a bound on the parameter γ_{PT} that controls the shape and strength of the signal, $\gamma_{PT} \leq 10^{-3}(\Delta V(\phi)/\dot{\phi}_{\text{Inf}})^2$. We include this limit in Fig. 8. All possible signals that can come from this scenario will be lower than the solid petrol blue line shown from 10^1 to 10^4 Hz. We have not discussed GWs produced by topological defects, which have been extensively studied recently albeit in different models (see, e.g., [9,10,19,74–82]) and could occur at much lower frequencies.

We have studied a minimal GUT setup that realizes a realistic symmetry breaking chain. Although a direct solution to the dark matter problem is not given in this context, we have mentioned in Sec. IID how the addition of a doublet of both $SU(2)_L$ and $SU(2)_R$ can provide a dark matter candidate, as some of us did in [39]. The fermion mass spectrum and perturbativity in the Higgs sector were not addressed but interestingly, both issues can be alleviated by introducing additional scalar fields at low energy. We have also made a number of simplifying assumptions about the mass spectrum and the contributions to the effective potential. Finally, the potential of a FOPT during the second

step of symmetry breaking, which would produce GW with lower frequencies, warrants exploration. This study should therefore be considered a first step in the investigation of phase transitions in more complicated and more realistic unified models, paving the way for a promising new direction of research at the intersection of GUT model building and cosmology.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: $SO(10)$ GROUP THEORY

The generators of $SO(10)$ are conveniently labeled by antisymmetric indices, so we denote them by $T^{\alpha\beta} = -T^{\beta\alpha}$ with $\alpha, \beta = 1, \dots, 10$.¹⁰ In the fundamental representation, they are antisymmetric 10×10 matrices,

$$(T^{\alpha\beta})_{ij} = -\frac{i}{\sqrt{2}}(\delta_{\alpha i}\delta_{\beta j} - \delta_{\alpha j}\delta_{\beta i}), \quad (\text{A1})$$

where $i, j = 1, \dots, 10$. We do not distinguish between upper and lower indices and imply summation over repeated indices. The generators satisfy

$$\text{Tr}(T^{\alpha\beta}T^{\gamma\delta}) = \delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}. \quad (\text{A2})$$

The scalar and gauge boson fields relevant to our discussion¹¹ transform under the adjoint representation 45 and are written as antisymmetric 10×10 matrices

⁹It has been pointed out that [68–70], once realistic coherence, duty-cycle, and experimental losses are included, the sensitivities of these resonant detectors can be significantly decreased compared to the proposal of [67], and that we include in the plots of Figs. 7 and 8.

¹⁰We closely follow the notation of [22].

¹¹Here we do not consider the SM fermions as well as the additional scalars that are needed for symmetry breaking but do not play a significant role in the FOPT.

as well,¹²

$$\phi = \frac{i}{\sqrt{2}} \phi_{\alpha\beta} T^{\alpha\beta}, \quad (\text{A3})$$

$$A^\mu = \frac{i}{\sqrt{2}} A_{\alpha\beta}^\mu T^{\alpha\beta}. \quad (\text{A4})$$

Their Lagrangian is

$$\mathcal{L} = -\frac{1}{4} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \frac{1}{4} \text{Tr}((D_\mu \phi)^\dagger (D^\mu \phi)) - V(\phi), \quad (\text{A5})$$

where the scalar potential is given in Eq. (3). The field-strength tensor and the covariant derivative are given by

$$F^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu - ig[A^\mu, A^\nu], \quad (\text{A6})$$

$$D^\mu \phi = \partial^\mu \phi - ig[A^\mu, \phi]. \quad (\text{A7})$$

In order to obtain the breaking pattern $SO(10) \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, we note that $SO(10)$ contains the subgroups $SO(6)$ and $SO(4)$, which are isomorphic to $SU(4)$ and $SU(2)_L \times SU(2)_R$, respectively. Defining the generators of $SU(4)$ as [83]

$$\begin{aligned} U_1 &= -\frac{1}{\sqrt{2}}(T^{36} + T^{45}), & U_8 &= \frac{1}{\sqrt{6}}(2T^{12} - T^{34} + T^{56}), \\ U_2 &= -\frac{1}{\sqrt{2}}(T^{46} - T^{35}), & U_9 &= -\frac{1}{\sqrt{2}}(T^{14} + T^{23}), \\ U_3 &= \frac{1}{\sqrt{2}}(T^{34} + T^{56}), & U_{10} &= \frac{1}{\sqrt{2}}(T^{13} - T^{24}), \\ U_4 &= \frac{1}{\sqrt{2}}(T^{16} + T^{25}), & U_{11} &= \frac{1}{\sqrt{2}}(T^{16} - T^{25}), \\ U_5 &= -\frac{1}{\sqrt{2}}(T^{15} - T^{26}), & U_{12} &= \frac{1}{\sqrt{2}}(T^{15} + T^{26}), \\ U_6 &= \frac{1}{\sqrt{2}}(T^{14} - T^{23}), & U_{13} &= \frac{1}{\sqrt{2}}(T^{36} - T^{45}), \\ U_7 &= \frac{1}{\sqrt{2}}(T^{13} + T^{24}), & U_{14} &= \frac{1}{\sqrt{2}}(T^{35} + T^{46}), \\ U_{15} &= \frac{1}{\sqrt{3}}(T^{12} + T^{34} - T^{56}), \end{aligned} \quad (\text{A8})$$

they satisfy

$$\text{Tr}(U_a U_b) = \delta_{ab}, \quad a, b = 1, \dots, 15. \quad (\text{A9})$$

The generator U_{15} commutes with both the $SO(4)$ generators and $U_{1,\dots,8}$, which generate the $SU(3)_c$ subgroup

of $SU(4)$. Consequently, a vev $\langle \phi \rangle \propto U_{15}$ leads to the desired symmetry breaking with $U(1)_{B-L}$ generated by U_{15} .

APPENDIX B: EFFECTIVE POTENTIAL

Based on the previous discussion of symmetry breaking at the GUT scale, we consider a classical (or background) field proportional to the generator U_{15} ,

$$\phi_c = \sqrt{2}i\varphi_c U_{15}, \quad (\text{B1})$$

where the factor $\sqrt{2}i$ ensures canonical normalization of the real scalar field φ_c . At the potential minimum,

$$\varphi_c = v \equiv \sqrt{3}\omega_{BL}. \quad (\text{B2})$$

Assuming that the symmetry breaking at the GUT scale is dominated by the single vev v , the effective potential can be approximated as a function of the single classical field φ_c .

The gauge boson masses after SSB are obtained by evaluating the commutators of the $SO(10)$ generators with $\langle \phi \rangle$ [22]. Likewise, we find the *field-dependent* mass matrix $M_g^2(\phi_c)$, a 45×45 matrix with elements

$$M_g^2(\phi_c)_{(\alpha\beta)(\gamma\delta)} = \frac{g^2}{2} \text{Tr}([T^{(\alpha\beta)}, \phi_c][T^{(\gamma\delta)}, \phi_c]), \quad (\text{B3})$$

where $(\alpha\beta)$ is an ordered index pair ($\alpha < \beta$) determining the row and analogously $(\gamma\delta)$ is an ordered pair determining the column. The block of M_g^2 containing the masses of the 24 gauge bosons that do not belong to any of the subgroups in our breaking chain is diagonal¹³ with the nonzero entries $m_{g1}^2(\phi_c) = \frac{1}{6}g^2\varphi_c^2$, see Table I where we collect the field-dependent masses of all particles relevant for our discussion. For the masses of the $SU(4)$ gauge bosons, the corresponding block of M_g^2 is not diagonal. Hence, it is more convenient to determine their masses in the basis of the $SU(4)$ generators, i.e., to calculate

$$M_{SU(4)}^2(\phi_c)_{ab} = \frac{g^2}{2} \text{Tr}([U_a, \phi_c][U_b, \phi_c]) \quad (\text{B4})$$

using the $SU(4)$ generators given in Eq. (A8). This yields six massive gauge bosons with field-dependent masses squared $m_{g2}^2(\phi_c) = \frac{2}{3}g^2\varphi_c^2$.

For the scalars, the field-dependent mass-squared matrix is given by

$$M_s^2(\phi_c)_{(\alpha\beta)(\gamma\delta)} = \frac{\partial^2 V_0(\phi)}{\partial \phi_{(\alpha\beta)} \partial \phi_{(\gamma\delta)}} \Big|_{\phi=\phi_c}, \quad (\text{B5})$$

¹²Our expression for A^μ differs from that in [22] by a factor of $\sqrt{2}i$, which ensures real and canonically normalized components A_{ij}^μ .

¹³This changes when we add a nonzero vev breaking $SU(2)_R$. Then, off-diagonal entries appear, which can be removed by combining pairs of gauge bosons into mass eigenstates.

where $V_0(\phi)$ is the tree-level potential (3). Evaluating the eigenvalues of this matrix, we obtain the field-dependent scalar and NGB masses squared given in Table I.

The thermal corrections are given by the well-known formulas [60,84]

$$V_{\text{th}}(\phi_c, T) = \sum_{i=g,s,\chi} \frac{n_i}{2\pi^2} T^4 J_b(x), \quad x \equiv \frac{m_i(\phi_c)}{T},$$

$$J_b(x) = \Re \int_0^\infty dy y^2 \ln [1 - e^{-\sqrt{y^2+x^2}}], \quad (\text{B6})$$

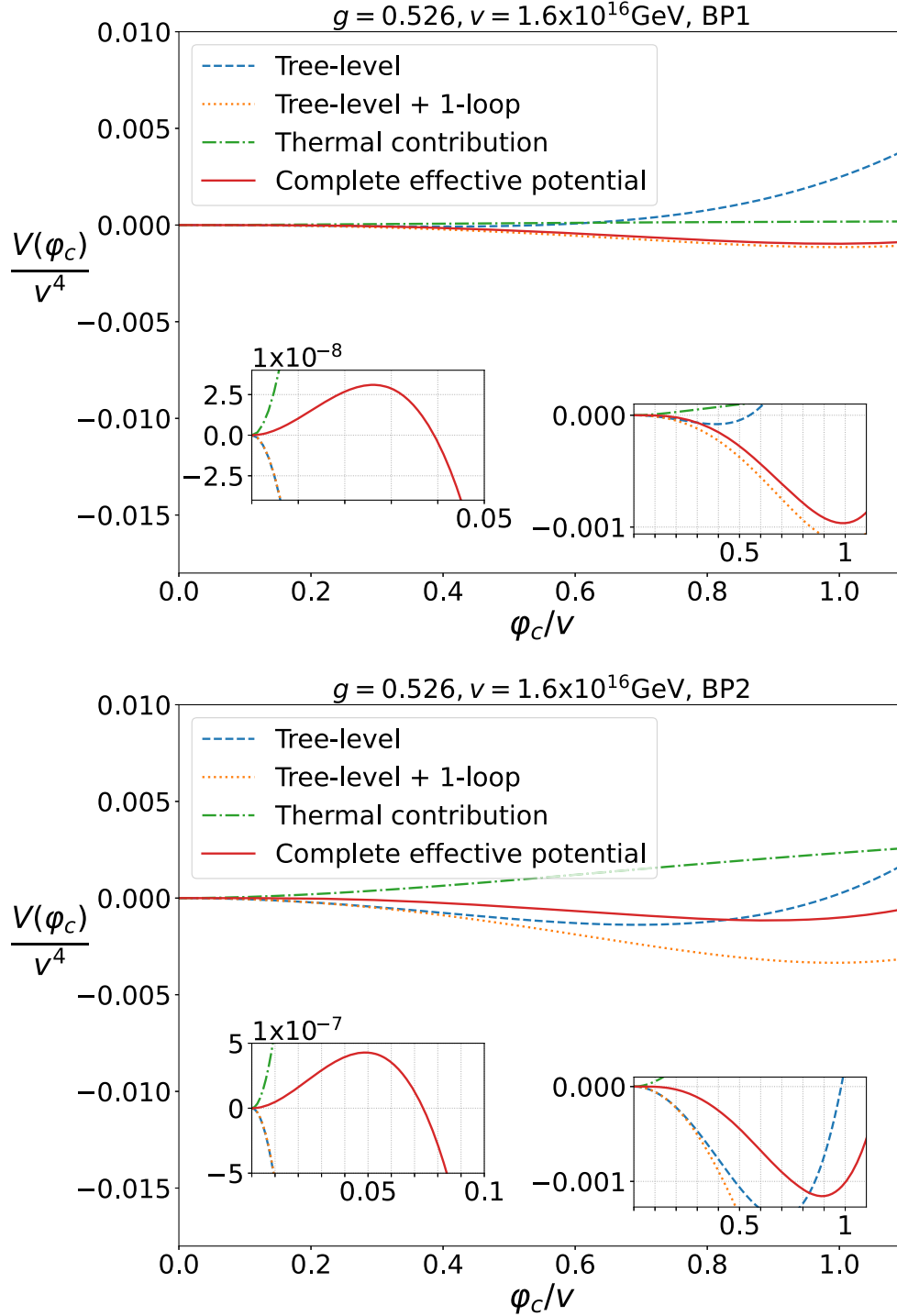


FIG. 9. Effective potential for BP1 and BP2 for the case $M_{\text{GUT}} = 1.6 \times 10^{16}$ GeV. The continuous red curve represents the complete one-loop effective potential $V(\phi_c, T_n)$ at the nucleation temperature.

where n_i represents the degrees of freedom of particles. Also, note that we have to sum over the individual field-dependent gauge and scalar boson masses $m_i(\phi_c)$ given in Table I. As we do not consider Yukawa couplings between the fermions and the scalar **45**, there is no fermionic contribution to the thermal corrections.

In Figs. 9 and 10 we present plots of the effective potential for the case of $M_{\text{GUT}} = 1.6 \times 10^{16}$ GeV.

APPENDIX C: FOPT AND GW CONVENTIONS

For completeness, we specify in this appendix the conventions we use for the FOPT parameters and the fits

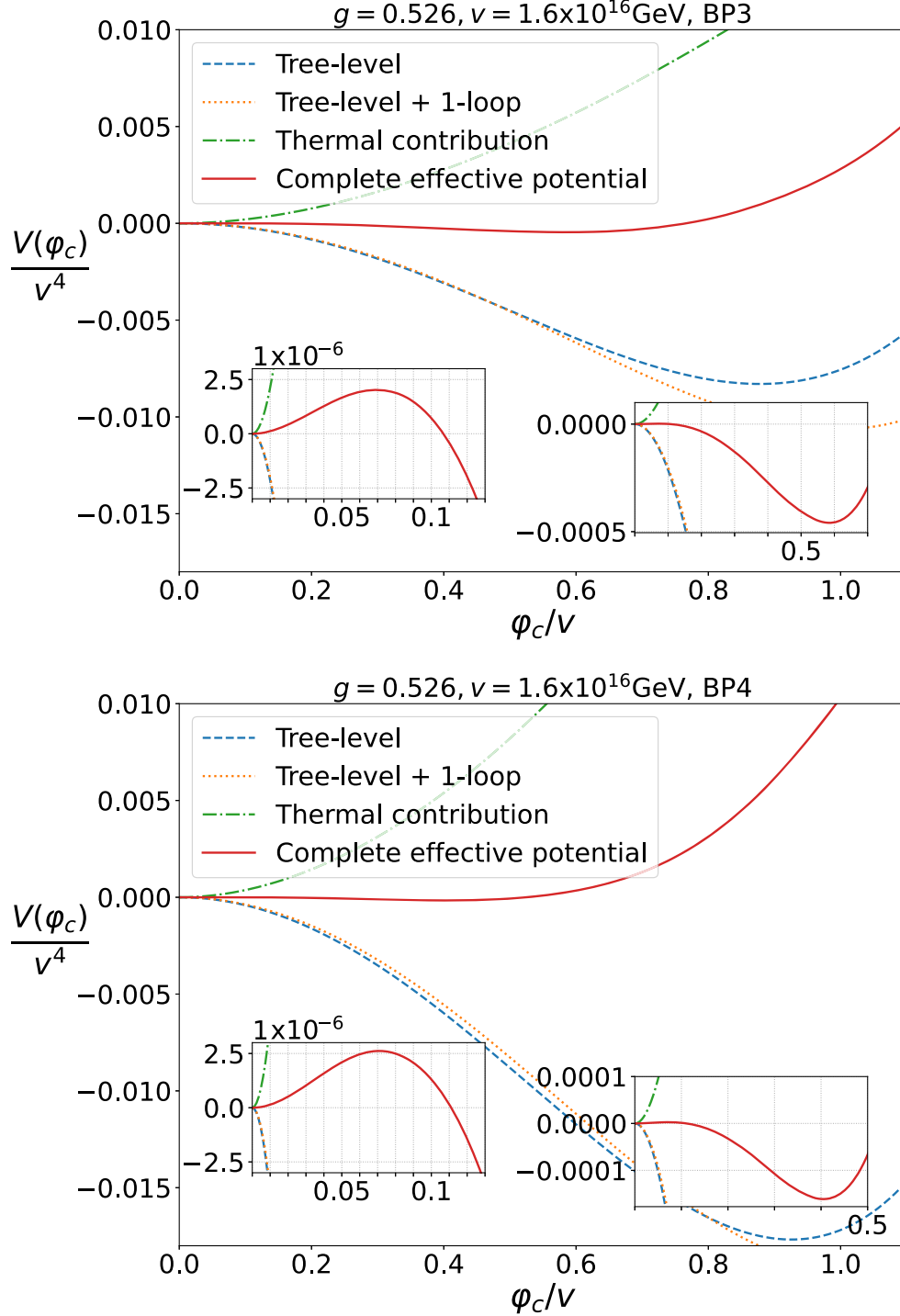


FIG. 10. Same as Fig. 9 for BP3 and BP4.

that we use for the density of GW. The 3D action describing the bubbles forming when the transition between the metastable and the true vacuum takes place is given by

$$S_3[\phi_c(r), T] = 4\pi \int_0^\infty r^2 dr \left[\frac{1}{2} \left(\frac{d\phi_c(r)}{dr} \right)^2 + V(\phi_c(r), T) \right]. \quad (\text{C1})$$

The strength of the phase transition can be parametrized by

$$\alpha = \frac{1}{\rho_{\text{rad}}} \left[\Delta V(\phi_c, T) - \frac{T}{4} \frac{\partial \Delta V(\phi_c, T)}{\partial T} \right] \Big|_{T=T_n}, \quad (\text{C2})$$

where $\Delta V(\phi_c, T)$ is the difference of the potential between the two minima (metastable and stable). The β parameter quantifies the rate at which the FOPT occurs,

$$\beta = H_* T \frac{d}{dT} \left(\frac{S_3}{T} \right) \Big|_{T=T_n \approx T_*}. \quad (\text{C3})$$

The GW density is governed by the parameters T_n , α , β , the sound speed, c_s , the bubble wall speed v_w , as well as the efficiency factors κ_ν , and ϵ . We employ these parameters in the well-known formulas [49,50] for sound wave and turbulence contributions [56,57] in the case where the barrier is mainly of thermal origin (i.e., it vanishes in the limit $T \rightarrow 0$) and the one-loop gauge boson contributions are non-negligible. This is known as the case of *nonrunaway bubbles*. In this case, the dominant contributions are sound waves and the magnetohydrodynamic (MHD) turbulence of the plasma.

The red-shifted sound wave contribution to the GW density observed today is

$$\begin{aligned} \Omega_{\text{sw}} h^2(f) &= 2.65 \times 10^{-6} H_* \tau_{\text{sw}} \left(\frac{\beta}{H_*} \right)^{-1} v_w \left(\frac{\kappa_\nu \alpha}{1 + \alpha} \right)^2 \left(\frac{g_*}{100} \right)^{-\frac{1}{3}} \\ &\times \left(\frac{f}{f_{\text{sw}}} \right)^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{7/2}, \end{aligned} \quad (\text{C4})$$

where

$$f_{\text{sw}} = 1.9 \times 10^{-5} \frac{1}{v_w} \left(\frac{\beta}{H_*} \right) \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz} \quad (\text{C5})$$

is the peak frequency as observed today. The factor $\tau_{\text{sw}} = \min[\frac{1}{H_*}, \frac{R_*}{\bar{U}_f}]$ is the time scale of the duration of the phase transition [58,85], and g_* and H_* , respectively, are the number of degrees of freedom in the thermal bath and the Hubble parameter at the time of GW production. For nonrunaway bubbles, the reheating temperature and the

thermal bath temperature, T_* , coincide with the nucleation temperature T_n .¹⁴ Thus, τ_{sw} can equal either $1/H_*$ or $R_*/\bar{U}_f$, where $H_* R_* = \max(v_w, c_s)(8\pi)^{1/3}(\beta/H_*)^{-1}$. The rms fluid velocity can be approximated as

$$\bar{U}_f^2 \approx \frac{3}{4} \left(\frac{\kappa_\nu \alpha}{1 + \alpha} \right).$$

We emphasize that Eq. (C4) is based on simulations that were restricted to values of $\alpha \lesssim 0.1$ and $\bar{U}_f \lesssim 0.05$ [86,87]. The efficiency factor κ_ν can be approximated by [88]

$$\kappa_\nu \simeq \begin{cases} \alpha(0.73 + 0.83\sqrt{\alpha} + \alpha)^{-1}, & v_w \sim 1, \\ v_w^{6/5} 6.9\alpha(1.36 - 0.037\sqrt{\alpha} + \alpha)^{-1}, & v_w \ll 1. \end{cases} \quad (\text{C6})$$

The MHD turbulence provides a smaller contribution to the GW signal,

$$\begin{aligned} \Omega_{\text{turb}} h^2(f) &= 3.35 \times 10^{-4} \left(\frac{\beta}{H_*} \right)^{-1} v_w \left(\frac{\epsilon \kappa_\nu \alpha}{1 + \alpha} \right)^{\frac{3}{2}} \left(\frac{g_*}{100} \right)^{-\frac{1}{3}} \\ &\times \frac{(f/f_{\text{turb}})^3 (1 + f/f_{\text{turb}})^{-\frac{11}{3}}}{1 + 8\pi \frac{f}{h_*}} \end{aligned} \quad (\text{C7})$$

with

$$h_* = 16.5 \frac{T_*}{10^8 \text{ GeV}} \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz} \quad (\text{C8})$$

and the peak frequency

$$f_{\text{turb}} = 2.7 \times 10^{-5} \frac{1}{v_w} \left(\frac{\beta}{H_*} \right) \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}. \quad (\text{C9})$$

In Eq. (C7) we have assumed that the turbulence efficiency factor κ_t can be written as $\kappa_t = \epsilon \kappa_\nu$, where ϵ is another efficiency factor. As simulations suggest that at most 5% to 10% of the bulk motion of the bubble wall is converted into vorticity [50] (which enters into the turbulence contribution), it is customary to assume a conservative value of $\epsilon = 0.05$ [86].

The bubble wall velocity v_w is determined by a micro-physical description of the interactions between the scalar field evolving through the bubble wall and the thermal plasma. As a precise computation is very challenging, we present the GW density profiles for the detonation velocity $v_w^d = (1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3})/(1 + \alpha)$ [89]. The values

¹⁴Recalling that $T_{\text{rh}} \approx T_n(1 + \alpha)^{1/4}$ [58], this is valid only for $\alpha \ll 1$.

$\epsilon = 0.05$ and $\bar{U}_f \sim 0.05$ were obtained in the regime of detonations and small α deflagrations [50,86,87,90].

APPENDIX D: MAGNETIC FLUX CONFINEMENT

As pointed out in Sec. II A, the symmetry breaking at the GUT scale that we consider in our scenario leads to the production of monopoles that can overclose the Universe. In this appendix, we provide an explicit example of how embedding $SO(10)$ into a larger group can address this feature through monopole confinement. As mentioned in the main text, this requires that the larger group is broken to $SO(10) \times U(1)_\psi$ at energies larger than the reheating temperature, so that the symmetry breaking does not produce observable topological defects. Specifically, the $SO(10)$ monopoles remain confined [attached to cosmic strings from $U(1)_\psi$ breaking] if $U(1)_\psi$ is broken at the same scale and if the monopoles carry nonvanishing $U(1)_\psi$ magnetic charge.

In order to obtain this, we first note that for our $SO(10)$ breaking chain (1), the monopoles come from the $U(1)_{B-L}$ factor, since they are classified via the homotopy groups [91]

$$\pi_2(\text{Spin}(10)/G_{3221}) \simeq \pi_1(G_{3221}) = \pi_1(U(1)_{B-L}) = \mathbb{Z}. \quad (\text{D1})$$

As the $\mathbb{Z}_4$ center of $\text{Spin}(10)$ is identified with a subgroup of $U(1)_\psi$ in the breaking

$$E_6 \rightarrow \frac{\text{Spin}(10) \times U(1)_\psi}{\mathbb{Z}_4}, \quad (\text{D2})$$

a monopole carries $U(1)_\psi$ magnetic charge if and only if it has nontrivial $\mathbb{Z}_4$ center charge (see for example [30]). However, the $B-L$ generator is a generator of the $SU(4)$ subgroup of $\text{Spin}(10)$, cf. Eq. (A8), and hence its monopole is center neutral. One way around this issue is starting from the group E_7 , which possesses a fundamental weight Λ_7 that does not lie in the root lattice $Q(E_7)$ [where $Q(G)$ represents the root lattice of the algebra G]. Hence, the breaking $E_7 \rightarrow E_6$ (for instance via a scalar in the 56) allows mixing between the additional E_7 Cartan direction and the $\mathfrak{so}(10)$ algebra, denoted by D_5 [92,93].¹⁵ The magnetic charge of a monopole then takes the form

$$m = t_{B-L} + \alpha \Lambda_7. \quad (\text{D3})$$

As $\Lambda_7 \notin Q(E_7)$, it is not contained in the sublattice $Q(D_5)$ either. Thus, we have $m \notin Q(D_5)$ for $\alpha \neq 0$. Such a magnetic charge lies in a nontrivial coset of the weight lattice relative to the D_5 root lattice. This implies that the monopole necessarily carries $U(1)_\psi$ magnetic charge and must attach to ψ strings when $U(1)_\psi$ is broken. This realizes universal monopole-string pairing, that is, the magnetic flux is created between the monopoles in the presence of $U(1)_\psi$.

Under these circumstances it only remains to ensure that the additional field needed to break $U(1)_\psi$ does not change the phase transition that we have studied. This can easily be realized by choosing parameters of the scalar potential such that the new field has only weak couplings to ϕ and thus no impact on the effective potential of Eq. (8) [and induces spontaneous $U(1)_\psi$ breaking via a crossover or second-order phase transition].

¹⁵Recall that the groups $SO(10)$ and $\text{Spin}(10)$ have the same algebra.

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