

Study of spin states in vacuum pair production in the Dirac-Heisenberg-Wigner formalism

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A general spin-resolved momentum distribution of electron-positron pairs produced in strong external fields is derived by combining the covariant spin projection operator and the Dirac-Heisenberg-Wigner formalism. The result shows that the spin-resolved and helicity-resolved momentum distributions given in previous literature are actually two special cases of it. By numerically studying pair production in a spatially homogeneous circularly polarized electric field with its rotation axis along the z axis, it is found that for any spin-direction unit vector, when the z component of the unit vector vanishes, the number densities of produced spin-up and spin-down particles are equal, while their momentum distributions have some asymmetry. When the z component of the unit vector is nonzero, there is a difference of 1–3 orders of magnitude in the number densities of spin-up and spin-down particles induced by angular momentum transfer in multiphoton absorption. Moreover, as the electric field strength increases or the field frequency decreases, the asymmetry between the spin-up and spin-down particle number densities decreases rapidly. These results offer an approach to study general spin states in vacuum pair production and enhance our understanding of angular momentum transfer from fields to matter in extreme environments.

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I. INTRODUCTION

In the presence of a strong external electromagnetic field, the quantum electrodynamics (QED) vacuum becomes unstable and decays into electron-positron pairs [1–5]. Since Dirac proposed the relativistic wave equation and predicted the existence of the positron [6], this topic has always been one of the most fascinating theoretical predictions in QED. Using the proper-time method, Schwinger calculated the electron-positron pair production rate in a strong constant field, which is $\sim \exp(-\pi E_{\text{cr}}/E_0)$ [3], where E_0 is the constant field strength, $E_{\text{cr}} = m^2 c^3 / (e\hbar) \sim 10^{16}$ V/cm is the so-called Schwinger critical field strength, and m and e are the magnitudes of the mass and charge of the electrons, respectively. Because of the exponential suppression of the production rate and the fact that the electric field strength presently achievable in the laboratory is much lower than the critical field strength E_{cr} , the phenomenon has not been verified directly in

experiments so far. Nevertheless, the development of high-intensity laser technology [7,8] provides new possibilities for observing electron-positron pair production in vacuum environments in the future.

In terms of theoretical research, significant progress has been made on the topic of vacuum pair production. Researchers have not only developed a variety of non-perturbative computational methods, but also revealed important physical phenomena, such as the dynamically assisted Schwinger mechanism [9–14], the effective mass signatures [15], the spiral structures of the momentum distribution [16–18], Ramsey interference [19,20], and so on. The widely used research methods include computational quantum field theory [21–23], the worldline instanton technique [24–28], the WKB approximation [29,30], the Dirac-Heisenberg-Wigner (DHW) formalism [31–34], and so on. The relationship or equivalence between different methods has also been explored [35–38], which assists in understanding the theoretical results from different perspectives. Recently, Aleksandrov *et al.* [39] corrected the quantum kinetic equations (QKEs) for studying electron-positron pair production in spatially homogeneous and time-dependent electric fields with multiple components and proved that they are equivalent to the DHW formalism. Moreover, spin-resolved and helicity-resolved momentum distribution functions are also discussed.

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In fact, the phenomena related to spin or helicity in electron-positron pair production from vacuum have also attracted widespread attention [40–48], leading to a series of important developments. In Ref. [43], Strobel and Xue studied the pair-production rate for rotating electric fields with the semiclassical WKB approach and found an asymmetry in the spin states. Blinne and Strobel [44] recovered this asymmetry using a combined projection operator in the framework of the DHW formalism. They proposed that the quantum state corresponding to the projection operator of a given physical quantity can be extracted from the Wigner function. Based on this, one can obtain the momentum distribution functions with additional observables. Subsequently, Blinne [49] revised the projection formula to ensure that the defined projected momentum distribution function is always real. However, the spin projection operator he proposed fails to yield spin asymmetry because it cannot isolate a single spin state. Li *et al.* [45] revealed that for a multicycle elliptically polarized field, the spin asymmetry roughly increases with the Keldysh adiabatic parameter [50]. By studying electron-positron pairs produced by two circularly polarized fields with different frequencies and a time delay, Hu *et al.* [47] found that a spiral structure exists in both the spin-up and spin-down momentum distributions, and the spin asymmetry degree can be significantly altered by adjusting the time delay between these two fields. In addition, Aleksandrov and Kudlis [48] analyzed helicity-resolved momentum distributions of produced particles by a circularly polarized electric field rotating counterclockwise about the z axis, and found that left-handed electrons (negative helicity) always prefer to propagate along the negative z axis, while right-handed electrons (positive helicity) always prefer to propagate along the positive z axis.

This paper focuses on the spin states of electron-positron pairs produced in a circularly polarized electric field. There are three main contents. First, a general expression for the momentum distribution of produced particles with different spin states is derived by combining the covariant spin projection operator with the DHW formalism. When the spin-direction unit vector is along the direction of particle momentum, we can obtain the same helicity-resolved momentum distribution as that in Ref. [48]. When the unit vector is taken along the z axis, the spin-resolved momentum distribution is obtained, and it is identical to that in Ref. [39]. It is also clarified that the spin-resolved momentum distribution derived from the semiclassical WKB approximation [43] and the DHW formalism [42,44,47] does not correspond to genuine pure spin states when the momentum of created particles perpendicular to the field direction is not zero, because it also incorporates information on chiral asymmetry. The second is the study of the spin-resolved momentum distribution with any spin-direction unit vector. Third, the effect of electric field strength and frequency on the spin asymmetry is examined.

The structure of this paper is as follows. In Sec. II, to make this work self-contained, we briefly review the DHW formalism and QKEs. In Sec. III, the theoretical formulas describing spin- and helicity-resolved momentum distribution functions are derived by combining the covariant spin projection operator with the DHW formalism. In Sec. IV, we investigate the spin-resolved momentum distribution and number density of particles produced in a circularly polarized electric field. A summary is given in Sec. V. Appendices A and B provide proof that the projection operator approach proposed in Sec. III is reasonable and reliable. In this paper, natural units ($\hbar = c = 1$) are used.

II. DIRAC-HEISENBERG-WIGNER FORMALISM AND QUANTUM KINETIC EQUATIONS

The study in this paper is mainly based on the DHW formalism, which has been widely applied to investigate electron-positron pair production from vacuum by strong background fields. The DHW formalism can not only conveniently provide information about the time evolution of the single-particle momentum distribution function [51,52], but also offer additional observables by combining with projection operators [44,49]. Since our study also utilizes QKEs to analyze the results, we provide a brief review of the DHW formalism and QKEs in this section.

A. DHW formalism

We start from the gauge-invariant density operator consisting of two Dirac field operators in the Heisenberg picture,

$$\hat{C}_{\alpha\beta}(\mathbf{x}, \mathbf{y}, t) = \exp \left(-ie \int_{-1/2}^{1/2} \mathbf{A}(\mathbf{x} + \xi \mathbf{y}, t) \cdot \mathbf{y} d\xi \right) \times \left[\hat{\Psi}_\alpha \left(\mathbf{x} + \frac{\mathbf{y}}{2}, t \right), \hat{\Psi}_\beta \left(\mathbf{x} - \frac{\mathbf{y}}{2}, t \right) \right], \quad (1)$$

where $\mathbf{A}$ is the vector potential, and $\mathbf{x}$ and $\mathbf{y}$ denote the center of mass and the relative coordinates, respectively. In this paper, $e > 0$ denotes the charge of the positron. Note that the temporal gauge and the mean-field approximation to the electromagnetic field are used.

The key quantity in the DHW formalism is the Wigner operator, which is defined as the Fourier transform of Eq. (1) with respect to $\mathbf{y}$. Its vacuum expectation is known as the Wigner function,

$$\mathcal{W}(\mathbf{x}, \mathbf{p}, t) = -\frac{1}{2} \int d^3 y e^{-i\mathbf{p} \cdot \mathbf{y}} \langle 0 | \hat{C}_{\alpha\beta}(\mathbf{x}, \mathbf{y}, t) | 0 \rangle. \quad (2)$$

Employing a set of complete bases $\{\mathbb{1}, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5, \sigma^{\mu\nu}\}$ of the Dirac algebra, the Wigner function can be decomposed into 16 real functions:

$$\mathcal{W}(\mathbf{x}, \mathbf{p}, t) = \frac{1}{4}(\mathbb{1}\mathbb{s} + i\gamma_5\mathbb{p} + \gamma^\mu\mathbb{v}_\mu + \gamma^\mu\gamma_5\mathbb{a}_\mu + \sigma^{\mu\nu}\mathbb{t}_{\mu\nu}), \quad (3)$$

where $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. $\mathbb{s}$, $\mathbb{p}$, $\mathbb{v}_\mu$, $\mathbb{a}_\mu$, and $\mathbb{t}_{\mu\nu}$ denote scalar, pseudoscalar, vector, axis vector, and tensor, respectively, which are also known as DHW functions. To make the description convenient, we can decompose the 4-vector and tensor into $\mathbb{v}^\mu = (\mathbb{v}^0, \mathbf{v})$, $\mathbb{a}^\mu = (\mathbb{a}^0, \mathbf{a})$, $(\mathbf{t}_1)^i = \mathbb{t}^{i0} - \mathbb{t}^{0i}$, and $\mathbf{t}_2 = \epsilon^{ijk}\mathbb{t}_{ij}\mathbf{e}_k$. Here, $\mathbf{e}_k$ is the unit vector. The partial differential equation system for the 16 Wigner function components described above can be obtained by taking Eq. (3) into the equation of motion satisfied by the Wigner function; more details can be found in Refs. [53,54]. When the system is in the initial vacuum state, the nonzero terms of the Wigner function are

$$\mathbb{s}_{\text{vac}} = -\frac{2m}{\omega(\mathbf{p})}, \quad \mathbf{v}_{\text{vac}} = -\frac{2\mathbf{p}}{\omega(\mathbf{p})}. \quad (4)$$

Here $\omega = \sqrt{m^2 + \mathbf{p}^2}$ denotes the energy of the particle and $\mathbf{p}$ represents the kinetic momentum of the particle.

In the following, we focus on the case of a spatially homogeneous and time-dependent electric field. Under this condition, $\mathbb{p}$, $\mathbb{v}^0$, $\mathbb{a}^0$, and $\mathbf{t}_2$ vanish. Using the method of characteristics and replacing the kinetic momentum $\mathbf{p} = \mathbf{q} - e\mathbf{A}(t)$ by the canonical momentum $\mathbf{q}$, the system of partial differential equations with 16 DHW functions can be transformed into an ordinary differential equation system consisting of the remaining ten nonzero DHW functions $W(\mathbf{q}, t) = (\mathbb{s}, \mathbf{v}, \mathbf{a}, \mathbf{t}_1)^T(\mathbf{q}, t)$,

$$\dot{W}(\mathbf{q}, t) = \mathcal{M}(\mathbf{q}, t)W(\mathbf{q}, t). \quad (5)$$

Here the dot above the letter denotes the derivative with respect to time and $\mathcal{M}$ is a 10×10 matrix:

$$\mathcal{M}(\mathbf{q}, t) = \begin{pmatrix} 0 & 0 & 0 & 2\mathbf{p}^T \\ 0 & 0 & -2\mathbf{p} \times & -2m \\ 0 & -2\mathbf{p} \times & 0 & 0 \\ -2\mathbf{p} & 2m & 0 & 0 \end{pmatrix}. \quad (6)$$

The single-particle momentum distribution function $\mathbf{f}(\mathbf{q}, t)$ is defined as

$$\mathbf{f}(\mathbf{q}, t) = \frac{1}{2\omega(\mathbf{q}, t)}[\varepsilon(\mathbf{q}, t) - \varepsilon_{\text{vac}}(\mathbf{q}, t)], \quad (7)$$

where $\varepsilon = m\mathbb{s} + \mathbf{p} \cdot \mathbf{v}$ represents the phase-space energy density. For numerical computation, a vector auxiliary function is introduced,

$$\mathbf{w}(\mathbf{q}, t) = -\mathbf{v}(\mathbf{q}, t) + [1 - \mathbf{f}(\mathbf{q}, t)]\mathbf{v}_{\text{vac}}(\mathbf{q}, t). \quad (8)$$

Then, the system of Eq. (5) has the following form:

$$\begin{aligned} \dot{\mathbf{f}} &= \frac{e\mathbf{E} \cdot \mathbf{w}}{2\omega}, \\ \dot{\mathbf{w}} &= \frac{2[(e\mathbf{E} \cdot \mathbf{p})\mathbf{p} - \omega^2 e\mathbf{E}]}{\omega^3}(\mathbf{f} - 1) \\ &\quad - \frac{(e\mathbf{E} \cdot \mathbf{w})\mathbf{p}}{\omega^2} - 2\mathbf{p} \times \mathbf{a} - 2m\mathbf{t}_1, \\ \dot{\mathbf{a}} &= -2\mathbf{p} \times \mathbf{w}, \\ \dot{\mathbf{t}}_1 &= \frac{2}{m}[m^2\mathbf{w} + (\mathbf{p} \cdot \mathbf{w})\mathbf{p}]. \end{aligned} \quad (9)$$

Here $\mathbf{E} = -\dot{\mathbf{A}}(t)$ is a spatially homogeneous and time-dependent electric field. The single-particle momentum distribution function $\mathbf{f}(\mathbf{q}, t)$ is then obtained by solving the system of Eq. (9) with the initial conditions $\mathbf{f}(\mathbf{q}, -\infty) = 0$ and $\mathbf{w}(\mathbf{q}, -\infty) = \mathbf{a}(\mathbf{q}, -\infty) = \mathbf{t}_1(\mathbf{q}, -\infty) = 0$. In addition, the number density of produced particles can be obtained by integrating $\mathbf{f}(\mathbf{q}, t)$ over the full momentum space at $t \rightarrow +\infty$, i.e.,

$$N = \lim_{t \rightarrow +\infty} \int \frac{d^3q}{(2\pi)^3} \mathbf{f}(\mathbf{q}, t). \quad (10)$$

B. QKEs

Under a spatially homogeneous and time-dependent electric field, the QKEs consist of an ordinary differential equation system with ten components. The full details of the derivation were given in Ref. [39]. Here we only give the results:

$$\begin{aligned} \dot{f} &= -2\mu_2 \cdot \mathbf{u}, \\ \dot{\mathbf{f}} &= 2\mu_1 \times \mathbf{f} - 2\mu_2 \times \mathbf{v}, \\ \dot{\mathbf{u}} &= 2\mu_1 \times \mathbf{u} + \mu_2(2f - 1) + 2\omega\mathbf{v}, \\ \dot{\mathbf{v}} &= 2\mu_1 \times \mathbf{v} - 2\mu_2 \times \mathbf{f} - 2\omega\mathbf{u}, \end{aligned} \quad (11)$$

where μ_1 and μ_2 are two functions orthogonal to each other,

$$\begin{aligned} \mu_1 &= \frac{e}{2\omega(\omega + m)}\mathbf{p} \times \mathbf{E}, \\ \mu_2 &= \frac{e}{2\omega^2(\omega + m)}[(\mathbf{p} \cdot \mathbf{E})\mathbf{p} - \omega(\omega + m)\mathbf{E}]. \end{aligned} \quad (12)$$

All components of the QKEs are 0 at the initial vacuum state. Note that the single-particle momentum distribution function here is $f = \frac{1}{2}\mathbf{f}$.

III. GENERAL SPIN-RESOLVED MOMENTUM DISTRIBUTION FUNCTION

Besides the full single-particle momentum distribution function, various additional observable quantum state information can be extracted from the Wigner function. Applying the projection operator $\mathcal{P}$ to the Wigner function results in a projected single-particle momentum distribution

function [44,49],

$$f_{\mathcal{P}} := \frac{1}{2\omega} \text{tr}[\mathcal{P} @ (\mathcal{W} - \mathcal{W}_{\text{vac}})(m\mathbb{1} + \boldsymbol{\gamma} \cdot \mathbf{p})], \quad (13)$$

where $\text{tr}(\cdots)$ denotes the trace of a square matrix, and

$$\mathcal{P} @ \mathcal{W} := \frac{1}{2}(\mathcal{P}\mathcal{W} + \mathcal{W}\gamma^0\mathcal{P}^\dagger\gamma^0). \quad (14)$$

Note that the projected momentum distribution function defined by Eq. (13) is always real; see Appendix A.

If one wants to obtain the spin-resolved single-particle momentum distribution function from the Wigner function, it is necessary to utilize the covariant spin projection operator [55,56],

$$\mathcal{P}_s = \frac{1}{2}(\mathbb{1} + s\gamma_5\gamma^\mu n_\mu), \quad (15)$$

where $s = +1$ and $s = -1$ represent spin up and spin down, respectively. The spin 4-vector $n^\mu = (n^0, \mathbf{n})$ is

$$n^0 = \frac{\mathbf{p} \cdot \boldsymbol{\eta}}{m}, \quad \mathbf{n} = \boldsymbol{\eta} + \frac{(\mathbf{p} \cdot \boldsymbol{\eta})\mathbf{p}}{m(\omega + m)}, \quad (16)$$

where $\boldsymbol{\eta}$ is the spin-direction unit vector in the rest frame of the particle. By using Eqs. (13) and (15), we can obtain the spin-resolved single-particle momentum distribution function

$$f_s = \frac{1}{2}\bar{\mathbf{f}} + s\frac{1}{4\omega}n_0(m\mathbf{a}_0 + \mathbf{p} \cdot \mathbf{t}_2) - s\frac{1}{4\omega}\mathbf{n} \cdot (m\boldsymbol{\alpha} + \mathbf{p} \times \mathbf{t}_1). \quad (17)$$

It can be seen that the spin effect is directly related to $\mathbf{a}_\mu$ and $\mathbf{t}_{\mu\nu}$ in the DHW functions.

For a spatially homogeneous and time-dependent electric field, $\mathbf{a}_0$ and $\mathbf{t}_2$ vanish, and Eq. (17) becomes

$$f_s = \frac{1}{2}\bar{\mathbf{f}} - s\frac{1}{4\omega}\mathbf{n} \cdot (m\boldsymbol{\alpha} + \mathbf{p} \times \mathbf{t}_1). \quad (18)$$

Furthermore, using one of the transformation relations between the DHW formalism and the QKEs [39],

$$\mathbf{f} = -\frac{1}{4\omega} \left[m\boldsymbol{\alpha} + \mathbf{p} \times \mathbf{t}_1 + \frac{(\mathbf{p} \cdot \boldsymbol{\alpha})\mathbf{p}}{\omega + m} \right], \quad (19)$$

Equation (18) can also be written as

$$f_s = \frac{1}{2}\bar{\mathbf{f}} + s\boldsymbol{\eta} \cdot \mathbf{f}. \quad (20)$$

This result coincides with that obtained from the projection operator approach in Ref. [39], as shown in Appendix B. At $t \rightarrow +\infty$, integrating over full-momentum space with

respect to f_s , one can obtain the spin-resolved particle number density

$$N_s = \frac{1}{2}N + s\boldsymbol{\eta} \cdot \boldsymbol{\Lambda}, \quad (21)$$

where $\boldsymbol{\Lambda} = \int \frac{d^3q}{(2\pi)^3} \mathbf{f}(\mathbf{q}, t)$.

From Eqs. (20) and (21), we know that f_s denotes the particle number density for a given canonical momentum $\mathbf{q}$ and a spin projection of $\pm 1/2$ along the unit vector $\boldsymbol{\eta}$. The corresponding N_s is the particle number density for a spin projection of $\pm 1/2$. Since $\boldsymbol{\eta}$ is a unit vector in the particle's rest frame, we cannot directly measure the spin projection in that frame. However, this can be achieved by performing the measurement in the laboratory frame and then applying a Lorentz boost to transform the results to the particle's rest frame. A schematic measurement procedure is as follows. The particles created from vacuum are first divided into discrete momentum bins. For each bin, the average 4-momentum and the three-dimensional polarization vector in the laboratory frame are measured using magnetic spectrometry (with corrections for spin precession) and Mott polarimetry combined with a spin rotator, respectively. From these quantities the corresponding spin 4-vector is constructed, which is then analytically boosted to the rest frame of the particles in that bin, yielding the rest-frame spin direction. By comparing this direction with a predefined unit vector $\boldsymbol{\eta}$, the expectation value of the spin projection along $\boldsymbol{\eta}$ is obtained, and from it the fraction of particles in that bin with spin aligned or antialigned is obtained, giving the spin-up and spin-down counts for that bin. The spin-resolved momentum distribution functions are obtained by dividing these counts by the phase-space volume of the bin. The total spin-resolved particle number densities follow from summing the counts over all bins and dividing by the real-space volume.

Below we consider two special cases of Eq. (18), where the spin-direction unit vector $\boldsymbol{\eta}$ is along the direction of the kinetic momentum $\mathbf{p}$ and the direction of the z axis, respectively.

A. Along the direction of kinetic momentum

If the spin-direction unit vector $\boldsymbol{\eta}$ is parallel to the direction of the kinetic momentum of the particle in the laboratory frame, i.e., $\boldsymbol{\eta} = \mathbf{p}/|\mathbf{p}|$, the spin polarization vector becomes

$$n^\mu = \left(\frac{|\mathbf{p}|}{m}, \frac{\omega\mathbf{p}}{m|\mathbf{p}|} \right). \quad (22)$$

According to Eq. (17), the spin-resolved momentum distribution function is expressed as

$$f_s = \frac{1}{2}\bar{\mathbf{f}} + s\frac{|\mathbf{p}|}{4\omega} \left(\mathbf{a}_0 + \frac{\mathbf{p} \cdot \mathbf{t}_2}{m} \right) - s\frac{\mathbf{p} \cdot \boldsymbol{\alpha}}{4|\mathbf{p}|}. \quad (23)$$

For spatially homogeneous and time-dependent electric fields, $\mathbf{a}_0$ and $\mathbf{t}_2$ vanish, and then

$$f_s = \frac{1}{2}\bar{\mathbf{f}} - s \frac{\mathbf{p} \cdot \boldsymbol{\alpha}}{4|\mathbf{p}|}. \quad (24)$$

Based on Eq. (19), $\mathbf{p} \cdot \boldsymbol{\alpha} = -4\mathbf{p} \cdot \mathbf{f}$ can be obtained, so in the representation form of QKEs, Eq. (24) can also be written as

$$f_s = f + s \frac{\mathbf{p} \cdot \mathbf{f}}{|\mathbf{p}|}. \quad (25)$$

Here we use the relation $f = \frac{1}{2}\bar{\mathbf{f}}$. Equation (25) is just the helicity-resolved momentum distribution function that was defined in Refs. [39,48]. For example, see Eqs. (14) and (15) in Ref. [48]. Note that positrons are used in our paper.

B. Along the direction of the z axis

Another special form of Eq. (18) that we consider is the spin-direction unit vector $\boldsymbol{\eta} = (0, 0, 1)$. The spin-resolved momentum distribution function becomes

$$\begin{aligned} f_s &= \frac{1}{2}\bar{\mathbf{f}} - s \frac{1}{4\omega} \left[m\boldsymbol{\alpha} + \mathbf{p} \times \mathbf{t}_1 + \frac{(\mathbf{p} \cdot \boldsymbol{\alpha})\mathbf{p}}{\omega + m} \right]_z \\ &= f + sf_z. \end{aligned} \quad (26)$$

This is the same as the spin-resolved distribution function (107) derived by selecting the z axis as the projection direction of spin in Ref. [39].

Moreover, in earlier studies [42–44,47], the spin-resolved momentum distribution function was defined using the projection operator

$$\begin{aligned} P_s &= \frac{1}{2}(\mathbb{1} + s\gamma_5) - s \frac{1}{2}\gamma_5 \frac{(q_z + \epsilon_\perp)\mathbb{1} + m\gamma^3}{\epsilon_\perp} \\ &= \frac{\epsilon_\perp + q_z + m}{2\epsilon_\perp} \mathbb{1} - \frac{q_z}{\epsilon_\perp} P_{c^s} - \frac{m}{\epsilon_\perp} P_{\mu_z^s}, \end{aligned} \quad (27)$$

where $\epsilon_\perp = \sqrt{q_z^2 + m^2}$, $P_{c^s} = \frac{1}{2}(\mathbb{1} + s\gamma_5)$ is the chiral projection operator, and $P_{\mu_z^s} = \frac{1}{2}(\mathbb{1} + s\gamma_5\gamma^3)$ is the magnetic moment projection operator along the z direction. It can be clearly seen that Eq. (27) contains not only the spin-dependent magnetic moment projection operator, but also the chiral projection operator. Therefore, when $q_z \neq 0$, the momentum distribution function obtained from Eq. (27) is not a purely spin-resolved momentum distribution function, but also incorporates information on chiral asymmetry. However, when $q_z = 0$, Eq. (27) becomes

$$P_s = \mathbb{1} - P_{\mu_z^s} = \frac{1}{2}(\mathbb{1} - s\gamma_5\gamma^3) = P_{\mu_z^{-s}}. \quad (28)$$

The projection operator contains only the magnetic moment projection operator and is exactly the covariant

spin projection operator obtained by taking the 4-vector $n^\mu = (0, 0, 0, 1)$. Consequently, the momentum distribution function obtained from Eq. (27) in earlier studies is a purely spin-resolved momentum distribution function only when $q_z = 0$.

In the next section, we will explore the spin-resolved momentum distribution and number density of particles produced by a circularly polarized field for any spin-direction unit vector numerically.

IV. NUMERICAL RESULTS AND ANALYSIS

The external field we use is the standing wave electric field formed by two counterpropagating laser beams along the z axis:

$$\mathbf{E}(t) = \frac{E_0}{\sqrt{2}} e^{-\frac{(\Omega t)^2}{\sigma^2}} \begin{pmatrix} \cos(\Omega t) \\ \sin(\Omega t) \\ 0 \end{pmatrix}, \quad (29)$$

where $E_0/\sqrt{2}$ and Ω are the amplitude and frequency of the electric field, respectively, and the dimensionless parameter σ controls the duration of the electric field pulse. In the numerical calculations, we set the initial time t_{in} and the final time t_{out} to a sufficiently large value to ensure that the external electric field $\mathbf{E}(t)$ asymptotically becomes 0 and the results of the calculations converge. In addition, we set the initial value of the vector potential to $\mathbf{A}(t_{\text{in}}) = 0$.

Before specific numerical calculations, we theoretically analyze the spin-resolved momentum distribution function in the polarization plane (q_x, q_y) at $q_z = 0$ with the assistance of QKEs. In this case, the momentum direction of created particles is perpendicular to the z axis, i.e., $\mathbf{q} = \mathbf{q}_\perp$, where $\perp$ represents perpendicular to the z axis. Since the electric field (29) we use is also perpendicular to the z axis, the quantities $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ in Eq. (12) can be written as $\boldsymbol{\mu}_1 = \mu_1 \mathbf{e}_z$ and $\boldsymbol{\mu}_2 = \mu_2 \mathbf{e}_\perp$, where $\mathbf{e}_z$ and $\mathbf{e}_\perp$ are unit vectors along the z axis and perpendicular to the z axis, respectively. By decomposing the vector functions $\mathbf{f}$, $\mathbf{u}$, and $\mathbf{v}$ into $\mathbf{f} = f_z \mathbf{e}_z + \mathbf{f}_\perp$, $\mathbf{u} = u_z \mathbf{e}_z + \mathbf{u}_\perp$, and $\mathbf{v} = v_z \mathbf{e}_z + \mathbf{v}_\perp$, the system of QKEs shown in Eq. (11) can be transformed into the following form:

$$\begin{aligned} \dot{f} &= -2\mu_2 \mathbf{e}_\perp \cdot \mathbf{u}_\perp, \\ \dot{f}_z &= -2\mu_2 \mathbf{e}_z \cdot (\mathbf{e}_\perp \times \mathbf{v}_\perp), \\ \dot{\mathbf{f}}_\perp &= 2\mu_1 \mathbf{e}_z \times \mathbf{f}_\perp - 2\mu_2 v_z \mathbf{e}_\perp \times \mathbf{e}_z, \\ \dot{u}_z &= 2\omega v_z, \\ \dot{\mathbf{u}}_\perp &= 2\mu_1 \mathbf{e}_z \times \mathbf{u}_\perp + \mu_2 (2f - 1) \mathbf{e}_\perp + 2\omega \mathbf{v}_\perp, \\ \dot{v}_z &= -2\mu_2 \mathbf{e}_z \cdot (\mathbf{e}_\perp \times \mathbf{f}_\perp) - 2\omega u_z, \\ \dot{\mathbf{v}}_\perp &= 2\mu_1 \mathbf{e}_z \times \mathbf{v}_\perp - 2\mu_2 f_z \mathbf{e}_\perp \times \mathbf{e}_z - 2\omega \mathbf{u}_\perp. \end{aligned} \quad (30)$$

Notice that $\dot{\mathbf{f}}_\perp \cdot \mathbf{f}_\perp + \dot{v}_z v_z + \dot{u}_z u_z = 0$. According to the initial conditions of the QKEs, we get $\mathbf{f}_\perp^2 + u_z^2 + v_z^2 = 0$, so

$f_x = f_y = u_z = v_z = 0$. Then the spin-resolved momentum distribution function given in Eq. (20) becomes

$$f_s = \frac{1}{2}\bar{f} + s\eta_z f_z. \quad (31)$$

This means that the spin-resolved momentum distribution in the polarization plane at $q_z = 0$ is only related to η_z and has nothing to do with η_x and η_y .

A. Spin-resolved momentum distributions

In this section, we discuss the spin-resolved momentum distribution function in two cases. The first one is $\eta_z = 0$ and the second one is $\eta_z \neq 0$.

First, we choose the spin-direction unit vector as $\boldsymbol{\eta} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$. According to Eq. (31), the momentum distributions for both spin-up and spin-down particles are equal to $\bar{f}/2$ in the polarization plane at $q_z = 0$. There is no spin asymmetry. Thus, we only focus on the distribution functions f_s in the momentum planes $(q_x = 0, q_y, q_z)$ and $(q_x, q_y = 0, q_z)$. The spin-resolved momentum distributions and spin asymmetry of positrons produced by the external rotating field (29) are exhibited in Fig. 1. Here the spin asymmetry degree κ is defined as $f_+ - f_-$ divided by the maximum value of the total momentum distribution function $\bar{f}$. Note that all contour plots in this paper use a continuous blue-white-yellow-red temperature colormap (low to high). In grayscale printouts, blue appears as dark gray, white as white, yellow as light gray, and red as medium gray.

As can be seen from Figs. 1(a) and 1(b), the patterns of the spin-up distribution function f_+ and the spin-down distribution function f_- are similar. The maximum values of both f_+ and f_- are about $\bar{f}/2$. Although the positions of the maxima have only a small shift (about $0.02m$) in the positive z -axis direction and the negative z -axis direction, respectively, there is still a maximum of about 4.6% spin asymmetry at $(q_y, q_z) \approx (1.03m, \pm 0.27m)$; see Fig. 1(c). Furthermore, the spin asymmetry degree is shifted significantly in the positive and negative z -axis directions, respectively. By analyzing the spin asymmetry degree κ and the momentum distribution function f_s , the following phenomena are observed. In the right region of the distribution function ($q_y > 0$), the spin-up (spin-down) positrons prefer to move towards the positive z -axis (negative z -axis) direction. In the left region of the distribution function ($q_y < 0$), the spin-up (spin-down) positrons prefer to move along the negative z -axis (positive z -axis) direction. This is different from the result in Ref. [48], where spin-up (or spin-down) positrons always prefer to move along the positive z -axis (or negative z -axis) direction. Moreover, both f_+ and f_- are asymmetric with respect to $q_z = 0$, as can be found in Eq. (20). This is because the total momentum distribution function $\bar{f}$ is symmetric about $q_z = 0$. However, according to the QKEs shown in Eq. (11), for the reflection transformation $q_z \rightarrow -q_z$, the sign of f_z remains the same, while f_x and f_y change sign [48]. It also explains why the spin asymmetry degree κ is an odd function of q_z . In addition, in Figs. 1(a)

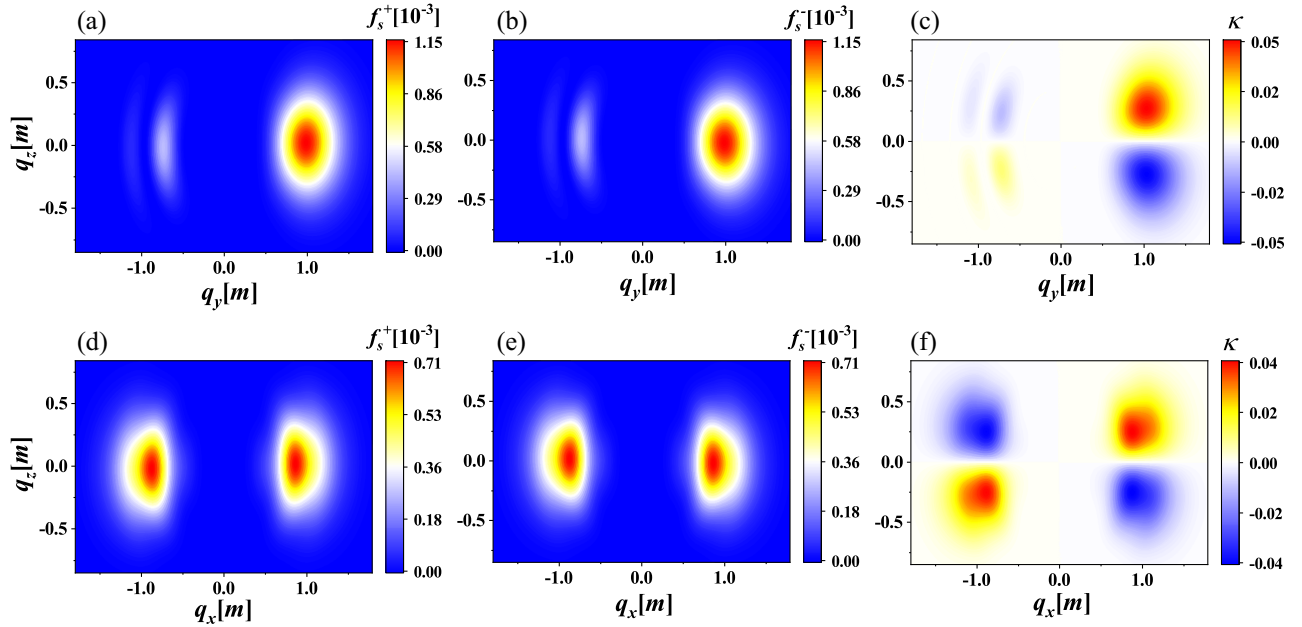


FIG. 1. Spin-resolved momentum distribution functions and the spin asymmetry degree κ of positrons for selecting $\boldsymbol{\eta} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$. Panels (a) and (d) correspond to the spin-up case, while panels (b) and (e) correspond to the spin-down case. Panels (c) and (f) denote the spin asymmetry degree. The first and second rows correspond to the momentum planes $(q_x = 0, q_y, q_z)$ and $(q_x, q_y = 0, q_z)$, respectively. The electric field parameters are $E_0 = 0.5E_{cr}$, $\Omega = 0.5m$, and $\sigma = 5$.

and 1(b), the maximum value of f_s is significantly shifted along the positive y -axis direction. This characteristic is the same as the total momentum distribution function. Since the time integral of $E_y(t)$ over the interval $[0, t_{\text{out}}]$ is a sufficiently large positive value, positrons (or electrons) produced mainly near $t = 0$ acquire a net positive (or negative) momentum due to the influence of this field.

For the momentum plane $(q_x, q_y = 0, q_z)$, the maximum value of the absolute value of the spin asymmetry degree is approximately 3.9%; see Fig. 1(f). Its position has a shift of about $0.26m$ along the z -axis direction. In the region where $q_x > 0$, the spin-up positrons prefer to move toward the positive z -axis direction, while in the region $q_x < 0$, they prefer to move along the negative z -axis direction. Positrons with opposite spin directions exhibit reversed directional preferences in their motion. The symmetry of the momentum distribution can be analyzed through the QKEs shown in Eq. (11). Since the components of the electric field (29) satisfy $E_x(-t) = E_x(t)$, $E_y(-t) = -E_y(t)$, and $E_z = 0$, under the transformations $t \rightarrow -t$ and $q_x \rightarrow -q_x$, we have $p_x \rightarrow -p_x$, $p_y \rightarrow p_y$, $p_z \rightarrow p_z$, and $\omega \rightarrow \omega$. Further, we can obtain the transformations of μ_1 and μ_2 as $\mu_{1x} \rightarrow -\mu_{1x}$, $\mu_{1y} \rightarrow \mu_{1y}$, $\mu_{1z} \rightarrow \mu_{1z}$, and $\mu_{2x} \rightarrow \mu_{2x}$, $\mu_{2y} \rightarrow -\mu_{2y}$, $\mu_{2z} \rightarrow -\mu_{2z}$. Applying these transformations to Eq. (11), we finally get $f \rightarrow f$, and $f_x \rightarrow -f_x$, $f_y \rightarrow f_y$, $f_z \rightarrow f_z$. Consequently, the total momentum distribution function at the final time $\mathbf{f} = 2f$ is symmetric about $q_x = 0$, whereas the spin-resolved momentum distribution functions $f_{\pm} = f \pm (\eta_x f_x + \eta_y f_y + \eta_z f_z)$ are not symmetric about $q_x = 0$, since f_x is antisymmetric about $q_x = 0$. Given that the initial vector potential $\mathbf{A}(t_{\text{in}})$ is set to zero in our numerical calculations, the actual symmetry axis should be the line $q_x^* = q_x - A_x(t_{\text{out}})/2 = 0$. Here q_x^* compensates the finite value of $A_x(t_{\text{out}})$ (the net electric field impulse). Additionally, Fig. 1(f) shows that the spin asymmetry degree κ appears to be antisymmetric about $q_x = 0$; however, it is not always antisymmetric about $q_x = 0$ for general electric field parameters, since $\kappa \sim 2\boldsymbol{\eta} \cdot \mathbf{f} = 2(\eta_x f_x + \eta_y f_y)$.

In addition, we also calculate the spin asymmetry degree for the case where $\eta_z = 0$, and η_x and η_y take other values. The results show that the maximum value of the absolute value of the spin asymmetry is usually within 6.5% in the momentum planes $(q_x = 0, q_y, q_z)$ and $(q_x, q_y = 0, q_z)$.

By choosing $\boldsymbol{\eta} = (1/\sqrt{2}, 0, 1/\sqrt{2})$, the spin-resolved momentum distribution functions and spin asymmetry degrees in the momentum planes $(q_x, q_y, q_z = 0)$, $(q_x = 0, q_y, q_z)$, and $(q_x, q_y = 0, q_z)$ are calculated and shown in Fig. 2.

The characteristics of the total momentum distribution in the momentum plane $(q_x, q_y, q_z = 0)$ have been studied previously (e.g., Refs. [51,57]). As can be seen in Figs. 2(a) and 2(b), the characteristics of the spin-up and spin-down momentum distributions f_s are similar to those of the total

momentum distribution. One of the most obvious characteristics is that the momentum distribution is mainly concentrated in the momentum plane $q_y > 0$. The reason is the same as in our previous discussion, i.e., it is influenced by the electric field component $E_y(t)$. Of course, it can also be explained by analyzing the distribution of turning points in the complex time plane [29,58]. Another obvious characteristic in Figs. 2(a) and 2(b) is the interference patterns of the spin-resolved momentum distributions in the negative y -axis direction. The reason for the appearance of interference patterns is still attributed to the effect of $E_y(t)$. Therefore, the difference between f_+ and f_- lies mainly in their numerical values, with f_+ being about 4.87 times that of f_- . However, in Ref. [47], the spin-up and spin-down momentum distribution functions differed by 2 orders of magnitude. This is because the difference between the values of f_+ and f_- is maximized at $\eta_z = \pm 1$, as can be seen from Eq. (31). In addition, if the direction of the unit vector $\boldsymbol{\eta}$ is reversed, i.e., having the form of $\boldsymbol{\eta} = (-1/\sqrt{2}, 0, -1/\sqrt{2})$, we can obtain $f_{s,\boldsymbol{\eta}} = f_{-s,-\boldsymbol{\eta}}$ from Eq. (18). This is simply the result of changing the observation direction of the spin. Interestingly, according to Eq. (31), this symmetry relation also holds if only η_z changes sign, i.e., $f_{s,\eta_z} = f_{-s,-\eta_z}$.

In Figs. 2(d) and 2(e), one can see that f_+ and f_- in the momentum plane $(q_x = 0, q_y, q_z)$ have the same shape and are both approximately symmetric about $q_z = 0$. The value of f_+ is about 4.87 times that of f_- . However, as observed in Fig. 2(h), spin-down positrons exhibit a pronounced directional preference in their motion within the momentum plane $(q_x, q_y = 0, q_z)$. For the spin asymmetry degree, the maximum values of κ in Figs. 2(c), 2(f), and 2(i) can reach about 66%, 66%, and 68%, respectively. This is also an important characteristic of spin asymmetry when the spin-direction unit vector $\boldsymbol{\eta}$ has a large z component.

B. Particle number density

According to Eq. (21), we can calculate the spin-resolved particle number density N_s for any unit vector $\boldsymbol{\eta}$, as shown in Fig. 3. Experimentally, this could be probed by projecting the spin vector obtained via a Lorentz boost from the laboratory frame onto different vectors $\boldsymbol{\eta}$ in the particle's rest frame, following the measurement scheme outlined in the paragraph below Eq. (21). Since N_s is antisymmetric with respect to $\boldsymbol{\eta}$, only the top view of the upper half of the sphere (i.e., $\eta_z = \sqrt{1 - \eta_x^2 - \eta_y^2}$) is shown here. One can see that N_+ (or N_-) reaches its maximum (or minimum) value at $\eta_z = 1$ and decreases (or increases) as η_z decreases. At $\eta_z = 0$, N_+ equals N_- . This is because when $E_z(t) \equiv 0$, $f_x(q_z) = -f_x(-q_z)$, and $f_y(q_z) = -f_y(-q_z)$, resulting in $N_s = N/2 + s\eta_z \Lambda_z = N/2$. For a general three-component electric field, the maximum or minimum value of the

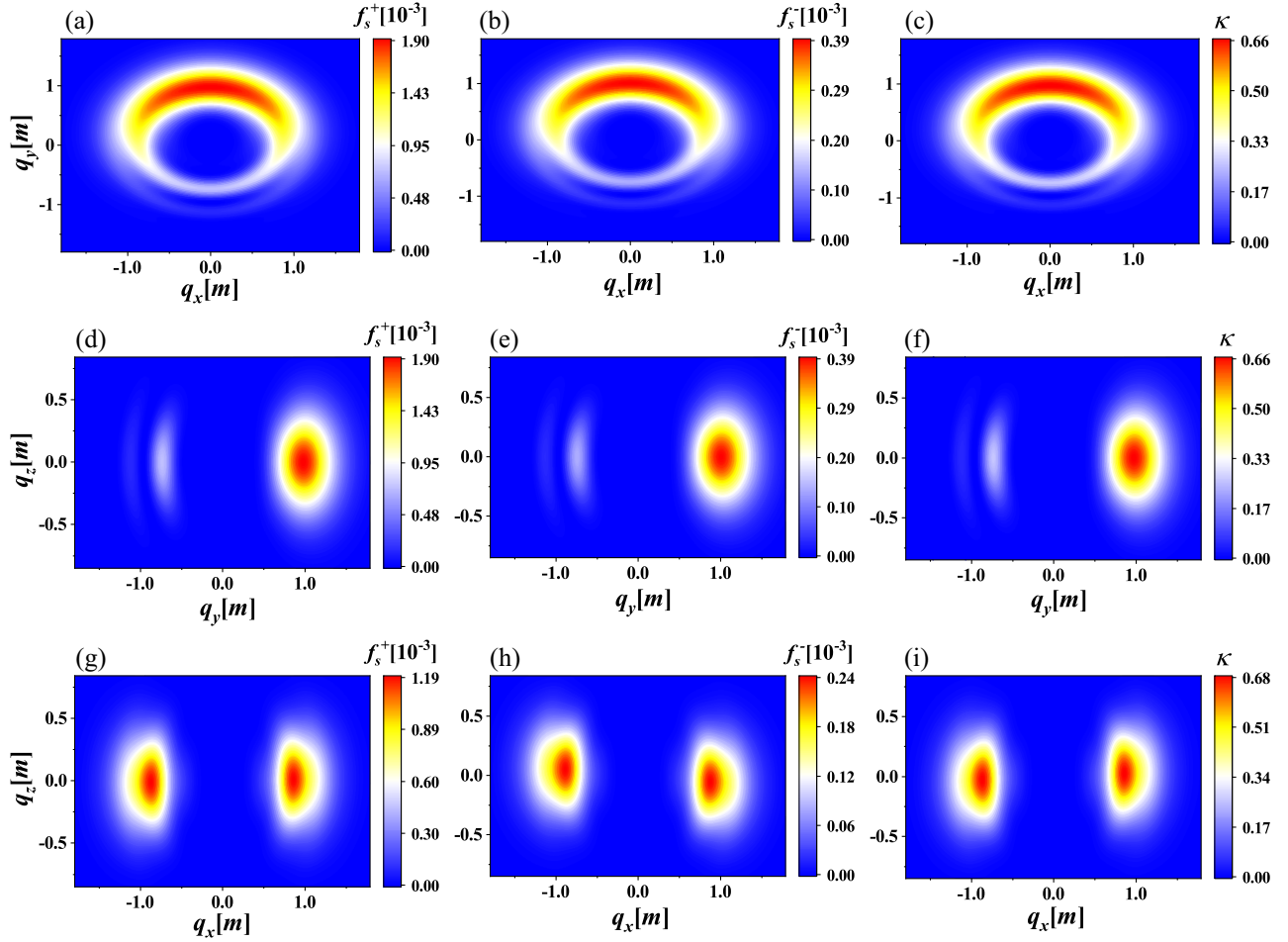


FIG. 2. Spin-resolved momentum distribution functions and the spin asymmetry degree κ of positrons for choosing $\boldsymbol{\eta} = (1/\sqrt{2}, 0, 1/\sqrt{2})$. Panels (a), (d), and (g) correspond to the spin-up case, while (b), (e), and (h) represent the spin-down case. Panels (c), (f), and (i) denote the spin asymmetry degree. The first, second, and third rows correspond to the momentum planes $(q_x, q_y, q_z = 0)$, $(q_x = 0, q_y, q_z)$, and $(q_x, q_y = 0, q_z)$, respectively. The electric field parameters are $E_0 = 0.5E_{\text{cr}}$, $\Omega = 0.5m$, and $\sigma = 5$.

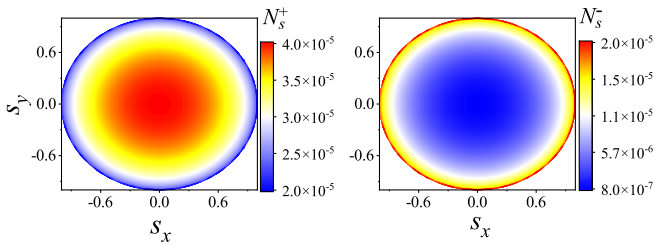


FIG. 3. Spin-up particle number density (left panel) and spin-down particle number density (right panel) for any spin-direction unit vector $\boldsymbol{\eta} = (\eta_x, \eta_y, \sqrt{1 - \eta_x^2 - \eta_y^2})$. The electric field parameters are $E_0 = 0.3\sqrt{2}E_{\text{cr}}$, $\Omega = 0.65m$, and $\sigma = 13\sqrt{2}$.

spin-resolved particle number density is obtained at $\boldsymbol{\eta}_\Lambda = \boldsymbol{\Lambda}/|\boldsymbol{\Lambda}|$; see Eq. (21). The physical phenomenon that we are more concerned with is the spin asymmetry. Therefore, we define the spin asymmetry degree

$$\kappa_N = \frac{N_+ - N_-}{N_+ + N_-}. \quad (32)$$

For the convenience of the following study on the effects of electric field intensity and frequency on κ_N , N_+ , and N_- here denote the spin-up and spin-down particle number density relative to $\boldsymbol{\eta}_\Lambda$, respectively.

The relationship between κ_N and the electric field amplitude E for the spin-direction unit vector $\boldsymbol{\eta} = (0, 0, 1)$ is shown in Fig. 4. The results demonstrate that when the electric field amplitude $E = 0.1E_{\text{cr}}$ and $\zeta = 5$ [where $\zeta = m\Omega/(eE)$ is the Keldysh adiabatic parameter], the number density of spin-up particles is approximately 90 times that of spin-down particles. One can also see that in the transition region from perturbative to nonperturbative processes where $\zeta \sim O(1)$, the spin asymmetry degree κ_N decreases rapidly with the electric field amplitude E . When $\zeta \ll 1$, the variation of κ_N with E becomes gradual.

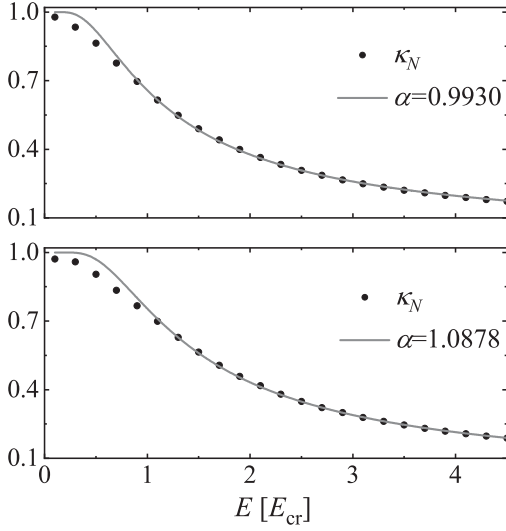


FIG. 4. Spin asymmetry degree κ_N as a function of the electric field amplitude $E = E_0/\sqrt{2}$ for the spin-direction unit vector $\boldsymbol{\eta} = (0, 0, 1)$. The solid gray line represents the fitted curve for κ_N . Other electric field parameters are $\sigma = 10\sqrt{2}$ and $\Omega = 0.5m$ for the upper panel, and $\sigma = 13\sqrt{2}$ and $\Omega = 0.65m$ for the lower panel.

The contribution of spin-down particles to the total particle number density becomes important. For example, when $E = 2.5E_{\text{cr}}$, the number density of spin-up particles is only about 2 times that of spin-down particles. The reason can be explained as follows. We know that there are two primary vacuum pair production mechanisms: the Schwinger tunneling mechanism (when $\zeta \ll 1$) and the multiphoton absorption mechanism (when $\zeta \gg 1$). For the circularly polarized electric field (29), photons have well-defined spin angular momentum with a projection along the z axis of $+1$. For a small field strength, pair production is dominated by multiphoton absorption. The spin angular momentum of absorbed photons is converted into the angular momentum of created electron-positron pairs, which prefers to induce the production of particles with spin $+1/2$ along the z axis. However, with the increase of the field strength, pair production becomes dominated by the Schwinger tunneling mechanism. For tunneling pair production, the angular momentum of the external field mainly transfers to the orbital angular momentum of created pairs due to the more pronounced acceleration of created particles by the strong field. Consequently, the dependence of tunneling probability on particle spin states is relatively weak. For an extremely high field strength, the number density of produced spin-up and spin-down particles is nearly comparable. Moreover, we fit the relationship between the spin asymmetry degree and the electric field strength, and obtain

$$\kappa_N = \frac{1 - \exp(-\pi\zeta^\alpha)}{1 + \exp(-\pi\zeta^\alpha)}, \quad (33)$$

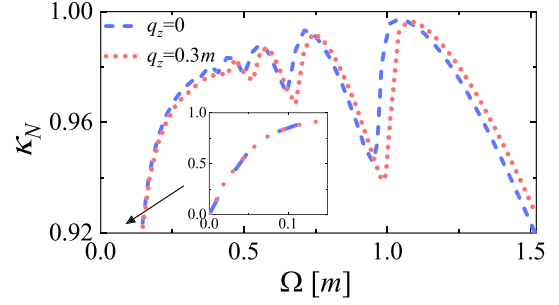


FIG. 5. Spin asymmetry degree κ_N at fixed q_z as a function of the electric field frequency Ω for the spin-direction unit vector $\boldsymbol{\eta} = (0, 0, 1)$. The blue dashed line and red dotted line correspond to $q_z = 0$ and $q_z = 0.3m$, respectively. Other electric field parameters are $E_0 = 0.1\sqrt{2}E_{\text{cr}}$ and $\sigma = 20\sqrt{2}\Omega/m$.

where α is a fitting parameter; see the gray solid line in Fig. 4. For the upper panel of Fig. 4, $\alpha = 0.9930$. The goodness of fit R^2 and 95% confidence interval are 0.9948 and [0.9720, 1.0140], respectively. For the lower panel of Fig. 4, $\alpha = 1.0878$. The goodness of fit R^2 and 95% confidence interval are 0.9924 and [1.0587, 1.1169], respectively. It can be seen that in the region dominated by tunneling pair production, the fitting results and numerical results show excellent agreement.

The relationship between the spin asymmetry degree κ_N at fixed q_z and the electric field frequency Ω for the spin-direction unit vector $\boldsymbol{\eta} = (0, 0, 1)$ is shown in Fig. 5. Note that the spin asymmetry degree at fixed q_z is obtained from the spin-resolved number density integrated over q_x and q_y at fixed q_z , according to Eq. (32). As shown in Fig. 5, in the low-frequency region ($\Omega < 0.4m$ and $\zeta < 4$), κ_N at fixed q_z increases with the increase of Ω . For linearly polarized fields [i.e., $\Omega = 0$ in Eq. (29)], the average spin angular momentum of photons is 0. Therefore, there is no net spin angular momentum transfer. The number density of spin-up particles is exactly equal to that of spin-down particles. Figure 5 also shows that for subcritical electric fields, the weakening of multiphoton absorption can decrease the spin asymmetry degree, providing another perspective to verify the conclusion drawn in Fig. 4. In the high-frequency region ($\Omega > 0.5m$ and $\zeta > 5$), κ_N at fixed q_z has some local maximum in the vicinity of the threshold for multiphoton absorption. These local maximum values increase as Ω increases. This is because particle pair production is most efficient at the threshold of multiphoton absorption, where the transfer of angular momentum is also more significant. Moreover, we find that the results for $q_z = 0$ and $q_z = 0.3m$ exhibit similar variation trends. According to the formula of the total energy of particles, it is easy to see that q_z plays a role similar to the particle mass. It only affects the frequency threshold of multiphoton absorption but does not change the trend of κ_N at fixed q_z as a function of Ω . Thus, a similar relationship between the spin asymmetry degree κ_N and the electric field frequency Ω

would be expected, although this remains to be verified numerically.

V. CONCLUSIONS AND DISCUSSIONS

In this study, a general spin-resolved momentum distribution function of electron-positron pairs produced from the vacuum in strong external fields was derived by combining the covariant spin projection operator and the DHW formalism. It was found that the helicity-resolved momentum distribution function is actually a special case of the spin-resolved momentum distribution function. It can be obtained by choosing the spin-direction unit vector along the momentum direction of produced particles. In particular, for a spatially homogeneous and time-dependent electric field, the helicity-resolved momentum distribution we derived is equivalent to the one obtained from the QKEs in [39,48].

The spin-resolved momentum distribution and number density of particles produced by a spatially homogeneous circularly polarized electric field rotating about the z axis for any spin-direction unit vector were also investigated numerically. The results show that when the z component of the spin-direction unit vector is zero, the number densities of produced particles with spin up and spin down are equal, but their momentum distributions exhibit asymmetry. The particles with specific spin polarization have directional preferences in their motion. In the positive (or negative) momentum direction q_x or q_y , spin-up positrons preferentially move toward the positive z axis (or negative z axis), while spin-down positrons show opposite directional preferences during their motion. Moreover, the z component of the spin-direction unit vector η_z also has a significant effect on spin asymmetry. There may even be a difference of 1–3 orders of magnitude between the spin-up and spin-down particle number densities when η_z is nonzero.

By examining the effects of the electric field strength and frequency on the spin-resolved particle number density, we have discovered the following results. The difference of 1–3 orders of magnitude between the spin-up and spin-down particle number densities is caused by angular momentum transfer during the multiphoton absorption process. For a circularly polarized electric field rotating counterclockwise (clockwise) about the z axis, both electrons and positrons produced primarily have spin values of $+1/2$ ($-1/2$) along the z axis. As the electric field strength increases or the field frequency decreases, the difference between the spin-up and spin-down particle number densities decreases rapidly during the transition phase from the perturbation to nonperturbation regime. This is because the influence of particle spin states on tunneling pair production is relatively weak. Interestingly, these results also suggest that spin asymmetry degree may serve as a tool for determining whether multiphoton absorption or tunneling dominates.

This study advances our understanding of vacuum pair production in a spatially uniform circularly polarized electric field. An important direction for future research

is to investigate pair production in more realistic fields and to explore the transfer of angular momentum from the field to matter under extreme conditions.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: PROOF THAT THE PROJECTED MOMENTUM DISTRIBUTION FUNCTION IS A REAL-VALUED FUNCTION

In this appendix, we prove that the projected momentum distribution function defined by Eq. (13) is a real-valued function. Since the Wigner function satisfies $\mathcal{W}^\dagger = \gamma^0 \mathcal{W} \gamma^0$, we have

$$\begin{aligned} (\mathcal{P} @ \mathcal{W})^\dagger &= \frac{1}{2} (\mathcal{P} \mathcal{W} + \mathcal{W} \gamma^0 \mathcal{P}^\dagger \gamma^0)^\dagger \\ &= \frac{1}{2} [\mathcal{W}^\dagger \mathcal{P}^\dagger + (\gamma^0)^\dagger \mathcal{P} (\gamma^0)^\dagger \mathcal{W}^\dagger] \\ &= \frac{1}{2} (\gamma^0 \mathcal{W} \gamma^0 \mathcal{P}^\dagger + \gamma^0 \mathcal{P} \mathcal{W} \gamma^0) \\ &= \gamma^0 (\mathcal{P} @ \mathcal{W}) \gamma^0. \end{aligned} \quad (\text{A1})$$

Based on this property, it can be proven that the complex conjugate of the projected momentum distribution function is equal to itself:

$$\begin{aligned} f_{\mathcal{P}}^* &= \frac{1}{2\omega} \text{tr} \{ [\mathcal{P} @ (\mathcal{W} - \mathcal{W}_{\text{vac}}) (m\mathbb{1} + \boldsymbol{\gamma} \cdot \mathbf{p})]^\dagger \} \\ &= \frac{1}{2\omega} \text{tr} \{ (m\mathbb{1} + \boldsymbol{\gamma} \cdot \mathbf{p})^\dagger [\mathcal{P} @ (\mathcal{W} - \mathcal{W}_{\text{vac}})]^\dagger \} \\ &= \frac{1}{2\omega} \text{tr} \{ (m\mathbb{1} - \boldsymbol{\gamma} \cdot \mathbf{p}) \gamma^0 [\mathcal{P} @ (\mathcal{W} - \mathcal{W}_{\text{vac}})] \gamma^0 \} \\ &= \frac{1}{2\omega} \text{tr} \{ \gamma^0 (m\mathbb{1} - \boldsymbol{\gamma} \cdot \mathbf{p}) \gamma^0 [\mathcal{P} @ (\mathcal{W} - \mathcal{W}_{\text{vac}})] \} \\ &= \frac{1}{2\omega} \text{tr} \{ (m\mathbb{1} + \boldsymbol{\gamma} \cdot \mathbf{p}) [\mathcal{P} @ (\mathcal{W} - \mathcal{W}_{\text{vac}})] \} \\ &= \frac{1}{2\omega} \text{tr} \{ [\mathcal{P} @ (\mathcal{W} - \mathcal{W}_{\text{vac}})] (m\mathbb{1} + \boldsymbol{\gamma} \cdot \mathbf{p}) \} \\ &= f_{\mathcal{P}}. \end{aligned} \quad (\text{A2})$$

APPENDIX B: RELATION BETWEEN TWO PROJECTION OPERATOR APPROACHES

By comparing Eq. (13) in this paper with Eq. (174) in Ref. [39], it seems that the projection $\frac{1}{2}\mathcal{P}_s @ \mathcal{W}$ is equal to $\mathcal{P}_s^{(e-)}\mathcal{W}$. However, we will demonstrate below that they are not equal. The two projection operator approaches become equal only after the final trace operation is completed.

To explore the relation between two projection operator approaches, we expand the Dirac field operator in terms of the instantaneous eigenstates of the Hamiltonian of the Dirac equation. For a spatially homogeneous and time-dependent electric field, in the temporal gauge, the Dirac field operator can be written as

$$\hat{\Psi}(\mathbf{x}, t) = \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}} \sum_{s=\pm 1} \left[\hat{b}_{\mathbf{q},s}(t) u_{\mathbf{q},s}(t) + \hat{d}_{-\mathbf{q},s}^\dagger(t) v_{-\mathbf{q},s}(t) \right], \quad (\text{B1})$$

where $\hat{b}_{\mathbf{q},s}(t)$ and $\hat{d}_{-\mathbf{q},s}^\dagger(t)$ are the time-dependent annihilation operator for electrons and creation operator for positrons, respectively, and

$$u_{\mathbf{q},s}(t) = \sqrt{\frac{\omega + m}{2\omega}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\omega+m}\chi_s \end{pmatrix}, \quad (\text{B2})$$

$$v_{-\mathbf{q},s}(t) = \sqrt{\frac{\omega + m}{2\omega}} \begin{pmatrix} -\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\omega+m}\chi_s \\ \chi_s \end{pmatrix}, \quad (\text{B3})$$

are the orthonormal instantaneous eigenstates of the Hamiltonian $\mathcal{H}_{\mathbf{q}}(t) = \gamma^0(\boldsymbol{\gamma}\cdot\mathbf{p} + m)$ of the Dirac equation. $\boldsymbol{\sigma}$ is the Pauli matrix, χ_s is the orthonormal eigenvector of $\boldsymbol{\sigma}\cdot\boldsymbol{\eta}$, and $\boldsymbol{\eta}$ is the spin-direction unit vector. Then, the Wigner function (2) becomes

$$\begin{aligned} \mathcal{W}(\mathbf{p}, t) &= -\frac{1}{2} \langle 0 | \left[\sum_{s'} (\hat{b}_{\mathbf{q},s'} u_{\mathbf{q},s'} + \hat{d}_{-\mathbf{q},s'}^\dagger v_{-\mathbf{q},s'}) , \sum_{s''} (\hat{b}_{\mathbf{q},s''}^\dagger \bar{u}_{\mathbf{q},s''} + \hat{d}_{-\mathbf{q},s''} \bar{v}_{-\mathbf{q},s''}) \right] | 0 \rangle \\ &= -\frac{1}{2} \sum_{s's''} \{ \langle 0 | [\hat{b}_{\mathbf{q},s'}, \hat{b}_{\mathbf{q},s''}^\dagger] | 0 \rangle u_{\mathbf{q},s'} \bar{u}_{\mathbf{q},s''} + \langle 0 | [\hat{b}_{\mathbf{q},s'}, \hat{d}_{-\mathbf{q},s''}] | 0 \rangle u_{\mathbf{q},s'} \bar{v}_{-\mathbf{q},s''} + \langle 0 | [\hat{d}_{-\mathbf{q},s'}, \hat{b}_{\mathbf{q},s''}^\dagger] | 0 \rangle v_{-\mathbf{q},s'} \bar{u}_{\mathbf{q},s''} \\ &\quad + \langle 0 | [\hat{d}_{-\mathbf{q},s'}, \hat{d}_{-\mathbf{q},s''}] | 0 \rangle v_{-\mathbf{q},s'} \bar{v}_{-\mathbf{q},s''} \}. \end{aligned} \quad (\text{B4})$$

Note that the independent variable t on the right-hand side of the above equation is omitted for convenience.

According to the projection operator approach presented in Ref. [39], we substitute Eq. (B4) into Eq. (C3) in Ref. [39] and obtain

$$\begin{aligned} \mathcal{P}_s^{(e-)}\mathcal{W} &= u_{\mathbf{q},s} \bar{u}_{\mathbf{q},s} \gamma^0 \mathcal{W}(\mathbf{p}, t) \gamma^0 u_{\mathbf{q},s} \bar{u}_{\mathbf{q},s} \\ &= -\frac{1}{2} \langle 0 | [\hat{b}_{\mathbf{q},s}, \hat{b}_{\mathbf{q},s}^\dagger] | 0 \rangle u_{\mathbf{q},s} \bar{u}_{\mathbf{q},s}. \end{aligned} \quad (\text{B5})$$

The momentum distribution function

$$\begin{aligned} f_s &= \frac{1}{\omega} \text{tr}[(m\mathbb{1} + \boldsymbol{\gamma}\cdot\mathbf{p}) \mathcal{P}_s^{(e-)} (\mathcal{W} - \mathcal{W}_{\text{vac}})] \\ &= \frac{1}{\omega} \text{tr}[\gamma^0 \mathcal{H}_{\mathbf{q}}(t) \mathcal{P}_s^{(e-)} \mathcal{W}] + \frac{1}{2} \\ &= -\frac{1}{2} \langle 0 | [\hat{b}_{\mathbf{q},s}, \hat{b}_{\mathbf{q},s}^\dagger] | 0 \rangle + \frac{1}{2} \\ &= \langle 0 | \hat{b}_{\mathbf{q},s}^\dagger \hat{b}_{\mathbf{q},s} | 0 \rangle. \end{aligned} \quad (\text{B6})$$

For the projection operator approach proposed in this paper, using Eq. (15), we can derive the relations $\mathcal{P}_s u_{\mathbf{q},s'} = \delta_{ss'} u_{\mathbf{q},s'}$ and $\mathcal{P}_s v_{-\mathbf{q},s'} = -s n^0 u_{\mathbf{q},s'} + \delta_{-ss'} v_{-\mathbf{q},s'}$. Therefore, the covariant spin projection operator can also be

represented by the relevant bispinors as $\mathcal{P}_s = u_{\mathbf{q},s} u_{\mathbf{q},s}^\dagger + v_{-\mathbf{q},-s} v_{-\mathbf{q},-s}^\dagger - s n^0 \sum_{s'} u_{\mathbf{q},s'} v_{-\mathbf{q},s'}^\dagger$. Applying the projection operator to the Wigner function, we obtain

$$\begin{aligned} \mathcal{P}_s \mathcal{W} \gamma^0 &= -\frac{1}{2} \sum_{s''} \{ \langle 0 | [\hat{b}_{\mathbf{q},s}, \hat{b}_{\mathbf{q},s''}^\dagger] | 0 \rangle u_{\mathbf{q},s} u_{\mathbf{q},s''}^\dagger \\ &\quad + \langle 0 | [\hat{b}_{\mathbf{q},s}, \hat{d}_{-\mathbf{q},s''}] | 0 \rangle u_{\mathbf{q},s} v_{-\mathbf{q},s''}^\dagger \\ &\quad + \langle 0 | [\hat{d}_{-\mathbf{q},-s}, \hat{b}_{\mathbf{q},s''}^\dagger] | 0 \rangle v_{-\mathbf{q},-s} u_{\mathbf{q},s''}^\dagger \\ &\quad + \langle 0 | [\hat{d}_{-\mathbf{q},-s}, \hat{d}_{-\mathbf{q},s''}] | 0 \rangle v_{-\mathbf{q},-s} v_{-\mathbf{q},s''}^\dagger \} \\ &\quad + \frac{s}{2} n^0 \sum_{s's''} \{ \langle 0 | [\hat{d}_{-\mathbf{q},s'}, \hat{b}_{\mathbf{q},s''}^\dagger] | 0 \rangle u_{\mathbf{q},s'} u_{\mathbf{q},s''}^\dagger \\ &\quad + \langle 0 | [\hat{d}_{-\mathbf{q},s'}, \hat{d}_{-\mathbf{q},s''}] | 0 \rangle u_{\mathbf{q},s'} v_{-\mathbf{q},s''}^\dagger \}. \end{aligned} \quad (\text{B7})$$

Since $(\mathcal{P}_s \mathcal{W} \gamma^0)^\dagger = \mathcal{W} \mathcal{P}_s \gamma^0$, we have $\mathcal{P}_s \mathcal{W} \gamma^0 + \mathcal{W} \mathcal{P}_s \gamma^0 = (\mathcal{P}_s \mathcal{W} + \mathcal{W} \mathcal{P}_s) \gamma^0 = 2 \text{Re}(\mathcal{P}_s \mathcal{W} \gamma^0)$. So,

$$\frac{1}{2} \mathcal{P}_s @ \mathcal{W} = \frac{1}{4} (\mathcal{P}_s \mathcal{W} + \mathcal{W} \mathcal{P}_s) = \frac{1}{2} \text{Re}(\mathcal{P}_s \mathcal{W} \gamma^0) \gamma^0. \quad (\text{B8})$$

The momentum distribution function is

$$\begin{aligned}
f_s &= \frac{1}{\omega} \text{tr} \left[\frac{1}{2} \mathcal{P}_s @ (\mathcal{W} - \mathcal{W}_{\text{vac}}) (m\mathbb{1} + \boldsymbol{\gamma} \cdot \mathbf{p}) \right] \\
&= \frac{1}{\omega} \text{tr} \left[\gamma^0 \mathcal{H}_{\mathbf{q}}(t) \frac{1}{2} \mathcal{P}_s @ \mathcal{W} \right] + \frac{1}{2} \\
&= -\frac{1}{4} \{ \langle 0 | [\hat{b}_{\mathbf{q},s}, \hat{b}_{\mathbf{q},s}^\dagger] | 0 \rangle - \langle 0 | [\hat{d}_{-\mathbf{q},-s}^\dagger, \hat{d}_{-\mathbf{q},-s}] | 0 \rangle \} + \frac{s}{4} n^0 \text{Re} \{ \sum_{s'} \langle 0 | [\hat{d}_{-\mathbf{q},s'}^\dagger, \hat{b}_{\mathbf{q},s'}^\dagger] | 0 \rangle \} + \frac{1}{2} \\
&= \frac{1}{2} \{ \langle 0 | \hat{b}_{\mathbf{q},s}^\dagger \hat{b}_{\mathbf{q},s} | 0 \rangle + \langle 0 | \hat{d}_{-\mathbf{q},-s}^\dagger \hat{d}_{-\mathbf{q},-s} | 0 \rangle \} - \frac{s}{2} n^0 \text{Re} \{ \sum_{s'} \langle 0 | \hat{b}_{\mathbf{q},s'}^\dagger \hat{d}_{-\mathbf{q},s'}^\dagger | 0 \rangle \} \\
&= \langle 0 | \hat{b}_{\mathbf{q},s}^\dagger \hat{b}_{\mathbf{q},s} | 0 \rangle.
\end{aligned} \tag{B9}$$

Note that in the final step of the derivation above, we used the relations $\langle 0 | \hat{d}_{-\mathbf{q},-s}^\dagger \hat{d}_{-\mathbf{q},-s} | 0 \rangle = \langle 0 | \hat{b}_{\mathbf{q},s}^\dagger \hat{b}_{\mathbf{q},s} | 0 \rangle$ and $\sum_{s'} \langle 0 | \hat{b}_{\mathbf{q},s'}^\dagger \hat{d}_{-\mathbf{q},s'}^\dagger | 0 \rangle = 0$, which are obtained from Eq. (105) in Ref. [39]. Although the derivation of the QKEs in Ref. [39] was carried out with the spin-direction unit vector $\boldsymbol{\eta}$ oriented along the z axis, the procedure remains valid for $\boldsymbol{\eta}$ pointing in an arbitrary direction. The generalization is achieved simply by substituting the Pauli matrix $\boldsymbol{\sigma}$ with the expression $\chi_s^\dagger \boldsymbol{\sigma} \chi_{s'}$. Therefore, according to Eq. (105) in Ref. [39], the general spin-resolved momentum distribution function ultimately becomes

$$f_s = \langle 0 | \hat{b}_{\mathbf{q},s}^\dagger \hat{b}_{\mathbf{q},s} | 0 \rangle = f + \mathbf{f} \cdot (\chi_s^\dagger \boldsymbol{\sigma} \chi_s) = f + s \mathbf{f} \cdot \boldsymbol{\eta}. \tag{B10}$$

From Eqs. (B5), (B7), and (B8), we can see that $\mathcal{P}_s^{(e^-)} \mathcal{W} \neq \frac{1}{2} \mathcal{P}_s @ \mathcal{W}$. However, despite this, the resulting momentum distribution functions are identical; see Eqs. (B6) and (B9). Moreover, if the spin direction is taken along the z axis, Eq. (B10) gives the spin-resolved momentum distribution functions. If the spin direction is aligned with the momentum direction, Eq. (B10) gives the helicity-resolved momentum distribution functions.

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