

Studying the $K\Lambda$ strong interaction using femtoscopic correlation functions and the $N^*(1535)$

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(Received 14 May 2025; accepted 29 July 2025; published 25 August 2025)

We investigate the $K\Lambda$ strong interaction dynamics around the energy region of $N^*(1535)$ resonance through femtoscopic correlation functions within the Koonin-Pratt formalism, where coupled-channel effects of πN , $K\Lambda$, $K\Sigma$ and ηN channels are taken into account, by means of which the $N^*(1535)$ resonance can be dynamically generated in the chiral unitary approach. Two-body scattering amplitudes are calculated using the chiral unitary approach, and the corresponding wave functions and correlation functions are obtained. Our analysis shows that the πN channel becomes important to the $K^+\Lambda$ correlation function through coupled channel effects, even if its mass threshold is far from the energy region of the $N^*(1535)$. Furthermore, with the picture that the $N^*(1535)$ is a dynamically generated state, we can get a good reproduction of the experimental data on the $K^+\Lambda$ correlation function. This work establishes femtoscopic correlation functions as a sensitive probe for coupled-channel dynamics and validates the Koonin-Pratt framework in multichannel hadronic systems, offering critical insights into some exotic hadron states and strong interaction mechanisms.

DOI: [10.1103/yrdv-jg6b](https://doi.org/10.1103/yrdv-jg6b)

I. INTRODUCTION

The investigation of $K\Lambda$ final state production offers significant opportunities for advancing our understanding about the light baryon resonance spectrum in the medium-energy region. Particularly it is worthy to note that this channel is around the energy of the nucleon excited resonance $N^*(1535)$ has been demonstrated to play a crucial role in numerous processes in hadron physics. This resonance state not only contributes substantially to the dynamics of $K\Lambda$ production but also serves as a key element to understand the complex structure of excited baryon states. With spin-parity quantum numbers $J^P = 1/2^-$, the $N^*(1535)$ is conventionally interpreted in the constituent quark model as the first orbital excitation of the nucleon [1]. However, its measured Breit-Wigner mass, $M_{N^*(1535)} = 1530 \pm 15$ MeV, unexpectedly exceeds that of the first radial excitation $N^*(1440)$ with Breit-

Wigner mass $M_{N^*(1440)} = 1440 \pm 30$ MeV [1–4], which is the so-called mass reversal problem in the classical quark model. Furthermore, despite its nominal u and d valence quark composition, this resonance exhibits an anomalously strong coupling to the ηN channel typically associated with strangeness. In the new measurement by the BESIII collaboration [5], the ratio of decay widths $\Gamma_{N^*(1535) \rightarrow N\eta} / \Gamma_{N^*(1535) \rightarrow N\pi}$ is determined to be $0.99 \pm 0.05 \pm 0.19$, which is much larger than the predictions based on flavor symmetry and phase space [6]. These distinctive characteristics indicate the presence of a nonconventional hadronic structure beyond the standard qqq configuration in $N^*(1535)$. If we consider $N^*(1535)$ as a meson-baryon dynamically generated state [7–11], or if we account for significant five-quark components in its wave function [12–16], the strong coupling of $N^*(1535)$ to the $N\eta$ channel can be naturally explained. Within the above hadronic molecule picture¹ for the $N^*(1535)$, its couplings to strangeness channels,² such as $K\Lambda$, $K\Sigma$, $N\eta'$, and $N\phi$, can be also strong [7,13,16]. For example, based on its strong

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¹Here, we do not distinguish between the meson-baryon dynamically generated state and the excited baryon state with significant five-quark components.

²These channels are above the mass threshold of $N^*(1535)$.

couplings to the strangeness channels, the role of $N^*(1535)$ was investigated in the reactions or decays: $pp \rightarrow pK^+\Lambda$ [17,18], $J/\psi \rightarrow \bar{p}p\eta$ [19], $\pi^-p \rightarrow n\phi$ and $pp \rightarrow pp\phi$ [20,21], $\pi^-p \rightarrow \eta n$ [22], $\pi^-p \rightarrow K^0\Lambda$ [23], $\gamma p \rightarrow \phi K^+\Lambda$ [24], $\pi N \rightarrow N\eta'$ and $J/\psi \rightarrow p\bar{p}\eta'$ [25], and $\gamma p \rightarrow p\eta\phi$ [26], where these available experimental data can be well reproduced. These above works partly support the hadronic molecule nature of $N^*(1535)$. On the other hand, within the framework of a hadronic molecule picture for the $N^*(1535)$ state, its production in the $\Lambda_c^+ \rightarrow \bar{K}^0\eta p$ decay has also been evaluated [27–30].

Even acknowledging the significance of the molecular components in the wave function of the $N^*(1535)$ resonance, many studies suggest that the three-quark components also play a crucial role in the structure of this state [31–34]. Under the quenched quark model, the results of Ref. [34] support that the $N^*(1535)$ is a three-quark state. Therefore, more works are still needed to study the nature of the $N^*(1535)$ state.

Recently, the femtoscopy techniques have emerged as a powerful tool for probing the emission source geometry, two-body hadronic wave functions, and effective hadron-hadron interactions through precision measurements of correlation functions in heavy-ion and p - p collisions [35–41]. Since high-quality measurements about the hadron-hadron scattering processes are not suitable for unstable particles in experimental studies, these correlation measurements provide a cleaner experimental environment with reduced resonance contamination compared to conventional approaches, serving as a complementary constraint to scattering experiments and an alternative way to investigate the invariant mass spectrum for a pair of particles [40,42–51]. Femtoscopic correlations between hadron pairs in momentum space have become a powerful tool for studying the hadron-hadron interaction and hadron dynamics, especially for these exotic hadrons with molecular nature and the near-threshold resonances [52–61]. In addition, the experimental measurements can be used to determine the free model parameters of the low-energy effective theories.

On the experimental side, the first measurements of the correlation function for three distinct charge combinations of $K^+\Lambda$, $K^-\Lambda$, and $K_S^0\Lambda$ were reported by the ALICE collaboration [62,63], enabling us to investigate their strong interactions and extract scattering parameters for all three charge configurations with high precision. On the theoretical side, the interactions of ηN , $K\Lambda$ and $K\Sigma$ were evaluated within the chiral unitary approach, and it was found that the corresponding correlation functions could illuminate the relation between the $N^*(1535)$ state and these coupled channels [64,65]. The theoretical results for the $K\Lambda$ correlation functions of Refs. [64,65] show clear cusps around the mass thresholds of ηN and $K\Sigma$ channels. However, these cusp structures have not been observed in the available experimental data [62,63]. It is worthy to

mention that the πN channel was not taken into account in the works of Refs. [64,65]. As noted in Refs. [41–43], the influence of this channel can significantly affect the shape of the correlation functions. Therefore, the πN channel is expected to play a crucial role in the $K^+\Lambda$ correlation function. Along this line, based on the previous works of Refs. [64,65], we study the $K\Lambda$ correlation function within the chiral unitary approach with the coupled channels, πN , $K\Lambda$, $K\Sigma$, and ηN , from where the $N^*(1535)$ is dynamically generated. Taking into account the Koonin-Pratt theoretical framework [66–68], we calculate the scattering amplitudes using the chiral unitary approach, from which we subsequently derive the wave functions and momentum correlation functions. It will be demonstrated that the resulting correlation functions are smooth due to the inclusion of the πN channel. In addition, the obtained properties of the $N^*(1535)$ state are the same as in the molecular picture.

This article is organized as follows: Sec. II A details the derivation of correlation functions within the Koonin-Pratt formalism. Sec. II B briefly introduces the chiral unitary method along with computational formulas for relevant observables. Sec. III A decomposes the correlation functions to analyze contributions from individual coupled channels. Section III B presents numerical results for the $K^+\Lambda$ correlation function. The study concludes with a brief summary in Sec. IV.

II. FORMALISM

The correlation function is defined as the ratio of the probability of simultaneously measuring two particles with momenta $\vec{p}_1$ and $\vec{p}_2$ to the product of single-particle probabilities

$$C(\vec{p}_1, \vec{p}_2) = \frac{P(\vec{p}_1, \vec{p}_2)}{P(\vec{p}_1)P(\vec{p}_2)}. \quad (1)$$

In actual experimental measurements, the correlation function is defined as [35–41]

$$C(p) = \mathcal{N} \frac{N_{\text{same}}(p)}{N_{\text{mixed}}(p)}, \quad (2)$$

where p is the relative momentum of the pair ($p = \frac{1}{2}|\vec{p}_1 - \vec{p}_2|$). $N_{\text{same}}(p)$ is the p distribution of pairs measured in the same event and $N_{\text{mixed}}(p)$ is the reference distribution from different (mixed) events. $\mathcal{N}$ is a normalization constant determined by the physical expectation that particles are uncorrelated at large relative momenta, which ensures $C(p) = 1$. Next, We will introduce the detailed theoretical framework for calculating the correlation function.

A. The Koonin-Pratt formalism for the correlation function in coupled channels

In this section, we will introduce and derive a common formalism of the correlation function. The definition of the correlation function for the i th channel in the coupled channels is given by [52,66–68],

$$C_i(\vec{p}) = \int_0^\infty d^3\vec{r} S(\vec{r}) |\Psi_i(\vec{r}, \vec{p})|^2, \quad (3)$$

where the $\Psi_i(\vec{r}, \vec{p})$ is the two-body scattering wave function of the i th hadron pair in coupled channels and $\vec{p}$ is the three momentum in the pair rest frame. $S(\vec{r})$ is the emitting source function that is usually parametrized as a normalized Gaussian function,

$$S(\vec{r}) = \frac{e^{-\frac{|\vec{r}|^2}{4R^2}}}{(4\pi R^2)^{3/2}}, \quad (4)$$

where R provides the range of the production source. Its typical value for pp collisions is about 1 fm. However, as the feed-down contributions mentioned in Sec. III B, there may be some long-lived resonances in the emitting progress, which will cause the particles to be emitted from farther apart. It means that the size of emitting source will be increased and an exponential tail appears in the source distribution [36,63]. Therefore, inspired by this picture, we need to use a Gaussian core and a non-Gaussian halo to build source function as in Ref. [63],

$$S_{\text{eff}}(\vec{r}) = \lambda_S [\omega_S S(\vec{r}, R_1) + (1 - \omega_S) S(\vec{r}, R_2)]. \quad (5)$$

Here we take $R_1 = 1.202_{-0.042}^{+0.043}$ fm, $R_2 = 2.330_{-0.045}^{+0.050}$ fm, $\lambda_S = 0.9806_{-0.0008}^{+0.0006}$, and $\omega_S = 0.7993_{-0.0027}^{+0.0037}$ as used in Ref. [63].

Furthermore, the two-body scattering wave function after the final state interaction can be obtained as,

$$\Psi_{ij}(\vec{r}, \vec{p}) = \delta_{ij} \Phi(\vec{r}, \vec{p}) + \Psi'_{ij}(\vec{r}, \vec{p}) = \delta_{ij} e^{i\vec{p}\cdot\vec{r}} + \Psi'_{ij}(\vec{r}, \vec{p}), \quad (6)$$

which consists of a plane wave $e^{i\vec{p}\cdot\vec{r}}$ and a scattered wave $\Psi'_{ij}(\vec{r}, \vec{p})$. Therefore, considering all coupled channels, the

total two-body wave function can be obtained,

$$|\Psi_i(\vec{r}, \vec{p})|^2 = \sum_j w_j |\Psi_{ij}(\vec{r}, \vec{p})|^2 = w_i |\Psi'_{ii}(\vec{r}, \vec{p}) + e^{i\vec{p}\cdot\vec{r}}|^2 + \sum_{j \neq i} w_j |\Psi'_{ij}(\vec{r}, \vec{p})|^2, \quad (7)$$

where w_j represents the production ratio of j th coupled channel particles generated in collisions to the number of i th channel. Due to the lack of experimental information for w_j , we set all the w_j to 1 and their corresponding emitting source size R to the same value for simplification [46,52,64]. Since the plane wave $e^{i\vec{p}\cdot\vec{r}}$ can be expanded to Legendre polynomials,

$$e^{i\vec{p}\cdot\vec{r}} = e^{i|\vec{p}||\vec{r}|\cos\theta} = \sum_{l=0}^{\infty} i^l (2l+1) j_l(|\vec{p}||\vec{r}|) P_l(\cos\theta), \quad (8)$$

where $j_l(x)$ is the l th order spherical Bessel function, and because the value of this integral $\int_{-1}^1 d\cos\theta P_l(\cos\theta)$ is nonzero only if $l = 0$, the following equation can be obtained,

$$\int_{-1}^1 d\cos\theta e^{i\vec{p}\cdot\vec{r}} = \int_{-1}^1 d\cos\theta j_0(|\vec{p}||\vec{r}|), \quad (9)$$

where $j_0(x) = \sin x/x$. Therefore, combining the Eq. (7) and Eq. (9), the correlation function $C_i(\vec{p})$ in Eq. (3) can be simplified as following,

$$C_i(\vec{p}) = 1 + \int_0^\infty d^3\vec{r} S_{12}(\vec{r}) \left(2\text{Re} \left[\Psi'_{ii}(\vec{r}, \vec{p}) j_0(|\vec{p}||\vec{r}|) \right] + \sum_j |\Psi'_{ij}(\vec{r}, \vec{p})|^2 \right). \quad (10)$$

Following the derivations of the Refs. [52,55], the analytic expression of correlation function can be obtained in the Koonin-Pratt formalism. First, the two-body scattering wave function $\Psi_{ij}(\vec{r}, \vec{p})$ in Eq. (6) can be derived from Lippmann-Schwinger equation,

$$\Psi_{ij}(\vec{r}, \vec{p}) = \delta_{ij} e^{i\vec{p}\cdot\vec{r}} + T_{ij}(\vec{r}, \vec{p}) \int_0^\infty \frac{d^4q}{(2\pi)^4} \frac{i(2M_j)^n e^{i\vec{q}\cdot\vec{r}}}{(q^2 - M_j^2 + i\epsilon)((P-q)^2 - m_j^2 + i\epsilon)} = \delta_{ij} e^{i\vec{p}\cdot\vec{r}} + T_{ij}(\vec{r}, \vec{p}) \tilde{G}_j(\vec{r}, \vec{p}), \quad (11)$$

where

$$\begin{aligned} \tilde{G}_j(\vec{r}, \vec{p}) &= (2M_j)^n \int_0^\infty \frac{d^3\vec{q}}{(2\pi)^3} \frac{(\omega_{M_j} + \omega_{m_j}) j_0(|\vec{q}||\vec{r}|)}{2\omega_{M_j}\omega_{m_j}[s - (\omega_{M_j} + \omega_{m_j})^2 + i\epsilon]} = (2M_j)^n \\ &\times \int_0^\infty \frac{d^3\vec{q}}{(2\pi)^3} \left[\frac{(\omega_{M_j} + \omega_{m_j}) j_0(|\vec{q}||\vec{r}|) - j_0(|\vec{q}_{j,\text{on}}||\vec{r}|)}{2\omega_{M_j}\omega_{m_j}[s - (\omega_{M_j} + \omega_{m_j})^2 + i\epsilon]} + j_0(|\vec{q}_{j,\text{on}}||\vec{r}|) \frac{\omega_{M_j} + \omega_{m_j}}{2\omega_{M_j}\omega_{m_j}[s - (\omega_{M_j} + \omega_{m_j})^2 + i\epsilon]} \right], \end{aligned} \quad (12)$$

where $|\vec{q}_{j,\text{on}}| = \lambda^{1/2}(s, M_j^2, m_j^2)/(2\sqrt{s})$, $\omega_{M_j} = \sqrt{M_j^2 + |\vec{q}|^2}$, and $s = P^2$ are the on-shell three momenta, the energy, and the invariant mass square of the j th hadron pair, respectively. And $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. M_j represents the fermionic mass in j th coupled channel which is just a convention due to fermion, where n is the number of fermions in loop function. The T_{ij} matrix is the two-body scattering amplitude that can be calculated by the chiral unitary approach.

Finally, the analytic equation for the correlation function $C_i(\vec{p})$ in the Koonin-Pratt formalism can be easily obtained,

$$C_i(\vec{p}) = 1 + \int_0^\infty d^3\vec{r} S_{12}(\vec{r}) \left(\sum_j |T_{ij}(\vec{r}, \vec{p}) \tilde{G}_j(\vec{r}, \vec{p})|^2 + 2\text{Re}[T_{ii}(\vec{r}, \vec{p}) \tilde{G}_i(\vec{r}, \vec{p}) j_0(|\vec{p}||\vec{r}|)] \right). \quad (13)$$

B. The chiral unitary approach for the $K\Lambda$ interaction within coupled channels

Within the chiral unitary approach, the unitary two-body scattering matrix T among the strangeness $S = 0$ and charge = 1 states $K^0\Sigma^+$, $K^+\Sigma^0$, $K^+\Lambda$, π^+n , π^0p , and ηp can be obtained with the Bethe-Salpeter equation [69–71],

$$T = (1 - VG)^{-1}V, \quad (14)$$

where V is the pseudoscalar meson-baryon interaction potential matrix obtained from the leading order chiral Lagrangian, as follows

$$V_{ij} = -\frac{C_{ij}}{4f_i f_j} (2\sqrt{s} - M_i - M_j) \sqrt{\frac{(M_i + E_i)(M_j + E_j)}{4M_i M_j}}, \quad (15)$$

where f_j , M_j , k_j , and E_j ($j = K^0\Sigma^+$, $K^+\Sigma^0$, $K^+\Lambda$, π^+n , π^0p , or ηp) are the meson decay constant, baryon mass, meson energy, and baryon energy of the j th channel, respectively. Here we take $f_\pi = 93$ MeV, $f_K = 1.22f_\pi$ and $f_\eta = 1.3f_\pi$, $m_{K^+} = 493.68$, $m_{K^0} = 497.61$, $m_{\pi^+} = 139.57$, $m_{\pi^0} = 134.98$, $m_\eta = 547.86$ MeV, and $M_p = 938.27$, $M_n = 939.57$, $M_{\Sigma^+} = 1189.37$, $M_{\Sigma^0} = 1192.64$, and $M_\Lambda = 1115.68$ MeV. The factor C_{ij} is symmetric and we take their values as used in Refs. [7,64]. Note that we have included the π^+n and π^0p channels, although their mass thresholds are far from the energy region of $N^*(1535)$.

Moreover, a critical distinction arises in the treatment of this loop function integrals, which exhibit logarithmic divergences requiring cutoff [72–74] or dimensional regularization [75,76]. The analytic form of the loop function

within the dimensional regularization is as follows:

$$G_j(s) = \frac{2M_j}{16\pi^2} \left\{ \alpha_j(\mu) + \ln \frac{M_j^2}{\mu^2} + \frac{s + \Delta}{2s} \ln \frac{m_j^2}{M_j^2} + \frac{p_j}{\sqrt{s}} [\ln(s - \Delta + 2p_j\sqrt{s}) - \ln(-s - \Delta + 2p_j\sqrt{s}) + \ln(s + \Delta + 2p_j\sqrt{s}) - \ln(-s + \Delta + 2p_j\sqrt{s})] \right\}, \quad (16)$$

with $\Delta = m_j^2 - M_j^2$, $p_j = \lambda^{1/2}(s, M_j^2, m_j^2)/(2\sqrt{s})$ and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. While μ is the scale of dimensional regularization and $\alpha_j(\mu)$ is the subtraction constant of j th channel. In this paper, we take $\mu = 1200$ MeV for all the coupled channels and $\alpha_j(\mu)^3$ are free parameters which are determined by fitting them to the experimental results of the $K\Lambda$ correlation function and the pole position of $N^*(1535)$ state.

With the obtained two-body scattering amplitudes, T_{ij} , one can calculate the scattering length and the effective range, which can be obtained as [7,64,65],

$$a_j = \frac{2M_j T_{jj}}{8\pi\sqrt{s}} \Big|_{s=s_{\text{th},j}}, \quad r_j = 2 \frac{\partial}{\partial p^2} \left[-\frac{8\pi\sqrt{s}}{2M_j T_{jj}} + ip_j \right] \Big|_{s=s_{\text{th},j}}, \quad (17)$$

where $s_{\text{th},j}$ is the threshold energy square of the j th channel.

In order to obtain the pole position of $N^*(1535)$ which is dynamically generated by the final state interaction, we need to extrapolate the energy to the complex plane. The analytical continuation of the loop function is given by

$$G_j(s) = \begin{cases} G_j^{(I)}(s), & \text{for } \text{Re}\{\sqrt{s}\} < (m_j + M_j) \\ G_j^{(II)}(s), & \text{for } \text{Re}\{\sqrt{s}\} \geq (m_j + M_j) \end{cases}, \quad (18)$$

where $G_j^{(II)}(s)$ is the loop function as in Eq. (16) and

$$G_j^{(II)}(s) = G_j^{(I)}(s) + i \frac{2M_j}{4\pi\sqrt{s}} p_j. \quad (19)$$

Then the pole position of $N^*(1535)$ satisfies $\det(1 - VG)|_{s=s_{\text{pole}}} = 0$ with $\sqrt{s_{\text{pole}}} = M_{N^*(1535)} - i\Gamma_{N^*(1535)}/2$. Once the pole position is determined, the coupling of $N^*(1535)$ resonance to each channel, g_i , can be obtained as,

³To minimize free parameters, we will take $\alpha_{K^+\Sigma^0} = \alpha_{K^0\Sigma^+} \equiv \alpha_{K\Sigma}$ and $\alpha_{\pi^+n} = \alpha_{\pi^0p} \equiv \alpha_{\pi N}$.

$$g_j g_j = \frac{r}{2\pi} \int_0^{2\pi} T_{ij}(\sqrt{s}) e^{i\theta} d\theta \Big|_{r \rightarrow 0}, \quad (20)$$

with $\sqrt{s} = \sqrt{s_{\text{pole}}} + r e^{i\theta}$. In general, the coupling constant g_j is complex.

With the obtained coupling constant, the partial decay width of the $N^*(1535)$ can be calculated by,

$$\Gamma_j = \frac{|g_j|^2}{4\pi} \frac{(E_j + M_j) \lambda^{1/2}(M_{N^*(1535)}^2, M_j^2, m_j^2)}{2M_{N^*(1535)}^2}, \quad (21)$$

where $E_j = (M_{N^*(1535)}^2 + M_j^2 - m_j^2)/(2M_{N^*(1535)})$ is the baryon energy of the j th channel.

III. RESULTS AND DISCUSSIONS

A. The πN channel effects in the $K^+\Lambda$ correlation function

With the two-body scattering amplitudes T obtained within the chiral unitary approach, one can easily obtain the $K^+\Lambda$ correlation function as in Eq. (13). We show the numerical results as a function of p_Λ , the relative momentum of the Λ in the $K^+\Lambda$ system, in Fig. 1, where the green-dashed curve stands for the results without including the π^+n and $\pi^0 p$ channels, and the orange-solid curve stands for the results considering the π^+n and $\pi^0 p$ channels. To investigate the significance of the πN channel, we adopt the model parameters from Refs. [64,65], which utilize a cutoff regularization scheme for the meson-baryon loop functions with $q_{\text{max}} = 630$ MeV. This specific choice of regularization scheme and parameter value ensures consistency with previous theoretical studies while providing a robust basis for evaluating the contributions of the π^+n and $\pi^0 p$ channels.

From Fig. 1, one can see that, without considering the π^+n and $\pi^0 p$ channels, the numerical results show a clear threshold cusp at $K\Sigma$ mass threshold as pointed out in

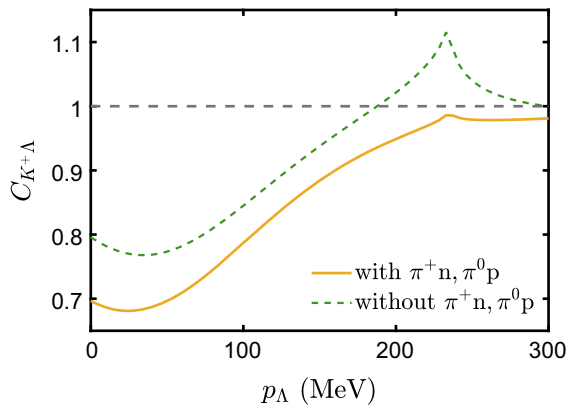


FIG. 1. The contributions of each coupled channel to the $K^+\Lambda$ correlation function. The dotted green line is the result without π^+n and $\pi^0 p$ channels, and the solid orange line is the result with π^+n and $\pi^0 p$. The dashed curve stands for unity for comparison.

Refs. [64,65], this is because of the singularity in the modified propagator $\tilde{G}$'s denominator [see Eq. (12)]. With the inclusion of π^+n and $\pi^0 p$ channels, the peak magnitudes of the $K^+\Lambda$ correlation function are reduced. In the whole energy region we considered, the obtained $K^+\Lambda$ correlation function is smaller than one. This indicates that the introduction of lower-energy π^+n and $\pi^0 p$ channels are important to the scattering amplitude through coupled-channel interactions, even if the threshold of the πN channel is far from the considered energy region. These findings qualitatively validate this theoretical formalism's predictive power for multichannel hadronic systems, establishing femtoscopic correlation functions as sensitive probes of coupled-channel effects.

B. Fitting results for the $K^+\Lambda$ correlation function

We first introduce the process of the $K^+\Lambda$ final state interaction in the relativistic heavy-ion collision [62,63,77,78] which is schematically illustrated in Fig. 2. The production process can be described through two distinct stages: (1) In the initial phase, kaons, pions, and hyperons are generated from the heavy-ion collision at extremely high energies, forming what is referred to as emitting source $[S(\vec{r})]$. (2) Subsequently, through final-state interactions, detectable hadron pairs K^+ and Λ were left and ultimately measured by the detector system. In this case, the $K^+\Lambda$ correlation function can be measured at low relative momenta.

Note that there are three distinct contributions involved in the measured $K^+\Lambda$ correlation functions: First, the genuine contributions arising from $K^+\Lambda$ pairs interacting via coupled-channel elastic and inelastic interactions (e.g., $K^0\Sigma^+, K^+\Lambda \rightarrow K^+\Lambda$). Second, the feed-down contributions from intermediate particle decay (e.g., $\Xi^0 \rightarrow \Lambda$). Third, the experimental artifacts from particle misidentification. In the following discussion, we only introduce the theoretical calculation process for the genuine contribution, while the other contributions need to be determined through fitting experimental data or direct experimental measurements.

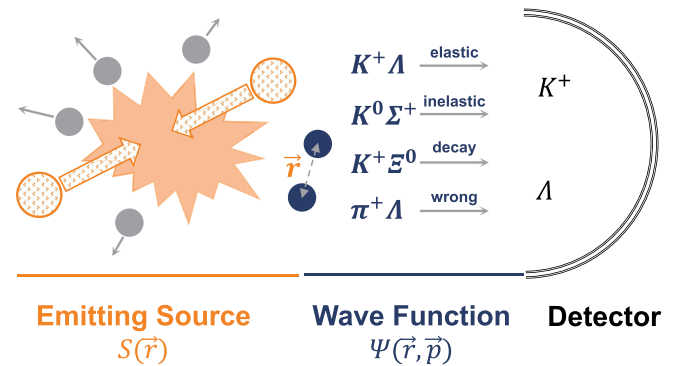


FIG. 2. The schematic diagram for the $K^+\Lambda$ final state interaction in the relativistic heavy-ion collision.

Therefore, to compare with the experimental measurements, the fitting function is defined as

$$C_{\text{fit}} = N' \times C_{\text{tot}} \times C_{\text{background}}, \quad (22)$$

where N' is the global normalization factor allowed to vary freely. The term $C_{\text{background}}$ accounts for residual experimental backgrounds, including hard scattering processes in collisions, unconsidered resonances in C_{tot} , and other effects, as reported in Ref. [63]. The theoretical component C_{tot} is expressed as

$$C_{\text{tot}} = 1 + \lambda_{\text{gen}}(C_{\text{gen}} - 1) + \sum_n \lambda_n(C_n - 1), \quad (23)$$

which includes all detectable $K^+\Lambda$ production mechanisms as mentioned above. The constant C_{gen} corresponds to the correlation function obtained in Eq. (13) and C_n represents the nongenuine contribution. Every contribution is weighted by a normalized parameter λ (i.e., $\lambda_{\text{gen}} + \sum_n \lambda_n = 1$). Given limited information about λ and the absence of experimental analysis of the decaying processes, we make an approximation that all the nongenuine contributions are as a flat constant term X (i.e., $\sum_n \lambda_n C_n = X$). Then the simplified fitting function becomes

$$C_{\text{fit}} = N \times (\lambda C_{\text{gen}} + 1 - \lambda) \times C_{\text{background}}, \quad (24)$$

where $N = N'(\lambda_{\text{gen}} + X)$, which is a normalization global factor close to one. $\lambda = \frac{\lambda_{\text{gen}}}{\lambda_{\text{gen}} + X}$ is a free parameter. In the fitting to the experimental measurements about the $K^+\Lambda$ correlation function, we take the following two fitting scenarios:

Scenario I: Subtraction constants $a(\mu)$ are fixed as in Ref. [7], while N and λ are free parameters. In this case, there are a total of 28 data points and two free parameters;

Scenario II: $a_j(\mu)$, N , and λ are fitted to the $K^+\Lambda$ correlation function. In addition, the $N^*(1535)$ pole position has also been fitted within $M_{N^*(1535)} = 1510 \pm 10$ MeV and $\Gamma_{N^*(1535)} = 105 \pm 25$ MeV, as quoted in the particle data group (PDG) [1]. In this case, there are 30 data points in total and six free parameters.

The fitted parameters and the corresponding obtained properties of $N^*(1535)$ with the central values of the fitted model parameters are summarized in Tables I and II, respectively. Note that the obtained $\chi^2/\text{d.o.f.} = 1.9$ for scenario I is reasonable small. For scenario II, the obtained $\chi^2/\text{d.o.f.}$ is 1.2, which is smaller than the one obtained in scenario I. And, the fitted model parameters N , λ , $a_{K\Lambda}$, and $a_{\pi N}$ are similar with those in scenario I within uncertainties. However, the values of $a_{K\Sigma}$ and $a_{\eta N}$ are much different for scenario II and I.

Based on the obtained properties of the $N^*(1535)$ state shown in Table II, we could conclude that both scenario I and scenario II can dynamically generate the $N^*(1535)$ state in the chiral unitary approach, and they also give very similar results for other properties of the resonance. In scenario I,

TABLE I. The fitted model parameters. Note that the subtraction constants for scenario I were fixed as in Ref. [7].

Scenario	I	II
N	0.99554 ± 0.00045	0.99633 ± 0.00032
λ	0.87 ± 0.01	0.91 ± 0.02
$a_{K\Sigma}$	-2.8	-3.17 ± 0.06
$a_{K\Lambda}$	1.6	1.86 ± 0.06
$a_{\eta N}$	0.2	0.76 ± 0.09
$a_{\pi N}$	2.0	1.91 ± 0.04
$\chi^2/\text{d.o.f.}$	1.9	1.2

TABLE II. The obtained properties for the $N^*(1535)$ resonance.

Scenario		I	II
$\sqrt{s_{\text{pole}}}$ (MeV)		$1531 - i36.5$	$1512 - i39.1$
Coupling	$g_{K^0\Sigma^+}$	$2.21 - i0.18$	$2.14 - i0.34$
	$g_{K^+\Sigma^0}$	$1.56 - i0.13$	$1.51 - i0.24$
	$g_{K^+\Lambda}$	$1.37 - i0.10$	$1.13 - i0.16$
	g_{π^+n}	$0.55 + i0.32$	$0.62 + i0.30$
	$g_{\pi^0 p}$	$0.40 + i0.22$	$0.44 + i0.20$
	$g_{\eta p}$	$-1.47 + i0.43$	$-1.63 + i0.58$
Partial Width (MeV)	Γ_{π^+n}	19.7	21.9
	$\Gamma_{\pi^0 p}$	9.9	11.1
	$\Gamma_{\eta p}$	41.2	39.6
Scattering Length (MeV ⁻¹)	$a_{K^0\Sigma^+}$	$0.28 - i0.14$	$0.26 - i0.12$
	$a_{K^+\Sigma^0}$	$0.19 - i0.13$	$0.18 - i0.11$
	$a_{K^+\Lambda}$	$0.33 - i0.14$	$0.29 - i0.17$
	a_{π^+n}	-0.08	-0.08
	$a_{\pi^0 p}$	0.02	0.02
	$a_{\eta p}$	$-0.25 - i0.17$	$-0.54 - i0.38$
Effective Range (MeV ⁻¹)	$r_{K^0\Sigma^+}$	$1.05 - i1.55$	$0.83 - i1.35$
	$r_{K^+\Sigma^0}$	$-1.46 - i3.69$	$-1.41 - i3.01$
	$r_{K^+\Lambda}$	$-0.52 - i0.38$	$0.47 - i0.89$
	r_{π^+n}	$-13.39 - i18.04$	$-13.39 - i17.29$
	$r_{\pi^0 p}$	30.34	21.26
	$r_{\eta p}$	$-8.60 + i6.77$	$-4.66 + i3.02$

the mass of $N^*(1535)$ resonance, $M_{N^*(1535)} = 1531$ MeV, is heavier than the one, $M_{N^*(1535)} = 1512$ MeV, obtained in scenario II. In the first case it is closer to the Breit-Wigner mass of $M_{N^*(1535)} = 1530 \pm 15$ MeV [1].

It is worthy to mention that the ratio of the coupling constants $|g_{K^0\Sigma^+}/g_{K^+\Sigma^0}| \approx |g_{\pi^+n}/g_{\pi^0 p}| \approx \sqrt{2}$ satisfies the isospin $I = 1/2$ decomposition,⁴ which support that the

⁴We have followed:

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle = \sqrt{\frac{2}{3}} \left| 11 \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle - \sqrt{\frac{1}{3}} \left| 10 \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle,$$

with the basis of $|I I_Z\rangle$.

isospin of $N^*(1535)$ is $1/2$. With the obtained total width $[\Gamma_{N^*(1535)} = -2\text{Im}(\sqrt{s_{\text{pole}}})]$ of $N^*(1535)$ and its partial decay widths to the πN and ηN channels, one could easily get the branching fractions, Scenario I:

$$\text{Br}[N^*(1535) \rightarrow \pi N] = \frac{\Gamma_{\pi^+ n} + \Gamma_{\pi^0 p}}{\Gamma_{N^*(1535)}} = 41\%, \quad (25)$$

$$\text{Br}[N^*(1535) \rightarrow \eta N] = \frac{\Gamma_{\eta p}}{\Gamma_{N^*(1535)}} = 56\%, \quad (26)$$

Scenario II:

$$\text{Br}[N^*(1535) \rightarrow \pi N] = \frac{\Gamma_{\pi^+ n} + \Gamma_{\pi^0 p}}{\Gamma_{N^*(1535)}} = 42\%, \quad (27)$$

$$\text{Br}[N^*(1535) \rightarrow \eta N] = \frac{\Gamma_{\eta p}}{\Gamma_{N^*(1535)}} = 51\%, \quad (28)$$

which are in agreement with these values, $\text{Br}[N^*(1535) \rightarrow \pi N] = (32\text{--}52)\%$ and $\text{Br}[N^*(1535) \rightarrow \eta N] = (30\text{--}55)\%$, as quoted in the PDG [1] within uncertainties. In addition, the obtained ratio of decay widths $\Gamma_{N^*(1535) \rightarrow \eta N} / \Gamma_{N^*(1535) \rightarrow \pi N} = \Gamma_{\eta p} / (\Gamma_{\pi^+ n} + \Gamma_{\pi^0 p})$ is 1.4 and 1.2 for scenario I and II, respectively. Both of the above values are larger than one and fall within two standard deviations of the branching ratio reported in Ref. [6].

On the other hand, the sum of the branching ratios of $\text{Br}[N^*(1535) \rightarrow \pi N]$ and $\text{Br}[N^*(1535) \rightarrow \eta N]$ is less than one, for both scenario I and II, this indicates that there are other decaying channels contribute to the total decay width of $N^*(1535)$, such as the $\pi\pi N$ and $\pi\Delta$ channels via the $\pi N \rightarrow \pi\pi N$ and $\pi N \rightarrow \pi\Delta$ transitions, respectively.

Then, we show the fitted numerical results for $K^+\Lambda$ correlation functions in both scenarios in Fig. 3, from where one can see that we can get a fairly well reproduction of the experimental measurements about the $K^+\Lambda$ correlation function within uncertainties. In Fig. 3, the gray shaded area indicates the experimental background uncertainty $C_{\text{background}}$ (more details can be found in Ref. [63]). The yellow dashed line represents the theoretical $K^+\Lambda$ genuine contribution C_{gen} , while the brown dotted line represents the theoretical total contribution C_{tot} that further incorporates the feed-down and particle misidentification contributions. The red line and the pink shaded area represent the theoretical fitted results C_{fit} including both the experimental background and experimental uncertainties.

For $p_\Lambda < 200$ MeV, the $K^+\Lambda$ correlation function is smaller than one, and it decreases with p_Λ decreasing. For $200 < p_\Lambda < 300$ MeV, the $K^+\Lambda$ correlation function is a little bit larger than one, while for $p_\Lambda > 300$ MeV, it is slightly smaller than one again. This indicates the repulsive interactions in the $K^+\Lambda$ system at low relative momenta. Indeed, with the fitted parameters, the obtained $K^+\Lambda$

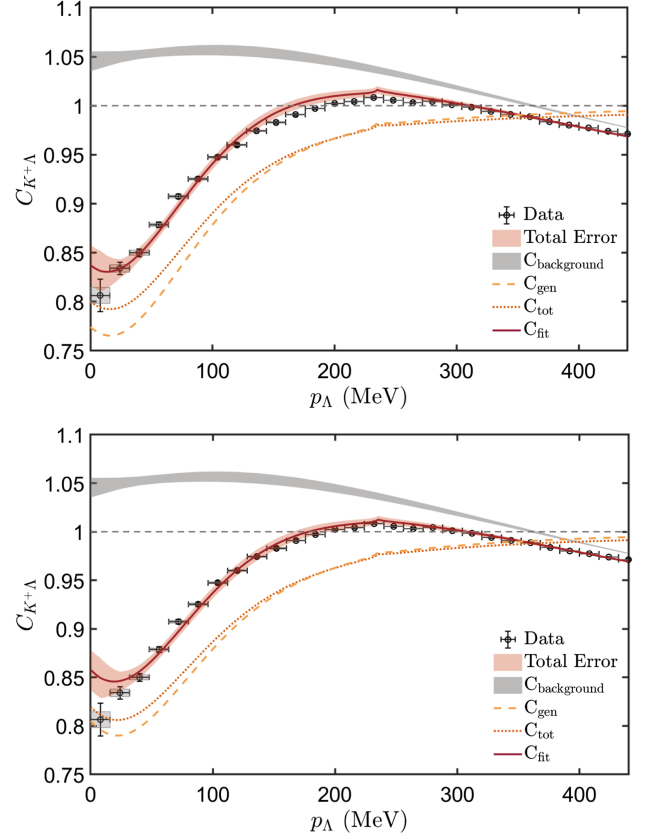


FIG. 3. The fitted results of the $K^+\Lambda$ correlation functions for scenario I (up) and II (down). The experimental data are taken from Ref. [63].

scattering length is $a_{K^+\Lambda} = 0.33 - i0.14$ and $0.29 - i0.17$ for scenario I and II, respectively, which is in agreement with the value obtained in Refs. [62–64] within errors. But, it is only half the experimental results of Refs. [62,63]. This discrepancy may be because of the differences in the amplitude used: our multicoupled-channel framework inherently incorporates intermediate hadronic loops, whereas the experimental analysis adopted a single-channel amplitude parametrization neglecting coupled-channel effects. Furthermore, the theoretical results show a small bump structure near $q \approx 235$ MeV, attributable to coupled-channel effects from $K^+\Sigma^0$ and $K^0\Sigma^+$ channels. It is worthy to mention that the bump structure shown in scenario II is smoother than the results obtained in scenario I.

Now we can evaluate the two-body S -wave $\pi N \rightarrow \pi N$ scattering amplitude in the chiral unitary approach with the fitted model parameters. In Fig. 4, the numerical results of the πN scattering amplitudes S_{11} with the fitted parameters of scenario I and II are shown, compared with the experimental data taken from Ref. [79]. One can see that, in the energies around 1450–1540 MeV, the real part of the πN scattering amplitude S_{11} can be well reproduced for both scenario I and II. However, we cannot get a good fit to the imaginary part of the πN scattering amplitudes above 1510 MeV for scenario II.

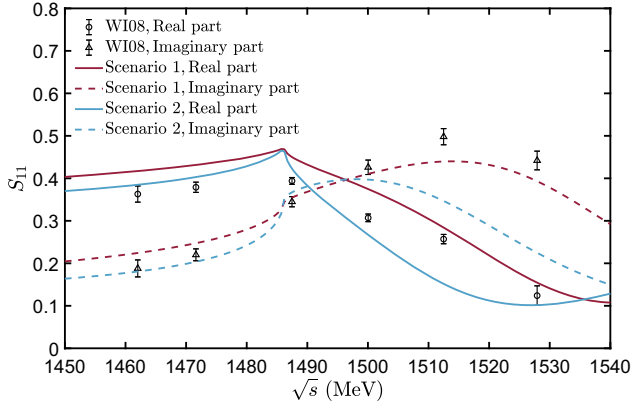


FIG. 4. Theoretical results for the πN elastic scattering amplitude S_{11} . The experimental data are taken from Ref. [79] of the WI08 solution.

IV. SUMMARY AND CONCLUSIONS

In this study, we employ the Koonin-Pratt formalism to analyze coupled-channel correlation functions. Focusing on the $K\Lambda$ system, we study the $K^+\Lambda$ correlation function with coupled-channel πN , $K\Lambda$, $K\Sigma$ and ηN S -wave interactions, where the $N^*(1535)$ resonance can be dynamically generated in the chiral unitary approach. With the two-body S -wave scattering amplitudes obtained within the chiral unitary approach, we then easily get the $K^+\Lambda$ correlation functions. From fitting our theoretical calculations to the experimental measurements, we can get a good reproduction of the experimental data on the $K^+\Lambda$

correlation function. It is also shown that the π^+n and π^0p channels are crucial to the $K^+\Lambda$ correlation function, even if their mass thresholds are far from the considered energy region. This work shows the importance of the coupled channel effects in the studying of the correlation functions.

Furthermore, based on the fitted model parameters, we have successfully determined the properties of the $N^*(1535)$ resonance which are also consistent with both the current experimental data and previous theoretical studies within the molecular nature of the state. It is expected that more precise experimental measurements about the correlation functions, including those for other coupled channels, will provide valuable insights into the investigation of exotic states whose fundamental nature remains unknown.

ACKNOWLEDGMENTS

We would like to thank Profs. Eulogio Oset, Raquel Molina and En Wang for useful discussions and a careful reading of the manuscript. This work is partly supported by the National Key R&D Program of China under Grant No. 2023YFA1606703, and by the National Natural Science Foundation of China under Grant Nos. 12435007 and 12361141819.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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