

Strangeonium spectrum with screening effects and interpretation of $h_1(1911)$ and $X(2300)$ observed by BESIII

Wei Hao^{1,*}, M. Atif Sultan^{2,3,†}, Li-Juan Liu^{1,‡} and En Wang^{1,§}

¹*School of Physics, Zhengzhou University, Zhengzhou 450001, China*

²*School of Physics, Nankai University, Tianjin 300071, China*

³*Centre for High Energy Physics, University of the Punjab, Lahore 54590, Pakistan*



(Received 18 December 2024; accepted 16 July 2025; published 13 August 2025)

Motivated by two new states $h_1(1911)$ and $X(2300)$ observed by BESIII, we have investigated the mass spectrum and the strong decay properties of the strangeonium mesons within the modified Godfrey-Isgur model by considering the screening effects. We have determined the free parameters using the masses and widths of the well-established $s\bar{s}$ states $\phi(1020)$, $\phi(1680)$, $h_1(1415)$, $f'_2(1525)$, and $\phi_3(1850)$. According to our results, $h_1(1911)$ and $X(2300)$ could be well explained as $h_1(2^1P_1)$ and $h_1(3^1P_1)$ $s\bar{s}$ states, respectively. Meanwhile, the possible assignments of $X(2000)$, $\eta_2(1870)$, and $\phi(2170)$ as 3^3S_1 , 1^1D_2 , and 2^3D_1 are also discussed. Furthermore, the masses and widths of the $2S$, $3S$, $1P$, $2P$, $3P$, $1D$, and $2D$ $s\bar{s}$ states are also given and compared with various theoretical predictions, which will be helpful for the observations and confirmations of these states in the future.

DOI: [10.1103/yqxf-5scf](https://doi.org/10.1103/yqxf-5scf)

I. INTRODUCTION

The strangeonium states lying between the light $q\bar{q}$ states and heavy $Q\bar{Q}$ states are related to the long-range interactions, and studying the $s\bar{s}$ meson system is crucial to deepening our understanding of the nonperturbative properties of quantum chromodynamics (QCD) [1–12]. However, for a long time, only several strangeonium states have been well established—i.e., $\phi(1020)$, $\phi(1680)$, $h_1(1415)$, $f'_2(1525)$, and $\phi_3(1850)$ [5]. Meanwhile, the mixing between the singlet and the octet caused by the SU(3) flavor symmetry breaking also complicates our understanding of the strangeonium states. Systematically studying the $s\bar{s}$ states can help us to distinguish between traditional $q\bar{q}$ states and non- $q\bar{q}$ states.

As a powerful platform to study the $s\bar{s}$ state, the BESIII Collaboration has performed a partial wave analysis of the process $J/\psi \rightarrow \phi\pi^0\eta$ in 2023, and observed two new states, $h_1(1900)$ and $X(2000)$, with statistical significances of 24.0σ and 16.9σ in the $\phi\eta$ invariant mass distribution [13]. The quantum numbers, mass, and width of $h_1(1900)$

are $J^{PC} = 1^{+-}$, $M = 1911 \pm 6 \pm 14$ MeV, and $\Gamma = 149 \pm 12 \pm 23$ MeV, and those of $X(2000)$ are $J^{PC} = 1^{--}$, $M = 1996 \pm 11 \pm 30$ MeV, and $\Gamma = 148 \pm 16 \pm 66$ MeV.

It should be pointed out that another state with mass around 2.0 GeV was also observed in the $\phi\eta'$ mass spectrum of the process $J/\psi \rightarrow \phi\eta\eta'$ by BESIII in 2019 [14]. Its mass and width are given by $M = 2002.1 \pm 27.5 \pm 15.0$ MeV and $\Gamma = 129 \pm 17 \pm 7$ MeV when assuming the spin-parity quantum numbers $J^P = 1^-$ for this state. However, when regarding the state as $J^P = 1^+$, the mass and width will be $M = 2062.8 \pm 13.1 \pm 4.2$ MeV and $\Gamma = 177 \pm 36 \pm 20$ MeV. Since the mass and width of this state observed in Ref. [14] are consistent with those of $X(2000)$ observed in Ref. [13] within the experimental uncertainties, the two states could be the same state.

Furthermore, recently BESIII has also reported another new state, $X(2300)$, by a partial wave analysis in the process $\psi(3686) \rightarrow \phi\eta\eta'$ with a covariant tensor approach, and determined the mass $M = 2316 \pm 9 \pm 30$ MeV and width $89 \pm 15 \pm 26$ MeV [15]. These newly observed states $h_1(1900)$, $X(2000)$, and $X(2300)$ further enrich the $s\bar{s}$ spectrum.

The spectrum of the $s\bar{s}$ states has been studied in many works. For instance, in Ref. [8], by using a modified Godfrey and Isgur model (MGI model), the ϕ family has been well studied. In this work, the new observed state $X(2000)$ in 2019 is explained as the $\phi(3S)$, and the state $\phi(2170)$ [also named $Y(2175)$] is assigned as the $\phi(2D)$ state. Furthermore, some states such as $f_2(2010)$, $f_2(2150)$, $X(2060)$, and $X(2500)$ can also be explained as strangeonium states based on a nonrelativistic linear potential quark model [5]. The spectrum

*Contact author: haowei@nankai.edu.cn

†Contact author: atifsultan.chep@pu.edu.pk

‡Contact author: liulijuan@zzu.edu.cn

§Contact author: wangen@zzu.edu.cn

of the $s\bar{s}$ states—especially the excited states—has not yet been well established, and the recently observed states could shed light on this issue.

As we know, many potential models have been developed to study the hadron spectrum [6,16–20]. In these models, the potential between quarks usually includes a Coulomb term at short distances and a linear confining term at large distances. Due to the fact that these models explain hadron states based on the fundamental assumptions of $q\bar{q}$ and qqq without coupling with other components, they are commonly referred to as quenched quark models. However, as more and more hadron states are discovered in experiments, many of them can no longer be well explained by quenched quark models. Except for developing some exotic state models such as multiquark state models, molecular state models, *etc.*, the unquenched model is also a good candidate in this direction. The unquenched effects could come from the contributions of the fluctuation caused by sea quarks and the vacuum polarization effects. The previous one causes the coupled-channel model [21–28], and the latter causes the screened potential model [29–39]. These effects can cause mass shifts with respect to the predictions of the quenched quark models. In fact, both the coupled channel model and the screened potential model have similar features under certain conditions [32]. The MGI model is developed by combining the GI model and screening effects [36–39], where the linear potential in the GI model is replaced with a screened potential, which is important for high excited states.

Thus, inspired by new observed states, we will systematically study the spectrum and decay properties of the strangeonium by using the MGI model and the 3P_0 model. We will investigate the possible assignments of the $s\bar{s}$ states, especially $h_1(1911)$ and $X(2300)$. Furthermore, we will calculate the masses and strong decay properties of other S -, P -, and D -wave mesons, and discuss the possible assignments of the states $X(2000)$, $\eta(1870)$, and $\phi(2170)$.

Recently, we noted that Ref. [40] has studied the mass spectrum and Okubo-Zweig-Iizuka (OZI)-allowed two-body decays of axial-vector mesons, including $X(2300)$, b_1 , h_1 , and h'_1 . They computed the mass spectrum and wave functions by diagonalizing the Hamiltonian matrix using a harmonic oscillator basis. Their conclusion that $h_1(1911)$ can be assigned as $h_1(2^1P_1)$ aligns with our results. However, while we interpret $X(2300)$ as $h_1(3^1P_1)$, its mass and width remain unexplained in Ref. [40].

This article is organized as follows: In Sec. II, we give a brief introduction of the modified GI model and the 3P_0 model. In Sec. III, the numerical results are presented. The summary is given in Sec. IV.

II. FORMALISM

A. Modified Godfrey-Isgur model

In this section, we briefly introduce the modified GI (MGI) model. First, for the GI model, the Hamiltonian of a meson system with $q\bar{q}$ is defined as [6]

$$\tilde{H} = (p^2 + m_q^2)^{1/2} + (p^2 + m_{\bar{q}}^2)^{1/2} + V_{\text{eff}}(\mathbf{p}, \mathbf{r}), \quad (1)$$

with

$$V_{\text{eff}}(\mathbf{p}, \mathbf{r}) = \tilde{H}_{q\bar{q}}^{\text{conf}} + \tilde{H}_{q\bar{q}}^{\text{so}} + \tilde{H}_{q\bar{q}}^{\text{hyp}}, \quad (2)$$

where $\tilde{H}_{q\bar{q}}^{\text{conf}}$ is spin-independent potential, $\tilde{H}_{q\bar{q}}^{\text{hyp}}$ is color-hyperfine interaction, and $\tilde{H}_{q\bar{q}}^{\text{so}}$ is spin-orbit interaction. The explicit expression of $\tilde{H}_{q\bar{q}}^{\text{conf}}$, $\tilde{H}_{q\bar{q}}^{\text{hyp}}$, and $\tilde{H}_{q\bar{q}}^{\text{so}}$ are given in Refs. [37,41,42].

The spin-independent potential $\tilde{H}_{q\bar{q}}^{\text{conf}}$ contains a constant term, a linear confining potential, and a one-gluon exchange potential

$$\tilde{H}_{q\bar{q}}^{\text{conf}} = \tilde{G}(r) + \tilde{S}(r), \quad (3)$$

$$\tilde{G}(r) = -\sum_k \frac{4\alpha_k}{3r} \left[\frac{2}{\sqrt{\pi}} \int_0^{\tau_{kij}r} e^{-x^2} dx \right], \quad (4)$$

$$\tilde{S}(r) = br \left[\frac{e^{-\sigma_{ij}^2 r^2}}{\sqrt{\pi}\sigma_{ij}r} + \left[1 + \frac{1}{2\sigma_{ij}^2 r^2} \right] \frac{2}{\sqrt{\pi}} \int_0^{\sigma_{ij}r} e^{-x^2} dx \right] + c, \quad (5)$$

where

$$\frac{1}{\tau_{kij}^2} = \frac{1}{\gamma_k^2} + \frac{1}{\sigma_{ij}^2}. \quad (6)$$

By considering the strengths of the various interactions depending on the center-of-mass (c.m.) momentum of the interaction quarks, the potentials should be revised by the momentum-dependent factors. Then, the Coulomb term, the contact, tensor, vector spin-orbit, and scalar spin-orbit potentials should be modified according to

$$\begin{aligned} \tilde{G}(r) &\rightarrow \left(1 + \frac{p^2}{E_1 E_2} \right)^{1/2} \tilde{G}(r) \left(1 + \frac{p^2}{E_1 E_2} \right)^{1/2}, \\ \tilde{V}_i(r) &\rightarrow \left(\frac{m_1 m_2}{E_1 E_2} \right)^{1/2+\epsilon_i} \tilde{V}_i(r) \left(\frac{m_1 m_2}{E_1 E_2} \right)^{1/2+\epsilon_i}, \end{aligned} \quad (7)$$

where ϵ_i is the parameter of each term, and i corresponds to the contact (c), tensor (t), vector spin-orbit [$\text{so}(v)$], and scalar spin-orbit [$\text{so}(s)$] terms [6].

Now, the effective potential of the Hamiltonian is expressed as

$$G_{\text{eff}}(r) = \left(1 + \frac{p^2}{E_1 E_2}\right)^{1/2} \tilde{G}(r) \left(1 + \frac{p^2}{E_1 E_2}\right)^{1/2} + \left[\frac{\mathbf{S}_1 \cdot \mathbf{L}}{2m_1^2} \frac{1}{r} \frac{\partial \tilde{G}_{11}^{\text{so}(v)}}{\partial r} + \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2m_2^2} \frac{1}{r} \frac{\partial \tilde{G}_{22}^{\text{so}(v)}}{\partial r} + \frac{(\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}}{m_1 m_2} \frac{1}{r} \frac{\partial \tilde{G}_{12}^{\text{so}(v)}}{\partial r} \right] \\ + \frac{2\mathbf{S}_1 \cdot \mathbf{S}_2}{3m_1 m_2} \nabla^2 \tilde{G}_{12}^c - \left(\frac{\mathbf{S}_1 \cdot \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{r}} - \frac{1}{3} \mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2} \right) \left(\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) \tilde{G}'_{12}, \quad (8)$$

with

$$S_{\text{eff}}(r) = \tilde{S}(r) - \frac{\mathbf{S}_1 \cdot \mathbf{L}}{2m_1^2} \frac{1}{r} \frac{\partial \tilde{S}_{11}^{\text{so}(s)}}{\partial r} - \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2m_2^2} \frac{1}{r} \frac{\partial \tilde{S}_{22}^{\text{so}(s)}}{\partial r}, \quad (9)$$

where $\mathbf{L}$ and $\mathbf{S}$ are the orbit and spin quantum numbers. The subscripts 1 and 2 in the above expression correspond to the quark and antiquark of a meson.

The modification of the GI model is achieved by introducing the screening effects into the initial GI model, which can be taken into account by making a replacement as

$$br \rightarrow V^{\text{scr}}(r) = \frac{b(1 - e^{\mu r})}{\mu}. \quad (10)$$

With this modification, the final expression for $\tilde{V}^{\text{scr}}(r)$ is shown as [37]

$$\tilde{V}^{\text{scr}}(r) = \frac{b}{\mu r} \left[r + e^{\frac{\mu^2}{4\sigma^2} + \mu r} \frac{\mu + 2r\sigma^2}{2\sigma^2} \left(\frac{1}{\sqrt{\pi}} \int_0^{\frac{\mu + 2r\sigma^2}{2\sigma}} e^{-x^2} dx - \frac{1}{2} \right) - e^{\frac{\mu^2}{4\sigma^2} - \mu r} \frac{\mu - 2r\sigma^2}{2\sigma^2} \left(\frac{1}{\sqrt{\pi}} \int_0^{\frac{\mu - 2r\sigma^2}{2\sigma}} e^{-x^2} dx - \frac{1}{2} \right) \right]. \quad (11)$$

B. The 3P_0 model

The two-body Okubo-Zweig-Iizuka (OZI)-allowed strong decay behaviors are discussed by the 3P_0 model, which was first proposed by L. Micu in 1969 [43] and then developed by A. Le Yaouanc *et al.* in the 1970s [44,45]. At present, it has been developed as an important tool for studying the strong decay properties of hadrons [35,46–50]. In the 3P_0 model, the S matrix of the decay $A \rightarrow BC$ is defined as

$$S = I - 2\pi i \delta(E_f - E_i) T. \quad (12)$$

The transition operator T in the above expression is shown as [22,23,51]

$$T = -3\gamma \sum_m \langle 1m1 - m | 00 \rangle \int d^3\mathbf{p}_3 d^3\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \mathcal{Y}_1^m \left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2} \right) \chi_{1,-m}^{34} \phi_0^{34} \omega_0^{34} b_3^\dagger(\mathbf{p}_3) d_4^\dagger(\mathbf{p}_4), \quad (13)$$

where the indicators 3 and 4 in the above formula correspond to the quark pairs generated by vacuum, and its strength is represented by the dimensionless parameter γ . The vectors $\mathbf{p}_3$ and $\mathbf{p}_4$ are the momenta of the quarks q_3 and $\bar{q}_4$, respectively. $\chi_{1,-m}^{34}$, ϕ_0^{34} , and ω_0^{34} are the spin, flavor, and color wave functions of the $q_3\bar{q}_4$, respectively. The solid harmonic polynomial $\mathcal{Y}_1^m(\mathbf{p}) \equiv |p|^1 Y_1^m(\theta_p, \phi_p)$ represents the momentum-space distribution of $q_3\bar{q}_4$. It should be pointed out that the meson-space wave functions of the initial state A , and the final states BC , are obtained by solving the MGI model in our calculations.

The partial wave amplitude $\mathcal{M}^{LS}(\mathbf{P})$ of the decay $A \rightarrow BC$ can be given by [52]

$$\mathcal{M}^{LS}(\mathbf{P}) = \sum_{\substack{M_{J_B}, M_{J_C}, \\ M_S, M_L}} \langle LM_L SM_S | J_A M_{J_A} \rangle \\ \times \langle J_B M_{J_B} J_C M_{J_C} | SM_S \rangle \\ \times \int d\Omega Y_{LM_L}^* \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\mathbf{P}), \quad (14)$$

where $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\mathbf{P})$ is the helicity amplitude and is defined as

$$\langle BC | T | A \rangle = \delta^3(\mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_C) \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\mathbf{P}). \quad (15)$$

TABLE I. Parameters of the modified Godfrey-Isgur model.

Parameter	Value
m_u	0.22
m_d	0.22
m_s	0.419
b	0.19318 GeV ²
c	-0.249 GeV
σ_0	1.8401 GeV
s	1.554
e_c	-0.168
e_t	0.025
$e_{so}(v)$	-0.035
$e_{so}(s)$	0.05
μ	0.05

Here, $|A\rangle$, $|B\rangle$, and $|C\rangle$ denote the mock meson states defined in Ref. [53].

With the above amplitude, the decay width $\Gamma(A \rightarrow BC)$ can be obtained by

$$\Gamma(A \rightarrow BC) = \frac{\pi |\mathbf{P}|}{4M_A^2} \sum_{LS} |\mathcal{M}^{LS}(\mathbf{P})|^2, \quad (16)$$

where $|\mathbf{P}| = \frac{\sqrt{[M_A^2 - (M_B + M_C)^2][M_A^2 - (M_B - M_C)^2]}}{2M_A}$, and M_A , M_B , and M_C are the masses of the mesons A , B , and C , respectively.

III. RESULTS AND DISCUSSIONS

In this work, only the pure $s\bar{s}$ states are investigated. For strangeonium, we use $\phi(1020)$, $\phi(1680)$, $h_1(1415)$, $f'_2(1525)$, and $\phi_3(1850)$, which can be regarded as good candidates of $s\bar{s}$ states [5], to fit the model parameters.¹ However, these states are not enough to fit all model parameters. So, we only fit a few parameters which have significant impact on the model's predictions, such as b , c , σ , s , and μ . All the parameters involved in the MGI model are listed in Table I. With these parameters, the spectrum and the strong decay properties of strangeonium are tabulated in Tables II–V, respectively. There are discrepancies between our parameters and those used in Ref. [40]. In Ref. [40], the parameters are fitted to the experimental data of the 1^{+-} states, while in this work, the parameters of the MGI model are fitted using established strangeonium

¹It is worth noting that the $h_1(1595)$ shares the same quantum numbers with the $h_1(1415)$, while Ref. [40] regards the $h_1(1595)$ as a candidate for the $2^1P_1 q\bar{q}$ ($q = u, d$), the partner of the $h_1(1900)$. We have reevaluated this assignment by treating the $h_1(1595)$ as the $1^1P_1 s\bar{s}$ state. Our analysis shows that the predicted mass (1460 MeV) of the $1^1P_1 s\bar{s}$ state aligns more closely with the $h_1(1415)$ than with the $h_1(1595)$. Therefore, in this work, we adopt the $h_1(1415)$ as the $1^1P_1 s\bar{s}$ state, consistent with Refs. [5,54].

states [e.g., $\phi(1020)$, $\phi(1680)$, $h_1(1415)$, $f'_2(1525)$, and $\phi_3(1850)$] as benchmarks. Additionally, the exact uncertainties of the predictions are difficult to determine, though they are typically estimated to be around 20%–30% [34,55].

A. S-wave $s\bar{s}$ states

The lowest S-wave vector $s\bar{s}$ state is $\phi(1020)$, which is observed in a bubble chamber experiment at Brookhaven in 1962 [60], which has been confirmed by a large number of experiments. Its mass and width are given by $M = 1019.461 \pm 0.016$ MeV and $\Gamma = 4.249 \pm 0.013$ MeV [56]. The state can be well explained as 1^3S_1 [5]. Our prediction of its mass is 1020 MeV which supports the assignment of $\phi(1020)$ as 1^3S_1 .

For the 2S-wave state $s\bar{s}$, currently there is no experimental data available for the state 2^1S_0 , and our predicted mass is 1542 MeV, as shown in Table II, which is close to the predicted mass 1578 MeV based on the GI model [57] and 1536 MeV based on Regge trajectories [58]. Compared with different model results, the mass range of the state $2^1S_0 s\bar{s}$ should be 1440–1630 MeV, and its main decay mode is $K^*\bar{K}$ with width 147 MeV, as shown in Table III. For the 2S-wave vector meson, $\phi(1680)$ is a good candidate of 2^3S_1 . The state was first discovered in $e^+e^- \rightarrow K_S K^\pm \pi^\mp$ with an observed mass 1680 ± 20 MeV and a width 150 ± 50 MeV [56,61]. Our predicted mass and width can well match the experimental values, especially the mainly decay modes KK , $K^*\bar{K}$, and $\eta\phi$.

By now, there is no observation of the $3^1S_0 s\bar{s}$ state in experiments. As shown in Table II, the predicted range mass is 1970–2144 MeV. Based on the 3P_0 model with wave functions obtained using the MGI model, we predict the width 158 MeV for this state, and the dominant decay modes are $K^*\bar{K}$, $K^*\bar{K}^*$, $K_0^*(1430)\bar{K}$, $K_2^*(1430)\bar{K}$, $\eta f_0(1370)$, and $\eta f_1(1420)$. For the 3^3S_1 state, the predicted mass range is 2092–2250 MeV from various models, as shown in Table II, and our predicted width is 118 MeV. By comparing the mass and decay width, we support assigning $X(2000)$ [$M = 1996 \pm 11 \pm 30$ MeV and $\Gamma = 148 \pm 16 \pm 66$ MeV] as a 3^3S_1 vector meson.

B. 1P-wave $s\bar{s}$ states

The four 1P-wave states are the states with the most abundant experimental information. For the states 1^3P_0 , 1^1P_1 , 1^3P_1 , and 1^3P_2 , our predicted masses are 1340 MeV, 1460 MeV, 1464 MeV, and 1523 MeV, respectively, as shown in Table II.

For the $f_0(1370)$ case, the amplitude analysis of the $K_S K_S$ system produced in the radiative decays of J/ψ gives its mass $M = 1350 \pm 9^{+12}_{-2}$ MeV and width $\Gamma = 231 \pm 21^{+28}_{-48}$ MeV [62]. Recently, the coupled-channel analysis of $\bar{p}p \rightarrow \pi^0\pi^0\eta$, $\pi^0\eta\eta$, and $K^+K^0\pi^0$ at 900 MeV/ c and of $\pi\pi$ scattering data gives its T -matrix pole $(1280.6 \pm 1.6 \pm 47.4) - i(205.2 \pm 1.7 \pm 20.7)$ MeV [63]. The study of

TABLE II. The mass spectrum (in MeV) of the strangeonium.

$n^{2S+1}L_J$	States	RPP [56]	This work	QL [5]	GI [6]	XWZZ [57]	EFG [58]	SIKY [59]	Pang [8]	VFV [54]
1^1S_0			657	797	960	657	743	690		956
1^3S_1	$\phi(1020)$	1019.461 ± 0.016	1020	1017	1020	1009	1038	1020	1030	1020
2^1S_0			1542	1619	1630	1578	1536	1440		1795
2^3S_1	$\phi(1680)$	1680 ± 20	1646	1699	1690	1688	1698	1740	1687	1726
3^1S_0			2022	2144		2125	2085	1970		
3^3S_1	$X(2000)$	$1996 \pm 11 \pm 30$	2092	2198		2204	2119	2250	2147	
1^3P_0	$f_0(1370)$	$1200 \rightarrow 1500$	1340	1373	1360	1355	1420	1180		
1^1P_1	$h_1(1415)$	1409^{+9}_{-8}	1460	1462	1470	1473	1485	1460		1511
1^3P_1	$f_1(1420)$	1426.3 ± 0.9	1464	1492	1480	1480	1464	1430		1508
1^3P_2	$f'_2(1525)$	1517.4 ± 2.5	1523	1513	1530	1539	1529	1480		1556
2^3P_0			1907	1971	1990	1986	1969	1800		
2^1P_1	$X(1911)$	$1911 \pm 6 \pm 14$	1934	1991	2010	2008	2024	2040		1973
2^3P_1			1948	2027	2030	2027	2016	2020		
2^3P_2			1969	2466		2480	2412	2540		
3^3P_0			2290	2434		2444	2364	2280		
3^1P_1	$X(2300)$	$2316 \pm 9 \pm 31$	2301	2435		2449	2398	2490		
3^3P_1			2314	2470		2468	2403	2480		
3^3P_2			2329	2466		2480	2412	2540		
1^3D_1			1832	1809	1880	1883	1845	1750	1869	
1^1D_2	$\eta_2(1870)$	1842 ± 8	1848	1825	1890	1893	1909	1830		1853
1^3D_2			1857	1840	1910	1904	1908	1810		
1^3D_3	$\phi_3(1850)$	1854 ± 7	1853	1822	1900	1897	1950	1830		1875
2^3D_1	$\phi(2170)$	2164 ± 6	2220	2272		2342	2258	2260	2276	
2^1D_2			2220	2282		2336	2321	2340		
2^3D_2			2229	2297		2348	2323	2330		
2^3D_3			2223	2285		2337	2338	2360		

the isoscalar scalar mesons and the scalar glueball from radiative decays of J/ψ gives its T -matrix pole $(1370 \pm 40) - i(195 \pm 20)$ MeV [64]. This information shows that the $f_0(1370)$ mass has a large uncertainty, but its average mass value is about 1200–1500 MeV, which is consistent with our prediction within error bars. Besides this, we have also calculated its strong decay properties in Table III, and the predicted total width is $\Gamma = 307$ MeV, and the dominant decay modes are $K\bar{K}$ and $\eta\eta$, which are in agreement with the experimental measurements. The ratio of the partial widths of two channels is $\Gamma_{\eta\eta}/\Gamma_{K\bar{K}} \approx 0.29$, which is close to the result 0.20 of Ref. [5].

In 2018, the BESIII Collaboration has observed $h_1(1415)$ with mass $1423 \pm 2.1 \pm 7.3$ MeV and width $90.3 \pm 9.8 \pm 17.5$ MeV in the process $J/\psi \rightarrow \eta' K\bar{K}\pi$ [65]. Later, BESIII has reported the $h_1(1415)$ in the partial wave analysis of $J/\psi \rightarrow \gamma\eta'\eta'$, and has determined the mass and width to be $1384 \pm 6^{+9}_{-0}$ MeV and $66 \pm 10^{+12}_{-10}$ MeV [66], respectively. Theoretically, the state $h_1(1415)$ can be explained as a dynamically generated resonance of the pseudoscalar-vector interaction [67,68], a mixed state of a singlet and an octet with mixing angle 29.5° [9], or a pure $s\bar{s}$ state in the constituent quark model [5,54]. Our predicted mass 1460 MeV is larger than the experimental

TABLE III. The decay widths (in MeV) of strangeonium. The data in {braces} indicates that the width is calculated by using the experimental initial-state mass.

$n^{2S+1}L_J$	1^3S_1	2^1S_0	2^3S_1	1^3P_0	1^1P_1	1^3P_1	1^3P_2
$K\bar{K}$	0.8 {0.8}	...	15.9 {15.2}	238.4	...	...	20.9 {20.3}
$K^*\bar{K}$	...	147.3	114.4 {127.7}	...	138.6 {81.6}	278.0 {210.5}	5.9 {5.3}
$\eta\eta$	...	...	...	17.1	...	...	0.7 {0.7}
$\eta\eta'$	...	...	...	...	...	...	0.0007 {0.0003}
$\eta\phi$	...	...	2.4 {4.0}	...	...	...	...
Total	0.8 {0.8}	147.3	132.7 {146.9}	255.5	138.6 {81.6}	278.0 {210.5}	27.5 {26.3}
Exp.	4.249 ± 0.013	...	150 ± 50	$200 \rightarrow 500$	78 ± 11	54.5 ± 2.6	86 ± 5

data but consistent with other theoretical results as shown in Table II. Thus, we adopt two choices of mass as input to calculate the strong decay properties. First, assuming $h_1(1415)$ as a $1^1P_1 s\bar{s}$ state with the predicted mass $M = 1460$ MeV, and the corresponding total decay width is predicted to be $\Gamma = 139$ MeV, which is larger than the average experimental value 78 ± 11 MeV [56]. Second, using the experimental mass $M = 1409$ MeV as input, the corresponding calculated width will now be $\Gamma = 82$ MeV, which is consistent with experimental data. It should be pointed out that with the assignment of the $1^1P_1 s\bar{s}$ state, the $h_1(1415)$ state can only decay into the $K\bar{K}^*$ channel.

There are some different explanations for the $f_1(1420)$, such as a candidate of the $1^3P_1 s\bar{s}$ state, a hybrid state [69], and a $K\bar{K}^*$ molecule state [70]. Besides this, in Ref. [71], by studying the production and decay of the $f_1(1285)$ into $\pi a_0(980)$ and $K^*\bar{K}$, the $f_1(1420)$ is not a genuine resonance. The experimental information of $f_1(1420)$ has not been updated for a long time. The latest experimental mass and width are given by the L3 Collaboration in 2007 with $M = 1434 \pm 5 \pm 5$ MeV, and by the DELPHI Collaboration in 2003 with $M = 1426 \pm 6$ MeV and $\Gamma = 51 \pm 14$ MeV [72]. Considering $f_1(1420)$ as a $1^3P_1 s\bar{s}$ state, we predict its mass $M = 1464$ MeV, which is larger than the experimental value 1426 MeV but is close to other theoretical results shown in Table II. The $K\bar{K}^*$ is the only dominant strong decay mode, which is the same 1^1P_1 state. The width of $f_1(1420)$ is predicted to be $\Gamma = 278$ MeV if the theoretical mass is used as input, and $\Gamma = 210$ MeV by using experimental mass, which are close to other theoretical results [5,73], but larger than the experimental data $\Gamma = 54.5 \pm 2.6$ MeV. However, as discussed in Refs. [5,73], this discrepancy could be due to the nearby $K\bar{K}^*(892)$ threshold effects, which can lead to strong suppression of the resonant width.

The $f'_2(1525)$ is widely regarded as the $1^3P_2 s\bar{s}$ state [5,6,73,74]. Our predicted mass 1523 MeV is consistent with experimental mass, but the predicted width is 30 MeV or 28 MeV, which are less than the experimental value, 86 ± 5 MeV. However, the branching fraction $\mathcal{B}(f'_2(1525) \rightarrow K\bar{K}) \approx 70\%$ is close to the measured value of 88% from RPP [56], and 70% from Ref. [5]. Besides, the branching fraction $\mathcal{B}(f'_2(1525) \rightarrow \eta\eta) \approx 10\%$ is also consistent with the results of Ref. [5]. Furthermore, the ratio $R_{\eta\eta/K\bar{K}} = \Gamma_{\eta\eta}/\Gamma_{K\bar{K}} = 14\%$ is also close to 13% from Ref. [5] and 11.5% from RPP [56]. It should be noticed that, although there is no experimental information available on the channel $f'_2(1525) \rightarrow K\bar{K}^*(892)$, it is also an important decay mode. Its branching fraction is predicted to be 20% in our model and 21% in Ref. [5].

It is noteworthy that the $f'_2(1525)$ and $f_0(1370)$ states may be interpreted as molecular states formed through vector-vector interactions [75–79]. To investigate this possibility, we repeated our analysis by omitting the

$f'_2(1525)$ from the input dataset. We found that the fitted parameters exhibit no substantial changes, and the resulting predictions remain consistent. Future precision measurements of the processes proposed in Refs. [76–79] could help to clarify the nature of these states.

C. 2P-wave $s\bar{s}$ states

We predicted the masses of the four 2P-wave $s\bar{s}$ states, which are 1907 MeV, 1934 MeV, 1948 MeV, and 1969 MeV for 2^3P_0 , 2^1P_1 , 2^3P_1 , and 2^3P_2 , respectively. Due to the screening effects, our predicted results are smaller than the other predictions, as shown in Table II. Comparing the mass information, one can find that the new observed state $h_1(1911)$ ($M = 1911 \pm 6 \pm 14$ MeV) can be a good candidate as an $h_1(2^1P_1) s\bar{s}$ state. Besides this, as shown in Table IV, the predicted total decay width is 112 MeV and 96 MeV, both of which are close to the measured data at $\Gamma = 149 \pm 12 \pm 23$ MeV. This further supports the $h_1(2^1P_1) s\bar{s}$ state assignment. The branching fractions of the dominant channels are $\mathcal{B}(h_1(2^1P_1) \rightarrow K\bar{K}^*) \approx 56\%$ and $\mathcal{B}(h_1(2^1P_1) \rightarrow K^*\bar{K}^*) \approx 36\%$, respectively.

D. 3P-wave $s\bar{s}$ states

Recently, $X(2300)$, with a mass of $2316 \pm 9 \pm 30$ MeV and a width of $89 \pm 15 \pm 26$ MeV, was first reported by the BESIII Collaboration. According to Tables II and IV, $X(2300)$ could be a good candidate for the $3^1P_1 s\bar{s}$ state with predicted mass 2301 MeV and width 112 MeV or 123 MeV. Its dominant decay modes are $K\bar{K}^*$, $K^*\bar{K}^*$, $K^*\bar{K}_{1B}$, and $K\bar{K}_2^*(1430)$ with branching fractions 35%, 12%, 22%, and 26%, respectively.

For the other 3P-wave $s\bar{s}$ states, our predicted masses and widths are $M(3^3P_0) = 2290$ MeV, $M(3^3P_1) = 2314$ MeV, $M(3^3P_2) = 2329$ MeV, $\Gamma(3^3P_0) = 102$ MeV, $\Gamma(3^3P_1) = 113$ MeV, and $\Gamma(3^3P_2) = 82$ MeV. The predicted masses are smaller than the predictions of the other models due to the screening effects. As shown in Table IV, the main decay modes of $3^3P_0 s\bar{s}$ state are $K\bar{K}(25\%)$, $K^*\bar{K}^*(16\%)$, $K^*\bar{K}_{1B}(13\%)$, $K\bar{K}_{1A}(29\%)$, and $K\bar{K}(1460)(14\%)$, and those of $3^3P_1 s\bar{s}$ state are $K\bar{K}^*(27\%)$, $K^*\bar{K}^*(18\%)$, $K^*\bar{K}_{1B}(17\%)$, $K\bar{K}_{1A}(10\%)$, and $K\bar{K}_2^*(1430)(19\%)$. The $3^3P_2 s\bar{s}$ state can mainly decay into $K\bar{K}^*(23\%)$, $K^*\bar{K}^*(17\%)$, $K\bar{K}_{1B}(13\%)$, and $K\bar{K}_2^*(1430)(17\%)$ final states.

E. 1D-wave $s\bar{s}$ states

The $1^3D_1 s\bar{s}$ state has not been established in experiments yet. The predicted mass within the MGI model is 1832 MeV, which is comparable with the mass range 1750–1880 MeV of the various quark models, as shown in Table II. With the wave functions obtained using the MGI model, the strong decay width is predicted to be

TABLE IV. The decay widths (in MeV) of the S -wave and P -wave strangeonium. The data in {braces} indicates that the width is calculated by using experimental initial-state mass.

$n^{2S+1}L_J$	3^1S_0	3^3S_1	2^3P_0	2^1P_1	2^3P_1	2^3P_2	3^3P_0	3^1P_1	3^3P_1	3^3P_2
$K\bar{K}$	...	11.2 {9.9}	51.8	...	...	0.1	24.8	...	...	1.2
$K^*\bar{K}$	49.7	43.3 {25.1}	...	58.7 {55.2}	58.7	15.8	...	39.4 {41.7}	31.3	19.2
$K^*\bar{K}^*$	12.0	0.4 {6.5}	62.2	38.1 {37.0}	44.1	21.1	16.4	13.4 {15.3}	20.2	14.4
$K_0^*(1430)\bar{K}$	54.3	...	...	0.6	0.1	...	...	0.3 {0.5}	0.6	...
$K_0^*(1430)\bar{K}^*$	...	...	...	...	...	...	...	...	...	0.5
$K_{1B}\bar{K}$	...	6.2 {9.0}	35.5	0.8 {0.7}	0.2	31.1	2.5	0.7 {0.7}	2.9	11.3
$K_{1B}\bar{K}^*$	...	...	...	...	...	...	12.6	24.9 {23.0}	19.2	6.1
$K_{1A}\bar{K}$	...	17.2 {6.7}	3.9	0.3 {0.1}	17.6	0.2	30.0	0.8 {0.9}	11.2	3.4
$K_{1A}\bar{K}^*$	...	...	...	...	...	...	...	1.2 {6.0}	5.7	2.2
$K_2^*(1430)\bar{K}$	20.2	24.4 {4.1}	...	3.6	11.4	7.4	...	29.4 {33.1}	21.1	14.3
$K_2^*(1430)\bar{K}^*$	...	...	...	...	...	...	...	...	...	7.6
$\eta\eta$	...	...	1.4	...	...	0.02	0.4	...	...	0.1
$\eta\eta'$	...	...	0.1	...	...	0.1	0.03	...	...	0.1
$\eta'\eta'$	...	...	...	...	...	0.01	0.3	...	...	0.1
$\eta\phi$	...	1.0 {0.3}	...	1.8 {1.5}	...	...	...	0.9 {0.9}	...	...
$\eta'\phi$	...	0.03 {0.01}	...	...	...	...	...	0.5 {0.5}	...	...
$K\bar{K}(1460)$	...	10.8 {1.2}	...	...	...	...	13.9	...	...	1.0
$\eta f_0(1370)$	6.0	...	...	...	0.05	...	...	...	0.1	...
$\eta h_1(1415)$	...	1.1 {0.8}	...	...	...	...	...	...	...	...
$\eta f_1(1420)$	4.9	...	...	...	...	...	...	...	0.1	...
$\eta f_2'(1525)$	...	...	...	...	...	...	...	...	0.2	0.2
Total	147.0	115.6 {63.5}	154.9	103.8 {94.5}	132.1	75.7	101.0	111.6 {122.7}	112.5	81.7
Exp.	...	$148 \pm 16 \pm 66$	...	$149 \pm 12 \pm 23$	...	...	...	$89 \pm 15 \pm 27$	...	...

178 MeV, and the dominant decay modes are $K\bar{K}$, $K^*\bar{K}$, $K^*\bar{K}^*$, $K_{1B}\bar{K}$, and $\eta\phi$.

The $\eta_2(1870)$ was first observed by the Crystal Ball Collaboration in the process $e^+e^- \rightarrow e^+e^-\eta\pi^0\pi^0$ with mass $1881 \pm 32 \pm 40$ MeV and width $221 \pm 92 \pm 44$ MeV [80]. Based on the results of multiple experimental groups, the current average experimental values of mass and width for $\eta_2(1870)$ with quantum numbers $I^G(J^{PC}) = 0^+(2^{--})$ is $M = 1842 \pm 8$ MeV and $\Gamma = 225 \pm 14$ MeV [56], which are consistent with our prediction for the 1^1D_2 state, as discussed in Ref. [46]. Our calculations show that this state mainly decays into $K^*\bar{K}$ and $K^*\bar{K}^*$, which should be helpful to search for this state in experiments.

There is no candidate for the 1^3D_2 state by now. The predicted mass of the 1^3D_2 state is 1857 MeV, close to that of the 1^1D_2 state. However, as shown in Table V, the 1^3D_2 state could decay into the $\eta\phi$ channel that is forbidden for 1^1D_2 due to the charge-parity conservation, which could be used to distinguish these two states in the future.

The $\phi_3(1850)$ state was first found in the process $K^-p \rightarrow K\bar{K}\Lambda$ of a high statistics bubble chamber experiment [81]. The predicted mass, 1853 MeV, and width, 98 MeV, for the 1^3D_3 state are consistent with the experimental data of $M = 1854 \pm 7$ MeV and $\Gamma = 87^{+28}_{-23}$ MeV. Besides, the decay ratio predicted in this work is

$$\frac{\Gamma(K\bar{K}^*)}{\Gamma(K\bar{K})} = 1.35, \quad (17)$$

which is also consistent with the LASS experimental data $0.55^{+0.85}_{-0.45}$ of Ref. [82].

F. 2D-wave $s\bar{s}$ states

For the 2D-wave states, combining our and other theoretical results as listed in Table II, we can estimate the mass ranges of the four 2D-wave states as 2220–2342 MeV, 2220–2340 MeV, 2229–2348 MeV, and 2223–2360 MeV.

The $\phi(2170)$ state was found in 2006 in the process $e^+e^- \rightarrow K^+K^-\pi\pi\gamma$ by the BABAR Collaboration with fitted mass $2175 \pm 10 \pm 15$ MeV and width $58 \pm 16 \pm 20$ MeV [83]. Subsequently, a large amount of experimental works have confirmed the existence of this state. The latest experimental results reported by BESIII in 2023 indicates that its mass is $2178 \pm 20 \pm 5$ MeV and width is $140 \pm 36 \pm 16$ MeV [84], and its internal structure are still in debate [85,86]. Now, the quantum numbers of this state are determined to be $I^G(J^{PC}) = 0^-(1^{--})$. As shown in Tables II and V, our predicted mass and width for the 2^3D_1 $s\bar{s}$ state are in agreement with the experimental data. Our results show that this state can decay into the modes $K\bar{K}(13\%)$, $K^*\bar{K}(8\%)$,

TABLE V. The decay widths (in MeV) of the D -wave strangeonium. The data in {braces} indicates that the width is calculated by using experimental initial-state mass.

$n^{2S+1}L_J$	1^3D_1	1^1D_2	1^3D_2	1^3D_3	2^3D_1	2^1D_2	2^3D_2	2^3D_3
$K\bar{K}$	50.1	...	...	17.1 {17.2}	25.7 {24.2}	...	...	3.1
$K^*\bar{K}$	59.8	92.9 {91.7}	123.3	22.7 {22.9}	15.7 {12.4}	27.2	38.9	0.02
$K^*\bar{K}^*$	5.1	24.2 {21.3}	14.3	55.7 {56.7}	51.6 {40.5}	21.1	26.5	26.9
$K_0^*(1430)\bar{K}$	...	...	...	...	...	6.4	0.2	...
$K_0^*(1430)\bar{K}^*$	...	...	...	...	...	...	...	...
$K_{1B}\bar{K}$	57.0	0.3 {0.2}	0.5	2.2 {2.3}	20.9 {18.1}	0.1	1.8	10.1
$K_{1B}\bar{K}^*$	...	...	...	...	6.2 {7.1}	11.5	12.9	7.1
$K_{1A}\bar{K}$	...	...	...	...	37.4 {17.9}	0.4	20.4	3.2
$K_{1A}\bar{K}^*$	...	...	...	...	...	...	...	...
$K_2^*(1430)\bar{K}$	...	...	...	...	22.2 {17.0}	40.3	27.4	13.4
$K_2^*(1430)\bar{K}^*$	...	...	...	...	...	...	...	...
$\eta\eta$	...	...	...	...	...	...	...	...
$\eta\eta'$	...	...	...	...	...	...	...	...
$\eta\phi$	2.9	...	12	0.2 {0.2}	0.2 {0.1}	...	1.0	0.1
$\eta'\phi$	...	...	...	...	0.1 {0.1}	...	0.1	0.1
$K\bar{K}(1460)$	...	...	...	...	23.9 {23.7}	...	...	3.7
$\eta f_0(1370)$	...	...	...	...	...	0.4	...	...
$\eta h_1(1415)$	...	...	...	...	0.8 {0.5}	...	0.2	0.4
$\eta f_1(1420)$	...	...	...	...	...	0.3	...	...
$\eta f_2'(1525)$	...	...	...	...	...	0.6	...	...
Total	174.9	117.3 {113.2}	144.2	97.9 {99.3}	204.7 {161.7}	108.4	129.3	68.1
Exp.	...	225 ± 14	...	87^{+28}_{-23}	106^{+24}_{-18}	...	...	...

$K^*\bar{K}^*(25\%)$, $K_{1B}\bar{K}(10\%)$, $K_{1B}\bar{K}^*(3\%)$, $K_{1A}\bar{K}(18\%)$, $K_2^*(1430)\bar{K}(11\%)$, $K\bar{K}(1460)(12\%)$, and $\eta h_1(1415)(1\%)$, while the $K^*\bar{K}^*$ mode is highly suppressed, and the decay to $\phi\eta$ and $\phi\eta'$ have been observed, according to the recent BESIII results. Indeed, many studies show that the $\phi(2170)$ state has a molecular nature [87–90].

We have also calculated the masses and the decay properties of the 2^1D_2 , 2^3D_2 , and 2^3D_3 states, which will be helpful in the future experiments. It is shown that the 2^1D_2 state can decay into $K^*\bar{K}$ (25%), $K^*\bar{K}^*$ (19%), $K_0^*(1430)\bar{K}$ (5%), $K_{1B}\bar{K}^*$ (10%), $K_2^*(1430)\bar{K}$ (36%), $\eta f_0(1370)$ (1%), $\eta f_1(1420)$ (1%), and $\eta f_2'(1525)$ (1%). The decay modes for the 2^3D_2 state are $K^*\bar{K}$ (30%), $K^*\bar{K}^*$ (21%), $K_{1B}\bar{K}$ (2%), $K_{1B}\bar{K}^*$ (10%), $K_{1A}\bar{K}$ (15%), $K_2^*(1430)\bar{K}$ (21%), and $\eta\phi$ (2%). For the 2^3D_3 state, it can decay into $K\bar{K}$ (4%), $K^*\bar{K}^*$ (39%), $K_{1B}\bar{K}$ (14%), $K_{1B}\bar{K}^*$ (10%), $K_{1A}\bar{K}$ (4%), $K_2^*(1430)\bar{K}$ (19%), $K\bar{K}(1460)$ (6%), and $\eta h_1(1415)$ (1%). Comparing the proportions of each channel, the decay channel $K\bar{K}$ plays an important role in distinguishing the states 2^3D_1 , 2^3D_3 from 2^1D_2 , 2^3D_2 , because the latter two states are forbidden to decay into $K\bar{K}$. Besides, the proportion of $K\bar{K}$ (13%) for state 2^3D_1 is larger than the one (4%) for the 2^3D_1 state. For 2^1D_2 and 2^3D_2 , our results show that the 2^3D_2 state can easily decay into the mode $K_{1A}\bar{K}$ (15%), which is forbidden for the 2^1D_2 state.

IV. SUMMARY

Inspired by new observed states $h_1(1911)$ and $X(2300)$, we have systematically investigated strangeonium mesons within the modified GI model and the 3P_0 model. In our calculations, meson space wave functions obtained using the MGI potential model are adopted to calculate the strong decay widths.

Our calculations indicate that $h_1(1911)$ can be regarded as a good candidate for $2^1P_1 s\bar{s}$ state with the predicted mass 1934 MeV and total width 106 MeV, which are consistent with the measured data $M = 1911 \pm 6 \pm 14$ MeV and $\Gamma = 149 \pm 12 \pm 23$ MeV. The branching fractions of the dominant decay modes are $\mathcal{B}(h_1(2^1P_1) \rightarrow K\bar{K}^*) \approx 56\%$ and $\mathcal{B}(h_1(2^1P_1) \rightarrow K^*\bar{K}^*) \approx 36\%$. The $X(2300)$ state with mass $2316 \pm 9 \pm 30$ MeV and width $89 \pm 15 \pm 26$ MeV can be well interpreted as a $3^1P_1 s\bar{s}$ state. The experimental information is in good agreement with our calculation of the mass 2301 MeV and width 112 MeV. Furthermore, the predicted dominant decay modes are $K\bar{K}^*$, $K^*\bar{K}^*$, $K^*\bar{K}_{1B}$, and $K\bar{K}_2^*(1430)$.

Furthermore, we have also calculated the spectrum and strong decay properties of S -wave, P -wave, and D -wave $s\bar{s}$ mesons. The possible assignments of $X(2000)$ and $\eta_2(1870)$ as 3^3S_1 and 1^1D_2 are discussed. For the $2D$ -wave state, our results support 2^3D_1 assignment for $\phi(2170)$, which is consistent with the conclusion of Refs. [5,8].

ACKNOWLEDGMENTS

This work is supported by the National Key R&D Program of China (No. 2024YFE0105200), the Natural Science Foundation of Henan under Grants No. 232300421140 and No. 222300420554, and the National Natural Science Foundation of China

under Grants No. 12475086, No. 12447208, and No. 12192263.

DATA AVAILABILITY

The data supporting this study's findings are available within the article.

-
- [1] Pei-Lian Liu, Shuang-Shi Fang, and Xin-Chou Lou, Strange quarkonium states at BESIII, *Chin. Phys. C* **39**, 082001 (2015).
 - [2] Bernhard Ketzer, Boris Grube, and Dmitry Ryabchikov, Light-meson spectroscopy with COMPASS, *Prog. Part. Nucl. Phys.* **113**, 103755 (2020).
 - [3] Nora Brambilla, Simon Eidelman, Christoph Hanhart, Alexey Nefediev, Cheng-Ping Shen, Christopher E. Thomas, Antonio Vairo, and Chang-Zheng Yuan, The XYZ states: Experimental and theoretical status and perspectives, *Phys. Rep.* **873**, 1 (2020).
 - [4] Eef van Beveren and George Rupp, Modern meson spectroscopy: The fundamental role of unitarity, *Prog. Part. Nucl. Phys.* **117**, 103845 (2021).
 - [5] Qi Li, Long-Cheng Gui, Ming-Sheng Liu, Qi-Fang Lü, and Xian-Hui Zhong, Mass spectrum and strong decays of strangeonium in a constituent quark model, *Chin. Phys. C* **45**, 023116 (2021).
 - [6] S. Godfrey and Nathan Isgur, Mesons in a relativized quark model with chromodynamics, *Phys. Rev. D* **32**, 189 (1985).
 - [7] L. Burakovsky and J. Terrance Goldman, Constraint on axial-vector meson mixing angle from nonrelativistic constituent quark model, *Phys. Rev. D* **56**, R1368 (1997).
 - [8] Cheng-Qun Pang, Excited states of ϕ meson, *Phys. Rev. D* **99**, 074015 (2019).
 - [9] Ya Gao, Yateng Zhang, Bo Zheng, Zhen-Hua Zhang, Wenbiao Yan, and Xiaohua Li, Evaluation of the θ_K and the mixing angle θ_{h_1} , [arXiv:1911.06967](https://arxiv.org/abs/1911.06967).
 - [10] Hai-Yang Cheng, Revisiting axial-vector meson mixing, *Phys. Lett. B* **707**, 116 (2012).
 - [11] L. Burakovsky and J. Terrance Goldman, Towards resolution of the enigmas of P wave meson spectroscopy, *Phys. Rev. D* **57**, 2879 (1998).
 - [12] De-Min Li, Bing Ma, and Hong Yu, Regarding the axial-vector mesons, *Eur. Phys. J. A* **26**, 141 (2005).
 - [13] M. Ablikim *et al.*, Study of the decay $J/\psi \rightarrow \phi\pi^0\eta$, *Phys. Rev. D* **110**, 112014 (2024).
 - [14] Medina Ablikim *et al.*, Observation and study of the decay $J/\psi \rightarrow \phi\eta\eta'$, *Phys. Rev. D* **99**, 112008 (2019).
 - [15] Medina Ablikim *et al.*, Observation of an axial-vector state in the study of $\psi(3686) \rightarrow \phi\eta\eta'$ decay, [arXiv:2410.05736](https://arxiv.org/abs/2410.05736).
 - [16] Lowell S. Brown and William I. Weisberger, Remarks on the static potential in quantum chromodynamics, *Phys. Rev. D* **20**, 3239 (1979).
 - [17] E. Eichten and F. Feinberg, Spin dependent forces in QCD, *Phys. Rev. D* **23**, 2724 (1981).
 - [18] S. N. Gupta and S. F. Radford, Quark quark and quark-anti-quark potentials, *Phys. Rev. D* **24**, 2309 (1981).
 - [19] James T. Pantaleone, S. H. Henry Tye, and Yee Jack Ng, Spin splittings in heavy quarkonia, *Phys. Rev. D* **33**, 777 (1986).
 - [20] Olga Lakhina and Eric S. Swanson, A canonical $D_s(2317)?$, *Phys. Lett. B* **650**, 159 (2007).
 - [21] Yu Lu, Muhammad Naeem Anwar, and Bing-Song Zou, Coupled-channel effects for the bottomonium with realistic wave functions, *Phys. Rev. D* **94**, 034021 (2016).
 - [22] J. Ferretti, G. Galata, E. Santopinto, and A. Vassallo, Bottomonium self-energies due to the coupling to the meson-meson continuum, *Phys. Rev. C* **86**, 015204 (2012).
 - [23] J. Ferretti, G. Galatà, and E. Santopinto, Interpretation of the $X(3872)$ as a charmonium state plus an extra component due to the coupling to the meson-meson continuum, *Phys. Rev. C* **88**, 015207 (2013).
 - [24] Pablo G. Ortega, Jorge Segovia, David R. Entem, and Francisco Fernandez, Molecular components in P -wave charmed-strange mesons, *Phys. Rev. D* **94**, 074037 (2016).
 - [25] Wei Hao, Yu Lu, and Bing-Song Zou, Coupled channel effects for the charmed-strange mesons, *Phys. Rev. D* **106**, 074014 (2022).
 - [26] Jing-Jing Yang, Wei Hao, Xiaoyu Wang, De-Min Li, Yu-Xiao Li, and En Wang, The mass spectrum and strong decay properties of the charmed-strange mesons within Godfrey-Isgur model considering the coupled-channel effects, *Eur. Phys. J. C* **83**, 1098 (2023).
 - [27] Wei Hao and Ruilin Zhu, Beauty-charm meson family with coupled channel effects and their strong decays, *Chin. Phys. C* **48**, 123101 (2024).
 - [28] Wei Hao, M. Atif Sultan, and En Wang, Spectrum and decay properties of the charmed mesons involving the coupled channel effects, *Phys. Rev. D* **111**, 094024 (2025).
 - [29] E. Laermann, F. Langhammer, I. Schmitt, and P. M. Zerwas, The interquark potential: $SU(2)$ color gauge theory with fermions, *Phys. Lett. B* **173**, 437 (1986).
 - [30] K. D. Born, E. Laermann, N. Pirch, T. F. Walsh, and P. M. Zerwas, Hadron properties in lattice QCD with dynamical fermions, *Phys. Rev. D* **40**, 1653 (1989).
 - [31] Adi Armoni, Beyond the quenched (or probe brane) approximation in lattice (or holographic) QCD, *Phys. Rev. D* **78**, 065017 (2008).
 - [32] Bai-Qing Li, Ce Meng, and Kuang-Ta Chao, Coupled-channel and screening effects in charmonium spectrum, *Phys. Rev. D* **80**, 014012 (2009).

- [33] Wei Hao, Guan-Ying Wang, En Wang, Guan-Nan Li, and De-Min Li, Canonical interpretation of the $X(4140)$ state within the 3P_0 model, *Eur. Phys. J. C* **80**, 626 (2020).
- [34] Xue-Chao Feng, Wei Hao, and li-Juan Liu, The assignments of the bottom mesons within the screened potential model and 3P_0 model, *Int. J. Mod. Phys. E* **31**, 2250066 (2022).
- [35] Wei Hao, Yu Lu, and En Wang, The assignments of the B_s mesons within the screened potential model and 3P_0 model, *Eur. Phys. J. C* **83**, 520 (2023).
- [36] Qin-Tao Song, Dian-Yong Chen, Xiang Liu, and Takayuki Matsuki, Higher radial and orbital excitations in the charmed meson family, *Phys. Rev. D* **92**, 074011 (2015).
- [37] Qin-Tao Song, Dian-Yong Chen, Xiang Liu, and Takayuki Matsuki, Charmed-strange mesons revisited: Mass spectra and strong decays, *Phys. Rev. D* **91**, 054031 (2015).
- [38] Jun-Zhang Wang, Dian-Yong Chen, Xiang Liu, and Takayuki Matsuki, Constructing J/ψ family with updated data of charmoniumlike Y states, *Phys. Rev. D* **99**, 114003 (2019).
- [39] Jun-Zhang Wang, Zhi-Feng Sun, Xiang Liu, and Takayuki Matsuki, Higher bottomonium zoo, *Eur. Phys. J. C* **78**, 915 (2018).
- [40] Ya-Rong Wang, Xiao-Hai Liu, Cheng-Qun Pang, and Hao Chen, New states $X(1910)$ and $X(2300)$ and higher light excited $J^{PC} = 1^{+-}$ mesons, *Phys. Rev. D* **111**, 054005 (2025).
- [41] Zheng-Ya Li, De-Min Li, En Wang, Wen-Cheng Yan, and Qin-Tao Song, Assignments of the $Y(2040)$, $\rho(1900)$, and $\rho(2150)$ in the quark model, *Phys. Rev. D* **104**, 034013 (2021).
- [42] Xue-Chao Feng, Zheng-Ya Li, De-Min Li, Qin-Tao Song, En Wang, and Wen-Cheng Yan, Mass spectra and decay properties of the higher excited ρ mesons, *Phys. Rev. D* **106**, 076012 (2022).
- [43] L. Micu, Decay rates of meson resonances in a quark model, *Nucl. Phys.* **B10**, 521 (1969).
- [44] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Naive quark pair creation model of strong interaction vertices, *Phys. Rev. D* **8**, 2223 (1973).
- [45] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Naive quark pair creation model and baryon decays, *Phys. Rev. D* **9**, 1415 (1974).
- [46] De-Min Li and En Wang, Canonical interpretation of the $\eta_2(1870)$, *Eur. Phys. J. C* **63**, 297 (2009).
- [47] Ting-Ting Pan, Qi-Fang Lü, En Wang, and De-Min Li, Strong decays of the $X(2500)$ newly observed by the BESIII Collaboration, *Phys. Rev. D* **94**, 054030 (2016).
- [48] Qi-Fang Lü, Ting-Ting Pan, Yan-Yan Wang, En Wang, and De-Min Li, Excited bottom and bottom-strange mesons in the quark model, *Phys. Rev. D* **94**, 074012 (2016).
- [49] Guan-Ying Wang, Shi-Chen Xue, Guan-Nan Li, En Wang, and De-Min Li, Strong decays of the higher isovector scalar mesons, *Phys. Rev. D* **97**, 034030 (2018).
- [50] Shi-Chen Xue, Guan-Ying Wang, Guan-Nan Li, En Wang, and De-Min Li, The possible members of the 5^1S_0 meson nonet, *Eur. Phys. J. C* **78**, 479 (2018).
- [51] J. Ferretti and E. Santopinto, Higher mass bottomonia, *Phys. Rev. D* **90**, 094022 (2014).
- [52] M. Jacob and G. C. Wick, On the general theory of collisions for particles with spin, *Ann. Phys. (N.Y.)* **7**, 404 (1959).
- [53] Cameron Hayne and Nathan Isgur, Beyond the wave function at the origin: Some momentum dependent effects in the nonrelativistic quark model, *Phys. Rev. D* **25**, 1944 (1982).
- [54] J. Vijande, F. Fernandez, and A. Valcarce, Constituent quark model study of the meson spectra, *J. Phys. G* **31**, 481 (2005).
- [55] Stephen Godfrey, K. Moats, and E. S. Swanson, B and B_s meson spectroscopy, *Phys. Rev. D* **94**, 054025 (2016).
- [56] S. Navas *et al.*, Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [57] Li-Ye Xiao, Xin-Zhen Weng, Xian-Hui Zhong, and Shi-Lin Zhu, A possible explanation of the threshold enhancement in the process $e^+e^- \rightarrow \Lambda\bar{\Lambda}$, *Chin. Phys. C* **43**, 113105 (2019).
- [58] D. Ebert, R. N. Faustov, and V. O. Galkin, Mass spectra and Regge trajectories of light mesons in the relativistic quark model, *Phys. Rev. D* **79**, 114029 (2009).
- [59] Shin Ishida and Kenji Yamada, Light quark meson spectrum in the covariant oscillator quark model with one gluon exchange effects, *Phys. Rev. D* **35**, 265 (1987).
- [60] L. Bertanza *et al.*, Possible resonances in the $\Xi\pi$ and $K\bar{K}$ systems, *Phys. Rev. Lett.* **9**, 180 (1962).
- [61] F. Mane, D. Bisello, J. C. Bizot, J. Buon, A. Cordier, and B. Delcourt, Study of $e^+e^- \rightarrow K_S^0 K^\pm \pi^\mp$ in the 1.4-GeV to 2.18-GeV energy range: A new observation of an isoscalar vector meson ϕ' (1.65-GeV), *Phys. Lett. B* **112**, 178 (1982).
- [62] M. Ablikim *et al.*, Amplitude analysis of the $K_S K_S$ system produced in radiative J/ψ decays, *Phys. Rev. D* **98**, 072003 (2018).
- [63] M. Albrecht *et al.*, Coupled channel analysis of $\bar{p}p \rightarrow \pi^0\pi^0\eta$, $\pi^0\eta\eta$ and $K^+K^-\pi^0$ at 900 MeV/c and of $\pi\pi$ -scattering data, *Eur. Phys. J. C* **80**, 453 (2020).
- [64] A. V. Sarantsev, I. Denisenko, U. Thoma, and E. Klempt, Scalar isoscalar mesons and the scalar glueball from radiative J/ψ decays, *Phys. Lett. B* **816**, 136227 (2021).
- [65] Medina Ablikim *et al.*, Observation of $h_1(1380)$ in the $J/\psi \rightarrow \eta' K \bar{K} \pi$ decay, *Phys. Rev. D* **98**, 072005 (2018).
- [66] M. Ablikim *et al.*, Partial wave analysis of $J/\psi \rightarrow \gamma\eta'\eta'$, *Phys. Rev. D* **105**, 072002 (2022).
- [67] Sheng-Juan Jiang, S. Sakai, Wei-Hong Liang, and E. Oset, The χ_{cJ} decay to $\phi K^* \bar{K}$, $\phi h_1(1380)$ testing the nature of axial vector meson resonances, *Phys. Lett. B* **797**, 134831 (2019).
- [68] L. Roca, E. Oset, and J. Singh, Low lying axial-vector mesons as dynamically generated resonances, *Phys. Rev. D* **72**, 014002 (2005).
- [69] Shin Ishida, Masuho Oda, Haruhiko Sawazaki, and Kenji Yamada, Is the $f_1(1420)$ our first hybrid meson?, *Prog. Theor. Phys.* **82**, 119 (1989).
- [70] Ronald S. Longacre, The $E(1420)$ meson as a $K\bar{K}\pi$ molecule, *Phys. Rev. D* **42**, 874 (1990).
- [71] V. R. Debastiani, F. Aceti, Wei-Hong Liang, and E. Oset, Revising the $f_1(1420)$ resonance, *Phys. Rev. D* **95**, 034015 (2017).

- [72] J. Abdallah *et al.*, Measurement of inclusive $f_1(1285)$ and $f_1(1420)$ production in Z decays with the DELPHI detector, *Phys. Lett. B* **569**, 129 (2003).
- [73] T. Barnes, N. Black, and P.R. Page, Strong decays of strange quarkonia, *Phys. Rev. D* **68**, 054014 (2003).
- [74] W. Roberts and B. Silvestre-Brac, Meson decays in a quark model, *Phys. Rev. D* **57**, 1694 (1998).
- [75] L.S. Geng and E. Oset, Vector meson-vector meson interaction in a hidden gauge unitary approach, *Phys. Rev. D* **79**, 074009 (2009).
- [76] Lianrong Dai and Eulogio Oset, Tests on the molecular structure of $f_2(1270)$, $f'_2(1525)$ from $\psi(nS)$ and $\Upsilon(nS)$ decays, *Eur. Phys. J. A* **49**, 130 (2013).
- [77] Ju-Jun Xie and E. Oset, $\bar{B}^0$ and $\bar{B}_s^0$ decays into J/ψ and $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, $K_2^*(1430)$, *Phys. Rev. D* **90**, 094006 (2014).
- [78] H. Nagahiro, L. Roca, E. Oset, and B. S. Zou, Role of meson loops in the $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ decays into $V\gamma$, *Phys. Rev. D* **78**, 014012 (2008).
- [79] Luciano M. Abreu, Lianrong Dai, and Eulogio Oset, J/ψ decay to ϕ, ω, K^{*0} plus $f_0(1370)$, $f_0(1710)$, $K_0^*(1430)$, $f_2(1270)$, $f'_2(1525)$ and $K_2^*(1430)$: Role of the D -wave for tensor production, *Phys. Lett. B* **843**, 137999 (2023).
- [80] K. Karch *et al.*, Analysis of the $\eta\pi^0\pi^0$ final state in photon-photon collisions, *Z. Phys. C* **54**, 33 (1992).
- [81] S. Al-Harrah *et al.*, Observation of a $K\bar{K}$ enhancement at 1.85-GeV in the reaction $K^-P \rightarrow K\bar{K}\Lambda$ at 8.25-GeV/c, *Phys. Lett. B* **101**, 357 (1981).
- [82] D. Aston *et al.*, Spin parity determination of the $\phi_J(1850)$ from K^-p interactions at 11-GeV/c, *Phys. Lett. B* **208**, 324 (1988).
- [83] Bernard Aubert *et al.*, A structure at 2175-MeV in $e^+e^- \rightarrow \phi f_0(980)$ observed via initial-state radiation, *Phys. Rev. D* **74**, 091103 (2006).
- [84] Medina Ablikim *et al.*, Measurement of $e^+e^- \rightarrow \phi\pi^+\pi^-$ cross sections at center-of-mass energies from 2.00 to 3.08 GeV, *Phys. Rev. D* **108**, 032011 (2023).
- [85] Chen-Guang Zhao, Guan-Ying Wang, Guan-Nan Li, En Wang, and De-Min Li, $\phi(2170)$ production in the process $\gamma p \rightarrow \eta\phi p$, *Phys. Rev. D* **99**, 114014 (2019).
- [86] Yi-Wei Jiang, Wei-Han Tan, Hua-Xing Chen, and Er-Liang Cui, Strong decays of the $\phi(2170)$ as a fully strange tetraquark state, *Symmetry* **16**, 1021 (2024).
- [87] Brenda B. Malabarba, K. P. Khemchandani, and A. Martinez Torres, $\phi(2170)$ decaying to $\phi\eta$ and $\phi\eta'$, *Phys. Rev. D* **108**, 036010 (2023).
- [88] Brenda B. Malabarba, Xiu-Lei Ren, K. P. Khemchandani, and A. Martinez Torres, Partial decay widths of $\phi(2170)$ to kaonic resonances, *Phys. Rev. D* **103**, 016018 (2021).
- [89] A. Martinez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale, and E. Oset, The $X(2175)$ as a resonant state of the $\phi K\bar{K}$ system, *Phys. Rev. D* **78**, 074031 (2008).
- [90] L. Alvarez-Ruso, J. A. Oller, and J.M. Alarcon, On the $\phi(1020)$ $f_0(980)$ S -wave scattering and the $Y(2175)$ resonance, *Phys. Rev. D* **80**, 054011 (2009).