

**Proper constituent gluon mass as the final piece to construct hybrid mesons**

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In this Letter, we propose that a proper constituent gluon mass  $m_g = 450$  MeV can be applied to identify the hybrids composed of quarks and gluons. By investigating the spectra and decay widths of the light hybrids ( $q\bar{q}g$ ) with  $J^P = 1^{-+}$ , we find the  $\pi_1(1600)$  and  $\eta_1(1855)$  may not be explained as  $1^{-+}$  hybrids, simultaneously, and the  $\eta_1(1855)$  observed by BESIII may not be a hybrid. In addition, we predict an existence of a hybrid  $\eta_1(1640)$ , which can be verified by searching the  $a_1(1260)\pi$  channel. Moreover, we suggest the  $K_1(1270)\bar{K}$  and  $K_1(1270)\pi$  as the golden channels to search for isospin-0 and isospin- $\frac{1}{2}$  hybrids, respectively.

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**Introduction.** As the fundamental theory of strong interaction, the quantum chromodynamics (QCD) reveals all the possible interactions between quarks, antiquarks, and gluons. Thus, theoretically, it can present explanations on nearly all the phenomena in strong interaction fields. However, due to the complex mathematical structure of this theory, direct calculations that started from the first principle are usually too difficult to carry on. As a result, various phenomenological models obeying the basic requirements of QCD are successfully proposed, which not only stand for our understandings on the strong interactions, but also were adopted to reveal the inner structures of the new hadronic structures (see review papers [1–8] for more details).

Among these QCD-based phenomenological models, the potential model is one of the most successful proposals, where the quark model, one-gluon exchange potential that is directly derived from QCD Lagrangian, and a phenomenological description on color confinement are combined together. After considering different modifications, such as Goldstone boson exchange [9–13], hidden local symmetry [14,15], scalar meson exchange [16–18], semirelativistic or relativistic [19–22] unquenched effects [23–28], etc., a global description on the spectra of traditional hadrons (mesons and baryons) can be actually well obtained. Especially, for the low-lying states, their masses can be very nicely reproduced [9–11,15].

However, with the rapid development of experiments, there appears a large number of hadronic states that cannot be contained in the traditional quark model, although their existence is permitted by QCD. Among these states, one kind of the convincing ones might be those with exotic quantum numbers; for example, three  $J^{PC} = 1^{-+}$  states  $\pi_1(1400/1600)$  [29–42],  $\pi_1(2015)$  [43,44], and a new isoscalar state  $\eta_1(1855)$  that was observed by the BESIII Collaboration in the  $J/\psi \rightarrow \gamma\eta\eta'$  process [45,46]. The classification of them naturally becomes an issue in the community. Currently, there are three main opinions for their interpretations: one is the tetraquark state [47–51], the other is hybrid [52–61], or a mixture of tetraquark and hybrid [62].

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Obviously, from the view of the constituent quark model, to build and complete the hybrid mass spectra, the mass of the constituent gluon is the key factor. Actually, studies on the effective mass of gluons have already been carried out [63–87]. Whereas, in Refs. [85–87], authors proposed that the gluons can be massive via the Schwinger mechanism, i.e., the nonperturbative effect of the gauge field will bring a correction into the gluon propagator and result in a mass term, and they gave that  $m_g \approx 450$  MeV. Thus, in this Letter, after adding the constituent gluon mass into the potential model and taking the same parameters given by meson spectra [9–11], we recalculate the spectra of low-lying hybrids in the light sector with quantum number  $J^{PC} = 1^{-+}$  by treating the hybrid as a three-body system, and the results show a very consistent behavior as previous theoretical works, especially for the ground states [53,74,88–90]. Furthermore, we calculate their two-body strong decays at leading order, and the results can also match the corresponding ones in Refs. [54,57], which show that a proper constituent gluon mass may be the final piece to construct a hybrid.

*Model setup.* After taking the gluon as a constituent, our treatment on the hybrid then becomes a three-body problem, whose coupling scheme can be given by Fig. 1.

We adopt the Gauss expansion method [92] for the spectra calculations, which is often used to study few-body systems. The relevant Hamiltonian can be constructed as

$$\hat{H} = \frac{\hat{p}_{q,\bar{q}}^2}{2\mu_{q,\bar{q}}} + \frac{\hat{p}_{q\bar{q},g}^2}{2\mu_{q\bar{q},g}} + \hat{V}_{q\bar{q}} + \hat{V}_{qg} + \hat{V}_{\bar{q}g}, \quad (1)$$

with  $\hat{p}_{a,b}$  and  $\mu_{a,b}$  being the relative momentum and reduced mass between  $a$  and  $b$  clusters, respectively. For the interaction between  $q\bar{q}$ , the potential keeps the same as the chiral constituent quark model [9,12,13], i.e.,

$$V_{q\bar{q}}(\mathbf{r}) = V_{q\bar{q}}^{\text{CON}}(\mathbf{r}) + V_{q\bar{q}}^{\text{OGE}}(\mathbf{r}) + V_{q\bar{q}}^{\text{GBE}}(\mathbf{r}), \quad (2)$$

where  $V_{q\bar{q}}^{\text{CON}}$ ,  $V_{q\bar{q}}^{\text{OGE}}$ , and  $V_{q\bar{q}}^{\text{GBE}}$  denote the confinement, one-gluon exchange (OGE), and Goldstone boson exchange (GBE), respectively [9,12,13]. While the interaction

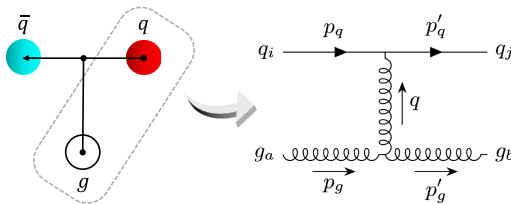


FIG. 1. The coupling scheme of hybrid (left) and the interaction between quark and gluon (right). Here, on the left, the T-type solid line means that the quark and antiquark couple first to form a  $(q\bar{q})$  cluster, then the gluon couples to this cluster to form the final hybrid.

between (anti)quark and gluon contains the confinement and one-gluon exchange (right panel in Fig. 1) only as

$$V_{qg}(\mathbf{r}) = V_{qg}^{\text{CON}}(\mathbf{r}) + V_{qg}^{\text{OGE}}(\mathbf{r}), \quad (3)$$

$$V_{qg}^{\text{CON}}(\mathbf{r}) = V_{q\bar{q}}^{\text{CON}}(\mathbf{r}) \quad \text{with} \quad \lambda_c \cdot \lambda_c^* \rightarrow i\lambda^d \cdot \mathbf{f}^d, \quad (4)$$

$$V_{\bar{q}g}(\mathbf{r}) = V_{qg}(\mathbf{r}) \quad \text{with} \quad \lambda_c \rightarrow -\lambda_c^*, \quad (5)$$

where  $\lambda$  denotes the Gell-Mann matrices, and  $\mathbf{f}$  are eight  $8 \times 8$  matrices, whose matrix elements are just the anti-symmetric structure constants of the SU(3) group, i.e.,  $(f_c)_{ab} = f_{cab}$ .  $V_{qg}^{\text{OGE}}(\mathbf{r})$  is derived directly from QCD Lagrangian. By adopting the nonrelativistic reduction in addition with Fourier transformation on the  $t$ -channel scattering amplitude between quark and gluon (right panel in Fig. 1) [93,94], its expression can be explicitly written in coordinate representation as

$$V_{qg}(\mathbf{r}) = \frac{\alpha_s}{2} \lambda_c \cdot \mathbf{f}_c \left[ \frac{1}{r} - \left( \frac{2\pi}{3m_g^2} + \frac{\pi}{2m_q^2} \right) \delta(\mathbf{r}) \right] + \frac{1}{2m_g^2 r^3} \left( \mathbf{s}_g \cdot (\mathbf{l}_g + \mathbf{s}_g) - 3 \frac{(\mathbf{s}_g \cdot \mathbf{r})(\mathbf{s}_g \cdot \mathbf{r})}{r^2} \right) - \frac{\mathbf{s}_q \cdot \mathbf{l}_q}{2m_q^2 r^3} - \frac{\mathbf{s}_g \cdot \mathbf{l}_q - \mathbf{s}_q \cdot \mathbf{l}_g}{m_g m_q r^3} - \frac{8\pi \mathbf{s}_g \cdot \mathbf{s}_q \delta(\mathbf{r})}{3m_g m_q} \Big],$$

with  $\alpha_s$  being the effective scale-dependent running coupling constant taking the famous form as in Ref. [9],  $\alpha_s(\mu) = \alpha_0 / \log(\mu^2 + \mu_0^2 / \Lambda_0^2)$ , where  $\mu$  is the reduced mass of two constituents, and the Dirac function is smeared as [9–13]  $\delta(\mathbf{r}) \rightarrow \mu e^{-\mu r / r_0} / (4\pi r_0 r)$ .

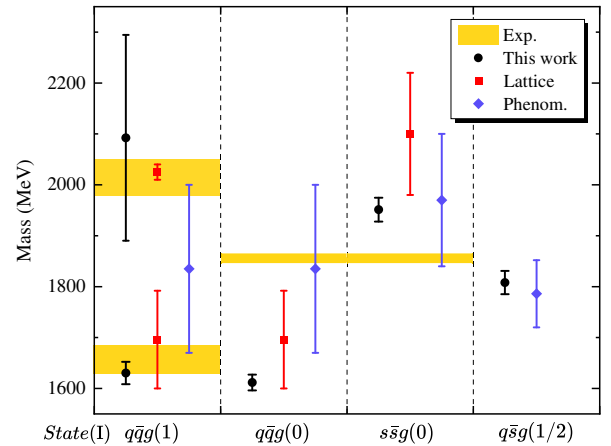


FIG. 2. Mass spectra of low-lying hybrid states with  $I(J^{PC}) = I(1^{-+})$ , where the error bar line means a collection of different results obtained by different models or works. Here, the black lines, yellow bands, red lines, and blue lines correspond to the results in our work, the experimental data from Ref. [91], the lattice QCD results referenced in [72–77,88], and in other phenomenological models [53,78–84,89,90], respectively.

Next, for the wave function of the  $q\bar{q}g$  hybrid, to keep the same with Refs. [54,57,95], its construction procedure is explicitly written as

$$\psi_{q\bar{q}g}^J = \left[ \left[ \psi_{q\bar{q}}^{\frac{1}{2}} \psi_{\bar{q}}^{\frac{1}{2}} \right]^{S_{q\bar{q}}} \left[ \left[ \psi_{q\bar{q},g}^{L_{q\bar{q},g}} \psi_g^1 \right]^{J_g} \psi_{q\bar{q}}^{L_{q\bar{q}}} \right]^{L_g} \right]^J \otimes \psi_{q\bar{q}g}^c \otimes \psi_{q\bar{q}g}^f. \quad (6)$$

Here,  $\psi_{q\bar{q}g}^f$  is the flavor wave function, which is nearly the same as the corresponding meson but inserting into one more gluon  $g$ . While  $\psi_{q\bar{q}g}^c$  is the color wave function as [93]  $|\psi_{q\bar{q}g}^c\rangle = \frac{\delta_{ad}(\lambda^a)_{bc}}{\sqrt{8}} \frac{f^{dbc}}{\sqrt{2}} |q^b\rangle \otimes |\bar{q}^c\rangle \otimes |g^d\rangle$ . For the orbital wave functions  $\psi_{q\bar{q},g}^{L_{q\bar{q},g}}$  and  $\psi_{q\bar{q}}^{L_{q\bar{q}}}$ , they are expanded in coordinate representation by a series of Gaussian bases as  $\psi_i^{L_i}(\mathbf{r}) = \sum_{n=1}^{n_{\max}} c_n^{L_i} N_{nL_i} e^{-\nu_n r^2} \mathcal{Y}_{L_i}(\mathbf{r})$ , where  $c_n^{L_i}$  are the coefficients to be determined by variational principle,  $\mathcal{Y}_{L_i}(\mathbf{r})$  denotes the solid spherical harmonics,  $N_{nL_i}$  is the normalization factor written as  $N_{nL_i} = \sqrt{2^{L_i+2} (2\nu_n)^{L_i+3/2} / (\sqrt{\pi} (2L_i+1)!!)}$ , and  $\nu_n = 1/r_n^2$  is parametrized with  $r_0$  and  $r_{\max}$  with  $r_n = r_0 (\frac{r_{\max}}{r_0})^{\frac{n-1}{n_{\max}-1}}$ . In this work, to be the same with meson spectra calculations [9–11], we take  $r_0 = 0.1$  fm,  $r_{\max} = 2$  fm, and  $n_{\max} = 8$ .

Then, for the strong decay of the  $q\bar{q}g$  hybrid, the simplest case is that the quark-antiquark pair generated from the constituent gluon combines with the remaining  $q\bar{q}$  to form the two mesons. Thus, the Hamiltonian is

$$\hat{H}_I = i\sqrt{4\pi\alpha_s} \frac{(\lambda^a)_{bc}}{2} \int d^3\vec{x} \bar{q}^c(\vec{x}) \gamma^\mu q^b(\vec{x}) A_\mu^a(\vec{x}), \quad (7)$$

with  $q^b$ ,  $\bar{q}^c$ , and  $A_\mu^a$  being the quark, antiquark, and gluon fields, respectively. Then, by field expansions and non-relativistic reduction [94,95], the leading order transition operator can be further rewritten as

$$\hat{T} = 3i\sqrt{\pi\alpha_s} (\lambda^a)_{bc} \sum_{s,s',m} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2 d^3\vec{k}}{\sqrt{2m_g} (2\pi)^6} \delta(\vec{p}_1 + \vec{p}_2 - \vec{k}) \times C_{1,m;1,-m}^{0,0} C_{1,-m;1/2,s'}^{1/2,s} d_{s'}^{c\dagger}(\vec{p}_1) b_s^{b\dagger}(\vec{p}_2) a_m^a(\vec{k}), \quad (8)$$

where  $a_m^a$  is the annihilation operator of the gluon, and  $b_s^{b\dagger}$  and  $d_{s'}^{c\dagger}$  are the creation operators of the quark and antiquark, respectively. Once acting it onto the wave functions of initial hybrid  $A$  and final mesons  $B$  and  $C$  to obtain the helicity amplitude  $\mathcal{M}_{A \rightarrow BC}^{\text{spin}}$ , the decay width can be finally calculated as

$$\Gamma_{A \rightarrow BC} = \frac{1}{1 + \delta_{BC}} \frac{p_B E_B E_C}{\pi M_A} \sum_{\text{spin}} \frac{|\mathcal{M}_{A \rightarrow BC}^{\text{spin}}|^2}{2J_A + 1}. \quad (9)$$

See Supplemental Material [96] for a detailed introduction to the interactions and wave functions.

*Numerical results and discussions.* We use the same quantum number configuration as our previous work [57] to present our numerical results, where to form a  $1^{-+}$  hybrid with lowest mass, the constituent gluon should be a transverse electric gluon, and the hybrid should be a gluon-excited state. Translating such configuration into the quantum numbers used in Eq. (6), it means  $S_{q\bar{q}} = 1$ ,  $L_{q\bar{q}} = 0$ ,  $L_{q\bar{q},g} = 1$ ,  $J_g = 1$ ,  $L_g = 1$ , and  $J = 1$ .

Then, we adopt three kinds of confinements to make our calculation, which are in screened form, linear form, and square form, respectively. These confinements actually represent different estimations on the unquenched effect [97,98]. However, different confinements should have the least impact on the ground state but have considerable effect on the excited one. Here, the adopted model parameters are summarized in Table I, which keeps the same values as given in Refs. [9–11], but just adds into the constituent gluon mass  $m_g = 450$  MeV as the final piece. Our spectra on low-lying light hybrids are presented by the black lines in Fig. 2.

As we can see from Fig. 2, for ground states, our results can match very well with lattice QCD and other phenomenological models. Especially, for the ground  $q\bar{q}g$  state with isospin-1, its position is just located at the experimental result of  $\pi_1(1600)$ . While for the ground  $q\bar{q}g$  and  $s\bar{s}g$  states with isospin-0, their masses can also match with other theoretical results, although they are a little far away from the experimental results of  $\eta_1(1855)$ . Actually, such phenomena are natural, since the physical isospin-0 hybrid in the light region is finally a mixture of  $q\bar{q}g$  and  $s\bar{s}g$  as

TABLE I. Parameters of the chiral quark model with three confinement potential forms: screened ( $n = 0$ ) [9], linear ( $n = 1$ ) [10], square ( $n = 2$ ) [11]. The Goldstone boson exchange interaction parameters are consistent. Masses of  $\pi$ ,  $\eta$ ,  $K$  adopt experimental values, while other parameters [ $m_\sigma = 3.42$  fm $^{-1}$ ,  $\Lambda_\pi = \Lambda_\sigma = 4.2$  fm $^{-1}$ ,  $\Lambda_\eta = \Lambda_K = 5.2$  fm $^{-1}$ ,  $\theta_p = -15^\circ$ ,  $g_{ch}^2/(4\pi) = 0.54$ ] are adopted from Ref. [9].

| [Screened, linear, square] |                           |                        |
|----------------------------|---------------------------|------------------------|
| Gluon mass                 | $m_g$ (MeV)               | 450                    |
| Quark masses               | $m_{u,d}$ (MeV)           | 313                    |
|                            | $m_s$ (MeV)               | [555, 525, 536]        |
| Confinement                | $a_c$ (MeV · fm $^{-n}$ ) | [430, 160, 101]        |
|                            | $\Delta$ (MeV)            | [181.1, -131.1, -78.3] |
|                            | $\mu_c$ (fm $^{-1}$ )     | [0.7, ..., ...]        |
|                            | $a_s$                     | 0.777                  |
| OGE                        | $\alpha_0$                | [2.12, 2.65, 3.67]     |
|                            | $\Lambda_0$ (fm $^{-1}$ ) | [0.113, 0.075, 0.033]  |
|                            | $\mu_0$ (MeV)             | 36.976                 |
|                            | $\hat{r}_0$ (MeV · fm)    | 28.17                  |
|                            | $\hat{r}_g$ (MeV · fm)    | 34.5                   |

$$\begin{bmatrix} \eta_1^{\text{low}} \\ \eta_1^{\text{high}} \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} q\bar{q}g \\ s\bar{s}g \end{bmatrix}. \quad (10)$$

Thus, there should exist two states  $\eta_1^{\text{low}}$  and  $\eta_1^{\text{high}}$ , whose masses are located between the masses of pure  $q\bar{q}g$  and  $s\bar{s}g$  hybrids. If we take  $\eta_1^{\text{high}}$  as  $\eta_1(1855)$ , as in our previous work [57], we can determine the mixing angle to be about  $28.8^\circ$ , with which the location of  $\eta_1^{\text{low}}$  state is then around 1640 MeV, which is denoted below as  $\eta_1(1640)$ . Finally, for the ground  $q\bar{s}g$  state, we predict that it is located around 1.8 GeV, which is consistent with Ref. [54].

However, for the first excited state of  $q\bar{q}g$  hybrid with isospin-1, our result exhibits a very large uncertainty. That is because, when treating the hybrid as a three-body problem, there are two possible exciting modes; one is between  $q\bar{q}$ , and the other is between gluon and  $q\bar{q}$  cluster. In addition, in our work, we adopt three kinds of confinements, which will have very different descriptions on the behaviors of excited states. As a result, the uncertainty of the first excited state of  $q\bar{q}g$  hybrid with isospin-1 is very large, although it can contain the location of  $\pi_1(2015)$ .

Nevertheless, it may indeed show that QCD, and the potential model derived from it, is a uniform theory. That means that, as long as we can reproduce the spectra of mesons and baryons, since the coupling constant in QCD is global, a generalization to the spectra of hybrids, at least for ground states, may be convincing. According to our further calculations, this primary conclusion may also be valid for decay widths, and our results on widths are collected in Table II.

As we can see from Table II, we treat the hybrid as a three-body problem and Refs. [54,57] treat it as a quasi-two-body problem, and the model parameters we use are totally different. As we get the similar spectra and adopt the same transition mechanism, most of our results are consistent with each other. Comparing to Ref. [54], the main difference comes from the  $b_1(1235)\pi$  channel of  $\pi_1(1600)$ , which results in the differences of  $K_1(1270)\pi$  and  $K_1(1400)\pi$  channels of  $K_1(1^{--})$ , since  $K_1(1270)$  and  $K_1(1400)$  are mixtures of  $K_1(^1P_1)$  and  $K_1(^3P_1)$  states, and  $b_1(1235)$  is also a  $^1P_1$  state. While comparing to Ref. [57], the main differences come from the  $a_1(1260)\pi$  and  $\pi(1300)\pi$  channels. According to our cross-check, for the  $a_1(1260)\pi$  channel, it is mainly due to the sensitivity on phase space caused by the node of Laguerre polynomials. While for the  $\pi(1300)\pi$  channel, it comes from the wave function of  $\pi(1300)$ , since in Ref. [57], we use just one single Laguerre polynomial to simulate the wave function, which is less accurate than this work in that the wave function of  $\pi(1300)$  is also given by the Gauss expansion method.

As a result, according to our numerical calculations, we keep similar conclusions as in Ref. [57] as, in the current decay mechanism, we cannot explain  $\pi_1(1600)$  and  $\eta_1(1855)$  as  $1^{--}$  hybrids simultaneously, and the  $\eta_1(1855)$  observed by BESIII in  $J/\psi \rightarrow \gamma\eta\eta'$  may not be a hybrid,

TABLE II. Numerical results and comparisons of decay widths (in  $\text{MeV} \times \alpha_s$  unit just as in Refs. [54,57,95]) for  $\pi_1(1600)$ ,  $\eta_1(1640)$ ,  $\eta'_1(1855)$ , and  $K_1(1^{--})$  in screened, linear, and square confinements (abbreviated as [Scr., Lin., Squ.]). For the mixing angle of  $\eta_1(1640)$  and  $\eta_1(1855)$ , as in Ref. [57], we take  $28.8^\circ$  for comparison.

| States          | Channels           | [Screened, linear, square] | [54,57]      |
|-----------------|--------------------|----------------------------|--------------|
| $\pi_1(1600)$   | $b_1(1235)\pi$     | [53.3, 69.7, 88.0]         | [244, 56.6]  |
|                 | $f_1(1285)\pi$     | [9.0, 11.5, 13.6]          | [15, 8.4]    |
|                 | $\rho\pi$          | [1.0, 1.3, 1.4]            | [2, ...]     |
|                 | Total              | [63.3, 82.5, 103.0]        | [261, 65]    |
| $\eta_1(1640)$  | $a_1(1260)\pi$     | [35.5, 45.8, 58.3]         | [55, 29.3]   |
|                 | $\pi(1300)\pi$     | [2.9, 1.5, 0.1]            | [5, 0.4]     |
|                 | Total              | [38.4, 47.3, 58.4]         | [60, 29.7]   |
| $\eta'_1(1855)$ | $a_1(1260)\pi$     | [11.0, 16.6, 24.5]         | [..., 18.1]  |
|                 | $f_1(1285)\eta$    | [6.4, 6.7, 8.6]            | [..., 5.6]   |
|                 | $\pi(1300)\pi$     | [4.3, 3.5, 2.4]            | [..., 1.1]   |
|                 | $K_1(1270)\bar{K}$ | [176.0, 160.8, 228.0]      | [157, 162.5] |
|                 | $K^*\bar{K}$       | [1.0, 1.4, 1.6]            | [2, ...]     |
|                 | Total              | [198.7, 189.0, 265.1]      | [159, 187.3] |
| $K_1(1^{--})$   | $K\pi$             | [0.6, 0.8, 1.3]            | [1, ...]     |
|                 | $K^*\pi$           | [1.3, 1.6, 2.3]            | [3, ...]     |
|                 | $K^*\eta$          | [0.2, 0.2, 0.4]            | [1, ...]     |
|                 | $K_1(1270)\pi$     | [36.5, 46.1, 46.5]         | [106, ...]   |
|                 | $K_1(1400)\pi$     | [0.0, 0.0, 2.2]            | [146, ...]   |
|                 | $h_1(1170)K$       | [6.8, 3.6, 5.8]            | [16, ...]    |
|                 | $K(1460)\pi$       | [1.4, 1.0, 0.5]            | [2, ...]     |
|                 | Total              | [46.8, 53.3, 59.0]         | [275, ...]   |

since under this situation its decay width to  $\eta\eta'$  is nearly zero. Then, to search for an isospin-0 hybrid located around 1855 MeV and an isospin- $\frac{1}{2}$  hybrid located around 1800 MeV, the golden channels may be  $K_1(1270)\bar{K}$  and  $K_1(1270)\pi$ , respectively. Furthermore, there may exist a partner of  $\eta'_1(1855)$ , whose mass is around 1640 MeV, and future experiments can search the  $a_1(1260)\pi$  channel to verify it. As for  $\pi_1(1600)$ , more precise analysis may be needed to see if it is really a broad structure.

In addition, as mentioned before, there already have many previous studies on the effective mass of a gluon. For example, in the 1980s, Cornwall *et al.* had already demonstrated that gluons can acquire a gauge-invariant dynamical mass with their values around  $500 \pm 200$  MeV [68–70]. In Refs. [78–84], the effective gluon mass was given around 0.5–1.0 GeV by the bag model, lattice QCD, and constituent gluon model. Recently, Refs. [85–87] systematically reviewed their works on the emergence of hadron mass and found an effective gluon mass on the order of  $m_g \approx \frac{1}{2}m_p$ , with  $m_p$  being the mass of the proton. Hence, we further use different gluon masses to study how they impact the spectra. The results show that, when the gluon mass is taken from 0.40 to 0.80 GeV, the mass positions of

ground  $1^{-+}$  hybrid mesons vary 1.58–1.90 GeV for the  $q\bar{q}g$  state with  $I = 1$ , 1.57–1.88 GeV for the  $q\bar{q}g$  state with  $I = 0$ , 1.91–2.19 GeV for the  $s\bar{s}g$  state, and 1.75 to 2.02 GeV for the  $q\bar{s}g$  state. The masses of ground  $1^{-+}$  hybrid mesons change less than the constituent gluon mass due to the nonlinear coupling constant  $\alpha_s$ . We also find our results still match the conclusions given by other models, which may prove our opinion in this work that proper constituent gluon mass is possibly the final piece to construct the hybrid meson spectra.

*Summary.* As a unified theory, QCD reveals an image of strong interactions. Especially, the global property of the coupling constants in it makes us believe that, if we can reproduce some necessary parts of the physical phenomena in the strong region, with the fixed parameters, maybe we can have a chance to get a whole picture.

As a trial, we take a look into the spectra of light hybrids with exotic quantum number  $1^{-+}$ , where the hybrid is treated as a three-body system and the gluon becomes a constituent. After taking the mass of the constituent gluon as  $m_g \approx \frac{1}{2}m_p \approx 450$  MeV [85–87], we make a numerical calculation on the spectra at first. The results show that, as long as we add into the constituent gluon mass as the final piece, by just using the same model parameters obtained from meson spectra [9–11], we can immediately get the same conclusion on the light  $1^{-+}$  hybrid spectra, especially for ground states [54,57].

Then, we calculate the partial decay widths of the ground light  $1^{-+}$  hybrids with a transition operator coming from the quark-gluon vertex. It shows that almost all the results are consistent with Refs. [54,57], which, in our view, also reflects the unification of potential model. Based on our numerical results, we find that we still cannot explain

$\pi_1(1600)$  and  $\eta_1(1855)$  as  $1^{-+}$  hybrids simultaneously due to the total width, and the  $\eta_1(1855)$  may not be a hybrid since its decay width to  $\eta\eta'$  at leading order is almost zero. In addition,  $K_1(1270)\bar{K}$  and  $K_1(1270)\pi$  channels may be the golden channels to search for the isospin-0 and isospin- $\frac{1}{2}$  hybrid, respectively. Furthermore, there exists another  $\eta_1(1640)$ , and future experiments such as BESIII can search the  $a_1(1260)\pi$  channel to verify it.

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*Data availability.* The data are not publicly available. The data are available from the authors upon reasonable request.

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