

Study of the molecular properties of the P_c and P_{cs} states

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In the present work, we investigate the molecular properties of the hidden charm pentaquark states P_c and P_{cs} with a coupled-channel framework that combines heavy quark spin symmetry and the local hidden gauge formalism. By solving the Bethe-Salpeter equation with the cutoff method, we obtain the pole trajectories, wave functions, and root-mean-square radii. For the hidden charm system, the full coupled-channel interactions respecting the heavy quark spin symmetry are essential to generate the P_c states, which significantly affect the poles' widths. The dominant bound channels are $\bar{D}\Sigma_c$ and $\bar{D}^*\Sigma_c$, which couple strongly to the lower decay channels. In contrast, for the hidden charm strange system, the full heavy quark spin symmetry treatment is not necessary, where the splitting pseudoscalar meson-baryon (PB) and vector meson-baryon (VB) sectors yield similar results. The main bound channels $\bar{D}\Xi_c$ and $\bar{D}^*\Xi_c$ couple strongly to $\bar{D}_s\Lambda_c$ and $\bar{D}_s^*\Lambda_c$, respectively, but weakly to the lower decay channels, different from the hidden charm case. The trajectories of the pole widths for the loosely bound channels $\bar{D}\Xi'_c$, $\bar{D}^*\Xi'_c$, and $\bar{D}^*\Xi_c^*$ exhibit distinct behaviors. Notably, all the primary bound channels have similar binding energies in the single-channel interactions due to equally attractive potentials. Furthermore, we also calculate the wave functions and root-mean-square radii of the corresponding poles. The wave functions are localized within $0 \sim 6$ fm and vanish fast beyond 4 fm. The root-mean-square radii, evaluated by two consistent methods, typically lie between 0.5 and 2 fm, comparable to the characteristic scale of molecular states. The root-mean-square radii depend on the pole trajectories and differ among the full coupled-channel case, the split PB and VB sectors, and the single-channel interactions.

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I. INTRODUCTION

The study of the internal structure of hadrons represents a cornerstone of modern nuclear and particle physics. For decades, the quark model [1,2] has provided a remarkably successful framework for classifying hadrons, postulating that the mesons are bound states of a quark and an antiquark ($q\bar{q}$), and the baryons are composed of three quarks (qqq). This simple and elegant scheme accounts for the vast majority of the observed hadronic spectrum, leading to the widespread belief that these are the only possible configurations. However, the fundamental theory of the strong interaction, quantum chromodynamics (QCD), does not inherently forbid the existence of more

complex, or “exotic” hadronic states, such as glueballs (bound states of gluons), hybrids (quarks and gluons), and multiquark states like tetraquarks ($qq\bar{q}\bar{q}$) and pentaquarks ($qqqq\bar{q}$). Thus, the search for these exotic states becomes a critical test of QCD in its nonperturbative regime, which catches much attention both in the experiments and theories [3–7].

The hunt for the pentaquark states has a long and controversial history. It was not until 2015 that the field witnessed a definitive breakthrough. In 2015, the LHCb Collaboration reported the unambiguous observation of two hidden charm pentaquark candidates, named $P_c(4380)^+$ and $P_c(4450)^+$, in the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decay [8]. Subsequently, in 2019, with the high-statistics data, the LHCb Collaboration discovered that the state $P_c(4450)^+$ was actually composed of two states with similar masses but different narrow widths, $P_c(4440)^+$ and $P_c(4457)^+$, while also a lower-mass new state $P_c(4312)^+$ was found [9], although there was some evidence supporting the $P_c(4380)^+$ signal with significance less than 5σ [9]. Moreover, in 2021, evidence for a new $P_c(4337)$ state

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was announced by the LHCb Collaboration in the $J/\psi p$ and $J/\psi \bar{p}$ invariant-mass distributions of the $B_s^0 \rightarrow J/\psi p \bar{p}$ decay with a significance around $3.1 \sim 3.7\sigma$ [10]. Extending these studies to the strange sector, in 2020, the LHCb Collaboration reported a new hidden charm pentaquark state $P_{cs}(4459)^0$ containing a strange quark in the $\Xi_b^- \rightarrow J/\psi \Lambda K^-$ decay [11]. In 2022, a narrow state $P_{cs}(4338)^0$ was observed in the $B^- \rightarrow J/\psi \Lambda \bar{p}$ decay [12]. Furthermore, in 2025, both the $P_{cs}(4459)^0$ and $P_{cs}(4338)^0$ were searched for in $\Upsilon(1S, 2S)$ inclusive decays by the Belle and Belle II Collaborations [13], where evidence of the $P_{cs}(4459)^0$ was found with a significance 3.3σ and no significant signal of the $P_{cs}(4338)^0$ was observed. These discoveries have transformed the pentaquark states from a speculative concept into a vibrant experimental field; see more discussions in the forthcoming reviews [4–7, 14–20].

Theoretically, the interpretation of these states has inspired wide debate. The proximity of these pentaquark states' masses to the thresholds of conventional charmed meson-baryon systems, such as the $\bar{D}^{(*)}\Sigma_c^{(*)}$ or $\bar{D}^{(*)}\Xi_c^{(*)}$ channels, led to a natural conclusion that these states are not compact pentaquark bound states in the traditional sense, but rather “molecular” states, analogous to the deuteron, implying that these pentaquark states might be bound states or resonances formed by a charmed baryon and a charmed antimeson through the long-distance strong interactions at the hadronic level [21–42]. This interpretation ties the study of pentaquark states intimately to the dynamics of the strong interaction near threshold and the nature of exotic hadronic molecules [6, 19, 43–50]. Even though this significant threshold proximity provides strong support for the hadronic molecular picture, numerous fundamental questions remain open, such as assigning them as the compact pentaquark states or hadrocharmonium states [51–58]. The nature of the binding mechanism is still under debate, whether it is predominantly driven by the long-range boson exchange, short-range QCD dynamics, or a combination thereof [19, 27, 59–63]. Thus, the molecular picture and compact multiquark states are also key unresolved issues [60, 64]. Furthermore, the existence of some of these pentaquark states were also questionable with the kinematic effects of the triangle singularity [65–72] or the cusp effects [73, 74], which are not supported by a deep learning framework [75]; see more discussions on the triangle singularity in Refs. [76, 77]. More progress and discussions on the theories can be found in the recent reviews [60, 62, 64, 78–83].

In the present work, to shed light on the internal structure of the P_c and P_{cs} states and provide a deeper understanding of their molecular properties, we systematically investigate the strong interactions of the systems $\bar{D}^{(*)}\Sigma_c^{(*)}$, $\bar{D}^{(*)}\Xi_c^{(*)}$ with their coupled channels and the molecular nature of these P_c , P_{cs} states in detail within a coupled-channel interaction approach, where the pole structures, wave

functions, and the radii of these resonances are evaluated. In the next section, we make a brief introduction of our formalism of the coupled-channel interaction approach. Following that, our study results for different cases are shown in detail. Finally, a short conclusion is made at the end.

II. THEORETICAL FRAMEWORK

Within the framework of the coupled-channel approach, the scattering amplitude T can be obtained by solving the on-shell Bethe-Salpeter equation in algebraic matrix form [84],

$$T = [1 - VG]^{-1}V, \quad (1)$$

where G is the diagonal matrix made of the loop functions with the propagator of the intermediate meson-baryon system, and V is the potential matrix for the coupled-channel interactions. In the isospin $I = 1/2$ and spin-parity $J^P = 1/2^-$ sector of the hidden charm system, where the P_c states appear, there are seven coupled channels, $\eta_c N$, $J/\psi N$, $\bar{D}\Lambda_c$, $\bar{D}\Sigma_c$, $\bar{D}^*\Lambda_c$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$. Following Ref. [85], the potential matrix elements V_{ij} for these seven coupled channels are shown in Table I, which took into account the constraint of the heavy quark spin symmetry (HQSS) [86, 87]; see more detail in Ref. [85]. Additionally, for the isospin $I = 1/2$ and spin-parity $J^P = 3/2^-$ sector, there are five coupled channels, $J/\psi N$, $\bar{D}^*\Lambda_c$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$, and $\bar{D}^*\Sigma_c^*$, where corresponding coupled-channel potentials are given in Eq. (31) of Ref. [85], not listed here.

Note that, in Table I, the low-energy constants μ_i (or μ_{ij}), λ_i are not specified by the HQSS; thus, they should be determined by the other models, such as the local hidden gauge (LHG) formalism [88–91], as done in Ref. [85]. Within the LHG framework and using the vector meson exchange mechanism, these low-energy constants after the S -wave projection are given by [85]

$$\begin{aligned} \mu_1 &= 0, & \mu_{23} &= 0, & \lambda_2 &= \mu_3, & \mu_{13} &= -\mu_{12}, \\ \mu_2 &= \frac{1}{4f^2}(k^0 + k'^0), & \mu_3 &= -\frac{1}{4f^2}(k^0 + k'^0), \\ \mu_{12} &= -\sqrt{6} \frac{m_\rho^2}{p_{D^*}^2 - m_{D^*}^2} \frac{1}{4f^2}(k^0 + k'^0), \end{aligned} \quad (2)$$

where $f_\pi = 93$ and $m_\rho = 775$ MeV are taken, k^0 and k'^0 represent the center-of-mass energies of the incoming and outgoing mesons in the transition process $MB \rightarrow M'B'$, respectively, given by $k^0 = \frac{s+m^2-M^2}{2\sqrt{s}}$, with m and M the masses of the meson and baryon in the corresponding channel, and s the total energy square of the system. Additionally, the transfer momentum squared $p_{D^*}^2$ is

TABLE I. Interaction potentials V_{ij} for the seven coupled channels in the $I = 1/2, J^P = 1/2^-$ sector.

$\eta_c N$	$J/\psi N$	$\bar{D}\Lambda_c$	$\bar{D}\Sigma_c$	$\bar{D}^*\Lambda_c$	$\bar{D}^*\Sigma_c$	$\bar{D}^*\Sigma_c^*$
μ_1	0	$\frac{\mu_{12}}{2}$	$\frac{\mu_{13}}{2}$	$\frac{\sqrt{3}\mu_{12}}{2}$	$-\frac{\mu_{13}}{2\sqrt{3}}$	$\sqrt{\frac{2}{3}}\mu_{13}$
0	μ_1	$\frac{\sqrt{3}\mu_{12}}{2}$	$-\frac{\mu_{13}}{2\sqrt{3}}$	$-\frac{\mu_{12}}{2}$	$\frac{5\mu_{13}}{6}$	$\frac{\sqrt{2}\mu_{13}}{3}$
$\frac{\mu_{12}}{2}$	$\frac{\sqrt{3}\mu_{12}}{2}$	μ_2	0	0	$\frac{\mu_{23}}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}\mu_{23}$
$\frac{\mu_{13}}{2}$	$-\frac{\mu_{13}}{2\sqrt{3}}$	0	$\frac{1}{3}(2\lambda_2 + \mu_3)$	$\frac{\mu_{23}}{\sqrt{3}}$	$\frac{2(\lambda_2 - \mu_3)}{3\sqrt{3}}$	$\frac{1}{3}\sqrt{\frac{2}{3}}(\mu_3 - \lambda_2)$
$\frac{\sqrt{3}\mu_{12}}{2}$	$-\frac{\mu_{12}}{2}$	0	$\frac{\mu_{23}}{\sqrt{3}}$	μ_2	$-\frac{2\mu_{23}}{3}$	$\frac{\sqrt{2}\mu_{23}}{3}$
$-\frac{\mu_{13}}{2\sqrt{3}}$	$\frac{5\mu_{13}}{6}$	$\frac{\mu_{23}}{\sqrt{3}}$	$\frac{2(\lambda_2 - \mu_3)}{3\sqrt{3}}$	$-\frac{2\mu_{23}}{3}$	$\frac{1}{9}(2\lambda_2 + 7\mu_3)$	$\frac{1}{9}\sqrt{2}(\mu_3 - \lambda_2)$
$\sqrt{\frac{2}{3}}\mu_{13}$	$\frac{\sqrt{2}\mu_{13}}{3}$	$\sqrt{\frac{2}{3}}\mu_{23}$	$\frac{1}{3}\sqrt{\frac{2}{3}}(\mu_3 - \lambda_2)$	$\frac{\sqrt{2}\mu_{23}}{3}$	$\frac{1}{9}\sqrt{2}(\mu_3 - \lambda_2)$	$\frac{1}{9}(\lambda_2 + 8\mu_3)$

kept for the nondiagonal elements, which is taken as $p_{D^*}^2 \approx m^2 + m'^2 - 2k^0 k'^0$.

As discussed in the last section, to systematically investigate the molecular nature of the P_c and P_{cs} states, we also study the strong interactions of the $\bar{D}^{(*)}\Xi_c^{(*)}$ channels in the hidden charm strange system compared with the ones of the $\bar{D}^{(*)}\Sigma_c^{(*)}$ channels. Note that Ref. [92] extended the framework of Ref. [85] to the charmed and strange sector with the results of Ref. [26]. Thus, as known from Ref. [92], there are nine coupled channels, $\eta_c\Lambda$, $J/\psi\Lambda$, $\bar{D}\Xi_c$, $\bar{D}_s\Lambda_c$, $\bar{D}\Xi'_c$, $\bar{D}^*\Xi_c$, $\bar{D}_s^*\Lambda_c$, $\bar{D}^*\Xi'_c$, and $\bar{D}^*\Xi_c^*$, in the isospin $I = 0$ and spin-parity $J^P = 1/2^-$ sector, of which the interaction potential matrix (V_{ij}) is shown in Table II with the constraint of the HQSS. Furthermore, for the isospin $I = 0$ and spin-parity $J^P = 3/2^-$ sector, the system consists of six coupled channels, $J/\psi\Lambda$, $\bar{D}^*\Xi_c$, $\bar{D}_s^*\Lambda_c$, $\bar{D}^*\Xi'_c$, $\bar{D}\Xi_c$, and $\bar{D}^*\Xi_c^*$, of which the corresponding potentials are given in Eq. (6) of Ref. [92].

Analogously, using the LHG formalism, the derived low-energy constants are determined as [92]

$$\mu_1 = \mu_3 = \mu_{24} = \mu_{34} = 0,$$

$$\mu_2 = \frac{\mu_{23}}{\sqrt{2}} = \mu_4 = \lambda = -\frac{1}{4f^2}(k^0 + k'^0),$$

$$\mu_{12} = -\frac{\mu_{13}}{\sqrt{2}} = \frac{\mu_{14}}{\sqrt{3}} = -\sqrt{\frac{2}{3}}\frac{m_V^2}{m_{D^*}^2}\frac{1}{4f^2}(k^0 + k'^0), \quad (3)$$

where we take $f_\pi = 93$ and $m_V = 800$ MeV, with k^0 and k'^0 as the ones above. It should be mentioned that, in the nondiagonal transition matrix elements involving D^* meson exchange, we introduce a reduction factor $\frac{m_V^2}{m_{D^*}^2}$ to approximately account for the exchange effect. Additionally, since the contribution of single-pion exchange to the potential is relatively small in the S -wave interactions, the pion exchange contribution is neglected in our formalism and, thus, $\mu_{24} = \mu_{34} = 0$.

Furthermore, in Eq. (1), the diagonal matrix G is constructed by the meson-baryon loop functions. Note that, in the prediction works [85,92], the loop functions

TABLE II. Interaction potentials V_{ij} for the nine coupled channels in the $I = 0, J^P = 1/2^-$ sector.

$\eta_c\Lambda$	$J/\psi\Lambda$	$\bar{D}\Xi_c$	$\bar{D}_s\Lambda_c$	$\bar{D}\Xi'_c$	$\bar{D}^*\Xi_c$	$\bar{D}_s^*\Lambda_c$	$\bar{D}^*\Xi'_c$	$\bar{D}^*\Xi_c^*$
μ_1	0	$-\frac{1}{2}\mu_{12}$	$-\frac{1}{2}\mu_{13}$	$\frac{1}{2}\mu_{14}$	$\frac{\sqrt{3}}{2}\mu_{12}$	$\frac{\sqrt{3}}{2}\mu_{13}$	$\frac{1}{2\sqrt{3}}\mu_{14}$	$\sqrt{\frac{2}{3}}\mu_{14}$
0	μ_1	$\frac{\sqrt{3}}{2}\mu_{12}$	$\frac{\sqrt{3}}{2}\mu_{13}$	$\frac{1}{2\sqrt{3}}\mu_{14}$	$\frac{1}{2}\mu_{12}$	$\frac{1}{2}\mu_{13}$	$\frac{5}{6}\mu_{14}$	$-\frac{\sqrt{2}}{3}\mu_{14}$
$-\frac{1}{2}\mu_{12}$	$\frac{\sqrt{3}}{2}\mu_{12}$	μ_2	μ_{23}	0	0	0	$\frac{1}{\sqrt{3}}\mu_{24}$	$-\sqrt{\frac{2}{3}}\mu_{24}$
$-\frac{1}{2}\mu_{13}$	$\frac{\sqrt{3}}{2}\mu_{13}$	μ_{23}	μ_3	0	0	0	$\frac{1}{\sqrt{3}}\mu_{34}$	$-\sqrt{\frac{2}{3}}\mu_{34}$
$\frac{1}{2}\mu_{14}$	$\frac{1}{2\sqrt{3}}\mu_{14}$	0	0	$\frac{1}{3}(2\lambda + \mu_4)$	$\frac{1}{\sqrt{3}}\mu_{24}$	$\frac{1}{\sqrt{3}}\mu_{34}$	$-\frac{2}{3\sqrt{3}}(\lambda - \mu_4)$	$\frac{1}{3}\sqrt{\frac{2}{3}}(\mu_4 - \lambda)$
$\frac{\sqrt{3}}{2}\mu_{12}$	$\frac{1}{2}\mu_{12}$	0	0	$\frac{1}{\sqrt{3}}\mu_{24}$	μ_2	μ_{23}	$\frac{2}{3}\mu_{24}$	$\frac{\sqrt{2}}{3}\mu_{24}$
$\frac{\sqrt{3}}{2}\mu_{13}$	$\frac{1}{2}\mu_{13}$	0	0	$\frac{1}{\sqrt{3}}\mu_{34}$	μ_{23}	μ_3	$\frac{2}{3}\mu_{34}$	$\frac{\sqrt{2}}{3}\mu_{34}$
$\frac{1}{2\sqrt{3}}\mu_{14}$	$\frac{5}{6}\mu_{14}$	$\frac{1}{\sqrt{3}}\mu_{24}$	$\frac{1}{\sqrt{3}}\mu_{34}$	$-\frac{2}{3\sqrt{3}}(\lambda - \mu_4)$	$\frac{2}{3}\mu_{24}$	$\frac{2}{3}\mu_{34}$	$\frac{1}{9}(2\lambda + 7\mu_4)$	$\frac{\sqrt{2}}{9}(\lambda - \mu_4)$
$\sqrt{\frac{2}{3}}\mu_{14}$	$-\frac{\sqrt{2}}{3}\mu_{14}$	$-\sqrt{\frac{2}{3}}\mu_{24}$	$-\sqrt{\frac{2}{3}}\mu_{34}$	$\frac{1}{3}\sqrt{\frac{2}{3}}(\mu_4 - \lambda)$	$\frac{\sqrt{2}}{3}\mu_{24}$	$\frac{\sqrt{2}}{3}\mu_{34}$	$\frac{\sqrt{2}}{9}(\lambda - \mu_4)$	$\frac{1}{9}(\lambda + 8\mu_4)$

took the form of the dimensional regularization scheme [93,94]; see more discussions in Ref. [85]. In the present work, in order to better understand the behaviors between different bound systems, we explore the three-momentum

cutoff method to the loop functions [95], where the analytical expression for the loop function $G_{ll}(s)$ of the l th channel is given by [96]

$$G_{ll}(s) = \frac{2M_l}{16\pi^2 s} \left\{ \sigma \left(\arctan \frac{s + \Delta}{\sigma \lambda_1} + \arctan \frac{s - \Delta}{\sigma \lambda_2} \right) - \left[(s + \Delta) \ln \left(\frac{q_{\max l}}{M_l} (1 + \lambda_1) \right) + (s - \Delta) \ln \left(\frac{q_{\max l}}{m_l} (1 + \lambda_2) \right) \right] \right\}, \quad (4)$$

with the definitions

$$\sigma = \sqrt{-\lambda(s, M_l^2, m_l^2)} = \sqrt{-[s - (M_l + m_l)^2][s - (M_l - m_l)^2]},$$

$$\Delta = M_l^2 - m_l^2, \quad \lambda_1 = \sqrt{1 + \frac{M_l^2}{q_{\max l}^2}}, \quad \lambda_2 = \sqrt{1 + \frac{m_l^2}{q_{\max l}^2}}, \quad (5)$$

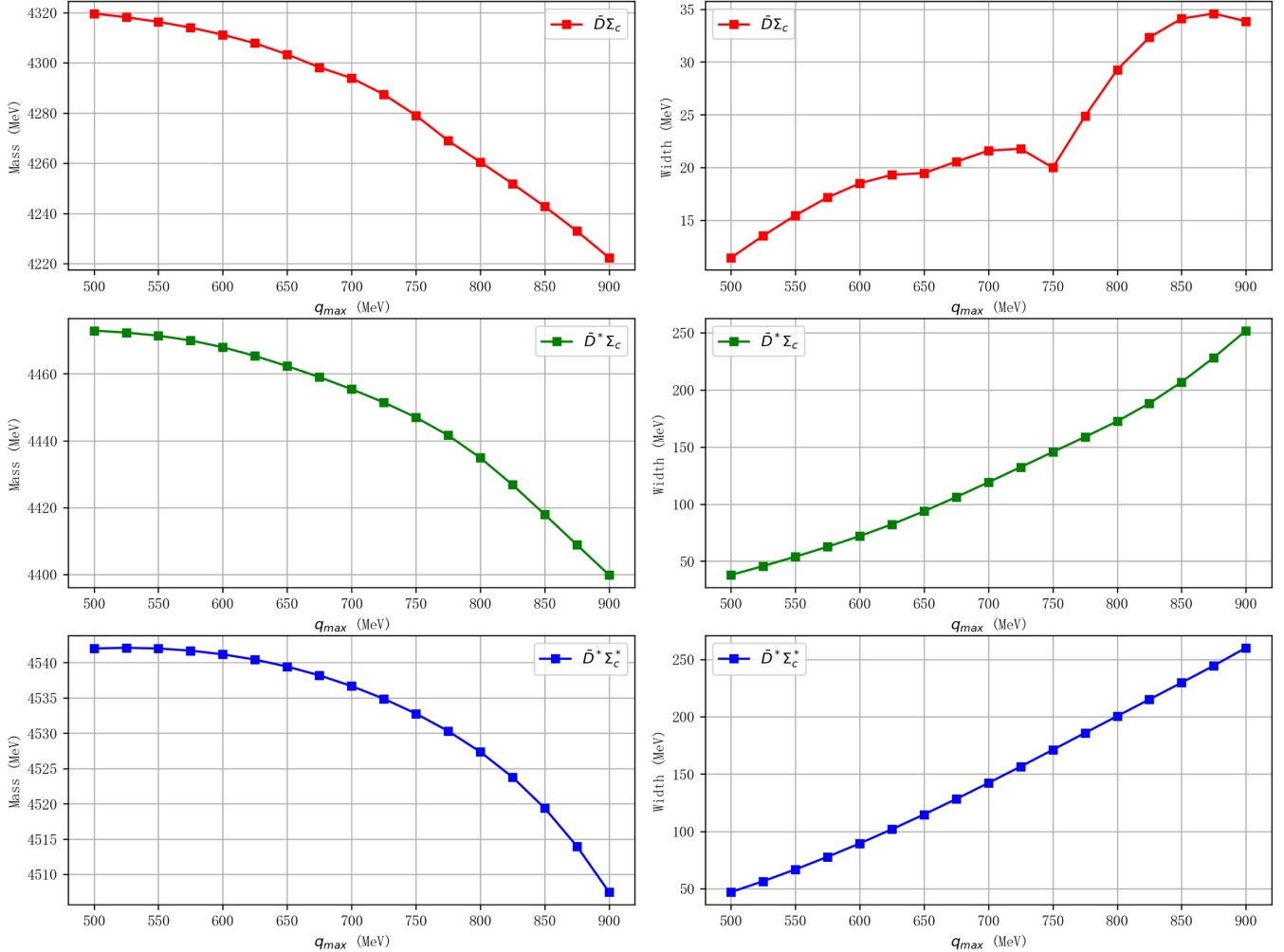


FIG. 1. Mass (left) and width (right) trajectories of the poles in the second Riemann sheets for the $I = 1/2, J^P = 1/2^-$ sector as a function of the cutoff $q_{\max}$ in the seven coupled-channel case.

TABLE III. Pole positions (M, Γ) in the second Riemann sheets for the $I = 1/2, J^P = 1/2^-$ sector with seven coupled channels.

$q_{\max}$ (MeV)	Mass (MeV)	Width (MeV)	Main channel	J^P	Experimental states
600	4311.22	18.50	$\bar{D}\Sigma_c$	$1/2^-$	$P_c(4312)$
700	4293.93	21.60	$\bar{D}\Sigma_c$	$1/2^-$	
800	4260.47	29.26	$\bar{D}\Sigma_c$	$1/2^-$	
600	4467.93	71.95	$\bar{D}^*\Sigma_c$	$1/2^-$	$P_c(4440)$
700	4455.39	119.08	$\bar{D}^*\Sigma_c$	$1/2^-$	
800	4434.89	172.42	$\bar{D}^*\Sigma_c$	$1/2^-$	
600	4541.17	89.41	$\bar{D}^*\Sigma_c^*$	$1/2^-$	$\dots$
700	4536.68	142.14	$\bar{D}^*\Sigma_c^*$	$1/2^-$	
800	4527.35	200.45	$\bar{D}^*\Sigma_c^*$	$1/2^-$	

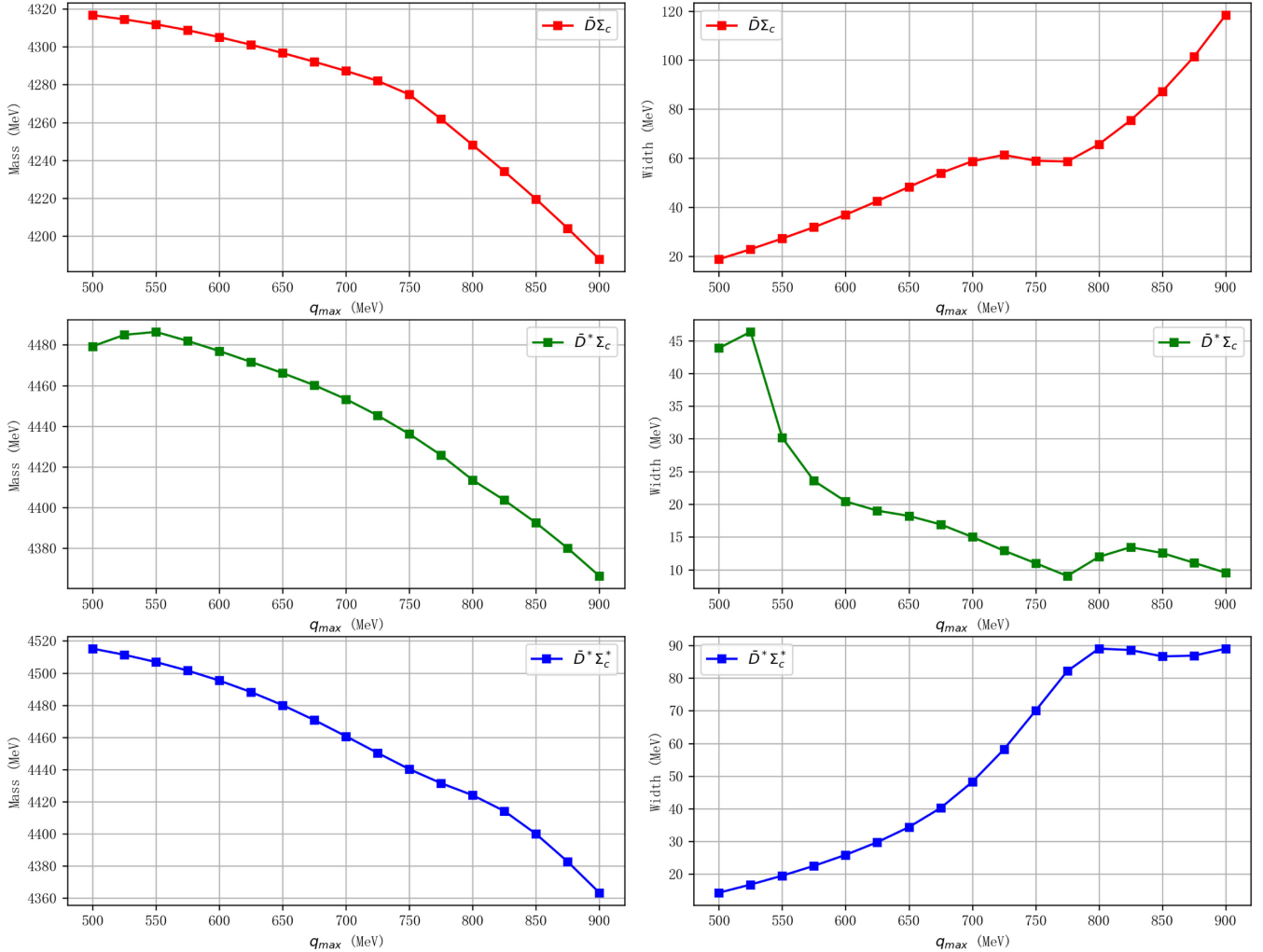
and the cutoff parameter $q_{\max}$ as free parameter; see the discussions later. Additionally, $\lambda(a, b, c)$ is the usual Källen triangle function $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. Using the cutoff scheme, it is also to be consistent with the

TABLE IV. Pole positions (M, Γ) in the second Riemann sheets for the $I = 1/2, J^P = 1/2^-$ sector with the splitting PB and VB sectors.

$q_{\max}$ (MeV)	Mass (MeV)	Width (MeV)	Main channel	Sector	J^P	Experimental states
600	4305.07	36.92	$\bar{D}\Sigma_c$	PB	$1/2^-$	$P_c(4312)$
700	4287.22	58.75	$\bar{D}\Sigma_c$	PB	$1/2^-$	
800	4248.12	65.69	$\bar{D}\Sigma_c$	PB	$1/2^-$	
600	4476.97	20.44	$\bar{D}^*\Sigma_c$	VB	$1/2^-$	$P_c(4440)$
700	4453.30	15.00	$\bar{D}^*\Sigma_c$	VB	$1/2^-$	
800	4413.54	11.96	$\bar{D}^*\Sigma_c$	VB	$1/2^-$	
600	4495.32	25.88	$\bar{D}^*\Sigma_c^*$	VB	$1/2^-$	$\dots$
700	4460.66	48.19	$\bar{D}^*\Sigma_c^*$	VB	$1/2^-$	
800	4424.03	89.01	$\bar{D}^*\Sigma_c^*$	VB	$1/2^-$	

evaluation of the wave functions of the resonances as discussed below.

To search for the poles of the T -matrix corresponding to the resonances by looking for the zeros of the determinant

FIG. 2. Mass (left) and width (right) trajectories of the poles in the second Riemann sheets for the $I = 1/2, J^P = 1/2^-$ sector as a function of the cutoff $q_{\max}$ in the splitting PB and VB sectors.

$\det[I - V \cdot G] = 0$ on the complex energy plane, the loop functions $G_{ll}(s)$ need to be analytically extrapolated from the first Riemann sheet to the second Riemann sheet [84,95]. The pole of the resonance is obtained as $\sqrt{s_{\text{pole}}} = M_{\text{pole}} - i\Gamma_{\text{pole}}/2$, implying that the real part of the pole M_{pole} corresponds to the mass of the resonance, and the imaginary part is one half of its decay width Γ_{pole} . With the analytical continuity condition, it was easy to obtain the relation [95]

$$\begin{aligned} G_{ll}^{(II)}(\sqrt{s} + i\epsilon) &= G_{ll}^{(I)}(\sqrt{s} + i\epsilon) - 2i \operatorname{Im} G_{ll}^{(I)}(\sqrt{s} + i\epsilon) \\ &= G_{ll}^{(I)}(\sqrt{s} + i\epsilon) + 2M_l \frac{i}{4\pi} \frac{p_{\text{cml}}}{\sqrt{s}}, \end{aligned} \quad (6)$$

with the three momentum in the center-of-mass frame

$$\begin{aligned} p_{\text{cml}} &= \frac{\lambda^{1/2}(s, M_l^2, m_l^2)}{2\sqrt{s}} \\ &= \frac{\sqrt{[s - (M_l + m_l)^2][s - (M_l - m_l)^2]}}{2\sqrt{s}}. \end{aligned} \quad (7)$$

To quantitatively characterize the coupling strengths between the poles and other channels, the scattering

amplitude can be rewritten by a Laurent series expansion near the pole s_{pole} on the complex energy plane [97,98],

$$T_{ij} = \frac{g_i g_j}{\sqrt{s} - \sqrt{s_{\text{pole}}}} + \gamma_0 + \gamma_1(s - s_{\text{pole}}) + \dots, \quad (8)$$

where g_i and g_j represent the effective coupling constants of the i th and j th channels, respectively, defined as

$$g_i g_j = \lim_{s \rightarrow s_{\text{pole}}} (\sqrt{s} - \sqrt{s_{\text{pole}}}) T_{ij}. \quad (9)$$

Using the Cauchy residue theorem, the square of the coupling constant g_i^2 can also be obtained by calculating the residues of the T_{ij} around the pole $s = s_{\text{pole}}$ on the complex energy plane [94,99], given by

$$g_i^2 = \frac{1}{2\pi i} \oint T_{ii} ds. \quad (10)$$

To investigate more properties of the resonances, we further study the wave function of the resonance at small distance to learn more about the sources of the resonance. As done in Ref. [100], the wave function $\phi(\vec{r})$ is defined via a Fourier transform,

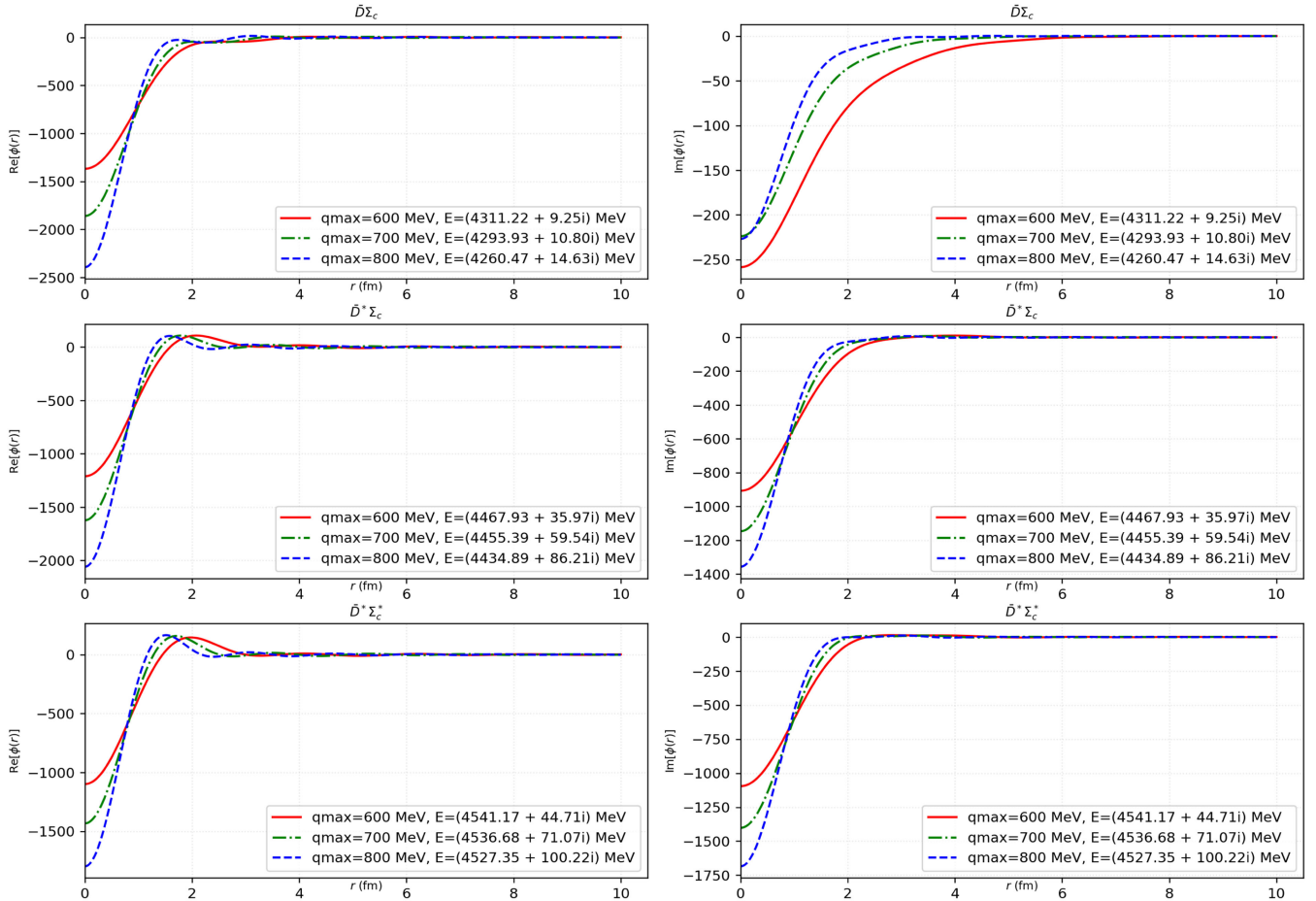


FIG. 3. Real (left) and imaginary (right) parts of the wave functions $\phi(r)$ of corresponding poles for the $I = 1/2, J^P = 1/2^-$ sector with different q_{max} in the seven coupled-channel case.

$$\phi(\vec{r}) = \int_{q_{\max}} \frac{d^3\vec{p}}{(2\pi)^{3/2}} e^{i\vec{p}\cdot\vec{r}} \langle \vec{p} | \psi \rangle. \quad (11)$$

After performing the angular integration over the momentum, the specific expression of the wave function is given by [99]

$$\phi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \frac{4\pi}{r} \frac{1}{C} \int_{q_{\max}} p dp \sin(pr) \times \frac{\Theta(q_{\max} - |\vec{p}|)}{E - \omega_1(\vec{p}) - \omega_2(\vec{p})} \frac{m_V^2}{\vec{p}^2 + m_V^2}, \quad (12)$$

where $\omega_i = (\vec{q}^2 + m_i^2)^{1/2}$, C is the normalization constant, and $E \equiv \sqrt{s_{\text{pole}}}$. Note that, in Eq. (12), we introduce an

additional form factor $f(\vec{q}) = \frac{m_V^2}{\vec{p}^2 + m_V^2}$ to regulate the dynamical behavior at short distances. One can take $m_V = m_\rho$ to account for the light vector meson exchanges in the main bound systems, where the P_c and P_{cs} states appear. Even if this additional form factor is removed, the line shapes of the wave functions will not substantially change. As one can see from the results later, the wave functions will go to zero after a few femtometers, which is, in fact, the confined size for a molecular state, coincided with the results of the radius defined below. With the wave functions obtained, one can evaluate the form factor $F(\vec{q}^2)$ of the corresponding resonance. By the definition, the form factor can be calculated from the wave function [100]

$$F(\vec{q}) = \int d^3\vec{r} \phi(\vec{r}) \phi^*(\vec{r}) e^{-i\vec{q}\cdot\vec{r}} = \int d^3\vec{p} \times \frac{\theta(\Lambda - p) \theta(\Lambda - |\vec{p} - \vec{q}|) f(\vec{q}) f(\vec{p} - \vec{q})}{[E - \omega_1(p) - \omega_2(p)] [E - \omega_1(\vec{p} - \vec{q}) - \omega_2(\vec{p} - \vec{q})]}, \quad (13)$$

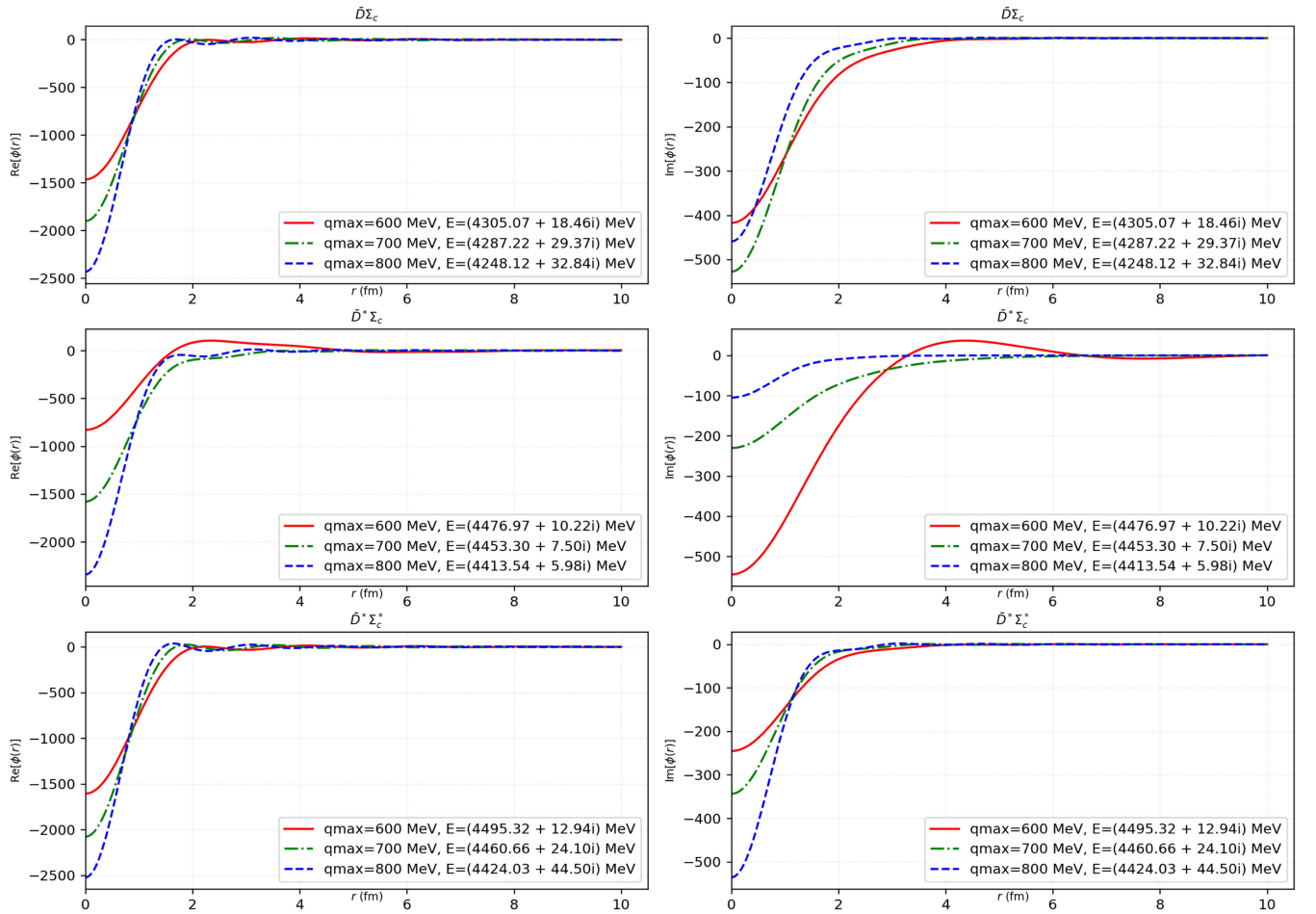


FIG. 4. Real (left) and imaginary (right) parts of the wave functions $\phi(r)$ of corresponding poles for the $I = 1/2, J^P = 1/2^-$ sector with different $q_{\max}$ in the splitting PB and VB sectors.

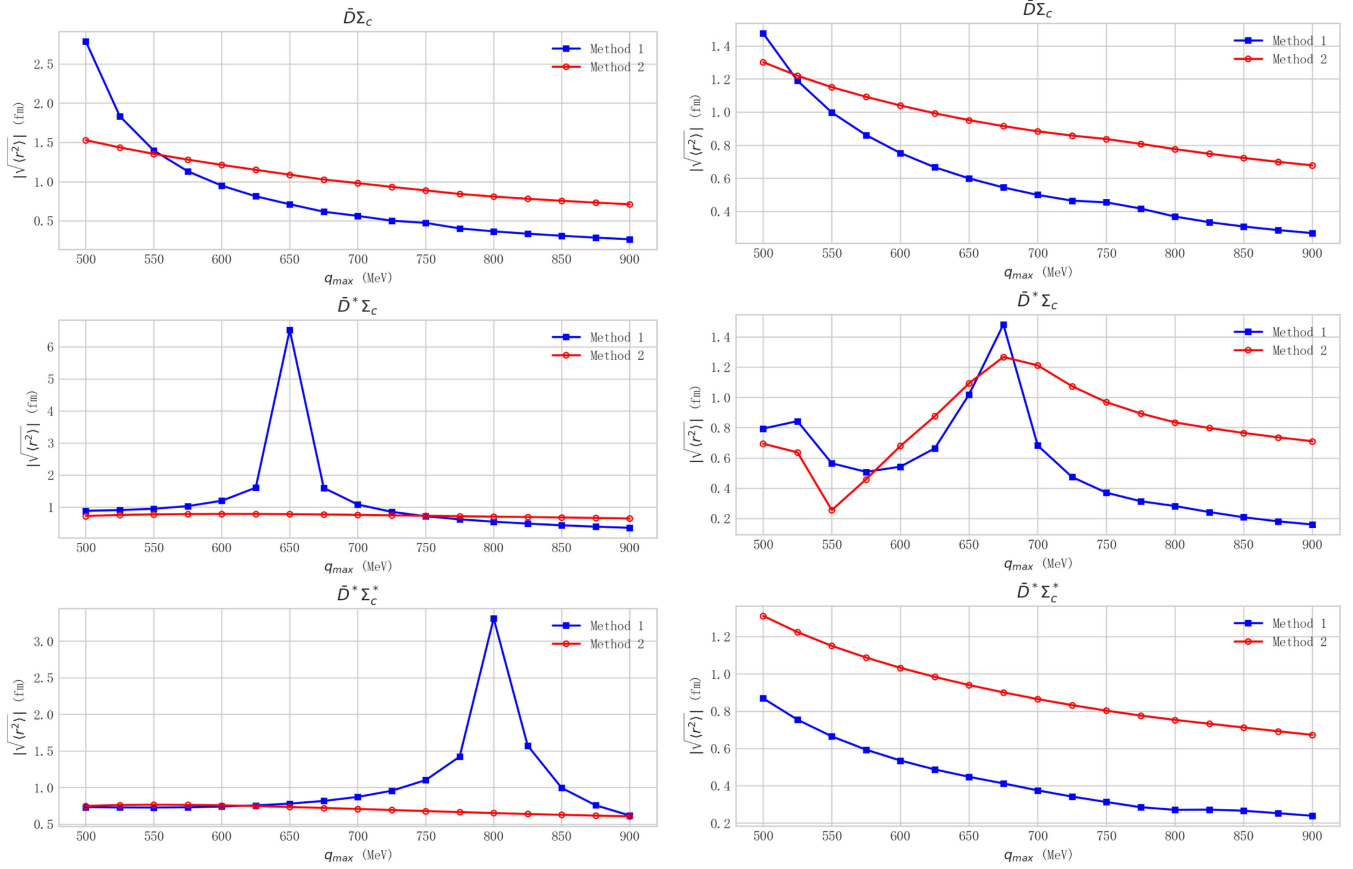


FIG. 5. rms radii of the corresponding poles for the $I = 1/2, J^P = 1/2^-$ sector as a function of the cutoff $q_{\max}$ in the seven coupled-channel case (left) and the splitting PB and VB sectors (right). Results from method 1 (blue) and method 2 (red) are compared.

where Λ is a cutoff parameter, taking $\Lambda = q_{\max}$ as the one in the loop functions for consistency. One should keep in mind that a normalization factor is introduced to keep $F(q = 0) \equiv 1$ in Eq. (13). In the limit of low-momentum transfer ($|\vec{q}| \rightarrow 0$), the form factor can be expanded as a Taylor expansion,

$$F(\vec{q}^2) \approx F(0) - \frac{1}{6} \langle r^2 \rangle \vec{q}^2 + \dots, \quad (14)$$

with the normalization condition $F(0) \equiv 1$. Accordingly, the mean-square radius of the state can be extracted from the derivative of the form factor with respect to $\vec{q}^2$ [101],

$$\langle r^2 \rangle = -6 \left[\frac{dF(q)}{dq^2} \right]_{q^2=0}, \quad (15)$$

where a soft step function should be chosen for the $\theta(\Lambda - p)$ and $\theta(\Lambda - |\vec{p} - \vec{q}|)$ functions to make the form

TABLE V. rms radii $|\sqrt{\langle r^2 \rangle}|$ of the corresponding poles for the $I = 1/2, J^P = 1/2^-$ sector using different methods in the seven coupled-channel case.

Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)
The radii of states calculated with Eq. (15)						
$\bar{D}\Sigma_c$	$1.187 + 0.248i$	1.212	$0.977 + 0.075i$	0.980	$0.809 + 0.027i$	0.809
$\bar{D}^*\Sigma_c$	$0.774 + 0.095i$	0.780	$0.748 + 0.070i$	0.751	$0.693 + 0.057i$	0.696
$\bar{D}^*\Sigma_c^*$	$0.756 - 0.012i$	0.756	$0.706 + 0.015i$	0.706	$0.649 + 0.033i$	0.650
Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)
The radii of states calculated with Eq. (16)						
$\bar{D}\Sigma_c$	$0.948 - 0.003i$	0.948	$0.562 - 0.027i$	0.562	$0.364 - 0.035i$	0.365
$\bar{D}^*\Sigma_c$	$0.005 + 1.191i$	1.191	$1.074 - 0.034i$	1.075	$0.538 - 0.016i$	0.538
$\bar{D}^*\Sigma_c^*$	$0.008 + 0.739i$	0.739	$0.012 + 0.871i$	0.871	$0.028 + 3.311i$	3.311

TABLE VI. rms radii $|\sqrt{\langle r^2 \rangle}|$ of the corresponding poles for the $I = 1/2, J^P = 1/2^-$ sector using different methods in the splitting PB and VB sectors.

Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)
The radii of states calculated with Eq. (15)						
$\bar{D}\Sigma_c$	$1.027 + 0.158i$	1.039	$0.879 + 0.087i$	0.883	$0.775 + 0.035i$	0.776
$\bar{D}^*\Sigma_c$	$0.429 + 0.526i$	0.678	$1.181 + 0.269i$	1.212	$0.834 + 0.017i$	0.834
$\bar{D}^*\Sigma_c^*$	$1.031 + 0.046i$	1.032	$0.864 + 0.021i$	0.865	$0.753 + 0.018i$	0.753
Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)
The radii of states calculated with Eq. (16)						
$\bar{D}\Sigma_c$	$0.752 - 0.005i$	0.752	$0.499 + 0.005i$	0.499	$0.368 - 0.010i$	0.368
$\bar{D}^*\Sigma_c$	$0.192 + 0.506i$	0.541	$0.658 - 0.177i$	0.682	$0.271 - 0.076i$	0.281
$\bar{D}^*\Sigma_c^*$	$0.535 + 0.009i$	0.535	$0.375 - 0.015i$	0.375	$0.270 + 0.005i$	0.271

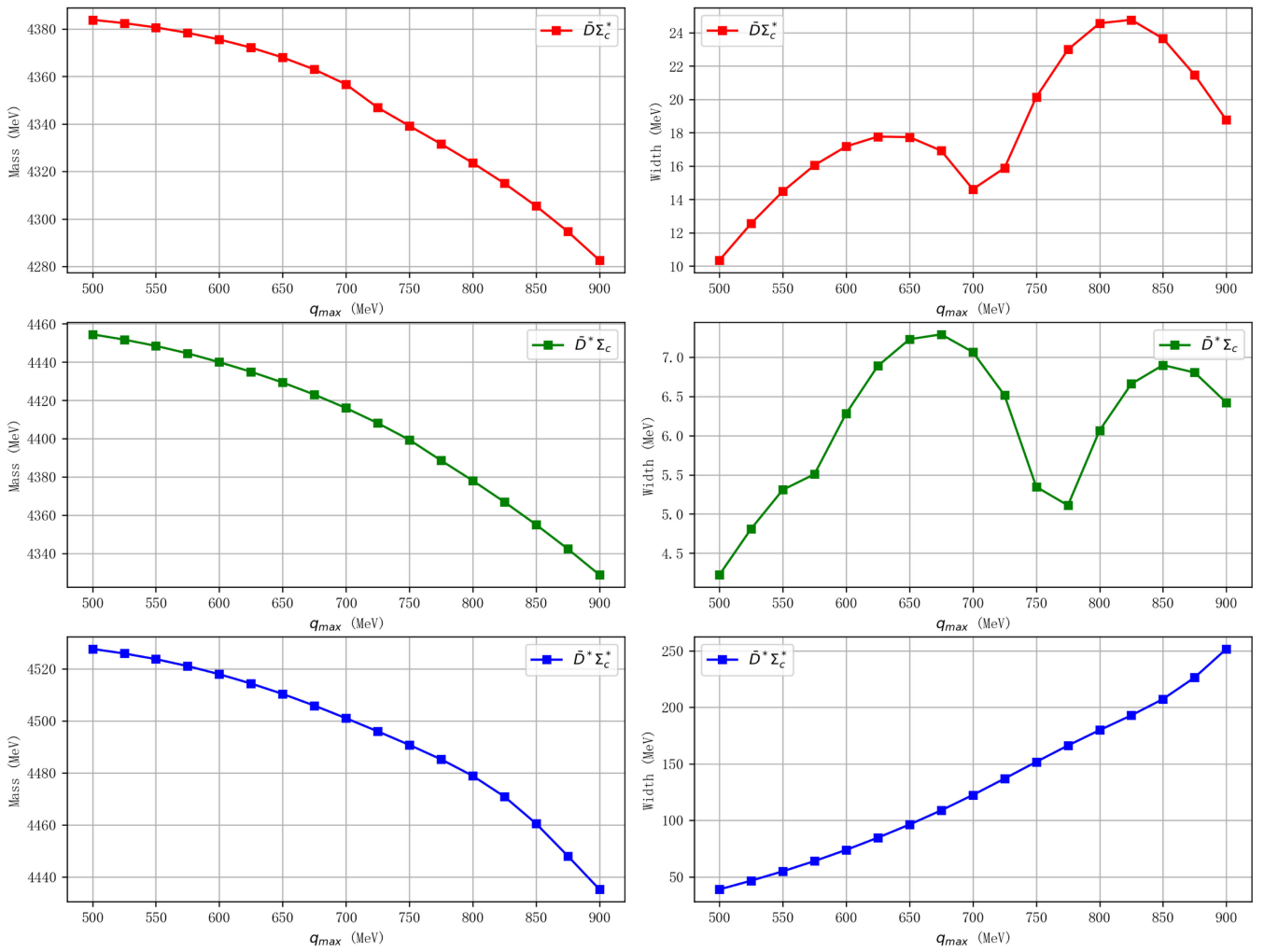
FIG. 6. Mass (left) and width (right) trajectories of the poles in the second Riemann sheets for the $I = 1/2, J^P = 3/2^-$ sector as a function of the cutoff $q_{\max}$ in the five coupled-channel case.

TABLE VII. Pole positions (M, Γ) in the second Riemann sheets for the $I = 1/2, J^P = 3/2^-$ sector with different $q_{\max}$ in the five coupled-channel case.

$q_{\max}$ (MeV)	Mass (MeV)	Width (MeV)	Main channel	J^P	Experimental state
600	4375.58	17.18	$\bar{D}\Sigma_c^*$	$3/2^-$	$P_c(4380)$
700	4356.65	14.61	$\bar{D}\Sigma_c^*$	$3/2^-$	
800	4323.58	24.57	$\bar{D}\Sigma_c^*$	$3/2^-$	
600	4439.97	6.28	$\bar{D}^*\Sigma_c$	$3/2^-$	$P_c(4457)$
700	4416.00	7.07	$\bar{D}^*\Sigma_c$	$3/2^-$	
800	4378.06	6.07	$\bar{D}^*\Sigma_c$	$3/2^-$	
600	4517.93	73.76	$\bar{D}^*\Sigma_c^*$	$3/2^-$	...
700	4501.03	122.34	$\bar{D}^*\Sigma_c^*$	$3/2^-$	...
800	4478.88	179.73	$\bar{D}^*\Sigma_c^*$	$3/2^-$	...

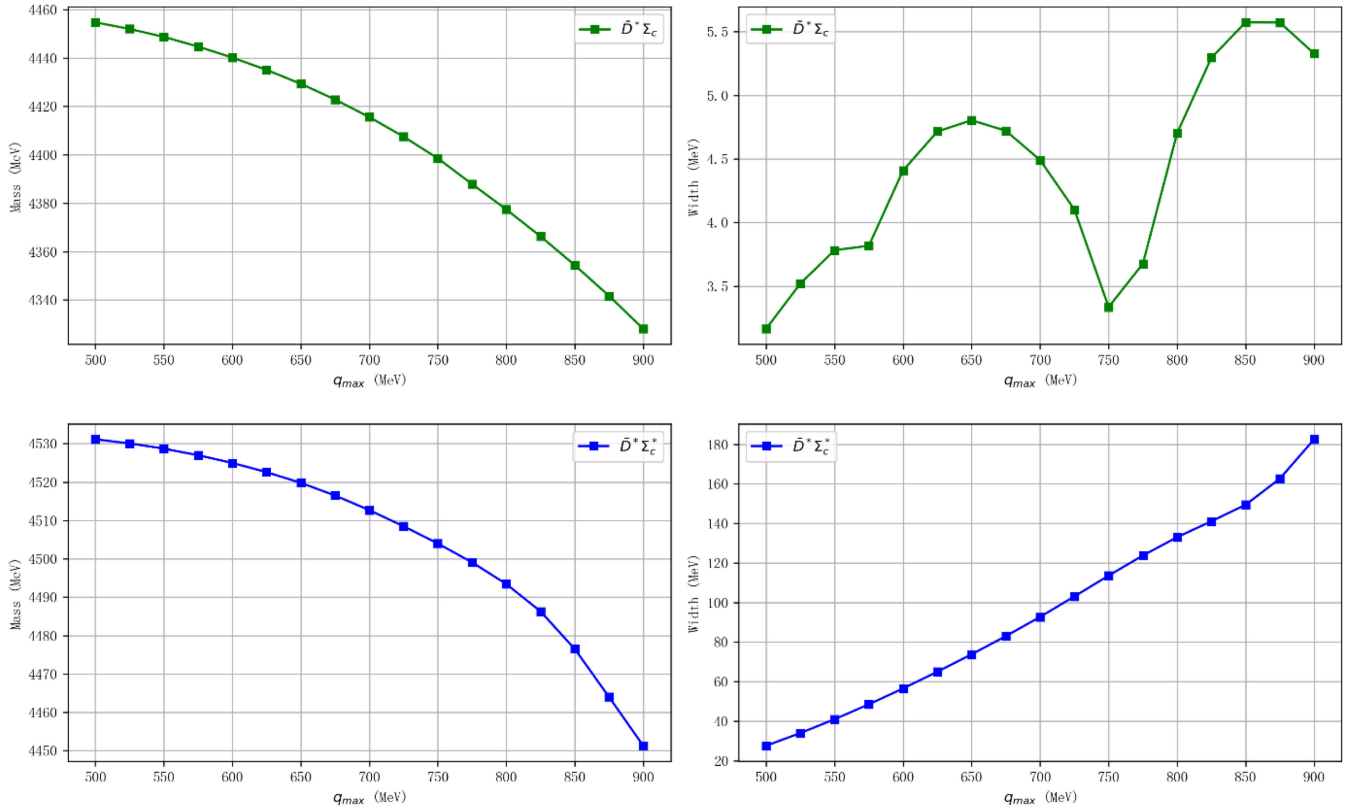
factor converged when $q^2 \rightarrow 0$. On the other hand, for the bound states, the mean-square radius can also be estimated using the derivative of the loop function G with respect to energy and the binding energy $B_{E,i}$ [102],

$$\langle r^2 \rangle_i = \frac{-g_i^2 \left[\frac{dG_i(s)}{d\sqrt{s}} \right]_{\sqrt{s}=\sqrt{s_{\text{pole}}}}}{4\mu_i B_{E,i}}, \quad (16)$$

TABLE VIII. Pole positions (M, Γ) in the second Riemann sheets for the $I = 1/2, J^P = 3/2^-$ sector with different $q_{\max}$ in the splitting VB sector.

$q_{\max}$ (MeV)	Mass (MeV)	Width (MeV)	Main channel	J^P	Sector	Experimental state
600	4440.14	4.41	$\bar{D}^*\Sigma_c$	$3/2^-$	VB	$P_c(4457)$
700	4415.56	4.49	$\bar{D}^*\Sigma_c$	$3/2^-$	VB	
800	4377.40	4.70	$\bar{D}^*\Sigma_c$	$3/2^-$	VB	
600	4524.98	56.54	$\bar{D}^*\Sigma_c^*$	$3/2^-$	VB	...
700	4512.69	92.73	$\bar{D}^*\Sigma_c^*$	$3/2^-$	VB	...
800	4493.45	133.01	$\bar{D}^*\Sigma_c^*$	$3/2^-$	VB	...

where the binding energy is obtained with $B_{E,i} = m_i + M_i - M_B$, the reduced mass $\mu_i = \frac{m_i M_i}{m_i + M_i}$, and g_i is the coupling constant defined above. $\langle r^2 \rangle_i$ is the mean-square distance of the bound state in the i th channel. In this case, the mean-square radius also depends on the binding energy for a certain pole with respect to the threshold. Thus, we have introduced two methods to evaluate the mean-square radius, which should be consistent with each other, in principle. However, when the pole is very close to the threshold, where the binding energy in the denominator becomes zero, $B_{E,i} \rightarrow 0$, and also the part $\frac{dG_i(s)}{d\sqrt{s}} \rightarrow \infty$, Eq. (16) may lead

FIG. 7. Mass (left) and width (right) trajectories of the poles in the second Riemann sheets for the $I = 1/2, J^P = 3/2^-$ sector as a function of the cutoff $q_{\max}$ in the splitting PB and VB sectors.

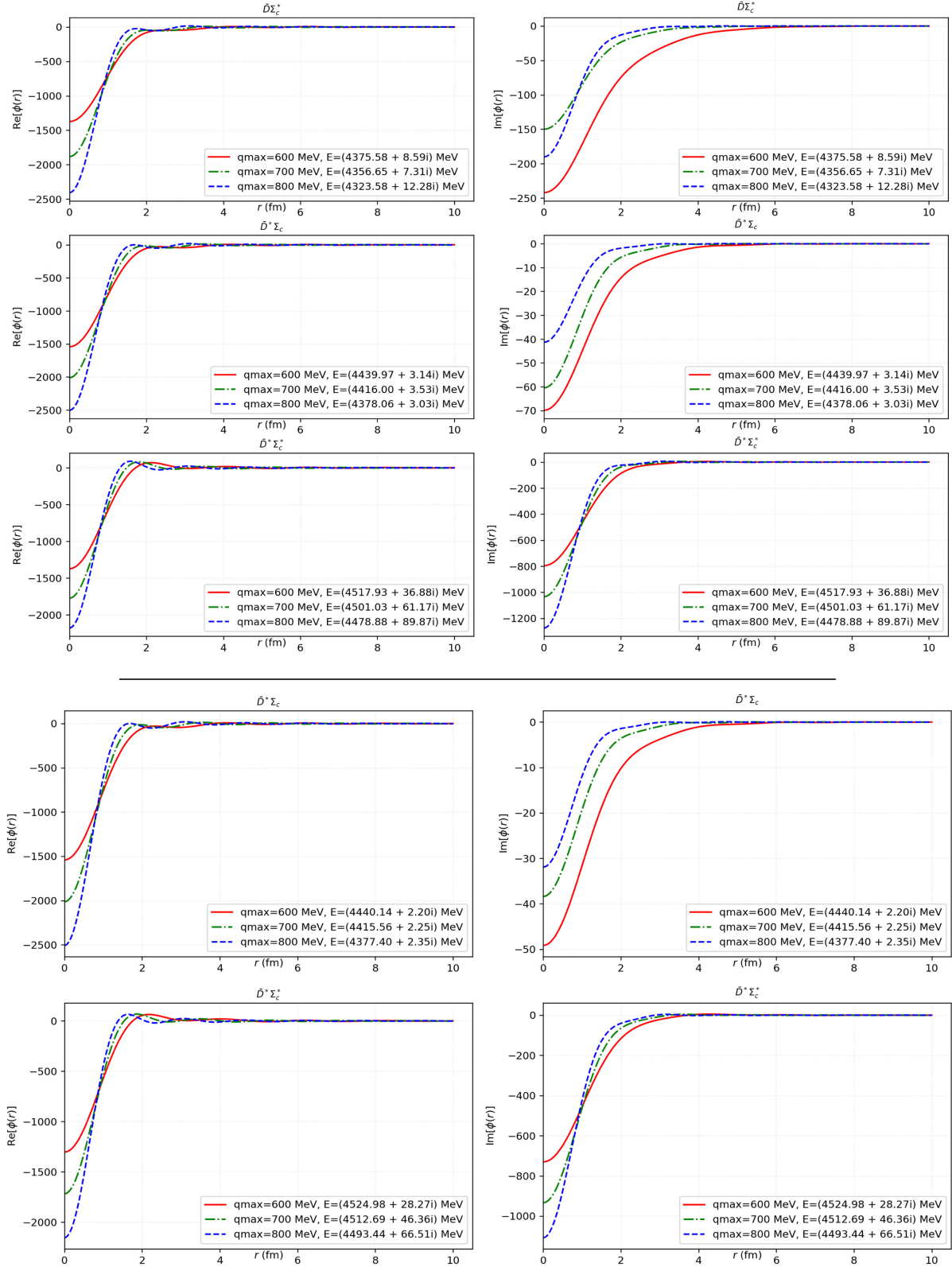


FIG. 8. Real (left) and imaginary (right) parts of the wave functions $\phi(r)$ of corresponding poles for the $I = 1/2, J^P = 3/2^-$ sector with different q_{max} . Results for the five coupled-channel case (top) and the splitting VB sector (bottom) are compared.

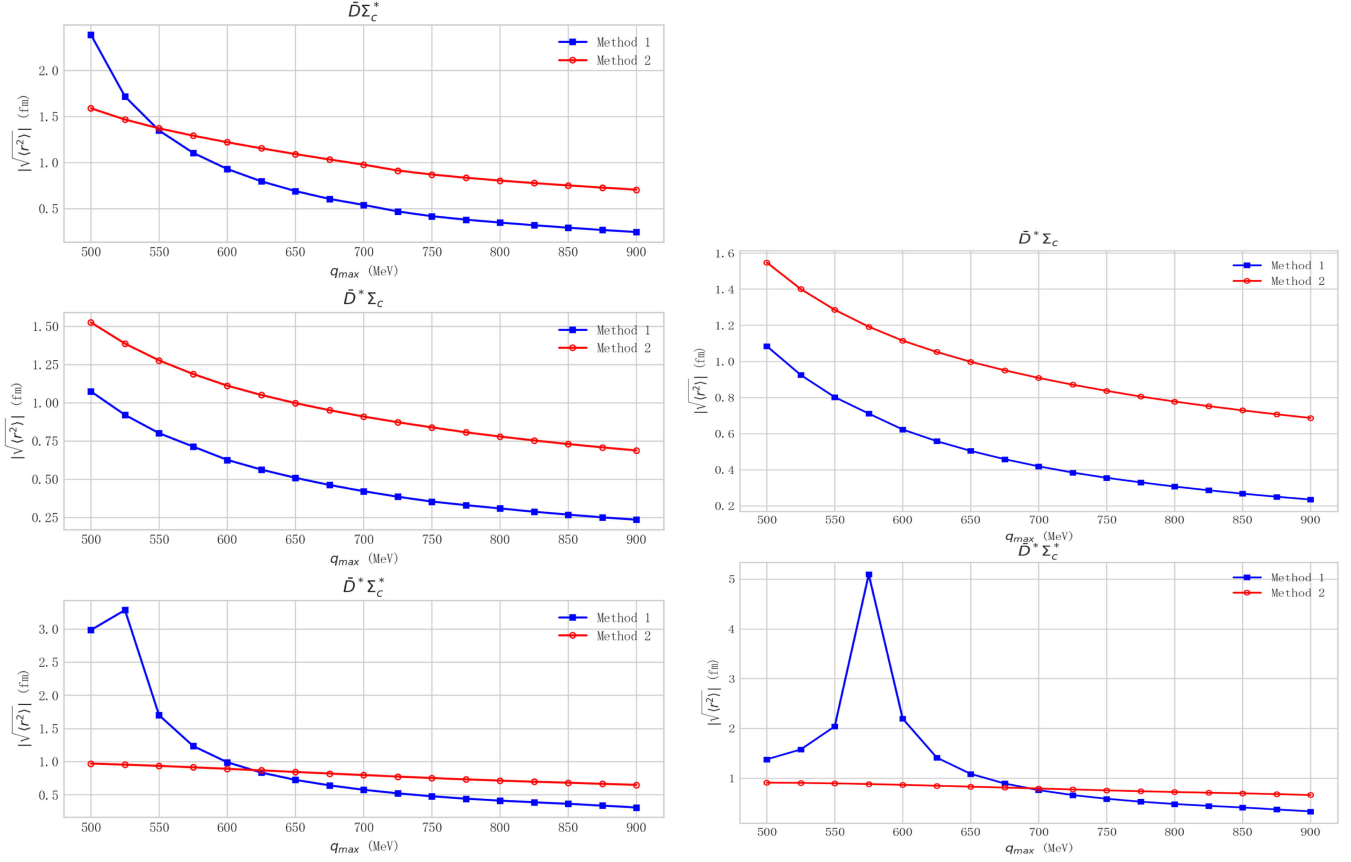


FIG. 9. rms radii of the corresponding poles for the $I = 1/2, J^P = 3/2^-$ sector as a function of the cutoff $q_{\max}$ in the five coupled-channel case (left) and the splitting VB sector (right). Results from method 1 (blue) and method 2 (red) are compared.

to numerical instability. In such cases, the results of Eq. (15) are more stable; see our results later. Moreover, the one of Eq. (15) should also depend on the derivative of the form factor well defined at $q^2 = 0$, where the form factor needs to be normalized to 1 and the soft step function is taken for its evaluation, as discussed above.

III. RESULTS

As discussed in the last section, to better investigate the properties of the bound systems, we take the three-momentum cutoff method to regularize the loop functions, and then obtain the poles of the P_c and P_{cs} states in coupled-channel interactions. Subsequently, we further

TABLE IX. rms radii $|\sqrt{\langle r^2 \rangle}|$ of the corresponding poles for the $I = 1/2, J^P = 3/2^-$ sector using different methods in the five coupled-channel case.

Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)
The radii of states calculated with Eq. (15)						
$\bar{D}\Sigma_c^*$	$1.198 + 0.236i$	1.221	$0.976 + 0.048i$	0.977	$0.804 + 0.022i$	0.805
$\bar{D}^*\Sigma_c$	$1.111 + 0.028i$	1.111	$0.909 + 0.009i$	0.909	$0.778 + 0.003i$	0.778
$\bar{D}^*\Sigma_c^*$	$0.885 + 0.095i$	0.890	$0.794 + 0.056i$	0.796	$0.711 + 0.044i$	0.713
Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)
The radii of states calculated with Eq. (16)						
$\bar{D}\Sigma_c^*$	$0.930 - 0.021i$	0.930	$0.540 - 0.001i$	0.540	$0.347 - 0.035i$	0.349
$\bar{D}^*\Sigma_c$	$0.625 - 0.016i$	0.625	$0.420 - 0.016i$	0.420	$0.307 - 0.012i$	0.307
$\bar{D}^*\Sigma_c^*$	$0.989 - 0.014i$	0.989	$0.574 - 0.024i$	0.575	$0.410 - 0.018i$	0.411

investigate the wave functions and mean-square radii of these states to discuss their internal properties in more detail. To obtain more dynamical information regarding these poles in the coupled-channel interactions, we split the full coupled-channel systems into the pseudoscalar meson-baryon (PB) and vector meson-baryon (VB) subsystems without the HQSS constraint and also compare the results with those of the single-channel interactions. Note that, in the present work, the only free parameter is the cutoff $q_{\max}$. In order to check the properties of different bound systems, we vary the values of $q_{\max}$, which also indicates the uncertainties of the results obtained. In principle, one can determine the value of the cutoff $q_{\max}$ with the experimental data or the mass(es) of the reproduced resonance(s), which is not the case in the present work; see the results and discussions later. Alternatively, one can compare the loop function of the cutoff method having the same result at threshold with the one using the dimensional regularization formula [103]; see more discussions in Ref. [104]. Note that the value of a_μ had been determined as $a_\mu(\mu = 1 \text{ GeV}) = -1.94$ in Ref. [37] with the mass of the $P_{cs}(4459)$ state. Thus, one can obtain the cutoffs $q_{\max} \simeq 350\text{--}410$ MeV for the main bound channels of $\bar{D}\Xi_c$, $\bar{D}\Xi'_c$, and $\bar{D}^*\Xi_c$ with this comparing approach. However, when we check the analytical properties of the loop functions with these cutoffs, we find that the artificial singularities [96] appear very close to the corresponding thresholds, which will affect the poles to become unstable. Whereas, for the case of the hidden charm systems, we get the cutoffs $q_{\max} \simeq 500\text{--}580$ MeV for the main bound channels of $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ by comparing with the dimensional regularization method taking $a_\mu(\mu = 1 \text{ GeV}) = -2.09$ [26], where the analytical properties of the loop functions with these cutoffs are better. Therefore, we take the cutoff $q_{\max}$ around the range of 500–900 MeV, which are below the chiral limit of 1 GeV, see our results later.

A. Results of coupled-channel interactions in hidden charm sector

We first investigate the hidden charm system binding the P_c states. For the isospin $I = 1/2$ and spin-parity $J^P = 1/2^-$ sector (see Table I for all seven coupled channels under the HQSS constraint), we calculated the pole positions on the second Riemann sheets by varying the cutoff $q_{\max}$ from 500 to 900 MeV. The obtained masses and width trajectories of the poles are shown in Fig. 1, and some parts of the results with $q_{\max} = 600, 700, 800$ MeV are listed in Table III. These poles primarily couple to the channels $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$, respectively.

The results of Fig. 1 indicate that the masses of the three main poles show a monotonic downward trend as the cutoff $q_{\max}$ increases, while their corresponding widths go up nearly linearly when the $q_{\max}$ increases. However, for the $q_{\max}$ varying range from 500 to 900 MeV, the changing of their masses and widths are quite different, where the mass of the first pole can be bound largely and the widths of last two poles increase strongly. For the pole of the $\bar{D}\Sigma_c$ channel, the mass drops down from 4320 to 4220 MeV, nearly having 100 MeV differences, while the width increases from about 11 to 34 MeV with 23 MeV differences, where there is a fluctuation near $q_{\max} = 750$ MeV due to the pole crossing the open channel threshold of the $D^*\Lambda_c$ to affect the decay properties. The mass of the second pole, mainly bound by the $\bar{D}^*\Sigma_c$ channel, decreases from about 4472 to 4400 MeV with 72 MeV dropping, whereas, the width changes about 210 MeV from 40 to 250 MeV. For the third one, mainly bound by the $\bar{D}^*\Sigma_c^*$ channel, the mass reduces about 34 MeV from 4542 to 4508 MeV and the width enhances about 200 MeV from 50 to 250 MeV, which is similar to the one of the second pole. These results show that the interaction of the $\bar{D}\Sigma_c$ channel is strong and can be bound strongly compared to the other two channels, $\bar{D}^*\Sigma_c$ and

TABLE X. rms radii $|\sqrt{\langle r^2 \rangle}|$ of the corresponding poles for the $I = 1/2, J^P = 3/2^-$ sector using different methods in the splitting VB sector.

Resonances	$q_{\max} = 600 \text{ MeV (fm)}$	$ \sqrt{\langle r^2 \rangle} _2 \text{ (fm)}$	$q_{\max} = 700 \text{ MeV (fm)}$	$ \sqrt{\langle r^2 \rangle} _2 \text{ (fm)}$	$q_{\max} = 800 \text{ MeV (fm)}$	$ \sqrt{\langle r^2 \rangle} _2 \text{ (fm)}$
The radii of states calculated with Eq. (15)						
$\bar{D}^*\Sigma_c$	$1.114 + 0.020i$	1.114	$0.908 + 0.006i$	0.908	$0.777 + 0.002i$	0.777
$\bar{D}^*\Sigma_c^*$	$0.846 + 0.179i$	0.865	$0.784 + 0.111i$	0.792	$0.717 + 0.081i$	0.721
Resonances	$q_{\max} = 600 \text{ MeV (fm)}$	$ \sqrt{\langle r^2 \rangle} _1 \text{ (fm)}$	$q_{\max} = 700 \text{ MeV (fm)}$	$ \sqrt{\langle r^2 \rangle} _1 \text{ (fm)}$	$q_{\max} = 800 \text{ MeV (fm)}$	$ \sqrt{\langle r^2 \rangle} _1 \text{ (fm)}$
The radii of states calculated with Eq. (16)						
$\bar{D}^*\Sigma_c$	$0.622 - 0.011i$	0.622	$0.418 - 0.008i$	0.418	$0.306 - 0.009i$	0.306
$\bar{D}^*\Sigma_c^*$	$2.194 - 0.017i$	2.194	$0.759 - 0.011i$	0.759	$0.481 - 0.001i$	0.481

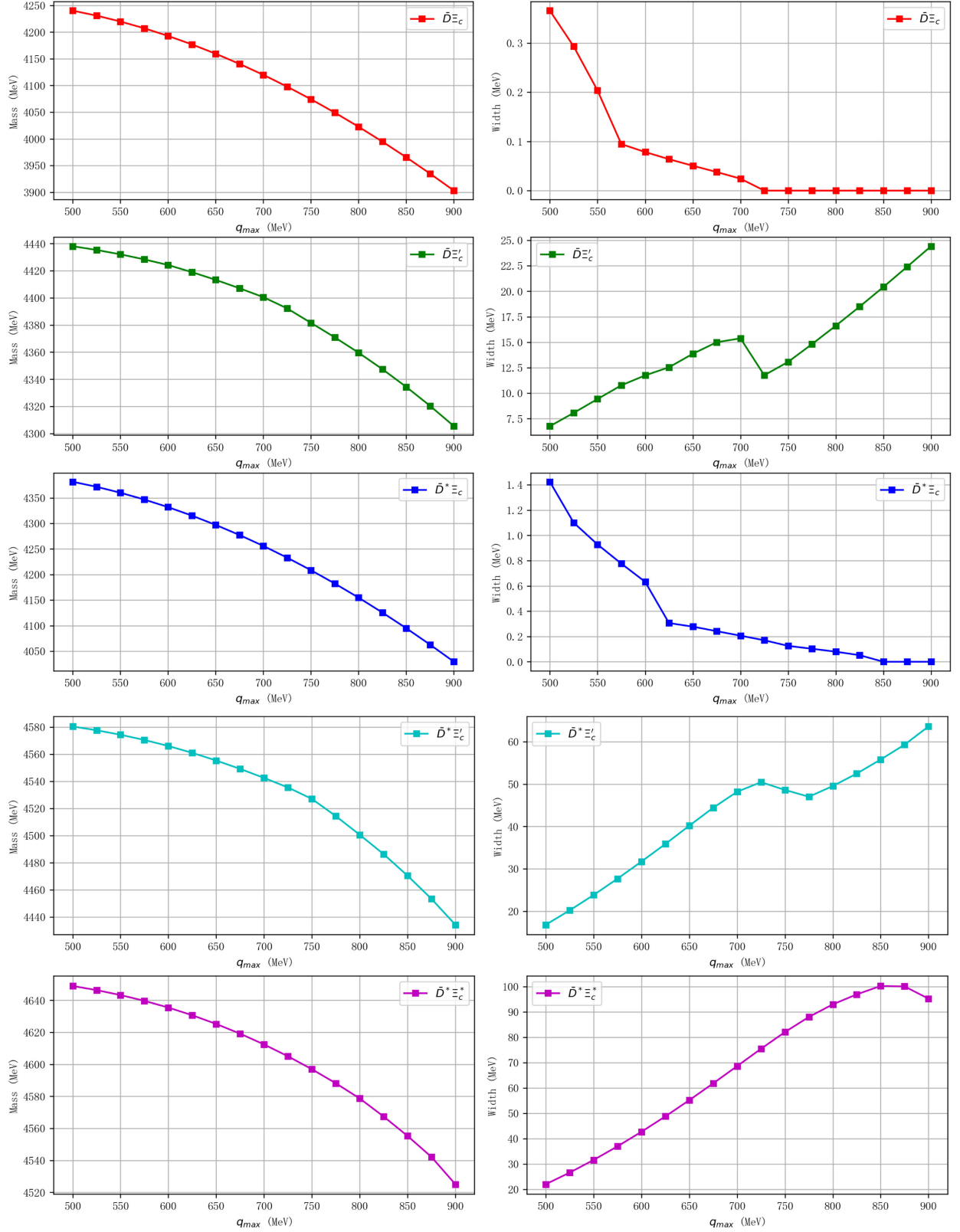


FIG. 10. Trajectories for the masses (left) and widths (right) of the poles in the second Riemann sheets for the $I = 0, J^P = 1/2^-$ sector in the nine coupled-channel case.

$\bar{D}^*\Sigma_c^*$, of which the poles have moved to above their thresholds for lower values of the cutoff $q_{\max}$. Therefore, from these results of Fig. 1, one also can easily find that it is difficult to get a “good” value for the cutoff $q_{\max}$ to obtain three main poles matching the masses and widths of the three P_c states well as the results obtained in Ref. [26] with one value of a_μ under the dimensional regularization scheme, as indicated in the results of the hidden charm and strange sector [42]. Indeed, as shown in Table III, they match the experimental findings of the P_c states which should be taken different value of the cutoff $q_{\max}$ with different uncertainties.

To check more details of these results, we split the coupled-channel system into the PB and VB subsystems to see the coupled-channel effect without the HQSS constraint, where the PB subsystem has three coupled channels, $\eta_c N$, $\bar{D}\Lambda_c$, and $\bar{D}\Sigma_c$, and there are four coupled channels in the VB subsystem, $J/\psi N$, $\bar{D}^*\Lambda_c$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$. The results are shown in Fig. 2 and Table IV. From Fig. 2, compared with Fig. 1, it looks like there is not much difference in the masses, where the line shape for the widths has been changed a lot. Now the width of the first pole, mainly from the $\bar{D}\Sigma_c$ channel, increases a lot from 20 to 120 MeV. Whereas the one for the second pole, contributed from the $\bar{D}^*\Sigma_c$ channel, has more reasonable values but decreases from 45 to 10 MeV with some fluctuations, which is more consistent with the narrow width of the experimentally observed $P_c(4440)$ state. For the third pole of the $\bar{D}^*\Sigma_c^*$ channel, the width enhances with about 72 MeV just a little smaller than the one in the full coupled-channel case before, but the mass is always below the threshold. From these results, one can find that the coupled-channel effect under the HQSS constraint just affects the widths of these poles when varying the cutoff $q_{\max}$ and does not change the bound properties of the strong interactions among the coupled channels.

As discussed in the last section, to reveal the molecular properties of the resonances, we continue to investigate the wave functions of these resonances at small distance. The results are shown in Figs. 3 and 4 for the seven coupled-channel case and the splitting PB and VB sectors, respectively. As the cutoff $q_{\max}$ increases from 600 to 800 MeV, the real parts of the wave functions at the origin $r = 0$ increase significantly. From these results of Figs. 3 and 4, one can see that the wave functions are nearly close to zero when $r > 4$ fm, at a reasonable size of the hadronic molecules, except for the second pole in the splitting PB and VB sectors, of which its imaginary parts can up to about 7 fm before going to zero. When we check in detail, for the second pole of the $\bar{D}^*\Sigma_c$, the one $(4476.97 + 10.22i)$ MeV has already moved to more than 15 MeV above the threshold for $q_{\max} = 600$ MeV, and thus, the tail of its wave function can spread higher to 7 fm; see the red line in the middle figures of Fig. 4. This is due to the loose bound of the system $\bar{D}^*\Sigma_c$, since its interaction would

lead to the pole moving to above the threshold when the cutoff is reduced, which is consistent with the finding in Ref. [101] for the $\pi\pi$ system of the isospin $I = 0$ sector and the $K\bar{K}$ system of the isospin $I = 1$ sector, as well as with the one in Ref. [105] for the $\pi\Sigma$ system of the isospin $I = 0$ sector.

Next, using the wave functions obtained, we calculated the radii of these poles. As discussed in the last section, there are two ways to evaluate the radii. One way utilizes the derivative of the G function with respect to the binding energy $B_{E,i}$, see Eq. (16), called “method 1.” The other one explores the form factor evaluated from the wave functions, see Eq. (15), labeled “method 2.” The results of the root-mean-square (rms) radii obtained by varying the cutoffs are shown in Fig. 5, some of which are also listed in Tables V and VI for the seven coupled-channel case and the splitting PB and VB sectors, respectively. From these results, one can see that the results with two methods are consistent with each other except for the situation of the pole closing to the threshold due to the binding energy $B_{E,i}$ going to zero. Indeed, the results of method 2 are more numerically stability, with the cutoff $q_{\max}$ smoothly changing. It is also found that the radii in most of the cases are less than 3 fm, which are consistent with the one estimated from the wave functions above.

Furthermore, we examine the properties of the P_c states in the $I = 1/2, J^P = 3/2^-$ sector. By varying the cutoff value $q_{\max}$, the trajectories of the pole positions for the main channels $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ under the five coupled-channel case are calculated, as shown in Fig. 6, where some results are presented in Table VII. The masses of these three poles exhibit a monotonic downward trend

TABLE XI. Pole positions (M, Γ) in the second Riemann sheets for the $I = 0, J^P = 1/2^-$ sector with the nine coupled-channel case.

$q_{\max}$ (MeV)	Mass (MeV)	Width (MeV)	Main channel	J^P	Experimental states
600	4192.91	0.08	$\bar{D}\Xi_c$	$1/2^-$	...
700	4119.91	0.02	$\bar{D}\Xi_c$	$1/2^-$	...
800	4022.66	0.00	$\bar{D}\Xi_c$	...	...
600	4331.89	0.63	$\bar{D}^*\Xi_c$	$1/2^-$	$P_{cs}(4338)$
700	4255.98	0.21	$\bar{D}^*\Xi_c$	$1/2^-$	...
800	4154.76	0.08	$\bar{D}^*\Xi_c$	$1/2^-$	...
600	4424.27	11.75	$\bar{D}\Xi'_c$	$1/2^-$	$P_{cs}(4459)$
700	4400.58	15.38	$\bar{D}\Xi'_c$	$1/2^-$	...
800	4359.59	16.61	$\bar{D}\Xi'_c$	$1/2^-$	...
600	4565.96	31.73	$\bar{D}^*\Xi'_c$	$1/2^-$	...
700	4542.46	48.20	$\bar{D}^*\Xi'_c$	$1/2^-$	...
800	4500.58	49.56	$\bar{D}^*\Xi'_c$	$1/2^-$	...
600	4635.46	42.75	$\bar{D}^*\Xi_c^*$	$1/2^-$	...
700	4612.43	68.66	$\bar{D}^*\Xi_c^*$	$1/2^-$	...
800	4578.71	93.02	$\bar{D}^*\Xi_c^*$	$1/2^-$	...

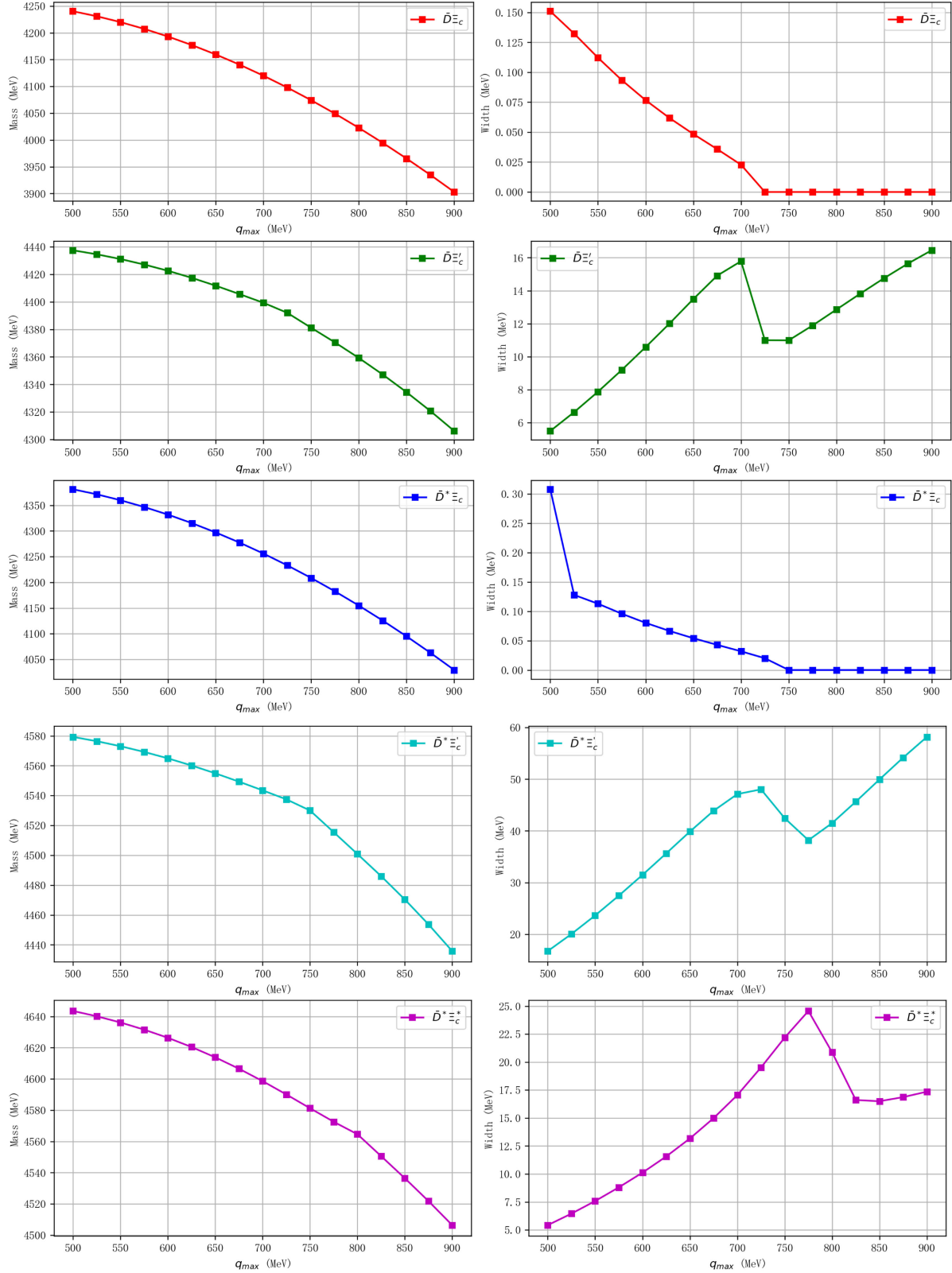


FIG. 11. Trajectories for the masses (left) and widths (right) of the poles in the second Riemann sheets for the $I = 0, J^P = 1/2^-$ sector in the splitting PB and VB sectors.

with the increase of the cutoff $q_{\max}$, which are all below the corresponding thresholds. Unlike the monotonic behaviors in the masses, the decay widths show the nonmonotonic fluctuations when varying with $q_{\max}$ for the first two poles. The fluctuations of the widths are due to the poles crossing the threshold of certain open channels. These results also indicate different bound behaviors for the three main channels.

As done in the $I = 1/2, J^P = 1/2^-$ sector, we also split the five coupled channels into two PB and VB subsystems to check the coupled-channel effect without the HQSS constraint, where the results are shown in Fig. 7 and some results are presented in Table VIII. Note that in this case only one PB channel is removed from the five coupled channels. Thus, after retaining the VB coupled channels, once again the masses of the later two poles are not much changed, and their widths are significantly reduced. This indicates again that the coupled channels between the PB and VB sectors under the HQSS constraint only affect the decay widths of the poles and not much to their masses. The width of the $\bar{D}^*\Sigma_c$ pole in the full coupled-channel case is more consistent with the narrow resonance of $P_c(4457)$ observed experimentally.

In Fig. 8, we display the results of the wave functions $\phi(\vec{r})$ of the three poles $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ with different cutoff values $q_{\max}$ in the $I = 1/2, J^P = 3/2^-$ sector. From the results of the top in Fig. 8 for the five coupled-channel case, one can see that the wave functions are mainly distributed within 0–6 fm and go to zero rapidly after $r > 4$ fm, which is consistent with the one found in the $I = 1/2, J^P = 1/2^-$ sector above. Note that, for the first pole of $\bar{D}\Sigma_c^*$, the imaginary part of its wave function for the pole $(4375.58 + 8.59i)$ MeV with $q_{\max} = 600$ MeV has a tail nearly up to 6 fm (see the red curve in the top of Fig. 8), which is due to its pole being close to the threshold for the loose bound system as found in Fig. 4 before. The wave functions after splitting into the VB subsystem do not change significantly, see the results on the bottom of Fig. 8 compared with the results on the top, and thus, the similar conclusion is not repeated here.

With the wave functions obtained, we also evaluate the radii of these poles. The results of the rms radii with varying cutoffs are shown in Fig. 9, some of which are shown in Tables IX and X for the five coupled-channel case and the splitting VB sector, respectively. From these results, one can see that the results with two methods are consistent with each other in most cases and the results of method 2 are more stable. As found in the $I = 1/2, J^P = 1/2^-$ sector, the radii in most of the cases are less than 3 fm, consistent with the results of the wave functions above. From Fig. 9, also compared with Tables IX and X, the coupled-channel effect from the HQSS constraint has some influence on the radii, but not much.

B. Results of coupled-channel interactions in hidden charm strange sector

Now we start to investigate the hidden charm strange sector to compare with what we have in the hidden charm sector obtained above. From the results of Ref. [92], we know that there are nine coupled channels under the HQSS constraint, as shown in Table II for the $I = 0, J^P = 1/2^-$ sector, where the bound systems are the $\bar{D}\Xi_c$, $\bar{D}\Xi'_c$, $\bar{D}^*\Xi_c$, $\bar{D}^*\Xi'_c$, and $\bar{D}^*\Xi_c^*$ channels. The trajectories of the masses and widths are shown in Fig. 10, where some results are presented in Table XI. From Fig. 10, one can see that the masses of all the poles decrease monotonically with the increase of the cutoff $q_{\max}$, which indicates that the strong attractive interactions among these systems dominate the generation of these poles. The poles corresponding to $\bar{D}\Xi_c$ and $\bar{D}^*\Xi_c$ exhibit extremely narrow widths and are located very close to the real axis on the complex energy plane, reflecting their relatively weak couplings to the low-energy open channels $\eta_c\Lambda$ and $J/\psi\Lambda$. The binding energies of these two poles, more than 100 MeV, are much larger than the other poles, which indicates the strong interactions for these two channels leading to deeply bound systems. Indeed, from the results of Refs. [37,92], the poles of the $\bar{D}\Xi_c$ and $\bar{D}^*\Xi_c$ channels are strongly coupled to the $\bar{D}_s\Lambda_c$ and $\bar{D}_s^*\Lambda_c$ channels, respectively, which lead to these systems becoming more bound. Because of their large binding energy, as shown in detail in Table XI, the $\bar{D}^*\Xi_c$ pole is assigned as the $P_{cs}(4338)$ state, while the $\bar{D}\Xi'_c$ pole

TABLE XII. Pole positions (M, Γ) in the second Riemann sheets for the $I = 0, J^P = 1/2^-$ sector with the splitting PB and VB sectors.

$q_{\max}$ (MeV)	Mass (MeV)	Width (MeV)	Main channel	Sector	J^P	Experimental state
600	4192.98	0.08	$\bar{D}\Xi_c$	PB	$1/2^-$	...
700	4119.94	0.02	$\bar{D}\Xi_c$	PB	$1/2^-$	...
800	4022.65	0.00	$\bar{D}\Xi_c$	PB	$1/2^-$	...
600	4331.75	0.08	$\bar{D}^*\Xi_c$	VB	$1/2^-$	$P_{cs}(4338)$
700	4255.99	0.03	$\bar{D}^*\Xi_c$	VB	$1/2^-$	...
800	4154.78	0.00	$\bar{D}^*\Xi_c$	VB	$1/2^-$	...
600	4422.56	10.58	$\bar{D}\Xi'_c$	PB	$1/2^-$	$P_{cs}(4459)$
700	4399.34	15.79	$\bar{D}\Xi'_c$	PB	$1/2^-$	...
800	4359.24	12.85	$\bar{D}\Xi'_c$	PB	$1/2^-$	...
600	4564.95	31.49	$\bar{D}^*\Xi'_c$	VB	$1/2^-$	...
700	4543.50	47.12	$\bar{D}^*\Xi'_c$	VB	$1/2^-$	...
800	4500.82	41.50	$\bar{D}^*\Xi'_c$	VB	$1/2^-$	...
600	4626.34	10.12	$\bar{D}^*\Xi_c^*$	VB	$1/2^-$	...
700	4598.59	17.05	$\bar{D}^*\Xi_c^*$	VB	$1/2^-$	...
800	4564.59	20.87	$\bar{D}^*\Xi_c^*$	VB	$1/2^-$	...

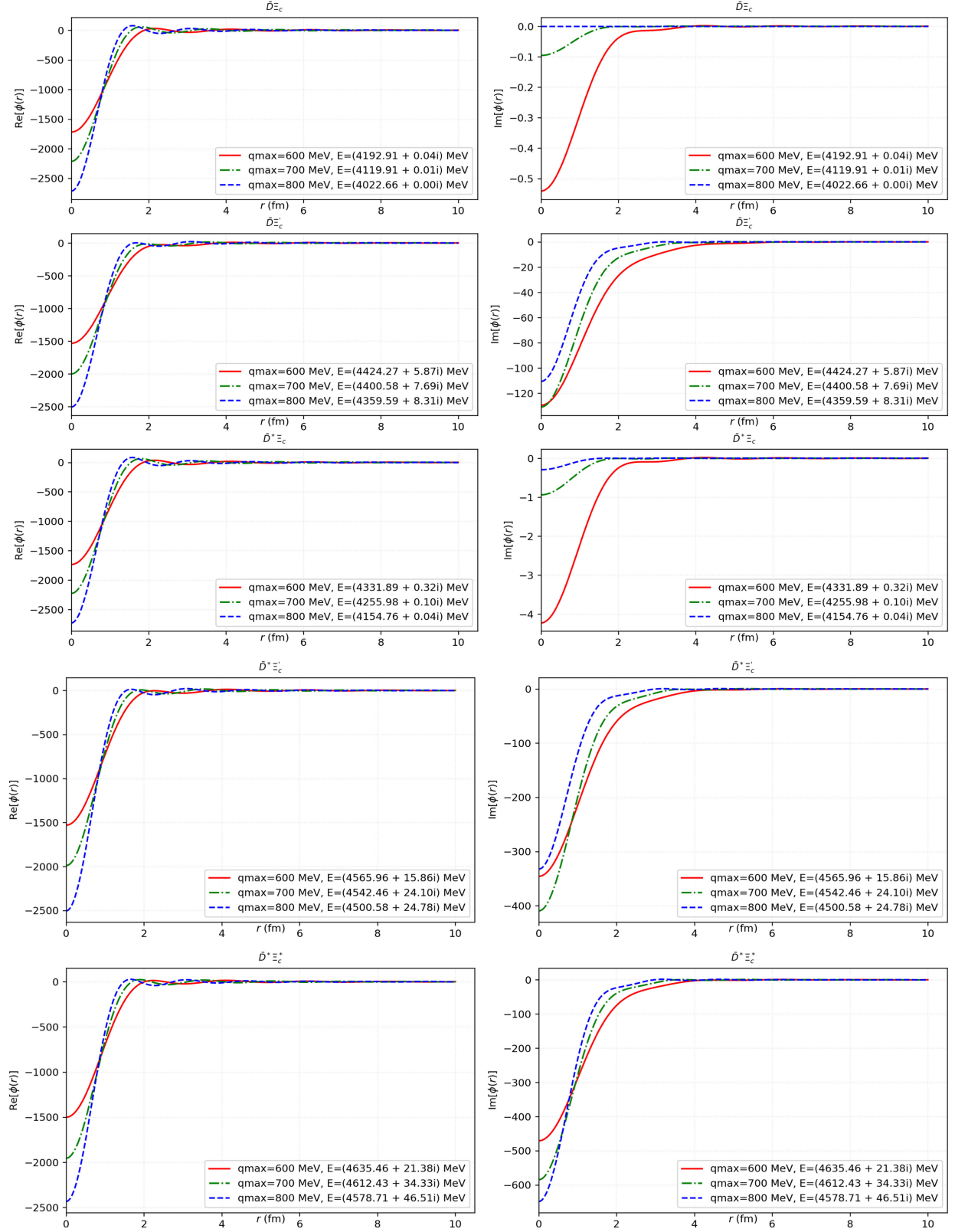


FIG. 12. Real (left) and imaginary (right) parts of the wave functions $\phi(r)$ of corresponding poles for the $I = 0, J^P = 1/2^-$ sector with different $q_{\max}$ in the nine coupled-channel case.

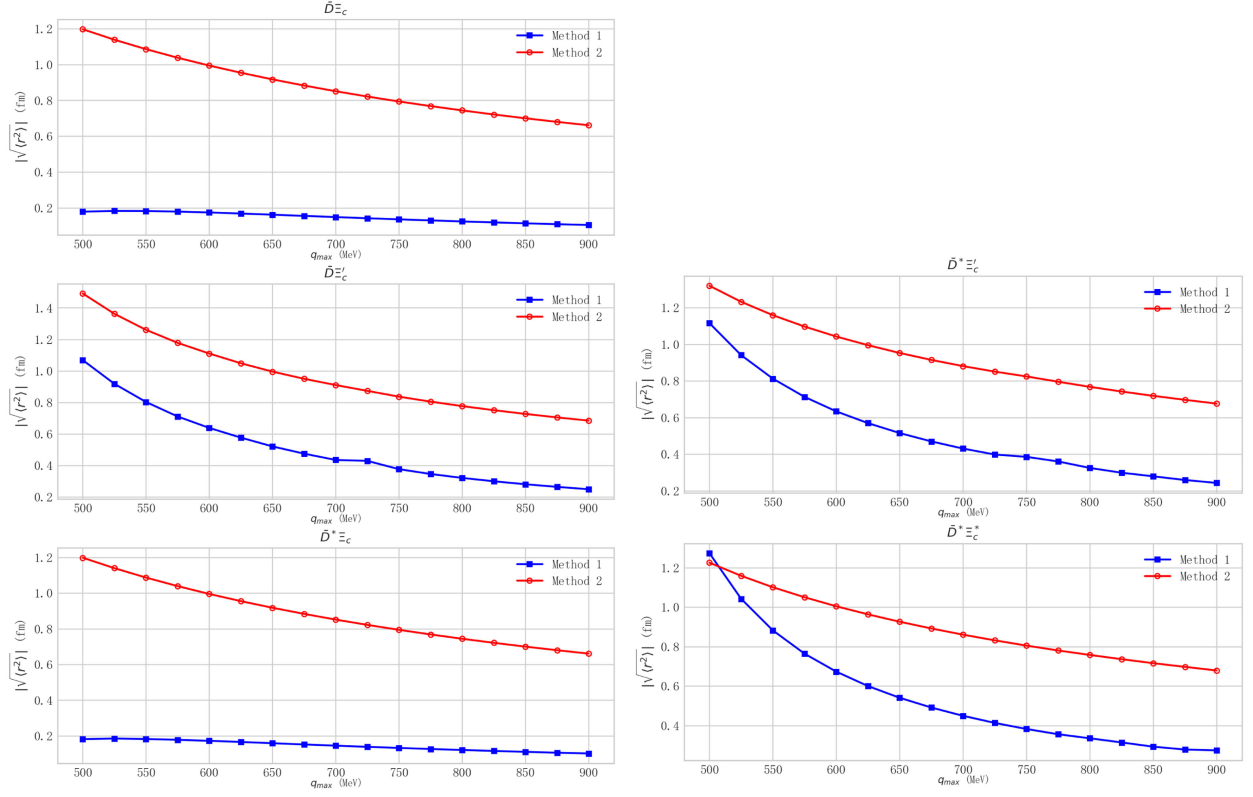


FIG. 13. rms radii of the corresponding poles for the $I = 0, J^P = 1/2^-$ sector as a function of the cutoff $q_{\max}$ in the nine coupled-channel case. Results from method 1 (blue) and method 2 (red) are compared.

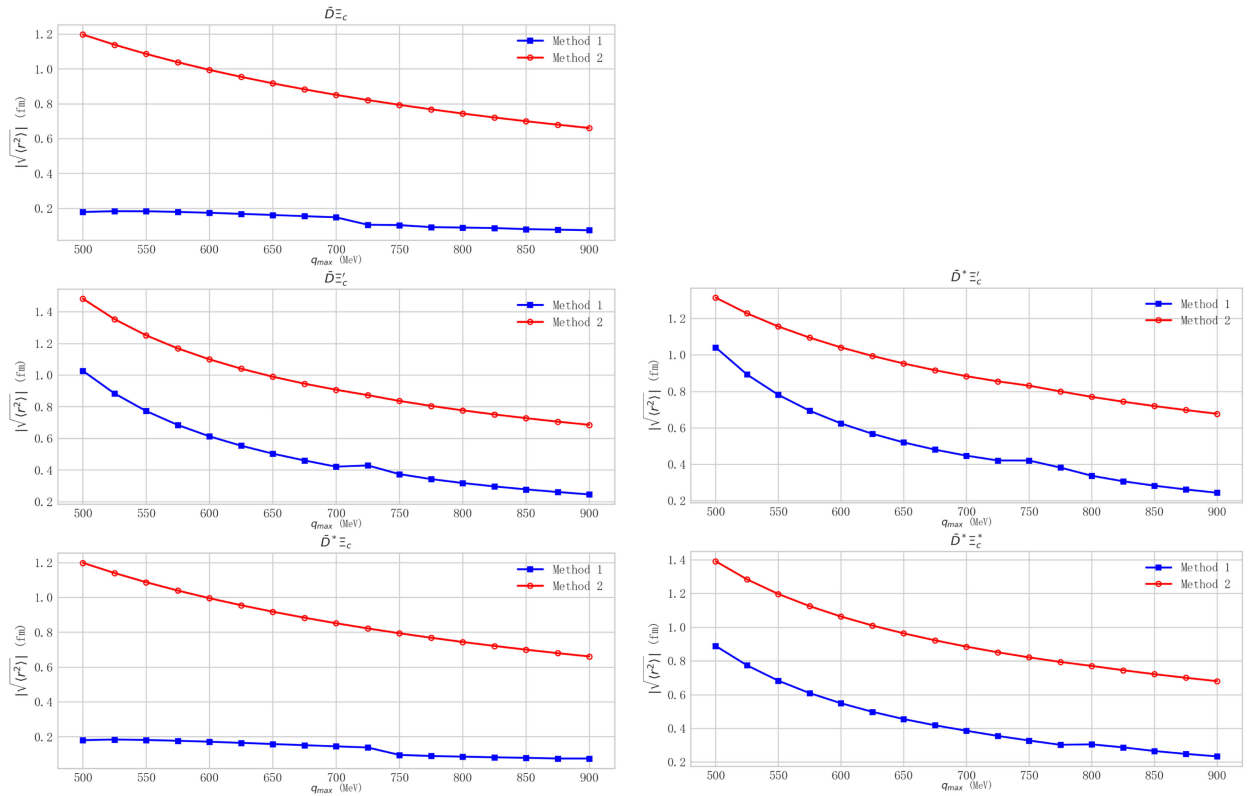


FIG. 14. rms radii of the corresponding poles for the $I = 0, J^P = 1/2^-$ sector as a function of the cutoff $q_{\max}$ in the splitting PB and VB sectors. Results from method 1 (blue) and method 2 (red) are compared.

TABLE XIII. rms radii $|\sqrt{\langle r^2 \rangle}|$ of the corresponding poles for the $I = 0, J^P = 1/2^-$ sector using different methods in the nine coupled-channel case.

Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)
The radii of states calculated with Eq. (15)						
$\bar{D}\Xi_c$	$0.994 - 0.000i$	0.994	$0.851 - 0.000i$	0.851	$0.744 + 0.000i$	0.744
$\bar{D}\Xi'_c$	$1.109 + 0.054i$	1.110	$0.910 + 0.020i$	0.910	$0.776 + 0.007i$	0.776
$\bar{D}^*\Xi_c$	$0.995 - 0.000i$	0.995	$0.851 - 0.000i$	0.851	$0.744 - 0.000i$	0.744
$\bar{D}^*\Xi'_c$	$1.038 + 0.100i$	1.043	$0.880 + 0.048i$	0.881	$0.768 + 0.018i$	0.768
$\bar{D}^*\Xi_c^*$	$0.998 + 0.117i$	1.004	$0.858 + 0.058i$	0.860	$0.757 + 0.034i$	0.757
Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)
The radii of states calculated with Eq. (16)						
$\bar{D}\Xi_c$	$0.174 - 0.000i$	0.174	$0.148 + 0.000i$	0.148	$0.123 + 0.000i$	0.123
$\bar{D}\Xi'_c$	$0.638 + 0.011i$	0.638	$0.435 + 0.009i$	0.435	$0.321 + 0.001i$	0.321
$\bar{D}^*\Xi_c$	$0.172 + 0.001i$	0.172	$0.145 + 0.000i$	0.145	$0.120 + 0.000i$	0.120
$\bar{D}^*\Xi'_c$	$0.634 + 0.009i$	0.634	$0.431 + 0.013i$	0.431	$0.325 + 0.009i$	0.325
$\bar{D}^*\Xi_c^*$	$0.673 + 0.010i$	0.673	$0.449 + 0.012i$	0.449	$0.334 + 0.019i$	0.334

is as the $P_{cs}(4459)$ state, as found in Ref. [42]. This conclusion is different from the general views [37,39–41,106], the $P_{cs}(4338)$ as the $\bar{D}\Xi_c$ state, the $P_{cs}(4459)$ as the $\bar{D}^*\Xi_c$ molecule, which are analogous to the case of hidden charm sector, the $P_c(4312)$ as the $\bar{D}\Sigma$ state, and the $P_c(4459)$ [$P_c(4440)$ and $P_c(4457)$] as the $\bar{D}^*\Sigma$ molecule. In Ref. [106], it was predicted that there were two $P_{cs}(4459)$ states with different spins, which had 6 MeV for the mass differences, similar to the one of the $P_c(4459)$ state in the hidden charm system.

As done in the hidden charm sector, to check the coupled-channel effect without the HQSS constraint, we also divide the coupled-channel system into the PB and VB subsystems. The trajectories of pole positions with varying $q_{\max}$ are shown in Fig. 11, where some of results are shown in Table XII. Most of these results are not much different than the full coupled-channels case, see Fig. 10 and Table XI, except for the width of the $\bar{D}^*\Xi_c^*$ pole being 4 times smaller, which indicates that the coupled-channel effect with the HQSS constraint is not very important to this sector, different from the case in the hidden charm sector before. Additionally, the width trajectories of the $\bar{D}\Xi'_c, \bar{D}^*\Xi'_c$ channels exhibit some fluctuations around $q_{\max} = 700$ –850 MeV due to the crossing of the thresholds of some open channels.

Next, we show the results of the real and imaginary parts of the wave functions for each pole obtained in Fig. 12, where one can see once again that the wave functions are mainly contributed within 0–6 fm and go to zero rapidly after $r > 4$ fm, as found in the hidden charm sector above. The results for the splitting PB and VB sectors are not repeated here, due to similar results.

With the wave functions obtained, we continue to calculate the radii of these poles and show the results of

the rms radii with varying the cutoffs in Fig. 13, where some of them are listed in detail in Table XIII. As found above, the results of method 2 are more stable than the results obtained with method 1, all of which are less than 1.5 fm. From Fig. 13, one can see that the results of method 1 for the channels $\bar{D}\Xi, \bar{D}^*\Xi$ are small, less than 0.2 fm, due to the large binding energies, see Eq. (16); meaning that Eq. (16) is not suited for the deeply bound systems. As shown in Fig. 14, one can see that the results in the splitting PB and VB sectors are not much different than the results of Fig. 13, which once again indicate that the coupled-channel effect from the HQSS constraint has little influence on the radii as found in the hidden charm sector.

Moreover, we examine the properties of the interactions of the $I = 0, J^P = 3/2^-$ sector in the hidden charm strange system, where the main bound systems are the $\bar{D}^*\Xi_c, \bar{D}\Xi_c^*, \bar{D}^*\Xi'_c, \bar{D}^*\Xi_c^*$ channels. For the six full coupled channels, the trajectories of the pole positions are shown in Fig. 15, some of which are presented in Table XIV. As shown in Fig. 15, the masses of these poles reduce for increasing of the cutoff $q_{\max}$, all of which are below the corresponding thresholds. Unlike the monotonic behaviors in the masses, the decay widths show the different line shapes for these poles. The fluctuations of the widths are due to the pole crossing the thresholds of certain open channels. These results also indicate different bound behaviors for three main channels, where the $\bar{D}^*\Xi_c$ channel is deeply bound compared to the others, as in the $I = 0, J^P = 1/2^-$ sector. Indeed, the $\bar{D}^*\Xi_c$ pole has an extremely narrow decay width, and as the mass decreases, the pole moves rapidly below all the thresholds with zero width. This also indicates that the $\bar{D}^*\Xi_c$ couples to the low-energy open channel $J/\psi\Lambda$ weakly. As found in Refs. [37,92], the poles of the $\bar{D}^*\Xi_c$ channel are strongly coupled to the $\bar{D}_s^*\Lambda_c$ channels, which leads to

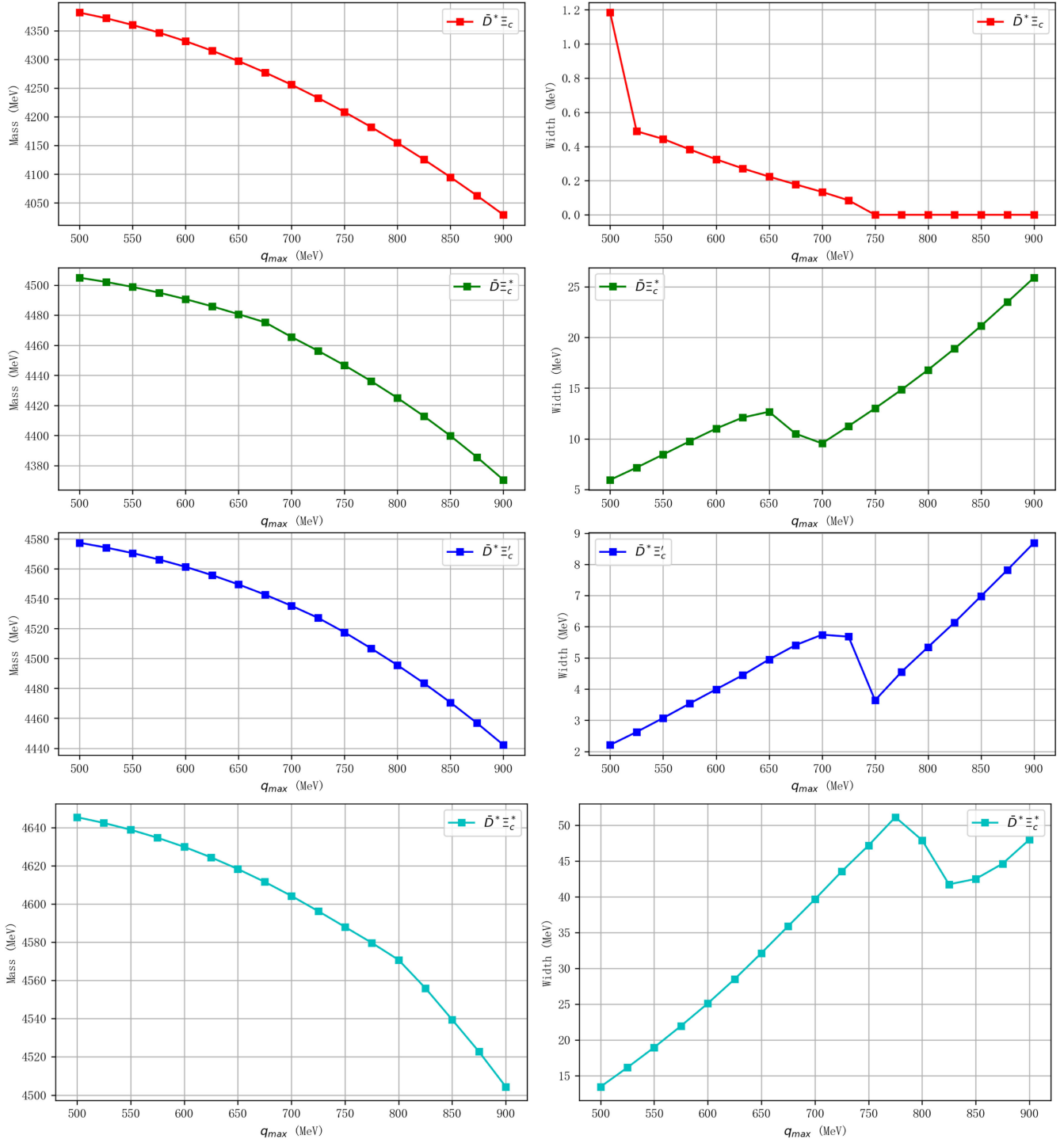


FIG. 15. Trajectories for the masses (left) and widths (right) of the poles in the second Riemann sheets for the $I = 0, J^P = 3/2^-$ sector in the six coupled-channel case.

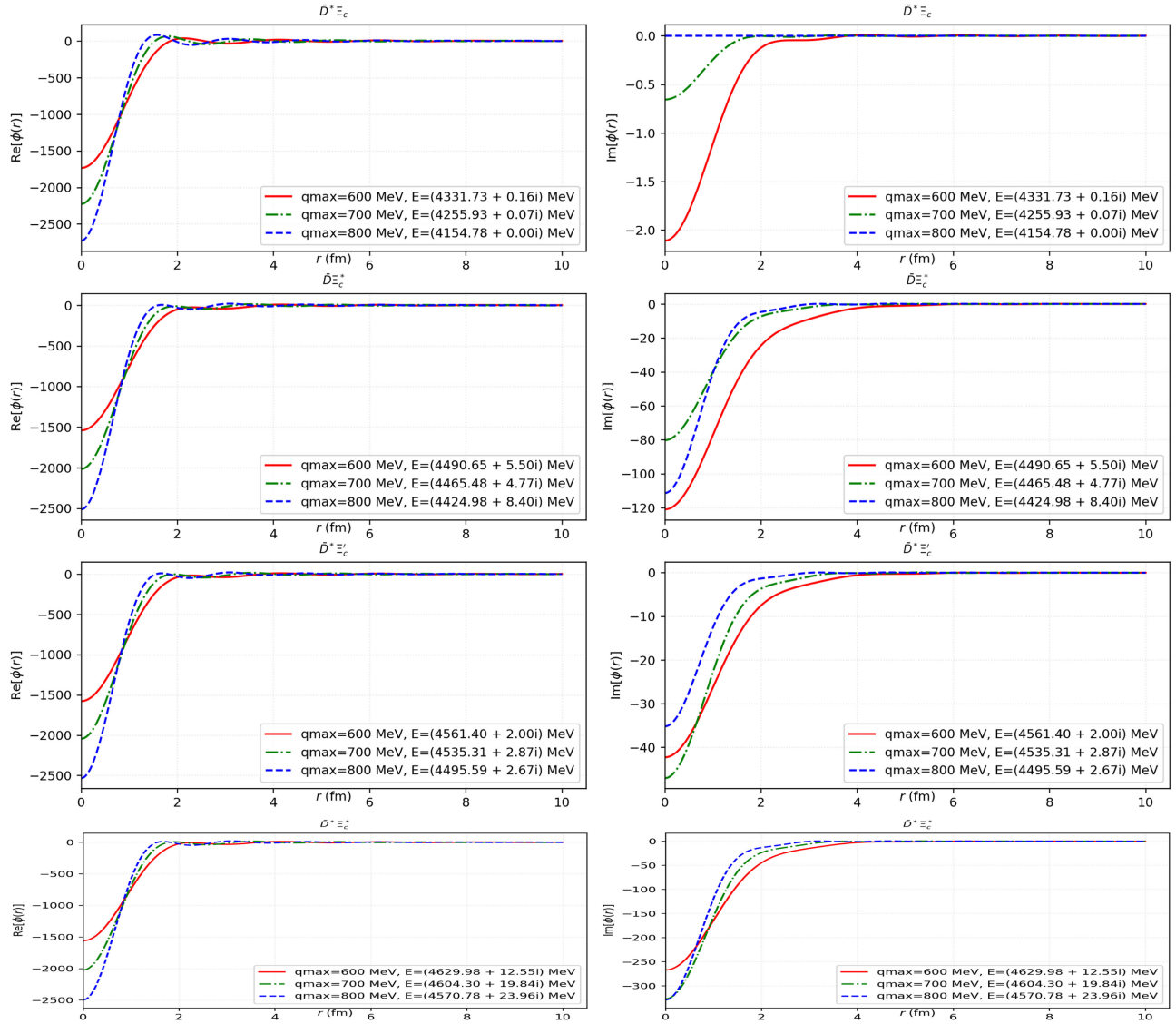
TABLE XIV. Pole positions (M, Γ) in the second Riemann sheets for the $I = 0, J^P = 3/2^-$ sector with the six coupled-channel case.

$q_{\max}$ (MeV)	Mass (MeV)	Width (MeV)	Main channel	J^P	Experimental state
600	4331.73	0.32	$\bar{D}^*\Xi_c$	$3/2^-$	$P_{cs}(4338)$
700	4255.93	0.13	$\bar{D}^*\Xi_c$	$3/2^-$	...
800	4154.78	0.00	$\bar{D}^*\Xi_c$	$3/2^-$	...
600	4490.65	11.01	$\bar{D}^*\Xi_c^*$	$3/2^-$	...
700	4465.48	9.55	$\bar{D}^*\Xi_c^*$	$3/2^-$	...
800	4424.98	16.80	$\bar{D}^*\Xi_c^*$	$3/2^-$	...
600	4561.40	3.99	$\bar{D}^*\Xi_c'$	$3/2^-$	...
700	4535.31	5.74	$\bar{D}^*\Xi_c'$	$3/2^-$	...
800	4495.59	5.35	$\bar{D}^*\Xi_c'$	$3/2^-$	...
600	4629.98	25.11	$\bar{D}^*\Xi_c^*$	$3/2^-$	...
700	4604.30	39.67	$\bar{D}^*\Xi_c^*$	$3/2^-$	...
800	4570.78	47.92	$\bar{D}^*\Xi_c^*$	$3/2^-$	...

the system becoming more bound. Note that, in this sector, there is only one PB channel, $\bar{D}^*\Xi_c^*$, and the results without it are not much different than Fig. 15 and not shown here, meaning that the influence of this channel can be ignored.

In Fig. 16, we present the results of the wave functions $\phi(\vec{r})$ of the four primary pole components in the six coupled-channel case with different cutoffs. From Fig. 16, as found in the $I = 0, J^P = 1/2^-$ sector above, the wave functions are mainly distributed within 0–6 fm and go to zero rapidly after $r > 4$ fm. It should be mentioned that the results with only the splitting VB sector are similar, and thus, they are not shown here.

Then, using the wave functions calculated, we further evaluate the radii of these poles. The results of the rms radii with varying the cutoffs are shown in Fig. 17, some of which are shown in Table XV. From Fig. 17, it is found that the results with two methods are not much different and the

FIG. 16. Real (left) and imaginary (right) parts of the wave functions $\phi(r)$ of corresponding pole for the $I = 0, J^P = 3/2^-$ sector with different $q_{\max}$ in the six coupled-channel case.

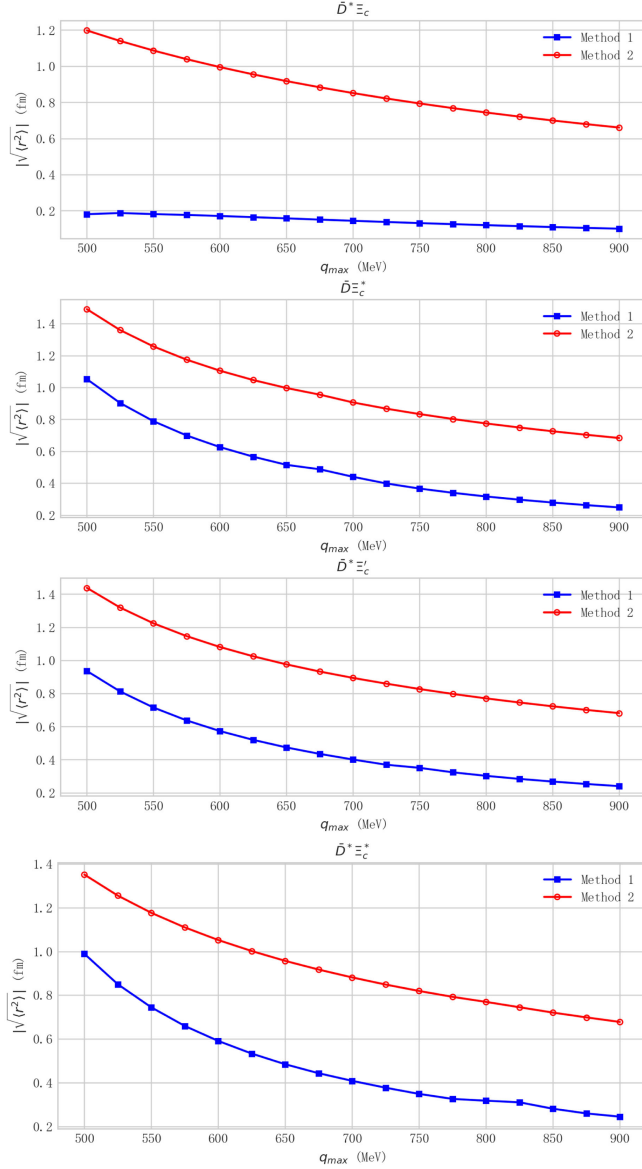


FIG. 17. rms radii of the corresponding poles for the $I=0, J^P=3/2^-$ sector as a function of the cutoff $q_{\max}$ in the six coupled-channel case. Results from method 1 (blue) and method 2 (red) are compared.

results of method 2 are more stable. As found in the $I=0, J^P=1/2^-$ sector, most of the radii are less than 1.5 fm, and the results of method 1 for the $\bar{D}^* \Xi_c$ are smaller than the others for its deep bound with large binding energy. Note that the similar results of the splitting VB sector are not shown any more, where only the radii of the $\bar{D}^* \Xi_c$ pole with method 1 are a bit smaller.

C. Results of single-channel interactions

In the last two subsections, first we utilize the coupled-channel formalism for the hidden charm system, where the

main bound channels in the $I=1/2, J^P=1/2^-$ sector ($\bar{D}\Sigma_c, \bar{D}^*\Sigma_c, \bar{D}^*\Sigma_c^*$) and in the $I=1/2, J^P=3/2^-$ sector ($\bar{D}\Sigma_c^*, \bar{D}^*\Sigma_c, \bar{D}^*\Sigma_c^*$) are studied in detail. For the hidden charm strange system, the main bound channels in the $I=0, J^P=1/2^-$ sector ($\bar{D}\Xi_c, \bar{D}^*\Xi_c, \bar{D}\Xi_c', \bar{D}^*\Xi_c', \bar{D}^*\Xi_c^*$) and in the $I=0, J^P=3/2^-$ sector ($\bar{D}^*\Xi_c, \bar{D}\Xi_c^*, \bar{D}^*\Xi_c', \bar{D}^*\Xi_c^*$) are also investigated. The strong interactions of these channels and their coupled channels with/without the HQSS constraint are studied carefully. In this subsection, we make further investigation into the properties of these bound systems with the single-channel interaction to check the coupled-channel effect in detail.

First, we investigate the hidden charm system. Note that, for the single-channel interaction, the isospin and spin parity are not necessary to specify for different sectors. As one can see from the coupled-channel interactions before and the interaction potentials of Table I, in the hidden charm system, there are only four bound channels, $\bar{D}\Sigma_c, \bar{D}\Sigma_c^*, \bar{D}^*\Sigma_c, \bar{D}^*\Sigma_c^*$, where the trajectories of the corresponding poles are shown in Fig. 18. As seen from Fig. 18, the masses of the four poles show a monotonic downward trend as $q_{\max}$ increases. Note that all these poles locate on the real axis of the first Riemann sheet below the corresponding thresholds, becoming pure bound states with zero decay width, since there is no coupled channel to decay for the single-channel interaction. Moreover, one also can easily find that the binding energies of all these poles are similar, and the mass differences of these poles are nearly the same, about 6 MeV, for the cutoff $q_{\max}$ varying from 500 to 900 MeV, due the similar interaction potentials; see the value of μ_3 in Eq. (2). When the coupled-channel effect is taken into account, the poles' trajectories are quite different, as found from the results before.

Next, the results of the wave functions $\phi(\vec{r})$ of the four poles are presented in Fig. 19 with different cutoffs. Note that now the wave functions $\phi(\vec{r})$ are also real due to the real bound poles with no width, see the results of Fig. 18. From Fig. 19, the wave functions are distributed within 0–6 fm and go to zero rapidly after $r > 4$ fm as normal. Furthermore, one also sees that the line shapes of these wave functions are similar for the same interaction dynamics, just under different channels.

With the wave functions calculated, the radii of these poles are obtained, where the results of the rms radii with varying the cutoffs are shown in Fig. 20. As shown in Fig. 20, unlike the unstable fluctuations observed in the coupled-channel cases above, the curves of method 1 and method 2 for the poles of the single-channel interactions appear extremely smooth and monotonic, since all of these poles are pure bound states with zero width below the corresponding threshold. Thus, the results with two methods are not much different and their line shapes are similar, where only the values reduce a little bit for the heavier channels. One also can see that all these radii are in the range from about 4 fm decreasing to 1 fm for the varying

TABLE XV. rms radii $|\sqrt{\langle r^2 \rangle}|$ of the corresponding poles for the $I = 0, J^P = 3/2^-$ sector using different methods in the six coupled-channel case.

Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _2$ (fm)
The radii of states calculated with Eq. (15)						
$\bar{D}^* \Xi_c$	$0.995 - 0.000i$	0.995	$0.851 - 0.000i$	0.851	$0.744 + 0.000i$	0.744
$\bar{D} \Xi_c^*$	$1.104 + 0.048i$	1.105	$0.906 + 0.011i$	0.906	$0.774 + 0.007i$	0.774
$\bar{D}^* \Xi_c'$	$1.081 + 0.013i$	1.081	$0.894 + 0.005i$	0.894	$0.770 + 0.002i$	0.770
$\bar{D} \Xi_c'^*$	$1.050 + 0.073i$	1.052	$0.881 + 0.034i$	0.881	$0.769 + 0.019i$	0.769
Resonances	$q_{\max} = 600$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 700$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)	$q_{\max} = 800$ MeV (fm)	$ \sqrt{\langle r^2 \rangle} _1$ (fm)
The radii of states calculated with Eq. (16)						
$\bar{D}^* \Xi_c$	$0.171 + 0.001i$	0.171	$0.144 + 0.000i$	0.144	$0.120 + 0.000i$	0.120
$\bar{D} \Xi_c^*$	$0.626 + 0.002i$	0.626	$0.440 - 0.003i$	0.440	$0.317 + 0.001i$	0.317
$\bar{D}^* \Xi_c'$	$0.573 + 0.000i$	0.573	$0.401 - 0.002i$	0.401	$0.302 - 0.002i$	0.302
$\bar{D} \Xi_c'^*$	$0.591 + 0.008i$	0.591	$0.408 + 0.007i$	0.408	$0.316 + 0.032i$	0.317

cutoffs, larger than what we have in the coupled-channel cases.

Second, we study the hidden charm strange system. In this system, there are six bound channels as found in the coupled-channel cases and seen from the interaction potentials, see Table II, which are the channels $\bar{D} \Xi_c$, $\bar{D} \Xi_c'$, $\bar{D}^* \Xi_c$, $\bar{D}^* \Xi_c'$, $\bar{D} \Xi_c^*$, and $\bar{D}^* \Xi_c^*$. The trajectories of their poles in the first Riemann sheet are shown in Fig. 21, which are analogous to the results of the hidden charm sector above with zero width, and they also have 6 MeV differences of the masses for these poles when the cutoff is

varied. Indeed, the binding energies of these poles are similar, too, for the same potentials; see the values of μ_2, μ_4 , and λ in Eq. (3).

Then, we show the results of the wave functions $\phi(\vec{r})$ of the six poles in Fig. 22 with different cutoffs, which are also real for the pure bound states with zero width. From the results of Fig. 22, one can see that these wave functions go to zero quickly after $r > 4$ fm as the others above.

Furthermore, the radii of these poles are shown in Fig. 23, where the results of two methods are consistent with each other. From Fig. 23, it can be found that all of

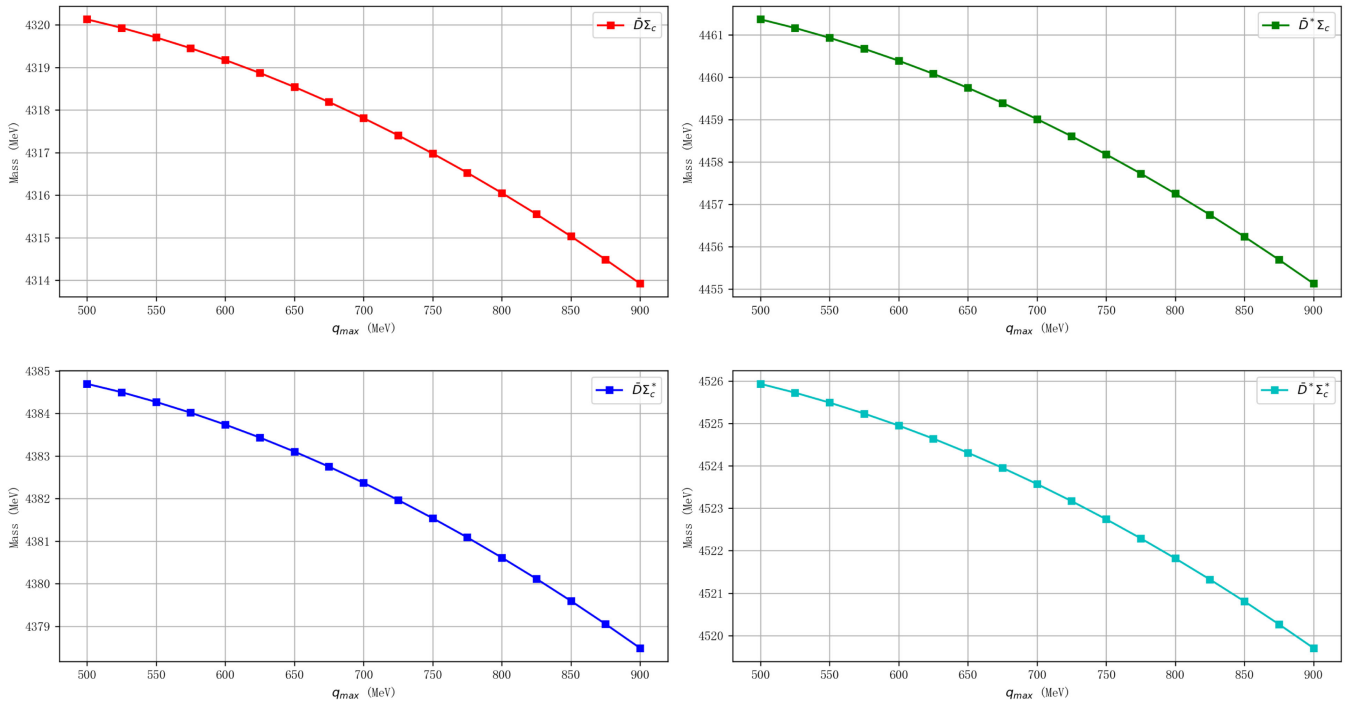


FIG. 18. Trajectories for the masses of the poles in the first Riemann sheet for the single-channel interactions of the hidden charm system.

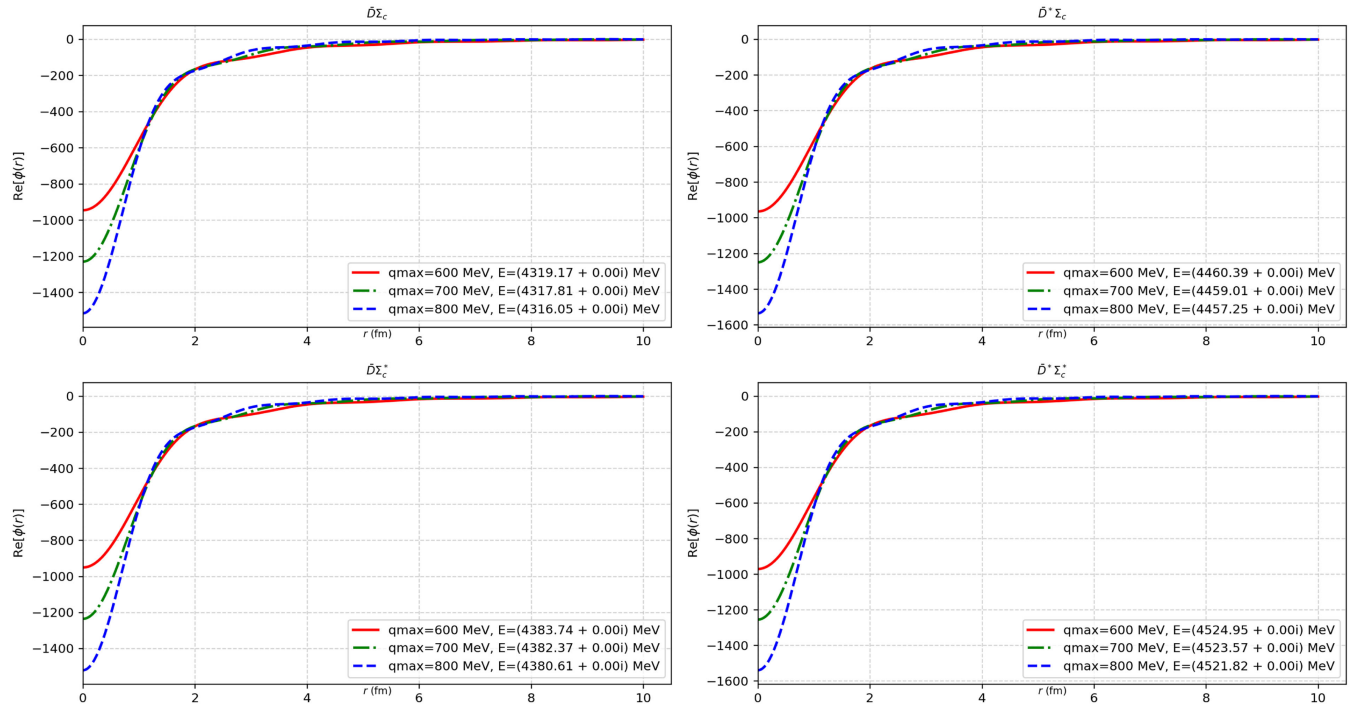


FIG. 19. Wave functions $\phi(r)$ of corresponding pole for the single-channel interactions of the hidden charm system with different q_{max} .

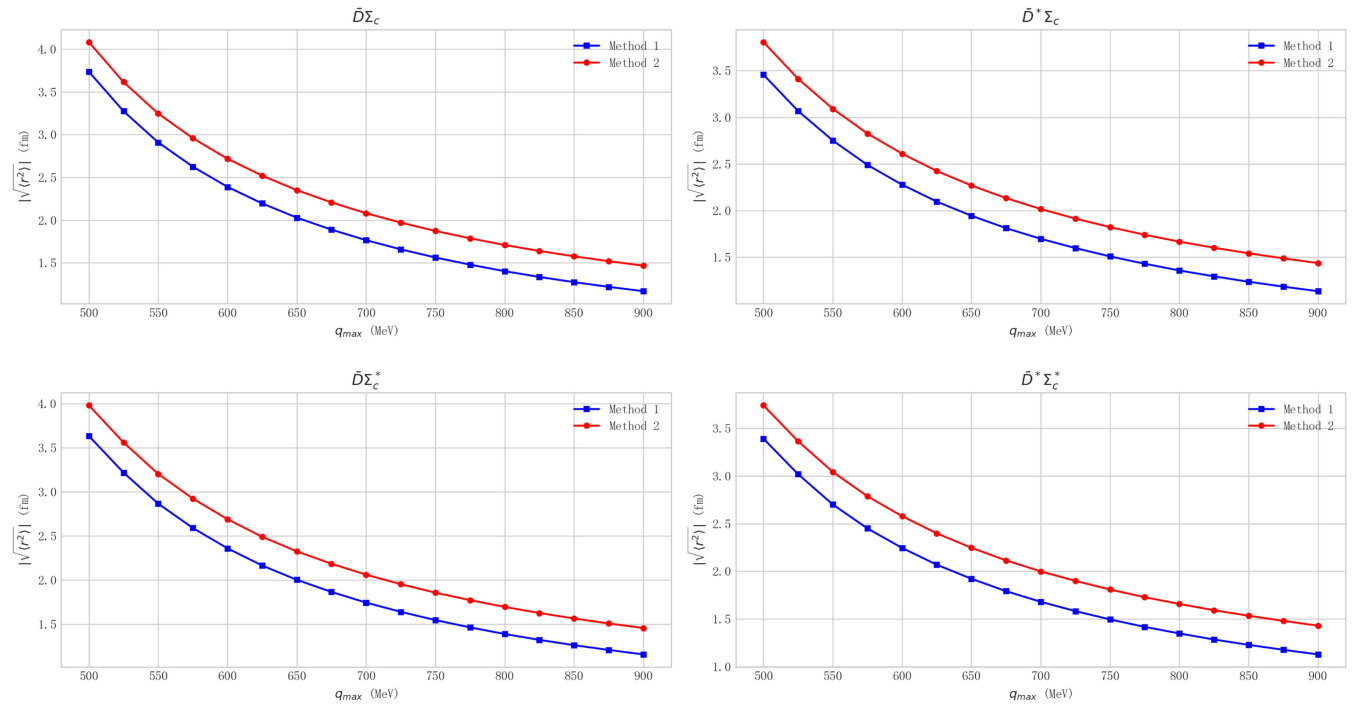


FIG. 20. rms radii of the corresponding poles for the single-channel interactions of the hidden charm system as a function of the cutoff q_{max} . Results from method 1 (blue) and method 2 (red) are compared.

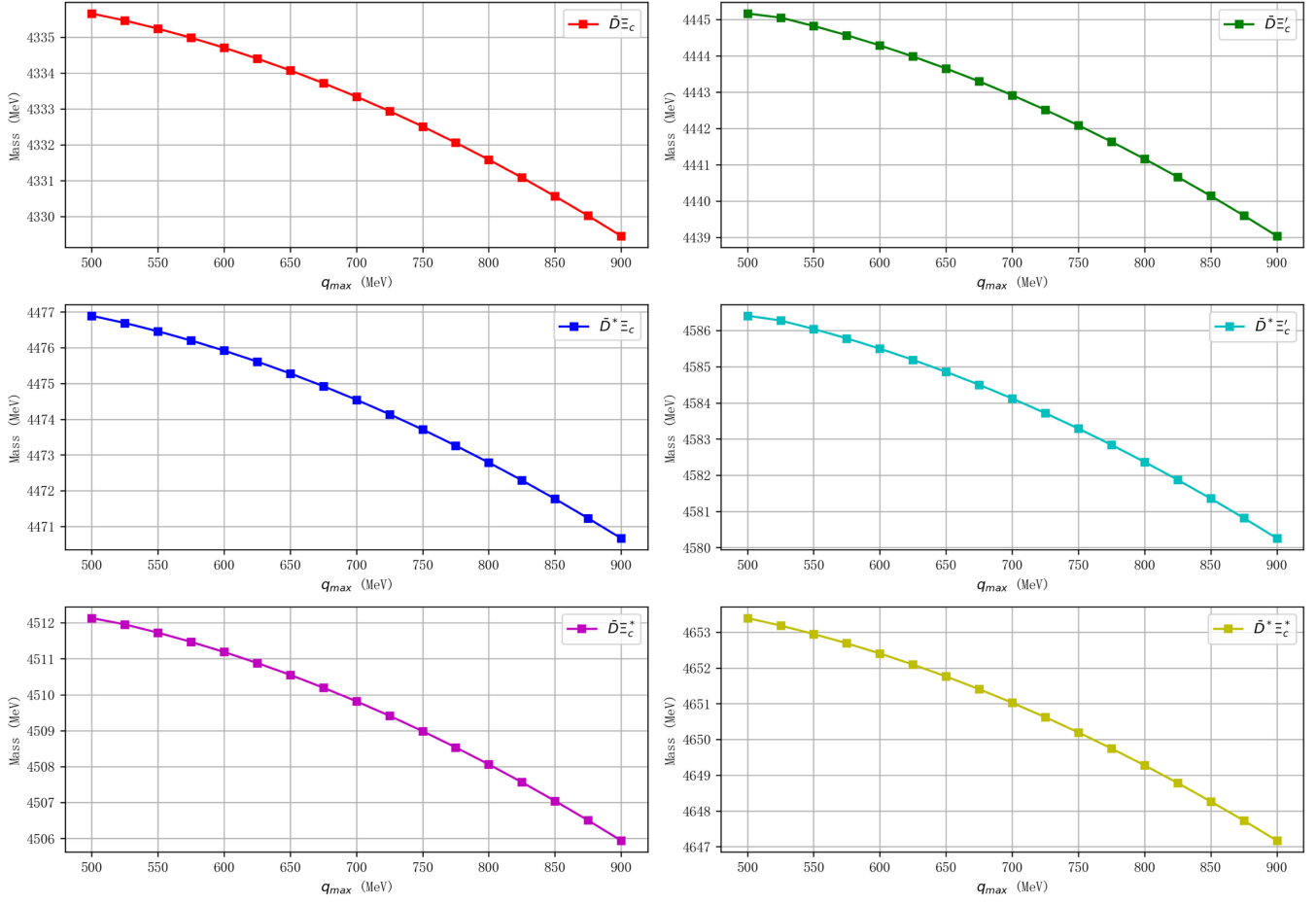


FIG. 21. Trajectories for the masses of the poles in the first Riemann sheet for the single-channel interactions of the hidden charm strange system.

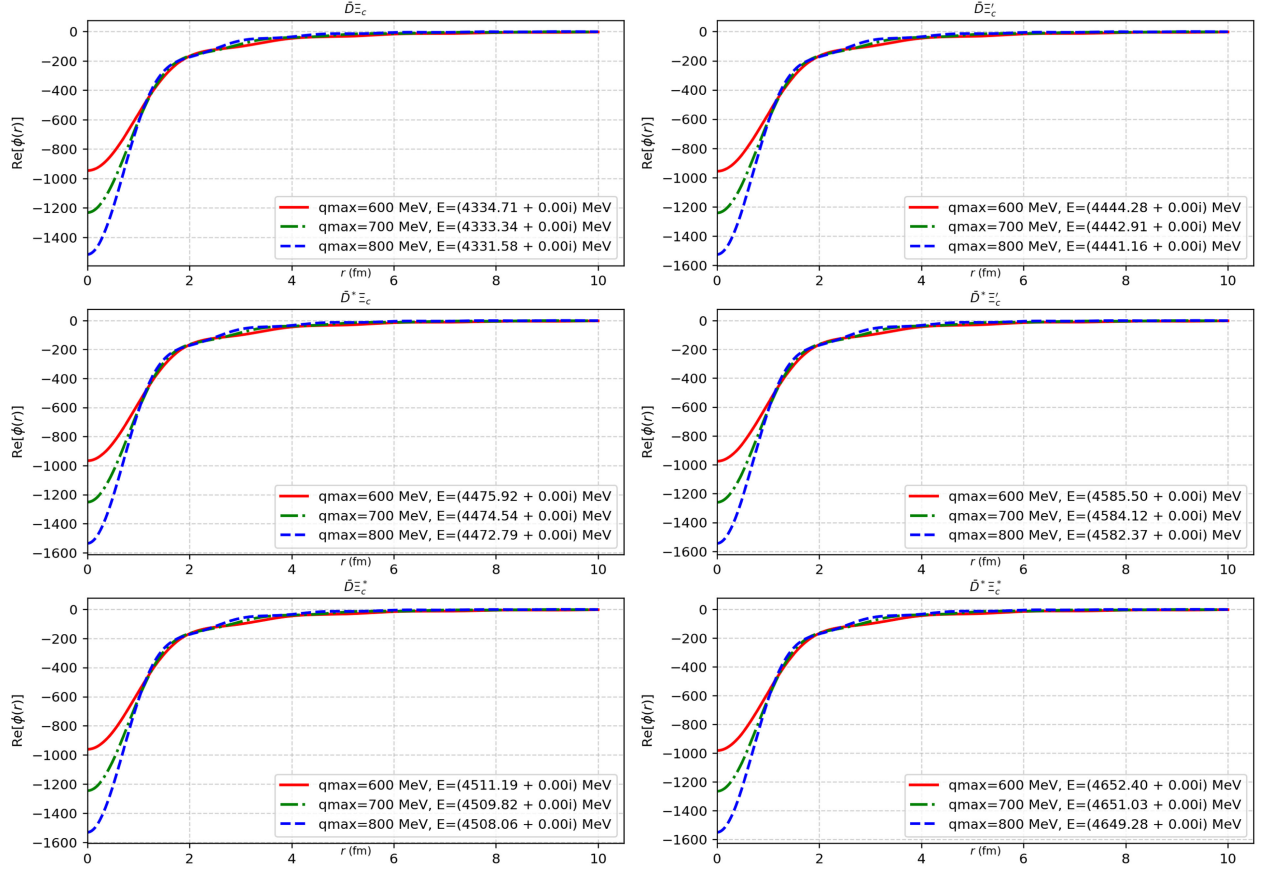


FIG. 22. Wave functions $\phi(r)$ of corresponding poles for the single-channel interactions of the hidden charm strange system with different $q_{\max}$.

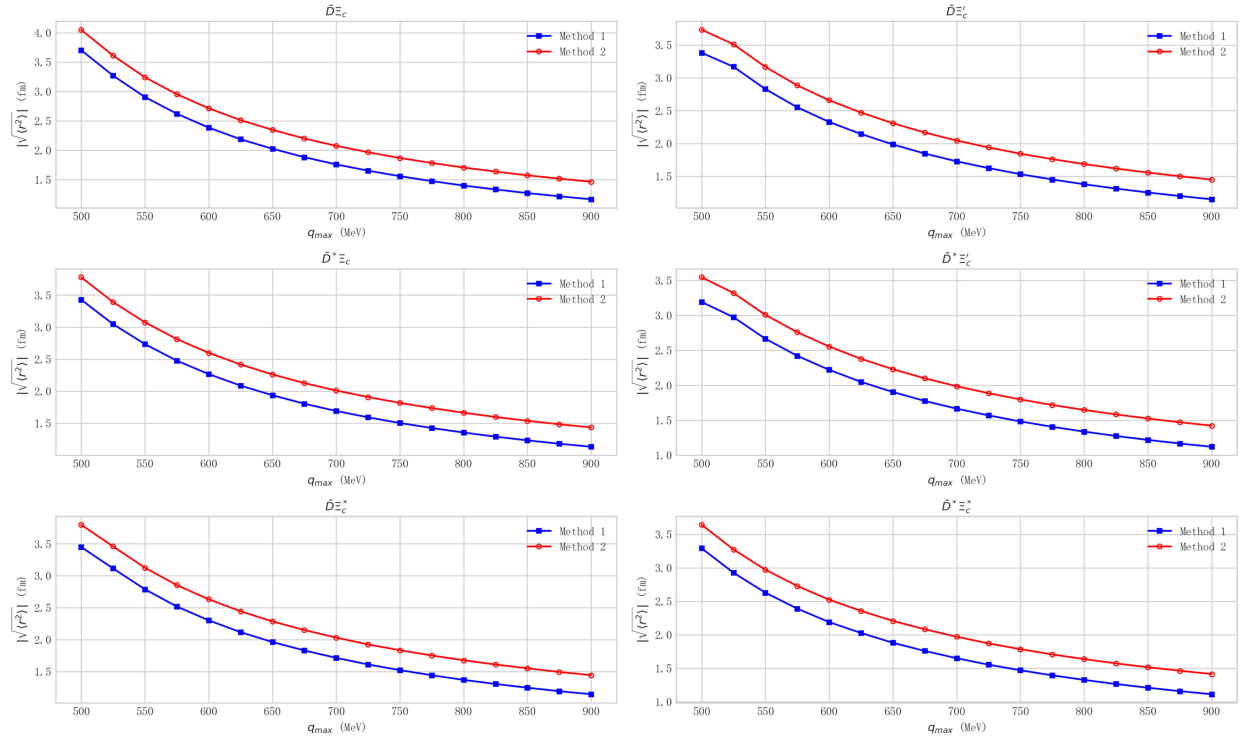


FIG. 23. rms radii of the corresponding poles for the single-channel interactions of the hidden charm strange system as a function of the cutoff $q_{\max}$. Results from method 1 (blue) and method 2 (red) are compared.

these radii are around the range of 1–4 fm, which are also larger than the results of the coupled-channel cases before and consistent with the results obtained in the hidden charm system; see the results of Fig. 20.

IV. CONCLUSIONS

In the present work, we systematically investigate the meson-baryon interactions for the molecular properties of the pentaquark states P_c and P_{cs} in the hidden charm and hidden charm strange systems using a coupled-channel approach, based on our former work that combined the heavy quark spin symmetry and the local hidden gauge formalism. By solving the Bethe-Salpeter equation with the momentum cutoff method, we obtain the poles' trajectories, wave functions, and root-mean-square radii.

For both the hidden charm and hidden charm strange systems, we investigate the interactions of the full coupled-channel systems under the constraint of the heavy quark spin symmetry, the splitting PB and VB sectors, and the single channels for varying the cutoffs, to understand more about the properties of the bound systems. To generate the P_c states, the full coupled-channel interactions with the heavy quark spin symmetry are important, which affect seriously the widths of the corresponding poles. Indeed, in the hidden charm system, the main bound channels are the ones $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, which couple to the lower decay channels strongly too; whereas it is not so necessary to reproduce the P_{cs} state using the full coupled-channel interactions with the heavy quark spin symmetry, showing that the results of the splitting PB and VB sectors are not much different. For the hidden charm strange system, the main bound channels $\bar{D}\Xi_c$, $\bar{D}^*\Xi_c$ couple strongly to the $\bar{D}_s\Lambda_c$, $\bar{D}_s^*\Lambda_c$, respectively, and not much to the lower decay channels, which are different from the case of the hidden charm system. The widths of the corresponding poles for the loose bound

channels, $\bar{D}\Xi_c'$, $\bar{D}^*\Xi_c'$, and $\bar{D}^*\Xi_c^*$, exhibit different behaviors of the trajectories. Note that all the main bound channels have similar binding energies for the single-channel interactions, since they have the same attractive interaction potentials.

Furthermore, we also calculated the wave functions and root-mean-square radii of the corresponding poles. The wave functions exhibit the effective range around 0–6 fm and go to zero fast for $r > 4$ fm. We use two methods to evaluate the root-mean-square radii, the results of which are consistent with each other in most of the cases. The root-mean-square radii are mostly of typical size between 0.5 and 2 fm, which is comparable to the characteristic scale of the molecular states. Indeed, the root-mean-square radii are dependent on the pole trajectories and have different results for the full coupled-channel case, the splitting PB and VB sectors, and the single-channel interactions. From these results, we can understand more about these bound hidden charm and hidden charm strange systems to hint at the molecular nature of the P_c and P_{cs} states.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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