

Efficient truncations of $SU(N_c)$ lattice gauge theory for quantum simulation

Anthony N. Ciavarella^{1,*}, Ivan M. Burbano^{1,2,3,†} and Christian W. Bauer^{1,2,‡}

¹*Physics Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

²*Department of Physics, University of California, Berkeley, Berkeley, California 94720, USA*

³*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*



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Quantum simulations of lattice gauge theories offer the potential to directly study the nonperturbative dynamics of quantum chromodynamics, but naive analyses suggest that they require large computational resources. Large N_c expansions are performed to order $1/N_c$ to simplify the Hamiltonian of pure $SU(N_c)$ lattice gauge theories. A reformulation of the electric basis is introduced with a truncation strategy based on the construction of local Krylov subspaces with plaquette operators. Numerical simulations show that these truncated Hamiltonians are consistent with traditional lattice calculations at relatively small couplings. It is shown that the computational resources required for quantum simulation of time evolution generated by these Hamiltonians is 17–19 orders of magnitude smaller than previous approaches, provided that the truncations in this work can reach lattice spacings in three-dimensional simulations comparable to the two-dimensional simulations performed.

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I. INTRODUCTION

The simulation of lattice gauge theories on quantum computers offers the potential to directly probe the real time dynamics of strongly coupled gauge theories [1,2]. Applying this to quantum chromodynamics (QCD) will enable direct simulations of the process of hadronization [3–6]. Simulations of real-time dynamics will also enable computations of observables relevant to understanding the quark-gluon plasma in the early universe, such as the shear viscosity [7,8]. Inelastic scattering amplitudes can also be computed for processes outside the reach of Luscher’s method [9–14]. The impact of quantum simulation on high energy physics is not restricted to the dynamics of QCD. Other $SU(N_c)$ gauge theories have been proposed as models for grand unification [15–17] or as theories of dark matter [18–22]. Quantum simulations of the non-perturbative aspects of these theories can aid in searches for new physics beyond the standard model.

The potential of quantum simulation to study real-time dynamics has motivated new developments in Hamiltonian lattice gauge theory [23–30]. Direct attempts to encode

lattice gauge theories onto quantum computers have impractically large resource requirements [31–33]. More efficient encodings of the electric basis of gauge theories can be found by exploiting Gauss’s law to reduce the number of degrees of freedom that need to be mapped onto a quantum processor [34–55]. Alternatively, one could use discrete subgroups to encode gauge fields [56–65]. Hybrid approaches that mix these two approaches have also been developed [66–72]. Other approaches using orbifold Hamiltonians [73–75] and quantum link models have also been explored [76–81].

These developments have enabled simulations of low dimensional lattice gauge theories on existing quantum computers. The dynamics of the Schwinger model has been simulated with lattice sizes reaching 112 sites [82–97]. Other $1 + 1D$ nonAbelian theories have been simulated as well [98–104]. The quantum simulation of higher dimensional gauge theories is complicated by the presence of plaquette operators. Limited simulations have been performed on two-dimensional lattices, restricted mostly to smaller sizes [8,35,37,105–113]. The first quantum simulation of an $SU(3)$ lattice gauge theory on a nontrivial two-dimensional lattice was performed by simplifying the Hamiltonian with the use of a large N_c expansion [48].

The large N_c expansion of gauge theories has a long history of providing insight into gauge theories [114–116]. This expansion correctly captures qualitative features of QCD, such as the Okubo-Zweig-Iizuka rule and the emergent Wigner $SU(4)$ symmetry of nuclear interactions [117,118]. It is also an essential ingredient in event generators of collider physics [119,120]. Applications of

*Contact author: anciavarella@lbl.gov

†Contact author: ivan_burbano@berkeley.edu

‡Contact author: cwbauer@lbl.gov

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the large N_c expansion to lattice gauge theory actions have shown that in various parameter regimes, the entire lattice can be described by a single plaquette [121–124]. Recent work has demonstrated how restricting the leading order large N_c piece of the Kogut-Susskind Hamiltonian can dramatically simplify the implementation of quantum simulation of lattice gauge theories [48].

In this work, it is shown how subleading terms in the large N_c expansion of the Kogut-Susskind Hamiltonian can be included in quantum simulations based on the large N_c limit. Truncations of the electric basis based on constructing local Krylov subspaces are introduced. Numerical simulations of SU(3) lattice gauge theory in $2 + 1D$ are performed to develop an understanding of what lattice spacings can be reached with this approach. The resources needed to perform a physically relevant quantum simulation with these truncations are estimated and found to be several orders of magnitude smaller than estimates based on direct encodings of lattice gauge theories.

II. FORMULATION OF SU(N_c) LATTICE GAUGE THEORY HAMILTONIANS IN LARGE N_c EXPANSION TO ORDER $1/N_c$

A. Electric basis choices

The Kogut-Susskind Hamiltonian describing a lattice gauge theory with gauge group SU(N_c) on a d -dimensional spatial lattice is given by

$$\hat{H} = \frac{g^2}{2} \sum_l \hat{E}_l^2 - \frac{1}{2g^2} \sum_p (\hat{\square}_p + \hat{\square}_p^\dagger), \quad (1)$$

where g is the gauge coupling, $\hat{E}_l^2$ is the electric energy operator on link l , and $\hat{\square}_p$ is the trace of the product of parallel transporters around plaquette p . Once an orientation of the lattice has been chosen, the Hilbert space for this system is spanned by electric basis states. In them, an irreducible representation (irrep) $\mathcal{R}$ is attached to each link, and a vector within said irrep is attached to each half-link, as shown in Fig. 1. In this work, we will orient horizontal links from left to right and vertical links from bottom to top. Plaquettes will be oriented counterclockwise. A more precise and pedagogical formulation of the Kogut-Susskind

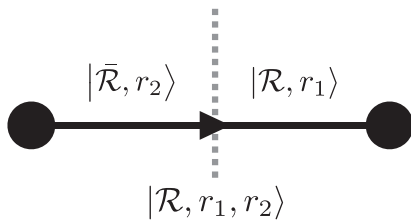


FIG. 1. A generic link is shown equipped with a state $|\mathcal{R}, r_1, r_2\rangle$. The target half-link is equipped with the vector $|\mathcal{R}, r_1\rangle$, while the source half-link is equipped with the vector $|\bar{\mathcal{R}}, r_2\rangle \equiv \langle \mathcal{R}, r_2|$.

Hamiltonian and electric/magnetic bases can be found in the Appendices of [71].

Irreducible representations of SU(N_c) can be specified by their Dynkin indices,

$$\mathcal{R} \triangleq (m_1, m_2, \dots, m_{N_c-1}), \quad (2)$$

or by the corresponding Young diagram,

$$\begin{aligned} (0, 0, \dots, 0, 0) &\equiv \mathbf{1} \\ (1, 0, \dots, 0, 0) &\equiv \square \\ (0, 0, \dots, 0, 1) &\equiv \blacksquare \\ (2, 0, \dots, 0, 0) &\equiv \square\square \\ (0, 0, \dots, 0, 2) &\equiv \blacksquare\blacksquare \\ (0, 1, \dots, 0, 0) &\equiv \begin{array}{|c|} \hline \square \\ \hline \end{array} \\ (0, \dots, 0, 1, 0) &\equiv \begin{array}{|c|} \hline \blacksquare \\ \hline \end{array} \\ (1, 0, \dots, 0, 1) &\equiv \blacksquare\square. \end{aligned}$$

Note that the antifundamental representation will be represented by $\blacksquare$ instead of a stack of $N_c - 1$ boxes. Components of the irrep $\mathcal{R}$, labeled by r , are represented by a Young tableaux that can be formed from the given Young diagram. The number of boxes in a given representation is given by

$$n(\mathcal{R}) = \sum_{k=1}^{\lfloor N_c/2 \rfloor} k(m_k + m_{N_c-k}) + \frac{N_c}{2} m_{N_c/2}, \quad (3)$$

where the last term is only added for even N_c .

Gauss's law requires that at each vertex of the lattice the irrep components of all neighboring links combine together to form a singlet. It is well known that the contractions at vertices have to obey so-called Mandelstam constraints [125,126], which is usually implemented by using a trivalent lattice that only contains vertices with at most 3 links attached to them [35,37,40,42,45]. A general hypercubic lattice can be converted to such a trivalent lattice using the mechanism of point splitting [71]. The Gauss law constraints at each vertex can be enforced by requiring that the representations of all half-links emanating from a given vertex combine into singlets under the gauge group, forming vertex states,

$$\begin{aligned} |v\rangle &\equiv |\phi(\mathcal{R}_A, \mathcal{R}_B, \mathcal{R}_C)\rangle \\ &= \sum_{r_a, r_b, r_c} \frac{C_{\mathcal{R}_A, r_a; \mathcal{R}_B, r_b}^{\mathcal{R}_C, r_c}}{\sqrt{\dim(\mathcal{R}_C)}} |\mathcal{R}_A, r_a\rangle |\mathcal{R}_B, r_b\rangle |\mathcal{R}_C, r_c\rangle, \end{aligned} \quad (4)$$

where $\mathcal{R}_A$, $\mathcal{R}_B$, and $\mathcal{R}_C$ are the representations on the links connecting to the trivalent vertex v when we orient them outward from the vertex, as shown in Fig. 2, and

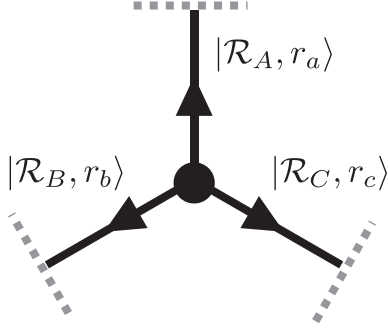


FIG. 2. A generic vertex is shown in the electric basis. The linear combination (4) of these states yields the basis of [37].

$C_{\mathcal{R}_A, r_a; \mathcal{R}_B, r_b}^{\mathcal{R}_C, r_c}$ is a $SU(N_c)$ Clebsch-Gordan coefficient. Note that in $SU(N_c)$, there may be multiple ways to form a representation, and the corresponding multiplicity index in the Clebsch-Gordan coefficient has been suppressed. The Hilbert space is then a subspace of the space spanned by the tensor product of all these vertex states

$$\mathcal{H} = \text{span}\left(\bigotimes_v |v\rangle\right). \quad (5)$$

The subspace corresponding to the physical Hilbert space has the gluing constraint (also referred to as an Abelian Gauss law) that half-links on a common link have the same representation. This is the representation chosen in [37].

An alternative representation of the gauge invariant Hilbert space was introduced in [48], using the fact that $SU(N_c)$ representations can be visualized using an arrow notation. In this notation, $\square$ and $\blacksquare$ are represented by oriented lines with arrows indicating the orientation. By convention, we use an arrow compatible with the orientation of the lattice for the irrep $\square$. Individual lines with arrows pointing in the same direction indicate that the corresponding boxes are arranged next to each other (in a symmetric fashion), while arrows going through multiple lines indicate that the corresponding boxes are arranged on top of each other (in an antisymmetric fashion), with examples given in [48]. In this notation, the vertex factors in Eq. (4) represent how the arrows of the three representations $\mathcal{R}_A$, $\mathcal{R}_B$, and $\mathcal{R}_C$ combine or split at the given vertex, and gauge invariance requires conservation of oriented lines. A gauge invariant state in the absence of any charges therefore contains no source of oriented lines, and therefore all oriented lines form closed loops. Such a state can therefore be represented by specifying all loops, and for links with multiple lines in the same direction how these lines combine into symmetric and antisymmetric pieces. The Hilbert space is therefore given by

$$\mathcal{H} = \text{span}(|\{L_i, a_i\}\rangle), \quad (6)$$

where L_i specifies the i th loop, and a_i how the arrows on link l combine. While this basis is highly nonlocal, it

proves to be useful when performing an expansion in inverse powers of $1/N_c$, as will be discussed next.

B. Simplifications from the $1/N_c$ expansion

So far this discussion has been completely general and has not relied on an expansion in $1/N_c$. Several important simplifications happen when performing this expansion, which we will now discuss. First, to leading order in $1/N_c$ the Casimir of a given representation is given by

$$C(\mathcal{R}) = n(\mathcal{R}) \frac{N_c}{2} (1 + \mathcal{O}(1/N_c)), \quad (7)$$

and the dimension of representations of $SU(N_c)$ scale with the number of boxes as

$$d(\mathcal{R}) \sim N_c^{n(\mathcal{R})}. \quad (8)$$

A second important simplification follows from the large N_c scaling of matrix elements of the plaquette operators. Any physical gauge-invariant state can be built from the electric vacuum state by applying plaquette operators. This allows states to inherit a large N_c suppression from the scaling of the plaquette matrix elements used to create the state. As shown in [48], at leading order in large N_c , the only unsuppressed states are those where loops enclose only a single plaquette. Working strictly to leading order in all terms would therefore require only specifying the number of boxes determining the representation at each plaquette, giving rise to a Hilbert space spanned by

$$\mathcal{H} = \text{span}(|\{n(\mathcal{R}_p)\}\rangle). \quad (9)$$

The corresponding Hamiltonian is very simple, reflecting the well-known factorization of Wilson loops at large N_c [127], and, as shown in Appendix B, is exactly solvable, leading to a theory with vanishing correlation length. Given that taking the continuum limit of the lattice theory requires tuning the lattice theory to the limit of infinite correlation length, one clearly needs to go beyond the strict $1/N_c$ limit. In fact, in [48] it was further shown that the presence of a loop suppresses the state by a factor proportional to N_c^{1-m} , with m being the number of plaquettes enclosed by the loop.

There are several different effects at $\mathcal{O}(1/N_c)$ that need to be taken into account. First, both the electric and magnetic terms in the Hamiltonian receive corrections at that order. The Casimir of the different representations, which are required for the electric contribution to the Hamiltonian, need to be included at subleading order. The same is true for transitions between representations mediated by the magnetic contribution to the Hamiltonian, described by Clebsch-Gordan coefficients. Closed form, analytical expressions exist for the Casimirs of any $SU(N_c)$ group, and are summarized in Appendix C.

Such expressions do not exist for Clebsch-Gordan coefficients, and need to be worked out for the various transitions required. Some general relations that allow the relevant combinations of Clebsch-Gordan coefficients used in this work to be derived are given in Appendix D.

Second, in order to obtain nonzero correlation lengths, working to $\mathcal{O}(1/N_c)$ requires introducing states where loops enclose $m = 2$ plaquettes. This can be understood by the fact that each contraction of a fundamental and antifundamental index of a representation is suppressed by one power of $1/N_c$. Using the same argument, one can obtain a power counting in $1/N_c$ of matrix elements of the plaquette operator. Since a plaquette operator contains Wilson line operators in the fundamental representation on the four links enclosing the plaquette, a transition can create or annihilate up to 4 lines in the state it acts upon. Transitions in which fewer than four lines are created or annihilated are suppressed by powers of $1/N_c$, and the suppression is given by the difference from four of the number of lines annihilated or created. Examples of transitions allowed at this order are shown in Figs. 4–7.

III. TRUNCATING THE ELECTRIC BASIS STATES

In the discussion up to this point, we have retained the full Hilbert space of the gauge boson at each link. Since this Hilbert space is infinite-dimensional, it has to be truncated to make numerical simulations feasible. This truncation restricts the full Hilbert space to a subspace. In practice, one wishes to describe the expectation of observables in a low-energy subspace. For this reason, the electric basis truncation is formed from states exhibiting low energy density on the lattice. Eigenstates of the electric Hamiltonian are characterized by the representations of the gauge fields on each link. Gauss' law, which has to be enforced on states in the physical Hilbert space, is most easily represented using such representations of the gauge field. The most commonly used truncation of the Hilbert space is therefore based on the electric energy density, including only those states that have an electric energy density below a certain value in the lattice. This truncation can be shown to be very efficient in the strong coupling limit, while its efficiency at weaker coupling needs to be studied more carefully [37,103]. In the weak coupling limit, rigorous error bounds show there is an exponential convergence as the electric basis truncation is raised [128]. In this work, we will choose to truncate based on the electric energy density and study the efficiency of this truncation by comparing results in the first few truncations.

The precise definition of the restriction of the energy density used can vary. A common definition is to limit the electric energy on each link, which restricts the representations of the electric basis states at each link. Since the electric energy in a given link is given by the Casimir of the representation of the link, which, as shown in Eq. (7), is determined only by the number of oriented lines on the link

in the large N_c limit, one can construct the subspace by including all gauge invariant states with at most k oriented lines on each link. This corresponds to the $1/N_c$ limit of the basis used in [37]. This basis includes all possible gauge-invariant states. However, a large number of states are suppressed by large powers of $1/N_c$, as one can form arbitrarily large loops from the states contained in this subspace. For this reason, it is useful to consider alternatives to this truncation that allow for an easier implementation of the constraints arising from the $1/N_c$ expansion.

An alternative to the previously considered construction [37] of a truncated subspace in this work is inspired by the Krylov subspace construction. Krylov subspaces are spanned by the vectors generated by repeatedly applying an operator to an initial vector. These subspaces are the basis of the Lanczos algorithm and have proven to be useful in traditional lattice calculations [129–131]. In this work, a localized version of Krylov subspaces is used by constructing states through repeated applications of the plaquette operators, which make up the magnetic Hamiltonian on the electric vacuum state. To limit the local energy densities that can be generated, we restrict the number of times a plaquette operator can be applied in a given local region. First, we require that we can apply a given plaquette operator at most n_p times. Since applications of neighboring plaquette operators both act on the shared link, we furthermore restrict the total number of link operators arising from these plaquette operators at each link to n_l . The two numbers satisfy $n_l \geq n_p$, and the combination (n_p, n_l) defines a plaquette-based truncation.

As is the case for any Krylov subspace construction, this latter construction is defined through the action of operators on sets of states that can be obtained through repeated applications on a starting state, in our case applying plaquette operators to the electric vacuum state. As discussed, the matrix elements of the plaquette operators depend on Clebsch-Gordan coefficients, which can be expanded in powers of $1/N_c$. The precise definition of the truncated subspace, therefore, depends on how this $1/N_c$ expansion is implemented. In this work, we set all transitions between states that are suppressed by two or more powers of $1/N_c$ to zero, while keeping all transitions that arise at $\mathcal{O}(1)$ or $\mathcal{O}(1/N_c)$. An example of a disallowed transition is given in Fig. 3.

Note that, due to transitions at $1/N_c$, the number of oriented lines on any given link can be smaller than the number of link operators applied. One can therefore add a restriction on the total number of oriented lines on each link of the states reached by the application of the plaquette operators, such that each truncation is labeled by the three numbers (n_p, n_l, k) .

The truncation introduced here implies a restriction on the local energy density on the lattice. By construction, all states in this truncation have irreps with at most k oriented

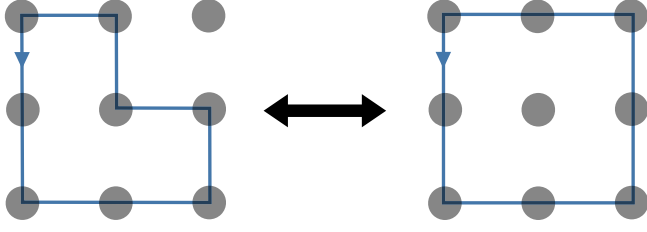


FIG. 3. This transition is of $\mathcal{O}(1/N_c)$. This can be seen because it involves the deletion of a single joined line due to the action of a plaquette operator. However, creating this field configuration from the electric vacuum requires multiple $\mathcal{O}(1/N_c)$ transitions. Accordingly, in this work, we do not attempt to correctly include the matrix elements associated with this transition.

lines on each link. However, there are constraints on the local energy density in a two-plaquette system as well. Consider the total number of oriented lines in any adjacent pair of plaquettes, arising from loops that are contained within the chosen pair. We specify this number and label the largest possible number on the lattice by Λ . The simplest $(1, 1, 1)$ truncation has at most a single plaquette in each pair excited, corresponding to $\Lambda = 4$. This is the truncation used in [48]. The $(1, 2, 1)$ truncation adds a single loop encircling both plaquettes, corresponding to $\Lambda = 6$. The $(1, 2, 2)$ truncation adds states with one loop encircling each plaquette, therefore contributing at $\Lambda = 8$. However, there are other states contributing to $\Lambda = 8$, which arise from the $(2, 2, 2)$ truncation. Thus, one might expect that the $(1, 2, 2)$ truncation might not be a systematic expansion, which will be confirmed in our numerical analysis later.

The next question is how to represent such truncations efficiently in a particular representation of the Hilbert space, and the states discussed so far allow for a particular convenient representation. For the truncations up to $(2, 2, 2)$ that will be used in this work, states can be written as

$$|\{\mathcal{R}_p\}, \{\mathcal{L}_l\}\rangle = \mathcal{N}(\{\mathcal{R}_p\}, \{\mathcal{L}_l\}) \prod_l |\mathcal{L}_l\rangle_l \langle \mathcal{L}_l|_l \prod_p \hat{\square}_p^{(\mathcal{R}_p)} |0\rangle, \quad (10)$$

where $|0\rangle$ is the electric vacuum, $\hat{\square}_p^{(\mathcal{R}_p)}$ is the plaquette operator at plaquette p with parallel transporters in the $\mathcal{R}_p$ representation, $|\mathcal{L}_l\rangle_l \langle \mathcal{L}_l|_l$ is the projection of the state on link l onto the representation $\mathcal{L}_l$, and $\mathcal{N}(\{\mathcal{R}_p\}, \{\mathcal{L}_l\})$ is a normalization constant. Therefore, basis states are specified

by assigning a representation to each plaquette and a representation to each link. The representations on the links are constrained to be an element of the Clebsch-Gordan decomposition of the representations on the neighboring plaquettes. Similar encodings of state labels onto plaquettes have been explored for $U(1)$ [66,132] and $SU(2)$ lattice gauge theories [49,107,108]. Note that the basis introduced in Ref. [49] was restricted to $2 + 1D$ honeycomb lattices with open boundary conditions due to the existence of redundant basis states. In particular, unlike the cubic case, in these lattices all vertices are of the form shown in Fig. 2. The truncation in $1/N_c$ in this work resolves this issue by removing the coupling to the redundant basis states. However, when the truncation is raised above $(2, 2, 2)$, the coupling to redundant basis states is reintroduced, causing this basis to break down. Furthermore, all transitions required in these truncations can be represented by specifying the representations of all of the neighboring plaquettes, as well as the projection $\mathcal{L}_l$ on the links of the plaquette.

A. $(n_p = 1, n_l = 1, k = 1)$ ($\Lambda = 4$) Truncation

The harshest truncation of the Hilbert space allows each plaquette operator to be applied only once and each link to have only a single plaquette operator applied to it. At this truncation, once the representations on the plaquettes have been specified, the representations on the links will be as well. Therefore, this truncation requires a single qutrit per plaquette with basis states labeled by $|\mathbf{1}\rangle$, $|\square\rangle$, and $|\blacksquare\rangle$. This qutrit basis will contain unphysical states (such as neighboring plaquettes both in the $|\square\rangle$ state) that do not correspond to a state in the gauge theory. However, the dynamics of the theory will not couple to these unphysical states. The existence of unphysical states will be a generic feature of the bases used in this work. With this basis, the truncated $SU(N_c)$ Hamiltonian at leading order in large N_c is given by

$$\hat{H} = \sum_p g^2 \left(N_c - \frac{1}{N_c} \right) (|\square\rangle_p \langle \square|_p + |\blacksquare\rangle_p \langle \blacksquare|_p) - \frac{1}{2g^2} \left(\prod_{\hat{n}} |\mathbf{1}\rangle_{p+\hat{n}} \langle \mathbf{1}|_{p+\hat{n}} \right) (|\square\rangle_p \langle \mathbf{1}|_p + |\blacksquare\rangle_p \langle \mathbf{1}|_p + \text{H.c.}), \quad (11)$$

where p labels plaquettes on the lattice and $p + \hat{n}$ specifies neighboring plaquettes in the $\hat{n}$ direction. The transitions allowed here are represented in Fig. 4. Note that at this

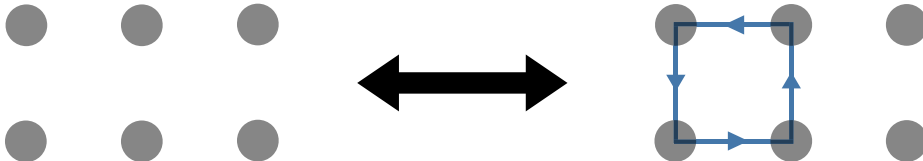


FIG. 4. The transition allowed in the $(1, 1, 1)$ truncation.

truncation, it is possible to enforce the C symmetry (charge conjugation) locally and replace the qutrits with qubits. This is because at this truncation the operators $\hat{C}_p = (|\square\rangle_p - |\blacksquare\rangle_p)(\langle\square|_p - \langle\blacksquare|_p)$ commute with the Hamiltonian, indicating that states where a plaquette is in the state $|\square\rangle_p - |\blacksquare\rangle_p$ are decoupled from the electric vacuum at this truncation. Explicitly, with the basis assignment

$$\begin{aligned} |0\rangle_p &= |\mathbf{1}\rangle_p, \\ |1\rangle_p &= \frac{1}{\sqrt{2}}(|\square\rangle_p + |\blacksquare\rangle_p), \end{aligned} \quad (12)$$

the Hamiltonian for the even C sector becomes

$$\begin{aligned} \hat{H} &= \sum_p g^2 \left(N_c - \frac{1}{N_c} \right) |1\rangle_p \langle 1|_p \\ &\quad - \frac{1}{\sqrt{2}g^2} \left(\prod_{\hat{n}} |0\rangle_{p+\hat{n}} \langle 0|_{p+\hat{n}} \right) \hat{X}_p, \end{aligned} \quad (13)$$

where $\hat{X}_p$ is the Pauli X operator acting on the qubit at plaquette p . Note that compared to our previous $SU(3)$ Hamiltonian [48], there is no term from the plaquette proportional to $|1\rangle_p \langle 1|_p$ as that was a unique feature of $SU(3)$. In the formulation in this work, this term will only be present in higher truncations of the theory.

B. ($n_p = 1, n_l = 2, k = 1$) ($\Lambda = 6$) Truncation

The next nontrivial truncation is the $(1, 2, 1)$ truncation. This truncation allows neighboring plaquettes to be excited with their shared link in the 1 representation. In a two-plaquette system, there could be up to $\Lambda = 6$ excited links. At leading order in large N_c , this is still equivalent to the $(1, 1, 1)$ truncation. At order $1/N_c$, the plaquette operator will be modified to allow neighboring plaquettes to be excited, as shown in Fig. 5. In this truncation, states can still be represented using a qutrit per plaquette. If neighboring plaquettes are both in the $|\blacksquare\rangle$ ($|\square\rangle$) state it will be interpreted as a loop of electric flux flowing (counter) clockwise around the two plaquettes with their shared link unexcited. The Hamiltonian is given by

$$\begin{aligned} \hat{H}^{(1,2,1),1/N_c} &= \hat{H}_E^{(1,2,1),1/N_c} + \hat{H}_B^{(1,2,1),1/N_c}, \\ \hat{H}_E^{(1,2,1),1/N_c} &= g^2 \left(N_c - \frac{1}{N_c} \right) \sum_p \left[(|\square\rangle_p \langle\square|_p + |\blacksquare\rangle_p \langle\blacksquare|_p) \right. \\ &\quad \times \left. \frac{1}{4} \sum_{\hat{n}} (|\square\rangle_p \langle\square|_p |\square\rangle_{p+\hat{n}} \langle\square|_{p+\hat{n}} + |\blacksquare\rangle_p \langle\blacksquare|_p |\blacksquare\rangle_{p+\hat{n}} \langle\blacksquare|_{p+\hat{n}}) \right], \\ \hat{H}_B^{(1,2,1),1/N_c} &= -\frac{1}{2g^2} \sum_p \left(\prod_{\hat{n}} |\mathbf{1}\rangle_{p+\hat{n}} \langle\mathbf{1}|_{p+\hat{n}} \right) (|\square\rangle_p \langle\mathbf{1}|_p + |\blacksquare\rangle_p \langle\mathbf{1}|_p + \text{H.c.}) \\ &\quad - \frac{1}{2g^2 N_c} \sum_p \sum_{\hat{k}} |\square\rangle_{p+\hat{k}} \langle\square|_{p+\hat{k}} \prod_{\hat{n} \neq \hat{k}} (|\mathbf{1}\rangle_{p+\hat{n}} \langle\mathbf{1}|_{p+\hat{n}}) (|\square\rangle_p \langle\mathbf{1}|_p + \text{H.c.}) \\ &\quad - \frac{1}{2g^2 N_c} \sum_p \sum_{\hat{k}} |\blacksquare\rangle_{p+\hat{k}} \langle\blacksquare|_{p+\hat{k}} \prod_{\hat{n} \neq \hat{k}} (|\mathbf{1}\rangle_{p+\hat{n}} \langle\mathbf{1}|_{p+\hat{n}}) (|\blacksquare\rangle_p \langle\mathbf{1}|_p + \text{H.c.}). \end{aligned} \quad (14)$$

The form of the plaquette operator at this truncation can be derived using the methods in Appendix D. At this truncation, $\hat{C}_p$ is no longer a conserved quantity. However, it is still possible to construct a local basis for the even C sector using a single qubit per plaquette. We can define (a nonlocal) loop basis by

$$|\{\mathcal{L}_i\}\rangle = \mathcal{N}(\{\mathcal{L}_i\}) \left(\prod_i \prod_{p \in B_i} |\square\rangle_p + \prod_i \prod_{p \in B_i} |\blacksquare\rangle_p \right), \quad (15)$$

where $\{\mathcal{L}_i\}$ is a set of nonintersecting closed loops $\mathcal{L}_i$, B_i is the minimal region of the lattice with $\partial B_i = \mathcal{L}_i$, and

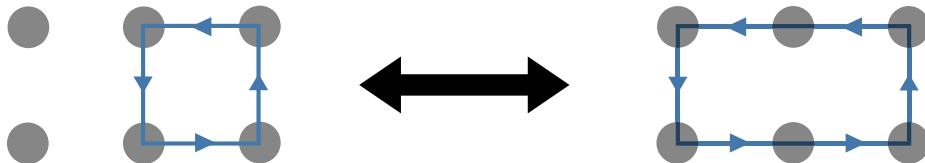


FIG. 5. The new transition allowed in the $(1, 2, 1)$ truncation.

$\mathcal{N}(\{\mathcal{L}_i\})$ is a normalization constant. Any even C state that can be generated through the dynamics of the electric vacuum or a state with localized excitations can be expressed in this basis. This loop basis can be represented locally by assigning a qubit to each plaquette. If the plaquette is an element of B_i , then the qubit is in the $|1\rangle$ state and otherwise the qubit is in the $|0\rangle$ state. Regions of qubits in the $|1\rangle$ state correspond to there being an even superposition of flux flowing clockwise and counterclockwise around the boundary of the region. With this encoding, the Hamiltonian is

$$\begin{aligned} \hat{H}^{(1,2,1)} = & \sum_p g^2 \left(N_c - \frac{1}{N_c} \right) |1\rangle_p \langle 1|_p - \frac{g^2 (N_c - \frac{1}{N_c})}{4} \sum_{\hat{n}} |1\rangle_p \langle 1|_p |1\rangle_{p+\hat{n}} \langle 1|_{p+\hat{n}} \\ & - \frac{1}{\sqrt{2}g^2} \left(\prod_{\hat{n}} |0\rangle_{p+\hat{n}} \langle 0|_{p+\hat{n}} \right) \hat{X}_p - \frac{1}{2g^2 N_c} \sum_{\hat{k}} |1\rangle_{p+\hat{k}} \langle 1|_{p+\hat{k}} \left(\prod_{\hat{n} \neq \hat{k}} |0\rangle_{p+\hat{n}} \langle 0|_{p+\hat{n}} \right) \hat{X}_p. \end{aligned} \quad (16)$$

C. ($n_p = 1, n_l = 2, k = 2$) Truncation

The next (n_p, n_l, k) truncation will still allow each plaquette operator to only be applied a single time but will allow neighboring plaquette operators to be applied and excite higher irreps. At leading order in large N_c , these shared links can support higher irreps labeled by $\blacksquare$ (the adjoint irrep), $\square\square$ (two indices combined symmetrically), and $\square\square$ (two indices combined antisymmetrically), along with their conjugate representations. Note that for $SU(3)$ $\blacksquare$ is the same as the $\square\square$ representation, but for larger N_c this is a different representation. To represent electric basis states, a qutrit will be assigned to each plaquette, and a qubit will be assigned to each link. When all qubits are in the zero state, the qutrit basis assignment is the same as the previous section, specifying if there is a loop of electric flux flowing around the plaquette. When neighboring plaquettes are excited, there is an ambiguity in the state of their shared link. If both plaquettes have a loop of flux in the same orientation, their shared link can be in the $\mathbf{1}$ or $\blacksquare$ irrep. However, at leading order in large N_c the $\mathbf{1}$ irrep is not present. If the plaquettes have opposite orientations, the shared link can be in the $\square\square$ or $\square\square$, or the conjugate of these representations, depending on the orientation of the link. The qubit on the shared link will be used to resolve this ambiguity. If the qubit is in the $|0\rangle$ state, the shared link will be in the $\mathbf{1}$ or $\square\square$, depending on the neighboring qutrits, and if the qubit is in the $|1\rangle$ state, the shared link will be in the $\blacksquare$ or $\square\square$ irrep. With this basis, the $SU(N_c)$ Hamiltonian at this truncation on a two-dimensional lattice is given by

$$\begin{aligned} \hat{H}^{(1,2,2)} &= \hat{H}_E^{(1,2,2)} + \hat{H}_B^{(1,2,2)}, \\ \hat{H}_E^{(1,2,2)} &= g^2 \sum_p \left(N_c - \frac{1}{N_c} \right) (|\square\square\rangle_p \langle \square\square|_p + |\blacksquare\rangle_p \langle \blacksquare|_p) \\ &\quad + g^2 \sum_{(p,p')} \left[-\frac{N_c - \frac{1}{N_c}}{2} (|\square\square\rangle_p \langle \square\square|_p |\square\square\rangle_{p'} \langle \square\square|_{p'} + |\blacksquare\rangle_p \langle \blacksquare|_p |\blacksquare\rangle_{p'} \langle \blacksquare|_{p'}) |0\rangle_l \langle 0|_l \right. \\ &\quad + \frac{1}{2N_c} (|\square\square\rangle_p \langle \square\square|_p |\square\square\rangle_{p'} \langle \square\square|_{p'} + |\blacksquare\rangle_p \langle \blacksquare|_p |\blacksquare\rangle_{p'} \langle \blacksquare|_{p'}) |1\rangle_l \langle 1|_l \\ &\quad - \frac{1 + \frac{1}{N_c}}{2} (|\square\square\rangle_p \langle \square\square|_p |\blacksquare\rangle_{p'} \langle \blacksquare|_{p'} + |\blacksquare\rangle_p \langle \blacksquare|_p |\square\square\rangle_{p'} \langle \square\square|_{p'}) |0\rangle_l \langle 0|_l \\ &\quad \left. + \frac{1 - \frac{1}{N_c}}{2} (|\square\square\rangle_p \langle \square\square|_p |\blacksquare\rangle_{p'} \langle \blacksquare|_{p'} + |\blacksquare\rangle_p \langle \blacksquare|_p |\square\square\rangle_{p'} \langle \square\square|_{p'}) |1\rangle_l \langle 1|_l \right], \\ \hat{H}_B^{(1,2,2)} &= \frac{1}{2g^2} \sum_p \left(\prod_{\hat{n}} \hat{C}_{\square\square}^{p+\hat{n}} |\square\square\rangle_p \langle \mathbf{1}|_p + \prod_{\hat{n}} \hat{C}_{\blacksquare}^{p+\hat{n}} |\blacksquare\rangle_p \langle \mathbf{1}|_p + \text{H.c.} \right), \\ \hat{C}_{\mathbf{R}}^{p+\hat{n}} &= |\mathbf{1}\rangle_{p+\hat{n}} \langle \mathbf{1}|_{p+\hat{n}} |0\rangle_l \langle 0|_l + \sqrt{1 - \frac{1}{N_c^2}} |\mathbf{R}\rangle_{p+\hat{n}} \langle \mathbf{R}|_{p+\hat{n}} |1\rangle_l \langle 0|_l \\ &\quad + \sqrt{\frac{1}{2} \left(1 - \frac{1}{N_c} \right)} |\bar{\mathbf{R}}\rangle_{p+\hat{n}} \langle \bar{\mathbf{R}}|_{p+\hat{n}} |0\rangle_l \langle 0|_l + \sqrt{\frac{1}{2} \left(1 + \frac{1}{N_c} \right)} |\bar{\mathbf{R}}\rangle_{p+\hat{n}} \langle \bar{\mathbf{R}}|_{p+\hat{n}} |1\rangle_l \langle 0|_l, \end{aligned} \quad (17)$$

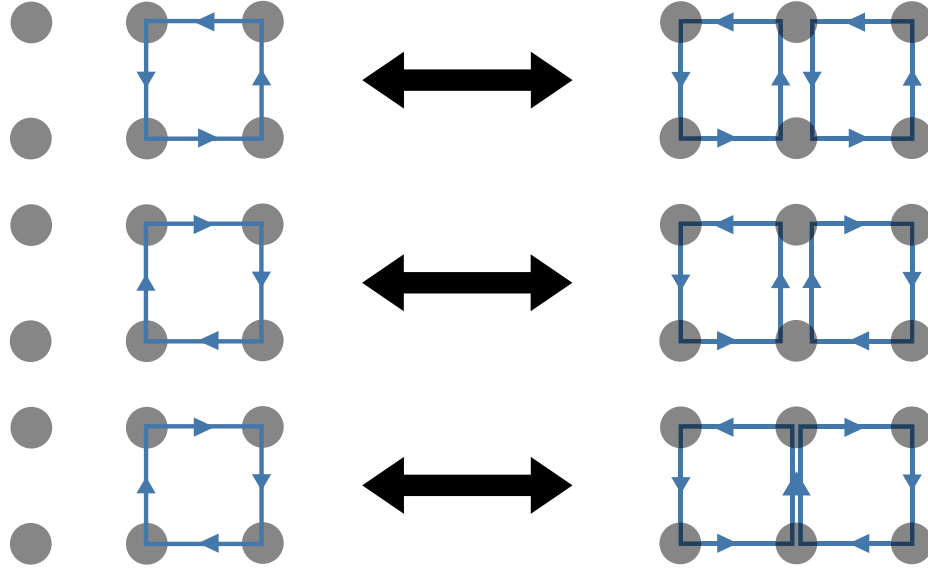


FIG. 6. New transitions allowed in the (1, 2, 2) truncation. The state depicted on the top right corresponds to a state of the form $|\square\rangle_p |1\rangle_l |\square\rangle_{p+\hat{n}}$. From (D1), we see this transition has a prefactor $\sqrt{1 - \frac{1}{N_c}}$, once we recall that the adjoint representation $\blacksquare$ has dimension $N_c^2 - 1$. Similarly, the state on the middle right corresponds to $|\square\rangle_p |1\rangle_l |\blacksquare\rangle_{p+\hat{n}}$, and the associated transition has prefactor $\sqrt{\frac{1}{2}(1 + \frac{1}{N_c})}$, once we recall that $\square\square$ has dimension $N_c(N_c + 1)/2$. Finally, the state on the lower right corresponds to $|\square\rangle_p |0\rangle_l |\blacksquare\rangle_{p+\hat{n}}$, and the associated transition has prefactor $\sqrt{\frac{1}{2}(1 - \frac{1}{N_c})}$, once we recall that $\square\square$ has dimension $N_c(N_c - 1)/2$.

where p and p' are neighboring plaquettes, and l is their shared link. Note that the matrix elements of the plaquette operator at this truncation can be derived using the results from the authors' previous work [48], as described in Appendix D. The new transitions included in this truncation are depicted in Fig. 6. The Hamiltonian in higher spatial dimensions takes a similar form, except the plaquette operators have

additional projectors to ensure no link carries more than two excitations.

The $1/N_c$ corrections at this truncation require including states with neighboring plaquettes excited and their shared link in the 1 irrep. In the loop notation, these states look like loops of electric flux extending over multiple plaquettes. At this truncation, these states can still be represented using the basis described in Sec. III C. The Hamiltonian becomes

$$\begin{aligned} \hat{H} &= \hat{H}_E^{(1,2,2)} + \hat{H}_B^{(1,2,2),1/N_c}, \\ \hat{H}_B^{(1,2,2),1/N_c} &= \hat{H}_B^{(1,2,2)} - \frac{1}{2g^2 N_c} \sum_p \sum_{\hat{k}} \left(|\square\rangle_{p+\hat{k}} \langle \square|_{p+\hat{k}} |0\rangle_l \langle 0|_l \prod_{\hat{n} \neq \hat{k}} C_{\square}^{p+\hat{n}} |\square\rangle_p \langle 1|_p + \text{H.c.} \right) \\ &\quad - \frac{1}{2g^2 N_c} \sum_p \sum_{\hat{k}} \left(|\blacksquare\rangle_{p+\hat{k}} \langle \blacksquare|_{p+\hat{k}} |0\rangle_l \langle 0|_l \prod_{\hat{n} \neq \hat{k}} C_{\blacksquare}^{p+\hat{n}} |\blacksquare\rangle_p \langle 1|_p + \text{H.c.} \right), \end{aligned} \quad (18)$$

where $\hat{k}$ is a unit vector in the k th direction, and l is the shared link for plaquettes p and $p + \hat{k}$. At this truncation, it is still possible to construct a loop basis for the even C sector similar to the one used for the (1, 2, 1) truncation. However, at this truncation, it is possible for disjoint loops to share links, which prevents an analogous construction of a local basis for the even C sector.

D. ($n_p = 2$, $n_l = 2$, $k = 2$) ($\Lambda = 8$) Truncation

The next truncation allows each plaquette operator to be applied twice and each link to have at most two plaquette operators applied to it. In a two-plaquette system, this would allow up to $\Lambda = 8$ units of electric flux. This will allow states that have plaquettes with a loop of flux in the $\square$, $\blacksquare$,

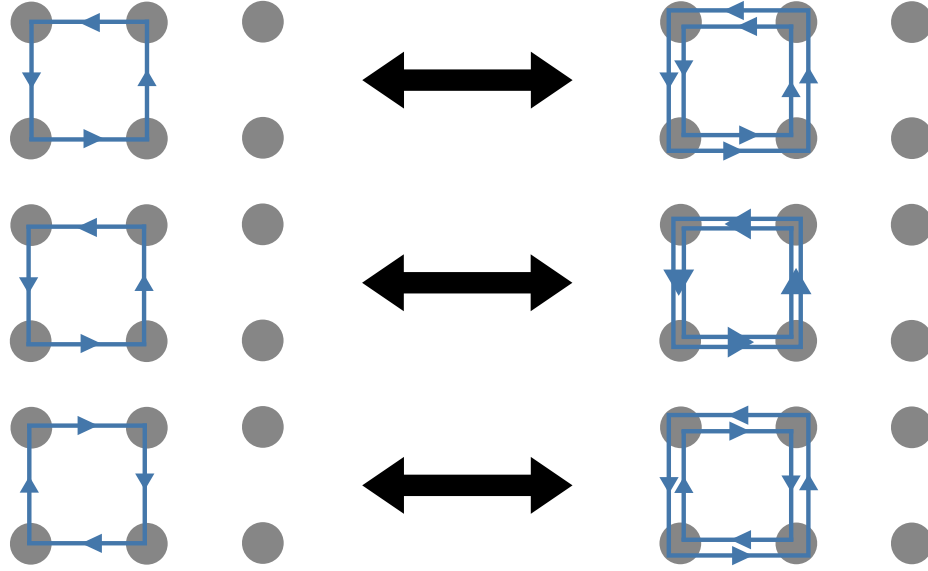


FIG. 7. New transitions allowed in the (2, 2, 2) truncation.

$\blacksquare$, $\square$, or $\blacksquare$ representation, as shown in Fig. 7. Basis states at this truncation can be represented similarly to the (1, 2, 2) truncation, with the qutrits replaced by a qu8. The qubits on the links will be used to resolve ambiguities as before. The Hamiltonian at leading order in large N_c is given by

$$\begin{aligned}
 \hat{H}^{(2,2,2)} &= \hat{H}_E^{(2,2,2)} + \hat{H}_B^{(2,2,2)} \\
 \hat{H}_E^{(2,2,2)} &= \hat{H}_E^{(1,2,2)} + \sum_p 2N_c g^2 |\blacksquare\square\rangle_p \langle \blacksquare\square|_p + 2g^2 \left(N_c + 1 - \frac{2}{N_c} \right) \left(|\square\square\rangle_p \langle \square\square|_p + |\blacksquare\blacksquare\rangle_p \langle \blacksquare\blacksquare|_p \right) \\
 &\quad + 2g^2 \left(N_c - 1 - \frac{2}{N_c} \right) \left(|\square\square\rangle_p \langle \square\square|_p + |\blacksquare\blacksquare\rangle_p \langle \blacksquare\blacksquare|_p \right) \\
 \hat{H}_B^{(2,2,2)} &= \hat{H}_B^{(1,2,2)} - \frac{1}{2g^2} \sum_p \prod_{\hat{n}} |\mathbf{1}\rangle_{p+\hat{n}} \langle \mathbf{1}|_{p+\hat{n}} \left(|\square\square\rangle_p \langle \square\square|_p + |\blacksquare\blacksquare\rangle_p \langle \blacksquare\blacksquare|_p + |\blacksquare\square\rangle_p \langle \square\blacksquare|_p + |\square\blacksquare\rangle_p \langle \blacksquare\square|_p \right. \\
 &\quad \left. + |\square\square\rangle_p \langle \square\square|_p + |\blacksquare\blacksquare\rangle_p \langle \blacksquare\blacksquare|_p + \text{h.c.} \right).
 \end{aligned} \tag{19}$$

The $1/N_c$ corrections to this Hamiltonian are obtained by replacing $\hat{H}_B^{(1,2,2)}$ with $\hat{H}_B^{(1,2,2),1/N_c}$.

In the case of $SU(3)$, $\square = \blacksquare$ and $\blacksquare = \square$, which means one only needs a qu6 for each plaquette instead of a qu8. Explicitly for $SU(3)$, the Hamiltonian at this truncation is

$$\begin{aligned}
 \hat{H}^{(2,2,2)} &= \hat{H}_E^{(2,2,2)} + \hat{H}_B^{(2,2,2)} \\
 \hat{H}_E^{(2,2,2)} &= \hat{H}_E^{(1,2,2)} + \sum_p 6g^2 |\blacksquare\square\rangle_p \langle \blacksquare\square|_p + g^2 \frac{20}{3} \left(|\square\square\rangle_p \langle \square\square|_p + |\blacksquare\blacksquare\rangle_p \langle \blacksquare\blacksquare|_p \right) \\
 \hat{H}_B^{(2,2,2)} &= \hat{H}_B^{(1,2,2)} - \frac{1}{2g^2} \sum_p \prod_{\hat{n}} |\mathbf{1}\rangle_{p+\hat{n}} \langle \mathbf{1}|_{p+\hat{n}} \left(|\blacksquare\blacksquare\rangle_p \langle \square\square|_p + |\blacksquare\square\rangle_p \langle \square\blacksquare|_p + |\blacksquare\square\rangle_p \langle \blacksquare\square|_p \right. \\
 &\quad \left. + |\square\square\rangle_p \langle \square\square|_p + |\blacksquare\blacksquare\rangle_p \langle \blacksquare\blacksquare|_p + \text{h.c.} \right).
 \end{aligned} \tag{20}$$

IV. NUMERICAL COMPARISON

To understand how these truncations perform in practice, the ground states of these SU(3) Hamiltonians were computed for small lattices in $2 + 1D$ using the density matrix renormalization group (DMRG) [133] implemented using the C++ and Julia *iTensor* libraries [134]. These calculations were performed using open boundary conditions. Calculations were performed for each truncation, including the $1/N_c$ corrections. Details of the DMRG are given in Appendix E. Since the continuum limit is located at $g = 0$, higher truncations should allow a closer approach to the continuum limit, and as the truncation is raised, the truncated Hamiltonian should approximate the untruncated theory better at small couplings g . To understand how a small lattice spacing can be achieved, the mass of the 0^{++} glueball was computed. In $3 + 1D$, the mass could be matched to a traditional lattice calculation of the glueball mass to set the lattice spacing. In traditional lattice calculations, particle masses are extracted by fitting correlation functions of operators with the appropriate quantum numbers. In this work, the glueball mass in $2 + 1D$ will be computed by fitting the vacuum correlation function of $\hat{\mathcal{O}}(p) = |\square\rangle_p \langle \square|_p + |\blacksquare\rangle_p \langle \blacksquare|_p$ to an exponential. Explicitly, the set of data points used in the fit was

$$\{(\|\vec{r}_1 - \vec{r}_2\|, \langle \Omega | \hat{\mathcal{O}}(\vec{r}_1) \hat{\mathcal{O}}(\vec{r}_2) | \Omega \rangle - \langle \Omega | \hat{\mathcal{O}}(\vec{r}_1) | \Omega \rangle \langle \Omega | \hat{\mathcal{O}}(\vec{r}_2) | \Omega \rangle) : \|\vec{r}_1 - \vec{r}_2\| \geq 1\}, \quad (21)$$

where $|\Omega\rangle$ is the vacuum state found using DMRG. For points that are a distance $d \equiv \|\vec{r}_1 - \vec{r}_2\|$ apart, this correlation function asymptotes to $ae^{-m_G d}$, which allows one to extract the glueball mass, m_G . An example of a correlation function is shown in Fig. 8 for the (1, 1, 1) truncation with $g = 0.8$ on an 8×8 lattice. The points at fixed separations are not the same due to the use of the open boundary

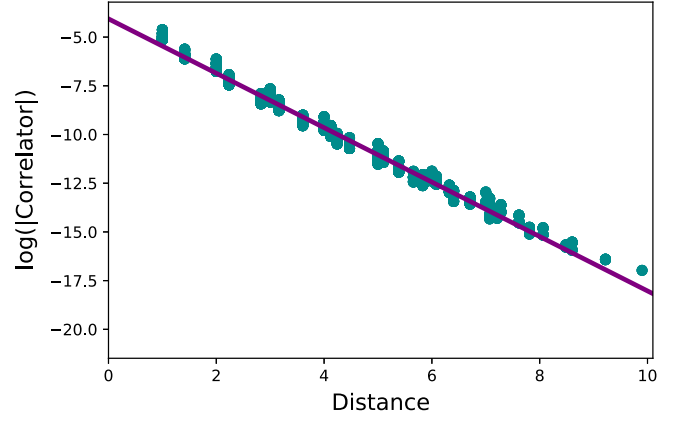


FIG. 8. The blue points show the correlation function defined in Eq. (21) for the (1, 1, 1) truncation with $g = 0.8$ on an 8×8 lattice. The purple line is the fit to an exponential function.

conditions in a finite volume. It is interesting to note that the points at noninteger separations (corresponding to lattice sites connected by a diagonal line) are still along the exponential fit. This is a reflection of the restoration of rotational symmetry, which is necessary to approach the continuum limit. The results of this calculation for all truncations are shown in Fig. 9 alongside a calculation of the glueball mass performed with the traditional action [135]. Surprisingly, these low-lying truncations (with the exception of the (1, 2, 2) truncation) appear to be consistent with the traditional action to around $g \approx 0.4$. These calculations were done on different lattice sizes, so exact agreement would not be expected due to finite volume effects. Finite volume effects are not expected to be large in this calculation as the finite volume corrections to a mass scale as $\mathcal{O}(e^{-mL})$ [9].

The (1, 2, 2) truncation fails to agree with the untruncated calculation at all and has the lattice spacing freezing out at a larger value than the other truncations. At first sight,

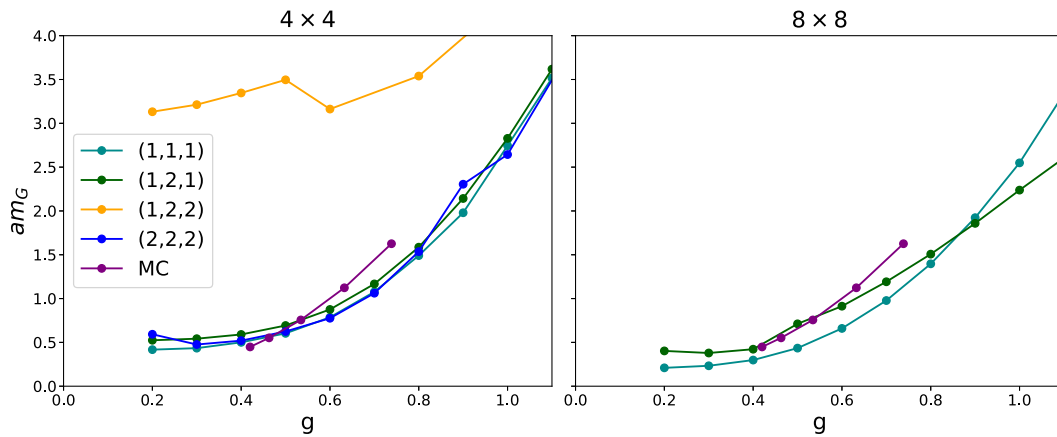


FIG. 9. Mass of the 0^{++} glueball computed at different truncations as a function of g . The left panel shows the DMRG calculations performed on a 4×4 lattice and the right panel shows the DMRG calculations performed on an 8×8 lattice. The Monte Carlo data was taken from Table XVIII of reference [135].

this is unexpected as the $(1, 2, 2)$ truncation has more states included than the $(1, 2, 1)$ truncation, so it would be expected to be in better agreement with the untruncated theory. It is unclear if similar behavior will occur in truncations with larger (n_P, n_L, k) , where $n_P \neq n_L$. Note that a strong coupling expansion of the vacuum state performed with the $(1, 1, 1)$ truncation will agree with an expansion performed with the untruncated Hamiltonian to first order. The $(2, 2, 2)$ truncation agrees to second order while the $(1, 2, 2)$ truncation is only in agreement to first order. Naively, improving the performance of the truncation of the theory at weak coupling would also improve agreement in the strong coupling regime. From this perspective, it would not be unexpected for the $(1, 2, 2)$ truncation to fail to improve over the $(1, 1, 1)$ truncation. However, this does not explain why it appears to perform worse than the $(1, 1, 1)$ truncation.

The unexpected behavior of the $(1, 2, 2)$ truncation can be better understood by taking a more general view of Hamiltonian truncations. Instead of viewing truncations as a projection on the Hilbert space, they can be treated as a deformation of the untruncated Hamiltonian that decouples parts of the Hilbert space. Explicitly, a deformed version of the Kogut-Susskind Hamiltonian can be given by

$$\hat{H}(\vec{\epsilon}) = \frac{g^2}{2} \hat{H}_E + \frac{1}{2g^2} \sum_i \epsilon_i \hat{H}_i, \quad (22)$$

where $\hat{H}_E$ is the full electric Hamiltonian, and $\hat{H}_i$ are different partitions of the magnetic Hamiltonian. The $\hat{H}_i$ are chosen so that the n th truncation corresponds to $\epsilon_i = 1$ for $i \leq n$ and $\epsilon_i = 0$ for $i > n$. Generically, $\vec{\epsilon}$ specifies some location in theory space with specific points corresponding to various truncations and the untruncated Hamiltonian. The untruncated Hamiltonian is given by all $\epsilon_i = 1$, and as $g \rightarrow 0$ there is a critical point corresponding to continuum Yang-Mills. Ultimately, one would like to perform a lattice

simulation in the neighborhood of this critical point. However, it may be the case that certain points in theory space have behavior that takes them farther away from this critical point than one may initially expect. To show that this is the case for the $(1, 2, 2)$ truncation, a numerical simulation of

$$\begin{aligned} \hat{H}(\vec{\epsilon}) = & \hat{H}^{(1,1,1)} + \epsilon_1(\hat{H}^{(1,2,2)} - \hat{H}^{(1,1,1)}) \\ & + \epsilon_2(\hat{H}^{(2,2,2)} - \hat{H}^{(1,2,2)}) \end{aligned} \quad (23)$$

was performed for a 5×1 lattice with periodic boundary condition (PBC). The vacuum state was computed using exact diagonalization. This system size was too small to extract a correlation length by fitting, so an effective mass was computed instead. Explicitly, the correlation function $G(x) = \langle \psi | \hat{O}(x) \hat{O}(0) | \psi \rangle - \langle \psi | \hat{O}(0) | \psi \rangle^2$ was computed, where $|\psi\rangle$ is the vacuum state. The correlations in the system were estimated from the effective mass $m_{\text{eff}} = -\log(|\frac{G(2)}{G(1)}|)$. The effective mass as a function of ϵ_1 and ϵ_2 for $g = 0.5$ is shown in Fig. 10. These calculations show that the $(1, 2, 2)$ truncation is close to a point in theory space where the effective mass is divergent. Appendix A shows how dropping all $1/N_c$ terms from the electric energy and plaquette operator in the $(1, 2, 2)$ truncation gives a Hamiltonian with zero correlation length for any coupling g . As $g \rightarrow 0$, the effective mass should go to zero in the untruncated theory. The failure of the effective mass to decrease indicates that this is not a useful truncation of the Kogut-Susskind Hamiltonian. To verify that this is only a feature of the $(1, 2, 2)$ truncation, the vacuum was computed for a plaquette chain of length 5 with PBC. The overlap of the vacuum state for each truncation with the vacuum state of the zero correlation length Hamiltonian in Appendix A as a function of g is shown in Fig. 11. As this figure shows, only the $(1, 2, 2)$ has a large overlap with the zero correlation truncation for all values of g . This indicates

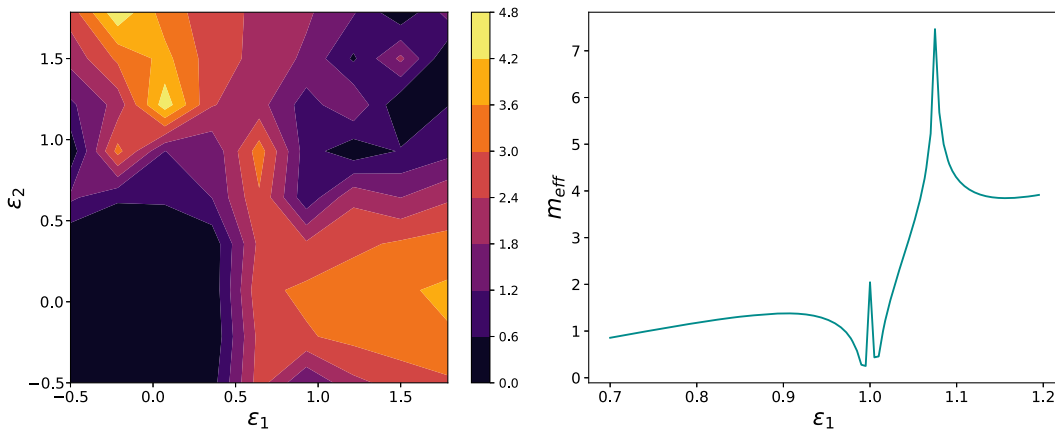


FIG. 10. Effective mass computed on a 5×1 lattice with $g = 0.5$. The left panel shows the effective mass as a function of ϵ_1 and ϵ_2 . The right panel shows the effective mass as a function of ϵ_1 with $\epsilon_2 = 0$.

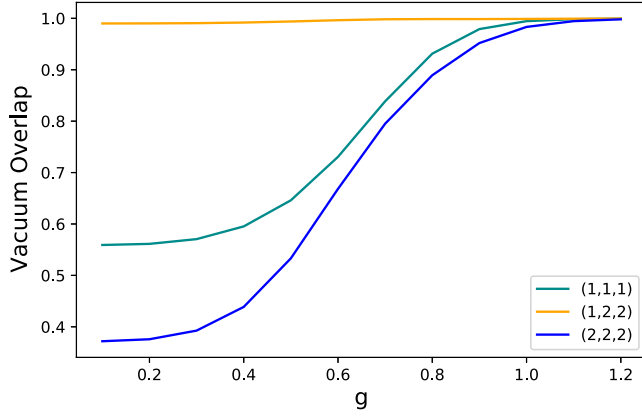


FIG. 11. Overlap of the vacuum state with the vacuum state of the zero correlation Hamiltonian as a function of g for a plaquette chain of length 5 with PBC.

that only the $(1, 2, 2)$ truncation is perturbatively close to this limit for all values of g .

We finally consider how these truncated Hamiltonians can be used to perform an actual calculation. One may expect that simulations with different truncations would be performed to estimate errors due to truncating the Hamiltonian. However, this is not necessary, as ultimately we are interested in deviations between our truncated Hamiltonian and the continuum limit, not deviations from the untruncated Kogut-Susskind Hamiltonian. The truncations of the Hamiltonian performed here are compatible with all of the symmetries of the Hamiltonian and preserve locality. This means that our truncated Hamiltonian can be described by a Symanzik effective action whose leading order piece is the Yang-Mills action plus irrelevant operators multiplied by powers of the lattice spacing. There are no additional relevant operators in the action, as any other relevant operator would break the symmetries respected by the truncated Hamiltonian. Note that this argument does not mean that the discarded pieces of the Hamiltonian consist of only irrelevant operators. These components will likely contain both relevant and irrelevant operators. As a result, the coupling in the Yang-Mills piece of the Symanzik action will not be given by g from the truncated Hamiltonian. The coupling in the Symanzik action will be renormalized with dependence on the truncation used. This is different from the untruncated case, where the Symanzik action has the same coupling as the Hamiltonian. Connecting simulations done with a truncated Hamiltonian to continuum physics will require a detailed understanding of how the truncation renormalizes the coupling in the Yang-Mills piece.

The lattice spacing a is inversely proportional to the correlation length of the system and goes to zero as the correlation length diverges. As Fig. 9 shows, it appears that the correlation length of the truncated Hamiltonians freezes out, which sets a limit on how small a lattice spacing can be

reached. From the perspective of the Symanzik action, the freezing out of the correlation length prevents the coefficients of the irrelevant operators from becoming arbitrarily small, as they do in the continuum limit. As the only deviations from the continuum limit are suppressed by powers of a , simulations with the truncated Hamiltonian can be performed with various values of a up to the minimum value allowed by the truncation, and then extrapolations can be performed to smaller a . This is the same as how traditional lattice calculations are performed in practice. From this perspective, increasing the truncation of the Hilbert space solely serves the role of making smaller lattice spacings accessible to simulation. However, performing these calculations with controlled errors requires a more detailed understanding of the effective actions that describe these truncated Hamiltonians.

V. COMPUTATIONAL RESOURCES

To simulate the real-time dynamics of this theory, it will be necessary to construct a circuit to implement the time evolution operator. Time evolution will be implemented using a first-order Trotterization formula, which approximates time evolution by

$$e^{-i\Delta t \sum_j \hat{H}_j} = \prod_j e^{-i\Delta t \hat{H}_j} + O(\Delta t^2), \quad (24)$$

where each of the $e^{-i\Delta t \hat{H}_j}$ can be efficiently implemented on quantum hardware. Note that the circuits developed in this section could, in principle, be used to implement higher-order Trotter formulas. In this work, the Hamiltonian will be Trotterized by

$$e^{-i\hat{H}t} \approx e^{-i\hat{H}_E t} e^{-i\hat{H}_B t}. \quad (25)$$

All terms in $\hat{H}_E$ commute with each other, so it does not matter in which order they are implemented. In the truncated Hamiltonian, plaquette operators with a shared link do not commute, and the order in which their time evolution is implemented matters. To implement the plaquette operators, the lattice should be split into separate sublattices, where on each sublattice, the plaquettes have no shared links. Each sublattice can then be evolved in parallel.

The simplest truncation to simulate in two dimensions is the $(1, 1, 1)$ truncation in the even C sector. The electric terms can be implemented with a single Z rotation per plaquette, and the plaquette terms can be implemented using previously developed circuits [48]. The implementation of a single plaquette term will require 16 CNOT gates, 2 H gates, and 16 R_z gates. A circuit of the same size can be used to implement the plaquette operator for the $(1, 2, 1)$ truncation in two dimensions with the qubit encoding. An additional 2 CNOT gates and R_z gate will be needed per

link to implement the electric energy operator in the $(1, 2, 1)$ truncation.

In three spatial dimensions with the $(1, 1, 1)$ truncation, the electric term can be implemented with a single R_z again, but the plaquette term is complicated by each plaquette on a square three-dimensional lattice having 12 neighbors instead of 4, as in the two-dimensional lattice. This means that implementing the time evolution generated by a single plaquette term requires implementing $\exp(i\theta \hat{P}_0^{\otimes 12} \hat{X}_p)$. This can be done using ancilla qubits with textbook techniques [136]. One can use Toffoli gates to encode the state of the control qubits onto ancilla qubits. The X rotation on the active qubit will be controlled on a single ancilla qubit. The control information on the ancillas will then be uncomputed with Toffoli gates. For a rotation controlled by 12 qubits, this will require 11 ancilla qubits and a total of 22 Toffoli gates. The rotation controlled by a single ancilla qubit will require 2 CNOT gates, 2 H gates, and 2 R_z gates. With the standard Toffoli circuit, the gate count for a single plaquette operator is 134 CNOT gates, 46 H gates, and 156 R_z gates. Note that cancellations can likely be found to reduce this gate count, but in this work, we are interested in only an order-of-magnitude estimate of the resources necessary for performing time evolution. With the qubit encoding of the $(1, 2, 1)$ truncation, the plaquette operator can be implemented using 13 Givens rotations, which gives the gate count in Table I.

The next truncation that allows one to approach closer to the continuum limit in two spatial dimensions is the $(2, 2, 2)$ truncation. At this truncation, there is a single qubit assigned to each link and a qu6 assigned to each plaquette. For simulations on qubit hardware, each qu6 can be replaced by 3 qubits. The electric term of the Hamiltonian consists of single plaquette terms and terms involving a plaquette, a link, and a neighboring plaquette. The evolution generated by the single plaquette term can be implemented by decomposing it into a sum over tensor products of Pauli $\hat{Z}$ terms and evolving each one separately. This will require 8 CNOT gates and 8 R_z gates per plaquette. The electric Hamiltonian terms with support on two plaquettes and a link will take a similar form, except acting on 7 qubits instead of 3. Using the same strategy, the time evolution generated by these terms can be implemented using 128 CNOT gates and 128 R_z gates per link. The plaquette term is somewhat more complicated to

implement. In this work, the plaquette operator will be decomposed into a sum over generators of gauge-invariant rotations, and each rotation will be implemented independently. This approach was previously suggested for performing quantum simulations of $SU(3)$ lattice gauge theory on a chain of plaquettes [37]. Each rotation couples two basis states and can be implemented with standard techniques for Givens rotations. These Givens rotations act on 19 qubits, and a Gray code can be implemented with CNOT gates to turn the Givens rotation into a single qubit rotation controlled by 18 qubits. The Gray code requires at most 36 CNOT gates to implement and undo. The controlled rotation of the qubit will use 18 ancilla qubits and 34 Toffoli gates to encode/decode control information on the ancillas and 2 H gates, 2 R_z gates, and 2 CNOT gates to implement the rotation controlled by the ancilla. Therefore, the total gate count for a single Givens rotation is 242 CNOT gates, 70 H gates, and 240 R_z gates. The number of Givens rotations that need to be performed depends on the order of the large N_c expansion used. Table II shows the number of Givens rotations required for different truncations in large N_c . The large N_c expansion reduces the number of Givens rotations needed in the time evolution circuit by roughly 95% from the theory with no large N_c truncation. The gate counts for this truncation are shown in Table I.

In three spatial dimensions at the $(2, 2, 2)$ truncation, the electric energy operators can be implemented using the same strategy as in 2D, and the gate counts will be the same per link. As with the $(1, 1, 1)$ truncation, the implementation of plaquette operators is complicated by each plaquette having 3 neighboring plaquettes per link. Due to this truncation, if two of the plaquettes neighboring a single link are excited, there will be no nonzero matrix elements for the plaquette operator. Note that the matrix elements of the plaquette operator only depend on the number of neighboring plaquettes excited on each link, not which neighboring plaquette is excited. This information can be encoded onto a set of three ancilla qubits using 3 CNOT gates per neighboring plaquette. The Givens rotations needed for implementing the time evolution operator will be controlled by these ancillas and the resulting circuit will be identical to the 2D case. The ancilla qubits will then be uncomputed after the Givens rotations. This approach to implementing the time evolution operator while keeping

TABLE I. Number of qubits and gates per plaquette required to implement a single first-order Trotter step for different truncations of the Hamiltonian. Note that in 2D there are two links per plaquette and in 3D there is a single link per plaquette.

	$(1, 1, 1)$ 2D	$(1, 1, 1)$ 3D	$(1, 2, 1)$ 2D	$(1, 2, 1)$ 3D	$(2, 2, 2)$ 2D at order $1/N_c$	$(2, 2, 2)$ 3D at order $1/N_c$
Qubits	1	1	1	1	5	4
CNOT	16	134	20	1, 744	249, 274	249, 226
H	2	46	2	598	72,030	72, 030
R_z	17	157	19	2, 030	247, 216	247, 096

TABLE II. The number of Givens rotations present when keeping terms in the plaquette order to different powers of $1/N_c$. The number of Givens rotations needed without a large N_c truncation was taken from Ref. [37].

	(2, 2, 2) Truncation
1	517
$1/N_c$	1029
No Large N_c truncation	19, 594

terms up to order $1/N_c$ has the gate cost shown in Table I. Note that it is likely more sophisticated methods of constructing Givens rotations that can reduce this gate count further [137].

While the gate count for the (1, 1, 1) truncation is small enough to be implemented on NISQ devices, the (2, 2, 2) truncation (and a closer approach to the continuum limit) will likely require using an error-corrected quantum computer. This changes the dominant cost of the circuit from CNOT gates to R_z gates. In an error-corrected quantum computer, R_z gates are implemented through the use of T gates. The number of T gates required to synthesize an R_z gate to accuracy ϵ is $1.15 \log_2(\frac{1}{\epsilon})$ [138]. Using the results from Table I and assuming R_z are synthesized to an accuracy of $\epsilon = 10^{-10}$, the implementation described in this section require 9.4×10^8 T gates per Trotter step on a 10×10 lattice and 2.8×10^{10} T gates per Trotter step on a $10 \times 10 \times 10$ lattice with the (2, 2, 2) truncation with $1/N_c$ corrections. Previous work has estimated that physically interesting simulations of the dynamics of lattice QCD could be performed on a $10 \times 10 \times 10$ lattice with a spacing of $a \approx 0.1$ fm [7]. The two-dimensional simulations performed in the previous section show that the truncations introduced in this work can reach vacuum correlation lengths of roughly two lattice sites.

In three spatial dimensions, the lattice spacing can be set by matching the correlation length to the predicted scalar glueball mass of 1730 MeV [139–141]. If similar correlation lengths can be reached in a three-dimensional system, this means the truncations in this work can reach a lattice spacing of $a \approx 0.057$ fm. The number of T gates required for a simulation was estimated to be 3.01×10^{49} in a previous work [32]. Follow-up work using qubitization methods instead of Trotterization reduced this gate

count to 1.7×10^{27} [33]. The (2, 2, 2) truncation reduces the T gate count to $\mathcal{O}(10^{10})$ per Trotter step. This lattice spacing may also be accessible to the (1, 2, 1) truncation, which would have a T gate count of $\mathcal{O}(10^8)$ per Trotter step. The T gate counts for all other truncations are given in Table III. Note that these previous estimates include matter as well as the gauge fields; however, the dominant cost of the circuits comes from implementing the gauge fields. Therefore, it is reasonable to use them for an order-of-magnitude estimate for the gate count. Those previous resource estimates were based on rigorous upper bounds for errors in the time evolution circuit [142]. At long times, these bounds fail to capture the suppression of Trotterization errors due to a form of many-body localization [143]. An estimate of this form will not be done in this work, as numerical studies have demonstrated that these bounds are extremely loose at shorter times as well and lead to overestimates of resource requirements by orders of magnitude [144]. For example, estimates of this kind suggest $\mathcal{O}(10^{13})$ T gates are necessary to simulate the Schwinger model on 128 sites [145], but in reality, 112 site simulations have been successfully performed on NISQ hardware [96,97]. Additionally, by treating the Trotterized time evolution operator as a temporal lattice, one can approach the $\Delta t \rightarrow 0$ limit as the spatial lattice spacing is taken to 0. This should be a more economical approach to simulation than approaching $\Delta t \rightarrow 0$ for each lattice spacing as proposed in Ref. [146].

Recent work has also estimated the number of T gates needed to simulate the harshest truncation of SU(3) lattice gauge theory in the electric basis without a large N_c expansion [54]. The (1, 1, 1) and (1, 2, 1) truncations introduced in this work should reproduce physics with an accuracy similar to this truncation. With the use of a large N_c truncation, the T gate cost of a Trotter step on a 10×10 lattice in the (1, 2, 1) truncation is an order of magnitude smaller than that for a 3×3 lattice with no large N_c expansion [54]. In three spatial dimensions, the T gate cost of a Trotter step in the (1, 1, 1) truncation on a $10 \times 10 \times 10$ lattice is comparable to the cost of a Trotter step with no large N_c expansion on a $3 \times 3 \times 3$ lattice. Note that the techniques used to optimize circuits in Ref. [54] could be used to improve the gate counts for the circuits in this work.

TABLE III. Number of qubits required to represent the state of the system and T gates required to implement a single first order Trotter step for different truncations of the Hamiltonian on a 10×10 and $10 \times 10 \times 10$ lattice. For the T gate count, it is assumed that R_z gates are synthesized to an accuracy of $\epsilon = 10^{-10}$.

	(1, 1, 1) 10×10	(1, 1, 1) $10 \times 10 \times 10$	(1, 2, 1) 10×10	(1, 2, 1) $10 \times 10 \times 10$	(2, 2, 2) 10×10 at order $1/N_c$	(2, 2, 2) $10 \times 10 \times 10$ at order $1/N_c$
Qubits	100	3, 000	100	3, 000	500	12, 000
T	64, 600	1.79×10^7	72, 700	2.3×10^8	9.4×10^8	2.8×10^{10}

VI. DISCUSSION

The large N_c expansion introduced in Ref. [48] has been extended to include $1/N_c$ corrections. At this truncation in large N_c , states can have extended loops of flux as well as loops around single plaquettes. The Hilbert space was truncated using a local Krylov basis construction. The form of the lowest lying truncations were given. The scalar glueball mass was computed in these truncated Hamiltonians with tensor networks, which showed that these truncations may be able to reach smaller lattice spacings than one would initially expect. In traditional lattice calculations, the use of smeared actions (such as those generated by Wilson flows) reduces the size of lattice artifacts [147]. In the Hamiltonian formulation, the analogous procedure can be performed using the similarity renormalization group, and applying it to these truncations should enable reaching even smaller lattice spacings [103]. Explicit strategies for constructing the time evolution operator on a quantum computer were given for the low-lying truncations in $2+1D$ and $3+1D$. The resource requirements for performing time evolution are orders of magnitude smaller than previous approaches that used a direct encoding of the gauge fields onto qubits. The circuit requirements for moderately sized $2+1D$ systems are within the 5-year roadmaps of multiple quantum computing companies [148,149]. Note that it is likely possible to reduce requirements further with more efficient circuit constructions. It may also be possible to use native gates of specific quantum computing platforms to reduce resources further. Future work will also explore how to use the $1/N_c$ suppression of some terms in the Hamiltonian to more efficiently implement the time evolution operator. By reducing the resources required for simulation, this work brings quantum simulations relevant to high-energy physics closer to reality. To reach the goal of simulating full QCD on a quantum computer, it will be necessary to include fermionic matter in this formulation. It is expected that the simplifications obtained in this work will carry over to the inclusion of matter.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [150], embargo periods may apply.

APPENDIX A: VANISHING CORRELATIONS IN THE (1, 2, 2) TRUNCATION

In the main text, it was shown that the (1, 2, 2) truncation of the Hamiltonian completely fails to reproduce the glueball mass. As explained in this section, this is due to this truncation being close to a point in theory space where the correlation length is 0 for all values of g . If one considers the (1, 2, 2) truncation with the $1/N_c$ splitting in the Casimirs dropped and the plaquette term in the $N_c \rightarrow \infty$ limit, each plaquette decouples from the others.

To determine the form of the Hamiltonian (for the even C sector) truncated in this manner, we will first consider a lattice composed of a single plaquette. For a lattice of this size, gauge-invariant states can be labeled by the representation $\mathbf{R}$ flowing around the plaquette. Using the known expressions for matrix elements of the plaquette operator, we find

$$(\hat{\square} + \hat{\square}^\dagger)|\mathbf{1}\rangle = |\square\rangle + |\blacksquare\rangle. \quad (\text{A1})$$

We can represent the state of a single plaquette with a qubit with basis states,

$$\begin{aligned} |0\rangle &= |\mathbf{1}\rangle \\ |1\rangle &= \frac{1}{\sqrt{2}}(|\square\rangle + |\blacksquare\rangle). \end{aligned} \quad (\text{A2})$$

With these basis states, the Hamiltonian is

$$\hat{H} = N_c g^2 \hat{P}_1 - \frac{1}{g^2 \sqrt{2}} \hat{X}, \quad (\text{A3})$$

where $\hat{P}_i = |i\rangle\langle i|$.

For a two-plaquette system with open boundary conditions (and truncating as described above), gauge invariant states can be specified using the representation on the three vertical links, $|\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3\rangle$. The physical states that need to be represented at this truncation can once again be determined by applying plaquette operators to the electric vacuum state $|\mathbf{1}, \mathbf{1}, \mathbf{1}\rangle$. A single application of either of the plaquette operators can create the states

$$\begin{aligned} (\hat{\square}_1 + \hat{\square}_1^\dagger)|\mathbf{1}, \mathbf{1}, \mathbf{1}\rangle &= |\square, \blacksquare, \mathbf{1}\rangle + |\blacksquare, \square, \mathbf{1}\rangle, \\ (\hat{\square}_2 + \hat{\square}_2^\dagger)|\mathbf{1}, \mathbf{1}, \mathbf{1}\rangle &= |\mathbf{1}, \square, \square\rangle + |\mathbf{1}, \blacksquare, \blacksquare\rangle. \end{aligned} \quad (\text{A4})$$

Applying neighboring plaquette operators produces the state

$$\begin{aligned}
 (\hat{\square}_2 + \hat{\square}_2^\dagger) (\hat{\square}_1 + \hat{\square}_1^\dagger) |1, 1, 1\rangle &= (\hat{\square}_2 + \hat{\square}_2^\dagger) (|\square, \blacksquare, 1\rangle + |\blacksquare, \square, 1\rangle) \\
 &= \sqrt{1 - \frac{1}{N_c^2}} |\square, \blacksquare, \square\rangle + \frac{1}{N_c} |\square, 1, \square\rangle + \sqrt{1 - \frac{1}{N_c^2}} |\blacksquare, \blacksquare, \blacksquare\rangle + \frac{1}{N_c} |\blacksquare, 1, \blacksquare\rangle \\
 &\quad + \sqrt{\frac{1}{2} \left(1 + \frac{1}{N_c}\right)} |\blacksquare, \square, \square\rangle + \sqrt{\frac{1}{2} \left(1 - \frac{1}{N_c}\right)} |\blacksquare, \square, \square\rangle \\
 &\quad + \sqrt{\frac{1}{2} \left(1 + \frac{1}{N_c}\right)} |\square, \blacksquare, \blacksquare\rangle + \sqrt{\frac{1}{2} \left(1 - \frac{1}{N_c}\right)} |\square, \blacksquare, \blacksquare\rangle.
 \end{aligned} \tag{A5}$$

In the large N_c limit this state becomes

$$\begin{aligned}
 (\hat{\square}_2 + \hat{\square}_2^\dagger) (\hat{\square}_1 + \hat{\square}_1^\dagger) |1, 1, 1\rangle &= |\square, \blacksquare, \square\rangle + |\blacksquare, \blacksquare, \blacksquare\rangle + \frac{1}{\sqrt{2}} |\blacksquare, \square, \square\rangle + \frac{1}{\sqrt{2}} |\blacksquare, \square, \square\rangle \\
 &\quad + \frac{1}{\sqrt{2}} |\square, \blacksquare, \blacksquare\rangle + \frac{1}{\sqrt{2}} |\square, \blacksquare, \blacksquare\rangle.
 \end{aligned} \tag{A6}$$

At this truncation, there are no additional states for a lattice of this size. Gauge-invariant states of this lattice can be specified using a qubit per plaquette. When one qubit is in the 0 state, the state assignment for the other qubit is identical to the one-plaquette cases. At leading order in $1/N_c$, the Casimirs of the representations $\blacksquare$, $\square$, and $\square$ are the same, so it is not necessary to specify which of these irreps is on the vertical link. The state $|1, 1\rangle$ corresponds to

$$|1, 1\rangle = \frac{1}{2} \left(|\square, \blacksquare, \square\rangle + |\blacksquare, \blacksquare, \blacksquare\rangle + \frac{1}{\sqrt{2}} |\blacksquare, \square, \square\rangle + \frac{1}{\sqrt{2}} |\blacksquare, \square, \square\rangle + \frac{1}{\sqrt{2}} |\square, \blacksquare, \blacksquare\rangle + \frac{1}{\sqrt{2}} |\square, \blacksquare, \blacksquare\rangle \right). \tag{A7}$$

This basis construction scales to larger two-dimensional lattices, using a single qubit per plaquette. With this basis, the Hamiltonian is given by

$$\hat{H}_2 = \sum_p \left(N_c g^2 \hat{P}_1^p - \frac{1}{g^2 \sqrt{2}} \hat{X}^p \right). \tag{A8}$$

From this expression, it is clear that each plaquette in the lattice is independent of all of the others, and the system is completely uncorrelated.

The $(1, 2, 2)$ Hamiltonian differs from this by terms in the electric Hamiltonian that are $\mathcal{O}(\frac{g^2}{N_c})$ and terms in the magnetic Hamiltonian that are $\mathcal{O}(\frac{1}{g^2 N_c})$. The energy gap between eigenstates of $\hat{H}_2$ is given by $2\sqrt{N_c^2 g^4/4 + 1/(2g^4)}$, so $\hat{H}^{(1,2,2)}$ can be treated as being perturbatively close to $\hat{H}_2$ for any value of g . Note that this argument only applies to the $(1, 2, 2)$ truncation, as the other truncations will have additional terms in the Hamiltonian that are comparable in size to the gap of $\hat{H}_2$. As the following argument will show, being perturbatively close to $\hat{H}_2$ is sufficient to guarantee that the correlation length in the $(1, 2, 2)$ truncation stays small at all values of g .

For a generic lattice Hamiltonian

$$\hat{H} = \hat{H}_0 + \varepsilon \hat{V}, \tag{A9}$$

where $\hat{H}_0$ only acts on single sites, $\hat{V}$ couples nearest neighbor sites, and ε is a small number. It can be shown in perturbation theory that the leading contribution to a connected vacuum correlation function is

$$G(x) = \langle \psi | \mathcal{O}(x) \mathcal{O}(0) | \psi \rangle - \langle \psi | \mathcal{O}(0) | \psi \rangle^2 = \varepsilon^{|x|} + \mathcal{O}(\varepsilon^{|x|+1}). \tag{A10}$$

This follows from applying traditional perturbation theory to the eigenstates of $\hat{H}_0$. Any term below order $|x|$ will only contribute to the connected piece of the correlation function. As a result, the effective mass obtained from such a correlation function is $m_{\text{eff}} = \log(\frac{1}{\epsilon})$.

From this result, it can be argued that for any value of g , the glueball mass for the (1, 2, 2) truncation will be large. This is an indication that this is not a useful truncation for attempting to approach the continuum limit, as the lattice spacing will be set by the inverse of the glueball mass. If the glueball mass does not approach zero (as is the case in this truncation), then the continuum limit cannot be approached.

APPENDIX B: STRICTLY LEADING ORDER IN $1/N_c$: AN EXACTLY SOLVABLE MODEL

We now turn to the decoupling of plaquettes mentioned around (9). This is obtained when only the leading N_c dependence of the electric energy and plaquette operators is kept and is a general feature of the Kogut-Susskind Hamiltonian, unrelated to truncation schemes of the electric basis. We flag to the reader that this is a different phenomenon than the decoupling studied in Appendix A. For a single plaquette, one can define electric basis states $|n\rangle$ by a sum over all representations whose Young tableaux have n boxes. Starting from the electric vacuum, these are the only states that can be obtained through time evolution at this order. Different representations will have coefficients proportional to the number of ways they can be built from lower-weight representations. Explicitly, the lowest lying basis states are

$$\begin{aligned} |0\rangle &= |1\rangle \\ |1\rangle &= \frac{1}{\sqrt{2}} (|\square\rangle + |\blacksquare\rangle) \\ |2\rangle &= \frac{1}{\sqrt{8}} \left(|\square\square\rangle + \left| \begin{array}{|c|} \hline \square \\ \hline \end{array} \right\rangle + |\blacksquare\blacksquare\rangle + \left| \begin{array}{|c|} \hline \blacksquare \\ \hline \end{array} \right\rangle + 2|\square\blacksquare\rangle \right) \end{aligned} \quad (\text{B1})$$

With this basis, one can show that the Hamiltonian for a single plaquette is

$$\hat{H} = g^2 N_c \sum_n n \hat{P}_n - \frac{1}{\sqrt{2}g^2} \sum_n \sqrt{n} \hat{\mathcal{X}}_{n,n+1}, \quad (\text{B2})$$

where $\hat{P}_n = |n\rangle\langle n|$, and $\hat{\mathcal{X}}_{n,n+1} = |n\rangle\langle n+1| + |n+1\rangle\langle n|$. This Hamiltonian is a displaced harmonic oscillator. This can be seen by introducing the usual raising and lowering operators and corresponding $\hat{x}$ and $\hat{p}$ operators. In particular,

$$\begin{aligned} \sum_n n \hat{P}_n &= a^\dagger a, \\ \sum_n \sqrt{n} \hat{\mathcal{X}}_{n,n+1} &= a + a^\dagger, \end{aligned} \quad (\text{B3})$$

which allows the Hamiltonian to be written as

$$\hat{H} = \lambda(a^\dagger a) - \frac{1}{\sqrt{2}g^2}(a + a^\dagger), \quad (\text{B4})$$

with

$$\lambda \equiv g^2 N_c. \quad (\text{B5})$$

Defining

$$\hat{x} = \sqrt{\frac{1}{2\lambda}}(a + a^\dagger), \quad \hat{p} = i\sqrt{\frac{\lambda}{2}}(a^\dagger - a), \quad (\text{B6})$$

we can write

$$\begin{aligned} \hat{H} &= \frac{\hat{p}^2}{2} + \frac{\lambda^2}{2} \hat{x}^2 - \frac{\sqrt{\lambda}}{g^2} \hat{x} - \frac{\lambda}{2}, \\ \hat{H} &= \frac{\hat{p}^2}{2} + \frac{\lambda^2}{2} \left(\hat{x} - \frac{\sqrt{\lambda}}{g^2 \lambda^2} \right)^2 - \frac{\lambda}{2} - \frac{1}{2g^4 \lambda}, \end{aligned} \quad (\text{B7})$$

The spectrum of this Hamiltonian can be obtained immediately, since this is just a harmonic oscillator with a shifted ground state. One can read off the eigenvalues from Eq. (B7),

$$E_n = \lambda n - \frac{1}{2g^4 \lambda}. \quad (\text{B8})$$

The corresponding eigenstates are obtained from the states $|n\rangle$ by using the displacement operator

$$D(\alpha) = \exp[\alpha(a^\dagger - a)], \quad (\text{B9})$$

which satisfies

$$aD(\alpha) = D(\alpha)(a + \alpha), \quad a^\dagger D(\alpha) = D(\alpha)(a^\dagger + \alpha). \quad (\text{B10})$$

Defining

$$\begin{aligned} (a^\dagger - \alpha)(a - \alpha)|n, \alpha\rangle &= a^\dagger a - \alpha(a^\dagger + a) - \alpha^2 \\ &= (n - \alpha^2)|n, \alpha\rangle, \end{aligned} \quad (\text{B11})$$

one can show that

$$\begin{aligned}
a^\dagger a - \alpha(a^\dagger + a)|n, \alpha\rangle &= [(\alpha^\dagger - \alpha)(a - \alpha) - \alpha^2]|n, \alpha\rangle \\
&= D(\alpha)a^\dagger D^\dagger(\alpha)D(\alpha)aD^\dagger(\alpha)|n\rangle \\
&\quad - \alpha^2|n, \alpha\rangle \\
&= D(\alpha)a^\dagger a|n\rangle - \alpha^2|n, \alpha\rangle \\
&= (n - \alpha^2)|n, \alpha\rangle.
\end{aligned} \tag{B12}$$

This implies that the Hamiltonian in (B4) is diagonalized by the displaced states

$$|n, \lambda, g\rangle \equiv D\left(-\frac{1}{\sqrt{2\lambda}g^2}\right)|n\rangle. \tag{B13}$$

On a larger lattice, it can be shown that to the leading order in large N_c , plaquette matrix elements are

$$\begin{aligned}
d(m_1, \dots, m_{N-1}) &= \prod_{p=1}^{N-1} \left[\frac{1}{p!} \prod_{q=p}^{N-1} \left(\sum_{z=q-p+1}^p (1 + m_z) \right) \right], \\
C(m_1, \dots, m_{N-1}) &= \frac{1}{2N} \sum_{k=1}^{N-1} \left[Nk(N-k)m_k + k(N-k)m_k^2 + \sum_{l=0}^{k-1} 2l(N-k)m_l m_k \right].
\end{aligned} \tag{C1}$$

Using these expressions, one can immediately obtain the results for the representations used in this work, which we present in tabulated form, both for general N and for $N = 3$.

Irrep	Dimension		Casimir	
	General N	$N = 3$	General N	$N = 3$
$(0, 0, \dots, 0, 0)$	1	1	0	0
$(1, 0, \dots, 0, 0)$	N	3	$(N^2 - 1)/(2N)$	4/3
$(0, 0, \dots, 0, 1)$	N	3	$(N^2 - 1)/(2N)$	4/3
$(2, 0, \dots, 0, 0)$	$N(N+1)/2$	6	$(N^2 + N - 2)/N$	10/3
$(0, 0, \dots, 0, 2)$	$N(N+1)/2$	6	$(N^2 + N - 2)/N$	10/3
$(0, 1, 0, \dots, 0)$	$N(N-1)/2$	3	$(N^2 - N - 2)/N$	4/3
$(0, \dots, 0, 1, 0)$	$N(N-1)/2$	3	$(N^2 - N - 2)/N$	4/3
$(1, 0, \dots, 0, 1)$	$N^2 - 1$	8	N	3

APPENDIX D: PLAQUETTE MATRIX ELEMENTS

The explicit form of the $SU(N_c)$ Hamiltonians in this work requires the computation of matrix elements of the plaquette operator between gauge-invariant states. In previous work, it was noted that the state of a plaquette with a single external link at each vertex (as shown in Fig. 12) can be specified by basis states, where the irreps on each link are given [37,48],

$$\left| \psi \begin{pmatrix} C_1 & R_2 & C_2 \\ R_1 & & R_3 \\ C_4 & R_4 & C_3 \end{pmatrix} \right\rangle.$$

independent of the state of the neighboring plaquettes [48]. Using this result, it follows that in this limit, the Kogut-Susskind Hamiltonian is given by a displaced harmonic oscillator at each plaquette. These harmonic oscillators are not coupled, leading to a system with zero correlation length.

APPENDIX C: DIMENSION AND CASIMIR OF GENERAL $SU(N)$ REPRESENTATIONS

Using the results given in the appendix of [151], the dimension and Casimir of a general $SU(N)$ irreducible representation with Dynkin indices $(m_1, \dots, m_{N-1})$ can be written in the compact notation,

A limited number of plaquette matrix elements were also computed in the supplemental material of Ref. [48],

$$\left\langle \psi \begin{pmatrix} \bar{\mathbf{A}} & \mathbf{1} & \mathbf{1} \\ \mathbf{A} & & \mathbf{1} \\ \mathbf{A} & \mathbf{1} & \mathbf{1} \end{pmatrix} \middle| \hat{\square} \middle| \psi \begin{pmatrix} \bar{\mathbf{A}} & \mathbf{R} & \mathbf{1} \\ \mathbf{B} & & \bar{\mathbf{R}} \\ \mathbf{A} & \bar{\mathbf{R}} & \mathbf{1} \end{pmatrix} \right\rangle = \sqrt{\frac{\dim(\mathbf{B})}{\dim(\mathbf{A})\dim(\mathbf{R})}}, \tag{D1}$$

where the plaquette operator has parallel transporters in representation $\mathbf{R}$ and $\mathbf{B} \in \mathbf{R} \otimes \mathbf{A}$ [48]. Other plaquette matrix elements were computed by applying neighboring plaquette operators to create different initial states and by using the fact that neighboring plaquette operators commute. The same technique can be used to determine the plaquette matrix elements used in the low-lying truncations in this work. At the (1, 2, 2) truncation, at most one plaquette operator will be applied to each neighboring plaquette.

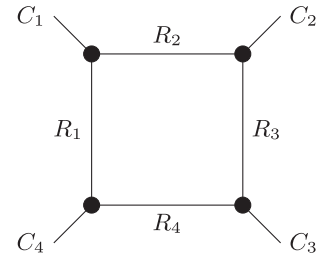


FIG. 12. A single plaquette with one external link at each vertex.

Combined with the fact that plaquette operators commute, it can be seen that the matrix elements at this truncation are given by products of the expression in Eq. (D1). This is the same method that was used to compute plaquette matrix elements in the supplemental material of Ref. [48].

APPENDIX E: DMRG DETAILS

The DMRG calculations in this work were performed by representing the state as a matrix product state (MPS). The plaquette at coordinates (x, y) on an $L \times L$ lattice corresponds to the tensor at position $y + L(x - 1)$, i.e., the MPS winds through the lattice like a typewriter. For the $(1, 1, 1)$ truncation, 60 sweeps of the DMRG were performed, and the maximum allowed bond dimension was 300 with a precision cutoff of 10^{-16} . For the $(1, 2, 1)$ truncation, 40 sweeps of the DMRG were performed and the maximum allowed bond dimension was 250 with a precision cutoff of 10^{-16} . In both the $(1, 1, 1)$ and $(1, 2, 1)$ calculations, the bond dimension did not reach the maximum allowed value for the couplings used. For the higher truncations, the link qubits left of and below a plaquette were blocked with the plaquette qudit into a single site in the MPS. DMRG for the higher truncations was performed using 25 sweeps and a maximum bond dimension of 100. The $(1, 2, 2)$ calculations did not ever reach this maximum bond dimension. A typical truncation error for the $(2, 2, 2)$ simulations at this maximum bond dimension was 10^{-8} . The DMRG was initialized with the system in the electric vacuum state. The number of sweeps of the DMRG was chosen such that the ground state energy had converged to at least 3 decimal places for the smallest value of g used. To ensure that a sufficient bond dimension was used for the $(2, 2, 2)$ truncation, the DMRG was run with multiple different maximum bond dimensions. The resulting glueball mass for a 4×4 lattice with $g = 0.2$ is shown in Fig. 13. As this figure shows, the glueball mass has converged to sufficient accuracy with a maximum bond dimension of 100. Similar behavior was found for other values of g . For the 8×8 calculations, discrepancies in the correlation function were found between the Julia and C++ implementations when the

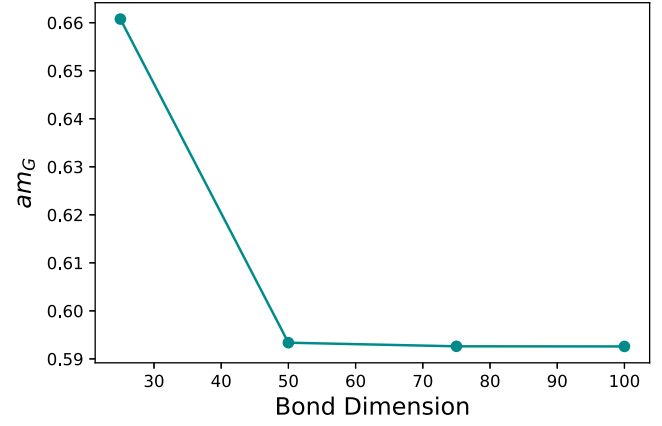


FIG. 13. Glueball mass computed in the $(2, 2, 2)$ truncation on a 4×4 lattice with $g = 0.2$ as a function of maximum bond dimension.

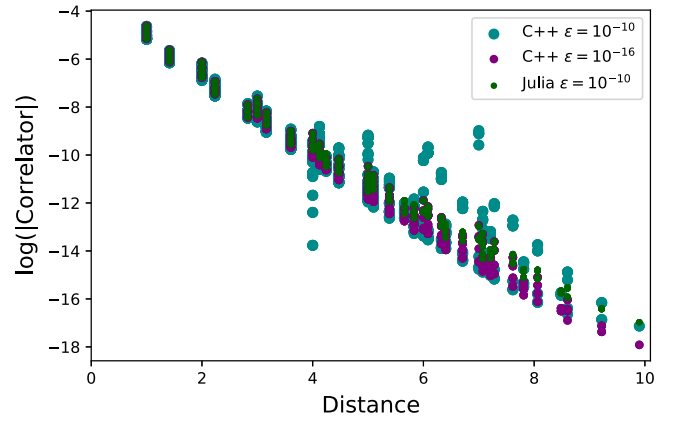


FIG. 14. Correlation function for the $(1, 1, 1)$ truncation on an 8×8 lattice with $g = 0.8$ computed in C++ and Julia.

same cutoff precision was used. Fig. 14 shows that for the $(1, 1, 1)$ truncation on an 8×8 lattice with $g = 0.8$, the correlation function computed in C++ and Julia with a cutoff precision of $\epsilon = 10^{-10}$ differ for distances larger than 4. When the C++ cutoff is decreased to $\epsilon = 10^{-16}$, the size of these disagreements decreases, indicating that this is likely a numerical stability issue in the C++ implementation.

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Correction: An incorrect version of the caption to Fig. 3 was used for publication and has now been replaced with the correct version.