

Gravitational form factors and mechanical properties of the nucleon in a meson dominance approach

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We analyze the gravitational form factors and mechanical properties of the nucleon, focusing both on some general issues, as well as on modeling with meson dominance. We show that the lattice QCD results for the nucleon gravitational form factors at $m_\pi = 170$ MeV, available for spacelike momentum transfer squared up to 2 GeV, are explained in a natural way within the meson dominance approach. We carry out the proper Raman spin decomposition of the energy-momentum tensor and in each spin channel use a minimum number of resonances consistent with the perturbative QCD short-distance constraints. These constraints are related to the superconvergence sum rules, following from the asymptotic perturbative QCD falloff of the form factors. The value of the nucleon D -term following from the fits is $-3.0(4)$. Next, we obtain the two-dimensional transverse gravitational densities of the nucleon in the transverse coordinate b . With the superconvergence sum rules, we derive new sum rules for these densities at the origin and for their derivatives, involving logarithmic weighting in the corresponding spectral density integrals. From analysis of the threshold behavior in the timelike region and the properties of the $\pi\pi \rightarrow N\bar{N}$ reaction, we infer the behavior of the transverse densities at asymptotically large coordinates. We also carry out the meson dominance analysis of the two- and three-dimensional mechanical properties of the nucleon (the pressure and stress) and explore their connection to the spectral densities via dispersion relations.

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I. INTRODUCTION

The internal structure of the nucleon has been known to exist since the 1950's, when by means of electron scattering it was found that it has a finite extension, nontrivial magnetic moments, or charge and current distributions [1], which in a relativistically invariant formulation can be associated with matrix elements of electromagnetic currents [2]. A remarkable analysis of Frazer and Fulco [3–5] allowed them to establish a connection between the electromagnetic nucleon structure and the $1^{--}\rho$ resonance in the charge form factors of both the pion and the nucleon, leading to the notion of vector-meson dominance [6] and the subsequent current-field identities [7,8]. In the Breit frame, these matrix elements are identified as spatial charge and current distributions

[9,10], providing a physically intuitive picture of a nucleon resembling a droplet made of charged particles. This image has been ameliorated by introducing the transverse distributions, which correspond to viewing the nucleon in the perpendicular direction of a hadron boosted to the infinite momentum frame [11–16]. Then, remarkably, the transverse density of quarks of a particular flavor is manifestly positive-definite [13,14,17], allowing for a probabilistic interpretation.

The considerations of the hadronic matrix elements of the stress-energy-momentum (SEM) tensor follow a similar path to the better-known EM case, naturally extending to study the energy, momentum, and mass distributions in terms of the corresponding hadronic form factors. Matrix elements of SEM were first described by Kobzarev and Okun [18]. The idea of tensor-meson dominance was introduced and applied by Freund [19] and by Sharp and Wagner [20] when describing the tensor-meson exchange in the NN interaction. The analytic properties were described by Pagels [21], who suggested saturation by the 2^{++} mesons in the dispersion relations (see also Ref. [22]). Current-field identities were proposed in Refs. [23–26], and a connection to gravity [27,28] was also given. An early mass-radius estimate of the

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nucleon [29], based on the sidewise dispersion relations, yielded 0.7 fm.

While many of the relevant issues were essentially understood in the pre-QCD era, they remained dormant due to the lack of an experimental or lattice motivation. The situation has changed since Xiang-Dong Ji established the possibility of relating the deeply virtual Compton scattering (DVCS) to both EM current and the SEM tensor via the moments of the corresponding generalized parton distributions (GPDs) [30]. Later on, Polyakov and Weiss discovered the D -term [31], and Polyakov suggested extending the nucleon picture by invoking its mechanical properties, such as mass, pressure, or shear force distributions, in an analogy to the classical elasticity theory [32] (see Ref. [33] for an alternative viewpoint), analyzed also in Refs. [34–38]. The mass and spin decomposition in terms of quarks and gluons has been addressed often—see Refs. [39–43]—with the proviso that the separation depends on the renormalization scale and hence is scheme dependent. Asymmetric SEM in the polarized nucleon was considered in Ref. [44].

The depiction of the nucleon as a stable droplet in mechanical equilibrium, as well as the relation of the D -term to moments of the pressure and the shear forces, constituting a genuine dynamical hadronic property, have triggered a lot of activity over the last several years, extending the venerable EM current program and methods onto the SEM sector (see, e.g., Ref. [45] and references therein for a review, and Ref. [46] for a transparent up-to-date perspective).

Numerous schemes and models have been proposed to describe the gravitational form factors (GFFs). Leading chiral corrections were addressed in the heavy baryon [47–50] or covariant baryon [51,52] approaches. Large- N_c scaling was established in Ref. [53]. Estimates were made in the chiral quark soliton model [54], the Skyrme model [55], the MIT bag model [56], the holographic model [57], and in AdS/QCD [58–61]. The QCD sum rules have also been applied [62,63]. Valence quarks in the light front formulation were considered in Ref. [64], and a full QCD light front viewpoint was presented in Ref. [65]. Importantly, within perturbative QCD (pQCD), Tong, Ma, and Yuan [66,67] have determined the leading asymptotic behavior of GFFs. A flavor decomposition using light-cone sum rules was presented in Ref. [68]. Quark and gluon decomposition of the nucleon GFFs was done in Ref. [69], and a soliton calculation with dilaton fields was presented in Ref. [70].

On the experimental and phenomenological side, some constraints have been obtained via DVCS from CLAS at JLab [71,72], as well as from the GlueX [73] data for the J/ψ photoproduction [74]. Along these lines, an estimate of the proton mass radius [75] was obtained, while Refs. [76,77] questioned whether the J/ψ near-threshold photoproduction could be directly used to study the

nucleon structure. In Ref. [78], a determination of GFFs from the Compton form factors was made. Holographic QCD modeling was carried out in Ref. [79]. A systematic Bayesian extraction of the proton mass radius based on the photoproduction of vector charmoniums was made in Ref. [80].

The recent renewed interest in the gravitational structure of hadrons is spurred on by state-of-the-art lattice QCD calculations involving all the parton species [81–85] at low values of the pion mass, $m_\pi = 170$ MeV, close to the physical point. Similar studies for the gluon contribution to the trace anomaly form factor were carried out in Ref. [86] at higher values of m_π . These studies vastly improve on the previous lattice analyses of the quark GFFs of the nucleon [87–92].

In this paper, we analyze the GFFs of the nucleon within a purely hadronic scheme, incorporating the quark-hadron duality principle via the meson¹ dominance, supplemented by short-distance constraints from pQCD. These constraints imply superconvergence sum rules for the spectral densities associated with GFFs. The framework provides large- N_c -motivated functional expressions in the form of sums over meson pole masses, taken from the Particle Data Group tables, and the residues fitted to the data. This large- N_c scheme, constrained with pQCD, has been successfully used in previous works for a variety of EM, SEM, and axial form factors, both for the pion and for the nucleon [94]. Here, we extend the method to describe the MIT lattice QCD data [81], using the insights of our previous analyses for the pion [95–97]. We use the data for the full form factors, involving the quark and gluon contributions. That way, the method is not sensitive to the renormalization scale and scheme, as it deals with features of conserved currents.

We discuss in detail the two- and three-dimensional spatial distributions and the mechanical properties of the nucleon coming from our approach. We also relate them to the spectral densities via dispersion relations and obtain novel sum rules for the values of the above distributions and their derivatives at the origin. From the threshold behavior in the timelike region and the analyticity and unitarity properties of the $\pi\pi \rightarrow N\bar{N}$ amplitudes, we find the forms of the spatial distributions at asymptotically large coordinates.

Recently, a paper by Cao, Guo, Li, and Yao [98] appeared, presenting a sophisticated Roy-Steiner equation analysis incorporating coupled-channel unitarity, analyticity, and crossing for all channels, including $\pi\pi$, $K\bar{K}$, and $N\bar{N}$. The lack of experimental information for timelike momenta above the $N\bar{N}$ threshold requires us to

¹In this paper, we use the definition of a meson in a broader sense, denoting a baryon-number-zero bound state or resonance, incorporating the quark model $q\bar{q}$ states, as well as glueballs or hybrids (see, e.g., Sec. 8.1.1 of Ref. [93]).

incorporate contributions from meson dominance to describe the data of Ref. [81].

II. GRAVITATIONAL FORM FACTORS OF THE NUCLEON

A. Stress-energy-momentum operator

We assume the definition of SEM following from the coupling to gravity, which provides a symmetric tensor from the outset. In the case of QCD, it coincides with the Belinfante-Rosenfeld symmetrized SEM [99,100] (see also Appendix E of Ref. [101]), reading

$$\Theta^{\mu\nu} = \frac{i}{4} \bar{\Psi} [\gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu] \Psi - F^{\mu\lambda} F_{\lambda\alpha}^\nu + \frac{1}{4} g^{\mu\nu} F^{\sigma\lambda} F_{\sigma\lambda} + \Theta_{\text{GF-EOM}}^{\mu\nu}, \quad (1)$$

where in the quantized case, one has in addition the gauge-fixing and the equation-of-motion terms. Under a Lorentz transformation $x^\mu \rightarrow \Lambda_\alpha^\mu x^\alpha$, it transforms covariantly, $\Theta^{\mu\nu}(x) \rightarrow \Lambda_\alpha^\mu \Lambda_\beta^\nu \Theta^{\alpha\beta}(\Lambda^{-1}x)$, but not irreducibly. A naive decomposition into traceless and traceful pieces,

$$\Theta^{\mu\nu} = \Theta_{S,n}^{\mu\nu} + \Theta_{T,n}^{\mu\nu} \equiv \frac{1}{4} g^{\mu\nu} \Theta + \left[\Theta^{\mu\nu} - \frac{1}{4} g^{\mu\nu} \Theta \right], \quad (2)$$

with $\Theta = \Theta_\mu^\mu$, does not comply with a separate conservation of the two components, since $\partial_\mu \Theta_{S,n}^{\mu\nu} = -\partial_\mu \Theta_{T,n}^{\mu\nu} \neq 0$. In particular, this becomes a problem in the application of the meson dominance. A consistent decomposition where the two tensor components are conserved separately was proposed long ago by Raman [102], $\Theta^{\mu\nu} = \Theta_S^{\mu\nu} + \Theta_T^{\mu\nu}$, with

$$\Theta_S^{\mu\nu} = \frac{1}{3} \left[g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right] \Theta \Rightarrow \partial_\mu \Theta_S^{\mu\nu} = \partial_\mu \Theta_T^{\mu\nu} = 0. \quad (3)$$

It was recently also used in Refs. [60,103]. The two irreducible tensors have well-defined angular momentum, $J^{PC} = 0^{++}$ and 2^{++} , correspondingly. Although this decomposition involves an apparent nonlocality, it does not lead to issues in our approach.

B. Matrix elements

With the symmetric SEM operator, the three gravitational form factors of the nucleon are defined through the matrix element in on-shell nucleons,

$$\begin{aligned} \langle p', s' | \Theta_{\mu\nu}(0) | p, s \rangle = & \bar{u}(p', s') \left[A(t) \gamma_{\{\mu} P_{\nu\}} + B(t) \frac{i P_{\{\mu} \sigma_{\nu\} \rho} q^\rho}{2m_N} \right. \\ & \left. + D(t) \frac{q_\mu q_\nu - g_{\mu\nu} q^2}{4m_N} \right] u(p, s), \end{aligned} \quad (4)$$

where m_N is the nucleon mass, and the Dirac spinors are normalized as $\bar{u}(p, s) u(p, s) = 2m_N$, $P = (p' + p)/2$, $q = p' - p$, $t = q^2$, $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$, and $a_{\{\mu} b_{\nu\}} = \frac{1}{2} (a_\mu b_\nu + a_\nu b_\mu)$. Clearly, $P \cdot q = 0$ and $P^2 = m_N^2 - t/4$. The $A(t)$ form factor is chirally even, whereas $B(t)$ and $D(t)$ are chirally odd.

Through the use of the Gordon identity

$$2m \bar{u}' \gamma^\alpha u = \bar{u}' (2P^\alpha + i\sigma^{\alpha\rho} q_\rho) u, \quad (5)$$

the definition (4) can be equivalently written in different ways, which, due to the mass term, may mix the chiral properties. For instance,

$$\begin{aligned} \langle p', s' | \Theta_{\mu\nu}(0) | p, s \rangle &= \frac{1}{m_N} \bar{u}' \left[A(t) P_\mu P_\nu + J(t) i P_{\{\mu} \sigma_{\nu\} \rho} q^\rho + D(t) \frac{q_\mu q_\nu - g_{\mu\nu} q^2}{4} \right] u. \end{aligned} \quad (6)$$

The form factors A , B , and J are related:

$$J(t) = \frac{A(t) + B(t)}{2}. \quad (7)$$

Another form of SEM that can be written is

$$\begin{aligned} \langle p', s' | \Theta_{\mu\nu}(0) | p, s \rangle &= \bar{u}' \left[2J(t) \gamma_{\{\mu} P_{\nu\}} - B(t) \frac{P_\mu P_\nu}{m_N} + D(t) \frac{q_\mu q_\nu - g_{\mu\nu} q^2}{4m_N} \right] u. \end{aligned} \quad (8)$$

Normalizations are $A(0) = 1$ and $J(0) = \frac{1}{2}$ (this last condition is referred to as Ji's sum rule [30]). These imply $B(0) = 0$, which is known as the vanishing of the ‘‘anomalous gravitomagnetic moment’’ of the nucleon [18]. Finally, the value $D(0)$ is the D -term [31], which is a dynamical quantity to be extracted from *ab initio* calculations or data (similarly to the magnetic moments or the axial coupling constant).

The trace of SEM, corresponding to the divergence of the dilation current, $J_\mu^D = \Theta_{\mu\nu} x^\nu$, and the trace anomaly of QCD, $\Theta_\mu^\mu = \partial^\mu J_\mu^D$, are both related to the scalar gravitational form factor $\Theta(t)$:

$$\begin{aligned} \langle p', s' | T_\mu^\mu(0) | p, s \rangle &= \bar{u}(p', s') \Theta(t) \bar{u}(p, s), \\ \Theta(t) &= \frac{1}{m_N} \left[\left(m_N^2 - \frac{t}{4} \right) A(t) - \frac{3}{4} t D(t) + \frac{1}{2} t J(t) \right], \end{aligned} \quad (9)$$

with the condition $\Theta(0) = m_N$. From there, one has

$$D(0) = \frac{4m_N}{3} [m_N A'(0) - \Theta'(0)]. \quad (10)$$

The extension of the Raman decomposition introduced in Sec. II A to the nucleon case is obtained by explicitly choosing a mutually orthogonal basis in the tensor components. For the matrix element $\Theta^{\mu\nu} \equiv \langle p', s' | T^{\mu\nu}(0) | p, s \rangle$, it takes the form

$$\begin{aligned}\Theta^{\mu\nu} &= \Theta_S^{\mu\nu} + \Theta_T^{\mu\nu}, \\ \Theta_S^{\mu\nu} &= \frac{1}{3} Q^{\mu\nu} \bar{u}' \Theta(t) u, \\ \Theta_T^{\mu\nu} &= \Theta^{\mu\nu} - \frac{1}{3} Q^{\mu\nu} \bar{u}' \Theta(t) u,\end{aligned}\quad (11)$$

where the transverse tensor is defined as

$$Q^{\mu\nu} \equiv g^{\mu\nu} - q^\mu q^\nu / q^2, \quad (12)$$

fulfilling $Q_\mu^\mu = 3$ and $q^\mu Q_{\mu\nu} = 0$. Explicitly,

$$\begin{aligned}\Theta_T^{\mu\nu} &= \frac{\bar{u}' u}{m_N} \left[P^\mu P^\nu - \frac{P^2}{3} Q^{\mu\nu} \right] A(t), \\ &+ \frac{1}{m_N} \bar{u}' \left[i P^{\{\mu} \sigma^{\nu\} \rho} q_\rho - \frac{t}{6} Q^{\mu\nu} \right] u J(t).\end{aligned}\quad (13)$$

Thus, as already stressed, the Raman decomposition implements a separate conservation of the scalar and tensor parts—namely, $q_\mu \Theta_S^{\mu\nu} = q_\mu \Theta_T^{\mu\nu} = 0$. We note that the apparent nonlocality in Eq. (12) does not plague the considered quantities, as we only use the properties $q_\mu Q^{\mu\nu} = 0$ and $Q_\mu^\mu = 3$. The separate pole terms in $1/q^2$ cancel in the total sum by the condition $\Theta(0) = m_N A(0)$. Moreover, the A and J terms of Eq. (13) are separately traceless and conserved. Equation (13) can be cast in a form involving mutually orthogonal, traceless, and conserved tensors—namely,

$$\begin{aligned}\Theta_T^{\mu\nu} &= \frac{\bar{u}' u}{m_N} \left[P^\mu P^\nu - \frac{P^2}{3} Q^{\mu\nu} \right] \left(A(t) + \frac{t}{2P^2} J(t) \right) \\ &+ \frac{1}{m_N} \bar{u}' \left[i P^{\{\mu} \sigma^{\nu\} \rho} q_\rho - \frac{t}{2P^2} P^\mu P^\nu \right] u J(t),\end{aligned}\quad (14)$$

which has interesting unitarity properties in the timelike region, corresponding to the helicity nonflip and flip amplitudes (see Sec. II E).

From what has been said, Θ is the form factor of the scalar 0^{++} (trace anomaly) channel, while A and J correspond to the tensor channel 2^{++} . On the other hand, D mixes the spin quantum numbers and in that sense is less fundamental. From Eq. (9),

$$D(t) = \frac{1}{3} \left[\frac{4P^2}{t} A(t) + 2J(t) - \frac{4m_N}{t} \Theta(t) \right]. \quad (15)$$

The decomposition described above is needed for the meson saturation model, as it determines which mesons

should be used for a particular form factor. We will therefore use Θ , A , and J as the basic form factors and obtain D from Eq. (15). Alternatively, one can use

$$-tD(t) = \frac{1}{3} \left[4m_N^2 \left(\frac{\Theta(t)}{m_N} - A(t) \right) - tB(t) \right], \quad (16)$$

which is the quantity appearing in the analysis of the mechanical properties.

With identity (15), the decomposition (8) can also be written in the form involving terms which are of good spin and separately conserved, which will become useful in Sec. III C—namely,

$$\begin{aligned}\langle p', s' | \Theta_{\mu\nu}(0) | p, s \rangle &= \bar{u}' \left[2J(t) \left(\gamma_{\{\mu} P_{\nu\}} - \frac{1}{3} m_N Q_{\mu\nu} \right) \right. \\ &- B(t) \frac{1}{m_N} \left(P_\mu P_\nu - \frac{P^2}{3} Q_{\mu\nu} \right) \\ &\left. + \Theta(t) \frac{1}{3} Q_{\mu\nu} \right] u.\end{aligned}\quad (17)$$

The tensors multiplying the spin-2 form factors are conserved and traceless.

C. Asymptotics from pQCD

Within pQCD, Tong, Ma, and Yuan [66,67] derived the asymptotic formulas for the GFFs on the nucleon at large $-t$, which are

$$\begin{aligned}A(t) &\sim + \frac{\alpha(t)^2}{(-t)^2}, & J(t) &\sim + \frac{\alpha(t)^2}{(-t)^2} \\ B(t) &\sim - \frac{\alpha(t)^2}{(-t)^3}, & D(t) &\sim - \frac{\alpha(t)^2}{(-t)^3}.\end{aligned}\quad (18)$$

Here, $\alpha(t)$ is the running strong coupling constant, which at the leading order reads

$$\alpha(t) = \frac{4\pi}{\beta_0 \ln(-t/\Lambda_{\text{QCD}}^2)}, \quad (19)$$

with $\beta_0 = (11N_c - 2N_f)/3 = 9$ for three active flavors. In Eq. (18), $\pm$ indicates the overall sign [we do not attempt to give the values of the constants, which depend on the specific nucleon partonic distribution amplitudes (DAs)]. As expected, the chirally odd form factors $B(t)$ and $D(t)$ fall off with an extra power of t as compared to the chirally even ones, $A(t)$ and $J(t)$. Actually, it has been found that the nontrivial relation $B(t) = 3D(t)$ holds asymptotically to leading order in pQCD [66,67].

For the scalar (trace anomaly) form factor $\Theta(t)$, the situation is somewhat more complicated. From Eq. (9), $m_N \Theta(t) = m_N^2 A(t) + \frac{t}{4} (B(t) - 3D(t)) = \mathcal{O}(m_N^2 \alpha(t)^2 / t^2)$. However, the pQCD hard kernel behaves as $\alpha(t)^3 / (-t)^2$,

and the presence of the soft end-point singularities in the nucleon DAs alters this behavior. In particular, Eq. (5.15) in Ref. [67], applying the DAs from Ref. [104], reads

$$\Theta(t) \sim -\frac{\alpha(t)^3}{(-t)^{3/2}} \log^2\left(\frac{-t}{\Lambda_c^2}\right), \quad (20)$$

where $\Lambda_c \simeq 200$ MeV is a scale associated with DAs. Note the negative sign asymptotically. As concluded in Ref. [67], since $\Theta(0) = m_N > 0$, Eq. (20) requires a change of sign in $\Theta(-t)$ in the spacelike domain $-t > 0$. Equations (18) are consistent, up to a weak logarithmic dependence, with the generic QCD counting rules [105,106].

D. Dispersion relations and sum rules

Quite generally, the nucleon form factors $F(t)$ satisfy analytic properties: They are real in the spacelike region, $t = -Q^2 \leq 0$, and possess a branch cut discontinuity starting at $s \geq 4m_\pi^2$, corresponding to the $\pi\pi \rightarrow N\bar{N}$ process below the $N\bar{N}$ threshold $s = 4m_N^2$. Hence, up to possible subtractions, a form factor $F(-Q^2)$ satisfies the dispersion relation

$$F(-Q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im } F(s)}{s + Q^2}, \quad (21)$$

where $Q^2 = -t$ is the spacelike momentum transfer squared.

The integrals in Eq. (21) converge if asymptotically $\text{Im } F(s)$ tends to zero sufficiently fast, which is the case in QCD. With the conditions in Eq. (18), one finds immediately [by expanding $1/(s + Q^2) = 1/Q^2 - s/Q^4 + \dots$] the so-called superconvergence sum rules [95,107–109],

$$\lim_{Q^2 \rightarrow \infty} Q^{2n} F(Q^2) = 0 \Rightarrow \int_{4m_\pi^2}^{\infty} ds s^{n-1} \text{Im } F(s) = 0, \quad (22)$$

$$n = 1, 2, \dots$$

This set of sum rules must be satisfied by the spectral density $\text{Im } F(s)/\pi$ of a given form factor. In particular, from Eq. (18) it follows that from pQCD, A and J satisfy the sum rules with $n = 1, 2$, while B and D with $n = 1, 2, 3$. The presence of α , carrying additional damping, is needed for the highest n value. Indeed, the timelike large- s behavior corresponding to Eq. (18) is

$$\begin{aligned} \text{Im } A(s) &\sim +\frac{1}{s^2 L^3}, & \text{Im } J(s) &\sim +\frac{1}{s^2 L^3}, \\ \text{Im } B(s) &\sim +\frac{1}{s^3 L^3}, & \text{Im } D(s) &\sim +\frac{1}{s^3 L^3}, \end{aligned} \quad (23)$$

with the meaning $\text{Im } A(s) \equiv \text{Im } A(s + i\epsilon)$. In the derivation above, we have used the analytic continuation into the complex $t = s$ plane, $-t = \exp(-i\theta)|t|$, whence $\alpha(s + i\epsilon) = 4\pi/(\beta_0(L - i\pi))$, with the shorthand notation $L = \ln(s/\Lambda^2)$.

For the case of Θ ,

$$\text{Im } \Theta(s) \sim -\frac{1}{s^{3/2}} \frac{L_c^2}{L^3}, \quad (24)$$

where $L_c = \log(s/\Lambda_c^2)$. Therefore, $\text{Im } \Theta(s)$ satisfies the sum rule (22) with $n = 1$ only.

Since all the considered GFFs of the nucleon satisfy Eq. (22) with $n = 1$ —namely, $\int_{4m_\pi^2}^{\infty} ds \text{Im } F(s) = 0$ —all the functions $\text{Im } F(s)$ cannot have a definite sign in the timelike domain $4m_\pi^2 \leq s < \infty$. This feature is analogous to the case of the gravitational spectral densities of the pion [95].

The charge-type sum rules have the form

$$F(0) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im } F(s)}{s}, \quad (25)$$

whereas the derivatives with respect to t at the origin are given by

$$\left. \frac{d^n F(t)}{dt^n} \right|_{t=0} = \frac{n!}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im } F(s)}{s^n}, \quad (26)$$

from where, as is well known, it follows that precise modeling at low t requires a detailed knowledge of the spectral densities near the 2π production threshold.

E. Spectral densities at threshold

For our later analysis, it is pertinent to determine some qualitative features of the spectral densities. In what follows, we will show that in the region $4m_\pi^2 \leq t \leq 4m_K^2$ (between the 2π and $K\bar{K}$ thresholds), the signs of the spectral functions corresponding to the Raman form factors Θ , J , $A + tJ/2P^2 = m_N^2/P^2[A + tB/4m_N^2]$, and B are well defined and in fact positive. This is consistent with positivity in the spacelike region at small $t = -Q^2$.

As already mentioned, the spectral functions corresponding to the nucleon GFFs start at $t = 4m_\pi^2$. They reflect the leading singularity in the $\pi\pi \rightarrow N\bar{N}$ process, which is not observable below the $N\bar{N}$ threshold $t = 4m_N^2$, but can be related to the observable pion form factors via unitarity and analyticity. The unitarity relations for the GFF were derived in Ref. [98] and can be written in several useful equivalent forms, for instance

$$\begin{aligned}
\text{Im } \Theta(t) &= \frac{3\sigma_\pi f_{0,+}(t)\Theta_\pi^*(t)}{2(4m_N^2 - t)}, \\
\text{Im } J(t) &= \frac{3t^2\sigma_\pi^5}{64\sqrt{6}} f_{2,-}(t)A_\pi^*(t), \\
\text{Im } A(t) + \frac{2t \text{Im } J(t)}{4m_N^2 - t} &= \frac{3t^2\sigma_\pi^5 m_N}{16(4m_N^2 - t)} f_{2,+}(t)A_\pi^*(t), \quad (27)
\end{aligned}$$

where $\sigma_\pi = \sqrt{1 - 4m_\pi^2/t}$. Note that the combinations corresponding to the mutually orthogonal Raman-like decomposition of Eq. (14), involving the form factors $J(t)$ and $A(t) + tJ(t)/2P^2 = m_N^2/P^2[A(t) + tB(t)/4m_N^2]$, comply naturally with the spin flip and spin nonflip $N\bar{N}$ amplitudes. From the above relations, we immediately get

$$\begin{aligned}
\text{Im } B(t) &= \frac{3t^2 m_N \sigma_\pi^5}{16(4m_N^2 - t)} \\
&\times \left[\frac{m_N \sqrt{2} f_{2,-}(t)}{\sqrt{3}} - f_{2,+}(t) \right] A_\pi^*(t). \quad (28)
\end{aligned}$$

From Watson's theorem for the pion GFFs, one has

$$\begin{aligned}
\Theta_\pi(t) &= |\Theta_\pi(t)| e^{i\delta_{00}(t)}, \\
A_\pi(t) &= |A_\pi(t)| e^{i\delta_{02}(t)}, \quad (29)
\end{aligned}$$

where δ_{00} and δ_{02} are the isoscalar S- and D-wave elastic scattering phase shifts for the $\pi\pi \rightarrow \pi\pi$ process, respectively. While the elastic scattering takes place strictly below the $\pi\pi$ production threshold—i.e., for the process $2\pi \rightarrow 4\pi$ starting at $s = 16m_\pi^2$ —it has been found over the years that in practice, the inelasticity takes off at the $K\bar{K}$ threshold $s = 4m_K^2$.

Clearly, since $\text{Im } \Theta(t)$ is a real quantity, $f_{0,+}(t) = \pm |f_{0,+}(t)| e^{i\delta_{00}(t)}$, where the sign can be fixed by analyzing the $\pi N \rightarrow \pi N$ data in the physical region $s \geq (m_\pi + m_N)^2$ and analytically continuing to the unphysical region using unitarity and crossing. The net result is a rapid change of $|f_{0,+}(s)|$ around and above the threshold region, due to a subthreshold singularity at $s = 4m_\pi^2 - m_\pi^4/m_N^2$, stemming from the nucleon pole exchange contribution.

In the case of the D-wave form factors A and J , the result involves both the spin flip and nonflip $N\bar{N}$ scattering amplitudes. Similarly to the S-wave case, up to a sign, they have a phase identical to the isoscalar D-wave $\pi\pi$ scattering phase shift, $\delta_{20}(t)$, such that $f_{2,\pm}(s) = \pm |f_{2,\pm}(s)| e^{i\delta_{20}(s)}$. The values around the threshold have been determined in the venerable Karlsruhe-Helsinki (KH80) analysis [110] (numerical values are tabulated in Ref. [111]) and reviewed with uncertainties by the Bonn group [112]. The data analysis supports the plus sign in all channels, such that in the interval $4m_\pi^2 \leq t \leq 4M_K^2$, one has

$$\begin{aligned}
\text{Im } \Theta(t) &= \frac{3\sigma_\pi |f_{0,+}(t)| |\Theta_\pi(t)|}{2(4m_N^2 - t)} > 0, \\
\text{Im } J(t) &= \frac{3t^2\sigma_\pi^5}{64\sqrt{6}} |f_{2,-}(t)| |A_\pi(t)| > 0, \\
\text{Im } A(t) + \frac{2t \text{Im } J(t)}{4m_N^2 - t} &= \frac{3t^2 m_N \sigma_\pi^5}{32\sqrt{6}} |f_{2,+}(t)| |A_\pi(t)| > 0, \\
\text{Im } A(t) &= \frac{3t^2\sigma_\pi^5 |A_\pi(t)|}{16(4m_N^2 - t)} \left[m_N |f_{2,+}(t)| - \frac{t}{2\sqrt{6}} |f_{2,-}(t)| \right] > 0, \\
\text{Im } B(t) &= \frac{3t^2 m_N \sigma_\pi^5 |A_\pi(t)|}{16(4m_N^2 - t)} \left[m_N \sqrt{\frac{2}{3}} |f_{2,-}(t)| - |f_{2,+}(t)| \right] > 0, \quad (30)
\end{aligned}$$

where the last inequality follows from the data analysis and the Roy-Steiner solutions, yielding numerically $m_N \sqrt{2} |f_{2,-}(t)| / \sqrt{3} - |f_{2,+}(t)| > 0$. Finally, close to the threshold, one has

$$\begin{aligned}
\text{Im } \Theta(t) &\sim +(t - 4m_\pi^2)^{\frac{1}{2}}, \\
\text{Im } A(t), \text{Im } J(t), \text{Im } B(t) &\sim +(t - 4m_\pi^2)^{\frac{5}{2}}, \quad (31)
\end{aligned}$$

where the corresponding proportionality constants are *positive* and, besides the $\pi\pi \rightarrow N\bar{N}$ amplitudes, also

involve the pion form factors at the threshold $s = 4m_\pi^2$. Whereas their normalization is $A_\pi(0) = 1$ and $\Theta_\pi(0) = 2m_\pi^2$, from the meson dominance formulas of our previous work [96], one gets

$$\begin{aligned}
A_\pi(4m_\pi^2) &= \frac{m_{f_2}^2}{m_{f_2}^2 - 4m_\pi^2} \sim 1 + \frac{4m_\pi^2}{m_{f_2}^2}, \\
\Theta_\pi(4m_\pi^2) &= 2m_\pi^2 + \frac{4m_\pi^2 m_\sigma^2}{m_\sigma^2 - 4m_\pi^2} \sim 6m_\pi^2. \quad (32)
\end{aligned}$$

F. Problem of the high-energy completion

Quite generally, one might hope that the use of the dispersion relations where $\text{Im} F(s)$ is known up to a sufficiently large value of s , such as the $N\bar{N}$ threshold $s_N = 4m_N^2$, allows one to determine accurately the form factor in the spacelike region. Taking for definiteness D , one can write

$$D(-Q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{4m_N^2} ds \frac{\text{Im} D(s)}{s + Q^2} + \frac{1}{\pi} \int_{4m_N^2}^{\infty} ds \frac{\text{Im} D(s)}{s + Q^2}, \quad (33)$$

where the spectral integral has been divided into a “better known” part below s_N and a “less known” part above s_N . Unitarity provides the absorptive part $\text{Im} D(s)$ up to the $K\bar{K}$ threshold in terms of the $\pi\pi \rightarrow N\bar{N}$ amplitudes $f_{+,0}(s)$ and $f_{\pm,2}(s)$ and the pion form factors in the timelike region, $A_\pi(t)$ and $\Theta_\pi(t)$. However, the high- s part is not known accurately. Alternatively, one could use subtracted dispersion relations, but then the predictive power is diminished. For instance, $D(0)$ would be taken as a low-energy constant, but its value would not be predicted.

The recent work of Ref. [98] based on dispersion relations and incorporating a full Roy-Seiner analysis with all the pertinent channels, including pions, kaons, and nucleons, has indeed reported a slow convergence of the dispersive integrals, which has some impact in the space-like region. This is relevant when confronting with the lattice QCD data [81] for $Q^2 \leq 2 \text{ GeV}^2$. To overcome this contingency, additional narrow resonances have been included in the analysis of Ref. [98].

With this guidance, in the next section, we retake the old idea of the meson dominance to describe the lattice data with a minimal but realistic amount of information.

III. MESON DOMINANCE APPROACH

Over the years, the idea of resonance saturation has been extended to all possible quantum number channels. In our case, we are concerned with the scalar- and tensor-meson dominance. We first sketch the basic features of the approach and then go into a detailed analysis, exploring the predictive power of the meson dominance approach for both GFFs and the mechanical properties of the nucleon.

A. Basic features of meson dominance

The meson dominance approach, motivated by the large- N_c arguments, is known to work remarkably well for hadronic form factors at intermediate spacelike momenta (see, e.g., Ref. [94] and references therein).² A form factor

²On the contrary, the determination of the timelike behavior of the form factors usually requires abundant data in a wide interval and, from the dispersive point of view, an extrapolation to the unphysical region. Fortunately, the impact of the timelike details on the spacelike region considered here is mild.

F is approximated as, in general, an infinite sum over narrow resonances in the appropriate channel,

$$F(-t) = \sum_i \frac{c_i}{m_i^2 - t}. \quad (34)$$

Here, m_i denotes the resonance mass and c_i its coupling. Alternatively, the spectral strength is given by

$$\frac{1}{\pi} \text{Im} F(-t) = \sum_i c_i \delta(m_i^2 - t). \quad (35)$$

The short-distance (or large $-t$) constraints (also known as the counting rules) impose relations between c_i 's and m_i 's. In particular, if $\lim_{t \rightarrow -\infty} (-t)^n F(t) = 0$, one obtains the constraints

$$\sum_i c_i m_i^{2k} = 0, \quad k = 0, \dots, n-1, \quad (36)$$

which here are just a manifestation of the superconvergence sum rules (22).

The previous formulas require a *minimal* number of resonances consistent with normalization and the pQCD sum rules. A natural extension is to take an infinite number of resonances as required by large N_c and quark-hadron duality for *two-point functions*. Actually, for this case, pQCD provides a log-increasing function of Q^2 and quark-hadron duality and consistent with an infinite Regge spectrum of excited resonances coupled to the current with an asymptotically constant amplitude. For three-point functions, this is far from trivial, since typically in the meson theory, one has *increasing* functions of the logarithm, whereas in pQCD one obtains *decreasing* ones. This was explicitly discussed and illustrated in Refs. [113–115], where different schemes are suggested to comply with pQCD.

B. Current-field identities

In the particular SEM case, the current-field identity [24–26] is the simplest way to express the scalar- and tensor-meson dominance,

$$\Theta^{\mu\nu} = \sum_S \frac{1}{3} f_S (\partial^\mu \partial^\nu - g^{\mu\nu} \partial^2) S + \sum_T f_T m_T^2 T^{\mu\nu}, \quad (37)$$

where S and $T^{\mu\nu}$ are the scalar 0^{++} and tensor 2^{++} meson fields, respectively, with $T^{\mu\nu} = T^{\nu\mu}$ and $T^\mu_\mu = 0$. On-shell, the fields have masses m_S and m_T , respectively, and $\partial^\mu T_{\mu\nu} = 0$ (for the complete Lagrangian, see, e.g., Ref. [116,117]). Defining the corresponding sources as J_S and $J_T^{\mu\nu}$, one gets the equations of motion

$$(\partial^2 + m_S^2) S = J_S, \quad (38)$$

$$(\partial^2 + m_T^2) T^{\mu\nu} = J_T^{\mu\nu} + \text{dist.}, \quad (39)$$

where the distributional contributions are deltas and derivatives located at $x = 0$. Passing to the momentum space, we get formally (up to polynomials in q)

$$\begin{aligned} \langle A | \Theta^{\mu\nu} | B \rangle &= \sum_S \frac{f_S}{3} \frac{g^{\mu\nu} q^2 - q^\mu q^\nu}{m_S^2 - q^2 - i\epsilon} \langle A | J_S | B \rangle \\ &+ \sum_T f_T \frac{m_T^2}{m_T^2 - q^2 - i\epsilon} \langle A | \sum_\lambda \epsilon_\lambda^{\mu\nu} \epsilon_{\alpha\beta}^\lambda J_T^{\alpha\beta} | B \rangle, \end{aligned} \quad (40)$$

where $\epsilon_\lambda^{\mu\nu}$ is the spin-2 polarization tensor, which is symmetric, $\epsilon_\lambda^{\mu\nu} = \epsilon_\lambda^{\nu\mu}$; traceless, $g_{\mu\nu} \epsilon_\lambda^{\mu\nu} = 0$; and transverse, $q_\mu \epsilon_\lambda^{\mu\nu} = 0$. Clearly, the separate conservation of the scalar and tensor contributions is manifest according to the Raman decomposition.

C. t -channel unitarity

From a field theory point of view, the meson dominance formula should not be taken literally, as it does not incorporate the notion of subtractions or the pQCD high-momentum behavior. Besides, it is well known that higher-spin fields have certain problems, particularly due to the role played by the off-shell behavior of the propagators [116,117]. The simplest way to avoid these issues is to use the meson dominance for the absorptive parts in the dispersion relations, where by construction the meson resonances are on the mass shell (see, e.g., Ref. [95] for the pion GFF case). Then, one can fix the minimal number of subtractions needed to satisfy the short-distance constraints (counting rules) imposed by pQCD.

It is far more practical to compute the absorptive part of the form factor in the timelike region, where for the graviton we have $g \rightarrow R \rightarrow N\bar{N}$. At $q^2 \rightarrow s + i\epsilon$, we then have

$$\frac{1}{\pi} \text{Im} \langle N\bar{N} | \Theta^{\mu\nu} | 0 \rangle = \sum_R \langle N\bar{N} | R \rangle \langle R | \Theta^{\mu\nu} | 0 \rangle \delta(m_R^2 - s) \quad (41)$$

and later will reconstruct the dispersive part from the dispersion relation with suitable subtraction constants. The vacuum-to-hadron transition amplitudes are

$$\langle S | \Theta^{\mu\nu} | 0 \rangle = \frac{1}{3} f_S q^2 Q^{\mu\nu}, \quad \langle T | \Theta^{\mu\nu} | 0 \rangle = f_T m_T^2 \epsilon_\lambda^{\mu\nu}, \quad (42)$$

where the extra factor of $1/3$ in the scalar case is conventional, chosen such that $\langle S | \Theta | 0 \rangle = f_S q^2$. The *on-shell* couplings of the resonances to the $N\bar{N}$ continuum are conventionally taken as [118]

$$\begin{aligned} \langle S | N\bar{N} \rangle &= g_{SNN}, \\ \langle T | N\bar{N} \rangle &= \epsilon_\lambda^{\alpha\beta} \bar{v}(-p') [g_{TNN} P^{\{\alpha\gamma\beta\}} + f_{TNN} P^\alpha P^\beta] u(p), \end{aligned} \quad (43)$$

with v denoting the $\bar{N}$ spinor. Thus, we get

$$\begin{aligned} \frac{1}{\pi} \text{Im} \langle N\bar{N} | \Theta^{\mu\nu} | 0 \rangle &= \sum_S \frac{g_{SNN} f_S}{3} \delta(m_S^2 - q^2) m_S^2 Q^{\mu\nu} \\ &+ \sum_{T,\lambda} \epsilon_\lambda^{\alpha\beta} P^{\{\alpha\gamma\beta\}} \epsilon_\lambda^{\mu\nu} g_{TNN} f_T m_T^2 \delta(m_T^2 - q^2) \\ &+ \sum_{T,\lambda} \epsilon_\lambda^{\alpha\beta} P_\alpha P_\beta \epsilon_\lambda^{\mu\nu} f_{TNN} f_T m_T^2 \delta(m_T^2 - q^2), \end{aligned} \quad (44)$$

which naturally complies with a separate conservation for each term, yielding zero when contracted with q^μ . The sum over the tensor polarizations is given by [119,120]

$$\sum_\lambda \epsilon_\lambda^{\alpha\beta} \epsilon_\lambda^{\mu\nu} = \frac{1}{2} (Q^{\mu\alpha} Q^{\nu\beta} + Q^{\nu\alpha} Q^{\mu\beta}) - \frac{1}{3} Q^{\mu\nu} Q^{\alpha\beta}. \quad (45)$$

The on-shell condition $P \cdot q = 0$ implies $P_\alpha Q^{\alpha\beta} = P^\beta$ and $\bar{v}' q u = 0$ yields $\gamma_\alpha Q^{\alpha\beta} = \gamma^\beta$, hence we obtain

$$\begin{aligned} \sum_\lambda \epsilon_\lambda^{\alpha\beta} P_\alpha P_\beta \epsilon_\lambda^{\mu\nu} &= P^\mu P^\nu - \frac{1}{3} P^2 Q^{\mu\nu}, \\ \sum_\lambda \epsilon_\lambda^{\alpha\beta} P^{\{\alpha\gamma\beta\}} \epsilon_\lambda^{\mu\nu} &= P^{\{\mu\gamma\nu\}} - \frac{1}{3} Q^{\mu\nu} P^\gamma, \end{aligned} \quad (46)$$

which exactly reproduces the tensor structure in Eq. (17). Therefore, in the narrow resonance, large- N_c -motivated approach, we get

$$\begin{aligned} \frac{1}{\pi} \text{Im} J(s) &= \frac{1}{2} \sum_T g_{TNN} f_T m_T^2 \delta(m_T^2 - q^2), \\ \frac{1}{\pi} \text{Im} B(s) &= -2m_N \sum_T f_{TNN} f_T m_T^2 \delta(m_T^2 - q^2), \\ \frac{1}{\pi} \text{Im} \Theta(s) &= \sum_S g_{SNN} f_S m_S^2 \delta(m_S^2 - q^2), \end{aligned} \quad (47)$$

and $\text{Im} A(s) = 2 \text{Im} J(s) - \text{Im} B(s)$, where, as expected, the corresponding spectral strengths get contributions exclusively from the proper spin states: A and B from spin-2 and Θ from spin-0.

D. Minimal hadronic saturation

In practice, to avoid proliferation of parameters, one tries to limit the number of mesons (recall footnote 1) to the lowest possible number consistent with the constraints.³ Note,

³For instance, with two mesons included and the $1/(-t)^2$ asymptotic behavior, the condition $c_1 + c_2 = 0$ cancels the $1/(-t)$ behavior, yielding $F(t) = F(0) m_1^2 m_2^2 / [(m_1^2 - t)(m_2^2 - t)]$. The construction based on Eqs. (34) and (36) provides a natural interpretation for the widely used multipole fits—for instance, the dipole form for the nucleon electromagnetic form factors—since $1/(m^2 - t)^2$ is numerically close to $1/[(m_1^2 - t)(m_2^2 - t)]$ for a suitably chosen m in the considered range of $-t$.

however, that more precise modeling in the timelike region typically utilizes more resonances and incorporates their width, in an effort to reproduce the spectra. An example is the Gounaris-Sakurai model [121,122] for the pion charge form factor, where as many as four ρ resonances are used, including the width effects. However, in the spacelike region considered here with the aim to understand the lattice QCD data, the meson dominance modeling is not sensitive to the details of the timelike region.

In this work, we use the asymptotic forms of Eq. (18) as an inspiration to build the meson saturation model. We do not take into account the log corrections, as they would require more precise modeling of the spectral densities, not relevant for the low values of $0 < -t < 2 \text{ GeV}^2$, and also would increase the number of free parameters. Ignoring the log corrections from $\alpha(t)$ weakens the asymptotic convergence and causes a lack of the sum rule (22) with the highest value of n in each channel.⁴ As stated above, in a minimum model, one uses the lowest possible number of resonances needed to implement the large $-t$ (the short-distance) constraints, as well as the behavior at $t = 0$. This number is determined by $B(t)$. Since $B(0) = 0$ and at $t \rightarrow -\infty$ we have $B(t) \sim 1/(-t)^3$, the lowest number of resonances is four, which leads to the form

$$B(t) = \frac{c_B t}{(1 - t/m_{f_2}^2)(1 - t/m_{f_2'}^2)(1 - t/m_{f_2''}^2)(1 - t/m_{f_2'''}^2)}, \quad (48)$$

where $f_2 \dots f_2'''$ are the four lowest-mass 2^{++} mesons.

We assume, accordingly, that the A and J channels couple to the same four resonances, but with different couplings, such that $A(0) = 1$, $J(0) = \frac{1}{2}$, and asymptotically Eqs. (18) hold. Thus, we take⁵

$$\begin{aligned} \text{I: } m_{f_2} &= 1.275, & m_{f_2'} &= 1.517, \\ \text{II: } m_{f_2} &= 1.275, & m_{f_2'} &= 1.430, \end{aligned}$$

with set I corresponding to the first four entries in the PDG (Particle Data Group) [123] summary tables, and set II corresponding to the PDG listings. Comparing the fits made with sets I and II allows for an estimate of a “systematic” error in our analysis. It turns out that in the range of the spacelike lattice data, this error is much smaller (relatively, a few percent) from the statistical error origi-

⁴For the electromagnetic case, models including the log corrections, thus satisfying all the superconvergence sum rules, were explored in Ref. [109].

⁵For a similar construction of the ansatz for the case of the nucleon electromagnetic form factors, see, e.g., Ref. [94].

$$\begin{aligned} A(t) &= \frac{1 - c_A t + c_2 t^2}{(1 - t/m_{f_2}^2)(1 - t/m_{f_2'}^2)(1 - t/m_{f_2''}^2)(1 - t/m_{f_2'''}^2)}, \\ J(t) &= \frac{1 - c_J t + c_2 t^2}{2(1 - t/m_{f_2}^2)(1 - t/m_{f_2'}^2)(1 - t/m_{f_2''}^2)(1 - t/m_{f_2'''}^2)}. \end{aligned} \quad (49)$$

Equation (7) and the above formulas are consistent with Eq. (48) with $c_B = c_J - c_A$ [the occurrence of the same c_2 in both formulas (49) follows from $B(t) \sim 1/t^3$].

In the scalar channel, where $\Theta(0) = m_N$, we take for simplicity two resonances,

$$\Theta(t) = \frac{m_N}{(1 - t/m_\sigma^2)(1 - t/m_{f_0}^2)}, \quad (50)$$

where the power agrees with the counting rules for the corresponding pQCD kernel. Here, m_σ is an effective mass of the sigma meson and f_0 is the scalar resonance $f_0(980)$. Again, more precise modeling would require more realistic and complicated spectral functions, which is expected to be irrelevant in the spacelike momentum region explored in lattice QCD simulations.

Naturally, Eqs. (49) and (50) can be written, via pole expansion of a meromorphic function, as sums of simple poles of the form (34), where the residues “conspire” in such a way that the short-distance constraints are satisfied. Some properties of the fitted functions are presented in the Appendix.

E. Fitting procedure

We treat m_σ , c_A , c_J , and c_2 as free model parameters. The mass of f_0 is fixed at $m_{f_0} = 980 \text{ MeV}$, while for the tensor masses we probe two sets, largely differing in the second and third excited states (all masses in GeV),

$$\begin{aligned} m_{f_2''} &= 1.565, & m_{f_2'''} &= 1.936, \\ m_{f_2''} &= 1.517, & m_{f_2'''} &= 1.565, \end{aligned} \quad (51)$$

nating from the uncertainty in the data. For this reason, we combine the systematic and statistical errors in the results presented below.

Since the lattice data correspond to $m_\pi = 170 \text{ MeV}$, we use correspondingly an increased nucleon mass, $m_N = 970 \text{ MeV}$ [124]. We do not change the values of the resonance masses, as they are expected to be at the level of a few percent. Accordingly, our results for the nucleon GFFs correspond to $m_\pi = 170 \text{ MeV}$. Based on our experience for the pion [95], we anticipate that the results of the chiral evolution to the physical point $m_\pi = 140 \text{ MeV}$ are not very significant, at a level of a few percent for the values of the model parameters.

F. Results

The joint fit to the available MIT lattice data for A , J , and D [for which we use Eq. (15)] yields for set I

$$\begin{aligned} c_A &= 0.62(5) \text{ GeV}^{-2}, & c_J &= 0.87(6) \text{ GeV}^{-2}, \\ c_2 &= 0.15(5) \text{ GeV}^{-4}, & m_\sigma &= 0.64(4) \text{ GeV}, \end{aligned} \quad (52)$$

and for set II

$$\begin{aligned} c_A &= 0.83(6) \text{ GeV}^{-2}, & c_J &= 1.12(7) \text{ GeV}^{-2}, \\ c_2 &= 0.25(5) \text{ GeV}^{-4}, & m_\sigma &= 0.64(4) \text{ GeV}, \end{aligned} \quad (53)$$

where the errors are statistical. The change of the c_i coefficients between the two sets follows the features explained in the Appendix. For both sets, the values of $\chi^2/\text{d.o.f.}$ are ~ 0.4 , indicating that the nature of the lattice data errors involves systematic uncertainty and/or correlations. In our fits, the values of c_A , c_J , and c_2 change to compensate for the effect of different masses of the m_{f_2} states in parametrizations I and II in the range of the data. The value of m_σ is in agreement within the uncertainties with the fit for the pion case in Ref. [95], where we obtained $m_\sigma = 0.65(3) \text{ GeV}$ at $m_\pi = 170 \text{ MeV}$.

The correlation matrix, for the rows and columns in the sequence of parameters given above, is, for case I,

$$\rho = \begin{pmatrix} 1 & 0.7 & -0.9 & 0.3 \\ 0.7 & 1 & -0.8 & 0.1 \\ -0.9 & -0.8 & 1 & -0.1 \\ 0.3 & 0.1 & -0.1 & 1 \end{pmatrix} \quad (54)$$

(for case II, it is similar). We note that m_σ is weakly correlated to the other parameters, as this correlation occurs only by their common appearance in D . On the other hand, the c_i coefficients are strongly correlated. The correlations are taken into account in the error propagation to the form factors, shown in the figures below with the bands, indicating the 68% confidence levels.

In Fig. 1, we plot the form factors A , J , and B . We get a proper description of the MIT lattice data [81]. The bands in the figures indicate the combined systematic error from the two choices of the tensor-meson spectrum (51), and from the statistical error reflecting the uncertainties in the fitted lattice data. We note that the systematic error for A and J , estimated by comparing sets I and II, is significantly smaller from the statistical error (a few percent), reflecting the fact that at the available range of $-t$ one cannot resolve the details of the model.

For the case of B , the values for the data were obtained via the relation (7), with the errors added in quadrature. We note the smallness of B , yet it is significantly larger from zero at $-t > 0$ compared to the error bars. The LHPC Collaboration [91] computed the quark parts of the GFFs

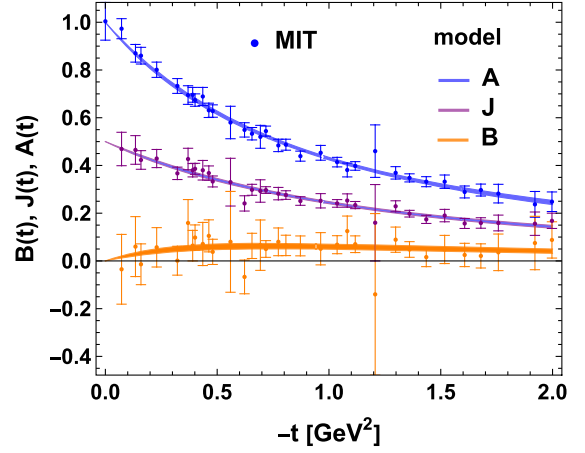


FIG. 1. Gravitational form factors of the nucleon, A , J , and B , plotted as functions of the spacelike momentum transfer $-t$. The MIT lattice QCD data for A and J are taken from Ref. [81], while those for B follow from the relation $B = 2J - A$ with errors added in quadrature. The meson dominance model fit is shown with the error bands, whose widths reflect the uncertainty in the parameters (see the text for details). The MIT data and the fits are for $m_\pi = 170 \text{ MeV}$.

and found a similar trend. Interestingly, the observed smallness was predicted long ago by Renner [125] within the quark model. The feature is reproduced in the chiral soliton model [126] and less accurately in the Skyrme model [55].

The scalar GFF $\Theta(t)$ is plotted in Fig. 2, where the data were obtained from Ref. [81] via the combination of Eq. (9)

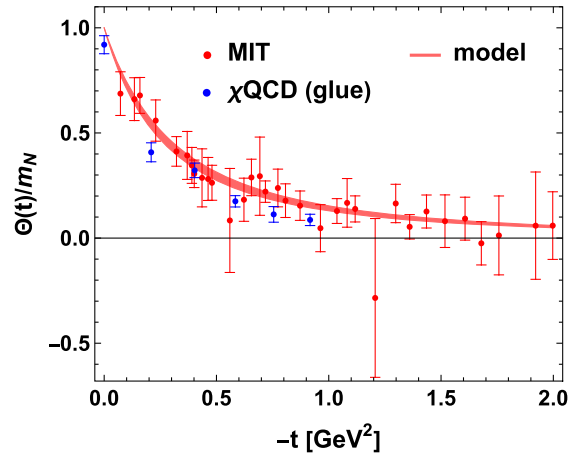


FIG. 2. Scalar (trace anomaly) gravitational form factor Θ of the nucleon, divided with the nucleon mass m_N and plotted as a function of the spacelike momentum transfer $-t$. The red data points labeled MIT are obtained from the data [81] via the relation (9), with errors added in quadrature. The meson dominance model fit is shown with the error band, whose width reflects the uncertainty of the effective σ mass (see text for details). The MIT data and the fit are for $m_\pi = 170 \text{ MeV}$. We also show (blue points), for a rough comparison, the glue contribution to the trace anomaly form factor at $m_\pi = 253 \text{ MeV}$ from the χ QCD Collaboration [86].

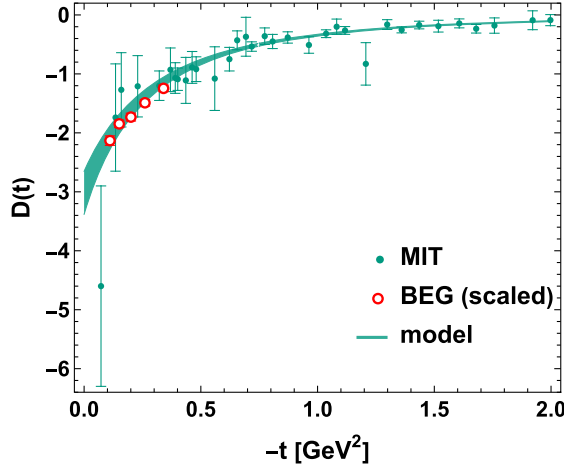


FIG. 3. Gravitational form factor D of the nucleon, plotted as a function of the spacelike momentum transfer $-t$. The MIT lattice QCD data are from Ref. [81]. The meson dominance model fit is shown with an error band whose width reflects the uncertainty in the parameters (see the text for details). The MIT lattice data and the fit are for $m_\pi = 170$ MeV. The points labeled BEG (open circles) are from DVCS obtained for the quark part in Ref. [72] and rescaled as described in the text.

with errors added in quadrature. The minimum meson dominance formula (50) fits the data properly, and as already remarked, with m_σ compatible within uncertainty to the case of the pion GFFs [95]. The present lattice errors are too large and the range of $-t$ too small to test more precisely the asymptotic behavior of Eq. (20).

Figure 3 shows the form factor D . In this case, the model description follows from Eq. (15). Again, we can see a proper description of the MIT lattice data, with the error band getting broader as $-t$ is decreased. For comparison, we also show the physical data from deeply virtual Compton scattering (DVCS) on the proton measured at CLAS [71] and extracted in Ref. [72] for the quark part of $D(t)$. The data points were obtained from Fig. 11 of Ref. [45] by utilizing the relation $D_{u+d} = \frac{18}{25} C_H(t)$, cf. Eq. (40) in Ref. [45]. To compare to $D(t)$ including the gluons, we assume that the form factors for the quarks and gluons have the same shape⁶ in $-t$, which allows for the scaling $D(t) = (\langle x \rangle_q + \langle x \rangle_g) / \langle x \rangle_q D_q(x)$. We take the world average $\langle x \rangle_g \simeq 0.41$ at $\mu = 2$ GeV, using the compilation of Fig. 8 in Ref. [128]. Thus, extracted DVCS data lie within the band corresponding to our fit to the MIT data.

Note that in our model, at spacelike momenta, A , J , B , and Θ are positive-definite, whereas D is negative-definite.

⁶This in general may not be the case, as the form factors $\bar{c}_a(t)$, dependent on the renormalization scheme and scale, may be different for various parton species. However, in models where all the gluons are radiatively generated by the QCD evolution from a low quark model scale, the shapes of the gluon and quark form factors are equal [127].

Hence, all form factors have a definite sign. Except for Θ , which according to Eq. (20) should change the sign, the obtained sign behavior is compatible with the asymptotics of Eq. (18).

G. D -term

The obtained value of the D -term corresponding to the plot in Fig. 3 is

$$D(0) = -3.0(4). \quad (55)$$

The mechanical radius, as defined in Ref. [129], is

$$\langle r^2 \rangle_{\text{mech}} = \frac{6D(0)}{\int_0^\infty d(-t)D(t)} = [0.72(5) \text{ fm}]^2, \quad (56)$$

which is a value comparable to other estimates (see Ref. [130] for a recent review).

IV. TRANSVERSE DENSITIES

A. Motivation

While not directly accessible in experiment, the transverse two-dimensional densities of hadrons, evaluated in the infinite-momentum frame, were promoted in Refs. [11–16] as the proper spatial distributions, dependent only on the intrinsic features of the probed state. The main advantage is a possibility, in the case where positivity holds, of a probabilistic interpretation of the parton distributions. In particular, the transverse density of quarks of a particular flavor is manifestly positive-definite [13,14,17]. Analogously, the transverse Θ^{++} distribution is also positive-definite (in the light-cone gauge) for any hadronic state, as shown generally in Ref. [97]:

$$\int dx^- \Theta^{++}(b, x^-)|_{x^+=0} > 0. \quad (57)$$

Moreover, the transverse densities are related to the transverse mechanical properties, as elaborated below, which is of attractive interpretational merit.

B. Definitions

We use the light-cone coordinate system in the convention $a^\pm = 1/\sqrt{2}(a^0 \pm a^3)$. The infinite-momentum Breit frame (IMBF), also known as the Drell-Yan frame [131], is defined as

$$\begin{aligned} P^1 &= P^2 = 0, & P^- &= \frac{m_N^2 - \frac{1}{4}t}{2P^+}, \\ q^+ &= q^- = 0, \end{aligned} \quad (58)$$

with $P^+ = p^+ = p'^+ \rightarrow \infty$. In this frame, the Dirac spinors are

$$u(p, s) = \sqrt{p^+} \begin{pmatrix} 1 \\ \sigma_3 \end{pmatrix} \chi_s, \quad (59)$$

with χ_s denoting a two-component Pauli spinor and $\bar{u}'(p', s')u(p, s) = 2P^+ \delta_{ss'}$. IMBF is used in these formulas and the following subsections.

The matrix elements in the transverse coordinate space are defined via the two-dimensional Fourier transforms of the basic SEM matrix elements—namely,

$$T^{\mu\nu}(b) = \int \frac{d^2 q_\perp}{(2\pi)^2 2P^+} e^{-i\mathbf{b} \cdot \mathbf{q}_\perp} \langle p', s | \Theta^{\mu\nu}(0) | p, s \rangle. \quad (60)$$

For the $++$ component, one finds

$$T^{++}(b) = \frac{P^{+2}}{m_N} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b} \cdot \mathbf{q}_\perp} A(-q_\perp^2); \quad (61)$$

hence, it depends only on the A form factor. We may thus define

$$\begin{aligned} A(b) &= \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b} \cdot \mathbf{q}_\perp} A(-q_\perp^2) \\ &= \int_0^\infty \frac{q_\perp dq_\perp}{2\pi} J_0(bq_\perp) A(-q_\perp^2), \end{aligned} \quad (62)$$

with an obvious normalization $\int_0^\infty 2\pi b db A(b) = 1$. According to Eq. (61), $A(b)$ can be interpreted as the distribution of P^{+2}/m_N in the transverse coordinate plane.

For the trace anomaly form factor, we obtain

$$\begin{aligned} \Theta(b) &\equiv 2T^{+-}(b) - T^{11}(b) - T^{22}(b) \\ &= \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b} \cdot \mathbf{q}_\perp} \Theta(-q_\perp^2), \end{aligned} \quad (63)$$

with $\int_0^\infty 2\pi b db \Theta(b) = m_N$.

The transverse coordinate spin density along the z axis, $J(b)$, is evaluated as [cf. Eq. (76) of Ref. [132] and the discussion of the associated symmetric prescription for $\Theta^{\mu\nu}$]

$$\begin{aligned} J(b) &= \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}} \left[J(-q_\perp^2) + q_\perp^2 \frac{dJ(-q_\perp^2)}{dq_\perp^2} \right] \\ &= \frac{b}{4\pi} \int_0^\infty dq_\perp J_1(bq_\perp) [q_\perp^2 J(-q_\perp^2)], \end{aligned} \quad (64)$$

where in getting to the last line we have integrated by parts and used the relation for the Bessel functions, $\frac{d}{dz} J_0(z) = -J_1(z)$.

The transverse density of D is related to the transverse (mechanical) components of SEM, discussed in detail in Sec. V.

C. Dispersive sum rules and the $b \rightarrow 0$ limit

For the transverse density defined via

$$\begin{aligned} F(b) &= \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b} \cdot \mathbf{q}_\perp} F(-q_\perp^2) \\ &= \frac{1}{2\pi} \int q_\perp dq_\perp J_0(bq_\perp) F(-q_\perp^2), \end{aligned} \quad (65)$$

the dispersion relation yields, after carrying out the $q_\perp$ integration (which can be interchanged with the integration over s for convergent integrals), the following formula [133]:

$$F(b) = \frac{1}{2\pi^2} \int_{4m_\pi^2}^\infty ds K_0(b\sqrt{s}) \text{Im } F(s). \quad (66)$$

For the special case of the J form factor, with the definition (64), one finds instead

$$J(b) = \frac{b}{4\pi^2} \int_{4m_\pi^2}^\infty ds \sqrt{s} K_1(b\sqrt{s}) \text{Im } J(s). \quad (67)$$

Because at asymptotically large arguments the Bessel functions $K_n(z) \sim e^{-z} \sqrt{\pi/(2z)}$, integrals in Eqs. (66) and (67) are convergent.

At $b \rightarrow 0$, the integration kernel in (66) is singular, since $K_0(b\sqrt{s}) \sim -\frac{1}{2} \log(b^2 s) + \text{const} + \mathcal{O}(b^2 \log b)$. However, the $\log(b^2)$ term and the constant are canceled via the superconvergence sum rule (22) for $n = 1$. This yields a model-independent general sum rule

$$F(b = 0) = -\frac{1}{4\pi^2} \int_{4m_\pi^2}^\infty ds \log s \text{Im } F(s), \quad (68)$$

holding for $F = A, B, \Theta$, and D . Equation (68) may be viewed as a spectral sum rule, which due to the $-\log s$ weight⁷ collects contributions from the whole physical region in s , and not—as one might naively expect—predominantly from the low- s values. At lower s , this weight is positive, and at large s it is negative, but since the spectral strength has no definite sign as implied by the superconvergence sum rule, it is not possible to make *a priori* statements concerning the sign of $F(b = 0)$.

Moreover, there is an important methodological warning following from Eq. (68) which we have not seen discussed in the literature. The formula indicates a strong sensitivity also to the high- s part of the spectrum, which at present is largely unknown and dependent on the particular model used. Another way to see this more directly is from Eq. (65) and noticing that the tails of the form factors in Figs. 1–3

⁷One does not need to keep a dimensionful scale here, since $\log(s/\mu^2) = \log s - \log \mu^2$, and the second term is canceled thanks to the sum rule (22).

extending outside the range of the data, and still much below the—in principle—known pQCD asymptotics, are strong. This uncertainty must be kept in mind when obtaining the transverse densities from models or fits to data, including the mechanical properties discussed in Sec. V.

For the curvature at $b = 0$, an analogous reasoning to the one leading to Eq. (68) can be carried out with the sum rule (22) for $n = 2$, which results in

$$F(b) = F(b=0) - \frac{b^2}{16\pi^2} \int_{4m_\pi^2}^{\infty} ds s \log s \operatorname{Im} F(s) + \mathcal{O}(b^4 \log b). \quad (69)$$

According to the discussion in Sec. II C, with proper pQCD constraints, it holds for $F = A, B$, and D . Similarly, for the b^4 contribution, we obtain

$$F(b) = F(b=0) + F'(b=0)b^2 - \frac{b^4}{256\pi^2} \int_{4m_\pi^2}^{\infty} ds s^2 \log s \operatorname{Im} F(s) + \mathcal{O}(b^6 \log b), \quad (70)$$

valid for $F = B$ and D , where the sum rule (22) for $n = 3$ holds. The higher-order terms in b contain the also $\log b$ pieces.

For J , using $K_1(b\sqrt{s}) \sim 1/(b\sqrt{s}) + b\sqrt{s}\frac{1}{4}\log(b^2s) + \text{const.} + \mathcal{O}(b^2 \log b)$, we find

$$J(b) = \frac{b^2}{16\pi^2} \int_{4m_\pi^2}^{\infty} ds s \log s \operatorname{Im} J(s) + \mathcal{O}(b^4 \log b). \quad (71)$$

Equations (69)–(71) are even more sensitive to the high-lying part of the spectrum than Eq. (68) due to extra powers of s in the integrand, hence the uncertainty discussed above is enhanced.

The regular behavior of $F(b=0)$ from Eq. (68) is in contrast to the case of the pion GFFs, where the long $q_\perp$ tails [pQCD predicts $A^\pi(t) \sim D^\pi(t) \sim \alpha(t)/(-t)$ and $\Theta^\pi \sim \alpha(t)^2$ [66]] make the integrals in the Fourier transforms only conditionally convergent, and thus the transverse densities of A and Θ for the pion become singular at $b = 0$ [97,134] and of a definite sign. The appearance of this singularity is clear from the present discussion. The pion gravitational form factors A and D , as well as the pion electromagnetic form factor, have the corresponding spectral densities behaving as $\sim 1/(s \log^2 s)$. Then, the integrand of (68) is asymptotically $1/(s \log s)$, which leads to divergence at $b \rightarrow 0$.

We conclude the general considerations of this subsection by remarking that for the nucleon electromagnetic form factor, $F_1(t) \sim \alpha^2/(-t)^2$, Eqs. (68) and (69) are valid, while for $F_2(t) \sim \alpha^2/(-t)^3$, Eq. (70) also applies.

D. Large- b asymptotics

Using Eq. (31) for the threshold behavior of the spectral densities at the 2π branch point and formulas (66) and (67), we arrive at the following behavior at $b \rightarrow \infty$:

$$\begin{aligned} A(b), B(b) &\sim +e^{-2m_\pi b}/b^4, \\ J(b) &\sim +e^{-2m_\pi b}/b^3, \\ \Theta(b) &\sim +e^{-2m_\pi b}/b^2, \end{aligned} \quad (72)$$

where $+$ indicates a positive constant. The powers of b are of a kinematic origin, related to the threshold behavior. The common positive sign stems from a combination of unitarity and analyticity driven by the data analysis (see discussion in Sec. II E). The powers of b are the same as for the pion [cf. Eq. (63) of Ref. [97]], which follows from the formalism brought up in Sec. II E. The P-wave case of the electromagnetic form factor of the pion, using the timelike experimental data, was analyzed in Ref. [114].

E. Meson dominance results

We now return to our meson dominance model. Inserting the spectral strength of the form (35) into (66) or (67), we find simple expressions involving sums of the Bessel functions $K_n(bm_i)$. The results for the basic form factors are presented in Fig. 4. For all cases, $\int 2\pi b db F(b) = F(t=0)$. The bands in the figures of this subsection indicate the errors discussed in Sec. III E, but they do not include the model uncertainties discussed in Sec. IV C, as they are difficult to assess.

In Ref. [97], within a discussion for the pion, we have shown that $A(b)$ is positive-definite at an operator level—i.e., independent of the hadron state in which the matrix element is taken. The present model for A complies with

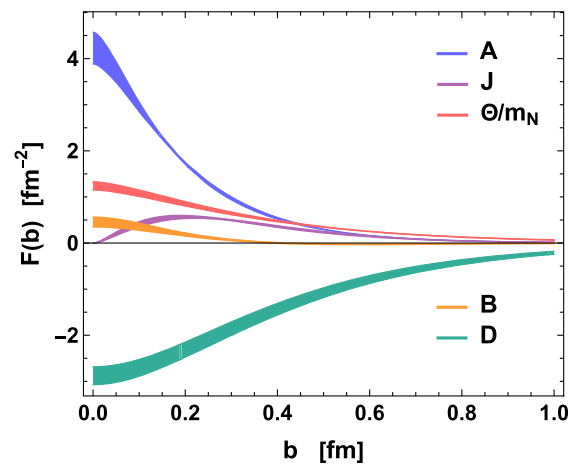


FIG. 4. Transverse densities of $A, J, \Theta/m_N, B$, and D , plotted as functions of the transverse coordinate b , following from the meson dominance model fitted to the MIT lattice data at $m_\pi = 170$ MeV.

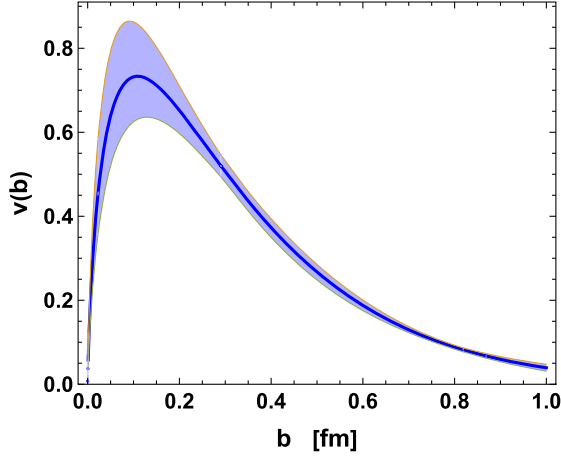


FIG. 5. Tangential velocity profile, defined in analogy to classical mechanics in Eq. (73), plotted as a function of the transverse coordinate b .

that general result. We note that in our model, the transverse density $A(b)$ is positive-definite, as required by the positivity condition.

Classically, the angular momentum is $\mathbf{J} = \mathbf{r} \times \mathbf{p} = m\mathbf{r} \times \mathbf{v}$. Following this analogy, we define the tangential velocity profile in the nucleon as

$$v(b) = \frac{J(b)}{b\Theta(b)}. \quad (73)$$

As is seen from Fig. 5, reflecting the behavior of the involved transverse densities, $v(b)$ grows linearly from 0 at low b , reaches a maximum around 0.1 fm, and then decreases approaching 0.

V. TRANSVERSE MECHANICAL PROPERTIES

A. Definitions and properties

The formulas displayed in this section mirror the derivations by Polyakov and Schweitzer [129], but for the two-dimensional transverse densities [135]. The transverse ($i, j = 1, 2$) part of SEM is

$$\begin{aligned} T^{ij}(b) &= \delta^{ij} p(b) + \left(\frac{b^i b^j}{b^2} - \frac{1}{2} \delta^{ij} \right) s(b) \\ &= \frac{1}{4m_N} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b} \cdot \mathbf{q}_\perp} (q_\perp^i q_\perp^j - \delta_{ij} q_\perp^2) D(-q_\perp^2). \end{aligned} \quad (74)$$

The transverse pressure is therefore given by

$$p(b) = -\frac{1}{16\pi m_N} \int_0^\infty q_\perp dq_\perp J_0(q_\perp b) q_\perp^2 D(-q_\perp^2), \quad (75)$$

whereas for the stress we get

$$s(b) = -\frac{1}{8\pi m_N} \int_0^\infty q_\perp dq_\perp J_2(q_\perp b) q_\perp^2 D(-q_\perp^2). \quad (76)$$

Since $J_0(z) = 1 - z^2/4 + \dots$, at the origin, the pressure is positive and concave, as long as $D(-q_\perp^2)$ is negative. Similarly, $J_2(z) = z^2/8 + \dots$ implies vanishing and convex stress near $b = 0$.

From Eq. (75), it is obvious that

$$\int_0^\infty db 2\pi b p(b) = 0, \quad (77)$$

as required by a classical analog of the stability condition. One can also show that

$$\int_0^\infty db 2\pi b s(b) = -\frac{1}{2m_N} \int_0^\infty q_\perp dq_\perp D(-q_\perp^2). \quad (78)$$

From the conservation of a static SEM, $\partial/\partial b^i T^{ij} = 0$, the equation

$$p'(b) + \frac{1}{2} s'(b) + \frac{s(b)}{b} = 0 \quad (79)$$

follows. As a check, this relation can be verified with the identity between the Bessel functions, $J'_0(z) + J'_2(z) + 2J_2(z)/z = 0$, applied to Eqs. (75) and (76). Integration of (79) by parts yields

$$p(0) = \int_0^\infty \frac{s(b)}{b} db. \quad (80)$$

Other relations can be derived, stemming from the fact that $p(b)$ and $s(b)$ are defined via integrals with common function $D(-q_\perp^2)$ and the two Bessel functions:

$$\begin{aligned} \int_0^\infty p(b) db &= \frac{1}{2} \int_0^\infty s(b) db, \\ \int_0^\infty b^2 p(b) db &= -\frac{1}{6} \int_0^\infty b^2 s(b) db, \\ &\dots \end{aligned} \quad (81)$$

In particular, the following relation with the D -term holds:

$$\begin{aligned} D(0) &= 2m_N \int_0^\infty 2\pi b db b^2 p(b) \\ &= -\frac{m_N}{2} \int_0^\infty 2\pi b db b^2 s(b). \end{aligned} \quad (82)$$

The spectral representation is

$$\begin{aligned} p(b) &= \frac{1}{16\pi^2 m_N} \int_{4m_\pi^2}^\infty ds K_0(b\sqrt{s}) s \operatorname{Im} D(s), \\ s(b) &= -\frac{1}{8\pi^2 m_N} \int_{4m_\pi^2}^\infty ds K_2(b\sqrt{s}) s \operatorname{Im} D(s), \end{aligned} \quad (83)$$

from where, in conjunction with the superconvergence sum rules, it follows that

$$p(b=0) = -\frac{1}{32\pi^2 m_N} \int_{4m_\pi^2}^{\infty} ds s \log s \operatorname{Im} D(s),$$

$$\left. \frac{ds(b^2)}{db^2} \right|_{b=0} = \frac{1}{64\pi^2 m_N} \int_{4m_\pi^2}^{\infty} ds s^2 \log s \operatorname{Im} D(s). \quad (84)$$

Note the sensitivity to the high- s timelike region.

The asymptotic behavior, reflecting Eq. (47) and obtained straightforwardly via Eq. (83), is

$$p(b) \sim -e^{-2m_\pi b}/b^2,$$

$$s(b) \sim +e^{-2m_\pi b}/b^2. \quad (85)$$

Note that it is governed by the behavior of $\operatorname{Im} \Theta(s)$, which is dominant at the 2π threshold.

Finally, using the fact that $2T^{+-}(b) = \epsilon(b)$ is the transverse energy density, one has the natural relation

$$\theta(b) = \epsilon(b) - 2p(b). \quad (86)$$

Note that we intuitively expect $\epsilon(b) > 0$, although $p(b)$ changes sign and $\theta(b)$ need not be positive (it was not in the pion case [97]). We have checked that indeed $\epsilon(b)$ is positive in the present meson dominance analysis.

B. Meson dominance results

The transverse pressure and stress evaluated in the meson dominance model of the previous sections are plotted in Fig. 6. We also show the combination $p(b) + s(b)/2$, whose positivity is the local criterion for classical stability [129] for the two-dimensional case. We see from the figure that this criterion is satisfied. In Fig. 7, we display the

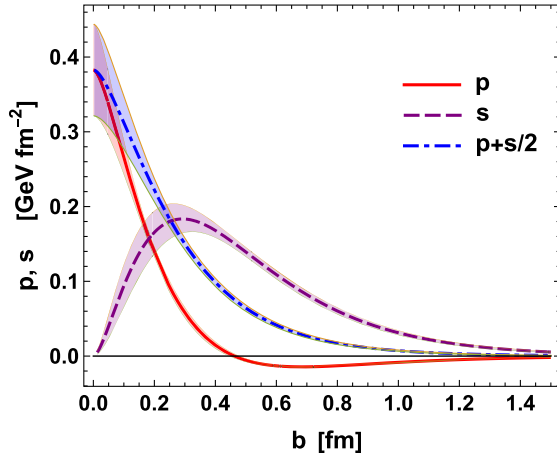


FIG. 6. Pressure p and stress s in the nucleon, evaluated in the meson dominance model fitted to the MIT lattice data at $m_\pi = 170$ MeV. The combination $p + \frac{1}{2}s$ is positive, as required by stability.

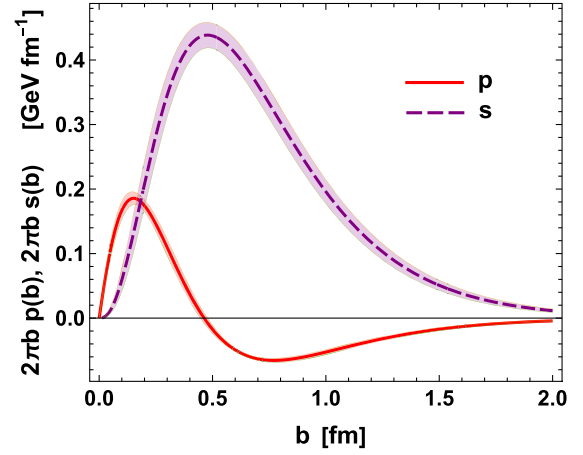


FIG. 7. Pressure p and stress s in the nucleon in the meson dominance model, multiplied with $2\pi b$. The pressure changes sign from + to - as b increases, and $\int_0^\infty 2\pi b p(b) db = 0$, as required by stability.

pressure and stress multiplied with $2\pi b$. It is clearly seen that the positive pressure inside the nucleon is compensated by the negative pressure outside, in agreement with Eq. (77) and expectations from classical stability conditions.

Finally, we note that the pressure can be written as

$$p(b) = \frac{m_N}{6} A(b) + \frac{1}{24m_N} \frac{1}{b} \frac{d}{db} b \frac{d}{db} B(b) - \frac{1}{6} \Theta(b). \quad (87)$$

This decomposition is shown in Fig. 8. We can see that the contribution of A , which by the general argument is positive, is repulsive and short range, the contribution of B is small, whereas the contribution of Θ is attractive and long range. Certainly, the long range of the Θ part reflects the smallness of the σ mass, while the short range of A is associated with the larger mass of f_2 . Note that this very

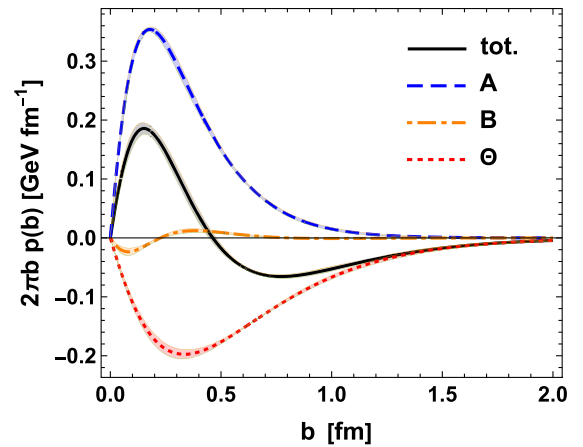


FIG. 8. Various contributions to the transverse pressure (multiplied with $2\pi b$) according to Eq. (87). The repulsive part comes from A , the confining part from Θ , and the contribution of B is small and changes the sign.

simple interpretation depends crucially on the proper spin decomposition of SEM introduced in Sec. II A. Qualitatively similar results were obtained in Refs. [69,70], where the 0^{++} component is associated with the gluon contribution to the trace anomaly.

C. Transverse radii

The two-dimensional transverse radii are obtained from the densities of Sec. IV as

$$\langle b^2 \rangle_F = \frac{\int_0^\infty 2\pi b b^2 F(b) db}{\int_0^\infty 2\pi b F(b) db} = \frac{4}{F(0)} \frac{dF(t)}{dt} \Big|_{t=0}, \quad (88)$$

whereas the mechanical radius is

$$\langle b^2 \rangle_{\text{mech}} = \frac{\int_0^\infty 2\pi b b^2 [p(b) + \frac{1}{2}s(b)] db}{\int_0^\infty 2\pi b [p(b) + \frac{1}{2}s(b)] db} = \frac{4D(0)}{\int_0^\infty d(-t)D(t)}. \quad (89)$$

In our meson dominance model, we find

$$\begin{aligned} \langle b^2 \rangle_A &= 4 \left(-c_A + \frac{1}{m_{f_2}^2} + \frac{1}{m_{f_2'}^2} + \frac{1}{m_{f_2''}^2} + \frac{1}{m_{f_2'''}^2} \right) \\ &= [0.34(1) \text{ fm}]^2, \end{aligned} \quad (90)$$

where numerically c_A approximately cancels the contribution $1/m_{f_2}^2 + 1/m_{f_2'''}^2$. Similarly,

$$\langle b^2 \rangle_\Theta = 4 \left(\frac{1}{m_\sigma^2} + \frac{1}{m_{f_0}^2} \right) = [0.60(3) \text{ fm}]^2, \quad (91)$$

where the comparatively large size reflects the smallness of the effective σ mass. Finally,

$$\langle b^2 \rangle_{\text{mech}} = [0.48(3) \text{ fm}]^2, \quad (92)$$

with the hierarchy $\langle b^2 \rangle_A < \langle b^2 \rangle_{\text{mech}} < \langle b^2 \rangle_\Theta$.

VI. RADIAL DENSITIES

A. Definitions

For $F = A, B$, or Θ , the three-dimensional spatial distributions [129] are defined in the Breit frame as

$$\begin{aligned} F(r) &= \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} F(-\Delta^2) \\ &= \frac{1}{2\pi^2} \int d\Delta \Delta^2 j_0(\Delta r) F(-\Delta^2), \end{aligned} \quad (93)$$

where $\Delta^2 = -t$, and $j_0(z) = \sin(z)/z$ is the spherical Bessel function. We recall the statement of Ref. [135] that for the nucleon (for other spin states, this does not

hold [136]), the three-dimensional and the transverse distributions are connected via the Abel transform. To see this explicitly, we notice that the Abel transform of the kernel in (93) is

$$2 \int_b^\infty r \Delta \frac{j_0(\Delta r)}{\sqrt{r^2 - b^2}} dr = \pi J_0(\Delta b), \quad (94)$$

which yields (65). Therefore, the “tomographic information” contained in the transverse densities of the nucleon describes fully the three-dimensional distributions, and vice versa.

The case of J is a bit more involved, as the spatial distribution contains both the monopole and quadrupole terms [132]:

$$\begin{aligned} J^i(r) &= s^i \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} \left[J(t) + \frac{2t}{3} \frac{dJ(t)}{dt} \right]_{t=-\Delta^2} \\ &\quad + s^j \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} \left(\Delta^i \Delta^j - \frac{1}{3} \delta^{ij} \Delta^2 \right) \frac{dJ(t)}{dt} \Big|_{t=-\Delta^2}. \end{aligned} \quad (95)$$

It can be written as

$$J^i(r) = s^i J^{\text{mon}}(r) + s^j \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) J^{\text{quad}}(r), \quad (96)$$

where the monopole and quadrupole terms are given by

$$J^{\text{mon}}(r) = -\frac{2}{3} J^{\text{quad}}(r) = \frac{r}{6\pi^2} \int_0^\infty d\Delta \Delta^3 j_1(\Delta r) J(-\Delta^2). \quad (97)$$

Note the simple proportionality relation with the factor of $-2/3$ between the monopole and quadrupole parts, which is a generic feature when $J(t)$ is nonsingular, allowing for integration by parts.

B. Spectral representation

Using the dispersion relation (21) in Eq. (93), in analogy to (66), we get the spectral representation of the three-dimensional distributions,

$$F(r) = \frac{1}{4\pi^2} \int_{4m_\pi^2}^\infty ds \frac{e^{-r\sqrt{s}}}{r} \text{Im } F(s). \quad (98)$$

Notice as a check that the Abel transform of (98) yields (66).

Expanding $\exp(-r\sqrt{s})$ at low r and using the superconvergence sum rule (22) for $n = 1$, we get

$$F(r=0) = -\frac{1}{4\pi^2} \int_{4m_\pi^2}^\infty ds \sqrt{s} \text{Im } F(s), \quad (99)$$

which indicates an even stronger sensitivity to large s than in the two-dimensional case of (66). With the overall sign in (99), the positivity of $F(0)$ means that there are parts of the corresponding spectrum with negative strength. When Eq. (22) holds for $n = 2$, then

$$F(r) = F(r=0) - \frac{r^2}{24\pi^2} \int_{4m_\pi^2}^{\infty} ds s^{3/2} \text{Im} F(s) + \mathcal{O}(r^3), \quad (100)$$

and additionally with $n = 3$,

$$F(r) = F(r=0) + F'(r=0)r^2 + \frac{r^4}{480\pi^2} \int_{4m_\pi^2}^{\infty} ds s^{5/2} \text{Im} F(s) + \mathcal{O}(r^5). \quad (101)$$

For the monopole angular momentum form factor

$$J^{\text{mon}}(r) = \frac{1}{12\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{e^{-r\sqrt{s}}(r\sqrt{s} + 1)}{r} \text{Im} J(s). \quad (102)$$

With the superconvergence sum rules, the lowest term in the low- r expansion is

$$J^{\text{mon}}(r) = \frac{r^2}{36\pi^2} \int_{4m_\pi^2}^{\infty} ds s^{3/2} \text{Im} J(s) + \mathcal{O}(r^3). \quad (103)$$

C. Radial pressure and shear forces

From the form of Eqs. (99)–(101) and (103), we expect the three-dimensional distributions near the origin to be sensitive to the details of the high- s spectrum, hence to the particular model used. This is the same issue that occurred in the transverse densities, but now the model dependence is even stronger.

In Fig. 9, we plot the three-dimensional densities for the nucleon from the meson dominance model. We note that at the origin they have a large uncertainty, as discussed above. The density $A(r)$ is the largest, as the corresponding form factor has a longest tail in $-t$, caused by the large values of the tensor-meson masses. Except for $B(r)$, which changes the sign, all the other densities have a definite sign. The corresponding radial densities are plotted in Fig. 10.

The three-dimensional pressure and shear are evaluated as [129]

$$p(r) = -\frac{1}{12\pi^2 m_N} \int d\Delta \Delta^4 j_0(\Delta r) D(-\Delta^2),$$

$$s(r) = -\frac{1}{8\pi^2 m_N} \int d\Delta \Delta^4 j_2(\Delta r) D(-\Delta^2). \quad (104)$$

They satisfy all the relations spelled out in Ref. [129], in particular $\int_0^\infty 4\pi r^2 p(r) dr = 0$ and $-m_N \int_0^\infty 4\pi r^4 p(r) dr = -\frac{4}{15} m_N \int_0^\infty 4\pi r^4 s(r) dr = D(0)$.

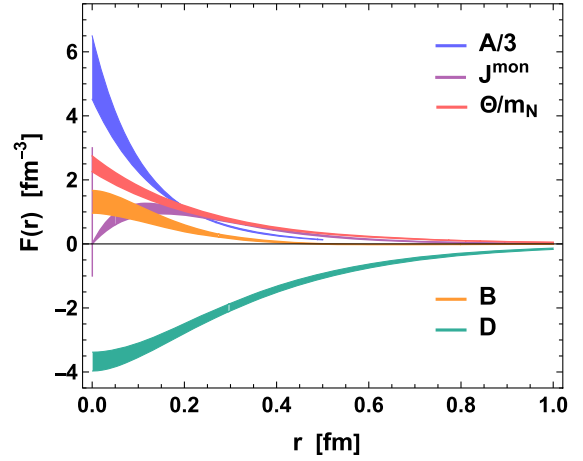


FIG. 9. Three-dimensional distributions of A (divided by 3), J^{mon} , Θ/m_N , B , and D , plotted as functions of the radial coordinate r , following from the meson dominance model fitted to the MIT lattice data at $m_\pi = 170$ MeV.

We plot these mechanical distributions in Fig. 11. Higher central pressure than that obtained from the chiral soliton model [54] means that the nucleon is a spatially more compact object in the meson dominance approach. In Fig. 12, we plot $4\pi r^4 m_N p(r)$, which integrates to $D(0)$.

Using the same methods as in Sec. VA, we get the sum rules

$$p(r=0) = -\frac{1}{24\pi^2 m_N} \int_{4m_\pi^2}^{\infty} ds s^{3/2} \text{Im} D(s),$$

$$\left. \frac{ds(r)}{dr^2} \right|_{r=0} = \frac{1}{240\pi^2 m_N} \int_{4m_\pi^2}^{\infty} ds s^{5/2} \text{Im} D(s). \quad (105)$$

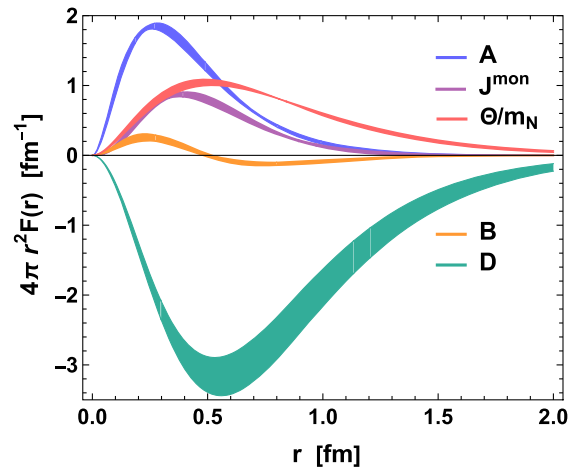


FIG. 10. Radial densities of A , J^{mon} , Θ/m_N , B , and D , plotted as functions of the radial coordinate r , following from the meson dominance model fitted to the MIT lattice data at $m_\pi = 170$ MeV.

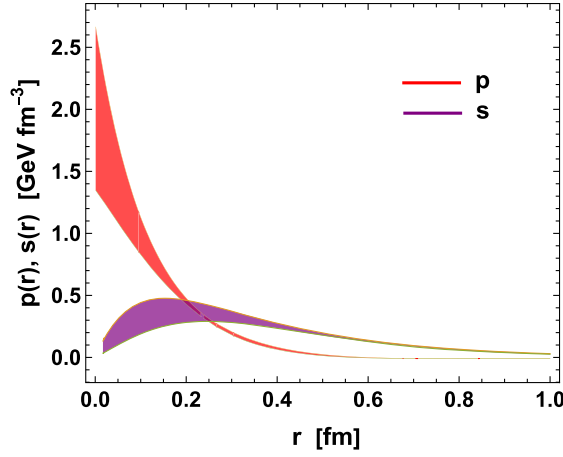


FIG. 11. Same as in Fig. 9, but for the pressure and stress. The pressure changes sign from + to - as r increases, at $r \simeq 0.5$ fm, and $\int_0^\infty 4\pi r^2 p(r) dr = 0$, as required by stability.

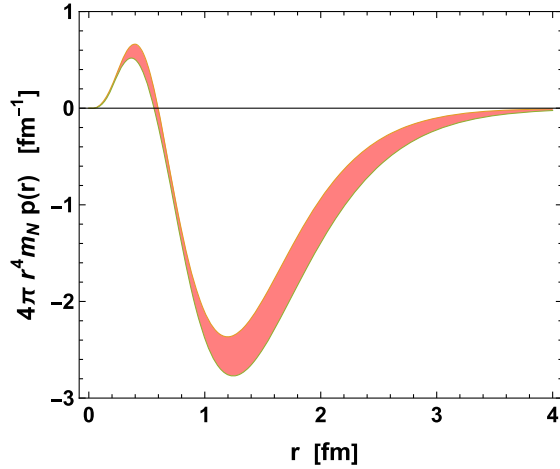


FIG. 12. Same as in Fig. 11, but for the pressure multiplied with $4\pi r^4 m_N$, which upon integration yields $D(0)$.

The large- r asymptotics at $r \rightarrow \infty$, following from the threshold behavior of Eq. (47), is

$$\begin{aligned} p(r) &\sim -e^{-2m_\pi r}/r^{5/2}, \\ s(r) &\sim +e^{-2m_\pi r}/r^{5/2}, \end{aligned} \quad (106)$$

where the tail of $\Theta(r) \sim +e^{-2m_\pi r}/r^{5/2}$ determines the falloff. For completeness, we mention that $A(r)$, $B(r) \sim +e^{-2m_\pi r}/r^{9/2}$ and $J^{\text{mon}}(r) \sim +e^{-2m_\pi r}/r^{7/2}$. We notice that the power of $1/r$ in the tails of Eq. (106) differs from the result obtained in the chiral soliton model [54], where the falloff was faster, $\sim e^{-2m_\pi r}/r^6$. This is probably caused by the large- N_c limit taken first in that approach. Our asymptotics robustly reflects the kinematic behavior at the 2π production threshold.

D. Radii and the D -term

The three-dimensional mean squared radii are

$$\langle r^2 \rangle_F = \frac{\int d^3r r^2 F(r)}{\int d^3r F(r)} = \frac{6}{F(0)} \frac{dF(t)}{dt} \Big|_{t=0}. \quad (107)$$

In our meson dominance model,

$$\begin{aligned} \langle r^2 \rangle_A &= 6 \left(-c_A + \frac{1}{m_{f_2}^2} + \frac{1}{m_{f_2'}^2} + \frac{1}{m_{f_2''}^2} + \frac{1}{m_{f_2'''}^2} \right) \\ &= [0.51(1) \text{ fm}]^2 \end{aligned} \quad (108)$$

and

$$\langle r^2 \rangle_\Theta = 6 \left(\frac{1}{m_\sigma^2} + \frac{1}{m_{f_0}^2} \right) = [0.90(4) \text{ fm}]^2. \quad (109)$$

For the angular momentum density,

$$\begin{aligned} \langle r^2 \rangle_J &= \frac{\int d^3r r^2 J^i(r)}{\int d^3r J^i(r)} = \frac{10}{J(0)} \frac{dJ(t)}{dt} \Big|_{t=0} \\ &= 20 \frac{dJ(t)}{dt} \Big|_{t=0}, \end{aligned} \quad (110)$$

with only the monopole term contributing. In the meson dominance model,

$$\begin{aligned} \langle r^2 \rangle_J &= 10 \left(-c_J + \frac{1}{m_{f_2}^2} + \frac{1}{m_{f_2'}^2} + \frac{1}{m_{f_2''}^2} + \frac{1}{m_{f_2'''}^2} \right) \\ &= [0.57(3) \text{ fm}]^2. \end{aligned} \quad (111)$$

The D -term is related to the mean square radii as follows:

$$D(0) = \frac{2m_N^2}{9} (\langle r^2 \rangle_A - \langle r^2 \rangle_\Theta). \quad (112)$$

The mechanical radius, as defined in Ref. [129], is

$$\langle r^2 \rangle_{\text{mech}} = \frac{6D(0)}{\int_0^\infty d(-t)D(t)} = [0.72(5) \text{ fm}]^2, \quad (113)$$

and the energy radius, defined via T^{00} [129], is

$$\begin{aligned} \langle r^2 \rangle_E &= \langle r^2 \rangle_A - \frac{3}{2m_N^2} D(0) \\ &= \frac{2}{3} \langle r^2 \rangle_A + \frac{1}{3} \langle r^2 \rangle_\Theta = [0.67(2) \text{ fm}]^2 \end{aligned} \quad (114)$$

(cf. various estimates for $\langle r^2 \rangle_{\text{mass}} \equiv \langle r^2 \rangle_E$ collected in Ref. [130]). To summarize, we find the hierarchy

$$\langle r^2 \rangle_A < \langle r^2 \rangle_J < \langle r^2 \rangle_E < \langle r^2 \rangle_{\text{mech}} < \langle r^2 \rangle_\Theta, \quad (115)$$

spanning the sizes from 0.5 fm to 0.9 fm. Note that the largest value of $\langle r^2 \rangle_\Theta$ reflects the fact that the trace anomaly lattice form factor of Fig. 2 is the longest range, and hence has the highest slope at the origin, essentially reflecting the smallness of the effective σ mass. This is in contrast to the extraction by Kharzeev [75] from the GlueX J/ψ photo-production data [73], where a relatively small value of $\langle r^2 \rangle_\Theta \sim [0.55 \text{ fm}]^2$ was found (see also the discussion in Refs. [76,77]).

VII. CONCLUSIONS

In this paper, we have discussed several general issues concerning GFFs of the nucleon and its mechanical properties, involving dispersion relations, superconvergence sum rules, asymptotic behavior, or positivity, as well as offered a natural fit of the available lattice data within the meson dominance approach.

On general grounds, GFFs of the nucleon can be scrutinized from the point of view of analyticity realized through the long-ago-established meson dominance principle, and implemented through the pertinent current-field identities. While GFFs are admittedly complicated structures for the timelike momenta, displaying resonance profiles corresponding to the exchanged mesons and the corresponding backgrounds, the spacelike region, which has lately been the main concern both for the theory and experiment, is largely independent of these details. We have shown that the meson dominance works well for the lattice QCD data for the nucleon in the available spacelike domain, mirroring the success for the pion [95].

Since the pQCD constraints impose known power falloffs based on the leading two-gluon exchange process, $A, J \sim \alpha^2/(-t)^2$, $B, D \sim \alpha^2/(-t)^3$, these requirements have been incorporated into our model in terms of a minimum number of resonances. The Q^2 range covered by the lattice QCD simulation, $0 \leq Q^2 \leq 2 \text{ GeV}^2$, does not probe precisely the asymptotic falloff; therefore, the corresponding superconvergence sum rules, or the spectral representation of the spatial densities, are only weakly constrained by the present data.

Estimates for various radii, exhibiting a certain hierarchy, and for the value of the D -term, have been made within the meson dominance model. A large value of the mass (trace-anomaly) radius, reflecting the smallness of the effective σ mass, has been found from the fit. A very similar best-fit value, $m_\sigma \sim 0.65 \text{ GeV}$, has been found for both the pion and the nucleon.

For the spatial distributions, we have derived sum rules for their values and derivatives at the origin in terms of the spectral integrals with the application of the superconvergence sum rules. We argue that these quantities are weakly

constrained, since we do not have sufficient knowledge on the corresponding spectral strengths.

From the threshold behavior in the timelike region for $\pi\pi \rightarrow N\bar{N}$, using Watson's theorem, we have derived the leading asymptotic behavior at large values of the coordinate for the spatial densities, including the pressure and shear forces. These formulas involve the obvious two-pion Yukawa tail, multiplied with a power falloff. The formulas obtained from our study exhibit the signs following from the $\pi - N$ elastic scattering data, which are positive for all the considered densities, except for the pressure, where the coefficient is negative.

The decomposition of SEM into the 0^{++} and 2^{++} parts offers a natural explanation of the spatial variance of pressure, whose attractive tail is associated with the small mass of the σ meson, while the inside repulsion is due to the large mass of the f_2 meson.

We also bring up the result [97] that the transverse distribution related to the T^{++} is positive at an operator level. This implies in general that $A(b) > 0$, which is satisfied in our model.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [81]; embargo periods may apply.

APPENDIX: FEATURES OF THE MINIMUM MESON DOMINANCE ANSATZ

Expansion of the meson dominance function for $A(t)$ from Eq. (49) at low t yields

$$A(t) = 1 + St + Rt^2 + \mathcal{O}(t^3), \quad (\text{A1})$$

with

$$S = -c_A + \frac{1}{m_{f_2}^2} + \frac{1}{m_{f_2'}^2} + \frac{1}{m_{f_2''}^2} + \frac{1}{m_{f_2'''}^2},$$

$$R = c_2 - \frac{c_A^2}{2} + \frac{1}{2} \left(\frac{1}{m_{f_2}^4} + \frac{1}{m_{f_2'}^4} + \frac{1}{m_{f_2''}^4} + \frac{1}{m_{f_2'''}^4} + S^2 \right). \quad (\text{A2})$$

Thus, for a given set of the four resonance masses, c_A and c_2 can be adjusted to fit the slope and the curvature of the function at the origin. At asymptotically large $-t$,

$$A(t) = \frac{c_2 m_{f_2}^2 m_{f_2'}^2 m_{f_2''}^2 m_{f_2'''}^2}{(-t)^2} + \mathcal{O}[1/(-t)^3]. \quad (\text{A3})$$

Analogous formulas hold for $J(t)$.

For $B(t)$, we have the features

$$B(t) = (c_A - c_J) \left[t + \left(\frac{1}{m_{f_2}^2} + \frac{1}{m_{f_2'}^2} + \frac{1}{m_{f_2''}^2} + \frac{1}{m_{f_2'''}^2} \right) t^2 \right] + \mathcal{O}(t^3) \quad (\text{A4})$$

and

$$B(t) = \frac{(c_J - c_A) m_{f_2}^2 m_{f_2'}^2 m_{f_2''}^2 m_{f_2'''}^2}{(-t)^3} + \mathcal{O}[1/(-t)^4]. \quad (\text{A5})$$

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