

Heat kernels and resummations: The spinor case

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Among the available perturbative approaches in quantum field theory, heat kernel techniques provide a powerful and geometrically transparent framework for computing effective actions in nontrivial backgrounds. In this work, resummation patterns within the heat kernel expansion are examined as a means of systematically extracting nonperturbative information. Building upon previous results for Yukawa interactions and scalar quantum electrodynamics, we extend the analysis to spinor fields, demonstrating that a recently conjectured resummation structure continues to hold. The resulting formulation yields a compact expression that resums invariants constructed from the electromagnetic tensor and its spinorial couplings, while preserving agreement with known proper-time coefficients. Beyond its immediate computational utility, the framework offers a unified perspective on the emergence of nonperturbative effects (such as Schwinger pair creation) in relation to perturbative heat kernel data, and provides a basis for future extensions to curved spacetimes and non-Abelian gauge theories.

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I. INTRODUCTION

Tunnelling phenomena occupy a central role in quantum mechanics and quantum field theory, providing direct access to processes that lie beyond conventional perturbation theory. From the decay of metastable states to barrier penetration, semiclassical configurations reveal exponential contributions to observables that perturbative expansions alone cannot capture. Such instanton-mediated effects span a wide range of physical settings: in strong-field quantum electrodynamics, the Schwinger effect manifests as electron-positron pair production via tunnelling in an external electric field [1,2], while in gravitational physics, Hawking radiation may be interpreted as particles tunnelling across a black hole horizon [3]. More broadly,

instantons govern processes such as vacuum decay, anomaly generation, and nonperturbative corrections to effective actions [4–6], illustrating how semiclassical trajectories in Euclidean spacetime encode exponentially suppressed contributions. Understanding the origin and structure of these effects remains essential for a complete description of quantum dynamics and motivates the search for further methods capable of extracting nonperturbative information, even those that at first sight appear to be purely perturbative.

Heat kernel techniques, a cornerstone of spectral geometry and quantum field theory, offer one such route. The heat kernel short-time asymptotic expansion encodes local geometric information through a hierarchy of coefficients that describe the properties of the underlying manifold and field content [7,8]. While this expansion is conventionally regarded as a perturbative tool, certain sequences of terms can be reorganized or *resummed*, giving rise to structures that reveal hidden nonperturbative behaviours [9–11]. This insight establishes a conceptual bridge between the semiclassical intuition of tunnelling processes and the analytic

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machinery of the heat kernel formalism, allowing non-perturbative features to emerge clearly from perturbative data.

At the methodological level, resummation has been applied in multiple contexts. In first-quantized settings, the heat kernel can be interpreted as a propagator in imaginary time, allowing for a resummation over the potential [12]. In quantum field theory, covariant perturbation theory provides systematic tools for backgrounds with rapidly varying curvatures or potentials, enabling, for instance, the computation of beta functions in a momentumlike scheme [13–18]. These general strategies naturally lead to more specific applications in curved spacetime: the resummation of the Ricci scalar [19,20] has played a key role in analysing how fermionic and scalar condensates respond to background geometry [21–25], while similar techniques have clarified Casimir self-interactions under spacetime-dependent boundary conditions [26–28]. For sufficiently simple backgrounds, closed expressions can even be obtained for the effective action [29–31] or the heat kernel itself [11,32].

Building on these ideas, we have recently initiated a program aimed at establishing the existence of further resummation patterns for the diagonal elements of the heat kernel and for the effective action of quantum fields interacting with nontrivial backgrounds [33–35]. In our previous works, we focused on certain strong-field resummations for scalar Yukawa interactions and for a quantum scalar field coupled to a vector background. In the present paper, we extend this analysis by exploring possible generalizations of the heat kernel resummation ansatz to systems involving spinor fields. Our goal is to outline a unified framework in which perturbative expansions, when appropriately reorganized, already encode the essential elements of some pieces of nonperturbative physics.

Unless otherwise stated, all computations are performed on a d -dimensional Euclidean background metric, with the understanding that a Wick rotation connects the results to their Minkowskian counterparts.

II. EFFECTIVE ACTION AND HEAT KERNEL METHODS

The generating functional for scattering amplitudes of a quantum field theory system is defined as

$$Z[J] := \int \mathcal{D}\varphi \exp(-S[\varphi] + J\varphi), \quad (1)$$

where $S[\varphi]$ denotes the classical action functional, and the path integral runs over all admissible field configurations. Here, φ is treated as a generic field, without specifying any internal or spacetime indices. From the generating functional of connected Green's functions, $W[J]$, defined by

$$e^{W[J]} := Z[J], \quad (2)$$

the effective action Γ is constructed through a Legendre transform,

$$\Gamma[\phi] := W[J] - \int d^d x J\phi, \quad (3)$$

where ϕ represents the expectation value of the quantum operator associated with φ . By definition, Γ satisfies thus an implicit equation in terms of the functional integral in Eq. (1), which is generally not solvable analytically. However, following the techniques presented by Schwinger and DeWitt [36], it is possible to show that, for systems whose action is quadratic,

$$S[\varphi] := \int d^d x \bar{\varphi} \mathcal{Q} \varphi, \quad (4)$$

with $\mathcal{Q}$ a differential operator, the functional integral becomes Gaussian and admits an explicit solution,

$$\Gamma[\phi] = S[\phi] + c_s \Gamma_1 := S[\phi] + c_s \log \text{Det} \mathcal{Q}. \quad (5)$$

Here, the first term corresponds to the classical action, while the second term defines the quantum part of the effective action, Γ_1 , whose coefficient c_s is a real-valued constant depending on the spin of the quantum field. Γ_1 is the primary object of study in this work and it is important to emphasize that, whenever the action incorporates interaction terms, i.e., it goes beyond the quadratic case, the result in Eq. (5) becomes a one-loop approximation, which is still a highly nontrivial quantity.

A convenient representation of the one-loop effective action is provided by the heat kernel operator [7],

$$K(\tau) := \exp(-\tau \mathcal{Q}), \quad (6)$$

allowing one to write

$$\Gamma_1 = - \int_0^\infty \frac{d\tau}{\tau} \text{Tr} K(x, x'; \tau) \quad (7)$$

$$= - \int_0^\infty \frac{d\tau}{\tau} \int d^d x K(x, x; \tau), \quad (8)$$

with x'^μ being an arbitrary reference point in spacetime. From the definition of the heat kernel, it can be shown that the elements $K(x, x'; \tau) := \langle x | K(\tau) | x' \rangle$ in this expression satisfy the following differential equation and initial condition,

$$(\partial_\tau + \mathcal{Q})K(x, x'; \tau) = 0, \quad K(x, x'; 0^+) = \delta(x - x'). \quad (9)$$

In general, Eq. (9) is solved perturbatively using a proper-time expansion of the heat kernel matrix elements,

$$K(x, x'; \tau) = \sum_{j=0}^{\infty} b_j(x, x') \tau^{j-d/2}. \quad (10)$$

This method provides a recursive procedure to compute the Gilkey-Seeley-DeWitt (GSDW) coefficients (b_j); it thus offers a perturbative approach to calculating both the heat kernel and, by extension, the effective action. While widely applicable, this approach does not come without some shortcomings: for starters, the recursive calculation of the GSDW coefficients becomes increasingly cumbersome to calculate for every new step, quickly turning into a computationally taxing problem for any except the most simple systems. More importantly, the perturbative nature of the expansion can obscure nonperturbative features of the system.

To circumvent these issues, as partially discussed in Sec. I, several works have developed resummed formulations of the heat kernel. In the context of first quantisation, the heat kernel acts as a propagator in imaginary time, for which a resummation of the potential has been established [37]. In curved spacetime, another contribution that has been resummed is the Ricci scalar for the case of a system consisting of a quantum field minimally coupled to a classical gravitational background [19,20,38]. More recently, resummed expansions for the fundamental invariants $\mathcal{F} := F_{\mu\nu}F^{\mu\nu}$ and $\mathcal{G} := \tilde{F}_{\mu\nu}F^{\mu\nu}$ in (scalar) Quantum Electrodynamics have been conjectured and partially proven [33,34]. The present work continues and extends these results, applying resummation techniques to a wider class of systems and demonstrating how nonperturbative information can be extracted directly from perturbative expansions.

III. RESUMMATION TECHNIQUES—THE CASE OF A SPINOR IN AN ELECTROMAGNETIC BACKGROUND

The starting point for our analysis comes from yet another classic resummation scheme. It has been previously shown that, for any system consisting of a quantum scalar field interacting with an at most quadratic potential background, i.e., for a system where it is possible to write

$$\mathcal{Q}_{\text{quad}} = -\partial^2 + \alpha + \beta_\mu(x - x')^\mu + \frac{1}{4}\gamma_{\mu\nu}^2(x - x')^\mu(x - x')^\nu, \quad (11)$$

where α , β , and γ are constant coefficients and x' is chosen to be the same as in Eq. (9), a fully explicit, closed expression for the heat kernel and the one-loop effective action can be derived [31]. This expression has been recently extended in [34] to include systems with arbitrary potential backgrounds, effectively resumming all the contributions made only of the potential and its first two derivatives out of the proper time expansion, thereby greatly simplifying the expressions of the corresponding generalised GSDW coefficients [see Eq. (24) below]. This general framework was then applied to a scalar QED system, where it was shown to allow the resummation of the $\mathcal{F}$ and $\mathcal{G}$ scalars defined above.

Now consider the action for a single quantum Dirac spinor Ψ interacting with a classical electromagnetic background,

$$S := - \int d^d x \bar{\Psi}(\gamma^\mu D_\mu + m)\Psi, \quad D_\mu := \partial_\mu + ieA_\mu, \quad (12)$$

where A_μ denotes the electromagnetic vector potential, e is the coupling constant (typically identified with the field's charge), m is the mass of the spinor, and γ^μ are the d -dimensional Dirac matrices (we will always assume that d is even, so that a unique representation of the γ matrices exists, but otherwise leave d arbitrary). The operator for which we wish to find a heat kernel resummation formula in this case is then given by¹

$$\begin{aligned} \mathcal{Q}_{\text{EM}} &:= (\gamma^\mu D_\mu + m)(-\gamma^\nu D_\nu + m) \\ &= -\partial^2 - ie(2A^\mu \partial_\mu + \partial_\mu A^\mu) + m^2 + e^2 A^2 - \frac{i}{2}e\sigma^{\mu\nu}F_{\mu\nu}, \end{aligned} \quad (13)$$

where we define $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$.

Let us first consider the case of a constant and homogeneous electromagnetic field, $F_{\mu\nu}(x) \rightarrow \bar{F}_{\mu\nu}$. In this case, we can find a straightforward expression for the vector potential

$$A_\mu(x) = -\frac{1}{2}\bar{F}_{\mu\nu}(x - x')^\nu. \quad (14)$$

For ease of notation, we will denote $\bar{x}^\mu := (x - x')^\mu$ from now on. By introducing (14) into (13), we arrive at²

$$\mathcal{Q}_{\text{h}} := -\partial^2 + ie\bar{F}^{\mu\nu}\bar{x}_\mu\partial_\nu + m^2 - \frac{i}{2}e\sigma^{\mu\nu}\bar{F}_{\mu\nu} + \frac{1}{4}e^2(\bar{F}^2)_{\mu\nu}\bar{x}^\mu\bar{x}^\nu, \quad (15)$$

which can be seen to be of a form similar to (11) with the definitions

$$\begin{aligned} \alpha_{\text{h}} &:= m^2 - \frac{i}{2}e\sigma^{\mu\nu}\bar{F}_{\mu\nu}, \\ \beta_{\text{h}}^\mu &:= 0, \\ (\gamma_{\text{h}}^2)_{\mu\nu} &:= e^2(\bar{F}^2)_{\mu\nu}. \end{aligned} \quad (16)$$

The term linear in derivatives in (15), absent from (11), does not modify the computation of the heat kernel. To demonstrate this, we shall initially disregard this term and proceed following the method of Brown and Duff [31] to obtain a closed-form expression for the heat kernel,

¹In the following we will omit the identity matrix in the spinor bundle.

²We treat $F^\mu{}_\nu$ as a matrix in its spacetime indices.

$$K_h(x, x'; \tau) := \frac{1}{(4\pi\tau)^{d/2}} \frac{e^{-\tau\alpha_h - \frac{1}{2}\bar{x}^\mu \mathcal{A}_{\mu\nu}^{-1}(\tau)\bar{x}^\nu - \mathcal{C}(\tau)}}{\det^{1/2}(\tau^{-1}\mathcal{A}(\tau))}, \quad (17)$$

where we have defined the functions

$$\begin{aligned} \mathcal{A}_{\mu\nu}(\tau) &:= \left[\frac{1}{\gamma_h} \tanh(\gamma_h \tau) \right]_{\mu\nu}, \\ \mathcal{C}(\tau) &:= \frac{1}{2} [\log(\cosh(\gamma_h \tau))]^\mu{}_\mu. \end{aligned} \quad (18)$$

By expanding the exponential, it is evident that the heat kernel in (17) contains only even powers of $\bar{x}$. When we act on this expansion with the linear-derivative term in (15), we generate contributions of the form $\bar{x}_\mu (F^{2k+1})^{\mu\nu} \bar{x}_\nu$, which vanish identically due to the antisymmetry of $F_{\mu\nu}$. Consequently, our solution in Eq. (17) satisfies the Eq. (9) both with and without the linear-derivative term. Since the heat kernel equation admits a unique solution, this term carries no additional information and can therefore be consistently neglected. In the coincidence limit, this result precisely reproduces the well-known Euler-Heisenberg expression.

For a general electromagnetic background, we would like to generalize Eq. (14) and write A^μ in terms of gauge-invariant quantities. We can do so by choosing the Fock-Schwinger gauge [39], defined by

$$\bar{x}^\mu A_\mu(x) = 0, \quad (19)$$

which allows us to write the potential A_μ in terms of $F_{\mu\nu}$ and its derivatives at the point x' , see [40],

$$A_\mu(x) = \sum_{k=0}^{\infty} \frac{1}{k!(k+2)} \bar{x}^{\mu_1} \dots \bar{x}^{\mu_k} \bar{x}^\rho \partial_{\mu_1 \dots \mu_k} F_{\rho\mu}(x'). \quad (20)$$

This relation can be plugged back into (13). By considering only the terms whose dependence on $\bar{x}$ is at most quadratic, we can once again draw similarities with (11), this time defining

$$\begin{aligned} \alpha &:= m^2 - \frac{i}{2} e \sigma^{\rho\lambda} F_{\rho\lambda}(x'), \\ \beta_\mu &:= -ie \left(\frac{1}{3} \partial^\rho F_{\mu\rho}(x') + \frac{1}{2} \sigma^{\rho\lambda} \partial_\mu F_{\rho\lambda}(x') \right), \\ (\gamma^2)_{\mu\nu} &:= e^2 (F^2)_{\mu\nu}(x') + ie (\partial^\rho \partial_{(\mu} F_{\nu)\rho}(x') + \sigma^{\rho\lambda} \partial_\mu \partial_\nu F_{\rho\lambda}(x')), \end{aligned} \quad (21)$$

where we have denoted idempotent symmetrization of indices by enclosing them in parenthesis. For this truncation, a closed form of the heat kernel can be found, in a manner similar to that leading to (17); as we shall see, this will provide a resummation when we consider the full expansion in Eq. (20). Note that this extends the result already found in [31] to the case $\partial_\rho F_{\mu\nu} \neq 0$.

Our claim is that, for the complete, untruncated case, the heat kernel takes the form

$$\begin{aligned} K_{\text{EM}}(x, x'; \tau) &:= \frac{1}{(4\pi\tau)^{d/2}} \frac{e^{-\tau\alpha - \frac{1}{2}\tilde{\sigma}^\mu(x, x') \mathcal{A}_{\mu\nu}^{-1}(x'; \tau) \tilde{\sigma}^\nu(x, x') - \mathcal{C}(x'; \tau)}}{\det^{1/2}(\tau^{-1}\mathcal{A}(x'; \tau))} \\ &\times \Omega(x, x'; \tau), \end{aligned} \quad (22)$$

where the auxiliary functions are defined as

$$\begin{aligned} \tilde{\sigma}_\mu(x, x') &:= \bar{x}_\mu + \mathcal{B}_\mu(x'; \tau), \\ \mathcal{A}_{\mu\nu}(x; \tau) &:= \left[\frac{1}{\gamma} \tanh(\gamma\tau) \right]_{\mu\nu}, \\ \mathcal{B}_\mu(x; \tau) &:= 2\beta^\nu [\gamma^{-2}(1 - \text{sech}(\gamma\tau))]_{\nu\mu}, \\ \mathcal{C}(x, \tau) &:= \beta^\mu [-\tau\gamma^{-2} + \gamma^{-3} \tanh(\gamma\tau)]_{\mu\nu} \beta^\nu \\ &\quad + \frac{1}{2} [\log(\cosh(\gamma\tau))]^\mu{}_\mu, \end{aligned} \quad (23)$$

and α , β , and γ are given by the expressions in Eq. (21). The function $\Omega(x, x'; \tau)$ admits a proper-time expansion³

$$\Omega(x, x'; \tau) = \sum_{j=0}^{\infty} a_j(x, x') \tau^j, \quad a_0(x, x') = 1. \quad (24)$$

The remarkable property of this resummation is that, as we are going to prove below, when the coincidence limit $x' \rightarrow x$ of the coefficients is taken, none of them depend on any of the electromagnetic invariants contained in the set

$$\mathcal{K} := \{(\sigma^{\rho\lambda} F_{\rho\lambda})^j, (F^j)^\mu{}_\mu, j > 0\}. \quad (25)$$

This result shows that our proposal (22) effectively removes from the coincidence limit of the heat kernel coefficients all dependence on the electromagnetic invariants that can be constructed solely with $F_{\mu\nu}$, i.e., those that contain no derivatives of it. In other words, we have at our disposal a nonperturbative resummation of the effects of an intense electromagnetic background. This property is crucial for applications to Schwinger pair production and related strong-field phenomena.

We shall sketch the proof of our claim by explicitly deriving the recurrence relation satisfied by the coefficients $a_j(x, x')$, which arises from substituting expression (22) into the heat kernel equation (9), and grouping all resulting terms in powers of the proper time τ . Denoting the k th Bernoulli number by B_k , the floor function by $[\cdot]$, and defining the effective potential $\mathfrak{S}$ as

³In flat space, the coefficient $a_0(x, x')$ coincides with the parallel transport operator $\mathcal{P}(x, x')$, the path-ordered exponential of the gauge connection along the geodesic joining x to x' [41]. In our case, the parallel transport becomes trivial along the gauge direction, yielding $\mathcal{P}(x, x') = 1$.

$$\mathfrak{S} := m^2 - \frac{i}{2} e \sigma^{\alpha\beta} F_{\alpha\beta} - i e \partial_\mu A^\mu + e^2 A^2 - \alpha - \bar{x}^\alpha \beta_\alpha - \frac{1}{4} \bar{x}^\alpha \bar{x}^\beta (\gamma^2)_{\alpha\beta}, \quad (26)$$

the result, for every $j \geq 0$, is

$$\begin{aligned} -(j+1 + \bar{x}_\alpha \partial^\alpha) a_{j+1}(x, x') &= (-\partial^2 + \mathfrak{S}) a_j(x, x') + 2A^\mu \partial_\mu a_j(x, x') \\ &+ \sum_{n=1}^{\lfloor \frac{j}{2} \rfloor} \frac{B_{2n}}{(2n)!} (4(2^{2n}-1) \beta^\alpha (\gamma^{2(n-1)})_{\alpha\beta} + 2^{2n} \bar{x}^\alpha (\gamma^{2n})_{\alpha\beta}) \partial^\beta a_{j+1-2n}(x, x') \\ &- \sum_{n=1}^{\lfloor \frac{j+1}{2} \rfloor} \frac{B_{2n}}{(2n)!} (4(2^{2n}-1) \beta^\alpha (\gamma^{2(n-1)})_{\alpha\beta} + 2^{2n} \bar{x}^\alpha (\gamma^{2n})_{\alpha\beta}) A^\beta a_{j+1-2n}(x, x'). \end{aligned} \quad (27)$$

There are three key ingredients to this proof. The first is that, given its particular form, $\mathfrak{S}$ and its first two derivatives vanish identically in the coincidence limit, i.e.,

$$\lim_{x' \rightarrow x} \mathfrak{S} = \lim_{x' \rightarrow x} \partial_\mu \mathfrak{S} = \lim_{x' \rightarrow x} \partial_\mu \partial_\nu \mathfrak{S} = 0. \quad (28)$$

This property ensures that, in the calculation of the coefficients a_j and their derivatives, only the contributions from third- or higher-order derivatives of $\mathfrak{S}$ will remain in the coincidence limit. These terms can be shown to depend only on invariants with at least one derivative of $F_{\mu\nu}$.

The second ingredient is the particular dependence of the coefficients α, β, γ , see (21), as well as the potential A^μ , on the electromagnetic field. While α has already been accounted for in the previous discussion (appearing only inside $\mathfrak{S}$), it is in principle possible that the coefficients in the second and third line of (27) could contribute terms belonging to the set $\mathcal{K}$. It is immediately evident that this is not the case for the terms contracted with β , since by definition the latter depends only on derivatives of F . As for A^μ , only its leading order is independent of derivatives of F , see (20); importantly, this term is linear in F . Given that γ can be seen to satisfy a structure of the form

$$\gamma^2 \sim F^2 + \text{terms with derivatives of } F, \quad (29)$$

we can readily conclude that the contributions to $\bar{x}^\alpha (\gamma^{2n})_{\alpha\beta} A^\beta$ will again depend either on derivatives of F , or will vanish identically due to the antisymmetry of $F_{\mu\nu}$. All summed, the reasoning from (28) up to this point implies no element of the set $\mathcal{K}$ appears explicitly in the recurrence relation. Their only possible appearances could come from the coefficients $a_k(x, x')$ themselves, or from contractions arising when taking derivatives of terms involving $\bar{x}$. However, the third ingredient, the fact that $a_0(x, x') = 1$, allows us to construct a proof by induction showing this does not occur either.

Indeed, an explicit ordering is induced by the recurrence relation, $a_0 \rightarrow a_1 \rightarrow \partial_\mu a_1 \rightarrow \partial_\mu \partial_\nu a_1 \rightarrow a_2 \dots$, where to calculate a given element all the previous are needed.

Taking the appropriate derivatives of Eq. (27), one can verify that the diagonal of a given term in this sequence would contain an object in $\mathcal{K}$ only if a previous one does. Since the first element manifestly does not, we conclude that none of the diagonal coefficients $a_k(x, x)$, nor any of the diagonal derivatives $\partial_{\mu_1} \dots \partial_{\mu_n} a_k(x, x)$, can depend on the invariants built exclusively from $F_{\mu\nu}$ and $\sigma_{\mu\nu}$.

The full dependence of the heat kernel on these objects is therefore completely contained within the global prefactor in (22), and is valid for an arbitrary even dimension d . In particular, this completes and expands the proof, initiated in [34], of the conjecture presented in [33] for $d = 4$, which states that the fundamental invariants $\mathcal{F}, \mathcal{G}$ can be fully resummed. Indeed, reducing our findings to the four-dimensional setup, one just need to recall the classical result that contracted powers of $F_{\mu\nu}$ can be written in terms of $\mathcal{F}, \mathcal{G}$ alone.

As a last comment, although we have chosen the Fock-Schwinger gauge for the proof, the result is gauge independent. In fact, in the coincidence limit, the heat kernel coefficients are made of geometric quantities; in the present case they depend only on the field strength tensor $F_{\mu\nu}$ and its covariant derivatives, which are manifestly gauge invariant.

IV. GENERALIZATIONS

A. A toy model

The operators associated with our system of a spinor in an electromagnetic background take the schematic form

$$\mathcal{Q}_{\text{sch}} = -\partial^2 + N^\mu \partial_\mu + \alpha + \beta_\mu \bar{x}^\mu + \frac{1}{4} \gamma_{\mu\nu}^2 \bar{x}^\mu \bar{x}^\nu. \quad (30)$$

So far, throughout the discussion we have taken advantage of the fact that the associated heat kernel is closely related to Eq. (17). Ultimately, this approach works because, in these cases, the coefficient N^μ can be written as $N^\mu = \bar{x}_\nu Y^{\nu\mu}$, where Y is an antisymmetric tensor, and β_μ vanishes. As a consequence, the heat kernel obtained for

vanishing N^μ is protected by the antisymmetry of $Y^{\mu\nu}$ against the generation of new covariant contributions.

However, when extending these resummation techniques to more general systems, it is essential to study the case where N^μ cannot be handled in such a way. To that effect, consider the following toy model for which

$$\mathcal{Q}_N = -\partial^2 + N^\mu \partial_\mu + \alpha, \quad (31)$$

with N^μ an arbitrary vector. Such an operator serves as a first approximation to a variety of theories, including the interaction of spinors with an axial potential [42] or torsion [43], as well as non-Abelian gauge field models.⁴ The heat kernel resummation formula we propose for this case takes the form

$$K_N(x, x'; \tau) := \frac{1}{(4\pi\tau)^{d/2}} e^{-\tau\alpha - \frac{1}{4\tau}(\tau N(x') - \bar{x})^2} \Omega_N(x, x'; \tau). \quad (32)$$

We may justify this formula by first considering the case where N^μ is a constant vector N_0^μ (which is for example physically motivated by the Schwinger effect in the presence of constant Lorentz-violating background fields [44]). This allows us to solve the heat kernel equation by finding its associated propagator, which satisfies

$$\mathcal{Q}_{N_0} G(x, x') = (-\partial^2 + N_0^\mu \partial_\mu + \alpha) G(x, x') = \delta(x, x'). \quad (33)$$

By performing a Fourier transform to momentum space, we can explicitly solve

$$(p^2 + iN_0^\mu p_\mu + \alpha) G(p) = 1 \Rightarrow G(p) = (p^2 + iN_0^\mu p_\mu + \alpha)^{-1} = \int_0^\infty d\tau e^{-\tau(p^2 + iN_0^\mu p_\mu + \alpha)}. \quad (34)$$

By comparison with the heat kernel equation, one can straightforwardly check that the integrand in Eq. (34) is the Fourier transform of the heat kernel, $K_{N_0}(p; \tau)$, and derive

$$\begin{aligned} K_{N_0}(x, x'; \tau) &= \int \frac{d^d p}{(2\pi)^d} e^{ip_\mu \bar{x}^\mu} K_{N_0}(p; \tau) \\ &= \frac{1}{(4\pi\tau)^{d/2}} e^{-\tau\alpha - \frac{1}{4\tau}(\tau N_0 - \bar{x})^2}. \end{aligned} \quad (35)$$

When N^μ is not constant, we can nonetheless perform an expansion around x'

$$N^\mu(x) = \sum_{k=0}^{\infty} \bar{x}^{\nu_1} \dots \bar{x}^{\nu_k} \partial_{\nu_1} \dots \partial_{\nu_k} N^\mu(x') = N^\mu(x') + O(\bar{x}). \quad (36)$$

Introducing K_N into its defining equation and organizing all terms by their powers of the proper time τ will then yield a recurrence relation

$$\begin{aligned} &\left[j + \bar{x}^\mu (\partial_\mu - \frac{1}{2}(N - N(x'))_\mu) \right] a_j^{(N)}(x, x') \\ &= \left[(N(x') - N)_\mu \left(\partial + \frac{1}{2} N(x') \right)^\mu + \partial^2 \right] a_{j-1}^{(N)}(x, x'). \end{aligned} \quad (37)$$

Following the same reasoning as in the previous section, one can show that the coefficients of the proper-time expansion do not depend on any invariants of the form

⁴Although this operator is non-Hermitian, it is elliptic, guaranteeing the existence of the heat kernel. A further $\partial_\mu N^\mu/2$ term should be added to obtain a Hermitian one.

$$\mathcal{K}_5 = \{(N^\mu N_\mu)^j, \quad j > 0\}. \quad (38)$$

Explicit calculations of the first few coefficients have been presented in [35].

B. Coupling to torsion

It is worth exploring whether these results can be further generalized to more complex systems. In doing so, however, we find that such generalizations cannot be performed naively and require an attentive consideration. To exemplify this, let us consider a system consisting of a quantum spinor field interacting with an axial vector field S^μ , which does not need to be a gauge field. This kind of system has been of interest for the study of some axion models [45], while also serving as a gateway to understanding the interaction between spinors and torsion in more general setups [43]. The action for such a system is given by

$$S_{\text{tor}} := - \int d^d x \bar{\Psi} (\gamma^\mu D_\mu^{\text{tor}} + m) \Psi, \quad (39)$$

$$D_\mu^{\text{tor}} := \partial_\mu + i\eta\gamma_5 S_\mu, \quad (40)$$

where η is a (pseudoscalar) coupling constant and γ_5 is the chiral element associated with the γ matrices (which in $d = 4$ is usually defined as $\gamma_5 := -i\gamma^0\gamma^1\gamma^2\gamma^3$). The associated operator we wish to study is then

$$\begin{aligned} \mathcal{Q}_{\text{tor}} &:= (\gamma^\mu D_\mu^{\text{tor}} + m)(-\gamma^\nu D_\nu^{\text{tor}} + m) \\ &= -\partial^2 + 2i\eta\gamma_5 \sigma^{\mu\nu} S_\mu \partial_\nu + m^2 - \eta^2 S^2 - i\eta\gamma_5 \partial_\mu S^\mu \\ &\quad - \frac{i}{2} \eta \sigma^{\mu\nu} S_{\mu\nu}, \end{aligned} \quad (41)$$

where we define $S_{\mu\nu} := \partial_\mu S_\nu - \partial_\nu S_\mu$ in analogy to the definition of $F_{\mu\nu}$. Limiting ourselves to the case where S^μ is constant, we see that the operator reduces to an expression resembling (31), with

$$\alpha := m^2 - \eta^2 S^2, \quad (42)$$

$$N^\mu := 2i\eta\gamma_5\sigma^{\mu\nu}S_\nu. \quad (43)$$

Unlike in the models of the previous section, this time N^μ is not a mere constant *scalar-valued* vector but a constant *matrix-valued* vector, reflecting its nontrivial spinor structure from the γ matrices product. The specific structure of N^μ can be used to show that

$$\{N^\mu, N^\nu\} = 8\eta^2(S^\mu S^\nu - S^2\eta^{\mu\nu})\mathbb{I}, \quad (44)$$

allowing us to search for an explicit solution to the heat kernel equation in a manner similar to the one used before. The result obtained,

$$K_{\text{tor}}(x, x'; \tau) = ie^{-\tau\alpha} \int \frac{d^d p}{(2\pi)^d} e^{-\tau p^2 + i p \cdot \bar{x}} \exp(\tau N^\mu p_\mu), \quad (45)$$

presents a major difference with respect to the case of a scalar-valued vector N^μ : the factor $\exp(\tau N^\mu p_\mu)$ is no longer the exponential of a scalar (which would allow the integral to be evaluated straightforwardly), but rather the exponential of a nontrivial matrix object. The integral can be explicitly done by performing a formal power series expansion and then reorganising the outcome. So far it has proven nontrivial to find a closed form for this heat kernel. In its current form, it is still possible to show that the resulting heat kernel proper-time expansion will once again effectively resum all the contributions from $S_\mu S^\mu$ (appearing as an effective curvature/mass correction term), but it remains to be seen whether further results can be derived.

V. CONCLUSIONS

The development of methods capable of probing quantum effects beyond perturbation theory remains a central pursuit in theoretical and mathematical physics. A wide range of strategies have been employed, ranging from lattice formulations [46] to semiclassical and instanton techniques [47–51], which capture nonperturbative contributions via tunnelling configurations and saddle points of the Euclidean action. Alongside these, effective-action and heat kernel methods [52] provide a complementary framework, in which nonperturbative information can be extracted from the spectral properties of differential operators. In particular, while the standard short-time expansion of the heat kernel yields an asymptotic perturbative series, appropriate resummation techniques can reveal analytic structures that encode nonperturbative physics. Unlike the

case of instanton methods, where analytic results can be obtained just for certain models, our results are rather general, depending only on the assumption of large fields.

Within the broader framework of quantum field theory in curved spacetime, such techniques acquire special significance. Originally conceived as an intermediate step toward a quantised theory of gravity, this field has evolved into a rich discipline in its own right, revealing phenomena that illuminate the interplay between gravitation and quantum mechanics across energy scales. The Hawking effect [53], which predicts spontaneous particle creation in strong gravitational fields, stands as its most emblematic example. Even in the simplest settings (for example, when a single quantum field interacts with a classical background) substantial conceptual and computational challenges persist. Standard approaches such as Feynman’s diagrammatic expansion [54] rely on infinite perturbative series, for which the partial sums’ convergence and interpretation become increasingly opaque beyond leading order, particularly in gravitational or strongly coupled regimes.

In this context, the heat kernel formalism provides a unifying framework where semiclassical perturbative and nonperturbative effects can be explored. This perspective motivates the program developed in recent years to identify and formalize resummation patterns that capture nonperturbative information directly from the heat kernel expansion. The results presented in this paper contribute to this effort and can be summarised in three main outcomes.

First, the resummation conjecture, originally posed in $d = 4$ by Navarro-Salas and Pla [33], has now been explicitly proved for both scalar and spinor quantum electrodynamics. The proof in the present manuscript yields a new resummed form of the heat kernel expansion for fermionic systems, effectively capturing all invariants built from fully contracted powers of the electromagnetic tensor $F_{\mu\nu}$, together with a “mass correction” term proportional to $\sigma^{\mu\nu}F_{\mu\nu}$. Our results generalize the original conjecture, inasmuch as they are valid in flat spaces of *arbitrary* dimensions d , with a restriction to even-dimensional spaces for fermionic systems. The peculiarity of $d = 4$ resides in the fact that any contracted power of the field strength can be written in terms of the invariants $\mathcal{F}$ and $\mathcal{G}$, which form actually the language employed in [33].

Second, although a detailed order-by-order derivation of the proper-time expansion coefficients was not carried out here, they can be obtained recursively from Eq. (27), yielding results consistent with earlier analyses [7,55,56]. This resummed heat kernel expansion thus provides an alternative and compact tool for analyzing one-loop effective actions and their associated phenomena, including particle-pair creation in the Schwinger process.

Third, the formalism developed here may admit extensions to more general settings. While such generalizations lie beyond the scope of the present work, possible directions include non-Abelian gauge theories and curved

spacetime backgrounds. These cases are expected to present additional conceptual and computational challenges, whose resolution may benefit from comparisons with other frameworks designed to study loop effects, such as numerical approaches [57], large- N expansions [58], large quantum nonlinear parameter resummations [59], and the worldline formalism [60]. In particular, recent developments concerning axial couplings [61] and gravitational setups [62] may provide further insight into the structure of these generalizations.

Beyond their formal aspects, resummed heat kernel methods open promising avenues for phenomenological applications. As previously discussed, the resummed kernel derived in Sec. III reproduces the known results for the Schwinger effect and at the same time, for the scalar case, suggests the existence of analogous “Schwinger-like” mechanisms for particle creation. Testing these ideas could be relevant in certain inflationary models or in scenarios involving ultralight dark matter [63,64]. Such directions underline the potential of resummed heat kernel formulations not only as analytic tools for nonperturbative physics, but also as bridges connecting semiclassical field theory with observational frontiers.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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