

Symmetry of screening masses of mesons in two-flavor lattice QCD at high temperatures

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We investigate spatial two-point correlation functions of mesonic operators in two-flavor lattice QCD at high temperatures. The simulated temperatures cover the range $T \in [147, 330]$ MeV, where the critical temperature is estimated around 165 MeV. To ensure a good control of the chiral symmetry we employ the Möbius domain-wall fermion action for two degenerate flavors of quarks. With a lattice cutoff $a^{-1} \sim 2.6$ GeV, the residual mass is reduced to 0.14 MeV. With the energy spectrum obtained from the screening mass at incremental values of the temperature range, we examine the $SU(2)_L \times SU(2)_R$ chiral symmetry, the anomalous axial $U(1)$, as well as an enhanced symmetry which exchanges the spin degrees of freedom. We also study how the data approaches the perturbative prediction given by twice the Matsubara frequency of free quarks.

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I. INTRODUCTION

The chiral phase transition of QCD, which occurred in the early Universe, gives a crucial input for understanding how hadrons formed and acquired their $O(1)$ GeV masses that we observe currently. While it is naturally assumed that the lightest two-quark flavors play a crucial role in the phase transition, the details of the critical phenomena depend on the symmetry of the up and down quarks above the critical temperature T_c [1].

In the massless limit of the up and down quarks, it is natural to assume that the standard $SU(2)_L \times SU(2)_R$ symmetry is recovered at some critical temperature in the range 150–200 MeV and no other symmetry exists. However, the axial $U(1)_A$, which is broken by anomaly, may effectively appear if nontrivial topological excitations of gluons are strongly suppressed above T_c . When such topological effects are described by instantons [2–6], which

is valid for pure Yang-Mills, QCD with heavy quarks, and QCD with a large number of color degrees of freedom, strong suppression of the quantum anomaly is not expected and the axial $U(1)_A$ is likely to remain broken at $T \sim T_c$. However, with the light dynamical quarks, the typical size of the topological excitation is described by the pion physics whose correlation length becomes larger than $1/T$, and this description of the topological excitations by instantons is no longer valid. The fate of the axial $U(1)_A$ anomaly at high T has been discussed in both theoretical works [7–13] as well as numerical works [14–20].

Another interesting possibility is that at high T , QCD may in fact have a larger symmetry. This symmetry is related to the antiperiodic boundary condition of quarks imposed in the temporal direction. At very high temperatures, the mesons would have large masses proportional to the temperature T , due to the Matsubara mass. In fact, in the perturbative evaluation in [21] the masses of all the mesons of different spins converge to $2\pi T$ at lowest order. From this point of view, we could consider the light quarks which form the meson to become sufficiently “heavy” at high temperatures leading to a spin-independent symmetry. Such a symmetry is analogous to the heavy quark symmetry [22,23] in $T = 0$ QCD. Note that this is

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not a symmetry of the original QCD Lagrangian but is an “emergent” symmetry in the large T limit. Although this is only an approximate symmetry within the perturbative framework in the large T limit, nonperturbative gluonic effects may enhance this symmetry even at $T \sim 2T_c$, this symmetry is called the chiral-spin symmetry [24–29].

Our goal in this work is to simulate QCD with chiral fermions and to investigate these symmetries at high temperatures, including how the meson masses approach the perturbative prediction $2\pi T$.¹ Lattice QCD simulations that have previously studied this issue, however, used fermion formulations that explicitly broke the $SU(2)_L \times SU(2)_R$ symmetry. For the Wilson fermion, the symmetry is broken down to a vectorlike $SU(2)_V$ and for the staggered fermion, only the $U(1)_{V'} \times U(1)_{A'}$ subgroup [which are different from the standard quark number and anomalous axial $U(1)_A$ symmetries] remains. In particular, we reported in [31–33] that the lattice artifacts in the probes of axial $U(1)_A$ anomaly are enhanced at high temperatures and it is difficult to disentangle the physical effect from the discretization errors.

In this work, we employ the domain-wall fermion formalism [34–36] with an improvement proposed in Refs. [37,38]. With this Möbius domain-wall fermion, which is a good approximation of the overlap fermion [39], we achieve a theoretically clean control of the symmetry of QCD. With a fixed lattice spacing $a \sim 0.075$ fm, the residual mass [32], which is an indicator of the explicit symmetry violation is reduced to 0.14(6) MeV. In addition, we set the quark masses so that the lightest bare quark mass $ma = 0.001$ is less than the physical mass value of the up and down quarks. The simulated temperatures span a range $T \in [147, 330]$ MeV, which corresponds to $[0.9, 2]T_c$, where the T_c is estimated from the peak of the chiral susceptibility [40]. The main runs are carried out with a fixed lattice size $L = 32$ except for the lowest two temperatures, where the pions behave as the pseudo-Nambu-Goldstone bosons; in this case we simulate two additional volumes, $L = 40$ and 48, to control the finite volume systematics. While this study is carried out for a fixed lattice spacing, our lattice spacing $a \sim 0.075$ fm is at the same level of the finest lattice in the literature. In addition to this, our use of the Möbius domain-wall fermion introduces an automatic $\mathcal{O}(a)$ improvement over discretization effects; while these cannot be estimated in this study, future work will be to make an extrapolation to the continuum limit.

On each ensemble of $\mathcal{O}(1000\text{--}10000)$ trajectories, we measure the two-point mesonic correlation functions in six different isotriplet channels every 50 trajectories. With the screening masses obtained from fitting the correlators, we

examine the symmetry at each simulated temperature and quark mass. In our main analysis of the mesonic correlators, the standard $SU(2)_L \times SU(2)_R$ chiral symmetry is examined by the difference between the vector V and axial vector A correlators. The tensor T_i and X_i channels (definitions will be given below), rather than scalar S (noisy) and pseudoscalar (PS) channels, are useful to estimate how much the axial $U(1)_A$ anomaly effect remains. In lattice QCD, the isospin triplet S channel has been difficult to precisely measure. As a possible reason, it was pointed out in [41] that the isospin triplet S channel may couple to many composite meson states. Therefore, study of the $U(1)_A$ as well as any other underlying quark symmetries using the S channel should be considered alongside an additional probe. Because of the Matsubara mass, the system may exhibit the chiral-spin $SU(2)_{CS}$ symmetry mentioned above. In our previous work at higher temperatures [42–44] we observed the emergence of this $SU(2)_{CS}$ symmetry from the correlator ratios among V , A , T_i , and X_i . A similar observation simulating quarks with a good chiral symmetry was reported in Refs. [45–47]. In this work, we revisit this enlarged symmetry group, computing the screening masses and simulating lower temperatures down to $T \sim 147$ MeV, which is $\sim 0.9T_c$.

The rest of the paper is organized as follows. In Sec. II, we discuss the general functional form of the mesonic correlators and the possible pattern of the symmetry at high temperatures. In particular, we address the enhancement of symmetry compared to the original QCD using the effective theory of heavy quarks due to the Matsubara mass. Our lattice setups as well as our methodology for how to extract the screening mass are addressed in Sec. III. Numerical results for our lattices are given in Sec. IV. Finally, we give a summary and discuss our numerical results in Sec. V. Some results in this work were already given in [48] and preliminary results for $N_f = 2 + 1$ simulations were presented in [49]. We also refer the readers to Refs. [50–52] for the related works from the JLQCD finite T project.

II. MESONIC TWO-POINT FUNCTIONS AND SYMMETRIES AT HIGH TEMPERATURES

In the limit of massless up and down quarks, the QCD action is invariant under the standard chiral transformation of the group $SU(2)_L \times SU(2)_R$, as well as that of the axial $U(1)_A$. The axial $U(1)_A$ symmetry is in general broken by quantum anomaly, but its magnitude at finite temperatures is nontrivial since it is tightly related to the topological excitations of gluons, which may be strongly suppressed at high temperatures.

In this work, we consider the mesonic two-point function in the z direction whose continuum limit is given by

$$C_\Gamma(z) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_0^\beta dt \langle O_\Gamma(x, y, z, t) O_\Gamma^\dagger(0, 0, 0, 0) \rangle, \quad (1)$$

¹In [30], they simulated QCD at very high temperatures $T \geq 1$ GeV and found that the meson masses approach $2\pi T$ from above, which is consistent with [21] although the channel dependence is still visible.

TABLE I. List of quark bilinear operators we compute the two-point correlation functions and the symmetries connecting them.

Γ	Reference name	Abbreviation	Symmetry correspondences
$\mathbb{I}$	Scalar	S	$\} U(1)_A$
γ_5	Pseudoscalar	PS	
γ_1, γ_2	Vector	V	$\} SU(2)_L \times SU(2)_R$
$\gamma_1\gamma_5, \gamma_2\gamma_5$	Axial vector	A	
$\gamma_4\gamma_3$	Tensor	T_t	$\} U(1)_A$
$\gamma_4\gamma_3\gamma_5$	Axial tensor	X_t	
			$\} SU(2)_{CS}$

where the quark bilinear operator $O_\Gamma = \bar{q}\Gamma T^a q$ is taken to be a flavor-triplet (the isospin generator is denoted by T^a). For Γ we choose $\gamma_5(PS)$, $\mathbb{I}(S)$, $\gamma_{1,2}(V)$, $\gamma_5\gamma_{1,2}(A)$, $\gamma_4\gamma_3(T_t)$, and $\gamma_5\gamma_4\gamma_3(X_t)$. The list of operators we measure and the symmetries connecting them are summarized in Table I.

Under the standard chiral $SU(2)_L \times SU(2)_R$ rotation,

$$\begin{aligned} q &\rightarrow \exp(i\alpha_a T^a + i\beta_a T^a \gamma_5) q, \\ \bar{q} &\rightarrow \bar{q} \exp(-i\alpha_a T^a + i\beta_a T^a \gamma_5), \end{aligned} \quad (2)$$

only V and A correlators mix while all the other channels remain unchanged up to isospin exchanges. Therefore, the $SU(2)_L \times SU(2)_R$ symmetry breaking can be detected by the difference between the V and A correlators.

For the axial $U(1)_A$ transformation,

$$q \rightarrow \exp(i\epsilon\gamma_5)q, \quad \bar{q} \rightarrow \bar{q} \exp(i\epsilon\gamma_5), \quad (3)$$

S and PS mix as well as the T_t and X_t channels, while A and V correlators are invariant. Thus, the S – PS and T_t – X_t pairs are the probes of the axial $U(1)_A$ breaking. As shown later, the S correlator is numerically noisy, and T_t and X_t are more useful to examine the axial $U(1)_A$ anomaly.

In addition to the above manifest symmetries in the QCD Lagrangian, we may expect emergence of larger symmetries due to antiperiodic boundary condition on the quarks in the temporal direction. This enhancement of symmetry is analogous to the heavy quark symmetry [22,23], which (approximately) appears in the effective theory of bottom or charm quarks.

In order to see this enhanced symmetry, let us consider a free quark propagator in the positive z direction,

$$\begin{aligned} \langle q(z)\bar{q}(0) \rangle(p_1, p_2) &= T \sum_{p_0} \int \frac{dp_3}{(2\pi)} \frac{e^{ip_3 z}}{i\gamma_0 p_0 + i\gamma_3 p_3 + i\gamma_1 p_1 + i\gamma_2 p_2 + m} \\ &= T \sum_{p_0} \int \frac{dp_3}{(2\pi)} \frac{-(i\gamma_0 p_0 + i\gamma_3 p_3 + i\gamma_1 p_1 + i\gamma_2 p_2 - m)e^{ip_3 z}}{p_0^2 + p_3^2 + p_2^2 + p_1^2 + m^2} \\ &= T \sum_{p_0} \frac{-(i\gamma_0 p_0 - \gamma_3 E(p_0, p_1, p_2) + i\gamma_1 p_1 + i\gamma_2 p_2 - m)e^{-E(p_0, p_1, p_2)z}}{2E(p_0, p_1, p_2)}, \end{aligned} \quad (4)$$

where $E(p_0, p_1, p_2) = \sqrt{p_0^2 + p_1^2 + p_2^2 + m^2}$ and the other components p_1, p_2 are fixed. Here the Matsubara mass $p_0 = (2n+1)\pi T$ takes discrete values due to the antiperiodicity in the imaginary temporal direction. In the large $T \gg m, p_1, p_2$ limit, the lowest $p_0 = \pm\pi T$ will dominate the signal and the correlator becomes

$$\begin{aligned} \langle q(z)\bar{q}(0) \rangle(p_1, p_2) &= T \left[\gamma_3 \frac{1 + i\text{sgn}(p_0)\gamma_0\gamma_3}{2} e^{-\pi T z} \right. \\ &\quad \left. + O(1/T) \right]. \end{aligned} \quad (5)$$

In the same way, the correlator in the temporal direction with a fixed p_1, p_2, p_3 in the large Matsubara mass limit is

expressed as

$$\begin{aligned} \langle q(t)\bar{q}(0) \rangle(p_1, p_2, p_3) &= -iT \left[\sum_{p_0} \frac{\gamma_0 p_0}{p_0^2} \exp(ip_0 t) \right. \\ &\quad \left. + O(1/T) \right]. \end{aligned} \quad (6)$$

These propagators are invariant under the following transformation:

$$\begin{aligned} q &\rightarrow \exp(i\Sigma^i \epsilon_i) q, \\ \bar{q} &\rightarrow \bar{q} \gamma_0 \exp(-i\Sigma^i \epsilon_i) \gamma_0, \end{aligned} \quad (7)$$

where Σ^i has three components,

$$\Sigma^i = (\gamma_k, -i\gamma_5\gamma_k, \gamma_5), \quad k = 1, 2. \quad (8)$$

We do not distinguish $k = 1$ and $k = 2$, since they are identical under the rotational symmetry along the z axis. The generators Σ^i form an $SU(2)$ algebra, which is identical to the chiral-spin $SU(2)_{CS}$ symmetry suggested in the literature [24–29].²

Note that any correction from gauge field potential only appears in the next-to-leading $1/T$ contribution. Thus, the chiral spin $SU(2)_{CS}$ symmetry emerges at sufficiently high temperatures. With this $SU(2)_{CS}$ symmetry, the mesonic operators (A_1, T_i, X_i) form a nontrivial triplet. These operators have different spins in the original four dimensions, but they are all in the spin-one representation of $SO(3)$ symmetry along the z axis. One can consider another set (V_2, PS, S) but the operators transform differently under the $SO(3)$ rotation and therefore, the mass degeneracy would not be that good.

From (5), it is obvious that in the free quark limit the meson screening mass is $2\pi T$. In fact, in the perturbative analysis [21] the screening mass is obtained as

$$M_\Gamma = 2\pi T + cg^2T,$$

with a numerical constant c coming from the one-loop correction where g is the gauge coupling. It is remarkable that the result does not depend on Γ . This result is analogous to the heavy quark symmetry [22,23] which is an approximate symmetry of the bottom or charm quarks at $T = 0$. As the spin and orbital angular momentum dependent interactions are proportional to the inverse of the heavy quark mass, the spectrum of the quarkonia and heavy-light mesons are insensitive to spin. The same may explain here that with a large Matsubara mass proportional to T , the operators (V_1, PS, S) with different spin form an $SU(2)_{CS}$ triplet.³ In Ref. [45], it was suggested that other combinations of tensors $\gamma_1\gamma_3(T_k)$ and $\gamma_1\gamma_3\gamma_5(X_k)$ form a quartet with A_4 and V_4 . Inclusion of this quartet into our analysis is of interest to future work; however, in this paper we will focus on the triplet.

It is an important numerical subject to quantify the emergent $SU(2)_{CS}$ symmetry at intermediate temperatures $T \sim T_c$ and see if it has any impact on the chiral phase transition. It is also interesting to investigate if an additional transition exists at $T > T_c$, triggered by this emergent symmetry. These three symmetries, accessible through the differences between mesonic screening masses from the

pairs of the triplet channels,⁴ constitute our motivation to simulate QCD at high temperatures in the range $[0.9, 2]T_c$, for a range of quark masses covering the physical point as a method to understand the symmetry structure of QCD at high temperatures.

III. LATTICE SETUPS

For gauge configuration generation, we perform hybrid Monte Carlo simulations⁵ with the tree-level improved Symanzik gauge action [57] and the Möbius domain-wall fermion action [37,38]. See Ref. [32] for details of our simulations. In this work, we focus on the gauge configurations at a fixed lattice spacing 0.075 fm generated with the bare coupling constant $\beta = 4.30$. A portion of the same gauge link ensembles are shared with our previous studies [33,50], in addition to these we also use an updated and newly generated series of ensembles on larger lattices at lower temperatures. The simulation parameters are summarized in Table II.

For the scale setting, we perform the Wilson flow and use the reference flow time $t_0 = (0.1539 \text{ fm})^2$ presented in [58] to determine the lattice cutoff $1/a = 2.643 \text{ GeV}$. Our main lattice size $L = 32$ corresponds to 2.37 fm, while we also simulate $L = 40$ and 48 lattices for the lowest two simulated temperatures to study the finite-size systematics near the critical temperature.

We vary the temperature by changing the temporal size of the lattice from 8 to 18, which corresponds the range $147 \text{ MeV} \leq T \leq 330 \text{ MeV}$. The critical temperature at the physical point is estimated to be $T_c \sim 165(3) \text{ MeV}$ from the peak position of the disconnected chiral susceptibility [40] where the error is statistical only.

We take 4–6 different quark masses depending on the temperatures. The lightest bare sea quark mass $m = 0.001$ corresponds to 2.6 MeV, which is estimated to be 71% of the physical quark mass $m_{\text{phys}} = 0.0014(2)$. In this study, we always take the valence and sea quark masses equal.

For the link variables in the Dirac operator, we perform the stout smearing [59] three times with the smoothing parameter $\rho = 0.1$. The size of the extra dimension for the Möbius domain-wall fermion action is set to $L_s = 16$. With this treatment, the residual mass or the chiral symmetry breaking of the lattice Dirac operator is suppressed to 0.14(6) MeV [32].

²In the hadron correlators, the relative momentum part in p_1 and p_2 must be integrated out but that ends up with a typical scale $1/z$ and the correction is $1/zT$ as clearly shown in [43].

³The $SO(3)$ rotational symmetry is broken also among (A_1, T_i, X_i) at very high temperatures, since the temporal direction shrinks as $1/T \rightarrow 0$. However, its effect is subleading.

⁴In our previous study [44], we also studied the ratio of differences of correlators $|C_A(z) - C_T(z)|/|C_A(z) - C_S(z)|$. However, we find that the denominator approaches to zero at high temperatures (presumably due to the large Matsubara mass and the analogy to “heavy quark symmetry” we discussed in Sec. II.) and the ratio is not a good indicator of the $SU(2)_{CS}$ symmetry. In this work, we focus on the screening mass differences and present the values in the physical unit (MeV) so that one can easily compare with the simulated temperature for each set of data.

⁵See Refs. [53–56] for details of our simulation codes.

TABLE II. Simulation parameters and meson screening mass in the physical unit (MeV) obtained from the cosh fit.

$L^3 \times L_t$	$L(\text{fm})$	$T [\text{MeV}]$	TL	am	$m [\text{MeV}]$	No. samples	m_S	m_{PS}	m_V	m_A	m_X	m_T
$48^3 \times 18$	3.6	147	2.7	0.00100	2.6	146		113(34)	951(111)	1106(182)	1120(201)	1068(154)
				0.00250	6.6	40	1097(288)	220(5)	1009(69)	1404(129)	1801(166)	1012(71)
				0.00375	9.9	40	1387(337)	245(5)	854(74)	1145(155)	1279(223)	1164(97)
				0.00500	13.2	83		135(7)	903(58)	1375(177)	1733(281)	1199(117)
$36^3 \times 18$	2.7	147	2.0	0.00100	2.6	146		166(22)	812(85)	941(132)	1483(206)	1391(115)
				0.00250	6.6	121	2605(533)	220(12)	767(53)	930(156)	1645(153)	1228(117)
				0.00375	9.9	122	1586(333)	240(11)	886(77)	1046(181)	1635(107)	1216(52)
				0.00500	13.2	131	1490(139)	288(6)	861(43)	1231(125)	1527(117)	1091(56)
$40^3 \times 16$	3.0	165	2.5	0.00100	2.6	165	642(251)	149(46)	1109(165)	1141(185)	994(268)	1453(305)
				0.00250	6.6	95	817(402)	245(18)	544(146)	536(201)	1499(196)	1161(146)
				0.00375	9.9	97		301(16)	1003(90)	1400(158)	1156(305)	1364(171)
				0.00500	13.2	95		273(20)	936(113)	1330(216)	1007(341)	721(95)
$32^3 \times 16$	2.4	165	2.0	0.00100	2.6	165		233(49)	883(93)	890(84)	1485(256)	1633(222)
				0.00250	6.6	116		362(53)	904(75)	978(133)	1268(431)	1212(176)
				0.00375	9.9	163	1624(538)	309(14)	934(90)	1047(131)	1552(178)	966(152)
				0.00500	13.2	143	444(187)	326(24)	932(84)	1106(159)	1679(207)	1216(89)
$32^3 \times 14$	2.4	189	2.3	0.00100	2.6	190	381(40)	353(39)	998(40)	998(41)	1187(62)	1161(61)
				0.00250	6.6	177		362(39)	1150(49)	1164(52)	1376(154)	1407(131)
				0.00375	9.9	137		402(27)	964(33)	996(44)	1318(146)	1212(135)
				0.00500	13.2	133	582(159)	413(29)	940(44)	1069(74)	1385(114)	1178(82)
$48^3 \times 12$	3.6	220	4.0	0.00100	2.6	220	793(43)	792(43)	1306(37)	1306(37)	1377(39)	1378(39)
				0.00250	6.6	97	913(21)	911(20)	1290(39)	1293(39)	1411(38)	1410(38)
				0.00375	9.9	114	1033(192)	706(47)	1189(78)	1235(93)	1441(48)	1431(54)
				0.00500	13.2	116	888(108)	752(109)	1234(33)	1236(35)	1378(58)	1316(48)
$40^3 \times 12$	3.0	220	3.3	0.00500	13.2	220	1534(351)	502(51)	1327(32)	1298(28)	1291(116)	1425(38)
				0.01000	26.4	244	1495(336)	696(35)	1222(24)	1275(32)	1302(106)	1444(99)
$32^3 \times 12$	2.4	220	2.7	0.00100	2.6	532	712(61)	701(58)	1281(38)	1283(39)	1402(58)	1408(58)
				0.00250	6.6	534	697(88)	797(85)	1235(22)	1234(23)	1294(46)	1307(51)
				0.00375	9.9	689	615(158)	696(77)	1244(23)	1251(24)	1570(103)	1374(104)
				0.00500	13.2	544	1350(360)	717(90)	1182(25)	1213(23)	1465(115)	1292(69)
				0.01000	26.4	622	1254(226)	686(36)	1227(57)	1160(76)	1345(101)	1302(80)
$24^3 \times 12$	1.8	220	2.0	0.00100	2.6	373	747(84)	741(82)	1283(27)	1280(29)	1427(28)	1434(28)
				0.00250	6.6	361	1637(334)	806(121)	1292(30)	1295(30)	1345(77)	1305(80)
				0.00375	9.9	331	687(120)	785(53)	1333(32)	1335(33)	1392(32)	1398(32)
				0.00500	13.2	363	2505(488)	736(134)	1219(31)	1245(36)	1327(66)	1469(55)
				0.01000	26.4	365	770(153)	713(54)	1271(29)	1307(38)	1421(33)	1450(29)
$32^3 \times 10$	2.4	264	3.2	0.00500	13.2	640	1319(35)	1266(26)	1566(18)	1566(18)	1666(32)	1631(26)
				0.00800	21.1	237	1252(19)	1251(19)	1572(15)	1574(15)	1626(19)	1627(20)
				0.01000	26.4	291	1525(121)	1117(94)	1560(19)	1581(27)	1630(49)	1654(28)
				0.01500	39.6	121	1458(123)	1236(100)	1596(21)	1599(21)	1666(28)	1693(29)
$32^3 \times 8$	2.4	330	4.0	0.00100	2.6	260	1810(11)	1809(11)	2026(9)	2026(9)	2083(13)	2083(13)
				0.00500	13.2	317	1796(19)	1791(18)	2038(12)	2038(12)	2094(24)	2094(24)
				0.01000	26.4	350	1783(15)	1781(15)	2033(8)	2033(8)	2082(9)	2082(9)
				0.01500	39.6	306	1828(24)	1791(22)	2027(10)	2028(10)	2064(17)	2063(17)
				0.02000	52.9	218	1807(15)	1796(14)	2013(18)	2014(18)	2069(15)	2066(15)
				0.04000	105.7	164	1824(27)	1770(20)	2021(16)	2024(16)	2080(16)	2073(15)

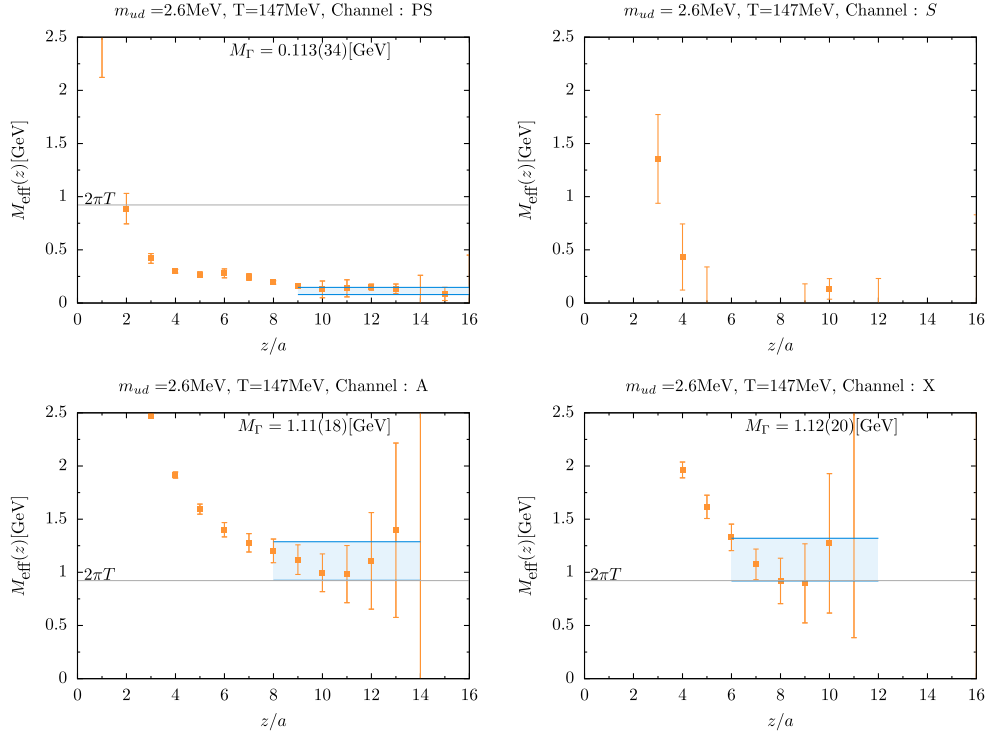


FIG. 1. Effective screening mass plots at $T = 147$ MeV. The data for PS , S , A , and X_t channels at the lightest quark mass $m = 0.001$ are presented.

For the correlator measurement, we use the four-dimensional effective Dirac operator,

$$D_{\text{DW}}^{4\text{D}}(m) = \frac{1+m}{2} + \frac{1-m}{2} \gamma_5 \tanh(L_s \tanh^{-1}(H_M)), \quad (9)$$

where the Möbius kernel H_M is

$$H_M = \gamma_5 \frac{2D_W}{2 + D_W}, \quad (10)$$

and D_W is the standard Wilson-Dirac operator with a negative mass $M = -1$. Here and in the following, we set $a = 1$ unless explicitly specified.

For each ensemble, 40–700 measurements were performed per 50 trajectories and our numerical data, after binning between 2–10 samples, did not show any significant autocorrelations. Using the rotational symmetry of the z -directional correlators, we applied averaging over three different spatial directions. For our sources, we did not apply any additional source smearing and used pointlike source and sink operators.

IV. NUMERICAL RESULTS

To obtain the screening masses from the correlation functions the effective mass is extracted from data by applying Newton's method to the cosh ansatz form of the spatial correlator. We then perform an uncorrelated

least-square fit by finding a suitable range for which the effective mass does not exceed 2σ from the error band. Regions where the error became 100% are omitted from the fits where possible. When the correlators are stable, the channels connected by symmetry transformations e.g., $V - A$, $PS - S$, and $X - T$ are fitted to the same range. We also confirm that the screening mass differences do not depend on the fitting range very much and the systematics are well-controlled compared to the statistical uncertainties.

We find that the S channel correlators are quite unstable, reflecting exceptionally large contribution from a few configurations. We decide to discard the S data when 1) the correlator goes negative or 2) there is no stable plateau in the fitting range of the symmetry partner PS .

Figures 1–3 present the typical effective mass plots with the cosh fitting ansatz at the lightest quark mass $m = 0.001$ at the lowest three simulated temperatures $T = 147$, 165, and 190 MeV.⁶ It is not difficult to find a reasonable plateau, in contrast to the study with staggered fermions in Ref. [19], where a sizable contamination from the excited states was reported. A possible reason for such a difference is the fact that the Möbius domain-wall fermion is free from the unphysical taste degrees of freedom as well as unwanted operator mixing among different multiplets of the $SU(2)_L \times SU(2)_R$ flavor group. From the data in the

⁶We also present the corresponding raw correlators in the Appendix.

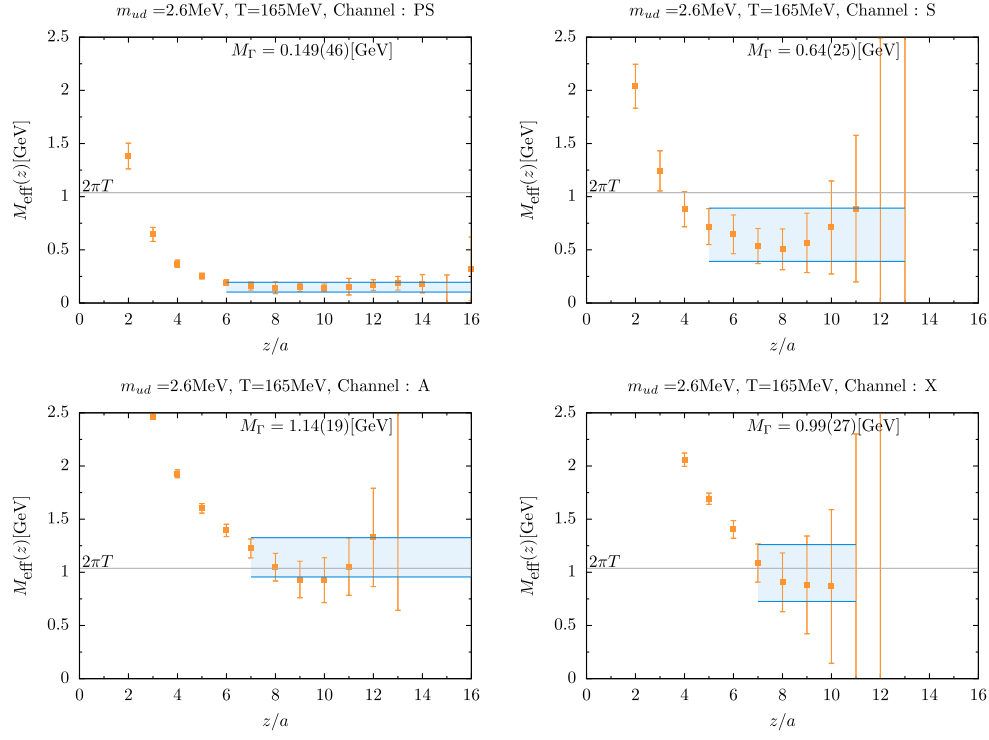
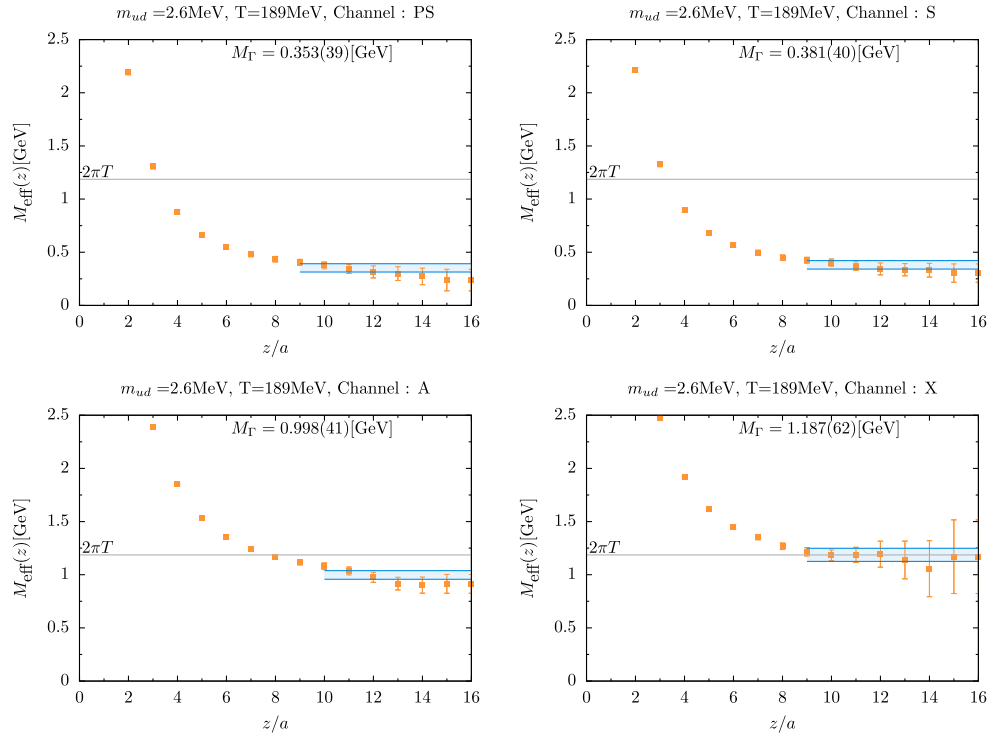
FIG. 2. The same plots as Fig. 1 but at $T = 165$ MeV.FIG. 3. The same plots as Fig. 1 but at $T = 190$ MeV.

TABLE III. The difference between the screening masses of the mesons in the physical unit (MeV).

$L^3 \times L_t$	$L(\text{fm})$	T [MeV]	TL	am	m [MeV]	No. samples	m_{PS-S}	m_{V-A}	m_{X-T}	m_{A-X}
$48^3 \times 18$	3.6	147	2.7	0.00100	2.6	146		-155(120)	52(233)	-8(196)
				0.00250	6.6	40	-877(289)	-395(119)	789(201)	-398(158)
				0.00375	9.9	40	-1141(335)	-291(155)	115(274)	-133(201)
				0.00500	13.2	83		-472(179)	534(381)	-361(297)
$36^3 \times 18$	2.7	147	2.0	0.00100	2.6	146		-129(66)	92(290)	-539(213)
				0.00250	6.6	121	-2385(525)	-163(139)	417(235)	-710(169)
				0.00375	9.9	122	-1346(331)	-160(171)	419(127)	-586(114)
				0.00500	13.2	131	-1203(141)	-370(127)	437(147)	-296(144)
$40^3 \times 16$	3.0	165	2.5	0.00100	2.6	165	-493(272)	-32(75)	-459(536)	148(377)
				0.00250	6.6	95	-572(414)	8(121)	338(307)	-963(184)
				0.00375	9.9	97		-397(143)	-208(447)	244(329)
				0.00500	13.2	95		-394(279)	287(304)	323(439)
$32^3 \times 16$	2.4	165	2.0	0.00100	2.6	165		-7(28)	-149(464)	-595(281)
				0.00250	6.6	116		-74(151)	56(542)	-290(445)
				0.00375	9.9	163	-1315(539)	-113(128)	586(247)	-505(248)
				0.00500	13.2	143	-118(194)	-174(138)	463(221)	-573(227)
$32^3 \times 14$	2.4	189	2.3	0.00100	2.6	190	-28(14)	0(4)	26(16)	-188(67)
				0.00250	6.6	177		-14(16)	-30(274)	-213(159)
				0.00375	9.9	137		-32(35)	107(265)	-321(140)
				0.00500	13.2	133	-169(159)	-129(75)	207(144)	-315(127)
$48^3 \times 12$	3.6	220	4.0	0.00100	2.6	220	-2(1)	-0(0)	-1(0)	-71(37)
				0.00250	6.6	97	-2(1)	-3(1)	0(1)	-118(47)
				0.00375	9.9	114	-327(206)	-46(35)	10(57)	-206(79)
				0.00500	13.2	116	-136(196)	-2(13)	62(86)	-141(62)
$40^3 \times 12$	3.0	220	3.3	0.00500	13.2	220	-1032(399)	14(18)	-22(96)	-141(98)
				0.01000	26.4	244	-799(347)	-48(20)	-2(79)	-87(79)
$32^3 \times 12$	2.4	220	2.7	0.00100	2.6	532	-11(7)	-2(2)	-5(4)	-119(80)
				0.00250	6.6	534	100(102)	1(2)	-13(32)	-60(36)
				0.00375	9.9	689	81(109)	-6(2)	197(195)	-320(98)
				0.00500	13.2	544	-633(444)	-31(17)	173(135)	-252(112)
$24^3 \times 12$	1.8	220	2.0	0.01000	26.4	622	-568(243)	67(79)	44(131)	-185(124)
				0.00100	2.6	373	-6(4)	4(4)	-7(5)	-148(38)
				0.00250	6.6	361	-831(225)	-3(2)	40(67)	-48(97)
				0.00375	9.9	331	98(74)	-1(1)	-6(6)	-56(43)
$32^3 \times 10$	2.4	264	3.2	0.00500	13.2	640	-53(53)	-1(1)	35(18)	-100(28)
				0.00800	21.1	237	-0(3)	-1(1)	-0(3)	-53(20)
				0.01000	26.4	291	-407(203)	-21(26)	-25(67)	-49(69)
				0.01500	39.6	121	-222(217)	-3(1)	-27(28)	-67(30)
$32^3 \times 8$	2.4	330	4.0	0.00100	2.6	260	-1(1)	-0(0)	-0(0)	-57(11)
				0.00500	13.2	317	-5(5)	-0(0)	0(0)	-56(26)
				0.01000	26.4	350	-2(1)	-0(0)	0(0)	-49(10)
				0.01500	39.6	306	-37(35)	-1(0)	1(1)	-37(13)
				0.02000	52.9	218	-12(6)	-2(0)	2(1)	-54(12)
				0.04000	105.7	164	-54(30)	-4(2)	6(3)	-55(11)

fitting range, which is shown by the shadow in these plots, we determine the meson screening masses for the different channels. The obtained values and their differences among various symmetry multiplets are summarized in Tables II and III.

In Fig. 4, we plot the temperature dependence of the obtained meson screening masses at our lightest mass $m = 0.001$ (~ 2.6 MeV). As discussed in Sec. II, the data at high T for all channels approach $2\pi T$, which is twice of the lowest mode Matsubara mass, shown by the gray line. At

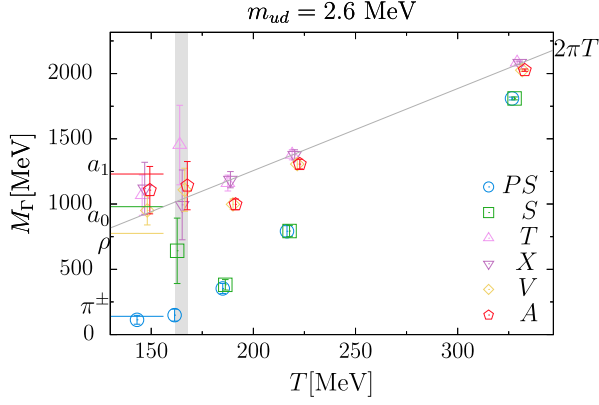


FIG. 4. T dependence of the screening masses for various meson channels at the lightest simulated mass $m = 0.001$ (~ 2.6 MeV). The gray line shows $2\pi T$, which is twice of the Matsubara mass. For a reference, we put the experimental results of $\pi^\pm$, ρ , a_0 , and a_1 mesons at zero temperature, indicated by bars at the vertical axis. The vertical gray band indicates our estimate for the critical temperature $T_c = 165(3)$ MeV from the chiral susceptibility. Note that the busy data points are slightly shifted horizontally.

lower temperatures around or below the critical temperature $T_c \sim 165(3)$ MeV (shown by the vertical gray band) there is a drastic change in the fluctuation of the masses. The scalar mass, in particular, shows inflated error bars at the lower temperatures, while quickly converging to a less noisy result, close to the PS channel data at higher temperatures. For reference, we put the experimental results of $\pi^\pm$, ρ , a_0 , and a_1 mesons at zero temperature, indicated by colored bars corresponding to the associated channels on the vertical axis. It is interesting to note that the lowest temperature, $T = 147$ MeV data, are already quite close to the experimental values at $T = 0$; this may indicate that the chiral symmetry breaking by the quark condensate is already strong enough to form hadrons. In the $T = 0$ limit, the T_t and X_t two-point functions transform in the same way as V and A correlators, respectively, and share the same intermediate states.

The same plot but with the temperature normalized by $T_c = 165$ MeV and the screening mass normalized by $2\pi T$ is shown in Fig. 5. For a comparison, the results by HotQCD Collaboration [19] which simulated $N_f = 2 + 1$ HISQ quarks are shown by the shadows [normalized by $2\pi T$ and the $(2 + 1)$ -flavor T_c estimate, respectively]. Although the value of T_c is different due to the absence/presence of the strange quark, the two results appear consistent and share many of the same features. The only exception is the S data at the lowest two simulated temperatures; as explained in [19], the shadow approaches the twice of the pion mass in the $T = 0$ limit, due to a lattice artifact of the staggered quark action. In our Möbius domain-wall fermion formulation, this never happens since the transition from the S triplet to two PS mesons is prohibited by the exact isospin symmetry.

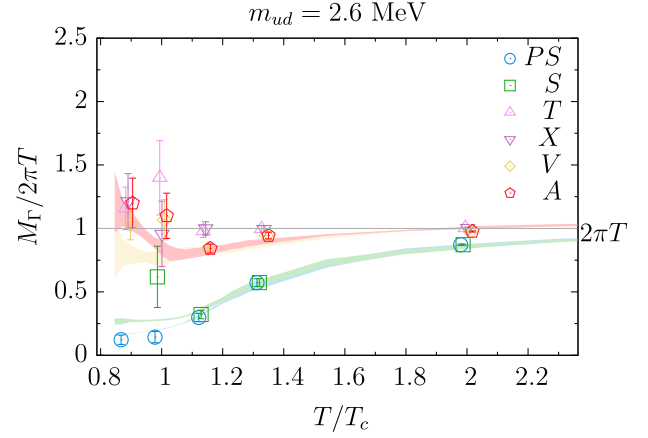


FIG. 5. The same figure as Fig. 4 but the temperature is normalized by $T_c = 165$ MeV and the screening mass is normalized by $2\pi T$. For a comparison, the results by HotQCD Collaboration [19] which simulated $N_f = 2 + 1$ HISQ quarks are shown by the shadows. Except for the S channels at the lowest two simulated temperatures, the two results look consistent with each other.

In Fig. 6, we plot the mass difference between A and V channels Δm_{V-A} , which probes the $SU(2)_L \times SU(2)_R$ chiral symmetry breaking, as a function of temperature T . Here the data from the largest spatial volume for each simulated temperature is plotted. Around the critical temperature shown by the vertical gray band, the mass difference Δm_{V-A} quickly vanishes. This is most clearly seen at our lightest mass $m = 0.001$, we find $\Delta m_{V-A}/m_A \sim 0.1\%$ at $T = 190$ MeV and 2.8% at $T = 165$ MeV. Further evidence of this rapid drop off is also seen in the literature with a rapid suppression of several order parameters associated with chiral symmetry shown in [60]. For the higher quark masses, Δm_{V-A} shows a milder T -dependence, which suggests that the crossover temperature becomes higher. Both values are consistent with zero to within a standard deviation.

A similar behavior but with larger error bars is found in Figs. 7 and 8 where the mass differences between X_t and T_t , Δm_{X-T} and that of PS and S , Δm_{PS-S} , are plotted. These are the probes for the axial $U(1)_A$ symmetry. We find that $\Delta m_{X-T}/m_X$ is 2.5% at $T = 190$ MeV and 12.7% at $T = 165$ MeV.⁷ The latter is consistent with zero within a larger error bar. This indicates that the chiral anomaly effect is quite suppressed above T_c .

⁷In recent study [61] with Möbius domain-wall quarks, violation of the axial $U(1)_A$ symmetry even at $T = 186$ MeV was reported. We notice, however, that their lattice spacings $a > 0.13$ fm are coarser than the one $a \sim 0.1$ fm we simulated in our previous work [31] where we find a larger violation of the Ginsparg-Wilson in the low-lying eigenmodes of the Dirac operator than the residual mass, which leads to overestimation of the axial $U(1)_A$ anomaly.

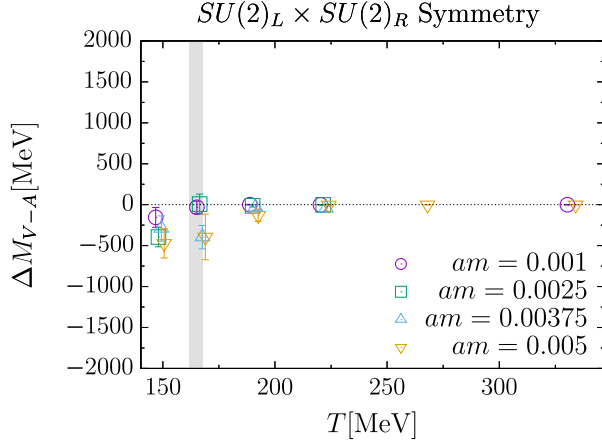


FIG. 6. Temperature T vs. the screening mass difference between V and A which probes the $SU(2)_L \times SU(2)_R$ symmetry. The vertical gray band indicates our estimate for the critical temperature $T_c = 165(3)$ MeV from the chiral susceptibility.

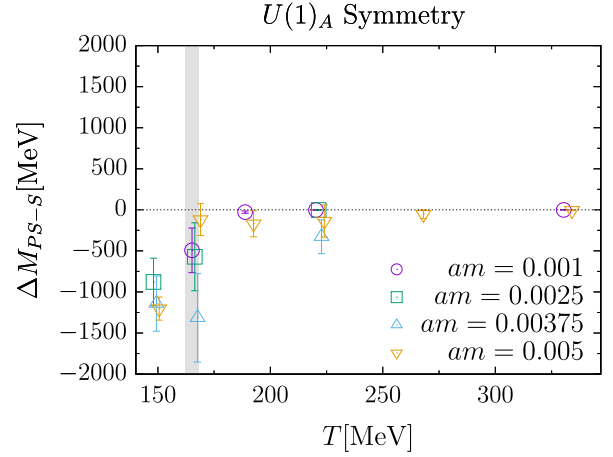


FIG. 8. Temperature T vs. the screening mass difference between PS and S which probes the $U(1)_A$ symmetry. The vertical gray band indicates our estimate for the critical temperature $T_c = 165(3)$ MeV from the chiral susceptibility.

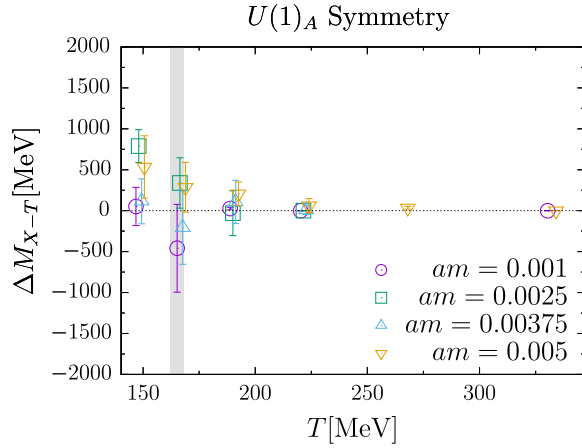


FIG. 7. Temperature T vs. the screening mass difference between X_t and T_t which probes the $U(1)_A$ symmetry. The vertical gray band indicates our estimate for the critical temperature $T_c = 165(3)$ MeV from the chiral susceptibility.

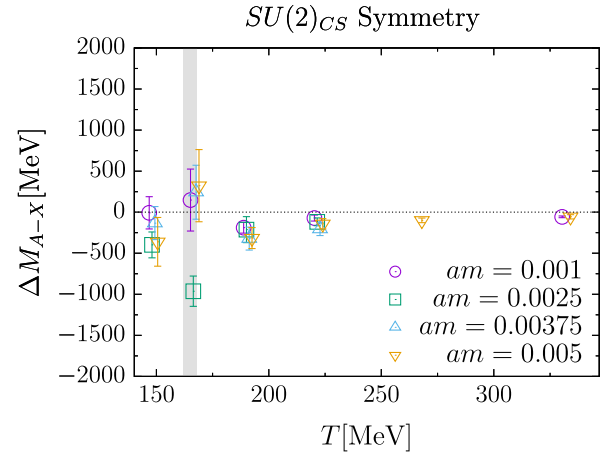


FIG. 9. Temperature T vs. the screening mass difference between A and X_t which probes the $SU(2)_{CS}$ symmetry. The vertical gray band indicates our estimate for the critical temperature $T_c = 165(3)$ MeV from the chiral susceptibility.

Figure 9 is a plot of the chiral-spin $SU(2)_{CS}$ symmetry, which is an approximate symmetry expected up to $O(1/T)$ corrections, as explained in Sec. II. This symmetry is examined by the mass difference Δm_{X-A} between X_t and A . The T dependence for the screening mass, in addition to the reduction in noise, is analogous to $SU(2)_L \times SU(2)_R$ and $U(1)_A$; however, we find a qualitative difference of $SU(2)_{CS}$ in Fig. 10. Here we plot the quark mass dependence of Δm_{V-A} (circles), Δm_{PS-S} (squares), Δm_{X-T} (upward triangles), and Δm_{A-X} (downward triangles) at $T = 330$ MeV. The former three probing $SU(2)_L \times SU(2)_R$ and $U(1)_A$ symmetries, are consistent with zero, and their uncertainty is ~ 1 MeV, which is less than 1% of T . In contrast to these other mass differences, the data Δm_{X-A} is clearly nonzero even at the lightest mass;

~ -40 MeV showing that the associated chiral-spin symmetry $SU(2)_{CS}$ is only approximate, $|\Delta m_{X-A}|/T \sim 0.12$. We note that at much higher temperatures than $T = 1$ GeV, it was reported in Ref. [30] that the mass difference between the PS and V channels goes down to $\sim 1\%$ of $2\pi T$ although, the difference is still statistically significant and deviates from the next-to-leading order perturbative QCD prediction, where the one-loop correction of each meson mass to $2\pi T$ is positive and channel independent (for related results and discussion see Refs. [62,63]).

Finally, let us discuss possible systematics in our numerical results. In Fig. 11, we plot the lattice size L dependence of all measured screening masses at $T = 147$, 165, and 220 MeV. The data at different L are consistent

within two standard deviations and we do not see any significant finite volume dependence. Due to our fixed lattice spacing at $a = 0.075$ fm, we cannot numerically estimate the discretization effects. Our choice is relatively

fine compared to other groups and we expect the automatic $O(a)$ improvement due to the chiral symmetry of the Möbius domain-wall fermion action. Note also, that our simulated up and down quark mass range covers the physical point, and the lightest quark mass is sufficiently close to the chiral limit.

V. SUMMARY AND DISCUSSION

We have simulated $N_f = 2$ lattice QCD at finite temperatures in the range $[0.9, 2]T_c$ with the Möbius domain-wall quark action and Symanzik gauge action setting the lattice spacing at $a \sim 0.075$ fm. Our simulated quark mass covers the physical point, and thus the chiral symmetry is well under control with the residual mass ~ 0.14 MeV. For the lowest two temperatures, we have simulated multiple volumes in order to check the finite size systematics.

The two-point correlation functions of the isovector operators with different spins have been measured in the spatial directions. From the standard cosh fit, we have extracted the screening masses for the six channels of the isospin triplet. The temperature dependence of the screening masses shows several remarkable features. First, the values at the lightest simulated quark mass $m = 0.001$ at $T = 0.9T_c$ are consistent with the experimental values at $T = 0$. This may indicate that the chiral symmetry is strongly broken, even for high temperatures below the pseudocritical point. Second, the scalar S , axial vector A ,

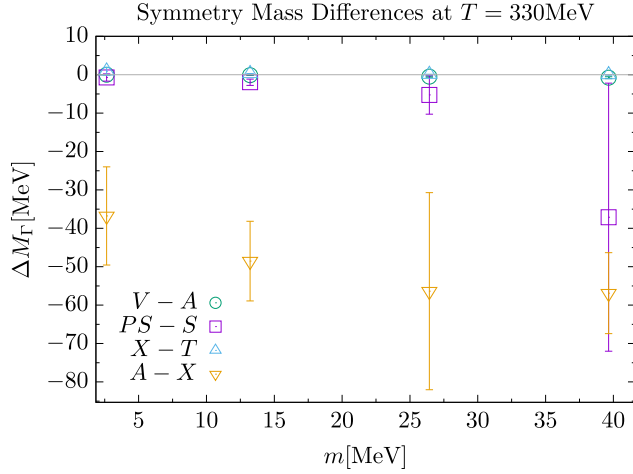


FIG. 10. Quark mass dependence of Δm_{V-A} (circles), Δm_{PS-S} (squares), Δm_{X-T} (upward triangles), and Δm_{A-X} (downward triangles) at $T = 330$ MeV. Compared to the former three probing $SU(2)_L \times SU(2)_R$ and $U(1)_A$ symmetries, which are consistent with zero with error bars ~ 1 MeV, the chiral-spin probe Δm_{A-X} is clearly nonzero; ~ -40 MeV and the associated chiral-spin symmetry $SU(2)_{CS}$ is only approximate, $|\Delta m_{X-A}|/T \sim 0.12$.

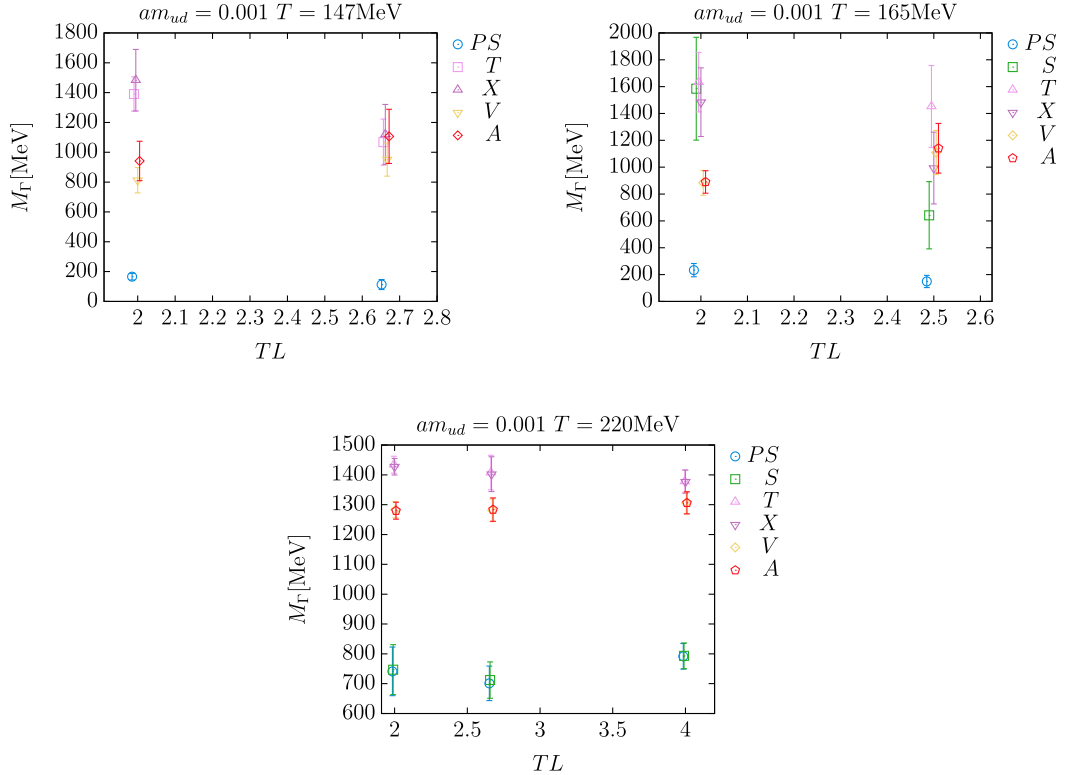


FIG. 11. The lattice size L vs. screening masses at $T = 147, 165$, and 220 MeV is plotted. We do not see any significant finite volume dependence beyond 2σ .

and axial tensor X_t masses all reduce above $T \sim T_c$ to values similar to their $SU(2)_L \times SU(2)_R$ or $U(1)_A$ partners PS , V , and T_t . Then above T_c , all the channels monotonically increase and appear to converge to twice the ground state Matsubara mass $2\pi T$.

Taking the difference of the screening masses between the aforementioned channel partners, we have examined various symmetries of QCD at high temperatures. The standard chiral $SU(2)_L \times SU(2)_R$ is restored at the critical temperature $T_c \sim 165$ MeV (observable from the degeneracy of the V and A channels), consistent with the critical temperature estimated from the chiral susceptibility in previous work. The axial $U(1)_A$ symmetry behaves similarly, with degeneracy between the T_t and X_t as well as PS and S at or close to T_c although the signals are noisier. The mass difference between these three channel pairs at $T \sim 2T_c$ is consistent with zero with quite small uncertainty less than 1 MeV, or 1% of the temperature. In contrast to the above two symmetries, the chiral spin $SU(2)_{CS}$ probed by A and X_t is only approximate at $T \sim 2T_c$, showing a breaking ~ 40 MeV, which is 12% of the temperature.

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the Multidisciplinary Cooperative Research Program in CCS, University of Tsukuba (xg17i032 and xg18i023), the supercomputer Fugaku provided by the RIKEN Center for Computational Science (hp200130, hp210165, hp210231, hp220279, and hp230323), Wisteria/BDEC-01 Odyssey at JCAHPC (HPCI: hp220093, hp230070, and hp240050 MCRP: wo22i038), and Polarie and Grand Chariot at Hokkaido University (hp200130). We used the Japan Lattice Data Grid (JLDG) for storing a part of the numerical data generated for this work. This work is supported in part by the Japanese Grant-in-Aid for Scientific Research (No. JP26247043, No. JP18H01216, No. JP18H04484, and No. JP22H01219), and by MEXT as “Priority Issue on Post-K computer” (Elucidation of the Fundamental Laws and Evolution of the Universe) and by the Joint Institute for Computational Fundamental Science (JICFuS).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: RAW CORRELATORS

In this appendix, we summarize the results for the raw correlator with the lightest quark mass $m = 0.001$ (2.6 MeV) at the simulated temperatures. Some data having negative values are neglected. Correlators for two volumes at the lowest temperatures in our study are shown in Fig. 12, while the largest aspect ratio plots for temperatures above T_c are shown in Fig. 13.

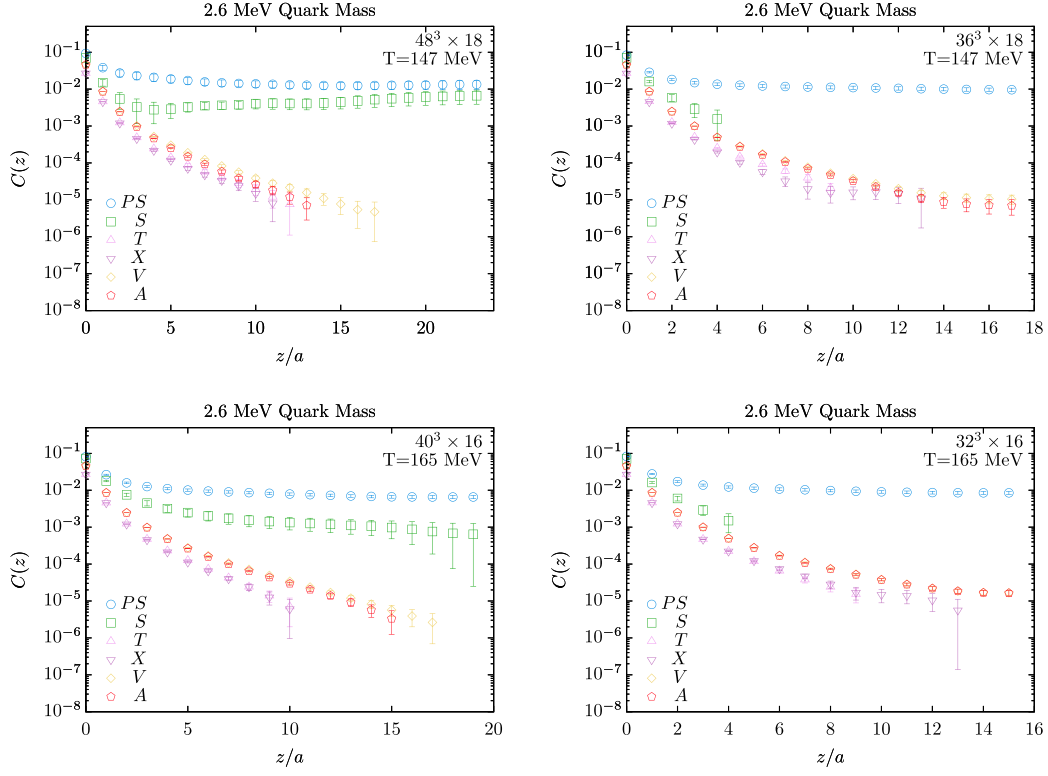


FIG. 12. Plots of the raw correlators for $T = 147$ MeV and 165 MeV. All plot of the correlators correlators are averaged over the three spatial directions and symmetrized with respect to the reflection of the axis $z \rightarrow -z$.

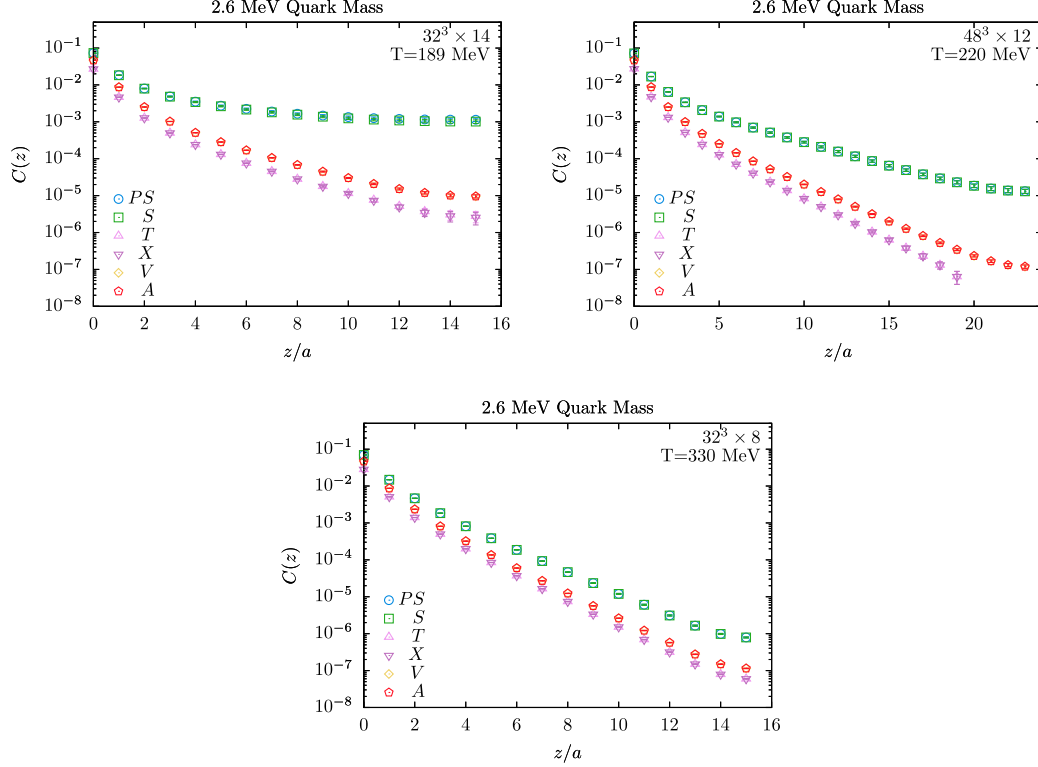


FIG. 13. The three higher-temperature raw correlator plots like Fig. 12. These plots are also symmetrized around the midpoint of the correlator.

APPENDIX B: SCREENING MASS PLOTS FOR $am > 0.0010$

As an additional check of the quark mass dependence on the meson screening mass, we present the same plots as Fig. 4 but with three different quark masses in Fig. 14.

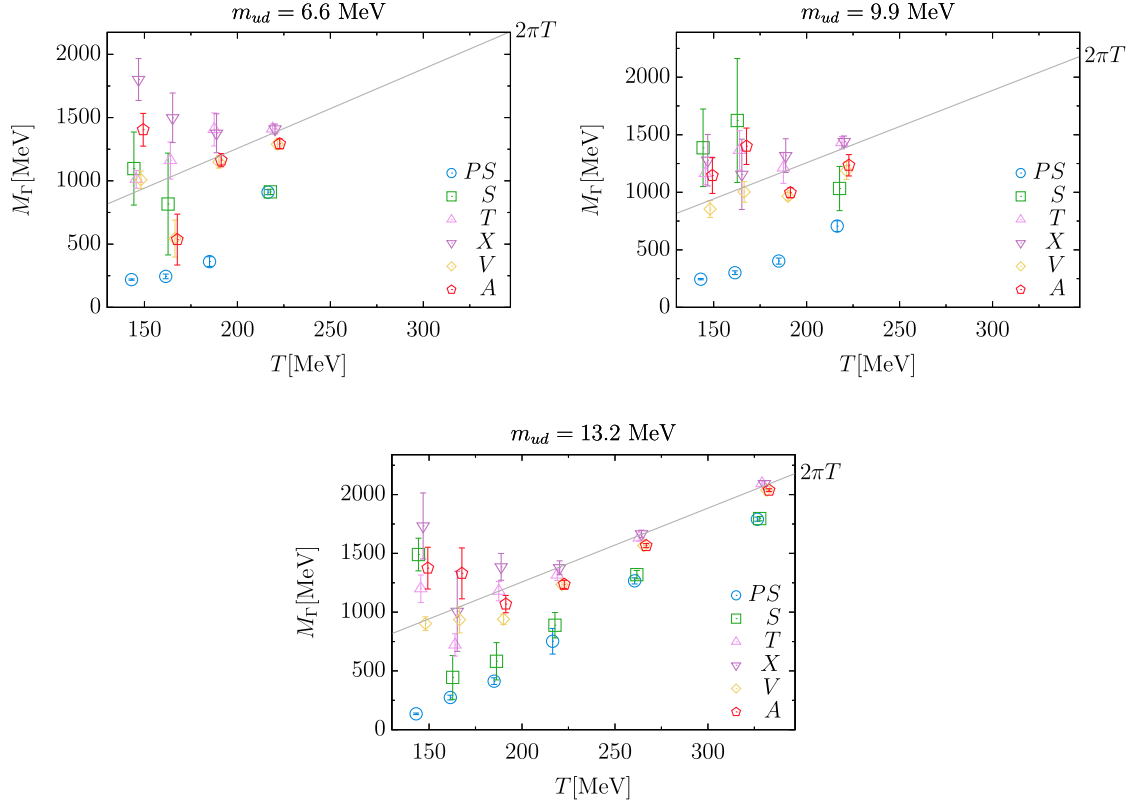


FIG. 14. These plots correspond to the three other quark mass ensembles above the physical point in our study $am = 0.0025(6.6 \text{ MeV})$, $0.00375(9.9 \text{ MeV})$, and $0.0050(13.2 \text{ MeV})$.

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