

Alternative left-right scenario: Unitarity, vacuum stability, and RG evolution

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We study the theoretical constraints on the scalar sector of the alternative left-right model (ALRM), an E_6 -motivated extension of the Standard Model based on the gauge group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)_{B-L}$, supplemented by a global $U(1)_S$ symmetry. We derive the complete set of tree-level perturbative unitarity constraints on the model, resulting in 14 independent conditions on the quartic scalar couplings. When combined with the boundedness-from-below conditions and the requirement of positive-definite scalar mass-squared eigenvalues, this yields significantly more stringent restrictions on the parameter space than either set alone. We then perform a one-loop renormalization group analysis, evolving the model parameters up to a high-energy cutoff scale, and requiring that the vacuum stability, the unitarity, and the perturbativity conditions are preserved throughout. The renormalization group evolution is found to restrict the allowed parameter space considerably beyond the tree-level bounds, with the constraints on the quartic couplings becoming more stringent as the cutoff scale is raised. Consequently, the physical scalar masses in the model acquire upper bounds. For the right-hand symmetry breaking scale, $v_R = 10$ TeV and requiring theoretical consistency up to 10^{16} GeV, we obtain $m_{H^\pm_1} \lesssim 6.5$ TeV, $m_{H^\pm_2} \lesssim 1.5$ TeV, and $m_{H^0_1} \simeq m_{A_1} \lesssim 1.5$ TeV, with all bounds scaling with v_R . These findings offer a predictive and falsifiable framework for searches of the extended Higgs sector of the ALRM at the current and future collider experiments.

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I. INTRODUCTION

While being remarkably successful in describing the dynamics of elementary particles at the electroweak scale, the Standard Model (SM) of particle physics fails to account for several critical experimental and theoretical shortcomings. The observation of nonzero neutrino masses, the overwhelming evidence for dark matter, and the baryon asymmetry of the Universe are among the most compelling experimental shortcomings that point towards the incompleteness of the SM. On the theoretical side, concerns regarding the stability of the electroweak vacuum and the

gauge hierarchy problem remain fundamental issues that the Standard Model has yet to address adequately. Among several attempts to resolve these difficulties, phenomenological models hold a central place. A strongly prevalent notion among particle physicists is that larger symmetries unify the dynamics at some higher energy scale, and break down naturally to the SM gauge group at the electroweak scale. Along this line of thought, several grand unified symmetries have been proposed and studied. Of particular interest in this work is the exceptional group E_6 , whose symmetry breaking proceeds through multiple steps. It was observed decades ago that this breaking chain admits an intermediate stage where the dynamics is left-right symmetric. This allowed a natural explanation for the otherwise puzzling observation of maximal parity violation in weak interactions, which is introduced as an input within the SM. As a bonus, this scenario also provides a natural mechanism for generating nonzero neutrino masses via seesaw mechanism.

The conventional left-right symmetric model (LRSB) features an additional $SU(2)_R$ gauge group, under which the right-handed up-type quarks and the right-handed

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down-type quarks pair up as doublets [1–4]. Similarly, the right-handed charged lepton is partnered with a right-handed neutrino. All these fields, including the right-handed neutrinos, are accommodated within the 27-plet of E_6 . The LRSM is subject to stringent constraints from flavor physics, primarily due to the presence of tree-level flavor-changing neutral current (FCNC) interactions mediated by the scalar sector. Consequently, current experimental data push the scale of left-right symmetry breaking into the multi-TeV regime, beyond the reach of the LHC but potentially accessible at future high-energy colliders. Although the conventional minimal LRSM admits a warm dark matter scenario based on a keV-scale right-handed neutrino, its realization relies on a highly constrained flavor structure and cosmological evolution, and the viable parameter space is rather restricted [5–9]. Reference [10] proposes an alternative decaying scalar dark matter scenario within the minimal LRSM that relies on an exceptionally large hierarchy between the scalar mass and the left-right symmetry breaking scale, making the framework somewhat fine-tuned. Similarly, although successful leptogenesis has been realized in the minimal LRSM through several mechanisms, including resonant leptogenesis, these scenarios generally require specific assumptions regarding the Yukawa sector, heavy-neutrino spectrum, and are therefore realized only in restricted regions of parameter space [8,11,12].

However, an alternative realization of the left-right gauge symmetry, known as the alternative left-right model (ALRM), emerges naturally from the breaking of the exceptional grand unified group E_6 . Under its maximal subgroup, $E_6 \supset SU(3)_C \otimes SU(3)_L \otimes SU(3)_H$, different embeddings of the $SU(2)$ subgroup of $SU(3)_H$ lead to distinct low-energy left-right theories. The conventional LRSM is obtained by identifying $SU(2)_H$ with the usual $SU(2)_R$, whereas the ALRM corresponds to an alternative embedding, denoted by $SU(2)_{R'}$ [13–16]. Consequently, the right-handed down-type quark d_R no longer forms an $SU(2)_{R'}$ doublet with the right-handed up-type quark u_R . Instead, the exotic quark d'_R assumes this role, leaving d_R as a singlet under both $SU(2)_L$ and $SU(2)_{R'}$. Thus, unlike the conventional LRSM, the ALRM does not arise from imposing an exact left-right parity symmetry as a fundamental symmetry of the intermediate left-right gauge theory. Rather, its left-right gauge structure originates from an alternative embedding of the unified E_6 symmetry, leading to a distinct phenomenological framework. To distinguish this behavior, we denote the gauge group of ALRM as $SU(2)_L \times SU(2)_{R'}$. An analogous rearrangement occurs in the leptonic sector, where the right-handed neutrino is treated as a singlet under all gauge symmetries. As discussed in detail in the subsequent sections, this rearrangement permits the introduction of a global symmetry that forbids the otherwise dangerous tree-level FCNC interactions. Remarkably, this same symmetry

provides a dark matter candidate, stabilized by the remnant R -parity surviving in the symmetry-broken phase.

The scalar sector of ALRM consists of one Higgs bidoublet Φ and two Higgs doublets denoted as $\chi_{L,R}$ (one each under $SU(2)_L$ and $SU(2)_{R'}$, respectively). The richer scalar field content of this scenario admits a stable vacuum configuration more readily than the SM. A systematic study of the scalar potential was carried out in Ref. [17], where the parameters of the tree-level potential were constrained by requiring boundedness from below (BFB). Expressing the physical scalar masses in terms of these parameters and the vacuum expectation values (VEVs), the additional requirement of positive-definite mass-squared eigenvalues imposes a further set of constraints, collectively restricting the viable parameter space of the model. Similar analyses of stable vacuum structure in multiscalar potentials [18–24] and in the conventional LRSM [25–30] have been extensively explored in the literature. Another key theoretical constraint on the Higgs sector arises from perturbative unitarity [31–34]. This requirement, coming from the unitarity of the S-matrix, implies that partial-wave scattering amplitudes must remain bounded in magnitude. These constraints are being used to put bounds [33,34] on the mass of Higgs boson. The perturbative unitarity constraints have not yet been examined in the context of the ALRM, and we aim to address them in this work.

Furthermore, the previous studies in the ALRM scenario have demonstrated the model's potential implications for neutrinoless double β decay and leptogenesis in Ref. [35]. It has also been shown that exotic fermions and Higgs bosons can indirectly affect rare top decays [36], as well as contribute to rare lepton-flavor-violating processes and the anomalous magnetic moment of the muon, a_μ [37,38]. Building on this, in the present work, we impose all tree level theoretical constraints, namely the potential being bounded from below, perturbative unitarity constraints, and the perturbativity limits of the couplings, and check their stability under one-loop renormalization group evolution. By tracking the running of the couplings and mass parameters, we identify the energy scales up to which these conditions continue to hold. Bringing these elements together in a single framework, we then determine the corresponding constraints on the parameter space of the ALRM.

The organization of the paper is as follows: in Sec. II, we present briefly the relevant details of the model framework, followed by a discussion of the theoretical constraints in Sec. III. This section also includes a numerical analysis identifying the region of parameter space compatible with the BFB and perturbative unitarity constraints. In Sec. IV, we study the renormalization group evolution (RGE) of the couplings in detail and numerically analyze its effect on these constraints. Finally, we conclude our findings in Sec. V.

II. MODEL FRAMEWORK

In this section, we briefly review the ALRM framework and refer the interested reader to Ref. [16] for a more detailed discussion. The ALRM framework is based on the gauge group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)_{B-L}$, supplemented by an additional global symmetry $U(1)_S$. The complete scalar and fermionic field content of the model, along with the corresponding gauge quantum

numbers, is given in Table I. The electric charges of the particle fields can be derived from the generalized Gell-Mann-Nishijima relation $Q = T_{3R'} + T_{3L} + Y_{B-L}$, where Y_{B-L} is the $U(1)_{B-L}$ charge, and T_{3L} and $T_{3R'}$ are the third-component isospin quantum numbers under $SU(2)_L$ and $SU(2)_{R'}$, respectively.

The scalar potential of the ALRM, which determines the Higgs spectrum and its mixings, is given by

$$\begin{aligned} V_{\text{scalar}} = & -\mu_1^2 \text{Tr}[\Phi^\dagger \Phi] - \mu_2^2 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) + \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2 + \lambda_2 \text{Tr}[\Phi^\dagger \tilde{\Phi}] \text{Tr}[\tilde{\Phi}^\dagger \Phi] \\ & + \lambda_3 [(\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2] + 2\lambda_4 (\chi_L^\dagger \chi_L)(\chi_R^\dagger \chi_R) + 2\alpha_1 \text{Tr}[\Phi^\dagger \Phi] (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) \\ & + 2\alpha_2 [\chi_L^\dagger \Phi \Phi^\dagger \chi_L + \chi_R^\dagger \Phi \Phi^\dagger \chi_R] + 2\alpha_3 [\chi_L^\dagger \tilde{\Phi} \tilde{\Phi}^\dagger \chi_L + \chi_R^\dagger \tilde{\Phi} \tilde{\Phi}^\dagger \chi_R] + \mu_3 [\chi_L^\dagger \Phi \chi_R + \chi_R^\dagger \Phi \chi_L], \end{aligned} \quad (1)$$

where $\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2$ and $\chi_{L,R} = i\sigma_2 \chi_{L,R}^*$. Here, we assume a left-right symmetry, and further consider all couplings to be real for simplicity. In the following, we shall introduce the notation $\alpha_{12} = \alpha_1 + \alpha_2$ and $\alpha_{13} = \alpha_1 + \alpha_3$, acknowledging the fact that the operator in α_1 term can be expressed as sum of the operators in α_2 and α_3 terms.

The spontaneous breaking of the ALRM gauge symmetry down to electromagnetism proceeds in two steps, driven by a vacuum configuration in which all neutral

scalar components, with the exception of ϕ_1^0 , acquire nonzero VEVs, as follows:

$$\begin{aligned} \langle \phi \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 0 & v_u \end{pmatrix}, \quad \langle \chi_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \\ \text{and } \langle \chi_R \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_R \end{pmatrix}. \end{aligned} \quad (2)$$

TABLE I. Particle content of the ALRM with their respective gauge structures. The generalized lepton number is defined as $L = S + T_{3R'}$ with S being $U(1)_S$ charge, and the R -parity as $R = (-1)^{3B+L+2s}$, where B is the baryon number and s is the spin.

Particles	$SU(3)_C$	$SU(2)_L$	$SU(2)_{R'}$	$U(1)_{B-L}$	$U(1)_S$	L	R
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	1	$\frac{1}{6}$	0	0	+
$Q_R = \begin{pmatrix} u_R \\ d'_R \end{pmatrix}$	3	1	2	$\frac{1}{6}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} + \\ - \end{pmatrix}$
d'_L	3	1	1	$-\frac{1}{3}$	-1	-1	-
d_R	3	1	1	$-\frac{1}{3}$	0	0	+
$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	1	2	1	$-\frac{1}{2}$	1	1	+
$L_R = \begin{pmatrix} n_R \\ e_R \end{pmatrix}$	1	1	2	$-\frac{1}{2}$	$\frac{3}{2}$	$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$	$\begin{pmatrix} - \\ + \end{pmatrix}$
n_L	1	1	1	0	2	2	-
ν_R	1	1	1	0	1	1	+
$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$	1	2	2*	0	$-\frac{1}{2}$	$\begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} - & + \\ - & + \end{pmatrix}$
$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}$	1	2	1	$\frac{1}{2}$	0	0	$\begin{pmatrix} + \\ + \end{pmatrix}$
$\chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$	1	1	2	$\frac{1}{2}$	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} - \\ + \end{pmatrix}$

The first step, spontaneous breakdown of $SU(2)_{R'} \otimes U(1)_{B-L} \otimes U(1)_S$ to the SM gauge group, proceeds in such a way that the generalized combination $L = S + T_{3R'}$, where S is the $U(1)_S$ charge, remains unbroken. This is achieved through the neutral component of an $SU(2)_{R'}$ doublet χ_R charged under $U(1)_S$ acquiring a nonzero VEV v_R . The second step, the breaking of the electroweak symmetry down to electromagnetism, is driven by the VEVs v_u and v_L of the bidoublet Φ and the $SU(2)_L$ doublet χ_L , respectively. The vanishing of $\langle \phi_1^0 \rangle$ is protected by the conservation of the generalized lepton number L , whereas ϕ_2^0 and χ_L^0 having $L = 0$ are allowed to acquire non-zero VEVs.

Consequently, ϕ_1^0 is prevented from mixing with any other scalar fields, resulting in two degenerate physical states, H_1^0 and A_1 , corresponding the scalar and pseudo-scalar degrees of freedom of ϕ_1^0 with mass, as follows:

$$m_{H_1^0}^2 = m_{A_1}^2 = 2v_u^2\lambda_2 - (\alpha_{12} - \alpha_{13})(v_L^2 + v_R^2) - \frac{v_L v_R \mu_3}{\sqrt{2}v_u}. \quad (3)$$

The pseudoscalar degrees of freedom of ϕ_2^0 , χ_L^0 , and χ_R^0 mix together to become the two would-be the Goldstone bosons absorbed by the neutral gauge bosons Z_L and Z_R , along with the orthogonal combination manifesting as a physical pseudoscalar particle A_2 with mass

$$m_{A_2}^2 = -\frac{\mu_3}{\sqrt{2}} \frac{v_u^2 v_R^2 + v_u^2 v_L^2 + v_L^2 v_R^2}{v_L v_R v_u}. \quad (4)$$

This immediately requires $\mu_3 < 0$. Furthermore, from Eq. (3), the splitting $\alpha_{12} - \alpha_{13}$ is required to be at most $\mathcal{O}(10^{-2})$, as we shall explicitly demonstrate in the numerical analysis of the subsequent sections. Similarly, the scalar degrees of freedom of ϕ_2^0 , χ_L^0 , and χ_R^0 mix together to produce three physical particles, denoted as h , H_2^0 , and H_3^0 . The lightest of these, h , emerges as the SM Higgs candidate, with its properties arranged to match those of the observed SM Higgs properties. The remaining two physical states, H_2^0 and H_3^0 , are considerably heavier, with masses being proportional to v_R . For a detailed discussion of the scalar sector, we refer the reader to Refs. [16,39]. In the charged scalar sector, the physical massive charged Higgs bosons $H_1^\pm$ and $H_2^\pm$, together with the two massless Goldstone bosons $G_1^\pm$ and $G_2^\pm$ which are absorbed by the W_L and W_R gauge bosons, respectively, are given by

$$\begin{pmatrix} \phi_1^\pm \\ \chi_L^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} H_1^\pm \\ G_1^\pm \end{pmatrix}, \quad \begin{pmatrix} \phi_2^\pm \\ \chi_R^\pm \end{pmatrix} = \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} H_2^\pm \\ G_2^\pm \end{pmatrix}, \quad (5)$$

where $\tan\beta = \frac{v_u}{v_L}$ and $\tan\zeta = \frac{v_u}{v_R}$. The masses of the charged Higgs bosons $H_1^\pm$ (R -parity even) and $H_2^\pm$ (R -parity odd) are given by

$$m_{H_1^\pm}^2 = -v^2 \left[(\alpha_{12} - \alpha_{13}) + \frac{\mu_3 v_R}{\sqrt{2}v_u v_L} \right], \quad (6)$$

and

$$m_{H_2^\pm}^2 = -v'^2 \left[(\alpha_{12} - \alpha_{13}) + \frac{\mu_3 v_L}{\sqrt{2}v_u v_R} \right], \quad (7)$$

respectively, where $v^2 = v_u^2 + v_L^2$ and $v'^2 = v_R^2 + v_u^2$. This again requires $(\alpha_{12} - \alpha_{13}) < \mathcal{O}(10^{-2})$, leaving the second term proportional to μ_3 as the dominant one. Since $v' \gg v$, and in most scenarios we consider $v_u \gg v_L$, the charged Higgs, $H_1^\pm$ is typically heavy, while $H_2^\pm$ lies in the sub-TeV to TeV range over a significant portion of the model parameter space. For appropriate parameter choices, the light charged Higgs boson can be long-lived, with its decay potentially yielding a disappearing track signature at the LHC [40], which is a distinctive collider signal of the model.

In the gauge boson sector, the breaking of left-right symmetry generates masses for the gauge bosons and induces mixing among them. Due to the conservation of the generalized lepton number, which prevents $\langle \phi_1^0 \rangle$ from being nonzero, the charged W_L and W_R bosons do not mix. The masses of these gauge bosons are given by

$$M_{W_L}^2 = \frac{g_L^2}{4}(v_u^2 + v_L^2), \quad \text{and} \quad M_{W_R}^2 = \frac{g_R^2}{4}(v_u^2 + v_R^2). \quad (8)$$

The charge-neutral gauge bosons, however, do undergo mixing, and the diagonalization of the mixing matrix leads to the physical mass eigenstates (A_μ, Z_μ, Z'_μ) . The $Z_L - Z_R$ mixing in Z' is strongly constrained experimentally, and in the limit where this mixing vanishes, the mass eigenvalues are obtained analytically, with the photon remaining massless. The masses of Z and Z' are given by

$$M_Z^2 = \frac{g_L^2(v_u^2 + v_L^2)}{4\cos^2\theta_W}, \quad \text{and} \quad M_{Z'}^2 = \frac{1}{4}g_{BL}^2\sin^2\phi_W v_L^2 + \frac{g_R^2(\cos^4\phi_W v_u^2 + v_R^2)}{4\cos^2\phi_W}, \quad (9)$$

respectively, where g_L , g_R , and g_{BL} denote the gauge couplings corresponding to $SU(2)_L$, $SU(2)_{R'}$, and $U(1)_{B-L}$, respectively; θ_W is the Weinberg mixing angle; and ϕ_W is the corresponding mixing angle in the right-handed gauge sector. In the hierarchy $v_R \gg v_u \gg v_L$, the term involving g_R dominates the Z' mass, and one can reasonably approximate $M_{Z'} \approx M_{W_R}/\cos\phi_W$. Since W_R does not decay exclusively to the SM particles, it evades

direct search constraints at colliders. The Z' , on the other hand, is subject to direct search bounds from the collider experiments, which translate into indirect constraints on W_R through the above relation. For a detailed discussion of the LHC constraints on the gauge bosons of the ALRM, we refer the reader to Ref. [41].

Finally, turning to the Yukawa interactions in the ALRM framework, we have

$$-\mathcal{L}_Y = Y_{Q1} \bar{Q}_L \chi_L d_R + Y_{Q2} \bar{Q}_L \tilde{\Phi} Q_R + Y_{Q3} \bar{Q}_R \chi_R d'_L + Y_{L1} \bar{L}_L \Phi L_R + Y_{L2} \bar{L}_L \tilde{\chi}_L \nu_R + Y_{L3} \bar{L}_R \tilde{\chi}_R n_L + \text{H.c.}, \quad (10)$$

where the flavor indices are suppressed, and the dual fields are defined as $\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2$ and $\tilde{\chi}_{L,R} = i\sigma_2 \chi_{L,R}^*$ with σ_2 being the second Pauli matrix. With the VEVs of scalars being as aforementioned, the fermion mass matrices are given by

$$M_u = \frac{v_u}{\sqrt{2}} Y_{Q2}, \quad M_d = \frac{v_L}{\sqrt{2}} Y_{Q1}, \quad M_\ell = \frac{v_u}{\sqrt{2}} Y_{L1}, \\ M_\nu = \frac{v_L}{\sqrt{2}} Y_{L2}, \quad M_{d'} = \frac{v_R}{\sqrt{2}} Y_{Q3}, \quad M_n = \frac{v_R}{\sqrt{2}} Y_{L3}. \quad (11)$$

In the present paper, our focus is the scalar potential and the theoretical constraints arising from the considerations of the vacuum stability and the perturbative unitarity. We devote the remainder of the article to this discussion.

$$\lambda_1 \geq 0, \lambda_4 \geq \lambda_3 \geq 0, \\ \lambda_1 + 2\lambda_2 \geq 0, \lambda_2 \leq 0, \\ \alpha_{12} - \alpha_{13} \geq 0,$$

A remark regarding $(\alpha_{12} - \alpha_{13})$ is in order here. The copositivity condition requires $(\alpha_{12} - \alpha_{13}) \geq 0$, while the positivity of $m_{H_1^0/A_1}^2$ and $m_{H_2^\pm}^2$ naively prefers $(\alpha_{12} - \alpha_{13}) \leq 0$ [Eqs. (3) and (7)]. On this basis, Ref. [17] concludes that $\alpha_{12} - \alpha_{13} = 0$. We investigate this further and find that a small positive value of $(\alpha_{12} - \alpha_{13})$ is in fact consistent with $m_{H_1^0/A_1}^2, m_{H_2^\pm}^2 > 0$, its allowed range depending on the other parameters of the model, in particular on μ_3 and $\tan\beta$. To demonstrate this, in Fig. 1, we plot $m_{H_2^\pm}^2$ as a function of $(\alpha_{12} - \alpha_{13})$ for fixed $\tan\beta = 10$ and $v_R = 10$ TeV, scanning μ_3 randomly over

III. THEORETICAL CONSTRAINTS

In this section, we discuss the theoretical constraints on the parameter space of the ALRM framework, starting with the tree-level vacuum stability constraints, followed by the constraints imposed by the perturbative unitarity.

A. Vacuum stability constraints

The stability of the vacuum of the ALRM scalar potential has been investigated at tree level in Ref. [17] using copositivity criteria. For the scalar potential to be bounded from below, it must remain positive in all field directions as the fields tend to infinity. This immediately requires the quartic couplings involving only a single field to be positive definite. However, the couplings corresponding to mixed quartic terms involving more than one field may be negative, provided the negative contribution is compensated by the single-field quartic couplings in all field directions. To illustrate this, note that the operator multiplying λ_2 can be expressed in terms of components as $\text{Tr}[\Phi^\dagger \tilde{\Phi}] \text{Tr}[\tilde{\Phi}^\dagger \Phi] = |\phi_1^0 \phi_2^0 - \phi_1^+ \phi_2^-|^2$, which involves products of two distinct fields, and whose negative contribution can be compensated by the single-field quartic terms such as $(\phi_1^0)^4$ and $(\phi_2^0)^4$, thus permitting $\lambda_2 < 0$. This leads to the copositivity conditions, which are necessary and sufficient conditions for the scalar potential to be bounded from below. A detailed and systematic derivation of these conditions for the ALRM scalar potential is presented in Ref. [17]. The constraints arising from the copositivity conditions, combined with the minimization conditions for the desired global vacuum, impose the following restrictions on the quartic couplings:

$$\alpha_{12} + \sqrt{\lambda_1 \left(\frac{\lambda_3 + \lambda_4}{2} \right)} \geq 0, \quad \alpha_{12} + \sqrt{\lambda_1 \lambda_3} \geq 0, \\ \alpha_{13} + \sqrt{\lambda_1 \left(\frac{\lambda_3 + \lambda_4}{2} \right)} \geq 0, \quad \alpha_{13} + \sqrt{\lambda_1 \lambda_3} \geq 0. \quad (12)$$

the range $[-5000, -10]$ GeV (left panel), and for fixed $\mu_3 = -1000$ GeV and $v_R = 10$ TeV, varying $\tan\beta$ between 2 and 50 (right panel). Negative values of $(\alpha_{12} - \alpha_{13})$ are excluded by the boundedness-from-below condition. It is evident from the figure that smaller values of $\tan\beta$ and larger $|\mu_3|$ permit a larger positive splitting $(\alpha_{12} - \alpha_{13})$.

B. Perturbative unitarity constraints

At tree level, another theoretical constraint on the ALRM parameter space emanates from the perturbative

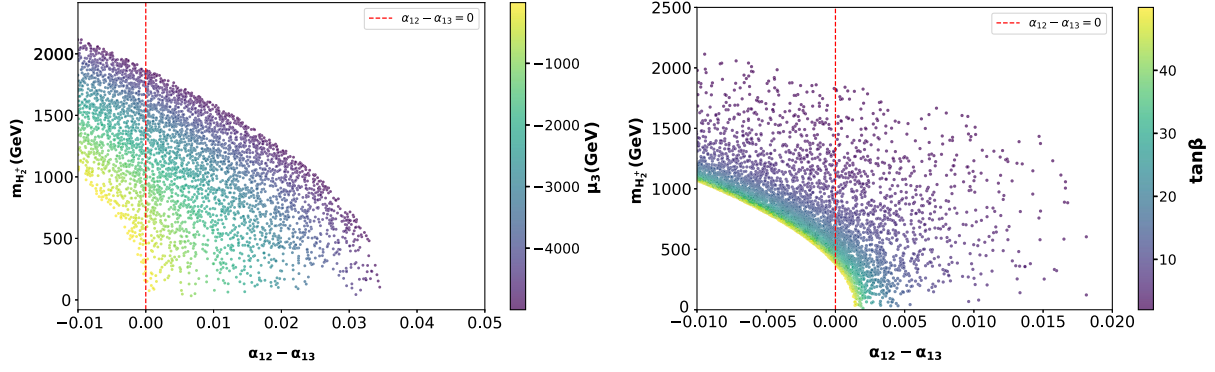


FIG. 1. Dependence of the charged Higgs mass $m_{H^\pm}$ on $(\alpha_{12} - \alpha_{13})$. Left: Impact of μ_3 for fixed $\tan\beta = 10$ and $v_R = 10$ TeV. Right: Sensitivity of $\tan\beta$ for fixed $\mu_3 = -1000$ GeV and $v_R = 10$ TeV. The negative values of $(\alpha_{12} - \alpha_{13})$ are disallowed for the potential to be bounded from below.

unitarity of the $2 \rightarrow 2$ scalar scattering amplitudes in the high-energy limit. It demands that the zeroth partial wave amplitude a_0 must satisfy $|\text{Re}(a_0)| \leq \frac{1}{2}$ [42]. In general, the invariant amplitude of the $2 \rightarrow 2$ scattering process from the partial wave decomposition could be given by

$$\mathcal{M}(s, \theta) = 16\pi \sum_{J=0}^{\infty} a_J(s)(2J+1) P_J(\cos \theta), \quad (13)$$

where P_J denotes the J th degree Legendre polynomial with J denoting the total orbital angular momentum of the final state. In addition, $\sqrt{s}$ denotes the c.m. energy and θ is the scattering angle. In the high-energy limit, the tree-level scalar scattering amplitudes arising from the quartic interactions are angle independent, such that only the $J=0$ partial wave contributes. Consequently, the unitarity condition reduces to $|\mathcal{M}| \leq 8\pi$. Finally, letting $\mathcal{M}$ denote all the coupled $2 \rightarrow 2$ scalar channels as a matrix, the perturbative unitarity constraints on the ALRM parameter space are obtained by requiring the eigenvalues of this scalar scattering matrix to satisfy

$$|\mathcal{M}_i| \leq 8\pi. \quad (14)$$

Given the extended scalar sector of the ALRM framework, characterized by nine independent parameters in the scalar potential, the constraints arising from the perturbative unitarity are highly nontrivial. In total, there are 136 independent $2 \rightarrow 2$ scattering processes involving scalar fields in the model. These processes can be systematically classified according to the total electric charge of the initial state: the charge-neutral, the singly charged, and the doubly charged sectors. We construct the matrix $\mathcal{M}$ in the basis of all

possible two-particle states and arrange it in a block-diagonal form, where each block corresponds to a definite charge sector. The full matrix decomposes into the neutral charged block of dimension 52, the singly charged blocks of dimension 32 each (for the positive and the negative charges, related by Hermitian conjugation), and the doubly charged blocks of dimension 10 each (again related by Hermitian conjugation). Altogether, these blocks account for the full 136-dimensional scattering matrix. Rotating to the diagonal basis, one obtains 136 eigenvalues corresponding to the coupled-channel system. Owing to the symmetry structure of the scalar potential, there is significant degeneracy among the eigenvalues of the scalar scattering matrix, and only 14 independent constraints from the perturbative unitarity were obtained¹:

$$e_{1,2}: |\alpha_{12,13}| \leq 4\pi, \quad (15)$$

$$e_{3,4,5}: |\lambda_{1,3,4}| \leq 4\pi, \quad (16)$$

$$e_6: |\alpha_{12} - 2\alpha_{13}| \leq 4\pi, \quad (17)$$

$$e_7: |2\alpha_{12} - \alpha_{13}| \leq 4\pi, \quad (18)$$

$$e_8: |\lambda_1 + 2\lambda_2| \leq 4\pi, \quad (19)$$

$$e_9: |\lambda_1 + 4\lambda_2| \leq 4\pi, \quad (20)$$

$$e_{10}: |2\lambda_4 - 3\lambda_3| \leq 4\pi, \quad (21)$$

¹The *Mathematica* files containing the scattering matrices and the corresponding eigenvalues are available at <https://github.com/hdekal/Unitarity-constraints-file>.

$$e_{11,12}: \left| \lambda_1 - 2\lambda_2 + \lambda_3 \pm \sqrt{(\lambda_1 - 2\lambda_2 - \lambda_3)^2 + 8(\alpha_{12} - \alpha_{13})^2} \right| \leq 8\pi, \quad (22)$$

$$e_{13,14}: \left| 5\lambda_1 + 2\lambda_2 + 3\lambda_3 + 2\lambda_4 \pm \sqrt{(5\lambda_1 + 2\lambda_2 - 3\lambda_3 - 2\lambda_4)^2 + 16(\alpha_{12} + \alpha_{13})^2} \right| \leq 8\pi. \quad (23)$$

The independent unitarity constraints on the quartic couplings are labeled as e_i ($i = 1, \dots, 14$) for ease of reference. For the paired eigenvalues, e_{11} and e_{13} correspond to the positive branch of the square root, while e_{12} and e_{14} correspond to the negative branch. These results have been independently verified using SARAH [43], a general-purpose framework for studying beyond the Standard Model (BSM) models, and anyPUB [44], a dedicated package for deriving perturbative unitarity constraints.

C. Numerical study

We perform a random scan over the parameter space to identify regions consistent with the vacuum stability, the perturbative unitarity, and the requirement that all physical scalar mass-squared eigenvalues are positive. Of the nine couplings in the scalar potential, four parameters (μ_1, μ_2, λ_1 , and λ_4) are traded for the vacuum expectation values v_R , $v_u = v \sin \beta$, $v_L = v \cos \beta$ (see Appendix A), and the SM-like Higgs mass m_h . This leaves six independent parameters:

$$\tan \beta, \lambda_2, \lambda_3, \alpha_{12}, \alpha_{13}, \text{ and } \mu_3. \quad (24)$$

In our scan, we fix $v_R = 10$ TeV and vary the remaining parameters over the ranges given as

$$\begin{aligned} \alpha_{12}, \alpha_{13} &\in [0, 4\pi], \\ \lambda_2 &\in [-4\pi, 0], \\ \lambda_3 &\in [0, 4\pi], \\ \tan \beta &\in [2, 50], \\ \mu_3 &\in [-10^4, -10] \text{ GeV}. \end{aligned} \quad (25)$$

The lower bound $\tan \beta = 2$ is chosen to reproduce the top quark mass with a Yukawa coupling $Y_{Q2} \sim 1$, using $m_t = \frac{1}{\sqrt{2}} Y_{Q2} v \tan \beta \simeq 172.5$ GeV. The upper bound $\tan \beta = 50$ corresponds to $v_L \sim 5$ GeV, ensuring that the bottom quark mass can be generated with a Yukawa coupling of order unity. The range of μ_3 is selected such that the charged Higgs mass $m_{H_2^\pm}$ lies within 100 GeV–2000 GeV. The remaining couplings are scanned over the full range allowed by the perturbativity. The parameters $\tan \beta$ and μ_3 play a special role through their implicit appearance in λ_1 and λ_4 , which are determined via the scalar

mass spectrum. Since this dependence involves the neutral scalar mass eigenstates in a nontrivial manner, we refer the reader to Refs. [16,17,39] for more details.

Throughout this work, we fix $v_R = 10$ TeV unless stated otherwise. This is justified because all parameters entering the theoretical constraints, except the derived quantities λ_1 and λ_4 , are independent of v_R . Moreover, the dependence of λ_1 and λ_4 on v_R always appears in combination with μ_3 , and its effect is largely absorbed by scanning over μ_3 . We have explicitly verified that varying v_R does not significantly affect the resulting constraints. The results of the scan over 10^6 points are presented in Fig. 2, showing the correlations among the parameters. Points satisfying the vacuum stability conditions of Eq. (12) are shown in purple, while those satisfying the perturbative unitarity bounds of Eqs. (15)–(23) are shown in blue. In both cases, we have made sure that the mass-square relations are positive definite.

In the most parameter planes, the perturbative unitarity constraints are found to be more restrictive than the BFB conditions, with the corresponding allowed regions lying almost entirely within the BFB-allowed parameter space. A notable exception is the λ_1 – λ_2 plane, where the two sets of constraints are complementary and exclude distinct regions of parameter space, leaving only a partial overlap between the allowed regions. This demonstrates the importance of imposing all theoretical constraints simultaneously, as considering either set independently would overestimate the viable parameter space of the model.

A strong correlation between α_{12} and α_{13} is observed, which can be understood from the vacuum stability and the positivity conditions that permit only a small positive splitting ($\alpha_{12} - \alpha_{13}$), as discussed above. The scan results indicate approximate upper bounds on the quartic couplings,

$$\begin{aligned} (\alpha_{12}, \alpha_{13}) &\lesssim 1.4, \quad \lambda_1 \lesssim 3.0, \quad |\lambda_2| \lesssim 1.5, \\ \lambda_3 &\lesssim 2.5, \quad \text{and} \quad \lambda_4 \lesssim 2\pi, \end{aligned} \quad (26)$$

which arise from the combined effect of the perturbative unitarity, the vacuum stability, and the positivity of scalar mass-squared eigenvalues.

Within this allowed region, the parameters are further restricted by correlations. For instance, $\lambda_4 \leq 2\pi$ when $\lambda_3 = 0$, while Eq. (23) implies the relation $3\lambda_3 + 2\lambda_4 \lesssim 4\pi$, leading to an upper bound

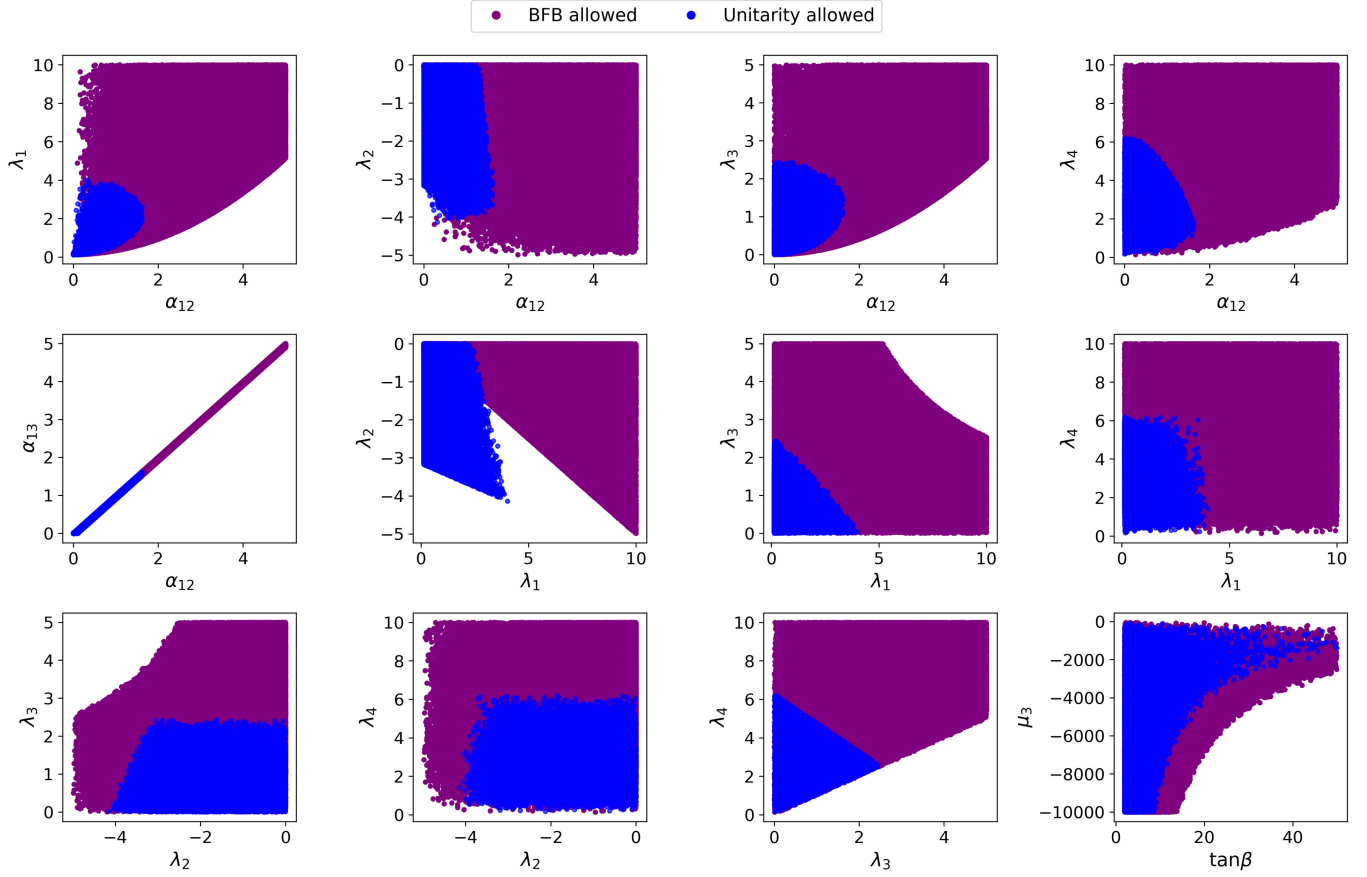


FIG. 2. Correlation plots showing the allowed region by the perturbative unitarity (blue) and the BFB conditions (purple). The scan range is as specified in Eq. (25), and we kept $v_R = 10$ TeV. In both cases, the positivity of all scalar mass-squared eigenvalues is maintained. This leads to absolute upper limits on the couplings: $(\alpha_{12}, \alpha_{13}) \lesssim 1.4$, $\lambda_1 \lesssim 3.0$, $|\lambda_2| \lesssim 1.5$, $\lambda_3 \lesssim 2.5$, and $\lambda_4 \lesssim 2\pi$.

$$\lambda_4^{\max} \simeq 2\pi - \frac{3}{2}\lambda_3, \quad (27)$$

which is clearly reflected in the λ_3 – λ_4 correlation plot in Fig. 2. In addition, the vacuum stability requires $\lambda_4 \geq \lambda_3$ [Eq. (12)], which further constrains $\lambda_3 \lesssim 2.5$ for the chosen ranges of μ_3 , $\tan\beta$, and v_R , with a corresponding value of $\lambda_4 \sim 2.5$. A pronounced correlation between $\tan\beta$ and μ_3 is also evident from Fig. 2. This inverse correlation arises from the relation $\lambda_4 - \lambda_3 = -\frac{\mu_3 \tan\beta}{\sqrt{2}v_R}$, together with the perturbative bounds on λ_3 and λ_4 .

Overall, the perturbative unitarity constraints, derived here for the first time in this framework, significantly

reduce the allowed parameter space beyond what is implied by the vacuum stability considerations alone.

IV. RENORMALIZATION GROUP EVOLUTION

The vacuum stability and the perturbative unitarity conditions derived in the previous section hold at tree level and at a fixed energy scale. To assess the theoretical consistency of the model across a wider range of energies, these conditions must be reexamined using the renormalization group evolution of the couplings. In the limit of large field values, the scalar potential is well approximated by its tree-level form with couplings replaced by their running counterparts, $C_i \rightarrow C_i(Q)$. The full set of running parameters comprises

$$C_i \in \begin{cases} \text{gauge couplings :} & g_3, g_L, g_R, g_{BL} \\ \text{Yukawa couplings :} & Y_{Q1}, Y_{Q2}, Y_{Q3}, Y_{L1}, Y_{L2}, Y_{L3} \\ \text{quadratic and trilinear couplings :} & \mu_1, \mu_2, \mu_3 \\ \text{quartic couplings :} & \lambda_1, \lambda_2, \lambda_3, \lambda_4, \alpha_{12}, \alpha_{13}. \end{cases} \quad (28)$$

Not all of these are independent. As discussed in Sec. III C, the parameters μ_1, μ_2 , and λ_4 are determined by the tadpole conditions (see Appendix A), and λ_1 is fixed in terms of the remaining parameters together with the requirement that the lightest CP -even scalar reproduces the observed SM Higgs mass, $m_h = 125$ GeV. The independent input parameters for the RGE analysis are therefore the gauge couplings, specified at the electroweak scale as discussed in Sec. IV A; the Yukawa couplings, expressed in terms of the physical fermion masses and the vacuum expectation values; and the scalar potential parameters

$$\lambda_2, \quad \lambda_3, \quad \alpha_{12}, \quad \alpha_{13}, \quad \mu_3, \quad (29)$$

all specified at the electroweak scale $v = 246$ GeV.

The RGEs of the model parameters are obtained by implementing the ALRM in `PyR@TE` [45,46], and the complete set of β -functions is given in Appendix B. The RG running is implemented in two stages, reflecting the two-step symmetry breaking pattern of the model. For simplicity, all the ALRM specific couplings are kept fixed below the scale v_R , while the full ALRM β -functions are employed above the symmetry breaking scale. At v_R , the gauge couplings are matched across the symmetry breaking threshold $SU(2)_{R'} \otimes U(1)_{B-L} \rightarrow U(1)_Y$, and the full ALRM β -functions are then used to evolve all couplings to higher energies.

At each scale above v_R , we verify simultaneously that: (i) the vacuum stability conditions of Eq. (12) are satisfied; (ii) all the perturbative unitarity bounds of Eqs. (15)–(23) are respected; and (iii) all the physical scalar mass-squared eigenvalues remain positive definite. The scale at which any one of these conditions is first violated defines a theoretical upper bound on the validity of the perturbative description. In parallel, we track the onset of any Landau pole, where a coupling diverges and the perturbative expansion breaks down entirely. The cutoff scale of the theory is then identified as the lesser of the scale at which a theoretical constraint is violated and the scale at which a Landau pole is encountered, whichever is reached first signals the boundary beyond which the model can no longer be considered perturbatively reliable without the introduction of new dynamics.

A. Gauge couplings

We shall first look at the running of the gauge couplings. Starting at the electroweak scale, we consider the g_L and g_Y running up to v_R . Beyond this, we have the g_Y given in terms of g_R and g_{BL} , as follows:

$$\frac{1}{g_Y^2} = \frac{1}{g_{BL}^2} + \frac{1}{g_R^2}. \quad (30)$$

To fix the freedom in the choice of gauge couplings, we relate g_R to g_L , and define their ratio as

$$r_g \equiv \frac{g_R(v_R)}{g_L(v_R)}. \quad (31)$$

Neglecting the mixing between $Z_L - Z_R$, the ratio between the heavy gauge boson masses is given by

$$\frac{M_{Z'}^2}{M_{W_R}^2} \approx r_g^2 (r_g^2 - \tan^2 \theta_W), \quad (32)$$

where θ_W is the weak mixing angle. It results in providing an upper limit for the ratio r_g as the ratio of the mass squares of gauge bosons must always remain a non-negative value. Henceforth, we get

$$r_g \geq \tan \theta_W \approx 0.55. \quad (33)$$

The other coupling g_{BL} is then determined at the scale v_R through the matching condition of Eq. (30), as follows;

$$\frac{1}{g_{BL}^2(v_R)} = \frac{1}{g_Y^2(v_R)} - \frac{1}{r_g^2 g_L^2(v_R)}, \quad (34)$$

which ensures consistency with the hypercharge relation at the symmetry breaking threshold. The ratio r_g is a free parameter of the matching procedure, subject to the requirement that the resulting values of g_R and g_{BL} remain within the perturbative regime not only at v_R but throughout their evolution up to the cutoff scale of the model.

Above v_R , the full ALRM gauge symmetry is restored and the couplings g_{BL} and g_R evolve independently according to their respective one-loop β -functions:

$$16\pi^2 \beta(g_{BL}) = \frac{13}{3} g_{BL}^3, \quad (35)$$

and

$$16\pi^2 \beta(g_R) = -\frac{17}{6} g_R^3. \quad (36)$$

Here, the opposite signs of the β -functions reflect qualitatively different high-energy behaviors of the two couplings: g_R is asymptotically free and decreases monotonically with increasing energy scale, while g_{BL} grows and may eventually approach its perturbative limit. This asymmetry, which is a direct consequence of the non-Abelian versus the Abelian nature of the respective gauge groups, plays an important role in determining the allowed range of r_g and hence the viable parameter space of the model at high energies. We examine the evolution of all gauge couplings up to energies of order 10^{16} GeV, which we use as a representative high energy cutoff scale and refer to as the grand unified theory (GUT) scale throughout the article for convenience. We emphasize, however, that gauge coupling unification is not imposed as a requirement of the model.

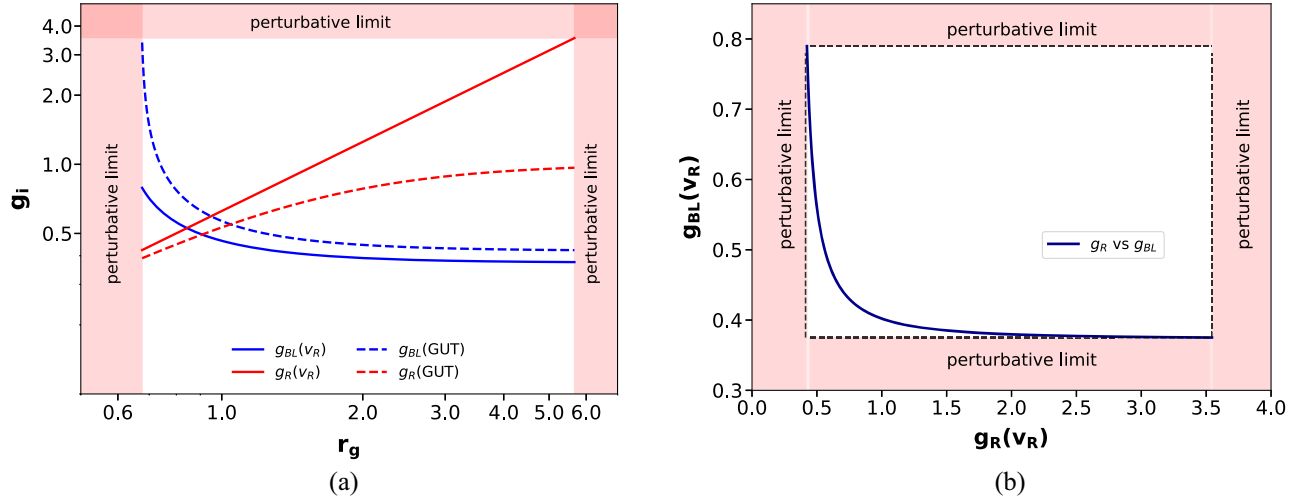


FIG. 3. (a) Dependence of g_{BL} and g_R on the ratio parameter $r_g = \frac{g_R(v_R)}{g_L(v_R)}$, shown at $v_R = 10$ TeV (solid lines) and at the GUT scale set as 10^{16} GeV (dashed lines). (b) g_R and g_{BL} at v_R . In both panels, the shaded regions are excluded by the perturbative limit ($g_i \leq \sqrt{4\pi}$).

For numerical analysis, we consider the values of the couplings g_L and g_Y as [47]

$$g_L(M_Z) = 0.65096 \pm 0.00004,$$

$$\text{and } g_Y(M_Z) = 0.35726 \pm 0.00002.$$

The results of our analysis are presented in Fig. 3. The left panel shows the variation of g_{BL} and g_R as functions of r_g , evaluated at v_R (solid lines) and at the GUT scale (dashed lines). The contrasting behavior of the two couplings is clearly visible: since g_R is asymptotically free, the dashed red line lies below the corresponding solid line, whereas g_{BL} , which grows with energy, has its dashed blue line lying above the solid one. The perturbative limit is approached by g_{BL} at the GUT scale for small values of $r_g \sim 0.677$, while $g_R(v_R)$ saturates the perturbative bound for large values $r_g \sim 5.66$. Furthermore, $g_R(v_R)$ increases with r_g while $g_{BL}(v_R)$ decreases, reflecting the inverse relationship between the two couplings imposed by the hypercharge matching condition of Eq. (30). Requiring both couplings to remain perturbative at all scales between v_R and the GUT scale yields the following allowed ranges:

$$0.423 < g_R(v_R) < \sqrt{4\pi},$$

$$0.375 < g_{BL}(v_R) < 0.789,$$

$$\text{and } 0.677 < r_g < 5.660. \quad (37)$$

The right panel of Fig. 3 displays the corresponding correlation between g_R and g_{BL} at the scale v_R , with the excluded regions shaded. The shape of the allowed band is a direct consequence of the matching condition Eq. (30), which constrains g_R and g_{BL} to lie on a curve of fixed $1/g_R^2 + 1/g_{BL}^2 = 1/g_Y^2$, so that an increase in g_R necessarily entails a decrease in g_{BL} and vice versa.

To assess the sensitivity of these results to the choice of symmetry breaking scale, we repeat the analysis for representative values of v_R ranging from 5 TeV to 100 TeV. The resulting allowed ranges of r_g , g_R , and g_{BL} are summarized in Table II. As v_R increases, the allowed band of r_g shifts slightly upward, reflecting the slow logarithmic running of g_L and g_Y between the electroweak and symmetry breaking scales. Nevertheless, the overall picture remains qualitatively unchanged across this range, and the allowed intervals for g_R and g_{BL} are only mildly sensitive to the precise value of v_R .

In Fig. 4, we present the perturbativity bounds on the masses of the heavy gauge bosons Z' (left panel) and W_R (right panel) as functions of r_g , shown for several representative values of v_R . The Z' mass receives contributions from both g_{BL} and g_R , while the W_R mass depends solely on g_R , as it arises entirely from the non-Abelian $SU(2)_{R'}$ sector. The red-shaded regions in each panel indicate the parameter space excluded by the perturbativity constraints on r_g summarized in Table II. The current exclusion limit from 13 TeV LHC searches for $Z' \rightarrow \ell^+ \ell^-$ ($\ell = e, \mu$) is $M_{Z'} \gtrsim 5.1$ TeV [48,49], derived under the assumption $g_R = g_L$ (i.e., $r_g = 1$). For $g_R \neq g_L$, this limit must be rescaled, since the Z' production cross section $\sigma(pp \rightarrow Z' \rightarrow \ell^+ \ell^-)$ depends on r_g .² After applying this rescaling, the excluded Z' mass range shifts to approximately [4.9, 6.1] TeV for $r_g \in [0.677, 5.66]$, as indicated by the purple dot-dashed line in the left panel of Fig. 4. This translates into a lower bound on r_g that depends on v_R ; for instance, for $v_R = 10$ TeV the constraint requires $r_g \gtrsim 1.1$.

Since the W_R boson decays exclusively into a SM up-type quark and an exotic down-type quark, or into a

²The impact of deviating from $g_L = g_R$ on gauge boson mass limits in the context of LRSM was studied in Ref. [50].

TABLE II. Allowed ranges of r_g , g_R , and g_{BL} at the scale v_R for the representative values of the $SU(2)_{R'}$ breaking scale, derived by requiring both couplings to remain perturbative up to the GUT scale.

v_R [TeV]	$r_g = \frac{g_R}{g_L}$	g_R	g_{BL}
5	[0.665, 5.635]	$[0.422, \sqrt{4\pi}]$	[0.370, 0.788]
10	[0.677, 5.660]	$[0.423, \sqrt{4\pi}]$	[0.375, 0.789]
50	[0.695, 5.730]	$[0.430, \sqrt{4\pi}]$	[0.382, 0.812]
100	[0.702, 5.761]	$[0.432, \sqrt{4\pi}]$	[0.385, 0.824]

charged lepton and a scotino, it does not give rise to the purely SM final states and therefore evades the direct LHC search constraints entirely. However, the W_R and Z' masses are related through Eqs. (8) and (9), so that the lower bound on $M_{Z'}$ translates into an indirect lower bound on M_{W_R} , as discussed in Ref. [41]. This indirect limit is shown as the dot-dashed curve in the right panel of Fig. 4, and is found to be numerically very close to the direct r_g constraint derived from $M_{Z'}$. In principle, the mass ratio $M_{W_R}/M_{Z'}$ carries a mild dependence on $\tan\beta$ through the gauge boson mass formulas; however, we have verified explicitly that this dependence is numerically negligible over the parameter space considered here and can safely be ignored.

B. Scalar quartic couplings

We now turn to the one-loop RGE running of the scalar quartic couplings, whose explicit β -functions are collected in Appendix B. The running of these couplings is driven by the contributions from the quartic sector itself, as well as from the gauge and the Yukawa sectors.

To illustrate the qualitative features of this evolution, we select three benchmark points (BPs) from the parameter

scan of Sec. III C, each satisfying the combined constraints of the vacuum stability, the perturbative unitarity, and the perturbativity, with $v_R = 10$ TeV fixed. The benchmark values of the independent quartic couplings, $\tan\beta$, and the trilinear coupling μ_3 are given in Table III. The Yukawa couplings entering the β -functions are fixed by the fermion mass spectrum; we take scotino masses $(m_{n_e}, m_{n_\mu}, m_{n_\tau}) = (300, 400, 500)$ GeV, exotic quark masses $(m_{d'}, m_{s'}, m_{b'}) = (1.5, 2, 3)$ TeV, and the remaining standard fermion masses at their measured values. The gauge couplings are fixed with $r_g = 1$ for simplicity, as discussed in Sec. IV A.

The resulting RGE evolution of the quartic couplings is shown in Fig. 5 for each of the three benchmark points. The shaded horizontal bands indicate the upper limits imposed by the combined vacuum stability and the perturbative unitarity constraints of Eq. (26), with values inside the bands being excluded; for clarity, only the most constraining bound is displayed for each coupling, as the remaining constraints are satisfied up to higher scales and do not limit the cutoff scale in the cases shown. The three benchmark points illustrate qualitatively distinct patterns of RGE evolution, which we now discuss in turn. The benchmark BP1 (Fig. 5, top left) remains theoretically consistent up to approximately 10^{16} GeV, at which point λ_1 reaches the upper boundary imposed by the combined vacuum stability and the unitarity constraints. The Landau pole lies roughly an order of magnitude above this scale. In the case of BP2 (Fig. 5, top right), the parameters λ_2 , λ_3 , and $\tan\beta$ are nearly unchanged relative to BP1, while α_{12} and α_{13} are approximately twice as large at the input scale. Through the SM Higgs mass condition that fixes λ_1 , this results in a substantially larger initial value of λ_1 , as is evident from the figure. This larger initial value drives a much faster running of λ_1 , which violates the theoretical

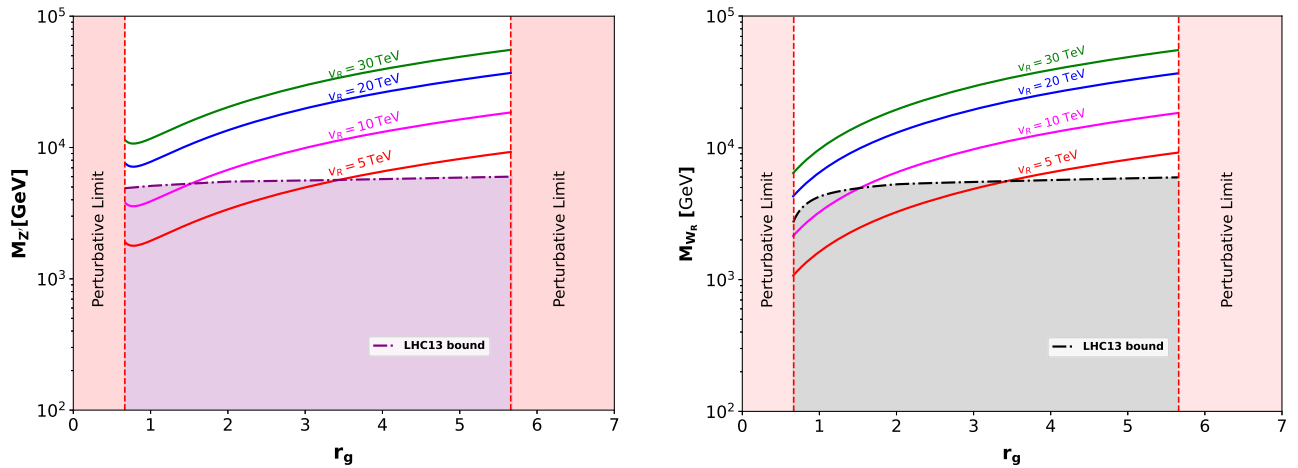


FIG. 4. Perturbativity bounds on the masses of the heavy gauge bosons Z' (left) and W_R (right) as functions of r_g , shown for the representative values of v_R . The red-shaded regions are excluded by the perturbativity constraints of Table II, and the purple dot-dashed line indicates the rescaled LHC exclusion limit from $Z' \rightarrow \ell^+ \ell^-$ searches [48,49]. The dot-dashed curve in the right panel shows the indirect lower bound on M_{W_R} derived from the $M_{Z'}$ constraint via Eqs. (8) and (9).

TABLE III. Input values of the scalar potential parameters at the scale $v_R = 10$ TeV for the three benchmark points used to illustrate the RGE running of the quartic couplings in Fig. 5.

BP	α_{12}	α_{13}	λ_2	λ_3	$\tan\beta$	μ_3 [GeV]
BP1	0.0272	0.0270	-0.1531	0.00425	15.41	-333.69
BP2	0.0627	0.0625	-0.1495	0.00461	15.83	-343.92
BP3	0.0182	0.0180	-0.1047	0.00425	17.81	-333.67

constraints already at around $10^{5.5}$ GeV, with the Landau pole reached shortly after at 10^6 GeV. The accelerated running of λ_1 in turn feeds into λ_2 through the $24\lambda_1\lambda_2$ term in $\beta(\lambda_2)$, causing λ_2 to also run faster than in BP1. In contrast, the RGEs of λ_3 and λ_4 are insensitive to λ_1 and λ_2 , and consequently follow trajectories very similar to BP1. On the other hand, in the case of BP3 (Fig. 5, bottom), the Landau pole lies slightly above 10^{16} GeV, yet the theoretical constraints are violated considerably earlier by two distinct couplings. First, λ_1 turns negative around 10^{10} GeV, violating the BFB condition, then λ_2 crosses zero and becomes positive at around 10^8 GeV, violating the copositivity condition required for a stable vacuum. The

remaining couplings follow a qualitatively similar trajectory to BP1, with λ_3 and λ_4 approaching the unitarity bound near 10^{15} GeV.

The benchmark analysis above serves to illustrate the high sensitivity of the RG evolution to the input values of the couplings at the scale v_R , and motivates a systematic study of the energy range over which the model remains theoretically consistent. We now perform such an analysis, scanning the input parameter space for $v_R \in \{5, 10, 15, 20\}$ TeV and determining, for each point, the highest scale up to which all theoretical constraints are simultaneously satisfied.

Specifically, for a given v_R , we perform a scan over the initial parameter space as defined in Eq. (25), retaining only those points that satisfy the vacuum stability conditions and perturbative unitarity constraints at the input scale. In the subsequent RG evolution, we require that all quartic couplings remain within their perturbative limits at every scale, with the additional sign conditions that all couplings except λ_2 remain non-negative, while λ_2 must remain strictly negative, as dictated by the copositivity and the vacuum conditions throughout the running. For each accepted point, the couplings are evolved from v_R upward,

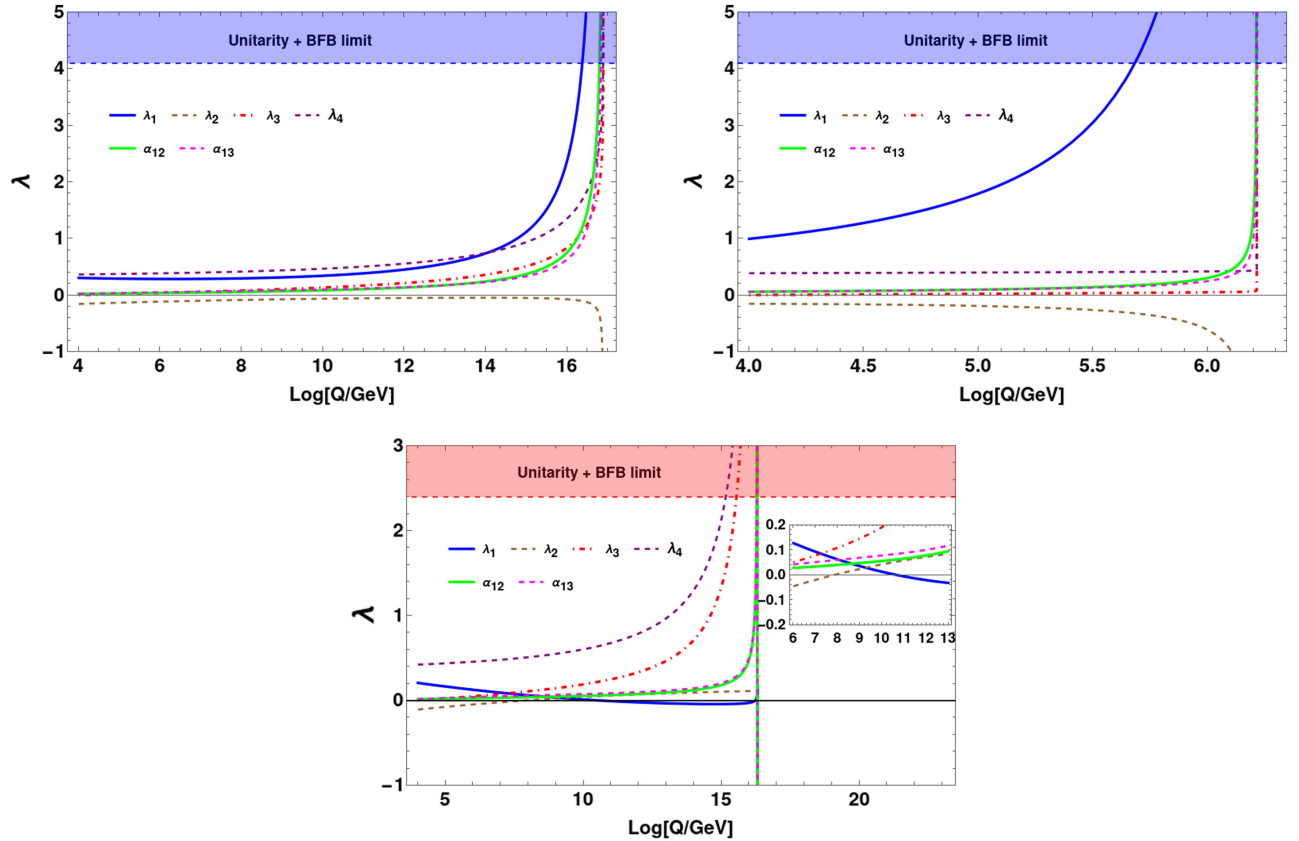


FIG. 5. RGE running of the scalar quartic couplings from the input scale $Q = v_R = 10$ TeV for BP1 (top left), BP2 (top right), and BP3 (bottom), as defined in Table III. The purple-shaded band indicates the theoretical upper bound on λ_1 , and the red-shaded band indicates the theoretical bound on λ_3 , both derived from the combined vacuum stability and perturbative unitarity conditions of Eq. (26). Couplings must remain below the upper band and above the lower band to satisfy all theoretical constraints.

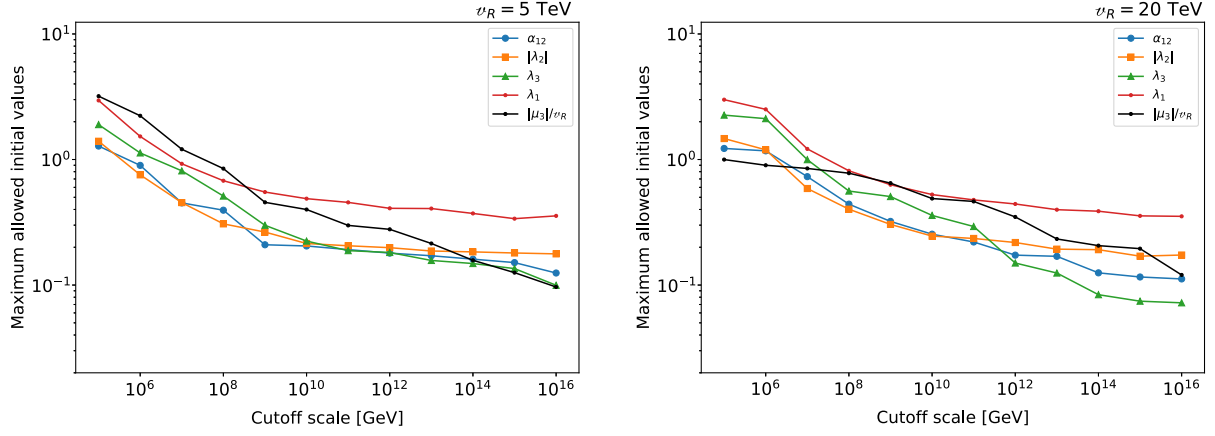


FIG. 6. Maximum allowed initial values of the scalar quartic couplings $\alpha_{12} \simeq \alpha_{13}$ (blue), $|\lambda_2|$ (orange), λ_3 (green), λ_1 (red), and $|\mu_3|/v_R$ (black), as functions of the cutoff scale, for $v_R = 5$ TeV (left) and $v_R = 20$ TeV (right). Each curve represents the upper boundary of the theoretically viable parameter space at the corresponding cutoff scale, obtained by requiring simultaneously the absence of Landau poles, the satisfaction of BFB conditions, and compliance with the perturbative unitarity bounds throughout the RGE evolution.

and their values are recorded at a series of the cutoff scales ranging from 10^5 GeV to 10^{16} GeV. The cutoff scale assigned to each point is the lowest scale at which any one of the following conditions is first violated: the vacuum stability bounds, the perturbative unitarity constraints, the sign conditions on the couplings, or the perturbativity limit. In this way, we map out the region of input parameter space that remains theoretically reliable up to any given energy scale, and identify how this region shrinks as the target cutoff scale is increased. A parameter point is retained as theoretically viable up to a chosen cutoff scale if it satisfies all of the following conditions throughout the RG evolution from v_R up to that scale: (1) no Landau pole is encountered below the cutoff scale, (2) the vacuum stability (BFB) conditions of Eq. (12) are satisfied at every scale up to the cutoff scale, and (3) all couplings remain within their perturbative limits and within the bounds imposed by the perturbative unitarity constraints of Eqs. (15)–(23) at every scale. The set of input values passing all three criteria defines the allowed initial parameter space consistent with theoretical reliability up to the chosen cutoff scale. For each cutoff scale, we extract the maximum allowed initial value of each coupling consistent with these criteria. The results are presented in Fig. 6, which shows the upper limits on $\alpha_{12} \simeq \alpha_{13}$ (blue), $|\lambda_2|$ (orange), λ_3 (green), λ_1 (red), and $|\mu_3|/v_R$ (black) as functions of the cutoff scale. Since the difference between α_{12} and α_{13} is negligible over the allowed parameter space, they are both represented by the same set of points in the presentation. The results are shown for $v_R = 5$ TeV (left panel) and $v_R = 20$ TeV (right panel) to illustrate the sensitivity to the symmetry breaking scale; both panels are found to be in close agreement, confirming that the theoretical bounds on the quartic couplings are only mildly sensitive to the precise value of v_R .

As expected on the general grounds, the maximum allowed values of all couplings decrease monotonically

as the cutoff scale is raised, reflecting the fact that a higher cutoff imposes more stringent requirements on the RGE trajectory. This behavior has direct phenomenological consequences: the choice of cutoff scale translates into concrete upper bounds on the scalar couplings at the electroweak scale, which in turn constrain the physical Higgs spectrum and the strength of scalar interactions accessible at the collider experiments. The interplay between the target validity scale and the low-energy phenomenology of the model is therefore an important consideration in any phenomenological study of the ALRM.

C. Constraints on scalar masses

The masses of the physical scalars in the ALRM are determined by the couplings of the scalar potential and the vacuum expectation values, so the theoretical constraints derived above translate directly into upper bounds on the scalar mass spectrum. From Eqs. (3), (6), and (7), one can see that for a fixed v_R , the scalar masses are governed primarily by μ_3 and $\tan \beta$, with subleading corrections from the quartic couplings. In particular, the mass splitting between $H_2^\pm$ and H_1^0/A_1 is controlled by λ_2 and the VEV v_u ; this splitting reaches at the most around 50 GeV when the cutoff scale is set to 10^{16} GeV, and grows to roughly an order of magnitude larger for the cutoff scale at 10^5 GeV.

The dominant contributions to the charged Higgs masses take the approximate forms as

$$m_{H_1^\pm}^2 \approx -\frac{\mu_3 v_R}{\sqrt{2}} \tan \beta, \quad \text{and} \quad m_{H_2^\pm}^2 \approx -\frac{\mu_3 v_R}{\sqrt{2}} \cot \beta, \quad (38)$$

where $\mu_3 < 0$ ensures the positive mass-squared eigenvalues. However, the vacuum stability and the perturbative unitarity constraints couple $\tan \beta$ and μ_3 ; the larger values

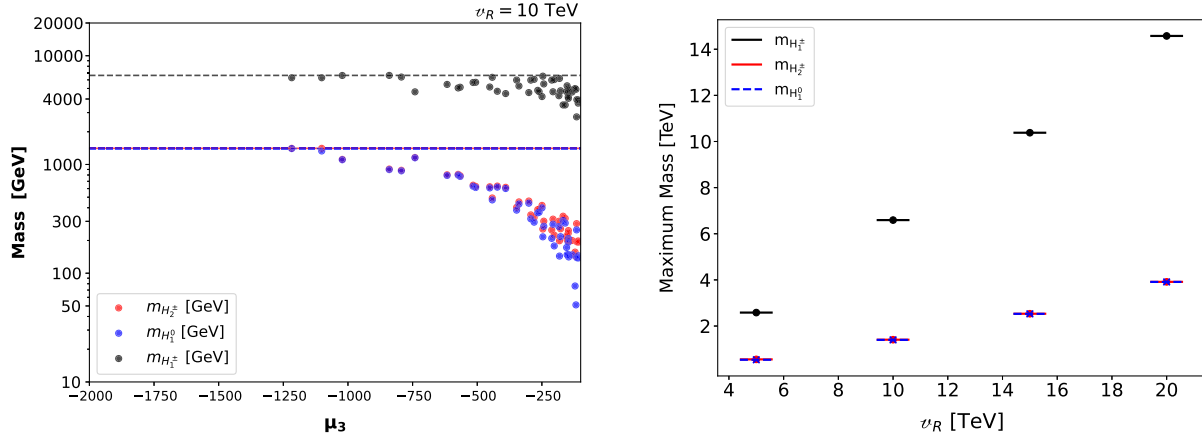


FIG. 7. Left: Mass spectrum of $m_{H_2^\pm}$ (red), $m_{H_1^0}$ (blue), and $m_{H_1^\pm}$ (black) as a function of μ_3 for $v_R = 10$ TeV, obtained from the full numerical scan with all theoretical constraints imposed up to 10^{16} GeV. Right: Maximum allowed scalar masses as a function of v_R , derived by requiring theoretical consistency up to 10^{16} GeV.

of $|\mu_3|$ restricting the maximum allowed $\tan\beta$ to approximately 8, as visible in Fig. 2. Ignoring the RGE constraints and working within the tree-level allowed parameter space alone, one would naively estimate upper bounds of

$$m_{H_1^\pm} \sim 20 \text{ TeV} \quad \text{and} \quad m_{H_2^\pm} \sim 4 \text{ TeV}. \quad (39)$$

The RGE analysis, however, significantly tightens these estimates. Requiring the model to remain theoretically consistent up to 10^{16} GeV, it restricts $|\mu_3|$ to approximately 1.3 TeV for $v_R = 10$ TeV, which reduces the maximum allowed masses to

$$m_{H_1^\pm} \sim 6.5 \text{ TeV} \quad \text{and} \quad m_{H_2^\pm} \sim 1.5 \text{ TeV}. \quad (40)$$

These analytical estimates are confirmed by the full numerical scan, whose results for $v_R = 10$ TeV are shown in the left panel of Fig. 7. The heavy charged Higgs $H_1^\pm$ (black points) reaches a maximum mass of approximately 6.5 TeV, while the degenerate neutral states H_1^0/A_1 (blue points) and the light charged Higgs $H_2^\pm$ (red points) are bounded by approximately 1.5 TeV. The dependence of these upper bounds on v_R is shown in the right panel of Fig. 7: larger values of v_R push all scalar masses upward, with $H_2^\pm$ entering the multi-TeV range while $H_1^\pm$ and H_1^0 become correspondingly heavier.

Taken together, these results demonstrate that the combination of the vacuum stability conditions and the perturbative unitarity constraints, when consistently imposed through the RGE, places stringent and calculable upper bounds on the scalar mass spectrum of the ALRM. A key feature of these bounds is that they are not merely order-of-magnitude estimates but are genuinely predictive: for a given choice of v_R and a specified validity scale, the maximum allowed masses of $H_2^\pm$, H_1^0 , and $H_1^\pm$ are concretely determined, as summarized in Fig. 7. These

bounds are therefore of direct relevance for ongoing and future searches for extended Higgs sectors at the LHC and at future high-energy colliders. We emphasize, however, that the numerical values of the upper bounds carry a dependence on v_R : larger symmetry breaking scales allow heavier scalar states, as illustrated in the right panel of Fig. 7, so that a complete phenomenological assessment requires specifying both the validity scale and the value of v_R .

V. SUMMARY AND CONCLUSIONS

We have performed a systematic study of the theoretical consistency of the alternative left-right model, E_6 -motivated extension of the Standard Model that naturally accommodates neutrino masses and dark matter while avoiding tree-level flavor-changing neutral currents. Three classes of theoretical constraints have been imposed on the scalar sector: the perturbative unitarity, the vacuum stability, and the perturbativity. We derived for the first time the complete set of perturbative unitarity constraints on the ALRM scalar potential, obtaining 14 independent conditions on the quartic couplings, verified independently using SARAH and anyPUB. Combined with the boundedness-from-below conditions and the requirement of positive-definite scalar mass-squared eigenvalues, these constraints yield significantly more stringent restrictions than by either set alone. The resulting absolute upper bounds on the quartic couplings are $(\alpha_{12}, \alpha_{13}) \lesssim 1.4$, $\lambda_1 \lesssim 3.0$, $|\lambda_2| \lesssim 1.5$, $\lambda_3 \lesssim 2.5$, and $\lambda_4 \lesssim 2\pi$, which are further restricted by their strong correlations.

We then performed a one-loop RGE analysis of all model parameters, implemented in two stages reflecting the two-step symmetry breaking of the ALRM. The RGE running reveals that the tree-level allowed parameter space is further and substantially restricted when theoretical consistency is demanded up to higher energy scales. The allowed ranges

of all quartic couplings shrink monotonically as the target validity scale is raised, with the constraints becoming particularly stringent when the model is required to remain well behaved up to a high-energy cutoff scale. The analysis also exposes the structure underlying this running: the cutoff scale is highly sensitive to the initial value of λ_1 , which is fixed by the SM Higgs mass condition and inherits a strong dependence on α_{12} and α_{13} , while λ_3 and λ_4 are comparatively insensitive owing to the structure of their β -functions. For the gauge sector, perturbativity up to a scale of 10^{16} GeV constrains the ratio $r_g = g_R/g_L$ to lie in the range $0.677 \lesssim r_g \lesssim 5.660$ for $v_R = 10$ TeV, with only mild dependence on v_R across the range of 5 TeV to 100 TeV. As a consequence of the restricted parameter space, the physical scalar masses of the ALRM are also bounded from above. For $v_R = 10$ TeV, requiring theoretical consistency up to a cutoff scale of 10^{16} GeV yields

$$\begin{aligned} m_{H_1^\pm} &\lesssim 6.5 \text{ TeV}, & m_{H_2^\pm} &\lesssim 1.5 \text{ TeV}, \\ m_{H_1^0} &\simeq m_{A_1} \lesssim 1.5 \text{ TeV}, \end{aligned} \quad (41)$$

with all bounds scaling with v_R . These upper limits provide a predictive window for searches for the extended Higgs sector of the ALRM at the LHC and future collider experiments, and allow the model to be systematically constrained or excluded when no such states are observed within the expected mass ranges.

In conclusion, our analysis demonstrates that the consistent combination of the vacuum stability, the perturbative unitarity, and one-loop RG evolution provides a powerful

framework for constraining the ALRM, restricting the parameter space far more stringently than tree-level considerations alone. These results provide a systematic theoretical foundation for future phenomenological studies of the model, including collider searches for its exotic scalar, gauge boson, and fermionic states.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: TADPOLE EQUATIONS

With $\langle \phi_2^{0R} \rangle = v_u$, $\langle \chi_L^{0R} \rangle = v_L$, $\langle \chi_R^{0R} \rangle = v_R$, where the superscript R denotes the real part of the corresponding field, and $\langle \psi \rangle = 0$ for all other field components, the nontrivial minimization conditions of the potential are given by

$$\left. \frac{\partial V}{\partial \phi_2^{0R}} \right|_{\text{vacuum}} = 0 \Rightarrow \frac{1}{\sqrt{2}} \mu_3 v_L v_R + \lambda_1 v_u^3 + v_u (\alpha_{12} (v_L^2 + v_R^2) - \mu_1^2) = 0, \quad (A1)$$

$$\left. \frac{\partial V}{\partial \chi_L^{0R}} \right|_{\text{vacuum}} = 0 \Rightarrow \frac{1}{\sqrt{2}} \mu_3 v_u v_R + \lambda_3 v_L^3 + v_L (\alpha_{12} v_u^2 + \lambda_4 v_R^2 - \mu_2^2) = 0, \quad (A2)$$

$$\left. \frac{\partial V}{\partial \chi_R^{0R}} \right|_{\text{vacuum}} = 0 \Rightarrow \frac{1}{\sqrt{2}} \mu_3 v_u v_L + v_R (\alpha_{12} v_u^2 + \lambda_3 v_R^2 + \lambda_4 v_L^2 - \mu_2^2) = 0. \quad (A3)$$

Solving these, we can express the coefficient of the quadratic terms, μ_1^2 and μ_2^2 , and one of the quartic couplings λ_4 as

$$\begin{aligned} \mu_1^2 &= \lambda_1 v_u^2 + \alpha_{12} (v_L^2 + v_R^2) + \frac{\mu_3 v_L v_R}{\sqrt{2} v_u}, \\ \mu_2^2 &= (v_L^2 + v_R^2) \lambda_3 + \alpha_{12} v_u^2, \\ \lambda_4 &= \lambda_3 - \frac{\mu_3 v_u}{\sqrt{2} v_L v_R}. \end{aligned} \quad (A4)$$

APPENDIX B: RENORMALIZATION GROUP EQUATIONS

With the convention

$$\beta(X) \equiv \mu \frac{dX}{d\mu} \equiv \frac{1}{(4\pi)^2} \beta^{(1)}(X),$$

the one-loop β functions of the various parameters are given by³

$$\beta^{(1)}(g_{BL}) = \frac{13}{3} g_{BL}^3, \quad (B1)$$

$$\beta^{(1)}(g_L) = -\frac{17}{6} g_L^3, \quad (B2)$$

$$\beta^{(1)}(g_R) = -\frac{17}{6} g_R^3, \quad (B3)$$

$$\beta^{(1)}(g_3) = -5g_3^3, \quad (B4)$$

$$\begin{aligned} \beta^{(1)}(Y_{Q1}) &= Y_{Q2} Y_{Q2}^\dagger Y_{Q1} + \frac{3}{2} Y_{Q1} Y_{Q1}^\dagger Y_{Q1} + 3\text{Tr}(Y_{Q1}^\dagger Y_{Q1}) Y_{Q1} + \text{Tr}(Y_{L2}^\dagger Y_{L2}) Y_{Q1} - \frac{5}{12} g_{BL}^2 Y_{Q1} \\ &\quad - \frac{9}{4} g_L^2 Y_{Q1} - 8g_3^2 Y_{Q1} \\ \beta^{(1)}(Y_{Q2}) &= 2Y_{Q2} Y_{Q2}^\dagger Y_{Q2} + \frac{1}{2} Y_{Q2} Y_{Q3} Y_{Q3}^\dagger + \frac{1}{2} Y_{Q1} Y_{Q1}^\dagger Y_{Q2} + 3\text{Tr}(Y_{Q2}^\dagger Y_{Q2}) Y_{Q2} + \text{Tr}(Y_{L1}^\dagger Y_{L1}) Y_{Q2} \\ &\quad - \frac{1}{6} g_{BL}^2 Y_{Q2} - \frac{9}{4} g_L^2 Y_{Q2} - \frac{9}{4} g_R^2 Y_{Q2} - 8g_3^2 Y_{Q2} \\ \beta^{(1)}(Y_{Q3}) &= Y_{Q2} Y_{Q2}^\dagger Y_{Q3} + \frac{3}{2} Y_{Q3} Y_{Q3}^\dagger Y_{Q3} + 3\text{Tr}(Y_{Q3}^\dagger Y_{Q3}) Y_{Q3} + \text{Tr}(Y_{L3}^\dagger Y_{L3}) Y_{Q3} - \frac{5}{12} g_{BL}^2 Y_{Q3} \\ &\quad - \frac{9}{4} g_R^2 Y_{Q3} - 8g_3^2 Y_{Q3} \\ \beta^{(1)}(Y_{L1}) &= 2Y_{L1} Y_{L1}^\dagger Y_{L1} + \frac{1}{2} Y_{L1} Y_{L3} Y_{L3}^\dagger + \frac{1}{2} Y_{L2} Y_{L2}^\dagger Y_{L1} + 3\text{Tr}(Y_{Q2}^\dagger Y_{Q2}) Y_{L1} - \frac{3}{2} g_{BL}^2 Y_{L1} - \frac{9}{4} g_R^2 Y_{L1} \\ &\quad + \text{Tr}(Y_{L1}^\dagger Y_{L1}) Y_{L1} - \frac{9}{4} g_L^2 Y_{L1} \\ \beta^{(1)}(Y_{L2}) &= Y_{L1} Y_{L1}^\dagger Y_{L2} + \frac{3}{2} Y_{L2} Y_{L2}^\dagger Y_{L2} + 3\text{Tr}(Y_{Q1}^\dagger Y_{Q1}) Y_{L2} + \text{Tr}(Y_{L2}^\dagger Y_{L2}) Y_{L2} - \frac{3}{4} g_{BL}^2 Y_{L2} - \frac{9}{4} g_L^2 Y_{L2} \\ \beta^{(1)}(Y_{L3}) &= Y_{L1} Y_{L1}^\dagger Y_{L3} + \frac{3}{2} Y_{L3} Y_{L3}^\dagger Y_{L3} + 3\text{Tr}(Y_{Q3}^\dagger Y_{Q3}) Y_{L3} + \text{Tr}(Y_{L3}^\dagger Y_{L3}) Y_{L3} - \frac{3}{4} g_{BL}^2 Y_{L3} - \frac{9}{4} g_R^2 Y_{L3} \\ \beta^{(1)}(\lambda_1) &= 32\lambda_1^2 + 16\lambda_1\lambda_2 + 16\lambda_2^2 + 8\alpha_{12}^2 + 8\alpha_{13}^2 - 9g_L^2\lambda_1 - 9g_R^2\lambda_1 + \frac{9}{8} g_L^4 + \frac{3}{4} g_L^2 g_R^2 + \frac{9}{8} g_R^4 \\ &\quad + 12\lambda_1 \text{Tr}(Y_{Q2}^\dagger Y_{Q2}) + 4\lambda_1 \text{Tr}(Y_{L1}^\dagger Y_{L1}) - 6\text{Tr}(Y_{Q2}^\dagger Y_{Q2} Y_{Q2}^\dagger Y_{Q2}) - 2\text{Tr}(Y_{L1}^\dagger Y_{L1} Y_{L1}^\dagger Y_{L1}) \\ \beta^{(1)}(\lambda_2) &= 24\lambda_1\lambda_2 + 16\lambda_2^2 - 4\alpha_{12}^2 + 8\alpha_{12}\alpha_{13} - 4\alpha_{13}^2 - 9g_L^2\lambda_2 - 9g_R^2\lambda_2 + \frac{3}{2} g_L^2 g_R^2 + 12\lambda_2 \text{Tr}(Y_{Q2}^\dagger Y_{Q2}) \\ &\quad + 4\lambda_2 \text{Tr}(Y_{L1}^\dagger Y_{L1}) + 3\text{Tr}(Y_{Q2}^\dagger Y_{Q2} Y_{Q2}^\dagger Y_{Q2}) + \text{Tr}(Y_{L1}^\dagger Y_{L1} Y_{L1}^\dagger Y_{L1}), \\ \beta^{(1)}(\lambda_3) &= 24\lambda_3^2 + 8\lambda_4^2 + 8\alpha_{12}^2 + 8\alpha_{13}^2 - 3g_{BL}^2\lambda_3 - 9g_L^2\lambda_3 + \frac{3}{8} g_{BL}^4 + \frac{3}{4} g_{BL}^2 g_L^2 + \frac{9}{8} g_L^4 + 12\lambda_3 \text{Tr}(Y_{Q1}^\dagger Y_{Q1}) \\ &\quad + 4\lambda_3 \text{Tr}(Y_{L2}^\dagger Y_{L2}) - 6\text{Tr}(Y_{Q1}^\dagger Y_{Q1} Y_{Q1}^\dagger Y_{Q1}) - 2\text{Tr}(Y_{L2}^\dagger Y_{L2} Y_{L2}^\dagger Y_{L2}) \\ \beta^{(1)}(\lambda_4) &= 24\lambda_3\lambda_4 + 8\lambda_4^2 + 4\alpha_{12}^2 + 8\alpha_{12}\alpha_{13} + 4\alpha_{13}^2 - 3g_{BL}^2\lambda_4 - \frac{9}{2} g_L^2\lambda_4 - \frac{9}{2} g_R^2\lambda_4 + \frac{3}{8} g_{BL}^4 + 6\lambda_4 \text{Tr}(Y_{Q1}^\dagger Y_{Q1}) \\ &\quad + 6\lambda_4 \text{Tr}(Y_{Q3}^\dagger Y_{Q3}) + 2\lambda_4 \text{Tr}(Y_{L2}^\dagger Y_{L2}) + 2\lambda_4 \text{Tr}(Y_{L3}^\dagger Y_{L3}) \end{aligned}$$

³These equations are presented in Ref. [51].

$$\begin{aligned}
\beta^{(1)}(\alpha_{12}) &= 12\alpha_{12}\lambda_1 + 8\alpha_{12}\lambda_3 + 4\alpha_{12}\lambda_4 + 12\alpha_{12}^2 - 8\alpha_{12}\alpha_{13} + 8\alpha_{13}\lambda_1 + 8\alpha_{13}\lambda_2 + 4\alpha_{13}\lambda_3 + 4\alpha_{13}\lambda_4 \\
&\quad + 4\alpha_{13}^2 - \frac{3}{2}\alpha_{12}g_{BL}^2 - 9\alpha_{12}g_L^2 - \frac{9}{2}\alpha_{12}g_R^2 - \frac{9}{8}g_L^4 + 6\alpha_{12}\text{Tr}(Y_{Q2}^\dagger Y_{Q2}) + 6\alpha_{12}\text{Tr}(Y_{Q1}^\dagger Y_{Q1}) \\
&\quad + 2\alpha_{12}\text{Tr}(Y_{L1}^\dagger Y_{L1}) + 2\alpha_{12}\text{Tr}(Y_{L2}^\dagger Y_{L2}) \\
\beta^{(1)}(\alpha_{13}) &= 8\alpha_{12}\lambda_1 + 8\alpha_{12}\lambda_2 + 4\alpha_{12}\lambda_3 + 4\alpha_{12}\lambda_4 + 4\alpha_{12}^2 - 8\alpha_{12}\alpha_{13} + 6\alpha_{13}\lambda_1 + 8\alpha_{13}\lambda_3 + 4\alpha_{13}\lambda_4 + 12\alpha_{13}^2 \\
&\quad - \frac{9}{2}\alpha_{12}g_L^2 + \frac{9}{2}\alpha_{12}g_R^2 - \frac{3}{2}\alpha_{13}g_{BL}^2 - \frac{9}{2}\alpha_{13}g_L^2 - 9\alpha_{13}g_R^2 + \frac{9}{8}g_L^4 + 6\alpha_{12}\text{Tr}(Y_{Q1}^\dagger Y_{Q1}) \\
&\quad - 6\alpha_{12}\text{Tr}(Y_{Q3}^\dagger Y_{Q3}) + 2\alpha_{12}\text{Tr}(Y_{L2}^\dagger Y_{L2}) - 2\alpha_{12}\text{Tr}(Y_{L3}^\dagger Y_{L3}) + 6\alpha_{13}\text{Tr}(Y_{Q2}^\dagger Y_{Q2}) \\
&\quad + 6\alpha_{13}\text{Tr}(Y_{Q3}^\dagger Y_{Q3}) + 2\alpha_{13}\text{Tr}(Y_{L1}^\dagger Y_{L1}) + 2\alpha_{13}\text{Tr}(Y_{L3}^\dagger Y_{L3}) - 6\text{Tr}(Y_{Q2}^\dagger Y_{Q2} Y_{Q3} Y_{Q3}^\dagger) \\
&\quad - 2\text{Tr}(Y_{L1}^\dagger Y_{L1} Y_{L3} Y_{L3}^\dagger) \\
\beta^{(1)}(\mu_1^2) &= -\frac{9}{2}g_L^2\mu_1^2 - \frac{9}{2}g_R^2\mu_1^2 + 2\mu_3^2 + 20\lambda_1\mu_1^2 + 8\lambda_2\mu_1^2 + 8\alpha_{12}\mu_2^2 + 8\alpha_{13}\mu_2^2 + 6\mu_1^2\text{Tr}(Y_{Q2}^\dagger Y_{Q2}) \\
&\quad + 2\mu_1^2\text{Tr}(Y_{L1}^\dagger Y_{L1}) \\
\beta^{(1)}(\mu_2^2) &= -\frac{3}{2}g_{BL}^2\mu_2^2 - \frac{9}{2}g_L^2\mu_2^2 + 4\mu_3^2 + 8\alpha_{12}\mu_1^2 + 8\alpha_{13}\mu_1^2 + 12\lambda_3\mu_2^2 + 8\lambda_4\mu_2^2 + 6\mu_2^2\text{Tr}(Y_{Q1}^\dagger Y_{Q1}) \\
&\quad + 2\mu_2^2\text{Tr}(Y_{L2}^\dagger Y_{L2}) \\
\beta^{(1)}(\mu_3) &= -\frac{3}{2}g_{BL}^2\mu_3 - \frac{9}{2}g_L^2\mu_3 - \frac{9}{2}g_R^2\mu_3 + 4\lambda_4\mu_3 + 16\alpha_{12}\mu_3 - 8\alpha_{13}\mu_3 + 3\mu_3\text{Tr}(Y_{Q2}^\dagger Y_{Q2}) \\
&\quad + 3\mu_3\text{Tr}(Y_{Q1}^\dagger Y_{Q1}) + 3\mu_3\text{Tr}(Y_{Q3}^\dagger Y_{Q3}) + \mu_3\text{Tr}(Y_{L1}^\dagger Y_{L1}) + \mu_3\text{Tr}(Y_{L2}^\dagger Y_{L2}) + \mu_3\text{Tr}(Y_{L3}^\dagger Y_{L3}). \tag{B5}
\end{aligned}$$

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