

Hiding new physics at the end of field space

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Is field space infinite? If not, it either loops back on itself or ends altogether. Periodic boundary conditions are of course familiar, but field space endpoints—which appear in real-world systems—are far less explored. In this paper we argue that boundaries in field space are generic, radiatively stable structures that allow for new physics at very low scales not ruled out by experiment. Such boundaries are delocalized in field space from the vacuum, so they can only be accessed by coherent fields or high multiplicity processes, both of which are weakly constrained observationally. Low multiplicity interactions do not detect the boundary and instead perceive a “mirage cutoff” that is parametrically higher than the true cutoff of the theory. Hence, field space boundaries are deformations of the standard model that are Lorentz invariant, local, unitary at low energies, and experimentally unconstrained. We comment on the possibility of field space boundaries on the long-range force carriers and the Higgs, as well as possible implications for the hierarchy problem.

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I. INTRODUCTION

Decades of null experimental results seemingly imply that physics beyond the standard model (SM) is either (i) too massive to be produced, (ii) too feebly interacting to be detected, or (iii) simply not present. In this paper we argue for a fourth option: (iv) a theory with kinematically accessible states and sizable couplings could still be invisible to current experiments if its interactions require a parametrically high multiplicity of particles. For a bosonic field, the limit of high multiplicity corresponds to new physics that is displaced in *field space* relative to the vacuum in which experimentalists and their experiments reside. One can then subtly modify the laws of nature, for instance by introducing boundaries for the electromagnetic fields or the Higgs field, which are experimentally unconstrained simply because humans have not created or observed sufficiently *large and coherent* fields. More generally, we will argue that field space endpoints are a novel Lorentz invariant, local, and unitary deformation of any effective field theory (EFT).

The notion of new physics lurking at high multiplicity may seem contrived. After all, shoehorning some ludicrously high dimension operator of the form ϕ^n for $n \gg 1$ serves little purpose other than to generate, via quantum

corrections, a tower of lower dimension operators that are more directly accessible by experiment.

Fortunately, a more sensible incarnation of this phenomenon exists and is in fact commonplace in EFT. For an example, consider a toy universe composed of a periodic scalar field ϕ with the oscillatory potential,

$$V_{\text{osc}}(\phi) = -\epsilon\Lambda^4 \cos\left(\frac{\phi}{\Lambda}\right), \quad (1)$$

where $\epsilon \ll 1$. An experimentalist in this world painstakingly constructs successively larger colliders, diligently extracting the coefficients of higher dimension operators, $\epsilon\phi^6/\Lambda^2$, $\epsilon\phi^8/\Lambda^4$, etc., each pointing to a scale of new physics, $\epsilon^{-1/2}\Lambda$, $\epsilon^{-1/4}\Lambda$, etc. These *mirage cutoffs* are parametrically higher than the true ultraviolet (UV) cutoff, which up to log corrections in ϵ , is of order Λ and only emerges at high multiplicity [1]. In the context of periodic scalars, the idea of burying new physics at high multiplicity was studied in recent interesting work [2].

An even more pedestrian incarnation of this phenomenon occurs in a finite, effectively one-dimensional solid—also known as a bar. As reviewed in Appendix A, the phonon field ϕ is *bounded in field space* because the bar itself is finite. Mechanically, this can be implemented by an infinite potential wall [3],

$$V_{\text{wall}}(\phi) = \begin{cases} 0 & , \quad |\phi| < \bar{\phi} \\ \infty & , \quad |\phi| > \bar{\phi} \end{cases}, \quad (2)$$

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which enforces $|\phi| < \bar{\phi}$ for some positive $\bar{\phi}$. While field space endpoints may seem exotic, we emphasize that they are next of kin to periodic boundary conditions.

Let us regulate $V_{\text{wall}}(\phi)$ by introducing a finite wall thickness Λ and considering an exponential potential,

$$V_{\text{exp}}(\phi) = \Lambda^4 \exp\left(\frac{\phi^2 - \bar{\phi}^2}{\Lambda^2}\right). \quad (3)$$

As a series expansion, the potential is

$$V_{\text{exp}}(\phi) = \epsilon \Lambda^4 \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\phi^2}{\Lambda^2}\right)^n, \quad (4)$$

where $\epsilon = \exp(-\bar{\phi}^2/\Lambda^2) \ll 1$. Since $V_{\text{exp}}(\phi)$ is basically a Wick rotation of $V_{\text{osc}}(\phi)$, it also exhibits a mirage cutoff.

Are the boundaries of field space robust under quantum corrections? For the periodic scalar the answer is yes, since $V_{\text{osc}}(\phi)$ is invariant under a discrete shift symmetry that constrains its radiative corrections. On the other hand, for the bounded scalar, $V_{\text{wall}}(\phi)$ must also be radiatively stable, purely on physical grounds. After all, solids do not spontaneously implode or explode in the presence of quantum effects. This leads to a natural question: in the language of EFT, how precisely does the length of a bar remain stable under quantum corrections, and can such a mechanism be generalized [4]?

We will argue here that periodic and bounded scalars are but a subset of a much broader class of EFTs that exhibit field space boundaries and mirage cutoffs. These EFTs are defined by a potential of the general form [9],

$$V(\phi) = \epsilon \Lambda^4 \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{\phi^2}{\Lambda^2}\right)^n, \quad (5)$$

where $c_n \sim \mathcal{O}(1)$ are *arbitrary*. This potential—which is just an entire function of the field times a small spurion—is radiatively stable and induces a boundary at

$$\bar{\phi} \sim \Lambda \sqrt{\log(1/\epsilon)}. \quad (6)$$

We will also show that $2n$ -point scattering violates unitarity when the energy per particle exceeds the multiplicity-dependent mirage cutoff,

$$\Lambda_n \sim \epsilon^{-1/2n} \Lambda, \quad (7)$$

which at finite n is much larger than the true cutoff Λ . The mirage dispels for $n \gg n_*$, where we have defined

$$n_* \sim \frac{1}{2} \log(1/\epsilon) \sim \frac{1}{2} \bar{\phi}^2/\Lambda^2 \gg 1, \quad (8)$$

at which point the true cutoff of the EFT emerges. Since field space boundaries are only visible at high multiplicity,

they are very weakly constrained observationally, allowing for boundaries of the Higgs and force carrier fields of the SM associated with exceedingly low cutoffs.

II. FIELD SPACE BOUNDARY

Even though the potential $V(\phi)$ in Eq. (5) is very general, it exhibits a universal behavior: a long, flat plateau interrupted by a steep rise or fall in the potential near $|\phi| \sim \bar{\phi}$. This pattern is easily checked by plotting $V(\phi)$ for random values of $c_n \sim \mathcal{O}(1)$. Intuitively, this structure forms because at small field values, $|\phi| \ll \bar{\phi}$, the potential is dominated by ϵ and thus nearly vanishing. Near $|\phi| \sim \bar{\phi}$, the field finally overpowers ϵ , yielding swift growth. Note that the equation for $\bar{\phi}$ in terms of ϵ in Eq. (6) is derived by identifying the largest term in Eq. (5) as a function of n at fixed ϕ and then solving for the value of ϕ for which that term is of order Λ^4 .

There is a more precise argument for this universality. Given that $c_n \sim \mathcal{O}(1)$, the Cauchy-Hadamard formula implies that the radius of convergence of $V(\phi)$ is infinite,

$$\lim_{n \rightarrow \infty} \frac{\log |c_n/n!|}{n} \sim -\lim_{n \rightarrow \infty} \log n = -\infty, \quad (9)$$

so $V(\phi)$ is entire [10]. By Picard's little theorem, $V(\phi)$ must take on all possible values in the complex plane, modulo perhaps a single point. Thus $V(\phi)$ must blow up for some complex value of ϕ . Note that for our purposes, this blow up need not exhibit a formally infinite singularity, but merely that $V(\phi)$ becomes much larger than Λ^4 . For a periodic scalar, this happens at imaginary values of the field, while for a generic $V(\phi)$ it is typically on the real axis. If $V(\phi)$ is monotonically growing, for example if $c_n > 0$, then this steep rise produces a potential wall that induces a field space boundary. Of course, there exist many potentials, for example with coefficients c_n of indefinite sign, that oscillate wildly between walls and cliffs at $|\phi| \sim \bar{\phi}$. Since these potentials are oscillatory we will not consider them further [18].

We emphasize that the above arguments do not rely on precise values for c_n , so field space boundaries are generic and robust under small corrections.

III. REGIME OF VALIDITY

What is the regime of validity of the EFT defined by $V(\phi)$? Clearly, this theory breaks down for coherent field configurations near the boundary, $|\phi| \sim \bar{\phi}$ [19]. On the other hand, if the background value of ϕ is far from the boundary, for example at $\phi = 0$, then the failure of the EFT is far more subtle because its only imprint is in the high multiplicity interactions of the fluctuations about this background. At low multiplicity, the center-of-mass energy E and energy per particle $E/2n$ are relatively similar, but at high

multiplicity they are completely different. Hence, the cutoff of the EFT is not just specified by a characteristic energy—we must very precisely distinguish the total energy versus energy per particle.

As shown in Appendix C, unitarity is violated when the energy per particle exceeds $E/2n \gg \Lambda_n$, where the mirage cutoff Λ_n is defined in Eq. (7). For low multiplicities, $n \ll n_*$, the mirage cutoff is parametrically higher than the true cutoff, so $\Lambda_n \gg \Lambda$. At high multiplicities, $n \gg n_*$, we see that $\Lambda_n \sim \Lambda$ and the physical cutoff of the EFT emerges. This makes sense because the $2n$ -point interaction in the potential is only activated when all $2n$ particles in the process are imbued with sufficient energy and multiplicity to probe the boundary. Since the energy of each particle cannot exceed Λ while remaining within the EFT, we conclude that this scale is the true cutoff. All divergent loop integrals should be cut off by Λ and moreover new physics must appear at Λ to restore unitarity, which will be very important for radiative stability. Lastly, since the true cutoff only emerges at $n_* \sim \frac{1}{2} \log(1/\epsilon) \sim \frac{1}{2} \bar{\phi}^2/\Lambda^2$, unitarity violation occurs when the center-of-mass energy exceeds $E \gg \bar{\phi}^2/\Lambda$.

Let us examine the break down of the EFT in some examples. For periodic scalars it is quite common to encounter a nonperturbatively generated potential, in which case the spurion $\epsilon \sim e^{-4\pi/\alpha} \ll 1$ is exponentially small. For example, for $\alpha \sim 1/137$ set by the fine structure constant, the true cutoff emerges at enormously high multiplicities defined by $n \gg n_* \sim 861$.

Meanwhile, for a physical bar we have that $\epsilon \sim \exp(-\bar{\phi}^2/\Lambda^2)$ for fixed $\bar{\phi}$. The limit of an infinitely steep transition corresponds to a vanishing cutoff $\Lambda \rightarrow 0$. While this is naively pathological, at fixed multiplicity this limit also sends the mirage cutoff to $\Lambda_n \rightarrow \infty$, so by Eq. (8) the boundary can only be detected by processes that are of formally infinite multiplicity. This entirely decouples the boundary from observables in the trivial vacuum, so it can only be accessed by coherent fields.

IV. RADIATIVE STABILITY

The form of $V(\phi)$ is stable under quantum corrections, which in certain cases follows from symmetry. The periodic potential, $V_{\text{osc}}(\phi)$, is invariant under $\phi \rightarrow \phi + 2\pi k\Lambda$ for integer k , while the wall potential $V_{\text{wall}}(\phi)$ is invariant under $\phi \rightarrow \phi + \mu\theta(\bar{\phi} - |\phi|)$ for infinitesimal μ .

Remarkably, any potential of the form of $V(\phi)$ is radiatively stable in a broadened sense: loops will correct the potential in a way that renormalizes its numerical coefficients by $\mathcal{O}(1)$ factors but otherwise preserves its general structure. As shown in Appendix C, a quadratic divergence generated by a loop of ϕ is regulated by Λ since that is the scale at which new physics emerges to restore unitarity. Hence, the one-loop diagram involving the $2n$ -point vertex $\epsilon\Lambda^4(\phi^2/\Lambda^2)^n$ generates a contribution to

the $2(n-1)$ -point vertex of the form $\epsilon\Lambda^4(\phi^2/\Lambda^2)^{n-2}$. Since a term of this form is already accounted for in $V(\phi)$, this one-loop correction simply renormalizes the corresponding numerical coefficient. The same logic then applies to the $2(n-2)$ -point vertex and so on and so forth.

Of course, the above argument is glib about $1/16\pi^2$ loop suppression and n -dependent enhancement from Wick contractions. Instead of estimating every conceivable diagram, let us compute all radiative corrections in one go using the Coleman-Weinberg (CW) potential,

$$V_{\text{CW}}(\phi) = \frac{\Lambda^2 M^2(\phi)}{32\pi^2} + \frac{M^4(\phi)}{64\pi^2} \log\left(\frac{M^2(\phi)}{\Lambda^2}\right) + \dots, \quad (10)$$

where $M^2(\phi) = V''(\phi)$ is the field-dependent mass for the hard scalar quanta and the ellipses denote contributions beyond one loop. As before, all divergences are regulated by Λ , which is the true cutoff at which new physics intervenes. The all-loop CW potential depends on the field-dependent interaction vertices of the hard quanta, which at two-, three-, four-point are $V''(\phi)$, $V'''(\phi)$, $V''''(\phi)$, and so on. Since these interactions are vanishingly small for $|\phi| \ll \bar{\phi}$, the all-loop CW potential does not lift the flat region of the potential, confirming that field space boundaries are radiatively stable.

Since $\epsilon \ll 1$, the leading radiative corrections all involve diagrams with a single insertion of the potential. Working at linear order in ϵ , the renormalized potential coming from power law divergences at all loops is

$$V_{\text{div}}(\phi) = \exp\left(\frac{\Lambda^2}{32\pi^2} \frac{d^2}{d\phi^2}\right) V(\phi) = \frac{\sqrt{8\pi}}{\Lambda} \int_{-\infty}^{\infty} d\phi' \exp\left(-\frac{8\pi^2(\phi' - \phi)^2}{\Lambda^2}\right) V(\phi'), \quad (11)$$

where $d^2/d\phi^2$ generates quadratically divergent loops that each evaluate to $\Lambda^2/16\pi^2$. Equation (11) implies that the renormalized potential $V_{\text{div}}(\phi)$ is just the bare potential $V(\phi)$ convoluted with a Gaussian *diffusion filter* that blurs any features in field space shorter than $\Lambda/4\pi$. Since the potential wall has thickness Λ , this preserves the location of the field space boundary. Note also that the Gaussian kernel will always map the entire function $V(\phi)$ to another entire function $V_{\text{div}}(\phi)$. In Appendix D we show in an explicit example how all loop power divergences preserve the size of the series coefficients, so $c_n \sim \mathcal{O}(1)$.

Bear in mind that the potential interactions in this EFT will generate derivative interactions of the form [20]

$$\epsilon\Lambda^4 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{c_{n,m}}{n!m!} \left(\frac{\phi^2}{\Lambda^2}\right)^n \left(\frac{\partial\phi\partial\phi}{\Lambda^4}\right)^m. \quad (12)$$

As usual, we should expect all operators consistent with symmetries—including derivatives—with all dimensionful

quantities written in units of the cutoff Λ and suppressed by ϵ [21]. Naively, these derivative interactions induce boundaries not just for ϕ , but also for $\partial\phi$, $\partial^2\phi$, etc. However, since the energy per particle is less than Λ within the EFT, the latter boundaries are not physical.

V. HIERARCHIES

Up until now we have considered the very simplest case in which the whole potential is $V(\phi)$. However, the plot thickens if we consider the extended potential, $U(\phi) = V(\phi) + \tilde{V}(\phi)$, where $\tilde{V}(\phi)$ is a set of arbitrary, possibly higher dimension operators. Importantly, the contribution from $V(\phi)$ generates a wall at $|\phi| \sim \bar{\phi}$, so this extended potential still exhibits a field space boundary. Furthermore, while $\tilde{V}(\phi)$ may have sizable interactions, all contributions to the CW potential involving $V(\phi)$ or its derivatives will be suppressed for $|\phi| \ll \bar{\phi}$, so like before the field space boundary is stable.

In general, there is another cutoff $\tilde{\Lambda}$ associated with the higher dimension operators and UV divergences in $\tilde{V}(\phi)$. Physically, $\tilde{\Lambda}$ is a proxy for the scale of new physics that UV completes $\tilde{V}(\phi)$. Since ϕ is a scalar, it suffers from a “hierarchy problem” [23] because its mass receives quadratically divergent corrections of the form $\tilde{\Lambda}^2/16\pi^2$.

If the resulting hierarchy problem is independent of the microphysics associated with the field space boundary, then $\tilde{\Lambda}$ and Λ are independent quantities. Furthermore, if $\tilde{\Lambda} \ll \Lambda$, then the true cutoff Λ of the boundary potential—as well as all of the associated mirage cutoffs—is parametrically higher than $\tilde{\Lambda}$. Even if the boundary potential $V(\phi)$ receives loop corrections cut off by $\tilde{\Lambda}$, these contributions are subleading to those cut off by Λ , so the field space endpoint is robust. On the other hand, if $\tilde{\Lambda} \gg \Lambda$ then one might worry that loops that are quadratically divergent in $\tilde{\Lambda}$ could destabilize the boundary potential $V(\phi)$. This is avoided, however, since whatever mechanism cancels the quadratically divergent contributions to ϕ mass—be it supersymmetry, compositeness, or simply fine-tuning—also cancels the corresponding divergences in $V(\phi)$, diagram by diagram.

A very speculative but also interesting possibility is that the boundary is linked to the hierarchy problem, in which case $\tilde{\Lambda} \sim \Lambda$. We return to this possibility later on.

VI. BEYOND THE STANDARD MODEL

Higgs Field. The Higgs field H plays a central role in the SM. Its vacuum expectation value (VEV) has been inferred to be $\langle H \rangle \sim 246$ GeV from nearly a century of experiments probing the masses and interactions of the quarks, leptons, photons, and electroweak gauge bosons [24]. The Higgs VEV is far and away the largest field configuration ever to be accessed experimentally by humans, albeit indirectly. The fluctuations of the Higgs field about its VEV are the

Higgs particle, whose existence was spectacularly confirmed at the LHC [25,26].

In spite of this, current experiments have not accessed coherent Higgs fluctuations about its VEV. The LHC has only sensitively probed interactions involving a single Higgs boson. More generally, low multiplicity interactions will at best reveal the local curvature of Higgs field space [27]. As long as the boundary in Higgs field space is larger than $\langle H \rangle$, it will be experimentally invisible, appearing only in high multiplicity Higgs interactions.

This all offers motivation to modify the field space of the Higgs and ask about its possible experimental repercussions. Our EFT analysis has shown that a gauge invariant potential of the general form

$$V(H) = \epsilon \Lambda^4 \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{|H|^2}{\Lambda^2} \right)^n, \quad (13)$$

induces a field space boundary and is radiatively stable. Concretely, consider $U(H) = V(H) + \tilde{V}(H)$, where

$$\begin{aligned} V(H) &= \Lambda^4 \exp\left(\frac{|H|^2 - \bar{H}^2}{\Lambda^2}\right) \\ \tilde{V}(H) &= \frac{\lambda}{4} |H|^4 - m^2 |H|^2, \end{aligned} \quad (14)$$

has an exponential wall with $\epsilon = \exp(-\bar{H}^2/\Lambda^2) \gg 1$, together with the usual SM potential. Electroweak symmetry breaking that makes direct use of a Higgs boundary does not appear to be viable [28], so we will assume hereafter that $\bar{H} > \langle H \rangle$ is safely above the VEV. Of course, such a boundary could still play a role in the strength of the electroweak phase transition, which is critical for models of baryogenesis.

Let us comment on possible connections between field space boundaries and the electroweak hierarchy problem. The Higgs mass parameter receives quadratically divergent corrections, $m^2 \sim \tilde{\Lambda}^2/16\pi^2$, indicating sensitivity to physical states at the UV cutoff $\tilde{\Lambda}$ associated with the SM. We saw earlier what happens when $\tilde{\Lambda}$ and Λ are independent scales. The hierarchy problem is independent of the boundary dynamics and consequently the field space endpoints are radiatively stable.

A more exotic option is that $\tilde{\Lambda} \sim \Lambda$ are linked. This offers a solution to the hierarchy problem reminiscent of large extra dimensions [29]: if the true cutoff of the SM is $\Lambda \sim 1$ TeV then the Higgs mass parameter m^2 is actually *natural*. However, unlike the scenario of large extra dimensions, the degrees of freedom associated with a boundary potential would be exceedingly difficult to observe. In particular, they would have evaded detection simply because they only appear in very high multiplicity interactions that are inaccessible to direct collider probes and indirect signatures from precision electroweak observables

or flavor violation. By the same token, new physics of this kind should couple exceedingly weakly to the Higgs bilinear $|H|^2$, so it strains the imagination to conceive of how those degrees of freedom could ever cancel the divergences of the Higgs at a mechanical level. Hence, such a scenario seems implausible.

Force fields. Next, consider the case of a boundary for the electromagnetic field. A version of this idea has existed for almost a century in the form of the Born-Infeld (BI) model, which is a nonlinear generalization of electromagnetism with the Lagrangian,

$$L(F) = -\Lambda^4 \sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu}/\Lambda^2)}. \quad (15)$$

As is well-known, BI theory regulates the Coulomb divergence of point charges by imposing a fundamental limit on the electromagnetic field strength. In BI theory, the parameter Λ plays dual roles as the boundary of field space *and* the energy scale at which photon scattering at any multiplicity violates unitarity.

Instead, let us consider the general Lagrangian for the electromagnetic field strength [30],

$$L(F) = \epsilon \Lambda^4 \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{F^2}{\Lambda^4} \right)^n. \quad (16)$$

In the SM, this Lagrangian should technically be defined in terms of the hypercharge or electroweak gauge bosons, but we can ignore this caveat for now. As shown in Appendix E, the mirage cutoff is $\Lambda_n \sim \Lambda \epsilon^{-1/4n}$, the boundary for the electromagnetic field is induced at $\bar{F} = \Lambda^2 \sqrt{\log(1/\epsilon)}$, and the critical multiplicity at which this endpoint can be detected corresponds to $n \gg n_* \sim \frac{1}{4} \log(1/\epsilon) \sim \frac{1}{4} \bar{F}^2 / \Lambda^4$ [31]. For processes at this multiplicity, unitarity is violated when the energy per particle exceeds $E/2n \gg \Lambda$, which corresponds to a center-of-mass energy $E \gg \frac{1}{2} \Lambda \log(1/\epsilon) \sim \frac{1}{2} \bar{F}^2 / \Lambda^3$.

What values of Λ and $\bar{F}$ are phenomenologically permitted? In principle, this setup is constrained cosmologically by the successful predictions of big bang nucleosynthesis, which are certainly untouched if the cutoff is higher than the temperature, $\Lambda \gtrsim 1$ MeV. It is a detailed question whether the opposite regime is ruled out.

Meanwhile, a leading constraint from collider physics is from light-by-light scattering induced by the interaction $\epsilon F^4 / \Lambda^4$. Measurements of relativistic heavy-ion collisions at the LHC imply a lower bound on the effective cutoff of the F^4 operator of order ~ 100 GeV [32–34], which translates into the constraints,

$$\Lambda \gtrsim \epsilon^{1/4} (100 \text{ GeV}) \quad \text{and} \quad \bar{F} \gtrsim \epsilon^{1/2} \log(1/\epsilon) (100 \text{ GeV})^2, \quad (17)$$

which are easily evaded by cranking down $\epsilon \ll 1$.

For example, consider the case of a preposterously low cutoff $\Lambda \sim 5$ GeV with a tiny spurion $\epsilon \sim e^{-2000}$, which can easily arise nonperturbatively. A cutoff at this scale is naively ruled out by experiment—after all, we have measured a litany of SM processes and particle masses that clearly exceed Λ . However, the crucial point is that Λ only emerges in $2n$ -point processes that exceed the critical multiplicity of $2n \gg 1000$ photons. This implies that the EFT breaks down for center-of-mass energies exceeding $E \gg 5$ TeV. Processes below this threshold energy are simply kinematically forbidden from producing a sufficient multiplicity of photons that have sufficient energy to violate unitarity in the EFT.

In this example the mirage cutoff associated with the F^4 operator is $\Lambda \epsilon^{-1/4} \sim 10^{218}$ GeV, which is obviously safe from the bounds in Eq. (17). We leave for future work how these models are constrained by limits on “soft bombs” at the LHC [35,36]. Furthermore, the bound on the electromagnetic field is $\bar{F} \sim (33 \text{ GeV})^2$. This is still much higher than the strongest electromagnetic fields ever experimentally observed, like those in magnetars or planned beam experiments [37]. Note that a field of this size is screened by Schwinger pair production, so a stable electromagnetic field at the boundary can only exist in models where $\bar{F}$ is below the e^+e^- mass threshold.

Similar logic applies to field space boundaries for the gluon field, which are implemented by a Lagrangian like Eq. (16) but for the gluon field strength. The leading constraint from the LHC applies to the anomalous gluon trilinear coupling G^3 , while an unscathed QCD phase transition would require Λ and $\bar{G}$ to be above the QCD scale. Last but not least, a boundary in the context of gravity can be induced by a version of Eq. (16) for the Riemann curvature. Since direct constraints on R^3 are exceedingly weak, Λ and $\bar{R}$ can again be very low.

VII. DISCUSSION

A boundary in field space introduces new physics, possibly at a very low scale, that is sequestered from the vacuum and invisible except through coherent field configurations and high multiplicity interactions. These are very weakly constrained by experiments, so field space boundaries are a viable Lorentz invariant, local, and unitary deformation of the SM. As one would expect from the very existence of finite solids, field space boundaries are radiatively stable and robust under small corrections. This work leaves several avenues for future exploration.

First and foremost is the challenge of devising a fully UV complete theory which exhibits a field space boundary and its accompanying features at high multiplicity. Seemingly related ideas have appeared in string theoretic models of inflation [12,38,39] that generate observationally relevant tails in high multiplicity correlation functions. It would be illuminating to explore whether these UV completions

preserve the phenomenon of the mirage cutoff, so the true cutoff is hidden even from low multiplicity derivative interactions.

Second, it would be interesting to explore the interplay between field space boundaries and symmetry. For example, the wall potential in Eq. (2) describes a scalar theory with a shift symmetry that is explicitly broken in a way that is *delocalized in field space*. The breaking of the shift symmetry is not soft—indeed it is achieved by an infinite tower of higher dimension operators. Hence, field space boundaries somehow achieve *hard breaking* that is radiatively stable. It would be interesting to apply this mechanism to other symmetries. For instance, perhaps Lorentz symmetry could be explicitly broken nonlocally on the field space boundary without inducing relevant Lorentz violating operators, in the spirit of extradimensional sequestering.

Finally, there is the question of whether field space boundaries can enrich any physics beyond the SM that directly utilizes large coherent fields. Here some possible targets of study are modifications of the QCD and electroweak phase transitions, baryogenesis, Higgs vacuum instability, and inflation model building.

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APPENDIX A: PHYSICAL BAR

A great deal of insight into field space boundaries can be gleaned from the physical example of a finite solid. This real-world system gives credence to the idea that field space endpoints are not actually all that exotic. In fact, they are quite ordinary.

To illustrate this point, let us review the EFT description of a thin, solid bar of finite extent [40]. This system exhibits longitudinal phonons which are effectively one-dimensional. These degrees of freedom are described by the comoving coordinates $\phi(x, t)$, which at a time t map the point x in physical space to a point ϕ in the internal space of labeled matter elements. Importantly, for an object of finite volume, the domain of allowed labels ϕ is bounded. We emphasize that this should not be viewed as a bound on the spacetime coordinates themselves.

The field space boundary is implemented by multiplying the EFT Lagrangian by a step function,

$$S = \int d^4x \theta(\bar{\phi} - \phi) L(\partial\phi). \quad (\text{A1})$$

Aside from the end points of the bar, ϕ is derivatively coupled. This makes sense because it is the Nambu-Goldstone boson of broken space translations. Since the value of ϕ is free to vary at the endpoints, we have implicitly chosen open boundary conditions. In contrast, clamping the bar would fix the value of ϕ at the ends. However, the case of open boundary conditions is of particular interest because it leads to a dynamical relaxation of certain relevant operators [40].

The EFT of a bar is valid up to a scale $\tilde{\Lambda}$ corresponding to the appearance of short distance modes associated with the finite thickness of the bar. Above this scale, this theory is UV completed into a fully three-dimensional solid which in turn has an EFT cutoff given by the atomic spacing.

The finite bar system offers some insight into the radiative stability of field space boundaries. Since the step function in Eq. (A1) explicitly breaks the shift symmetry of ϕ , it is natural to expect that radiative corrections will compound this breaking. However, it is important to realize that the original action is still *exactly invariant* under the nonlinear shift symmetry $\phi \rightarrow \phi + \mu\theta(\bar{\phi} - \phi)$ for infinitesimal parameter μ . To preserve this symmetry, all radiatively generated terms must be localized to the ends of the bar, so they are proportional to $\delta(\phi - \bar{\phi})$ and its derivatives. Physically, these terms encode the surface microphysics residing at the ends of the bar. Crucially, we see that the location of the endpoint in field space—namely, the end of the bar—is radiatively stable.

In a physically realistic situation, the ends of the bar will also taper off at some rate. We can model this effect by regulating the step function to

$$\theta(\bar{\phi} - \phi) = \lim_{\Lambda \rightarrow 0} \left(1 - \tanh\left(\frac{\bar{\phi} - \phi}{\Lambda}\right) \right), \quad (\text{A2})$$

where Λ is the tapering scale. Note that at momenta of order Λ , new physics comes into play which is dictated by the precise shape of the ends. In general, we expect that $\Lambda \lesssim \tilde{\Lambda}$ since the tapering length is bounded by the thickness of the bar.

APPENDIX B: BALLS, SPRINGS, STRINGS

Even though field space boundaries appear in physical systems, it is not easy to write down an explicit UV complete Lagrangian for these dynamics. Nevertheless, it is relatively straightforward to devise a many body system whose mean field approximation exhibits a field space boundary. Here we present a simple example of this constructed from balls, springs, and strings.

Consider an infinite one-dimensional infinite of balls of mass m , each connected to its neighbor by a spring with

constant k . The Lagrangian is

$$L = \sum_n \frac{1}{2} m \dot{x}_n^2 - \frac{1}{2} k (x_{n+1} - x_n - a)^2, \quad (\text{B1})$$

where a is the equilibrium distance between balls. Transforming to coordinates describing the deviations from equilibrium positions

$$x_n = na + y_n, \quad (\text{B2})$$

we obtain the resulting Lagrangian

$$L = \sum_n \frac{1}{2} m \dot{y}_n^2 - \frac{1}{2} k (y_{n+1} - y_n)^2. \quad (\text{B3})$$

At this point we add an additional force to the system. Imagine that each ball is tethered its associated equilibrium point by a string of fixed length $\bar{y}$, so field space is effectively bounded by $|y_n| < \bar{y}$. The Lagrangian is

$$L = \sum_n \frac{1}{2} m \dot{y}_n^2 - \frac{1}{2} k (y_{n+1} - y_n)^2 - U(y_n), \quad (\text{B4})$$

where we have defined the wall potential

$$U_{\text{wall}}(y_n) = \begin{cases} 0 & , \quad |y_n| < \bar{y} \\ \infty & , \quad |y_n| > \bar{y} \end{cases}, \quad (\text{B5})$$

which leashes each ball to its equilibrium point in a way that exerts no force unless the string becomes taut.

Moving to the continuum limit, we send

$$\begin{aligned} y_n &\rightarrow y(x) \\ y_{n+1} - y_n &\rightarrow a \frac{\partial}{\partial x} y(x) \\ \sum_n &\rightarrow \frac{1}{a} \int dx, \end{aligned} \quad (\text{B6})$$

so the Lagrangian becomes an integral over space,

$$L = \int dx \left[\frac{1}{2a} m \dot{y}^2 - \frac{1}{2} k a \left(\frac{\partial y}{\partial x} \right)^2 - \frac{1}{a} U_{\text{wall}}(y) \right]. \quad (\text{B7})$$

Next, we rescale the field to go to our final field variable,

$$\phi(x) = \sqrt{\frac{m}{a}} y(x), \quad (\text{B8})$$

and the final formula for the Lagrangian is

$$L = \int dx \left[\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} v^2 \left(\frac{\partial \phi}{\partial x} \right)^2 - V_{\text{wall}}(\phi) \right], \quad (\text{B9})$$

where we have defined the velocity squared $v^2 = ka^2/m$ and the potential $V_{\text{wall}}(\phi) = U_{\text{wall}}(y)/a$.

APPENDIX C: UNITARITY VIOLATION

The precise definition of the cutoff of an EFT is given by the energy scale at which scattering amplitudes violate unitarity. Here we work through an explicit calculation that examines this limit more carefully.

To start, let us include a source term J that couples to the boundary potential $V(\phi)$ via

$$J \times \epsilon \Lambda^4 \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{\phi^2}{\Lambda^2} \right)^n. \quad (\text{C1})$$

Here J is a proxy for some incoming degrees of freedom that merge to produce a spray of ϕ particles, so we are interested in the $2n$ -point process $J \rightarrow \phi^{2n}$ where $n \gg 1$. For example, J could represent a field decaying into ϕ , or a pair of fields annihilating into ϕ .

The rate for the $2n$ -point process is proportional to

$$R(J \rightarrow \phi^{2n}) = \frac{\kappa}{(2n)!} \int d\Phi_{2n} |A_{2n}|^2, \quad (\text{C2})$$

where κ is a constant and $d\Phi_{2n}$ is the phase space density for the final state of $2n$ particles,

$$d\Phi_{2n} = \delta^4 \left(p - \sum_{i=1}^{2n} p_i \right) \prod_{i=1}^{2n} d^4 p_i \delta(p_i^2) \theta(p_i^0), \quad (\text{C3})$$

and the corresponding scattering amplitude is

$$A_{2n} = \frac{(2n)!}{n!} \frac{\epsilon c_n}{\Lambda^{2n-4}}. \quad (\text{C4})$$

Since the amplitude is constant, we can use the explicit formula for the volume of phase space [41],

$$\int d\Phi_{2n} = \left(\frac{\pi}{2} \right)^{2n-1} \frac{E^{4n-4}}{(2n-1)!(2n-2)!} \sim (E/2n)^{4n}, \quad (\text{C5})$$

where E is the center-of-mass energy and the last line shows the leading scaling in the high multiplicity limit $2n \gg 1$. This formula encodes the intuition that the center-of-mass energy is finely partitioned into an approximate energy per particle $E/2n$, so phase space contracts accordingly at high multiplicity. Putting everything together, we obtain

$$\begin{aligned} R(J \rightarrow \phi^{2n}) &= \frac{4\kappa\epsilon^2\Lambda^8}{\pi E^4} \frac{c_n^2}{n!(n-1)!(2n-2)!} \left(\frac{\pi^{1/2} E}{4^{1/4} \Lambda} \right)^{4n} \\ &\sim \left(\frac{E/2n}{\epsilon^{-1/2n}\Lambda} \right)^{2n} \sim \left(\frac{E/2n}{\Lambda_n} \right)^{2n}, \end{aligned} \quad (\text{C6})$$

where we have defined the mirage cutoff,

$$\Lambda_n \sim \epsilon^{-1/2n} \Lambda, \quad (\text{C7})$$

which is in agreement with Eq. (7).

Let us unpack the physical significance of Eq. (C6). To maintain unitarity at $2n \gg 1$, the quantity in parentheses in Eq. (C6) must be smaller than one, so the energy per particle must be below the mirage cutoff, so

$$E/2n \lesssim \Lambda_n. \quad (\text{C8})$$

From the n dependence in Λ_n we see immediately that the dynamics depend sensitively on whether we consider low or high multiplicity processes, corresponding to $n \ll n_*$ and $n \gg n_*$, respectively, where

$$n_* \sim \frac{1}{2} \log(1/\epsilon) \sim \frac{1}{2} \bar{\phi}^2 / \Lambda^2 \gg 1, \quad (\text{C9})$$

and the induced boundary in field space is at

$$\bar{\phi} \sim \Lambda \sqrt{\log(1/\epsilon)}. \quad (\text{C10})$$

At low multiplicities, the perceived cutoff is parametrically higher than Λ , which is the phenomenon of the mirage cutoff. Meanwhile, at high multiplicities the mirage dispels since $\Lambda_n \sim \Lambda$ and the true cutoff emerges. In this regime the energy per particle is bounded by $E/2n \lesssim \Lambda$ to remain within the regime of validity of the EFT. In conclusion, we learn that Λ is the physical cutoff of the EFT, serving as a proxy for the energy scale of new physics that enters to unitarize scattering. For this reason, all divergent loop integrals in the EFT should be regulated by Λ as well.

Note that the unitarity bound in Eq. (C8) also implies that the center-of-mass energy cannot be arbitrarily large within the EFT. In particular, we find that

$$E \lesssim 2n\Lambda_n \sim \Lambda \log(1/\epsilon) \sim \bar{\phi}^2 / \Lambda, \quad (\text{C11})$$

where we have minimized the right-hand side of the inequality on n and then plugged in Eq. (6).

The true cutoff Λ is only visible in high multiplicity processes. One might then worry that perturbation theory—which is an asymptotic expansion—could break before attaining sufficiently high multiplicity. However, it should be noted that the degeneration of the asymptotic series occurs when there is a proliferation of Feynman diagrams contributing to n -point scattering. For processes involving the boundary potential, however, each insertion of a potential interaction is accompanied by an additional factor of the spurion $\epsilon \ll 1$. So scattering events with more than one insertion of a boundary potential interaction are dramatically suppressed, and hence the growth of Feynman diagrams is ameliorated.

APPENDIX D: POWER LAW DIVERGENT CORRECTIONS

We have argued that the general structure of the potential in Eq. (5) is radiatively stable. Here we demonstrate this in an explicit example by computing the *power law divergent* corrections to the exponential potential,

$$V_{\text{exp}}(\phi) = \epsilon \Lambda^4 \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\phi^2}{\Lambda^2} \right)^n, \quad (\text{D1})$$

working to all loop orders. Of course, power divergences are famously scheme dependent—for example, in dimensional regularization they vanish entirely. For our analysis here we will regulate quadratic divergences as $\Lambda^2/16\pi^2$, where Λ is a proxy for the scale at which unitarity is violated and new physics must enter. As we will see, in this scheme the power law divergent loop corrections renormalize the numerical coefficients of the potential while maintaining its form.

Since ϵ is small, it suffices to compute to leading order in this parameter. The radiative correction to the $2k$ -point vertex from the $2n$ -point vertex with $2(n-k)$ legs closed up into an $(n-k)$ -loop diagram is

$$\left[\frac{\phi^{2k}}{(2k)!} \right] \left[\frac{(2n)!}{n!} \frac{\epsilon}{\Lambda^{2n-4}} \right] \left[\frac{\Lambda^2}{16\pi^2} \right]^{n-k} \left[\frac{1}{2^{n-k}(n-k)!} \right], \quad (\text{D2})$$

where every loop is a quadratically divergent tadpole. Of the terms in square brackets, the first is the operator corresponding to the $2k$ -point interaction, the second is the $2n$ -point vertex, the third is the product of quadratically divergent loop integrals each taken to be $\Lambda^2/16\pi^2$, and the fourth is the symmetry factor of the $(n-k)$ -loop diagram. Summing Eq. (D2) for $n = k, \dots, \infty$, we obtain a closed formula for the exponential potential corrected by power law divergences at all loop orders,

$$V_{\text{exp,div}}(\phi) = \epsilon \Lambda^4 \sum_{n=0}^{\infty} \left(1 - \frac{1}{8\pi^2} \right)^{-n-1/2} \frac{1}{n!} \left(\frac{\phi^2}{\Lambda^2} \right)^n, \quad (\text{D3})$$

which agrees with Eq. (11). Note that the factor of $1/8\pi^2$ is scheme dependent because it is proportional to the squared ratio of the cutoff that regulates quadratic divergences divided by the cutoff that controls the higher dimension operators in the potential. For simplicity we have chosen both of these cutoffs to be exactly Λ . This is the scale of unitarity violation induced by the latter, which in turn fixes the scale of new physics that defines the former. In any case, since the renormalized potential has the same form as Eq. (D1), the field space boundary is radiatively stable.

APPENDIX E: ELECTROMAGNETIC BOUNDARIES

We asserted in Sec. VI that a boundary for the electromagnetic field can be spontaneously generated by a Lagrangian of the schematic form,

$$L(F) = \epsilon \Lambda^4 \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{F^2}{\Lambda^4} \right)^n. \quad (\text{E1})$$

Here we spell out an explicit construction.

To start, we assume that the Lagrangian is some unknown function $L(\sigma)$ of the scalar invariant

$$\sigma = \frac{F^2}{\Lambda^4} = \frac{2}{\Lambda^4} (\mathbf{B}^2 - \mathbf{E}^2). \quad (\text{E2})$$

Since this EFT is defined in terms of a Lagrangian rather than a potential, it is not guaranteed that the Hamiltonian is bounded from below, as required by consistency. The Hamiltonian for this general theory is,

$$H = \mathbf{E} \cdot \frac{\partial L}{\partial \mathbf{E}} - L = -\frac{4\mathbf{E}^2}{\Lambda^4} L'(\sigma) - L(\sigma), \quad (\text{E3})$$

after dropping total derivatives and terms proportional to the Gauss law constraint. A sufficient condition for a Hamiltonian that is bounded below is that $L(\sigma) \leq 0$ and $L'(\sigma) \leq 0$. These conditions are satisfied for any Lagrangian that is negative and monotonically decreasing, for example like $L(\sigma) \sim -\exp(\sigma)$. This class of functions includes the electromagnetic analog of Eq. (3),

$$L(F) = -\Lambda^4 \exp\left(\frac{F^2 - \bar{F}^2}{\Lambda^4}\right), \quad (\text{E4})$$

where $\bar{F}$ is the boundary of the electromagnetic field. Like before, we can series expand the Lagrangian,

$$L(F) = -\epsilon \Lambda^4 \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{F^2}{\Lambda^4} \right)^n, \quad (\text{E5})$$

where $\epsilon = \exp(-\bar{F}^2/\Lambda^4) \ll 1$ is a tiny spurion.

Next, let us determine the regime of validity of this EFT. In analogy with Eq. (C1), we couple a source J to the electromagnetic field boundary,

$$-J \times \epsilon \Lambda^4 \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{F^2}{\Lambda^4} \right)^n, \quad (\text{E6})$$

and consider the $2n$ -point process $J \rightarrow \gamma^{2n}$. While the associated rate is too cumbersome to calculate exactly, it can be easily estimated. Approximating the phase space integral by Eq. (C5) and the scattering amplitude by

$$A_{2n} \sim \frac{(2n)!}{n!} \frac{\epsilon c_n}{\Lambda^{4n-4}} \left(\frac{E}{2n} \right)^{2n}, \quad (\text{E7})$$

we obtain the rate

$$R(J \rightarrow \gamma^{2n}) \sim \left(\frac{E/2n}{\epsilon^{-1/4n} \Lambda} \right)^{8n} \sim \left(\frac{E/2n}{\Lambda_n} \right)^{8n}, \quad (\text{E8})$$

where we have defined the mirage cutoff for the electromagnetic field to be

$$\Lambda_n \sim \epsilon^{-1/4n} \Lambda. \quad (\text{E9})$$

Like before, we find that unitarity requires that the energy per particle be less than the mirage cutoff, so

$$E/2n \lesssim \Lambda_n. \quad (\text{E10})$$

Again, the n dependence in Λ_n defines a critical multiplicity for which n exceeds the value

$$n_* \sim \frac{1}{4} \log(1/\epsilon) \sim \frac{1}{4} \bar{F}^2/\Lambda^4. \quad (\text{E11})$$

Last not least, the remain within the regime of validity of the EFT, the center-of-mass energy is bounded by

$$E \lesssim 2n\Lambda \sim \frac{1}{2} \Lambda \log(1/\epsilon) \sim \frac{1}{2} \bar{F}^2/\Lambda^3, \quad (\text{E12})$$

where we have again minimized the right-hand side of the inequality as a function of n .

[1] This statement applies to the potential, but in general one must also grapple with low multiplicity derivative interactions that are set by the scale Λ without any additional suppression by ϵ . In this case the true cutoff Λ is accessible at low multiplicity and there is no mirage. For example, unsuppressed interactions of this kind appear in many UV

completions of the axion after integrating out an accompanying radial Higgs mode or tower of extradimensional excitations. Still, it is still striking that the axion potential exhibits a mirage cutoff.

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- [3] Alternatively, we can enforce a boundary on field space by multiplying the kinetic energy term by a step function $\theta(\bar{\phi} - \phi)$. The physical equivalence between an infinite wall in the potential energy versus a step function in the kinetic energy is analogous to what happens in Einstein versus Jordan frame in gravity. For a detailed review of the EFT of phonons in a physical bar, see Appendix A. In Appendix B we also present a simple model of balls, springs, and strings whose mean field approximation exhibits a boundary in field space.
- [4] Notably, boundaries in *coordinate space* have long appeared in the context of gravity as geometric cutoffs or “Dirichlet walls.” Since these boundaries are not dynamical branes, they need not be stabilized, but at the same time their consistency is not guaranteed. Nevertheless, under certain conditions, these endpoint objects have been found to be stable [5–7] and consistent with the initial value problem [8].
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- [9] Here we define our potential to be a series expansion in $(\phi^2/\Lambda^2)^n$ with coefficients that scale as $1/n!$, as opposed to a series expansion in $(\phi/\Lambda)^n$ with coefficients that scale as $1/n!$. We do this for two reasons. First, our phenomenological applications all involve field space boundaries for charged fields like the Higgs boson or the SM force carriers, which only appear in even powers in their self-interactions. The second reason has to do with a minor subtlety regarding the scale of unitarity violation. A general potential defined by a series expansion in $(\phi/\Lambda)^n$ with coefficients $1/n!$ will cover the familiar case of a periodic scalar, as well as the exponential runaway of a dilaton field. However, as shown in [2], these models exhibit scattering which at *any* multiplicity will never see a breakdown of the EFT at the scale Λ but rather at the slightly higher cutoff $\Lambda\sqrt{\log(1/\epsilon)}$, which can be realized in explicit UV models [2]. Of course, these factors of $\log(1/\epsilon)$ corrections are far less important than the powers of ϵ that generate the phenomenon of the mirage cutoff. Nevertheless, for simplicity we restrict here to a potential which is a series expansion in $(\phi^2/\Lambda^2)^n$ with coefficients $1/n!$, which as shown in Appendix C has a physical cutoff defined by unitarity violation in scattering that is precisely the scale Λ , with no additional factors of $\log(1/\epsilon)$. Note that in general, changing the power of ϕ in the series expansion has the same effect as changing the scaling of the series coefficient. So for example, a series expansion in $(\phi^k/\Lambda^k)^n$ with coefficients $1/n!$ is the same as a series expansion in $(\phi/\Lambda)^n$ with coefficients $(1/n!)^{1/k}$.
- [10] It may be concerning that this potential—like that of the axion—is an entire function. In particular, there is evidence that the generating functional of connected correlators $W[J]$ cannot be entire [11,12], which is related to the asymptotic nature of the perturbative series [13–16]. Furthermore, in certain explicit examples the operator product expansion can also be asymptotic [17]. Nevertheless, we note that these degenerations at high multiplicity all stem from the growth of diagrammatic topologies in *correlators*, while the potential defines individual *interaction vertices*. As shown in Appendix C, any contribution to a correlators involving multiple insertions of the potential interactions will be subleading since $\epsilon \ll 1$. Consequently, the growth in diagrams is tamed and any subtleties concerning the asymptotic nature of perturbation theory are not relevant.
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- [19] One should bear in mind that the EFT transitions from the weak field to strong field regime long before ϕ reaches $\bar{\phi}$. To see why, recall that the weak field regime by definition applies when perturbation theory in the field value is accurate. This implies that the n -point interaction in $V(\phi)$ must be more important than the $(n+1)$ -point interaction. From $V(\phi)$ it is clear that this transition occurs at the critical field value $|\phi| \sim \Lambda \ll \bar{\phi}$.
- [20] The potential terms generate derivative interactions in the same way as in ϕ^4 theory, where wave function renormalization appears at two loops.
- [21] What is the natural size for these derivatively coupled interactions? Consider, for example, the $(\partial\phi)^4$ operator, whose coefficient is famously constrained to be nonnegative by analyticity, unitarity, and causality [22]. In particular, this coefficient is equal to an energy weighted integral over the total cross-section for $\phi\phi \rightarrow \chi$, where χ denotes the heavy states in the UV completion. To preserve the phenomenon of the mirage cutoff, the coefficient of $(\partial\phi)^4$ must be suppressed by $\epsilon \ll 1$, which implies that χ must couple very weakly to a pair of ϕ particles. This is very different than what happens in traditional UV completions, like the linear

- sigma model completion of a periodic scalar. Conversely, an EFT with a mirage cutoff can only arise in a UV completion in which the heavy state χ only couples to a *high multiplicity* of ϕ particles.
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