

Exclusive photoproduction of $\chi_c\gamma$ pairs in the small- x kinematics

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In this manuscript we extend our previous analysis of the exclusive photoproduction of the $\chi_c\gamma$ pairs to the high-energy (small- x) kinematics, evaluate in detail the cross section in the color glass condensate framework, and discuss the potential of this channel as a novel complementary probe of the hadronic interactions. In the leading order in strong coupling α_s , the cross section is controlled by the dipole scattering amplitude. Using available phenomenological parametrizations of the latter, we estimated numerically the cross sections for different polarizations of χ_c mesons and final-state photons in the kinematics of the ultraperipheral collisions at LHC and the future Electron Ion Collider. We found that the expected number of produced and detected events is significant for experimental study. We suggest that the ratio of the cross sections with different spins $\sigma_{\gamma\chi_{c1}}/\sigma_{\gamma\chi_{c2}}$ can be used to test the validity of the heavy quark spin symmetry relations for χ_c , and in this way resolve a long-standing ambiguity observed in hadroproduction of χ_c mesons. Finally, we also analyzed the role of this process as a potential background to exclusive χ_c photoproduction, which has been recently suggested as an alternative channel for studies of odderons.

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I. INTRODUCTION

The color glass condensate (CGC) approach [1–5] has been established as a theoretical framework for systematic analysis of the hadronic processes in the small- x kinematics. It naturally incorporates saturation effects and allows us to obtain a self-consistent and phenomenologically successful description of various lepton-hadron and hadron-hadron processes [6–21]. The interactions of partons with the target in this approach are encoded in the nonperturbative n -point correlators of Wilson lines (multipole scattering amplitudes), whose dependence on rapidity obeys the evolution equations of Balitsky-Kovchegov, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, and Kovner (BK-JIMWLK) [22–32]. In the presence of an appropriate hard scale which controls the transverse size of the system, it is expected that the dominant contribution stems from the quark-antiquark component of the wave function, convoluted with a forward dipole scattering amplitude. On this premise, a number of processes has been used in phenomenological analyses in

order to fix the forward dipole scattering amplitude. However, the higher order multipoles may contribute in loop corrections even in the simplest and well-known processes, like deep inelastic scattering (DIS) [33,34]. A recent analysis of the next-to-leading order (NLO) corrections to various processes demonstrated that they could be numerically sizable [35–38], thus shedding doubts on phenomenological extractions based on the leading order picture.

While it is impossible to switch off completely the contributions of the higher twist correlators, it is possible to reveal their presence testing the universality of the forward dipole scattering amplitude in various processes. The exclusive $2 \rightarrow 3$ processes from this point of view are well suited for this analysis, since they have a different structure of the amplitude, and thus can be used as independent tools for phenomenological studies [39–47]. Due to high luminosity of the ongoing photoproduction experiments at the LHC (in ultraperipheral kinematics) [48] and the future experiments at the Electron Ion Collider (EIC) [49,50], measurement of such processes is feasible in the nearest future. A special interest in this program deserves the production of quarkonia-photon pairs. The heavy quark mass serves as a natural hard scale in processes involving their production and justifies perturbative description of the $Q\bar{Q}$ pair formation, as well as the use of the

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Non-Relativistic Quantum Chromodynamics (NRQCD) framework for their hadronization [51–54]. Due to strong suppression of C -parity exchanges in the t -channel, the largest cross sections have the quarkonia-photon pairs which include C -even quarkonia, like η_c and χ_c . The $\eta_c\gamma$ production has already been discussed in the literature [55]. In what follows we briefly summarize the main reasons that motivate the interest in the $\chi_c\gamma$ production:

- (i) Since the χ_c mesons are the P -wave charmonia, their internal structure and decay channels are completely different compared to those of J/ψ and η_c , and for this reason can serve as independent (complementary) probes of the dynamics of hadronic processes. While η_c differs from J/ψ only by spin alignment in the heavy quark mass limit, the χ_c mesons differ by both spin and orbital wave functions. Since the long-distance matrix elements (LDMEs) of different χ_c states are related by heavy quark spin symmetry (HQSS), the ratio of the cross sections with different spins and helicities has been considered in the literature as a possible tool for precision tests of NRQCD. At present there are indications that the χ_c -meson triplet is still not understood very well. The experimental results for the ratio $\sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$ in hadroproduction experiments disagrees significantly with HQSS-based expectations, and the phenomenologically successful descriptions of that channel either assume that HQSS is heavily broken for χ_c , or introduce sizable contributions of the color octet LDMEs [56–60]. The study of the ratio of cross sections $\sigma_{\gamma\chi_{c2}}/\sigma_{\gamma\chi_{c1}}$ in exclusive photoproduction experiments can decisively clarify the validity of HQSS because this channel does not get contributions from the color octets.
- (ii) The mesons χ_{c1} and χ_{c2} are much easier to study experimentally than η_c . Their decay width is more than an order of magnitude smaller than that of η_c , which implies a more pronounced peak in the invariant mass distribution of its decay products and stronger background suppression. The χ_{c1} and χ_{c2} decay predominantly into $\gamma J/\psi$ (with subsequent decay of J/ψ into lepton pairs), so the kinematics of χ_c mesons can be reconstructed with very good precision. In contrast, η_c decays predominantly into three light hadrons, and at least one of them is neutral (π^0 , K^0 , or η), which complicates reconstruction of η_c kinematics.
- (iii) The exclusive photoproduction of C -even mesons (without a final-state photon, via $\gamma p \rightarrow \chi_c p$ and $\gamma p \rightarrow \eta_c p$ processes) historically attracted attention as a possible tool for studies of odderons. While most of the previous studies focused on the η_c channel [61–66], recently it was demonstrated in [67] that a dominant contribution in that channel comes from the photon-photon fusion (the so-called Primakoff

mechanism), and for this reason χ_c photoproduction received renewed attention [68,69] as a sensitive tool for precision studies of odderons. The exclusive photoproduction of $\chi_c\gamma$ pairs in this context deserves interest as a potential background to odderon searches.

Similar to previous studies of $\eta_c\gamma$ pairs [55], the $\chi_c\gamma$ pairs may get contributions from radiative decays of higher excited states. Due to heavy mass of χ_c , the only relevant (meaningfully large) contribution comes from $\psi(2S) \rightarrow \chi_c\gamma$ radiative decays. In order to get rid of such contributions, in what follows we will assume that the invariant mass $M_{\gamma\chi_c}$ is larger than the mass of the $\psi(2S)$ meson.

In what follows we will focus on the high-energy (small- x) photon-proton collisions, which can be studied as a subprocess in ultraperipheral pA and AA collisions at the LHC, as well as in the future high-energy runs at the Electron Ion Collider, the Large Hadron electron Collider (LHeC) [70], and the Future Circular Collider (FCC-he) [71]. This kinematics presents interest because, as we will show below, the cross section grows rapidly with energy, thus facilitating experimental analysis. Previously the $\chi_c\gamma$ production has been already studied in [72] in the collinear factorization framework, which is valid at moderate energies $W \lesssim 100$ GeV. However, at very high energies that approach becomes unreliable because the hard partonic amplitude (coefficient functions) may get sizable loop corrections due to BFKL logarithms [73], which pose challenges for applicability of the approach. Technically, this problem manifests itself as a significant increase of the uncertainty of the predictions at $W \gtrsim 100$ GeV (see Fig. 7 in [72]). In the CGC framework this problem does not appear because large logarithms are already taken into account in parametrizations which satisfy the BK-JIMWLK evolution equation. Furthermore, at high energies the CGC framework allows one to separate clearly different stages of the process: the formation of the dipole, its interaction with the target (shock wave), and the final-state hadronization of a dipole into quarkonium. The subleading (next-to-eikonal) contributions are suppressed in this kinematics as inverse powers of the collision energy and may be disregarded. For this reason, the analysis of the process in the CGC framework is well motivated and justified.

The paper is structured as follows. In Sec. II we discuss in detail the kinematics of the process, and derive the theoretical results for the cross section in the color glass condensate framework. In Sec. III we provide tentative estimates for the cross sections, expected counting rates, and polarizations of produced quarkonia, obtained with a widely used phenomenological parametrization of the forward dipole scattering amplitude. We also estimate the contribution of this process and the $\psi(2S) \rightarrow \chi_c\gamma$ radiative decay to odderon-mediated $\gamma p \rightarrow \chi_c p$ suggested in [69]. Finally, in Sec. IV we draw conclusions.

II. THE THEORETICAL FRAMEWORK

As we explained in the Introduction, we will analyze the $\gamma p \rightarrow \chi_c \gamma p$ process in the high-energy (small- x) kinematics, using for description the CGC framework. We will focus on the kinematics of quasireal incoming photons, which dominate the spectrum of equivalent photons in ultraperipheral collisions and electroproduction experiments. In Secs. II A–II D below we will discuss the kinematics of the process and the light-cone wave functions of χ_c mesons with different helicities, introduce briefly the CGC framework, and derive the amplitude of the process in this approach. The final theoretical expressions for different helicity components are given in Sec. II D, in Eqs. (78)–(98) and (108)–(127). The latter equations will be used for numerical estimates in the next section.

A. Kinematics of the process

The evaluation of the invariant cross section can be realized in the so-called photon-proton collision frame,

where the 3-momenta of the incoming proton and photon are anticollinear to each other. For the sake of definiteness we will assume that the collision axis coincides with the axis z of our reference frame, and that the photon and proton propagate in its positive and negative directions respectively. The mass of the heavy quarkonium M_{χ_c} and the invariant mass of the photon quarkonium pair $M_{\gamma\chi_c}$ will be considered as hard scales, in agreement with [74]. The $\chi_c \gamma$ photoproduction may get contributions from radiative decays of higher excited states (so-called feed down contributions). Due to the heavy mass of χ_c , the only relevant (meaningfully large) contribution comes from $\psi(2S) \rightarrow \chi_c \gamma$ radiative decays. In order to get rid of such contributions, in what follows we will assume that the invariant mass $M_{\gamma\chi_c}$ is larger than the mass of the $\psi(2S)$ meson. The evaluation of the physical amplitude requires one to define the light-cone decomposition of various momenta. In what follows we will utilize the convention of Kogut-Soper [75], assuming that any 4-vector v^μ can be represented as

$$v^\mu = (v^+, v^-, \mathbf{v}_\perp), \quad v^\pm = \frac{v^0 \pm v^3}{\sqrt{2}}, \quad \mathbf{v}_\perp = v_x \hat{x} + v_y \hat{y}, \quad (1)$$

so its square and convolution with the Dirac matrix are given by

$$v^2 \equiv v^\mu v_\mu = 2v^+ v^- - \mathbf{v}_\perp^2, \quad \hat{v} \equiv \gamma^\mu v_\mu = \gamma^+ v^- + \gamma^- v^+ - \boldsymbol{\gamma}_\perp \cdot \mathbf{v}_\perp. \quad (2)$$

The light-cone decomposition of a particle's momenta will be chosen in agreement with earlier studies on meson-photon production [76,77],

$$q^\mu = \left(\sqrt{\frac{s}{2}}, 0, \mathbf{0}_\perp \right), \quad k^\mu = \left(\bar{\alpha}_{\chi_c} \sqrt{\frac{s}{2}}, \frac{(\mathbf{k}_\perp^\gamma)^2}{\bar{\alpha}_{\chi_c} \sqrt{2s}}, \mathbf{k}_\perp^\gamma \right) \quad (3)$$

$$P_{\text{in}}^\mu = \left(\frac{m_N^2}{\sqrt{2s}(1+\xi)}, (1+\xi)\sqrt{\frac{s}{2}}, \mathbf{0}_\perp \right), \quad P_{\text{out}}^\mu = \left(\frac{m_N^2 + \Delta_\perp^2}{\sqrt{2s}(1-\xi)}, (1-\xi)\sqrt{\frac{s}{2}}, \Delta_\perp \right), \quad (4)$$

$$\Delta^\mu = P_{\text{out}}^\mu - P_{\text{in}}^\mu = \left(\frac{2\xi m_N^2 + (1+\xi)\Delta_\perp^2}{\sqrt{2s}(1-\xi^2)}, -2\xi\sqrt{\frac{s}{2}}, \Delta_\perp \right), \quad (5)$$

$$p_{\chi_c}^\mu = \left(\alpha_{\chi_c} \sqrt{\frac{s}{2}}, \frac{(\mathbf{p}_\perp^{\chi_c})^2 + M_{\chi_c}^2}{\alpha_{\chi_c} \sqrt{2s}}, \mathbf{p}_\perp^{\chi_c} \right), \quad (6)$$

$$\mathbf{k}_\perp^\gamma := \mathbf{p}_\perp - \frac{\Delta_\perp}{2}, \quad \mathbf{p}_\perp^{\chi_c} := -\mathbf{p}_\perp - \frac{\Delta_\perp}{2}, \quad p_\perp \equiv (\mathbf{k}_\perp^\gamma - \mathbf{p}_\perp^{\chi_c})/2 \quad (7)$$

where q and k are the momenta of the incoming and outgoing (emitted) photons, $P_{\text{in}}, P_{\text{out}}$ are the momenta of the proton before and after the interaction, and p_{χ_c} is the momentum of the produced χ_c meson. The parameter α_{χ_c} stands for the fraction of the photon's light-cone momentum carried by χ_c , and a shorthand notation $\bar{\alpha}_{\chi_c} \equiv 1 - \alpha_{\chi_c}$ corresponds to a similar light-cone fraction carried by the scattered photon. For the proton which moves in the opposite direction, the parameter ξ controls the longitudinal momentum transfer to the proton during the process. The hard scale s can be related to the photon-proton invariant energy W of the process as

$$S_{\gamma N} \equiv W^2 = (q + P_{\text{in}})^2 = s(1 + \xi) + m_N^2, \quad \sqrt{s} = \sqrt{\frac{W^2 - m_N^2}{1 + \xi}}. \quad (8)$$

As will be demonstrated below, for high-energy collisions ($W \gg m_N$), the variable $\xi \ll 1$, so (8) can be approximated as $\sqrt{s} \approx W$. The light-cone decomposition in the target rest frame (r.f.) can be obtained from (3)–(7) making a longitudinal boost

$$v^+ \rightarrow \Lambda v^+, \quad v^- \rightarrow \Lambda^{-1} v^-, \quad \mathbf{v}_\perp \rightarrow \mathbf{v}_\perp$$

with $\Lambda = \sqrt{s}(1 + \xi)/m_N$, yielding

$$q_{\text{r.f.}}^\mu = \left(\frac{s(1 + \xi)}{m_N \sqrt{2}}, 0, \mathbf{0}_\perp \right), \quad k_{\text{r.f.}}^\mu = \left(\frac{\bar{\alpha}_{\chi_c} s(1 + \xi)}{\sqrt{2} m_N}, \frac{(\mathbf{k}_\perp^\gamma)^2}{\bar{\alpha}_{\chi_c} s \sqrt{2} (1 + \xi)} \frac{m_N}{m_N}, \mathbf{k}_\perp^\gamma \right), \quad (9)$$

$$P_{\text{in,r.f.}}^\mu = \left(\frac{m_N}{\sqrt{2}}, \frac{m_N}{\sqrt{2}}, \mathbf{0}_\perp \right), \quad P_{\text{out}}^\mu = \left(\frac{m_N^2 + \Delta_\perp^2 (1 + \xi)}{\sqrt{2} (1 - \xi)} \frac{1}{m_N}, \frac{m_N}{\sqrt{2} (1 + \xi)}, \Delta_\perp \right), \quad (10)$$

$$\Delta_{\text{r.f.}}^\mu = P_{\text{out}}^\mu - P_{\text{in}}^\mu = \left(\frac{2\xi m_N^2 + (1 + \xi) \Delta_\perp^2}{\sqrt{2} m_N} \frac{1 + \xi}{1 - \xi}, \frac{-\sqrt{2} \xi m_N}{(1 + \xi)}, \Delta_\perp \right), \quad (11)$$

$$p_{\chi_c, \text{r.f.}}^\mu = \left(\frac{\alpha_{\chi_c} s(1 + \xi)}{\sqrt{2} m_N}, \frac{(\mathbf{p}_\perp^{\chi_c})^2 + M_{\chi_c}^2}{\alpha_{\chi_c} s \sqrt{2} (1 + \xi)} \frac{m_N}{m_N}, \mathbf{p}_\perp^{\chi_c} \right). \quad (12)$$

The light-cone decomposition (3)–(7) and (9)–(12) satisfies the energy-momentum conservation and guarantees that all particles are on shell. As we discussed in [55,74], it is possible to express the variables ξ , $p_\perp$, $\Delta_\perp$, α_{χ_c} , which define the kinematics, in terms of the invariant mass $M_{\gamma\chi_c}^2$, momentum transfers to the proton (t) and to the photon (t'),

$$t = \Delta^2 = (P_{\text{out}} - P_{\text{in}})^2 = -\frac{1 + \xi}{1 - \xi} \Delta_\perp^2 - \frac{4\xi^2 m_N^2}{1 - \xi^2} \leq -\frac{4\xi^2 m_N^2}{1 - \xi^2}, \quad (13)$$

$$t' = (k - q)^2 = -2q \cdot k = -\frac{(\mathbf{p}_\perp - \Delta_\perp/2)^2}{(1 - \alpha_{\chi_c})} \leq 0, \quad (14)$$

$$M_{\gamma\chi_c}^2 = (k + p_{\chi_c})^2 = (q + P_{\text{in}} - P_{\text{out}})^2 = t - 2q \cdot \Delta = t + 2s\xi. \quad (15)$$

Resolving these equations, it is possible to obtain exact identities [55,74],

$$\xi = \frac{M_{\gamma\chi_c}^2 - t}{2(W^2 - m_N^2) - M_{\gamma\chi_c}^2 + t} > 0, \quad s = W^2 - m_N^2 + \frac{t - M_{\gamma\chi_c}^2}{2}, \quad 2\xi s = M_{\gamma\chi_c}^2 - t, \quad (16)$$

$$\Delta_\perp^2 = -t \frac{1 - \xi}{1 + \xi} - \frac{4\xi^2 m_N^2}{(1 + \xi)^2}, \quad \mathbf{p}_\perp \cdot \Delta_\perp = \frac{2\xi m_N^2}{(1 - \xi^2)} \frac{t'}{s}, \quad (17)$$

$$p_\perp^2 = -t' \left(\frac{1}{2} - \alpha_{\chi_c} \right) + \frac{1}{2} (\alpha_{\chi_c} M_{\gamma\chi_c}^2 - M_{\chi_c}^2) + \frac{t}{4} \left(\frac{1 - \xi}{1 + \xi} - 2\alpha_{\chi_c} \right) + \frac{\xi^2 m_N^2}{(1 + \xi)^2}, \quad (18)$$

$$\alpha_{\chi_c} = -\frac{1}{2\xi s} \left(t' - M_{\chi_c}^2 - \frac{2\xi m_N^2}{s(1 - \xi^2)} (M_{\gamma\chi_c}^2 + t' - t) \right) - \frac{2\xi m_N^2}{s(1 - \xi^2)}, \quad (19)$$

which fix the kinematics of the process in terms of the variables W , t , t' , $M_{\gamma\chi_c}$ up to a global rotation in the transverse plane. For high-energy processes we can simplify these relations using the hierarchy of scales W , $\sqrt{s} \gg M_{\chi_c}$, $M_{\gamma\chi_c} \gtrsim p_\perp$, $\Delta_\perp$. The

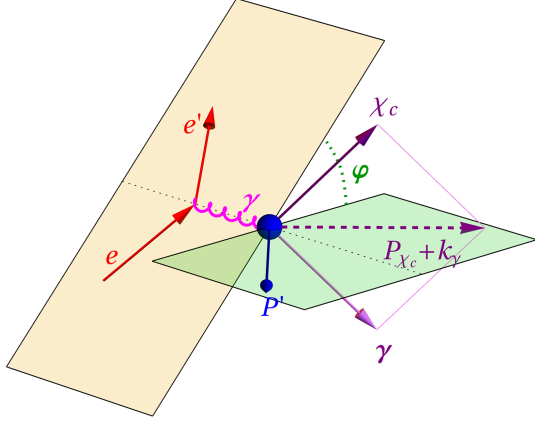


FIG. 1. The definition of the angle φ between leptonic and hadronic planes for the $ep \rightarrow e' p' \chi_c \gamma$ electroproduction channel, as seen from the target rest frame. The lepton scattering plane is formed by the three-momenta of incoming and scattered electrons (marked with labels e, e'), and the hadron scattering plane may be defined as a plane that includes the three-momenta of the recoiled proton P_{out} and the total three-momentum $p_{\chi_c} + k_\gamma$ of the produced $\gamma \chi_c$ pair. Due to momentum conservation, this plane also includes the 3-momentum of the virtual photon γ^* . In the photon-proton collision frame, the incoming photon propagates in the direction anticollinear to the incoming virtual photon γ^* , so the momentum of the recoil proton P_{out} is almost anticollinear to the momentum of the virtual photon γ^* .

parameter ξ defined in (16) is also very small, $\xi \sim M_{\gamma \chi_c}^2 / W^2 \ll 1$, and can be disregarded if accompanied by other terms which do not vanish in this limit. Similarly the smallness of the ratio $t'/s \sim M_{\gamma \chi_c}^2 / W^2 \sim \mathcal{O}(\xi) \ll 1$ in the right-hand side of (17) implies that the vectors $\mathbf{p}_\perp, \Delta_\perp$ are almost orthogonal to each other. As was discussed long ago in [78] and confirmed by various experimental studies, the cross sections of exclusive processes decrease rapidly as

a function of the invariant momentum transfer $t \sim \Delta_\perp^2$ to the target. Technically, this happens because a large momentum transfer to the target will not destruct it only if the momentum is shared equally between all partons, and each momentum exchange between partons is suppressed by (perturbative) propagators. As we will see below, such suppression also takes place in our process, and for this reason in the kinematics of interest the variables $\Delta_\perp^2, t$ are negligible compared to typical scales $\sim M_{\chi_c}^2$. In this kinematics, it is possible to rewrite (18) and (19) in a simpler form (see details in [55,74]),

$$-t' \approx \alpha_{\chi_c} M_{\gamma \chi_c}^2 - M_{\chi_c}^2, \quad p_\perp^2 = \bar{\alpha}_{\chi_c} [\alpha_{\chi_c} M_{\gamma \chi_c}^2 - M_{\chi_c}^2] \approx -\bar{\alpha}_{\chi_c} t', \quad M_{\gamma \chi_c}^2 \approx 2s\xi. \quad (20)$$

The physical constraint $p_\perp^2 \geq 0$ implies that the variable α_{χ_c} is bound by $\alpha_{\chi_c} \geq M_{\chi_c}^2 / M_{\gamma \chi_c}^2$. Furthermore, while parametrically $p_\perp$ is large, $p_\perp \sim \mathcal{O}(M_{\chi_c})$, due to suppression at the extremes of the interval $\alpha_{\chi_c} \in (M_{\chi_c}^2 / M_{\gamma \chi_c}^2, 1)$, numerically it remains relatively small, $p_T \lesssim 1$ GeV, in the kinematics where the cross section is meaningfully large for experimental study ($M_{\gamma \chi_c} \lesssim 5$ GeV). The cross section of the photoproduction subprocess may be represented as

$$\frac{d\sigma_{\gamma p \rightarrow \gamma \chi_c p}^{(T)}}{dt dt' dM_{\gamma \chi_c}} \approx \frac{|\mathcal{A}_{\gamma p \rightarrow \gamma \chi_c p}^{(\lambda; \sigma, H)}|^2}{128\pi^3 M_{\gamma \chi_c}} \quad (21)$$

where $\mathcal{A}_{\gamma p \rightarrow \gamma \chi_c p}^{(\lambda; \sigma, H)}$ is the process amplitude, and the superscript indices λ, σ, H are the helicities of the incoming photon, outgoing photon, and the produced charmonium, respectively. The cross section of the electroproduction is given by [9,79]

$$\begin{aligned} \frac{d\sigma_{ep \rightarrow e' \gamma \chi_c p}}{d\Omega} &= \epsilon \frac{d\sigma^{(L)}}{d\Omega} + \frac{d\sigma^{(T)}}{d\Omega} + \sqrt{\epsilon(1+\epsilon)} \cos \varphi \frac{d\sigma^{(LT)}}{d\Omega} + \sqrt{\epsilon(1+\epsilon)} \sin \varphi \frac{d\sigma^{(L'T)}}{d\Omega} \\ &+ \epsilon \cos 2\varphi \frac{d\sigma^{(TT)}}{d\Omega} + \epsilon \sin 2\varphi \frac{d\sigma^{(T'T)}}{d\Omega}, \end{aligned} \quad (22)$$

where φ is the angle between the lepton and proton scattering planes (see Fig. 1 for definition), the notation $d\Omega \equiv d \ln W^2 dQ^2 dt dt' dM_{\gamma \chi_c} d\varphi$ is used for the phase volume, and the variable $Q^2 = -q^2$ is the virtuality of the photon propagating from leptonic to hadronic parts. The superscript letters L, T, T' in the right-hand side of (17) distinguish the contributions of the longitudinal and transverse photons (as well as their possible interference). The parameter ϵ is the ratio of the longitudinal and transverse photon fluxes. The latter may be expressed in terms of

inelasticity parameter y and the invariant energy $\sqrt{s_{ep}}$ of the electron-proton collision as

$$\epsilon \approx \frac{1-y}{1-y+y^2/2}, \quad y = \frac{W^2 + Q^2 - m_N^2}{s_{ep} - m_N^2}. \quad (23)$$

In what follows we will focus on the kinematics of small Q^2 , which gives the dominant contribution to the total cross section. In this kinematics, the contribution of the

longitudinally polarized photons is negligible, and (22) reduces to

$$\frac{d\sigma_{ep \rightarrow eM_1 M_2 p}}{d\Omega} \approx \frac{d\sigma^{(T)}}{d\Omega} + \epsilon \cos 2\varphi \frac{d\sigma^{(TT)}}{d\Omega} + \epsilon \sin 2\varphi \frac{d\sigma^{(T'T)}}{d\Omega} \approx \frac{d\sigma^{(T)}}{d\Omega} (1 + c_2 \cos 2\varphi + s_2 \sin 2\varphi), \quad (24)$$

where $d\sigma^{(T)}/d\Omega$ is related to the photoproduction cross section (21) as [80–82]

$$\frac{d\sigma^{(T)}}{d\Omega} \approx \frac{\alpha_{\text{em}}}{\pi Q^2} \left(1 - y + \frac{y^2}{2} - (1 - y) \frac{Q_{\text{min}}^2}{Q^2} \right) \frac{d\sigma_{\gamma p \rightarrow \gamma \chi_{cP}}^{(T)}}{dt dt' dM_{\gamma \chi_c}}, \quad (25)$$

the superscript index (T) is used to remind one that the dominant contribution comes from the transversely polarized photon, $Q_{\text{min}}^2 = m_e^2 y^2 / (1 - y)$, m_e is the mass of the electron, and the angular harmonics c_2 , s_2 are related to different components of the amplitude $\mathcal{A}_{\gamma p \rightarrow \chi_{cP}}^{(\lambda, \sigma, H)}$ as

$$c_2 = \frac{2\epsilon \text{Re} \left(\mathcal{A}_{\gamma p \rightarrow \chi_{cP}}^{(-, +, H)*} \mathcal{A}_{\gamma p \rightarrow \chi_{cP}}^{(+, +, H)} \right)}{|\mathcal{A}_{\gamma p \rightarrow \chi_{cP}}^{(+, +)}|^2 + |\mathcal{A}_{\gamma p \rightarrow \chi_{cP}}^{(-, +)}|^2}, \quad s_2 = \frac{2\epsilon \text{Im} \left(\mathcal{A}_{\gamma p \rightarrow \chi_{cP}}^{(-, +, H)*} \mathcal{A}_{\gamma p \rightarrow \chi_{cP}}^{(+, +, H)} \right)}{|\mathcal{A}_{\gamma p \rightarrow \chi_{cP}}^{(+, +)}|^2 + |\mathcal{A}_{\gamma p \rightarrow \chi_{cP}}^{(-, +)}|^2}. \quad (26)$$

As we will see below, these coefficients are small, so the cross section (24) is dominated by the angular-independent term (the so-called equivalent photon approximation).

B. The polarization vectors and wave functions

For the polarization vectors of the photons, we will use the light-cone parametrization

$$\varepsilon_T^{(\lambda=\pm 1)}(\ell) = \left(\frac{\boldsymbol{\varepsilon}_\lambda \cdot \boldsymbol{\ell}_\perp}{\ell^-}, 0, \boldsymbol{\varepsilon}_\lambda \right), \quad \boldsymbol{\varepsilon}_\lambda = \frac{\hat{\mathbf{x}} + i\lambda \hat{\mathbf{y}}}{\sqrt{2}}, \quad (27)$$

where ℓ is the 4-momentum of the corresponding photon ($\ell = q$ or k) in this process. For the polarization vector of the axial vector quarkonium χ_{c1} , we will use a similar parametrization suggested in [69]

$$E_{T, \lambda}^{(\chi_{c1})}(p_{\chi_c}) = \left(\frac{\boldsymbol{\varepsilon}_T \cdot \mathbf{p}_\perp^{\chi_c}}{p_{\chi_c}^-}, 0, \boldsymbol{\varepsilon}_T \right), \quad E_L^{(\chi_{c1})}(p_{\chi_c}) = \left(\frac{(\mathbf{p}_\perp^{\chi_c})^2 - M_{\chi_c}^2}{2M_{\chi_c} p_{\chi_c}^-}, \frac{p_{\chi_c}^-}{M_{\chi_c}}, \frac{\mathbf{p}_\perp^{\chi_c}}{M_{\chi_c}} \right), \quad (28)$$

where the subindices L and $T = \pm 1$ distinguish the transverse and longitudinal polarizations. This parametrization automatically satisfies the transversality condition $p_{\chi_c} \cdot E^{(\chi_{c1})} = 0$. The polarization tensor $E_{\mu\nu}$ of the charmonium χ_{c2} can be constructed from polarization vectors defined in (28) using conventional Clebsch-Gordan coefficients as

$$E_{\mu\nu}^{(\chi_{c2})} = \begin{cases} \left(E_{T=\pm 1}^{(\chi_{c1})} \right)_\mu \left(E_{T=\pm 1}^{(\chi_{c1})} \right)_\nu, & H_{\chi_{c2}} = \pm 2, \\ \frac{\left(E_{T=\pm 1}^{(\chi_{c1})} \right)_\mu \left(E_L^{(\chi_{c1})} \right)_\nu + \left(E_L^{(\chi_{c1})} \right)_\mu \left(E_{T=\pm 1}^{(\chi_{c1})} \right)_\nu}{\sqrt{2}}, & H_{\chi_{c2}} = \pm 1, \\ \frac{\left(E_{T=\pm 1}^{(\chi_{c1})} \right)_\mu \left(E_{T=-1}^{(\chi_{c1})} \right)_\nu + \left(E_{T=-1}^{(\chi_{c1})} \right)_\mu \left(E_{T=\pm 1}^{(\chi_{c1})} \right)_\nu + 2 \left(E_L^{(\chi_{c1})} \right)_\mu \left(E_L^{(\chi_{c1})} \right)_\nu}{\sqrt{6}}, & H_{\chi_{c2}} = 0, \end{cases} \quad (29)$$

where $H_{\chi_{c2}}$ is the helicity projection of the corresponding charmonium. The wave function of the χ_c mesons in helicity basis in momentum space is given by

$$\Psi_{H, h\bar{h}} = \bar{u}_h(k_Q) \Gamma_{\chi_c}(k_Q, k_{\bar{Q}}) v_{\bar{h}}(k_{\bar{Q}}) \phi_{\chi_c}(z, \mathbf{k}_\perp^{(\text{rel})}), \quad \Gamma_{\chi_c}(k_Q, k_{\bar{Q}}) = \begin{cases} 1, & J = 0, \\ i\gamma_5 \gamma^\mu E_\mu^{(\chi_{c1})}, & J = 1, \\ \frac{\gamma^\mu K^\nu + \gamma^\nu K^\mu}{2} E_{\mu\nu}^{(\chi_{c2})} & J = 2, \end{cases} \quad (30)$$

where $h, \bar{h}$ are the helicities of the quark, z and $\mathbf{k}_\perp^{(\text{rel})}$ are the light-cone fraction and transverse momentum of the quark (with respect to the direction of the χ_c meson), and the momentum of relative motion $K^\mu \equiv k_Q^\mu - k_{\bar{Q}}^\mu$ is expressed in terms of these variables as

$$K^\mu = \left((2z-1)\alpha_{\chi_c} \sqrt{\frac{s}{2}}, \frac{2\mathbf{p}_\perp^{\chi_c} \cdot \mathbf{K}_\perp + (1-2z)[(\mathbf{p}_\perp^{\chi_c})^2 + M_{\chi_c}^2]}{\alpha_{\chi_c} \sqrt{2s}}, \mathbf{K}_\perp \right). \quad (31)$$

The K^- component of the vector K^μ was fixed from the orthogonality condition $K \cdot p_{\chi_c} = 0$, which follows from the fact that the quarks are (nearly) on-shell in the heavy quark mass limit. The transverse component of K^μ is related to $\mathbf{k}_\perp^{(\text{rel})}$ as

$$\mathbf{k}_\perp^{(\text{rel})} = \bar{z}\mathbf{k}_Q - z\mathbf{k}_{\bar{Q}} = \mathbf{K}_\perp - (2z-1)\mathbf{p}_\perp^{\chi_c}. \quad (32)$$

For χ_{c0} mesons, the Fourier transformation of (30) over the transverse coordinates allows one to obtain the light-cone wave function

$$\Psi_{h\bar{h}}^{(\chi_{c0})}(z, r) = -\frac{1}{z\bar{z}} [i h e^{-i h \theta} \delta_{h,\bar{h}} \partial_r + m(z - \bar{z}) \delta_{h,-\bar{h}}] \Phi_{\chi_{c0}}(z, \mathbf{r}_\perp), \quad (33)$$

where $\mathbf{r} = \{r \cos \theta, r \sin \theta\}$ is the vector of relative separation of the quark and antiquark in transverse plane, z is the fraction of the momentum carried by the quark, and the configuration space wave function Φ_{χ_c} is related to the function $\phi_{\chi_c}(z, \mathbf{k}_\perp^{(\text{rel})})$ by relation

$$\Phi_{\chi_{cJ}}(z, \mathbf{r}_\perp) = \int \frac{d^2 \mathbf{k}_\perp^{(\text{rel})}}{(2\pi)^2} \frac{\sqrt{z\bar{z}} \phi_{\chi_{cJ}}(z, \mathbf{k}_\perp^{(\text{rel})}) e^{i \mathbf{k}_\perp^{(\text{rel})} \cdot \mathbf{r}_\perp}}{(\mathbf{k}_\perp^{(\text{rel})})^2 - M_{\chi_{cJ}}^2 z(1-z) + m^2}, \quad J = 0, 1, 2. \quad (34)$$

The integrand of (34) includes an additional light-cone denominator whose origin will become obvious in the next section [see the denominators in the last lines of (69) and (70) below; also see Appendix of [55] for a more detailed discussion]. For the helicity components of the χ_{c1} and χ_{c2} we may obtain in a similar fashion

$$\Psi_{H=\pm 1, h\bar{h}}^{(\chi_{c1})}(z, r) = \frac{\sqrt{2}}{z\bar{z}} [-e^{iH\theta} (\bar{z} \delta_{h,H} \delta_{\bar{h},-H} + z \delta_{h,-H} \delta_{\bar{h},H}) \partial_r - i H m (z - \bar{z}) \delta_{h,H} \delta_{\bar{h},H}] \Phi_{\chi_{c1}}(z, r), \quad (35)$$

$$\Psi_{H=0, h\bar{h}}^{(\chi_{c1})}(z, r) = \frac{2i}{z\bar{z} M_{\chi_{c1}}} [h \delta_{h,-\bar{h}} \Delta_r^{(+)} + i m e^{-i h \theta} \delta_{h,\bar{h}} \partial_r] \Phi_{\chi_{c1}}(z, r), \quad (36)$$

where $\Delta_r^{(\pm)} = \partial_r^2 \pm (1/r) \partial_r$, $\bar{z} = 1 - z$, and

$$\Psi_{H=+2, h\bar{h}}^{(\chi_{c2})}(z, r) = \frac{2}{z\bar{z}} \left[e^{2i\theta} (z \delta_{h,+} \delta_{\bar{h},-} - \bar{z} \delta_{h,-} \delta_{\bar{h},+}) \Delta_r^{(-)} + i m e^{i\theta} \delta_{h,+} \delta_{\bar{h},+} \partial_r \right] \Phi_{\chi_{c2}}(z, r), \quad (37)$$

$$\Psi_{H=-2, h\bar{h}}^{(\chi_{c2})}(z, r) = \frac{2}{z\bar{z}} \left[-e^{-2i\theta} (\bar{z} \delta_{h,+} \delta_{\bar{h},-} - z \delta_{h,-} \delta_{\bar{h},+}) \Delta_r^{(-)} - i m e^{-i\theta} \delta_{h,-} \delta_{\bar{h},-} \partial_r \right] \Phi_{\chi_{c2}}(z, r), \quad (38)$$

$$\Psi_{H=+1, h\bar{h}}^{(\chi_{c2})}(z, r) = \frac{M_{\chi_{c2}}}{z\bar{z}} \left[i e^{i\theta} ((3z - 4z^2) \delta_{h,+} \delta_{\bar{h},-} + (3\bar{z} - 4\bar{z}^2) \delta_{h,-} \delta_{\bar{h},+}) \partial_r + m(z - \bar{z}) \delta_{h,+} \delta_{\bar{h},+} \right] \Phi_{\chi_{c2}}(z, r), \quad (39)$$

$$\Psi_{H=-1, h\bar{h}}^{(\chi_{c2})}(z, r) = \frac{M_{\chi_{c2}}}{z\bar{z}} \left[-i e^{-i\theta} ((3\bar{z} - 4\bar{z}^2) \delta_{h,+} \delta_{\bar{h},-} + (3z - 4z^2) \delta_{h,-} \delta_{\bar{h},+}) \partial_r + m(z - \bar{z}) \delta_{h,-} \delta_{\bar{h},-} \right] \Phi_{\chi_{c2}}(z, r), \quad (40)$$

$$\Psi_{H=0, h\bar{h}}^{(\chi_{c2})}(z, r) = \frac{\sqrt{2/3}}{z\bar{z}} [\delta_{h,-\bar{h}} (-3\Delta_r^{(+)} + 2m^2(z - \bar{z})) - i m e^{-i h \theta} \delta_{h,\bar{h}} \partial_r] \Phi_{\chi_{c2}}(z, r). \quad (41)$$

1. Relation to NRQCD LDMEs

In order to understand the relations of the light-cone distribution amplitudes to long distance matrix elements (LDMEs) of NRQCD, we need to analyze the behavior of the wave functions in the limit when K^μ becomes small. Since we are interested in the P -wave quarkonia, in the limit $K^\mu \rightarrow 0$ the corresponding vertices should vanish. This is obvious for χ_{c2} due to explicit K^μ in (30). To demonstrate that a similar property is valid for the states $J = 0$ and $J = 1$, we need to rewrite (30) in equivalent form, using the Dirac equation for the $\bar{u}, v$ spinors and the orthogonality of the polarization vector $E_H \cdot p_{\chi_c} = 0$, namely

$$\begin{aligned} \bar{u}(k_Q) \Gamma_{\chi_{cJ}} v(k_{\bar{Q}}) &= \frac{1}{2m} \bar{u}(k_Q) (m \Gamma_{\chi_{cJ}} + \Gamma_{\chi_{cJ}} m) v(k_{\bar{Q}}) = \frac{1}{2m} \bar{u}(k_Q) (\hat{k}_Q \Gamma_{\chi_{cJ}} - \Gamma_{\chi_{cJ}} \hat{k}_1) v(k_{\bar{Q}}) \\ &= \frac{1}{4m} \bar{u}(k_Q) (\hat{p}_{\chi_c} \Gamma_{\chi_{cJ}} - \Gamma_{\chi_{cJ}} \hat{p}_{\chi_c}) v(k_{\bar{Q}}) + \frac{1}{4m} \bar{u}(k_Q) (\hat{K} \Gamma_{\chi_{cJ}} + \Gamma_{\chi_{cJ}} \hat{K}) v(k_{\bar{Q}}) = \bar{u}(k_Q) \Gamma_{\chi_{cJ}}^{(\text{eff})} v(k_{\bar{Q}}), \end{aligned} \quad (42)$$

where $\Gamma_{\chi_{cJ}}^{(\text{eff})} = K^\mu (\gamma^\mu \Gamma_{\chi_{cJ}} + \Gamma_{\chi_{cJ}} \gamma^\mu) / 4m = \gamma_{\chi_{cJ}}^\mu K_\mu$, and the momentum-independent matrices $\gamma_{\chi_{cJ}}^\mu$ for different quarkonia are given by $\gamma_{\chi_{c0}}^\mu \equiv \gamma^\mu / 2m$, $\gamma_{\chi_{c1}}^\mu \equiv [\gamma^\mu, \hat{E}_H] \gamma^5 / 4m$, and $\gamma_{\chi_{c2}}^\mu = E^{\mu\nu} \gamma_\nu$. In the amplitudes of physical processes, the wave function (30) contributes in convolution with amplitude of hard partonic process. If the latter possess a hard scale which exceeds significantly the relative momentum K^μ ($\sim$ inverse charmonium radius), we can disregard completely the dependence on $z, \mathbf{k}_\perp^{(\text{rel})}$ in the hard partonic amplitude, and the convolution over $z, \mathbf{K}_\perp$ will affect only the arguments of the wave function, yielding

$$\begin{aligned} \int dz \int \frac{d^2 \mathbf{k}_\perp^{(\text{rel})}}{(2\pi)^2} \Phi_{\chi_c}(z, \mathbf{k}_\perp^{(\text{rel})}) &= \lim_{\lambda, \mathbf{r}_\perp \rightarrow 0} \left\langle 0 \left| \bar{\psi} \left(-\frac{\lambda}{2} n_+ - \frac{\mathbf{r}_\perp}{2} \right) \gamma_{\chi_{cJ}}^\mu i \overleftrightarrow{D}_\mu \mathcal{L} \left(-\frac{\lambda}{2} n_+ - \frac{\mathbf{r}_\perp}{2}, \frac{\lambda}{2} n_+ + \frac{\mathbf{r}_\perp}{2} \right) \Psi \left(\frac{\lambda}{2} n_+ + \frac{\mathbf{r}_\perp}{2} \right) \right| \chi_c(p) \right\rangle \\ &= \left\langle 0 \left| \bar{\psi}(0) \gamma_{\chi_{cJ}}^\mu \frac{i}{2} \overleftrightarrow{D}_\mu \Psi(0) \right| \chi_c(p) \right\rangle \end{aligned} \quad (43)$$

where $\overleftrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu + 2igA_\mu^a t_a + 2ieA_\mu$ is the covariant derivative, and we replaced K_μ with $i(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)$ in the configuration space. The square of the matrix element in the last line of (43) in the nonrelativistic limit reduces to NRQCD LDMEs. Indeed, using the explicit form of $\gamma_{\chi_{cJ}}^\mu$ for $J = 0, 1, 2$, we may recover the familiar NRQCD operators from [83–85]¹

$$|\langle 0 | \bar{\psi}(0) \gamma_{\chi_{cJ}}^\mu i \overleftrightarrow{D}_\mu \psi(0) | \chi_c(p) \rangle|^2 = \frac{1}{4m^2} \times \begin{cases} \left| \langle 0 | \bar{\psi} \left(\frac{i}{2} \overleftrightarrow{D}_\top \gamma_\top^\top \right) \chi | \chi_c(p) \rangle \right|^2 = \langle \mathcal{O}_{\chi_c}^{[1]}(^3P_0^{[1]}) \rangle, & J = 0, \\ \left| \langle 0 | \psi^\dagger \left(-\frac{i}{4} \overleftrightarrow{D}_\top [\gamma_\top^\top, \gamma_\top^\top] \right) \gamma_5 \chi | \chi_c(p) \rangle \right|^2 = \langle \mathcal{O}_{\chi_c}^{[1]}(^3P_1^{[1]}) \rangle, & J = 1, \\ \left| \langle 0 | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}_\top \gamma_\top^\nu \right) \chi | \chi_c(p) \rangle \right|^2 = \langle \mathcal{O}_{\chi_c}^{[1]}(^3P_2^{[1]}) \rangle, & J = 2, \end{cases} \quad (45)$$

where $\top$ implies a part of the vector which is transverse to $p_{\chi_c}^\mu$, namely $v_\top^\mu = (g^{\mu\nu} - p_{\chi_c}^\mu p_{\chi_c}^\nu / M_{\chi_c}^2) v^\nu$; $a_\top^{(\mu} b_\top^{\nu)} = (a_\top^\mu b_\top^\nu + a_\top^\nu b_\top^\mu) / 2 - a_\top \cdot b_\top (g^{\mu\nu} - p_{\chi_c}^\mu p_{\chi_c}^\nu / M_{\chi_c}^2) / 3$, the operators $\psi^\dagger, \chi$ are the Pauli spinor operators which create a quark and an antiquark, respectively. The presence of the gauge fields in covariant derivative $\overleftrightarrow{D}_\mu$ implies that we should consider the χ_c

¹In the rest frame of the proton, we may further simplify and rewrite these operators as [51]

$$|\langle 0 | \bar{\psi}(0) \gamma_{\chi_{cJ}}^\mu i \overleftrightarrow{D}_\mu \psi(0) | \chi_c(p) \rangle|^2 = \begin{cases} \left| \langle 0 | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D} \cdot \boldsymbol{\sigma} \right) \chi | \chi_c(p) \rangle \right|^2 = \langle \mathcal{O}_{\chi_c}^{[1]}(^3P_0^{[1]}) \rangle, & J = 0, \\ \left| \langle 0 | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D} \times \boldsymbol{\sigma} \right) \chi | \chi_c(p) \rangle \right|^2 = \langle \mathcal{O}_{\chi_c}^{[1]}(^3P_1^{[1]}) \rangle, & J = 1, \\ \left| \langle 0 | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^{(i} \sigma^{j)} \right) \chi | \chi_c(p) \rangle \right|^2 = \langle \mathcal{O}_{\chi_c}^{[1]}(^3P_2^{[1]}) \rangle, & J = 2. \end{cases} \quad (44)$$

meson as a mixture of $|\bar{Q}Q\rangle$, $|\bar{Q}Qg\rangle$, and $|\bar{Q}Q\gamma\rangle$ Fock components unless we work in the light-cone gauge $A^- = 0$, $A_a^- = 0$. In what follows we will use the latter gauge condition, both for photons and gluons.

In the heavy quark mass, it is expected that the color singlet LDMEs $\langle \mathcal{O}_{\chi_c}^{[1]}(^3P_J^{[1]}) \rangle$ should coincide, and in the potential models can be represented in terms of the slope of the radial wave functions $R'_{\chi_c}(0)$ as

$$\langle \mathcal{O}_{\chi_c}^{[1]}(^3P_J^{[1]}) \rangle = 6N_c(2J+1)|R'_{\chi_c}(0)|^2/4\pi. \quad (46)$$

The latter parameter may be related to the light-cone function $\phi_{\chi_{cJ}}(z, \mathbf{k}_\perp^{(\text{rel})})$ introduced in (30): after straightforward but tedious substitution of the variables, we may obtain [86]

$$3\sqrt{\frac{\pi}{2}}R'_{\chi_c}(0) = \sqrt{2M_{\chi_{cJ}}} \int \frac{dz d^2\mathbf{k}_\perp^{(\text{rel})}}{z(1-z)8\pi^2} \phi_{\chi_{cJ}}(z, \mathbf{k}_\perp^{(\text{rel})}) \left[M_{\chi_{cJ}}^2 \left(z - \frac{1}{2} \right)^2 + \mathbf{K}_\perp^2 \right]. \quad (47)$$

In what follows we will use the explicit parametrization from [69] which in the light-cone representation is given by

$$\Phi_{\chi_{cJ}}(z, r) = \mathcal{N}_{\chi_{cJ}} z(1-z) \exp \left(-\frac{\mathcal{R}_{\chi_{cJ}}^2}{8} \left(\frac{m^2}{z(1-z)} - 4m^2 \right) - \frac{2z(1-z)r^2}{\mathcal{R}_{\chi_{cJ}}^2} \right), \quad (48)$$

and $\mathcal{N}_{\chi_{cJ}}, \mathcal{R}_{\chi_{cJ}}$ are numerical constants given explicitly by $\mathcal{R}_{\chi_{c0}} \approx 1.54 \text{ GeV}^{-1}$, $\mathcal{R}_{\chi_{c1}} = \mathcal{R}_{\chi_{c2}} \approx 1.48 \text{ GeV}^{-1}$, $\mathcal{N}_{\chi_{c0}} \approx 1.15 \text{ GeV}^{-1}$, $\mathcal{N}_{\chi_{c1}} \approx 1.39 \text{ GeV}^{-1}$, $\mathcal{N}_{\chi_{c2}} \approx 0.60 \text{ GeV}^{-1}$.² The values of $|R'(0)|$ obtained with this parametrization are in reasonable agreement with the value $|R'(0)|^2 = 0.075 \text{ GeV}^5$ found in potential models and widely used in the literature for numerical estimates. For the momentum space wave function, we may obtain

$$\tilde{\Phi}_{\chi_{cJ}}(z, \mathbf{k}) \equiv \int d^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \Phi_{\chi_{cJ}}(z, \mathbf{r}) = \frac{\sqrt{z\bar{z}} \phi_{\chi_{cJ}}(z, \mathbf{k})}{k^2 - M_{\chi_{cJ}}^2 z\bar{z} + m^2} = \frac{\pi \mathcal{N}_{\chi_{cJ}} \mathcal{R}_{\chi_{cJ}}^2}{2} \exp \left(-\frac{\mathcal{R}_{\chi_{cJ}}^2}{8} \left(\frac{\mathbf{k}^2 + m^2(1-4z\bar{z})}{z\bar{z}} \right) \right). \quad (49)$$

C. Scattering processes in the CGC framework

In the CGC framework the interaction of the high-energy partons with the gluonic field of the target is described in eikonal approximation by the Wilson line $U(\mathbf{x}_\perp)$ defined as

$$U(\mathbf{x}_\perp) = P \exp \left(ig \int dx^- A_a^+(x^-, \mathbf{x}_\perp) t^a \right), \quad (50)$$

where we use notation $\mathbf{x}_\perp$ for the impact parameter of the parton, A_a^μ for the gluonic field of the target, and t_a are the usual color group generators in the irreducible representation which corresponds to the parton (**3**, $\bar{\mathbf{3}}$ or **8** for quark, antiquark, or gluon, respectively). The interaction encoded in the Wilson link (50) formally may be reformulated in terms of the CGC Feynman rules [12,87–89] provided in Table I.

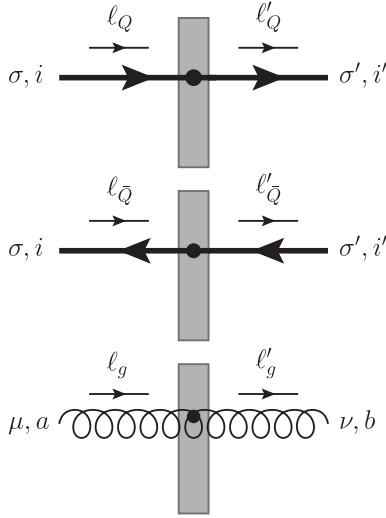
In the CGC picture it is assumed that the gluonic field A_a^μ is created by randomly distributed color sources, and probability to find a given density of color sources $\rho_a(x^-, \mathbf{x}_\perp)$ in the target is controlled by the weight functional $W[\rho]$ [1–3]. The component A_a^+ which appears in (50) may be related to density of these source ρ_a as $A_a^+(x) = -\frac{1}{\nabla_\perp^2} \rho_a(x^-, \mathbf{x}_\perp)$. For any physical observable, the expected values may be obtained averaging the observable $\mathcal{A}[\rho]$ found at fixed distribution of charges $\rho(x^-, \mathbf{x}_\perp)$ over all possible configurations $\rho(x^-, \mathbf{x}_\perp)$, namely

$$\langle \mathcal{A} \rangle = \int \mathcal{D}\rho W[\rho] \mathcal{A}[\rho], \quad (51)$$

where the angular brackets $\langle \dots \rangle$ stand for the above-mentioned averaging. The analytic evaluation of the integral $\mathcal{D}\rho$ is possible only for a few simple choices of $W[\rho]$, as, for example, Gaussian parametrization [1,2], and development of

²We disregard tiny $\sim 1\%$ differences of these constants for different helicity states (polarizations) of the same meson, since these differences are smaller than the precision of experimental data for $\chi_c \rightarrow \gamma\gamma$ used in [69] (around 3%–4%); besides, a comparable uncertainty may come from the theoretical NNLO corrections $\sim \alpha_s^2(m_c) \approx 10\%$.

TABLE I. The CGC Feynman rules [12,87–89] for the interaction of high-energy partons with the target encoded in the Wilson link (50). The solid vertical block stands for the shock wave (Wilson line). We use notations ℓ_i, ℓ'_i ($i = Q, \bar{Q}, g$) for the momenta of the corresponding parton before and after the interaction, σ, σ' for Dirac indices of the quarks, i, i' for the color indices in the fundamental representations **3** or $\bar{\mathbf{3}}$ of the color group, and a, b for the gluon color indices in the adjoint representation **8**. The matrices U and $\mathcal{U}$ are the Wilson lines in the fundamental and the adjoint representations. The prefactor $2\pi\delta(\ell_i^+ - \ell'^+)$ in the interaction vertices reflects conservation of the longitudinal momentum of the parton in eikonal approximation. We may also observe that interaction with a shock wave does not change transverse coordinates of the partons in the mixed (light-cone) representation, and using identities $\bar{u}_{h_1}(p)\gamma^+u_{h_2}(q) = \bar{v}_{h_1}(p)\gamma^+v_{h_2}(q) = 2\delta_{h_1,h_2}\sqrt{p^+q^+}$ from [75], we may see that the interaction remains diagonal in helicity basis.



The interaction vertex of the quark with the shock wave:

$$T_{\sigma,\sigma';i,i'}^Q = 2\pi\delta(\ell_Q^+ - \ell'^+_{\bar{Q}})\gamma_{\sigma,\sigma'}^+ \int d^2z e^{i(\ell_Q - \ell'_{\bar{Q}})z} [U(z) - 1]_{i',i}$$

The interaction vertex of the antiquark with the shock wave:

$$T_{\sigma,\sigma';i,i'}^{\bar{Q}} = -2\pi\delta(\ell_{\bar{Q}}^+ - \ell'^+_{\bar{Q}})\gamma_{\sigma,\sigma'}^+ \int d^2z e^{i(\ell_{\bar{Q}} - \ell'_{\bar{Q}})z} [U^\dagger(z) - 1]_{i,i'}$$

The interaction vertex of the gluon with the shock wave:

$$T_{\mu,\nu;a,b}^g = -2\pi\delta(\ell_g^+ - \ell'^+_{\bar{g}})g_{\mu\nu}\text{sgn}(\ell_g^+) \times \int d^2z e^{-i(\ell_g - \ell'_{\bar{g}})z} [\mathcal{U}^{\text{sgn}(\ell_g^+)}(z) - 1]_{b,a}$$

phenomenological models for the latter is very challenging. Fortunately, in the presence of hard scales which limit the number of partons in the initial state, all physical amplitudes may be represented as convolutions of process-dependent perturbative impact factors and universal (target-dependent) correlators of Wilson lines (multipole scattering amplitudes). For many exclusive processes, the dominant contribution is controlled by the lowest order dipole scattering amplitude,

$$\mathcal{N}(x, \mathbf{r}, \mathbf{b}) = 1 - S_2\left(Y = \ln\left(\frac{1}{x}\right), \mathbf{x}_q, \mathbf{x}_{\bar{q}}\right),$$

$$S_2(Y, \mathbf{x}_q, \mathbf{x}_{\bar{q}}) = \frac{1}{N_c} \langle \text{tr}(U(\mathbf{x}_q)U^\dagger(\mathbf{x}_{\bar{q}})) \rangle_Y \quad (52)$$

where Y is the rapidity of the dipole, $\mathbf{x}_q, \mathbf{x}_{\bar{q}}$ are the impact parameters of the quark, and the variables $\mathbf{r} \equiv \mathbf{x}_q - \mathbf{x}_{\bar{q}}$ and $\mathbf{b} \equiv \alpha_q \mathbf{x}_q + \alpha_{\bar{q}} \mathbf{x}_{\bar{q}}$ determine the transverse separation and the impact parameter of the dipole's center of mass. In phenomenological studies the dipole amplitude can be constrained from both exclusive and inclusive channels. Due to the optical theorem, the inclusive channels are sensitive only to the imaginary part of the amplitude, and in order to have a consistent description, for exclusive channels, the real part of the amplitude should be taken into account. As was demonstrated long ago in [90,91], if the amplitude grows as a function of invariant energy as

$\sim s^\alpha$, then the real part of the amplitude becomes proportional to the imaginary part, and thus the full amplitude of the exclusive process can be restored from the imaginary part using a set of identities

$$\mathcal{A} = (\beta + i) \text{Im } \mathcal{A}, \quad |\mathcal{A}|^2 = (1 + \beta^2) |\text{Im } \mathcal{A}|^2,$$

$$\beta \equiv \frac{\text{Re } \mathcal{A}}{\text{Im } \mathcal{A}} = \tan\left(\frac{\pi\alpha}{2}\right), \quad \alpha = \frac{\partial \ln \mathcal{A}}{\partial \ln s}. \quad (53)$$

Following existing conventions, we will assume that $\mathcal{N}(x, \mathbf{r}, \mathbf{b})$ corresponds to the dipole amplitude extracted from inclusive processes, tacitly assuming that a factor $(\beta + i)$ should be added in exclusive channels. In our evaluations we do not take into account the so-called skewedness factor R_g suggested in [92] on phenomenological grounds, since its original derivation relies heavily on the dominance of the two-gluon exchange in the t -channel and the collinear factorization. In the CGC framework the theoretical status of R_g is less clear: its original relation to the longitudinal momentum exchange in the t -channel implies that it may be interpreted as one of the subeikonal corrections. However, a systematic analysis of these corrections is a formidable problem far outside the scope of the present manuscript (see, e.g., [93,94] for details). Numerically, the factor R_g would increase the cross section by approximately 30%–40%, on par with uncertainty due to omitted next-to-leading order corrections.

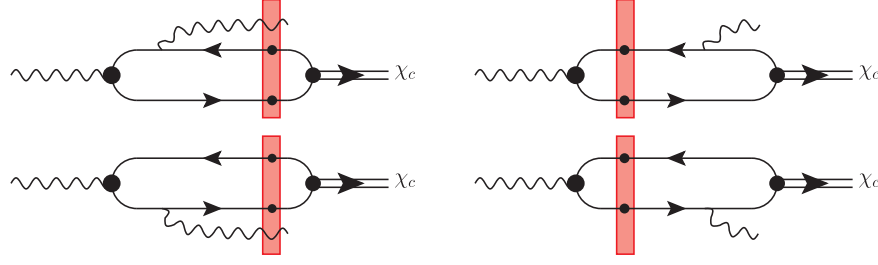


FIG. 2. The dominant mechanism of $\chi_c \gamma$ pair photoproduction in the CGC framework in the leading order in α_s . The red block stands for a shock wave. The diagrams in the first and the second row are related by charge conjugation (inversion of the quark line). In the left column we disregard interaction of the emitted photon with a shock wave as a tiny $\mathcal{O}(\alpha_{\text{em}})$ correction, and to emphasize this we do not put a “dot” on a photon line which crosses the shock wave.

D. Photoproduction of the $\chi_c \gamma$ pairs

The evaluation of the exclusive photoproduction of $\chi_c \gamma$ pairs resembles a similar evaluation of the $\eta_c \gamma$ photoproduction presented in [55] and differs only in the final state wave function, although it yields a significantly different result for the impact factors. Technically, it requires evaluation of four diagrams shown in Fig. 2. In what follows we will separate the contributions with photon emission before and after the interaction with a shockwave, so the amplitude can be represented as

$$\mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(\lambda; \sigma, H)} = \mathcal{A}_1^{(\lambda; \sigma, H)} + \mathcal{A}_2^{(\lambda; \sigma, H)} \quad (54)$$

where $\mathcal{A}_1^{(\lambda, \sigma)}$ and $\mathcal{A}_2^{(\lambda, \sigma)}$ correspond to the left and right columns of Fig. 2. Since the interaction of the emitted photon with the shock wave may be disregarded as a $\mathcal{O}(\alpha_{\text{em}})$ correction, both contributions are controlled by the dipole amplitude $\mathcal{N}(x, \mathbf{r}, \mathbf{b})$, convoluted with appropriate impact factors.

The detailed evaluation of these impact factors is straightforward and is explained in detail in subsequent subsections. Following earlier studies [33,34], in all evaluations below we will introduce the subindices 0,1,2 to distinguish the kinematic variables associated with quark, antiquark, and photon, respectively. For example, the variables

$$z_0 \equiv \frac{k_Q^+}{q^+}, \quad z_1 \equiv \frac{k_{\bar{Q}}^+}{q^+}, \quad z_2 \equiv \frac{k_\gamma^+}{q^+} = 1 - \alpha_{\chi_c} \quad (55)$$

are the fractions of the incoming photon's light-cone momentum q^+ carried by the quark, antiquark, and photon when crossing the shock wave, respectively, and $\mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2$

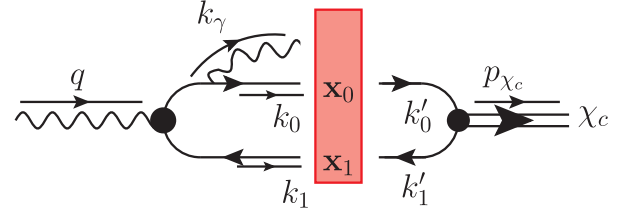


FIG. 3. The definitions of the parton momenta k_0, k_1, k'_0, k'_1 before and after interaction with the shock wave. The variables $\mathbf{x}_0, \mathbf{x}_1$ stand for the transverse coordinates of the quark and antiquark when they pass through the shock wave.

are their transverse coordinates (in configuration space). The light-cone momentum conservation implies that for the diagrams in the left column of Fig. 2, the variables z_0, z_1, z_2 should satisfy

$$z_0 + z_1 + z_2 = 1, \quad z_0 + z_1 = \alpha_{\chi_c}, \quad (56)$$

whereas for the diagrams in the right column there is a similar constraint

$$z_0 + z_1 = 1. \quad (57)$$

1. Evaluation of the amplitude $\mathcal{A}_1^{(\lambda; \sigma, H)}$

The amplitude $\mathcal{A}_1^{(\lambda; \sigma, H)}$ obtains contributions of two charge conjugate diagrams shown in the left column of Fig. 2. We will start our evaluation from the diagram shown in the upper left corner of Fig. 2, defining the partons' momenta as explained in Fig. 3 and using the CGC Feynman rules from Table I.

A straightforward evaluation yields

$$\begin{aligned} \mathcal{A}_{1,1} = & 4\pi\alpha_{\text{em}}e_c^2 \int d^2\mathbf{x}_0 d^2\mathbf{x}_1 \int \frac{d^4k_0}{(2\pi)^4} \frac{d^4k_1}{(2\pi)^4} \frac{d^4k'_0}{(2\pi)^4} \frac{d^4k'_1}{(2\pi)^4} (2\pi)^4 \delta(k_0 + k_1 + k_\gamma - q) (2\pi)^4 \delta(k'_0 + k'_1 - p_{\chi_c}) \\ & \times (2\pi) \delta(k_0^+ - k_0'^+) (2\pi) \delta(k_1^+ - k_1'^+) e^{i(k_0 - k'_0) \cdot \mathbf{x}_0} e^{i(k_1 - k'_1) \cdot \mathbf{x}_1} \phi_{\chi_c}(z, \mathbf{k}_\perp^{(\text{rel})}) \\ & \times \langle \text{Tr}[\hat{e}_\gamma(q) S(-k_1) \gamma_+ U^\dagger(\mathbf{x}_1) S(-k'_1) \Gamma_{\chi_c}(k'_1, k'_0) S(k'_0) \gamma_+ U(\mathbf{x}_0) S(k_0) \hat{e}_\gamma^*(k_\gamma) S(k_0 + k_\gamma)] \rangle \end{aligned} \quad (58)$$

where $\varepsilon_\gamma(q)$, $\varepsilon_\gamma^*(k_\gamma)$ are the polarization vectors of the photons, and a symbol $\hat{\cdot}$ was defined in (2). We also use notation $e_c = 2/3$ for the electric charge of the charm quark. The matrix Γ_{χ_c} was defined earlier in (30) for different spins and helicities of χ_c . The wave function Φ depends on the light-cone fraction $z_{\chi_c} = k_0^+/p_{\chi_c}^+$ and relative transverse momentum $\mathbf{k}_\perp^{(\text{rel})} = (z_0\mathbf{k}_1' - z_1\mathbf{k}_0')/(z_0 + z_1)$. The propagator of free fermions is defined in a standard way as

$$S(p) = i \frac{\hat{p} + m}{p^2 - m^2}. \quad (59)$$

The numerators of the propagators may be rewritten as

$$\begin{aligned} \hat{p} + m &= \left(\gamma^+ \frac{\mathbf{p}_\perp^2 + m^2}{2p^+} + \gamma^- p^+ - \boldsymbol{\gamma}_\perp \cdot \mathbf{p}_\perp + m \right) + \gamma^+ \left(p^- - \frac{\mathbf{p}_\perp^2 + m^2}{2p^+} \right) \\ &= \theta(p^+) \sum_h u_h(\tilde{p}) \bar{u}_h(\tilde{p}) + \theta(-p^+) \sum_{\bar{h}} v_{\bar{h}}(-\tilde{p}) \bar{v}_{\bar{h}}(\tilde{p}) + \gamma^+ \left(p^- - \frac{\mathbf{p}_\perp^2 + m^2}{2p^+} \right), \quad \tilde{p} = \left(p^+, \frac{\mathbf{p}_\perp^2 + m^2}{2p^+}, \mathbf{p}_\perp \right). \end{aligned} \quad (60)$$

The last term $\sim \gamma^+$ in (60) drops out when the quark is on-shell, or when the propagator is multiplied by γ^+ (due to identity $\gamma^+ \gamma^+ = 0$), so effectively (60) reduces to a helicity sum of quark or antiquark spinors. Since the matrices U , $U^\dagger$ in (58) do not have any spinorial indices, whereas all the other objects do not depend on color, the evaluation of traces over color and spinorial indices can be done independently, thus reducing (58) to a convolution of the dipole scattering amplitude (52) with the so-called (target-independent) impact factor

$$\mathcal{A}_{1,1} = \int d^2\mathbf{x}_0 d^2\mathbf{x}_1 \mathcal{N}(Y, \mathbf{x}_0, \mathbf{x}_1) R_{1,1}(Y, \mathbf{x}_0, \mathbf{x}_1, q, p_{\chi_c}, k_\gamma), \quad (61)$$

where $R_{1,1}$ is given by

$$\begin{aligned} R_{1,1}(Y, \mathbf{x}_0, \mathbf{x}_1, q, p_{\chi_c}, k_\gamma) &= 4\pi\alpha_{\text{em}} e_c^2 N_c \int \frac{d^4 k_0}{(2\pi)^4} \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_0'}{(2\pi)^4} \frac{d^4 k_1'}{(2\pi)^4} (2\pi)^4 \delta(k_0 + k_1 + k_\gamma - q) (2\pi)^4 \delta(k_0' + k_1' - p_{\chi_c}) \\ &\quad \times (2\pi) \delta(k_0^+ - k_0'^+) (2\pi) \delta(k_1^+ - k_1'^+) e^{i(k_{0\perp} - k_{0\perp}') \cdot \mathbf{x}_0} e^{i(k_{1\perp} - k_{1\perp}') \cdot \mathbf{x}_1} \phi_{\chi_{cJ}}(z, \mathbf{k}_\perp^{(\text{rel})}) \\ &\quad \times \langle \text{Tr}[\hat{\varepsilon}_\gamma(q) S(-k_1) \gamma_+ S(-k_1') \Gamma_{\chi_c}(k_1', k_0') S(k_0') \gamma_+ S(k_0) \hat{\varepsilon}_\gamma^*(k_\gamma) S(k_0 + k_\gamma)] \rangle. \end{aligned} \quad (62)$$

The integrals over the momenta $d^4 k_1$ and $d^4 k_1'$ in (62) can be calculated using δ functions, which effectively fix k_1 and k_1' to values

$$k_1 = q - k_\gamma - k_0, \quad k_1' = p_{\chi_c} - k_0'. \quad (63)$$

Due to the identity $\gamma^+ \gamma^+ = 0$, we may replace the numerators of propagators connected to the shockwave with on-shell spinors, as explained in the text under Eq. (60). Furthermore, using the well-known spinor identities [75,78]

$$\bar{u}_h(\ell) \gamma_+ u_{h'}(\ell') = \bar{v}_h(\ell) \gamma_+ v_{h'}(\ell') = 2\sqrt{\ell^+ \ell'^+} \delta_{h,h'} \quad (64)$$

where h, h' are helicities of partons, and conservation of the plus-component light cone, we may rewrite (62) as

$$\begin{aligned} R_{1,1}(Y, \mathbf{x}_0, \mathbf{x}_1, q, p_{\chi_c}, k_\gamma) &= 16\pi\alpha_{\text{em}} e_c^2 N_c (2\pi) \delta(q^+ - k_\gamma^+ - p_{\chi_c}^+) \\ &\quad \times \int \frac{dk_0^+ dk_0^- d^2\mathbf{k}_0}{(2\pi)^4} \frac{dk_0'^+ dk_0'^- d^2\mathbf{k}_0'}{(2\pi)^3} k_0^+ (q^+ - k_\gamma^+ - k_0^+) e^{i(k_{0\perp} - k_{0\perp}') \cdot \mathbf{x}_0} e^{i(k_{1\perp} - k_{1\perp}') \cdot \mathbf{x}_1} \\ &\quad \times \sum_{h, \bar{h}} \frac{[\bar{u}_h(\tilde{k}_0) \hat{\varepsilon}_\gamma^*(k_\gamma) S(k_0 + k_\gamma) \hat{\varepsilon}_\gamma(q) v_{\bar{h}}(\tilde{k}_1)] [\bar{v}_{\bar{h}}(\tilde{k}_1') \Gamma_{\chi_c}(k_1', k_0') u_h(\tilde{k}_0')] \phi_{\chi_{cJ}}(z, \mathbf{k}_\perp^{(\text{rel})})}{(k_0^2 - m^2)(k_1^2 - m^2) ((k_0')^2 - m^2)((k_1')^2 - m^2)}. \end{aligned} \quad (65)$$

For the sake of brevity, in (65) we keep notations k_1, k_1' , tacitly assuming that they are linear combinations of k_0, k_0' defined in (63). The dependence on k_0^- and $k_0'^-$ in the integrand of (65) is isolated in the first and the second terms of the last line; for

this reason integration over these variables becomes straightforward and can be realized using Cauchy's theorem and reduced to a sum of residues of the integrands at the poles, namely

$$\begin{aligned} & \int_{-\infty}^{+\infty} \frac{dk_0^-}{2\pi} \frac{1}{(2k_0^+ k_0^- - \mathbf{k}_{0\perp}^2 - m^2 + i0)(2(k_\gamma^+ + k_0^+)(k_\gamma^- + k_0^-) - (\mathbf{k}_\perp^\gamma + \mathbf{k}_{0\perp})^2 - m^2 + i0)} \\ & \times \frac{1}{(2(q^+ - k_\gamma^+ - k_0^+)(q^- - k_\gamma^- - k_0^-) - (\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 - m^2 + i0)} \\ & = - \frac{i\Theta(q^+ - k_\gamma^+ - k_0^+)}{8k_0^+(k_0^+ + k_\gamma^+)(q^+ - k_\gamma^+ - k_0^+)} \left[-\frac{q^+((\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2)}{2(q^+ - k_\gamma^+ - k_0^+)(k_\gamma^+ + k_0^+)} \right] \left[-\frac{k_{\gamma\perp}^2}{2k_\gamma^+} - \frac{k_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2}{2(q^+ - k_\gamma^+ - k_0^+)} \right], \end{aligned} \quad (66)$$

$$\begin{aligned} & \int_{-\infty}^{+\infty} \frac{dk_0^-}{2\pi} \frac{(k_\gamma^- + k_0^-)}{(2k_0^+ k_0^- - \mathbf{k}_{0\perp}^2 - m^2 + i0)(2(k_\gamma^+ + k_0^+)(k_\gamma^- + k_0^-) - (\mathbf{k}_\perp^\gamma + \mathbf{k}_{0\perp})^2 - m^2 + i0)} \\ & \times \frac{1}{(2(q^+ - k_\gamma^+ - k_0^+)(q^- - k_\gamma^- - k_0^-) - (\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 - m^2 + i0)} \\ & = - \frac{i\Theta(q^+ - k_\gamma^+ - k_0^+) \frac{(\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2}{2(k_\gamma^+ + k_0^+)}}{8k_0^+(k_0^+ + k_\gamma^+)(q^+ - k_\gamma^+ - k_0^+)} \left[-\frac{q^+((\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2)}{2(q^+ - k_\gamma^+ - k_0^+)(k_\gamma^+ + k_0^+)} \right] \left[-\frac{k_{\gamma\perp}^2}{2k_\gamma^+} - \frac{k_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2}{2(q^+ - k_\gamma^+ - k_0^+)} \right] \\ & - \frac{i\Theta(q^+ - k_\gamma^+ - k_0^+)}{8k_0^+(k_0^+ + k_\gamma^+)(q^+ - k_\gamma^+ - k_0^+)} \left[-\frac{k_{\gamma\perp}^2}{2k_\gamma^+} - \frac{k_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2}{2(q^+ - k_\gamma^+ - k_0^+)} \right], \end{aligned} \quad (67)$$

$$\begin{aligned} & \int_{-\infty}^{+\infty} \frac{dk_0^-}{2\pi} \frac{1}{(2k_0^+ k_0^- - \mathbf{k}_{0\perp}^2 - m^2 + i0)(2(p_{\chi_c}^- - k_0^-)(p_{\chi_c}^- - k_0^-) - (\mathbf{p}_\perp^{\chi_c} - \mathbf{k}_{0\perp})^2 - m^2 + i0)} \\ & = - \frac{\Theta(p_{\chi_c}^+ - k_0^+)}{4k_0^+(p_{\chi_c}^+ - k_0^+)} \frac{i}{p_{\chi_c}^- - \frac{k_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{p}_\perp^{\chi_c} - \mathbf{k}_{0\perp})^2 + m^2}{2(p_{\chi_c}^+ - k_0^+)}}. \end{aligned} \quad (68)$$

The Heaviside step functions $\Theta(\dots)$ in (66)–(68) introduce the upper cutoff for the integral over the plus-component k_0^+ and stem from the requirement that the integrands should have poles on both sides of the real axis ($\text{Im } k^- = 0$) for nonzero result. The identities (66) and (68) allow us to rewrite (62) as

$$\begin{aligned} R_{1,1}(Y, \mathbf{x}_0, \mathbf{x}_1, q, p_{\chi_c}, k_\gamma) &= \delta(q^+ - k_\gamma^+ - p_{\chi_c}^+) e^{i(q_\perp - \mathbf{k}_\perp^\gamma - \mathbf{p}_\perp^{\chi_c}) \cdot \mathbf{x}_1} \int_0^{q^+ - k_\gamma^+} dk_0^+ \\ & \times \left[\frac{2\pi\alpha_{\text{em}} e_c^2}{k_0^+ + k_\gamma^+} \int \frac{d^2 \mathbf{k}_{0\perp}}{(2\pi)^2} \frac{\bar{u}_h(k_0) \hat{\varepsilon}_\gamma^*(k_\gamma) (\hat{k}_0 + \hat{k}_\gamma + m) \hat{\varepsilon}_\gamma(q) v_{\bar{h}}(q - k_\gamma - k_0) e^{i\mathbf{k}_0^+ \cdot (\mathbf{x}_0 - \mathbf{x}_1)}}{\sqrt{k_0^+(q^+ - k_\gamma^+ - k_0^+)} \left[-\frac{q^+((\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2)}{2(q^+ - k_\gamma^+ - k_0^+)(k_\gamma^+ + k_0^+)} \right] \left[-\frac{k_{\gamma\perp}^2}{2k_\gamma^+} - \frac{k_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2}{2(q^+ - k_\gamma^+ - k_0^+)} \right]} \right. \\ & \left. + \frac{2\pi\alpha_{\text{em}} e_c^2}{k_0^+ + k_\gamma^+} \int \frac{d^2 \mathbf{k}_{0\perp}}{(2\pi)^2} \frac{\bar{u}_h(k_0) \hat{\varepsilon}_\gamma^*(k_\gamma) \gamma^+ \hat{\varepsilon}_\gamma(q) v_{\bar{h}}(q - k_\gamma - k_0) e^{i\mathbf{k}_0^+ \cdot (\mathbf{x}_0 - \mathbf{x}_1)}}{\sqrt{k_0^+(q^+ - k_\gamma^+ - k_0^+)} \left[-\frac{k_{\gamma\perp}^2}{2k_\gamma^+} - \frac{k_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2}{2(q^+ - k_\gamma^+ - k_0^+)} \right]} \right] \\ & \times \int \frac{d^2 \mathbf{k}'_{0\perp}}{(2\pi)^2} \frac{\bar{v}_{\bar{h}}(p_{\chi_c}^+ - k_0^+, \mathbf{p}_\perp^{\chi_c} - \mathbf{k}'_{0\perp}) \Gamma_{\chi_c}(p_{\chi_c} - k_0', k_0') u_h(k_0^+, \mathbf{k}'_{0\perp}) \phi_{\chi_c}(z, \mathbf{K}_\perp) e^{-i\mathbf{k}'_{0\perp} \cdot (\mathbf{x}_0 - \mathbf{x}_1)}}{2\sqrt{k_0^+(p_{\chi_c}^+ - k_0^+)} \left[p_{\chi_c}^- - \frac{k'_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{p}_\perp^{\chi_c} - \mathbf{k}'_{0\perp})^2 + m^2}{2(p_{\chi_c}^+ - k_0^+)} \right]}, \end{aligned} \quad (69)$$

where the δ -function $\delta(q^+ - k_\gamma^+ - p_{\chi_c}^+)$ in (69) reflects the smallness of the light-cone momentum transfer Δ^+ compared to the transverse momentum transfer to the target $\mathbf{\Delta}_\perp = \mathbf{q}_\perp - \mathbf{k}_\perp^\gamma - \mathbf{p}_\perp^{\chi_c}$ in the high-energy kinematics. The expressions in square brackets in the denominators of (69) coincide with the energy denominators of the light-cone perturbation theory

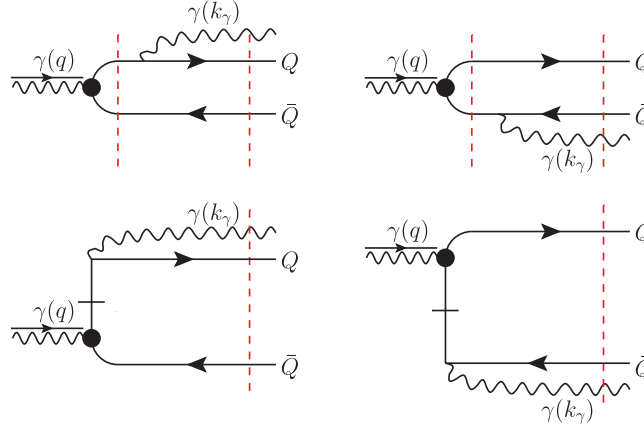


FIG. 4. The diagrams which correspond to the $\gamma \rightarrow \bar{Q}Q\gamma$ amplitude in the leading order in strong coupling α_s . The contributions of the two diagrams in the first row correspond to expressions in the second lines of (69) and (70). Similarly, the contributions of the diagrams in the last row are given by the expressions in the third lines of the same equations (69) and (70). The vertical fermion line with a short horizontal dash in the diagrams of the last row denotes instantaneous part of the quark propagator. The colored vertical dashed lines stand for the light-cone energy denominators of the canonical light-cone perturbation theory [75,78].

[75]. The analysis of the other diagrams is done similarly. The evaluation of the lower left diagram in Fig. 2 yields for the impact factor

$$\begin{aligned}
 R_{1,2}(Y, \mathbf{x}_0, \mathbf{x}_1, q, p_{\chi_c}, k_\gamma) = & -\delta(q^+ - k_\gamma^+ - p_{\chi_c}^+) e^{i(q_\perp - \mathbf{k}_\perp - \mathbf{p}_\perp) \cdot \mathbf{x}_1} \int_0^{q^+ - k_\gamma^+} dk_0^+ \\
 & \times \left[\frac{2\pi\alpha_{\text{em}} e_c^2}{q^+ - k_0^+} \int \frac{d^2 \mathbf{k}_{0\perp}}{(2\pi)^2} \frac{\bar{u}_h(k_0) \hat{\epsilon}_\gamma(q) (\hat{q} - \hat{k}_0 - m) \hat{\epsilon}_\gamma^*(k_\gamma) v_{\bar{h}}(q - k_\gamma - k_0) e^{ik_0^+ \cdot (x_0 - x_1)}}{\sqrt{k_0^+ (q^+ - k_\gamma^+ - k_0^+)} \left[q^- - \frac{q^+ (k_{0\perp}^2 + m^2)}{2(q^+ - k_\gamma^+) k_0^+} \right] \left[q^- - \frac{k_\gamma^2}{2k_\gamma^+} - \frac{k_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2}{2(q^+ - k_\gamma^+ - k_0^+)} \right]} \right. \\
 & + \left. \frac{2\pi\alpha_{\text{em}} e_c^2}{q^+ - k_0^+} \int \frac{d^2 \mathbf{k}_{0\perp}}{(2\pi)^2} \frac{\bar{u}_h(k_0) \hat{\epsilon}_\gamma(q) \gamma^+ \hat{\epsilon}_\gamma^*(k_\gamma) v_{\bar{h}}(q - k_\gamma - k_0) e^{ik_0^+ \cdot (x_0 - x_1)}}{\sqrt{k_0^+ (q^+ - k_\gamma^+ - k_0^+)} \left[q^- - \frac{k_\gamma^2}{2k_\gamma^+} - \frac{k_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{k}_{\gamma\perp} + \mathbf{k}_{0\perp})^2 + m^2}{2(q^+ - k_\gamma^+ - k_0^+)} \right]} \right] \\
 & \times \int \frac{d^2 \mathbf{k}'_{0\perp}}{(2\pi)^2} \frac{\bar{v}_{\bar{h}}(p_{\chi_c}^+ - k_0^+, \mathbf{p}'_{\perp} - \mathbf{k}'_{0\perp}) \Gamma_{\chi_c}(p_{\chi_c} - k_0', k_0') u_h(k_0^+, \mathbf{k}'_{0\perp}) e^{-ik'_{0\perp} \cdot (x_0 - x_1)} \phi_{\chi_c}(z, \mathbf{K}_\perp)}{2\sqrt{k_0^+ (p_{\chi_c}^+ - k_0^+)} \left[p_{\chi_c}^- - \frac{k_{0\perp}^2 + m^2}{2k_0^+} - \frac{(\mathbf{p}'_{\perp} - \mathbf{k}'_{0\perp})^2 + m^2}{2(p_{\chi_c}^+ - k_0^+)} \right]}. \quad (70)
 \end{aligned}$$

The result for (70) differs from (69) only by the structure of the second and the third lines. After integration over $\mathbf{k}'_{0\perp}$ in the last lines of (69) and (70), we may obtain the wave function of χ_c meson in the light-cone representation introduced earlier in (34). Likewise, the result of integration over $\mathbf{k}_{0\perp}$ in the second and the third lines of (69) and (70) can be interpreted as a light-cone wave function of the $\bar{Q}Q\gamma$ Fock component in the photon, which in the leading order is given by a set of diagrams shown in Fig. 4. These diagrams nearly coincide with the diagrams j, k, ℓ, m in [33] which contribute to the $\gamma \rightarrow \bar{Q}Q\gamma$ wave function, and for this reason the result of integration over $\mathbf{k}_{0\perp}$ differs from the expressions found in [33] only by color factor. The structure of (69) and (70) implies that the amplitude $\mathcal{A}_1$ can be rewritten as a convolution of the dipole amplitude $\mathcal{N}(x, \mathbf{r}, \mathbf{b})$, the wave function $\Psi_{\chi_c}^\dagger$ of the produced χ_c meson, and the wave function $\Psi_{\gamma \rightarrow \bar{Q}Q\gamma}$ of the $\bar{Q}Q\gamma$ Fock component of the incoming photon (the amplitude of the subprocess $\gamma \rightarrow \bar{Q}Q\gamma$), namely

$$\begin{aligned}
 \mathcal{A}_1^{(\lambda; \sigma, H)} = & \int_0^{\alpha_{\chi_c}} dz_0 \prod_{k=0}^2 (d^2 \mathbf{x}_k) \Psi_{\chi_c}^{(h, \bar{h}) \dagger} \left(\frac{z_0}{z_0 + z_1}, \mathbf{r}_{10} \right) \Psi_{\gamma \rightarrow \bar{Q}Q\gamma}^{(\lambda, \sigma, h, \bar{h})} (z_0, z_1 = \alpha_{\chi_c} - z_0, z_2 \equiv \bar{\alpha}_{\chi_c}, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2) \\
 & \times \mathcal{N}(x, \mathbf{r}_{10}, \mathbf{b}_{10}) \exp \left[-ip_{\perp}^{\chi_c} \cdot \left(\mathbf{b}_{10} - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}} \mathbf{r}_\gamma \right) - ik_{\perp}^\gamma \cdot \mathbf{x}_2 \right], \quad (71)
 \end{aligned}$$

where the dummy integration variable z_0 is related to k_0^+ using identities (55), the parameters λ, σ are the helicities of the photon before and after interaction, $h, \bar{h}$ are the helicities of the heavy quarks when they cross the shock wave, $\mathbf{r}_{10} \equiv \mathbf{x}_1 - \mathbf{x}_0$ is the relative distance between the quarks, $\mathbf{b}_{10} = (z_0 \mathbf{x}_0 + z_1 \mathbf{x}_1)/(z_0 + z_1)$ is the position (impact parameter) of the center of mass of the quark-antiquark pair, and $\mathbf{r}_\gamma = \mathbf{x}_2 - \mathbf{b}_{10}$ is the distance between the emitted photon and the center of mass of the dipole. The argument x of the dipole cross section $\mathcal{N}$ corresponds to the fraction of the target's momentum in the minus direction which is taken by heavy quarks, $x \approx \Delta^-/P^- = 2\xi/(1+\xi) \approx M_{\chi_c}^2/W^2$. As we mentioned earlier, the impact factors $R_{1,1}$ and $R_{1,2}$ are related to each other by charge conjugation (C -parity). In order to fully exploit this symmetry, it is convenient to introduce the variable ζ defined as a fraction of the photon's momentum carried by a virtual parton (quark or antiquark) which emits a secondary (final-state) photon afterwards, namely

$$z_0 = \zeta - \bar{\alpha}_{\chi_c}, \quad z_1 = 1 - \zeta, \quad z_2 = 1 - \alpha_{\chi_c} \quad (72)$$

for the contribution $R_{1,1}$, and

$$z_0 = 1 - \zeta, \quad z_1 = \zeta - \bar{\alpha}_{\chi_c}, \quad z_2 = 1 - \alpha_{\chi_c} \quad (73)$$

for the contribution $R_{1,2}$. While the expression (71) presents a lot of interest for conceptual understanding, unfortunately it is not feasible to use it directly for numerical phenomenological studies due to multidimensional integration of the oscillating function; besides the wave function $\Psi_{\gamma \rightarrow \gamma \bar{Q} Q}^{(\dots)}$ in light-cone (configuration) representation includes new special functions $\mathcal{I}, \mathcal{J}$ defined in [33,34] as nontrivial two-dimensional integrals. This suggests that evaluation of the amplitude should be done in the momentum space. However, evaluation of the Fourier image of $\mathcal{N}(x, \mathbf{r}_{10}, \mathbf{b}_{10})$ is not well defined due to saturation of the latter at large distances r_{10} and noncommutativity of the limits $r_{10} \rightarrow \infty$ and $b_{10} \rightarrow \infty$. At the same time, we may observe that for the product $\Psi_{\chi_{cJ}}^\dagger(\mathbf{r}_{10}) \mathcal{N}(x, \mathbf{r}_{10}, \mathbf{b}_{10})$ this problem does not exist, because $\Psi_{\chi_{cJ}}$ is proportional to Φ_{χ_c} and its derivatives, which are strongly suppressed at large distances. For this reason, in what follows we will introduce new functions,³

$$n_{\Phi_{\chi_c}}(x, \ell, s, z) = \int d^2\mathbf{b} d^2\mathbf{r} e^{-i\ell \cdot \mathbf{r}} e^{-is \cdot \mathbf{b}} \mathcal{N}(x, \mathbf{r}, \mathbf{b}) \Phi_{\chi_c}(\mathbf{r}, z), \quad (74)$$

$$n_{\Phi_{\chi_c}}^{(\pm)}(x, \ell, s, z) = \int d^2\mathbf{b} d^2\mathbf{r} e^{-i\ell \cdot \mathbf{r}} e^{-is \cdot \mathbf{b}} \mathcal{N}(x, \mathbf{r}, \mathbf{b}) e^{\pm i\theta} \partial_r \Phi_{\chi_c}(\mathbf{r}, z), \quad (75)$$

$$n_{\Phi_{\chi_c}}^{(2,\pm 2)}(x, \ell, s, z) = \int d^2\mathbf{b} d^2\mathbf{r} e^{-i\ell \cdot \mathbf{r}} e^{-is \cdot \mathbf{b}} \mathcal{N}(x, \mathbf{r}, \mathbf{b}) e^{\pm 2i\theta} \Delta_r^{(-)} \Phi_{\chi_c}(\mathbf{r}, z), \quad (76)$$

$$n_{\Phi_{\chi_c}}^{(2,0)}(x, \ell, s, z) = \int d^2\mathbf{b} d^2\mathbf{r} e^{-i\ell \cdot \mathbf{r}} e^{-is \cdot \mathbf{b}} \mathcal{N}(x, \mathbf{r}, \mathbf{b}) \Delta_r^{(+)} \Phi_{\chi_c}(\mathbf{r}, z), \quad (77)$$

and, using explicit expressions for the wave functions in helicity basis (33)–(41), will replace the corresponding products $\Psi_{\chi_{cJ}}^\dagger(\mathbf{r}_{10}) \mathcal{N}(x, \mathbf{r}_{10}, \mathbf{b}_{10})$ with (inverse) Fourier images of the functions $n_{\Phi_{\chi_c}}, n_{\Phi_{\chi_c}}^{(\pm)}, n_{\Phi_{\chi_c}}^{(2,0)}, n_{\Phi_{\chi_c}}^{(2,\pm 2)}$. After integration over the variables $\mathbf{r}_{10}$ and $\mathbf{b}_{10}$, we may obtain two additional δ functions

$$(2\pi)^4 \delta(s + \Delta_\perp) \delta\left(\ell - \mathbf{k}_0^\perp - \frac{\zeta - \bar{\alpha}_{\chi_c}}{\alpha} \Delta_\perp\right)$$

which allow us to get rid of the integrals over $d^2\mathbf{k}_0$ and d^2s . The final result of this procedure depends on the spin and helicity of the χ_{cJ} meson. We performed these evaluations in helicity basis, using explicit expressions for spinors from [75,78] and performing the algebraic simplifications using *Mathematica*. In view of the space constraints, we provide below the results only for components with “+” helicity of the incoming photon, tacitly assuming that the remaining

³For phenomenological parametrizations of $\mathcal{N}(x, \mathbf{r}, \mathbf{b})$ that do not depend explicitly on orientations of $\mathbf{r}, \mathbf{b}$, the integration over orientations of these vectors can be done analytically, yielding $(2\pi)^2 J_0(\ell r) J_0(sb)$ for $n_{\Phi_{\chi_c}}^{(2,0)}$, $(2\pi)^2 i J_1(\ell r) J_0(sb)$ for $n_{\Phi_{\chi_c}}^{(\pm)}$, and $-(2\pi)^2 J_2(\ell r) J_0(sb)$ for $n_{\Phi_{\chi_c}}^{(2,\pm 2)}$.

components may be recovered using $[\mathcal{A}_1^{(\lambda;\sigma,H)}]^* = \mathcal{A}_1^{(-\lambda;-\sigma,-H)}$. For the χ_{c0} meson, the helicity amplitudes read as

$$\begin{aligned} \mathcal{A}_{1,\chi_{c0}}^{(+;+)} &= 4\pi\bar{k}m \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} \\ &\times \left\{ \frac{\alpha_{\chi_c} \bar{\alpha}_{\chi_c} (\alpha_{\chi_c} - 2\bar{\zeta}) [(\zeta - \bar{\alpha}_{\chi_c})(\mathbf{P}_{(1)}^2 + m^2) - \bar{\alpha}_{\chi_c} m^2] n_{\Phi_{\chi_c}}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right. \\ &\left. - \frac{i\alpha_{\chi_c} \bar{\alpha}_{\chi_c}^2 \zeta n_{\Phi_{\chi_c}}^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}, z_{\chi_c})(P_{(1)} + iP_{(1)y})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right\}, \end{aligned} \quad (78)$$

$$\begin{aligned} \mathcal{A}_{1,\chi_{c0}}^{(+;-)} &= -4i\pi\bar{k}m\alpha_{\chi_c}^2 \bar{\alpha}_{\chi_c}^2 \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{\bar{\zeta}^3}{\zeta - \bar{\alpha}_{\chi_c}}} n_{\Phi_{\chi_c}}^{(+)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}, z_{\chi_c}) \\ &\times \frac{(P_{(1)x} + iP_{(1)y})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \end{aligned} \quad (79)$$

where we introduced shorthand notations

$$\mathbf{K}_{(1)} = \frac{\boldsymbol{\ell} \bar{\alpha}_{\chi_c} + \zeta(\alpha_{\chi_c} \mathbf{k}_{\perp}^{\gamma} - \bar{\alpha}_{\chi_c} \mathbf{p}_{\perp}^{\chi_c})}{\zeta - \bar{\alpha}_{\chi_c}} = \frac{\mathbf{P}_{(1)} \bar{\alpha}_{\chi_c}}{\zeta - \bar{\alpha}_{\chi_c}} + \frac{(\alpha_{\chi_c} \mathbf{k}_{\perp}^{\gamma} - \bar{\alpha}_{\chi_c} \mathbf{p}_{\perp}^{\chi_c})}{\alpha_{\chi_c}}, \quad (80)$$

$$\mathbf{P}_{(1)} = \boldsymbol{\ell} + \frac{\bar{\zeta}}{\alpha_{\chi_c}} (\alpha_{\chi_c} \mathbf{k}_{\perp}^{\gamma} - \bar{\alpha}_{\chi_c} \mathbf{p}_{\perp}^{\chi_c}), \quad z_{\chi_c} = \frac{\zeta - \bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}. \quad (81)$$

For the χ_{c1} meson, the corresponding amplitudes are given by

$$\begin{aligned} \mathcal{A}_{1,\chi_{c1}}^{(+;+,+1)} &= 4\pi\bar{k}\alpha_{\chi_c} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{2\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} \zeta (K_{(1)x} - iK_{(1)y}) \\ &\times \frac{(\bar{\zeta}\bar{\alpha}_{\chi_c} (P_{(1)x} + iP_{(1)y}) (2\zeta - \bar{\alpha}_{\chi_c}) n_{\Phi_{\chi_c}}^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}, z_{\chi_c}) - i\zeta m^2 n_{\Phi_{\chi_c}}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}, z_{\chi_c}) (\alpha_{\chi_c} - 2\bar{\zeta}))}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \end{aligned} \quad (82)$$

$$\begin{aligned} \mathcal{A}_{1,\chi_{c1}}^{(+;-,+1)} &= -4i\pi\bar{k}m^2\alpha_{\chi_c} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} n_{\Phi_{\chi_c}}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}, z_{\chi_c}) \\ &\times \frac{\zeta (K_{(1)x} + iK_{(1)y}) \sqrt{2\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 2\bar{\zeta})}}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \end{aligned} \quad (83)$$

$$\begin{aligned} \mathcal{A}_{1,\chi_{c1}}^{(+;+,0)} &= -\frac{8\bar{\alpha}_{\chi_c} \alpha_{\chi_c}^2}{M_{\chi_{c1}}} \pi\bar{k} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} \\ &\times \left\{ \frac{m^2 \bar{\alpha}_{\chi_c} \zeta (P_{(1)x} + iP_{(1)y}) n_{\Phi_{\chi_c}}^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right. \\ &\left. - i \frac{n_{\Phi_{\chi_c}}^{(2,0)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}, z_{\chi_c}) [(\zeta - \bar{\alpha}_{\chi_c})(\mathbf{P}_{(1)}^2 + m^2) + m^2 \bar{\alpha}_{\chi_c}]}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right\}, \end{aligned} \quad (84)$$

$$\begin{aligned} \mathcal{A}_{1\chi_{c1}}^{(+;- ,0)} &= -\frac{8m^2}{M_{\chi_{c1}}} \pi \bar{\kappa} \bar{\alpha}_{\chi_c}^2 \alpha_{\chi_c}^2 \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{\bar{\zeta}^3}{\zeta - \bar{\alpha}_{\chi_c}}} n_{\Phi_{\chi_c}}^{(+)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c}) \\ &\quad \times \frac{(P_{(1)x} + iP_{(1)y})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \end{aligned} \quad (85)$$

$$\begin{aligned} \mathcal{A}_{1\chi_{c1}}^{(+;+,-1)} &= 4\pi \bar{\kappa} \alpha_{\chi_c} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} n_{\Phi_{\chi_c}}^{(+)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c}) \\ &\quad \times \frac{(K_{(1)x} - iK_{(1)y})(P_{(1)x} + iP_{(1)y})\zeta(2\zeta - 1)\sqrt{2\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})}}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \end{aligned} \quad (86)$$

$$\begin{aligned} \mathcal{A}_{1\chi_{c1}}^{(+;-,-1)} &= 4\pi \bar{\kappa} \alpha_{\chi_c}^2 \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{2\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} \\ &\quad \times \frac{\zeta^2(K_{(1)x} + iK_{(1)y})(P_{(1)x} + iP_{(1)y})(\alpha_{\chi_c} - 2\bar{\zeta})n_{\Phi_{\chi_c}}^{(+)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \end{aligned} \quad (87)$$

$$\mathcal{A}_{1\chi_{c1}}^{(+;-,-1)} = 0. \quad (88)$$

Finally, for the χ_{c2} meson we obtained

$$\begin{aligned} \mathcal{A}_{1\chi_{c2}}^{(+;+,-2)} &= 8i\pi \bar{\kappa} \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{\bar{\zeta}^3}{\zeta - \bar{\alpha}_{\chi_c}}} \\ &\quad \times \frac{[m^2 \bar{\alpha}_{\chi_c} + (\mathbf{P}_{(1)}^2 + m^2)(\zeta - \bar{\alpha}_{\chi_c})]n_{\Phi_{\chi_c}}^{(2,-2)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \end{aligned} \quad (89)$$

$$\begin{aligned} \mathcal{A}_{1\chi_{c2}}^{(+;-,-2)} &= 8\pi \bar{\kappa} m^2 \bar{\alpha}_{\chi_c}^2 \alpha_{\chi_c}^2 \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{\bar{\zeta}^3}{\zeta - \bar{\alpha}_{\chi_c}}} n_{\Phi_{\chi_c}}^{(-)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c}) \\ &\quad \times \frac{(P_{(1)x} + iP_{(1)y})}{(\bar{\zeta} + \bar{\alpha}_{\chi_c})^2(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \end{aligned} \quad (90)$$

$$\begin{aligned} \mathcal{A}_{1\chi_{c2}}^{(+;+,-1)} &= 4iM_{\chi_{c2}} \pi \bar{\kappa} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} (K_{(1)x} - iK_{(1)y}) \sqrt{\frac{\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} \zeta \\ &\quad \times \left\{ \frac{(P_{(1)x} + iP_{(1)y})(\zeta - \bar{\alpha}_{\chi_c})(8\bar{\zeta}^3 + 4\zeta^2(\alpha_{\chi_c} - 4) - 6\zeta(\alpha_{\chi_c} - 2) + 3\alpha_{\chi_c} - 4)n_{\Phi_{\chi_c}}^{(-)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right. \\ &\quad \left. - i \frac{\zeta m^2 \alpha_{\chi_c} n_{\Phi_{\chi_c}}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})(\alpha_{\chi_c} - 2\bar{\zeta})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right\}, \end{aligned} \quad (91)$$

$$\begin{aligned}
\mathcal{A}_{1,\chi_{c2}}^{(+;-,+1)} &= 4\pi\bar{\kappa}M_{\chi_{c2}} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \zeta \sqrt{\frac{\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} (\alpha_{\chi_c} - 2\bar{\zeta}) \\
&\times \left\{ \frac{\alpha_{\chi_c}(\alpha_{\chi_c} - \bar{\zeta})m^2 n_{\Phi_{\chi_c}}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right. \\
&\left. + \frac{i\zeta(P_{(1)x} + iP_{(1)y})(\alpha_{\chi_c} - 2\bar{\zeta})^2 n_{\Phi_{\chi_c}}^{(-)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right\}, \tag{92}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{1,\chi_{c2}}^{(+;+,0)} &= -4\sqrt{\frac{2}{3}}\pi\bar{\kappa}\alpha_{\chi_c} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} \\
&\times \left\{ \frac{2m^2(\alpha_{\chi_c} - 2\bar{\zeta})n_{\Phi_{\chi_c}}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})[\bar{\alpha}_{\chi_c}(\zeta - \bar{\alpha}_{\chi_c})(\mathbf{P}_{(1)}^2 + m^2) + m(m\bar{\alpha}_{\chi_c}^2 - \zeta^2(K_{(1)x} - iK_{(1)y}))]}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right. \\
&- \frac{2im^2\zeta\alpha_{\chi_c}(P_{(1)x} + iP_{(1)y})\bar{\alpha}_{\chi_c}^2 n_{\Phi_{\chi_c}}^{(-)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \\
&\left. - \frac{3\zeta\alpha_{\chi_c}(P_{(1)x} + iP_{(1)y})(K_{(1)x} - iK_{(1)y})[\bar{\zeta}(2\zeta + \alpha_{\chi_c}) - 1]n_{\Phi_{\chi_c}}^{(2,0)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right\}, \tag{93}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{1,\chi_{c2}}^{(+;-,-0)} &= 4\sqrt{\frac{2}{3}}\pi\bar{\kappa}m^2\alpha_{\chi_c} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} \\
&\times \left\{ \frac{2\zeta m^3(K_{(1)x} + iK_{(1)y})(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 2\bar{\zeta})n_{\Phi_{\chi_c}}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right. \\
&- \frac{2i\alpha_{\chi_c}\bar{\alpha}_{\chi_c}^2 \zeta m^2(P_{(1)x} + iP_{(1)y})n_{\Phi_{\chi_c}}^{(+)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \\
&\left. - \frac{3\alpha_{\chi_c}\zeta^2(\alpha_{\chi_c} - 2\bar{\zeta})(K_{(1)x} + iK_{(1)y})(P_{(1)x} + iP_{(1)y})n_{\Phi_{\chi_c}}^{(2,0)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right\}, \tag{94}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{1,\chi_{c2}}^{(+;+,-1)} &= 4i\pi\bar{\kappa}M_{\chi_{c2}} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \zeta\bar{\zeta} \sqrt{\frac{\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} n_{\Phi_{\chi_c}}^{(+)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c}) \\
&\times \frac{(K_{(1)x} - iK_{(1)y})(P_{(1)x} + iP_{(1)y})(8\zeta^3 + 4\zeta^2(3\alpha_{\chi_c} - 4) - 6\zeta(\alpha_{\chi_c} - 2)\bar{\alpha}_{\chi_c} + (\alpha_{\chi_c} - 4)\bar{\alpha}_{\chi_c}^2)}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \tag{95}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{1,\chi_{c2}}^{(+;-,-1)} &= 16i\pi\bar{\kappa}M_{\chi_{c2}} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \zeta^2\bar{\zeta} \sqrt{\frac{\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} n_{\Phi_{\chi_c}}^{(+)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}, z_{\chi_c}) \\
&\times \frac{(K_{(1)x} + iK_{(1)y})(P_{(1)x} + iP_{(1)y})(\alpha_{\chi_c} - 2\bar{\zeta})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta}\zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c})\bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)}, \tag{96}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{1\chi_c2}^{(+;+,-2)} &= -8i\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \pi \bar{k} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\frac{\bar{\zeta}}{\zeta - \bar{\alpha}_{\chi_c}}} \\
&\times \left\{ \frac{n_{\Phi_{\chi_c}}^{(2,+2)}(x, \ell, -\Delta_\perp, z_{\chi_c}) [(\zeta - \bar{\alpha}_{\chi_c})(\mathbf{P}_{(1)}^2 + m^2) + m^2 \bar{\alpha}_{\chi_c}](\zeta - \bar{\alpha}_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta} \zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c}) \bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right. \\
&\left. + \frac{im^2 \zeta \alpha_{\chi_c} \bar{\alpha}_{\chi_c} (P_{(1)x} + iP_{(1)y}) n_{\Phi_{\chi_c}}^{(+)}(x, \ell, -\Delta_\perp, z_{\chi_c})}{(\mathbf{P}_{(1)}^2 + m^2)(\bar{\zeta} \zeta^2 \mathbf{K}_{(1)}^2 + (\zeta - \bar{\alpha}_{\chi_c}) \bar{\alpha}_{\chi_c} \mathbf{P}_{(1)}^2 + \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \zeta m^2)} \right\}, \quad (97)
\end{aligned}$$

$$\mathcal{A}_{1\chi_c2}^{(+;-,-2)} = 0. \quad (98)$$

2. Evaluation of the amplitude $\mathcal{A}_2^{(\lambda;\sigma,H)}$

The amplitude $\mathcal{A}_2^{(\lambda;\sigma,H)}$ corresponds to the contribution of the diagrams in the right column of Fig. 2 and can be evaluated using the procedure outlined in the previous Sec. IID 1. We may represent the amplitude as a convolution of the dipole amplitude and the impact factors $R_{2,i}$,

$$\mathcal{A}_{2,i} = \int d^2\mathbf{x}_0 d^2\mathbf{x}_1 \mathcal{N}(Y, \mathbf{x}_0, \mathbf{x}_1) R_{2,i}(Y, \mathbf{x}_0, \mathbf{x}_1, q, p_{\chi_c}, k_\gamma), \quad i = 1, 2. \quad (99)$$

The corresponding expressions for $R_{2,i}$ in (99) differ from the impact factors $R_{1,i}$ only by permutation of the χ_c and the incoming photon, namely

$$\begin{aligned}
R_{2,1}(Y, \mathbf{x}_0, \mathbf{x}_1, q, p_{\chi_c}, k_\gamma) &= 4\pi\alpha_{\text{em}} e_c^2 N_c \\
&\times \int \frac{d^4 k_0}{(2\pi)^4} \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_0'}{(2\pi)^4} \frac{d^4 k_1'}{(2\pi)^4} (2\pi)^4 \delta(k_0 + k_1 - q) (2\pi)^4 \delta(k_0' + k_1' - p_{\chi_c} - k_\gamma) \\
&\times (2\pi) \delta(k_0^+ - k_0'^+) (2\pi) \delta(k_1^+ - k_1'^+) e^{i(\mathbf{k}_{0\perp} - \mathbf{k}_{0\perp}') \cdot \mathbf{x}_0} e^{i(\mathbf{k}_{1\perp} - \mathbf{k}_{1\perp}') \cdot \mathbf{x}_1} \phi_{\chi_c\gamma}(z, \mathbf{k}_\perp^{\text{rel}}) \\
&\times \langle \text{Tr}[\Gamma_{\chi_c}(k_1', k_0' - k_\gamma) S(k_0' - k_\gamma) \hat{e}_\gamma^*(k_\gamma) S(k_0') \gamma_+ S(k_0) \hat{e}_\gamma(q) S(-k_1) \gamma^+ S(-k_1')] \rangle, \quad (100)
\end{aligned}$$

$$\begin{aligned}
R_{2,2}(Y, \mathbf{x}_0, \mathbf{x}_1, q, p_{\chi_c}, k_\gamma) &= 4\pi\alpha_{\text{em}} e_c^2 N_c \\
&\times \int \frac{d^4 k_0}{(2\pi)^4} \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_0'}{(2\pi)^4} \frac{d^4 k_1'}{(2\pi)^4} (2\pi)^4 \delta(k_0 + k_1 - q) (2\pi)^4 \delta(k_0' + k_1' - p_{\chi_c} - k_\gamma) \\
&\times (2\pi) \delta(k_0^+ - k_0'^+) (2\pi) \delta(k_1^+ - k_1'^+) e^{i(\mathbf{k}_{0\perp} - \mathbf{k}_{0\perp}') \cdot \mathbf{x}_0} e^{i(\mathbf{k}_{1\perp} - \mathbf{k}_{1\perp}') \cdot \mathbf{x}_1} \phi_{\chi_c\gamma}(z, \mathbf{k}_\perp^{\text{rel}}) \\
&\times \langle \text{Tr}[\Gamma_{\chi_c}(k_1' + k_\gamma, k_0') S(k_0') \gamma_+ S(k_0) \hat{e}_\gamma(q) S(-k_1) \gamma_+ S(-k_1') \hat{e}_\gamma^*(k_\gamma) S(-k_1' - k_\gamma)] \rangle. \quad (101)
\end{aligned}$$

Repeating the procedure outlined in the previous Sec. IID 1, we may rewrite the impact factors $R_{2,i}$ in terms of the photon wave function $\psi_{\gamma \rightarrow \bar{Q}Q}$ and the amplitude (“wave function”) $\Psi_{\bar{Q}Q \rightarrow \gamma\chi_c}^\dagger$ of the $\bar{Q}Q \rightarrow \chi_c \gamma$ subprocess, contracted over helicities $h, \bar{h}$ of the quarks and convoluted over the light-cone component k_Q^+ of the quark momentum. The final result for the amplitude $\mathcal{A}_2^{(\lambda;\sigma)}$ may be rewritten as

$$\begin{aligned}
\mathcal{A}_2^{(\lambda;\sigma)} &= \int_0^{\alpha_{\chi_c}} dz_0 \prod_{k=1}^3 (d^2\mathbf{r}_k) \Psi_{\bar{Q}Q \rightarrow \gamma\chi_c}^{(\sigma,h,\bar{h})\dagger}(z_0, z_1 = \alpha_{\chi_c} - z_0, z_2 \equiv \bar{\alpha}_{\chi_c}, \mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2) \psi_{\gamma \rightarrow \bar{Q}Q}^{(\lambda,h,\bar{h})}\left(\frac{z_0}{z_0 + z_1}, \mathbf{r}_{10}\right) \\
&\times \mathcal{N}(x, \mathbf{r}_{10}, \mathbf{b}_{10}) \exp \left[-i\mathbf{p}_\perp^{\chi_c} \cdot \left(\mathbf{b}_{10} - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}} \mathbf{r}_\gamma \right) - i\mathbf{k}_\perp^\gamma \cdot (\mathbf{r}_\gamma + \mathbf{b}_{10}) \right], \quad (102)
\end{aligned}$$

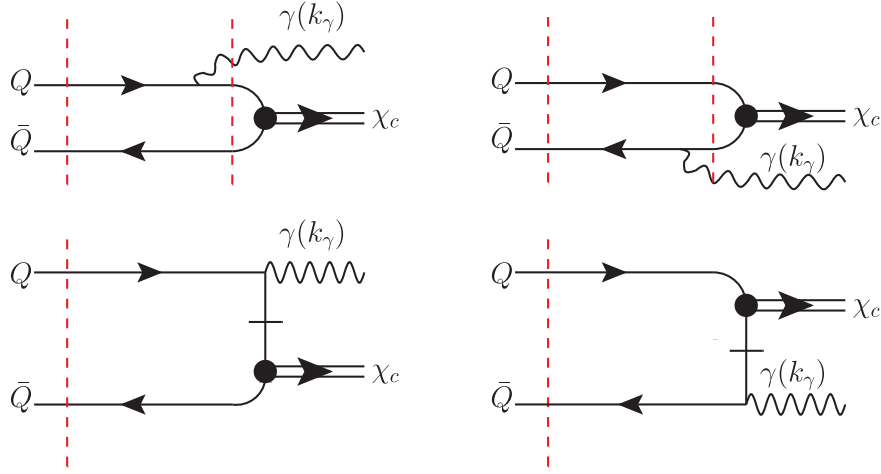


FIG. 5. The leading order diagrams which contribute to the amplitude of the subprocess $Q\bar{Q} \rightarrow \gamma\chi_c$. The vertical dashed lines denote the energy denominators of the light-cone perturbation theory [75,78].

where we use the same notations as in the previous section [see the text under Eq. (71)]. The photon wave function $\psi_{\gamma \rightarrow Q\bar{Q}}^{(\lambda, h, \bar{h})}$ in the leading order in α_s is given by [95,96]

$$\begin{aligned} \psi_{\gamma \rightarrow Q\bar{Q}}^{(\lambda, h, \bar{h})}(\zeta, r) &= -ie e_c \int \frac{d^2 \mathbf{k}_{Q\perp}}{(2\pi)^2} \frac{\bar{u}_h(k_Q^+, \mathbf{k}_{Q\perp}) \hat{\epsilon}_\lambda(q) v_{\bar{h}}(q^+ - k_Q^+, -\mathbf{k}_{Q\perp})}{q^- - \frac{q^+ (\mathbf{k}_{Q\perp}^2 + m^2)}{2k_Q^+ (q^+ - k_Q^+)}} e^{i\mathbf{k}_{Q\perp} \cdot \mathbf{r}_{10}} \Bigg|_{q^- = 0, \quad k_Q^+ = \zeta q^+} \\ &= \frac{\sqrt{2}}{2\pi} e e_f [-i\lambda e^{-i\lambda\phi_r} (\zeta \delta_{h,\lambda} \delta_{\bar{h}, -\lambda} - (1 - \zeta) \delta_{h, -\lambda} \delta_{\bar{h}, \lambda}) m K_1(mr) + m \delta_{h,\lambda} \delta_{\bar{h}, \lambda} K_0(mr)], \end{aligned} \quad (103)$$

The evaluation of the amplitude $\Psi_{\bar{Q}Q \rightarrow \gamma\chi_c}^{(\sigma, h, \bar{h})}$ in the leading order in the coupling α_s resembles a similar evaluation of the wave function $\Psi_{\gamma \rightarrow Q\bar{Q}}^{(\lambda, \sigma, h, \bar{h})}$ discussed earlier and requires evaluation of the diagrams shown in the right column of Fig. 5 using the light-cone rules from [78]. The evaluation is straightforward; however, the final expression is very lengthy, introduces a number of new special functions defined via two-dimensional integrals, and for this reason evaluation of (102) in configuration space is not feasible. As we mentioned earlier, the Fourier transformation of the dipole amplitude $\mathcal{N}(x, \mathbf{r}, \mathbf{b})$ presents certain challenges due to saturation at large r and noncommutativity of limits $r \rightarrow \infty$ and $b \rightarrow \infty$. However, the product of the dipole amplitude and wave function $\mathcal{N}(x, r, b) \psi_{\gamma \rightarrow Q\bar{Q}}^{(\lambda, h, \bar{h})}(\zeta, r)$ has a well-defined Fourier image: the functions K_0, K_1 suppress the product at large r , whereas a color transparency (suppression of $\mathcal{N}$ at small r) guarantees that the Fourier integrals remain convergent at small r . Similar to (74)–(77), we may introduce new functions

$$n_0(x, \ell, s) = \int d^2 \mathbf{b} d^2 \mathbf{r} e^{-i\ell \cdot \mathbf{r}} e^{-is \cdot \mathbf{b}} \mathcal{N}(x, \mathbf{r}, \mathbf{b}) K_0(mr), \quad (104)$$

$$n_1^{(\pm)}(x, \ell, s) = \int d^2 \mathbf{b} d^2 \mathbf{r} e^{-i\ell \cdot \mathbf{r}} e^{-is \cdot \mathbf{b}} \mathcal{N}(x, \mathbf{r}, \mathbf{b}) K_1(mr) e^{\pm i\phi_r}, \quad (105)$$

where the angle $\phi_r = \arg(r_x + ir_y)$ characterizes the azimuthal orientation of the vector $\mathbf{r}$, and express the product $\mathcal{N}(x, r, b) \psi_{\gamma \rightarrow Q\bar{Q}}^{(\lambda, h, \bar{h})}(\zeta, r)$ in terms of the (inverse) Fourier transforms of these functions. We found that the functions $n_0, n_1^{(\pm)}$ have a mild dependence on all their arguments and are strongly suppressed at large ℓ, s . For this reason, it is possible to significantly speed up the evaluations using caching (interpolation of preevaluated function values) on a modestly sized grid.

In order to reduce the charge conjugate contributions in the amplitude $\mathcal{A}_2^{(\lambda, \sigma, H)}$ to the common denominator, we will replace the dummy integration variable z_0 with a new variable ζ defined as a fraction of the incoming photon's momentum carried by the active fermion before emission of the secondary photon, namely

$$\zeta = \frac{k_0^+}{k_0^+ + k_1^+} = \frac{z_0}{z_0 + z_1}, \quad (106)$$

for the contribution with photon emission from the quark, and with

$$\zeta = \frac{k_1^+}{k_0^+ + k_1^+} = 1 - \frac{z_0}{z_0 + z_1}, \quad (107)$$

for the other contribution. In both cases, the variable ζ cannot be less than $k_\gamma^+/q^+ = \bar{\alpha}_{\chi_c}$. The evaluation of the amplitudes is straightforward and largely repeats the evaluation of (78)–(98). Due to space limitation, we will provide only the final results. For the χ_{c0} mesons, the corresponding helicity amplitudes are given by

$$\begin{aligned} \mathcal{A}_{2\chi_{c0}}^{(+;+)} = & -2m\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \bar{\kappa} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\sqrt{\zeta\bar{\zeta}^3}}{(\zeta - \bar{\alpha}_{\chi_c})^2} \tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)}) \\ & \times \left\{ \frac{\zeta n_0(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp)(\mathbf{P}_{(2)}^2 + m^2(z_c - \frac{1}{2})^2)}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \right. \\ & \left. + \frac{m\bar{\alpha}_{\chi_c}(mn_0(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp)(z_c - \frac{1}{2}) + i\zeta(P_{(2)x} + iP_{(2)y})n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp))}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \right\}, \quad (108) \end{aligned}$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c0}}^{(+;-)} = & 2i\bar{\kappa}m^2\alpha_{\chi_c} \bar{\alpha}_{\chi_c}^2 \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \sqrt{\zeta\bar{\zeta}^5} \\ & \times \frac{(P_{(2)x} - iP_{(2)y})n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp)\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)})}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)}, \quad (109) \end{aligned}$$

where we introduced the shorthand notations

$$\mathbf{K}_{(2)} = -\frac{\boldsymbol{\ell}\bar{\alpha}_{\chi_c}}{\zeta} + \frac{(\zeta - \bar{\alpha}_{\chi_c})}{\alpha_{\chi_c}^2 \zeta} (\alpha_{\chi_c} \mathbf{k}_\perp^\gamma - \bar{\alpha}_{\chi_c} \mathbf{p}_\perp^{\chi_c}) = -\frac{\mathbf{P}_{(2)}\bar{\alpha}_{\chi_c}}{\zeta} + \frac{(\alpha_{\chi_c} \mathbf{k}_\perp^\gamma - \bar{\alpha}_{\chi_c} \mathbf{p}_\perp^{\chi_c})}{\alpha_{\chi_c}}, \quad (110)$$

$$\mathbf{P}_{(2)} = \boldsymbol{\ell} + \frac{\bar{\zeta}}{\alpha_{\chi_c}^2} (\alpha_{\chi_c} \mathbf{k}_\perp^\gamma - \bar{\alpha}_{\chi_c} \mathbf{p}_\perp^{\chi_c}), \quad z_{\chi_c} = \frac{\zeta - \bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}. \quad (111)$$

For the χ_{c1} meson, the corresponding amplitudes are given by

$$\begin{aligned} \mathcal{A}_{2\chi_{c1}}^{(+;+,+1)} = & 2m\bar{\kappa} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)})\sqrt{2\zeta\bar{\zeta}^3}(K_x + iK_y)}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \\ & \times [(2\zeta - 1)(P_{(2)x} + iP_{(2)y})(\zeta - \bar{\alpha}_{\chi_c})n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp) - i\zeta mn_0(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp)(2\zeta + \alpha_{\chi_c} - 2)], \quad (112) \end{aligned}$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c1}}^{(+;-,+1)} = & -2im\bar{\kappa} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\sqrt{2\zeta\bar{\zeta}^3}(K_{(2)x} - iK_{(2)y})(2\zeta + \alpha_{\chi_c} - 2)\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)})}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \\ & \times [mn_0(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp)(\zeta - \bar{\alpha}_{\chi_c}) + i\zeta\alpha_{\chi_c}(P_{(2)x} + iP_{(2)y})n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp)], \quad (113) \end{aligned}$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c1}}^{(+;+,0)} = & i\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \bar{\kappa} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)})\sqrt{\zeta\bar{\zeta}}}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \\ & \times \left\{ mn_0(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp) \left[\bar{\alpha}_{\chi_c} \alpha_{\chi_c}^2 \bar{\zeta} (m^2(z_c - 1/2)^2 - \mathbf{P}_{(2)}^2) - \alpha_{\chi_c} \zeta \left(\mathbf{P}_{(2)}^2 + m^2 \left(z_c - \frac{1}{2} \right)^2 \right) \right] \right. \\ & \left. + in_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_\perp) (P_{(2)x} + iP_{(2)y}) \left(2m^2 \bar{\alpha}_{\chi_c} \alpha_{\chi_c}^2 \zeta \bar{\zeta} - \alpha_{\chi_c} \zeta^2 \left(\mathbf{P}_{(2)}^2 + m^2 \left(z_c - \frac{1}{2} \right)^2 \right) \right) \right\}, \quad (114) \end{aligned}$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c1}}^{(+;-;0)} &= \bar{\kappa} \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)}) \sqrt{\zeta \bar{\zeta}^3} (P_{(2)x} - iP_{(2)y})}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}} (\zeta - \bar{\alpha}_{\chi_c}) (\alpha_{\chi_c} - 4\zeta \bar{\zeta}) m^2)} \\ &\quad \times n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}) \left(\zeta \alpha_{\chi_c} \left(\mathbf{P}_{(2)}^2 + m^2 \left(z_c - \frac{1}{2} \right)^2 \right) - 2\bar{\zeta} m^2 \bar{\alpha}_{\chi_c} \right), \end{aligned} \quad (115)$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c1}}^{(+;+;-1)} &= -2\bar{\kappa} m \bar{\alpha}_{\chi_c} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)}) \\ &\quad \times \frac{[\sqrt{2\zeta \bar{\zeta}^5} (\zeta - \bar{\alpha}_{\chi_c}) (K_{(2)x} + iK_{(2)y}) (P_{(2)x} - iP_{(2)y}) (2\zeta - \bar{\alpha}_{\chi_c}) n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp})]}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}} (\zeta - \bar{\alpha}_{\chi_c}) (\alpha_{\chi_c} - 4\zeta \bar{\zeta}) m^2)}, \end{aligned} \quad (116)$$

$$\mathcal{A}_{2\chi_{c1}}^{(+;-;-1)} = 0. \quad (117)$$

Finally, for the χ_{c2} meson we obtained

$$\mathcal{A}_{2\chi_{c2}}^{(+;+;+2)} = 4m^2 \bar{\alpha}_{\chi_c}^2 \bar{\kappa} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)}) \sqrt{\zeta \bar{\zeta}^5} (P_{(2)x} + iP_{(2)y})^2 n_0(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp})}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}} (\zeta - \bar{\alpha}_{\chi_c}) (\alpha_{\chi_c} - 4\zeta \bar{\zeta}) m^2)}, \quad (118)$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c2}}^{(+;-;+2)} &= 4i\bar{\kappa} m \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\alpha_{\chi_c} \bar{\alpha}_{\chi_c} \tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)}) \sqrt{\zeta \bar{\zeta}^5} (P_{(2)x} + iP_{(2)y}) n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp})}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}} (\zeta - \bar{\alpha}_{\chi_c}) (\alpha_{\chi_c} - 4\zeta \bar{\zeta}) m^2)} \\ &\quad \times \left(\zeta \left(\mathbf{P}_{(2)}^2 + m^2 \left(z_c - \frac{1}{2} \right)^2 \right) - m^2 \bar{\alpha}_{\chi_c} \right), \end{aligned} \quad (119)$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c2}}^{(+;+;+1)} &= -\frac{\bar{\kappa}}{2\alpha_{\chi_c}} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)}) \sqrt{\zeta \bar{\zeta}} (K_{(2)x} + iK_{(2)y})}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}} (\zeta - \bar{\alpha}_{\chi_c}) (\alpha_{\chi_c} - 4\zeta \bar{\zeta}) m^2)} \\ &\quad \times (m^2(-4\bar{\zeta}^2 + \alpha_{\chi_c}^2 + 4\bar{\zeta} \alpha_{\chi_c}) + \alpha_{\chi_c}^2 \mathbf{P}_{(2)}^2) \{ \zeta m \alpha_{\chi_c} n_0(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}) (2\zeta + \alpha_{\chi_c} - 2) \\ &\quad + i(\zeta - \bar{\alpha}_{\chi_c}) (8\bar{\zeta}^3 + 4\bar{\zeta}^2 (\alpha_{\chi_c} - 4) - 6\bar{\zeta} (\alpha_{\chi_c} - 2) + 3\alpha_{\chi_c} - 4) (P_{(2)x} + iP_{(2)y}) n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}) \}, \end{aligned} \quad (120)$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c2}}^{(+;-;+1)} &= -\frac{\bar{\kappa}}{2\alpha_{\chi_c}} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)}) \sqrt{\zeta \bar{\zeta}} (K_{(2)x} + iK_{(2)y}) (2\zeta + \alpha_{\chi_c} - 2)}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}} (\zeta - \bar{\alpha}_{\chi_c}) (\alpha_{\chi_c} - 4\zeta \bar{\zeta}) m^2)} \\ &\quad \times (m^2(-4\bar{\zeta}^2 + \alpha_{\chi_c}^2 + 4\bar{\zeta} \alpha_{\chi_c}) + \alpha_{\chi_c}^2 \mathbf{P}_{(2)}^2) \\ &\quad \times (m \alpha_{\chi_c} n_0(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}) (\zeta - \bar{\alpha}_{\chi_c}) + i\zeta (2\zeta + \alpha_{\chi_c} - 2)^2 (P_{(2)x} + iP_{(2)y}) n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp})), \end{aligned} \quad (121)$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c2}}^{(+;+;0)} &= \frac{\bar{\kappa} \bar{\alpha}_{\chi_c}}{2\sqrt{6} \alpha_{\chi_c}} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)}) \sqrt{\zeta / \bar{\zeta}}}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2 \mathbf{K}_{(2)}^2 - \zeta \alpha_{\chi_c} \bar{\alpha}_{\chi_c} \mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}} (\zeta - \bar{\alpha}_{\chi_c}) (\alpha_{\chi_c} - 4\zeta \bar{\zeta}) m^2)} \\ &\quad \times \left\{ n_0(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}) (1 - 2z_c) \right. \\ &\quad \times (m^4(-4\bar{\zeta}^2 + \alpha_{\chi_c}^2 + 4\bar{\zeta} \alpha_{\chi_c}) (-4\bar{\zeta}^2 + \alpha_{\chi_c}^2 + 4\bar{\zeta} (1 - 2\zeta) \alpha_{\chi_c}) + 2m^2 \alpha_{\chi_c}^4 \mathbf{P}_{(2)}^2 + \alpha_{\chi_c}^4 (\mathbf{P}_{(2)}^2)^2) \\ &\quad + i n_1^{(-)}(x, \boldsymbol{\ell}, -\mathbf{\Delta}_{\perp}) \alpha_{\chi_c}^2 ((P_{(2)x} + iP_{(2)y})/m) \left[8\bar{\zeta} \bar{\zeta}^2 m^4 \bar{\alpha}_{\chi_c} (\zeta - \bar{\alpha}_{\chi_c}) \right. \\ &\quad \left. \left. + \bar{\zeta}^2 \alpha_{\chi_c} (m^2(-4\bar{\zeta}^2 + \alpha_{\chi_c}^2 + 2\bar{\zeta} \alpha_{\chi_c}) + \alpha_{\chi_c} (\alpha_{\chi_c} - 2\bar{\zeta}) \mathbf{P}_{(2)}^2) \left(m^2 \left(z_c - \frac{1}{2} \right)^2 + \mathbf{P}_{(2)}^2 \right) \right] \right\}, \end{aligned} \quad (122)$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c2}}^{(+;-,-,0)} = & \frac{\bar{\kappa}\alpha_{\chi_c}\bar{\alpha}_{\chi_c}}{2\sqrt{6}m} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{i\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)})(P_{(2)x} - iP_{(2)y})n_1^{(-)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp})\sqrt{\zeta\bar{\zeta}}}{(\zeta - \bar{\alpha}_{\chi_c})} \\ & \times \left\{ \frac{\zeta\alpha_{\chi_c}(m^2(-4\bar{\zeta}^2 + \alpha_{\chi_c}^2 + 2\bar{\zeta}\alpha_{\chi_c}) + \alpha_{\chi_c}(\alpha_{\chi_c} - 2\bar{\zeta})\mathbf{P}_{(2)}^2)(\mathbf{P}_{(2)}^2 + m^2(z_c - \frac{1}{2})^2)}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2\mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c}\bar{\alpha}_{\chi_c}\mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \right. \\ & \left. - \frac{8\bar{\zeta}^2 m^4 \bar{\alpha}_{\chi_c}(\zeta - \bar{\alpha}_{\chi_c})}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2\mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c}\bar{\alpha}_{\chi_c}\mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \right\}, \end{aligned} \quad (123)$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c2}}^{(+;+,-,1)} = & \bar{\kappa} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)})\sqrt{\zeta\bar{\zeta}}(m^2(-4\bar{\zeta}^2 + \alpha_{\chi_c}^2 + 4\bar{\zeta}\alpha_{\chi_c}) + \alpha_{\chi_c}^2\mathbf{P}_{(2)}^2)}{4\alpha_{\chi_c}(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2\mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c}\bar{\alpha}_{\chi_c}\mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \\ & \times \{mn_0(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp})[\bar{\alpha}_{\chi_c}^2(P_{(2)x} - iP_{(2)y})(\alpha_{\chi_c} - 4\bar{\zeta}) - \alpha_{\chi_c}(K_{(2)x} + iK_{(2)y})(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 2\bar{\zeta})] \\ & - in_1^{(-)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp})[2m^2\bar{\alpha}_{\chi_c}^2\alpha_{\chi_c} \\ & - (2\zeta - 1)(K_{(2)x} + iK_{(2)y})(P_{(2)x} - iP_{(2)y})(4\bar{\zeta}^2(2\zeta - 1) + \alpha_{\chi_c}^3 - 6\bar{\zeta}\alpha_{\chi_c}^2 - 3\bar{\zeta}^2(4\zeta - 3)\alpha_{\chi_c})]\}, \end{aligned} \quad (124)$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c2}}^{(+;-,-,1)} = & 2i\bar{\kappa} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \frac{\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)})n_1^{(-)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp})(\zeta\bar{\zeta})^{3/2}}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2\mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c}\bar{\alpha}_{\chi_c}\mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \\ & \times (K_{(2)x} - iK_{(2)y})(P_{(2)x} - iP_{(2)y})(\zeta - \bar{\alpha}_{\chi_c})(z_c - 1/2) \left(m^2(-4\bar{\zeta}^2 + \alpha_{\chi_c}^2 + 4\bar{\zeta}\alpha_{\chi_c}) + \alpha_{\chi_c}^2\mathbf{P}_{(2)}^2 \right), \end{aligned} \quad (125)$$

$$\begin{aligned} \mathcal{A}_{2\chi_{c2}}^{(+;+,-,2)} = & -4m\bar{\alpha}_{\chi_c}\bar{\kappa} \int_{\bar{\alpha}_{\chi_c}}^1 d\zeta \int \frac{d^2\ell}{(2\pi)^2} \tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P}_{(2)})\sqrt{\zeta\bar{\zeta}}^3(P_{(2)x} - iP_{(2)y}) \\ & \times \left\{ \frac{m\bar{\alpha}_{\chi_c}((P_{(2)x} - iP_{(2)y})n_0(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp})(\zeta - \bar{\alpha}_{\chi_c}) - i\zeta m\alpha_{\chi_c}n_1^{(-)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp}))}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2\mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c}\bar{\alpha}_{\chi_c}\mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \right. \\ & \left. + \frac{i\alpha_{\chi_c}\bar{\zeta}^2 n_1^{(-)}(x, \boldsymbol{\ell}, -\boldsymbol{\Delta}_{\perp})(\mathbf{P}_{(2)}^2 + m^2(z_c - \frac{1}{2})^2)}{(\bar{\zeta}(\zeta - \bar{\alpha}_{\chi_c})^2\mathbf{K}_{(2)}^2 - \zeta\alpha_{\chi_c}\bar{\alpha}_{\chi_c}\mathbf{P}_{(2)}^2 - \frac{\bar{\alpha}_{\chi_c}}{\alpha_{\chi_c}}(\zeta - \bar{\alpha}_{\chi_c})(\alpha_{\chi_c} - 4\zeta\bar{\zeta})m^2)} \right\}, \end{aligned} \quad (126)$$

$$\mathcal{A}_{2\chi_{c2}}^{(+;-,-,2)} = 0. \quad (127)$$

At the extremes of the integration domain $\zeta = 1$ and $\zeta = \bar{\alpha}_{\chi_c}$, the integrand is strongly suppressed by the wave function $\tilde{\Phi}_{\chi_c}(z_{\chi_c}, \mathbf{P})$, and in the heavy quark mass limit the dominant contribution comes from the central region $\zeta \sim 1 - \alpha_{\chi_c}/2$ ($z_{\chi_c} = 1/2$). If we construct a Taylor expansion near the point $z_{\chi_c} = 1/2, \mathbf{P} = 0$, the first nonvanishing term in the latter can be reduced to NRQCD LDMEs of χ_c mesons using the relations (46) and (47) from Sec. II B. The explicit expressions for the coefficients in front of LDMEs are extremely lengthy (require taking derivatives of the derivatives of the integrands) and will not be provided explicitly due to space limitations. The evaluation of the cross section was performed without making such expansion, taking explicitly the integrals over $d^2\ell d\zeta$ in momentum space.

III. NUMERICAL ESTIMATES

A. Differential cross sections

While the results found in the previous section are valid for any parametrization of the dipole amplitude, below we provide a tentative numerical estimate of the cross section using a particular parametrization of the dipole amplitude (we neither endorse nor refute this or any other parametrization of the dipole amplitude). For the sake of definiteness, we will use in what follows the CGC parametrization of the forward dipole scattering amplitude, whose parameters were fixed from fits of HERA data [16],

$$N(x, \mathbf{r}, \mathbf{b}) = \begin{cases} N_0 \left(\frac{r Q_s(x)}{2} \right)^{2\gamma_{\text{eff}}(r)}, & r \leq \frac{2}{Q_s(x)} \\ 1 - \exp(-\mathcal{A} \ln^2(\mathcal{B} r Q_s)), & r > \frac{2}{Q_s(x)} \end{cases}, \quad (128)$$

$$\mathcal{A} = -\frac{N_0^2 \gamma_s^2}{(1 - N_0)^2 \ln(1 - N_0)}, \quad \mathcal{B} = \frac{1}{2} (1 - N_0)^{-\frac{1 - N_0}{N_0 \gamma_s}}, \quad (129)$$

$$Q_s(x, \mathbf{b}) = \left(\frac{x_0}{x} \right)^{\lambda/2} T_G(b), \quad \gamma_{\text{eff}}(r) = \gamma_s + \frac{1}{\kappa \lambda \ln(1/x)} \ln \left(\frac{2}{r Q_s(x)} \right), \quad T_G(b) = \exp \left(-\frac{b^2}{4\gamma_s B_{\text{CGC}}} \right), \quad (130)$$

$$\gamma_s = 0.6492, \quad \lambda = 0.2023, \quad x_0 = 6.9 \times 10^{-4}, \quad B_{\text{CGC}} = 5.5 \text{ GeV}^{-2} \approx (0.463 \text{ fm})^2. \quad (131)$$

In the heavy quark mass limit, the binding energy of the quarkonium formally is a small parameter, $\sim \mathcal{O}(v^2) \sim \mathcal{O}(\alpha_s^2(m_c))$. In order to have consistency with this expectation, we will assume that the charm mass $m_c \approx M_{\chi_c}/2 \approx 1.8 \text{ GeV}$. We also will implement the corrections due to the real part of the amplitude (53), as discussed in Sec. IIC.

We would like to start a discussion from the threefold differential unpolarized cross section (21). In Fig. 6 we have shown this observable as a function of the invariant momentum transfer t to the target. Since the parameter $x \sim M_{\chi_c}^2/W^2$ is very small in the chosen kinematics, the variable t may be approximated as $t \approx -\Delta_\perp^2$. The pronounced dependence on this variable largely stems from the explicit $\Delta_\perp$ dependence which appears in the arguments of

the functions n, n_Φ (74)–(77), (104), (105) in the momentum space amplitudes (78)–(98) and (108)–(127). The dependence on $\Delta_\perp$, which appears in the prefactors in front of these functions, leads only to minor $\mathcal{O}(t/M_{\chi_c}^2)$ corrections. Indeed, we may observe that in all these prefactors the external vectors $\mathbf{p}_\perp, \Delta_\perp$ always contribute only on the linear combination

$$L = (\alpha_{\chi_c} \mathbf{k}_\perp^\gamma - \bar{\alpha}_{\chi_c} \mathbf{p}_\perp^{\chi_c}) = \mathbf{p}_\perp + \Delta_\perp \left(\frac{1}{2} - \alpha_{\chi_c} \right). \quad (132)$$

After integration over the dummy momentum ℓ , the absolute value of the amplitude may depend only on the square L^2 . Since the vectors $\mathbf{p}_\perp$ and $\Delta_\perp$ are mutually orthogonal in the high-energy kinematics [see Eq (17)], and

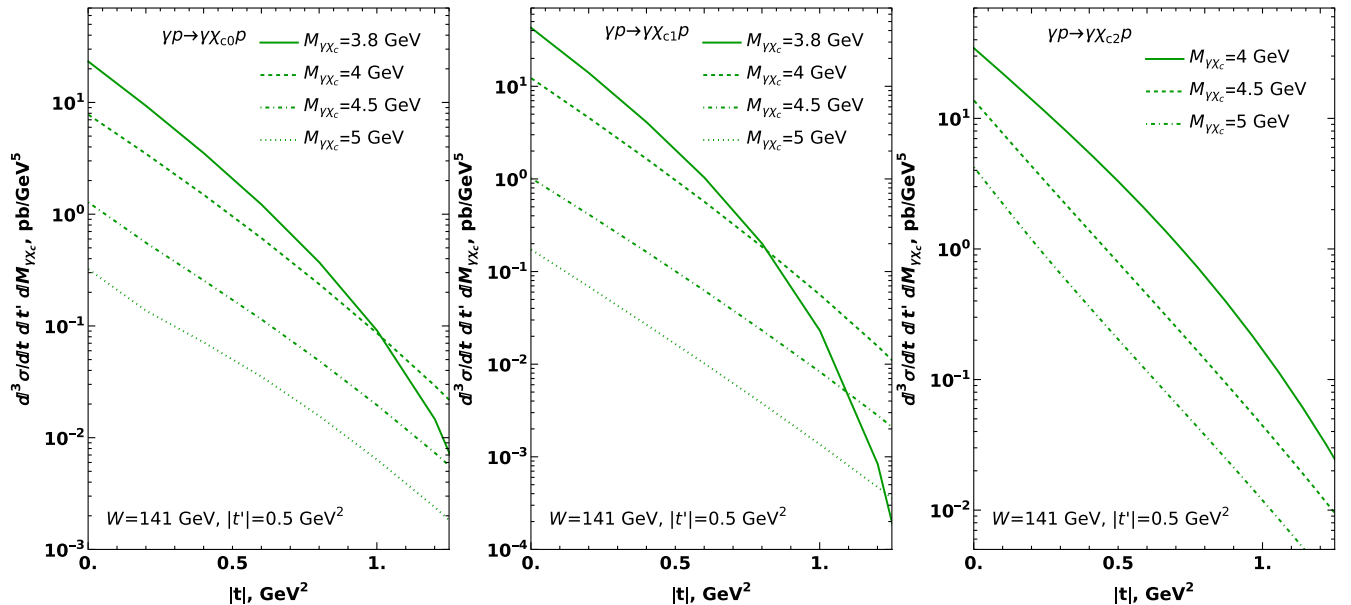


FIG. 6. The dependence of the photoproduction cross section (21) on the invariant momentum transfer to the target t . The left, central, and right columns correspond to the χ_{c0} , χ_{c1} , and χ_{c2} mesons, respectively. The t dependence at $M_{\chi_c} = 3.8 \text{ GeV}$ reflects the kinematic constraint $|t| \leq M_{\chi_c}^2 - M_c^2 - |t'|$.

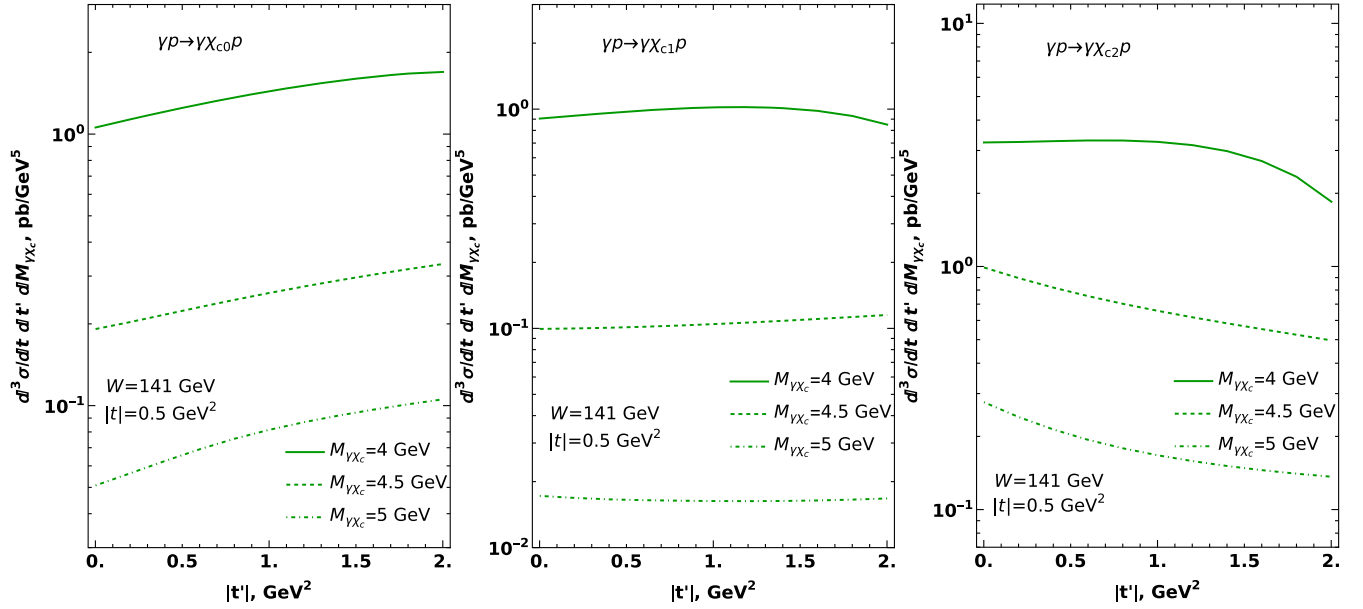


FIG. 7. The dependence of the photoproduction cross section (21) on the invariant momentum transfer to the photon [variable $t' = (q - k)^2$ defined in (15)]. The left, central, and right columns correspond to the χ_{c0} , χ_{c1} , and χ_{c2} mesons, respectively.

$p_\perp$ is parametrically large, $p_\perp^2 \sim M_{\chi_c}^2$, we may obtain

$$\begin{aligned}
 L^2 &\approx \bar{\alpha}_{\chi_c} [\alpha_{\chi_c} M_{\chi_c}^2 - M_{\chi_c}^2] - t \left(\frac{1}{2} - \alpha_{\chi_c} \right)^2 \\
 &\approx \bar{\alpha}_{\chi_c} [\alpha_{\chi_c} M_{\chi_c}^2 - M_{\chi_c}^2] (1 + \mathcal{O}(t/M_{\chi_c}^2)) \\
 &\approx |t'| (1 + \mathcal{O}(t/M_{\chi_c}^2)).
 \end{aligned} \quad (133)$$

Besides, the factor $(\frac{1}{2} - \alpha_{\chi_c})^2$ in front of t in (133) provides additional numerical suppression that justifies omission of this t dependence. For comparison, the t dependence that originates from the functions n, n_Φ is related to the implemented impact parameter dependence of the dipole amplitude and is controlled by the (inverse) proton size. In all the phenomenological parametrizations fitted to exclusive processes, this dependence is rapidly decreasing, in agreement with theoretical expectations [78]. Technically, this implies that the $\chi_c\gamma$ pairs predominantly have oppositely directed transverse momenta $\mathbf{p}_\perp^{\chi_c}, \mathbf{k}_\perp^\gamma$. For this reason,

in what follows we will focus on the kinematics $|t| = |t_{\min}|$, or the t -integrated observables that get the largest contribution from small- t kinematics.

The dependence on t' shown in Fig. 7 is very mild in the kinematics of small $|t'|$ because in the impact factors this variable contributes in linear combination with parametrically large scales $\sim M_{\chi_c}^2, M_{\chi_c}^2$, and, unless we consider the near-threshold kinematics $M_{\chi_c}^2 \sim M_{\chi_c}^2$, may be disregarded altogether. This variable also may be related to the transverse momenta $\pm \mathbf{p}_\perp$ of the final state χ_c and γ , and thus is correlated with the angle between these particles. As we will explain below, the latter observation allows one to understand the completely different t' dependence of various helicity components.

In Fig. 8 we provide predictions for the dependence on the invariant mass $M_{\chi_c\gamma}$. The latter variable plays the role of the hard scale that controls the scale of all the transverse momenta, and shows up explicitly in the impact factors provided in the previous section. This variable also

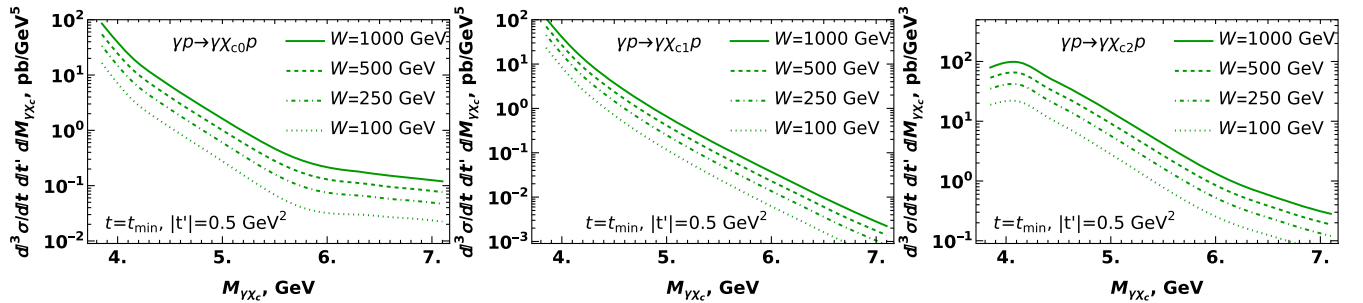


FIG. 8. The dependence of the photoproduction cross section (21) on the invariant mass $M_{\chi_c\gamma}$ of the quakonium-photon pair. The left, central, and right columns correspond to the χ_{c0} , χ_{c1} , and χ_{c2} mesons, respectively.

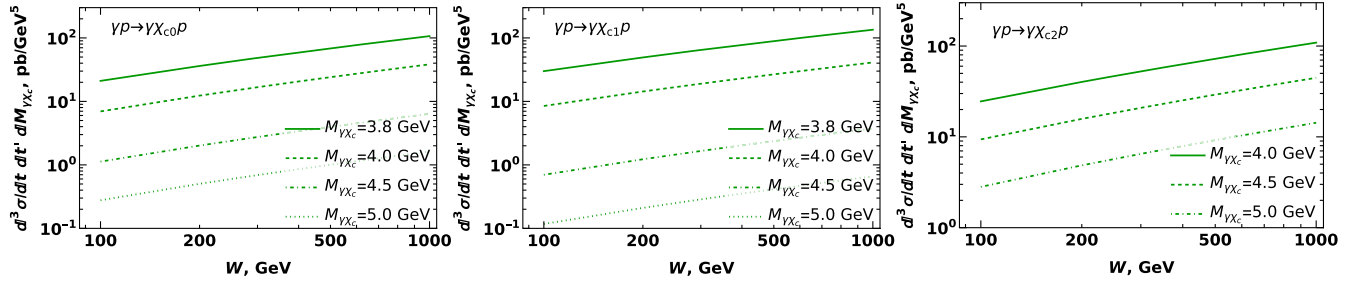


FIG. 9. The energy dependence of the $\gamma p \rightarrow \chi_c \gamma p$ cross section for different spins and invariant energies. The nearly linear dependence in double logarithmic coordinates suggests that the dependence may be described by power law $\sim W^\delta$, where δ has a very mild dependence on other kinematic variables, with typical values $\delta \sim 0.67\text{--}0.73$.

determines the effective value of the parameter $x \sim M_{\chi_c}^2/W^2$ in the dipole amplitude (128). Due to suppression of the latter at large x , the cross sections of all quarkonia are strongly suppressed at large M_{χ_c} . We also may observe that unless we consider a near-threshold kinematics $M_{\chi_c} \sim M_{\chi_c}$, the dependence on all variables largely factorizes.

The comparison of the predictions of the CGC and the collinear factorization frameworks at these energies should be done with due care because the predictions of the latter suffer from sizable uncertainties that may reach up to an order of magnitude (see, e.g., Fig. 7 in [72] for $\chi_c \gamma$).⁴ Nevertheless, it is possible to see that the cross sections shown in Figs. 6–8 are significantly larger than the collinear factorization results presented in [72]. It is known from the literature that the CGC and the collinear factorization are expected to agree only at large transverse momenta of quarks $k_\perp \gg Q_s$, whereas for small transverse momenta may differ by a factor of four for the inclusive heavy quark pair production (see, e.g., [97,98] for more details). In the kinematics that gives the largest contribution to $\chi_c \gamma$ production, the quarks have small transverse momenta controlled by the parameters $p_\perp, \Delta_\perp$ of order Q_s (see Sec. II A). Furthermore, since an exclusive cross section is proportional to the square of the amplitude, the discrepancy between the CGC and the collinear factorization frameworks increases by up to an order of magnitude. For this reason, the observed discrepancy is consistent with the findings of [97,98] and can be interpreted as a manifestation of collinear and k_T factorization breaking.

The dependence on the invariant energy shown in Fig. 9 can be described by the power law, $d\sigma(W) \sim W^{2\alpha}$, where the constant α has a very mild dependence on other variables. This behavior follows from a mild x dependence of the forward dipole amplitude (128) and reflects the increase of the gluonic density at higher energies.

⁴In the CGC framework the uncertainties are also present and may be enhanced due to large cancellations in the integrals over the variables $\zeta, \ell_\perp$ in the explicit expressions (78)–(98) and (108)–(127). However, these uncertainties are significantly smaller than in the collinear factorization picture.

Numerically, the values of the parameter α are consistent with the energy dependence implemented in the small- r asymptotic limit of (128), namely $\alpha \approx 2\lambda\langle\gamma_{\text{eff}}\rangle \approx (0.32, 0.37)$. The early onset of the saturation effects would reveal itself as a deviation from the power law behavior.

Finally, in Fig. 10 we have shown the relative contributions of different quarkonia helicities H_{χ_c} to the unpolarized cross section. In the kinematics of relatively small t' , the quarkonia and photons have small transverse momenta and thus are produced at relatively small angles with respect to the direction of the incoming photon. The helicities of all particles in this kinematics nearly coincide with the projection of the angular momentum onto the collision axis, and for this reason are subject to the helicity conservation rule,

$$H_{\gamma,\text{in}} \approx H_{\gamma,\text{out}} + H_{\chi_c}, \quad \text{small } t' \approx 0. \quad (134)$$

This selection rule allows one to understand the dominance of the $H_{\chi_c} = 0$ helicity component for χ_{c0}, χ_{c1} : since the helicities of the on-shell photons only take values ± 1 , and $|H_{\chi_c}| \neq 2$ for these mesons, only the helicity state $H_{\chi_c} = 0$ allows one to satisfy (134), provided that the photon helicity is not flipped during the process, $H_{\gamma,\text{in}} = H_{\gamma,\text{out}}$. We checked numerically that the contribution of the component with helicity flip of the photon, $H_{\gamma,\text{in}} = -H_{\gamma,\text{out}}$, is indeed strongly suppressed. At larger values of the variables $|t'|, M_{\chi_c}$, the angles between final-state particles and the collision axis also increases rapidly, so the helicities are no longer related to projections of the angular momentum onto the same axis, and thus the selection rule (134) becomes void. In order to understand the relative size of the contributions with helicity flip of the photon, in Fig. 11 we have shown the angular harmonics c_2 defined in (26). We can see that for the harmonics $H_{\chi_c} = 0$ and $H_{\chi_c} = 1$, which give the dominant contributions in the cross section, the asymmetry c_2 remains moderate. It increases as a function of $|t'|$, in agreement with the above-mentioned picture based on the increase of the angles; however, it does not exceed $\sim 30\%$ in the kinematics of interest. Since the

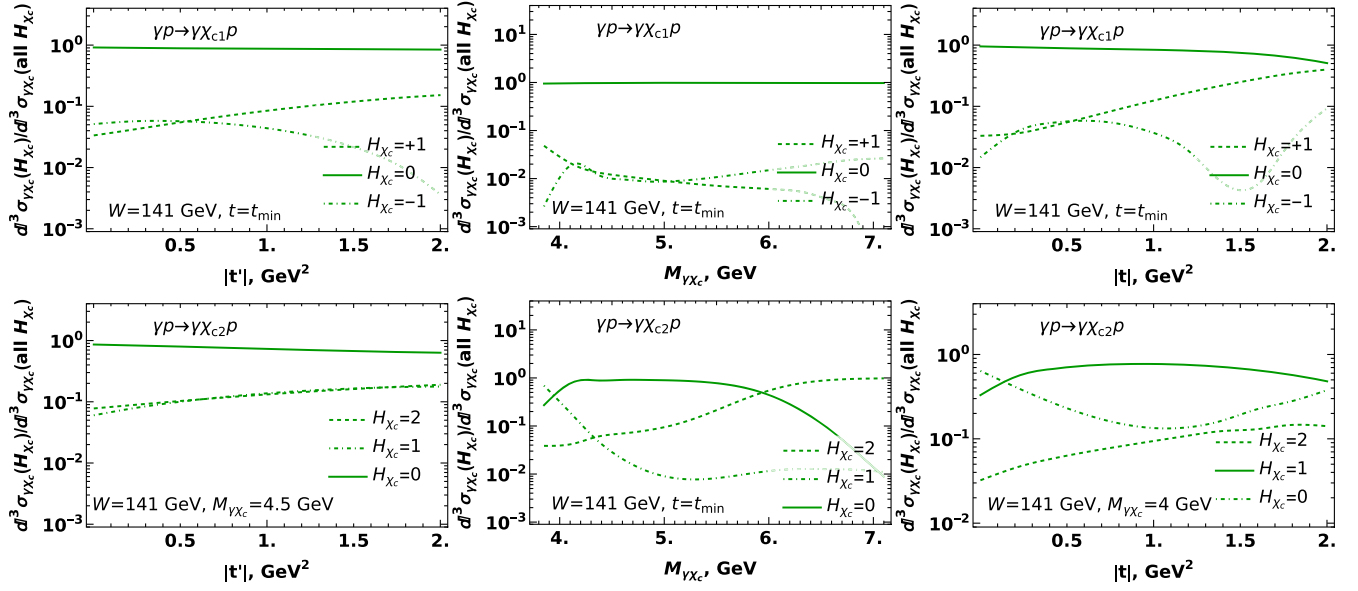


FIG. 10. The relative contributions of different helicity components of χ_c in the total cross section, as a function of kinematic variables $t, t', M_{\gamma\chi_c}$. The upper and lower rows correspond to χ_{c1} and χ_{c2} , respectively. For the sake of legibility, in the lower row (for χ_{c2}) we did not show the negligibly small contributions of the negative helicity H_{χ_c} .

complex phases of the amplitudes $\mathcal{A}_{\gamma p \rightarrow \chi_{c1} \gamma p}^{(+,+)}$ and $\mathcal{A}_{\gamma p \rightarrow \chi_{c2} \gamma p}^{(+,-)}$ are nearly identical, the harmonics s_2 is vanishingly small. An experimental confirmation of these theoretical expectations would constitute a strong evidence in favor of the expected dominance of the contribution without photon helicity flip.

B. Integrated cross sections and counting rates

While the threefold differential cross sections are well suited for the phenomenological studies, experimentally it may be easier to study the partially integrated (single-differential) cross sections. In Fig. 12 we have shown the cross sections $d\sigma/dM_{\gamma\chi_c}$ and $d\sigma/dt'$ for different charmonia, as well as the ratio of the cross sections of χ_{c1} and χ_{c2}

mesons. As we discussed in the previous section, for the threefold cross section the dependence on $W, t, M_{\gamma\chi_c}$ largely factorizes; for this reason the dependence of the integrated cross sections on these variables is similar to that of the unintegrated cross section. In the lower row of the same figure we have shown the ratio of the cross sections $d\sigma_{\gamma\chi_{c1}}/d\sigma_{\gamma\chi_{c2}}$ which can be measured with very good precision because both χ_{c1} and χ_{c2} have large branching fractions of radiative decays $\chi_c \rightarrow J/\psi \gamma$. Since the exclusive $\chi_c \gamma$ photoproduction does not get contributions from the color octet LDMEs, it can be used for tests of HQSS and resolve ambiguities observed in χ_c hadroproduction. The ratio does not coincide with the naively expected 3/5 due to strong polarization dependence of the cross section. As could be seen from Fig. 10 in the previous subsection, at

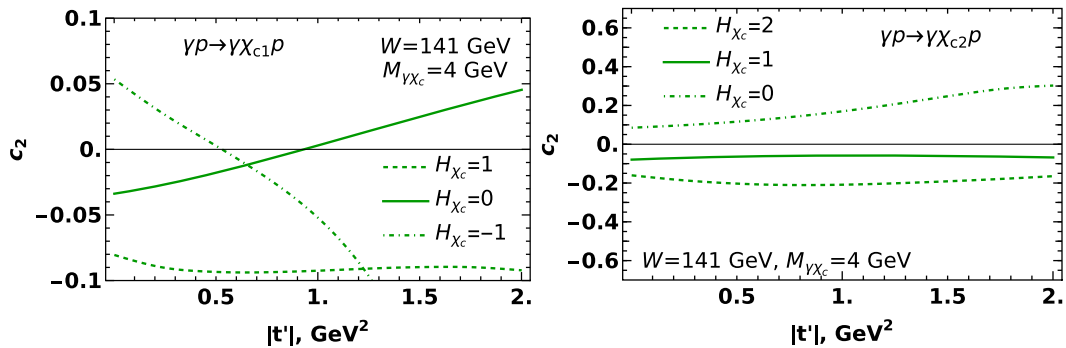


FIG. 11. The angular harmonics c_2 of electroproduction cross section (26), for different helicity components of χ_{c1} and χ_{c2} mesons, respectively. For the sake of legibility, in the right plot (for χ_{c2}) we did not show the asymmetries of the contributions with negative helicity H_{χ_c} , since the latter are negligibly small.

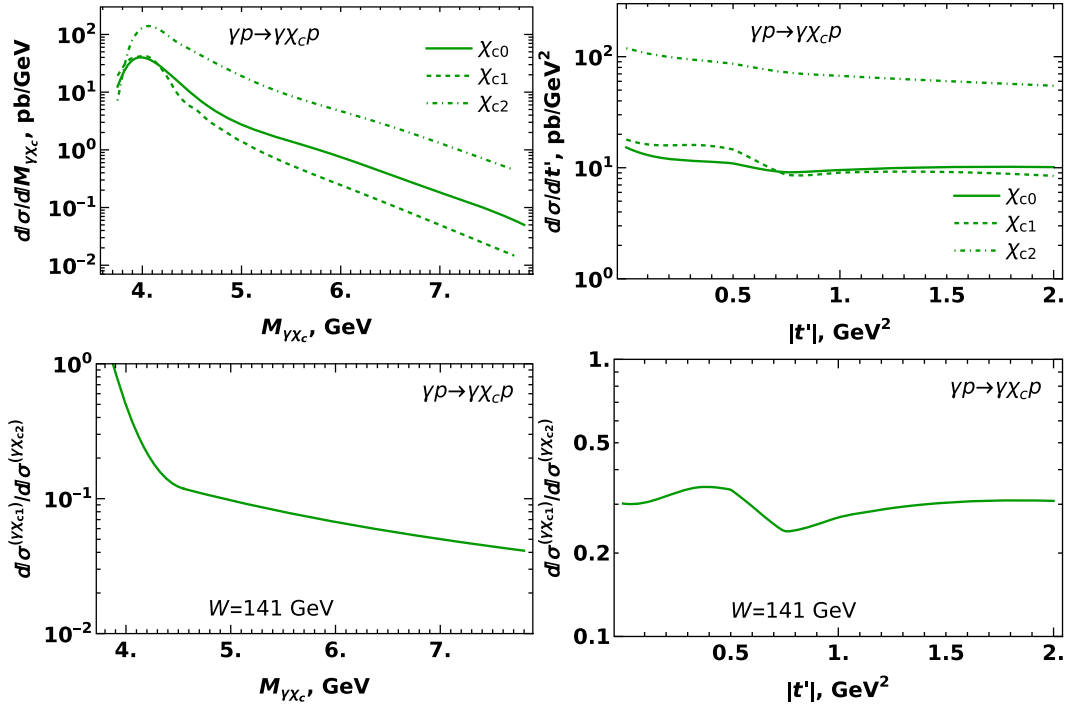


FIG. 12. Upper row: the single-differential cross sections $d\sigma/dM_{\gamma\chi_c}$, $d\sigma/dt'$ for different charmonia χ_{cJ} . Both plots correspond to the invariant energy $W = 141$ GeV. Lower row: ratio of the $\chi_{c1}\gamma$ and $\chi_{c2}\gamma$ photoproduction cross sections. The left and right plots correspond to the ratio of $d\sigma/dM_{\gamma\chi_c}$ and $d\sigma/dt'$ cross sections, respectively.

small $M_{\gamma\chi_c} \sim M_{\chi_c}$ (near-threshold kinematics), the cross sections of both χ_{c1} and χ_{c2} get the dominant contribution from the helicity component $H_{\chi_c} = 0$, and in view of the HQSS symmetry, the ratio of cross sections $d\sigma_{\gamma\chi_{c1}}/d\sigma_{\gamma\chi_{c2}}$ approaches unity. This ratio decreases as a function of $M_{\gamma\chi_c}$ due to increased relative contribution of the helicity component $H_{\chi_c} = 2$ in the cross section $\sigma_{\chi_{c2}}$ (in the denominator). The helicity component $H_{\chi_c} = 0$ also gives a large contribution in the cross sections integrated over $M_{\gamma\chi_c}$, namely the cross sections $d\sigma/dt'$ and $d\sigma/dt$: since $d\sigma/dM_{\gamma\chi_c}$ decreases rapidly as a function of $M_{\gamma\chi_c}$, the dominant contribution comes from the region $M_{\gamma\chi_c} \lesssim 4.5$ GeV. The dependence of the ratio on the variables t, t' is much milder.

The total (integrated) cross sections of the photoproduction process $\gamma p \rightarrow \chi_{cJ} \gamma p$ is given by

$$\sigma_{\text{tot}}^{\gamma c0}(W \approx 100 \text{ GeV}, M_{\gamma\chi_c} \geq 3.7 \text{ GeV}) \approx 19 \text{ pb}, \quad (135)$$

$$\sigma_{\text{tot}}^{\gamma c1}(W \approx 100 \text{ GeV}, M_{\gamma\chi_c} \geq 3.7 \text{ GeV}) \approx 18 \text{ pb}, \quad (136)$$

$$\sigma_{\text{tot}}^{\gamma c2}(W \approx 100 \text{ GeV}, M_{\gamma\chi_c} \geq 3.7 \text{ GeV}) \approx 78 \text{ pb}, \quad (137)$$

and scales with energy approximately as $\sim W^{0.7}$. In our estimates we introduced a cutoff on minimal invariant mass $M_{\gamma\chi_c}$ in order to avoid (a huge) background from the

radiative decays of $\psi(2S) \rightarrow \chi_c \gamma$.⁵ In the Tables II and III we provide tentative estimates for the total cross sections of the electroproduction $ep \rightarrow e\gamma\chi_c p$ and the ultraperipheral production $pp \rightarrow pp\gamma\chi_c$ in LHC kinematics. Since the spectrum of virtual photons is dominated by quasireal photons, for the sake of simplicity we disregarded the contributions of photons with large virtuality $Q^2 \gtrsim 1 \text{ GeV}^2$, and assumed that the photoproduction cross section does not depend on Q^2 at $Q^2 \lesssim 1 \text{ GeV}^2$ in view of the smallness of $\mathcal{O}(Q^2/M_{\chi_c}^2)$ corrections in the heavy quark mass limit. We found that varying the upper cutoff Q_{max}^2 between 1 and 2 GeV^2 changes the cross section within 10%.

C. Comparison with odderon-mediated χ_c photoproduction

The odderons, or C -odd gluon exchanges in the t -channel, remain one of the least understood components of the nonperturbative QCD [61–66], and the contribution of the odderons to the dipole scattering amplitude at present is largely unknown. A recent discovery of the odderons from comparison of pp and $p\bar{p}$ elastic cross sections measured at the LHC and Tevatron [100,101] reinvigorated

⁵Combining the experimental $\psi(2S)$ photoproduction cross section [99] and the branching fractions of $\psi(2S) \rightarrow \chi_{cJ} \gamma$ [79], we estimate that the background contribution from $\psi(2S)$ radiative decay is $\sigma_{\text{tot}}^{(\text{rad})} \approx 1.1 \text{ nb}$.

TABLE II. The total electroproduction cross sections, production, and counting rates for different χ_c mesons in EIC kinematics at electron-proton energy $\sqrt{s_{ep}} = 141$ GeV. The cross section roughly scales with energy as $\sim W^{0.7}$. For estimates of the production and counting rates, we used the instantaneous luminosity $\mathcal{L} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 10^{-5} \text{ fb}^{-1}$ and the integrated luminosity $\int dt \mathcal{L} = 100 \text{ fb}^{-1}$. The column “combined branching” corresponds to the product of branching fractions of $\chi_c \rightarrow J/\psi\gamma$ and $J/\psi \rightarrow \mu^+\mu^-$ decays.

	$\sigma_{\text{tot}}^{(ep)}$	Production rates		Decay channel	Combined branching	Counting rates	
		N	dN/dt			N_d	dN_d/dt
χ_{c0}	0.35 pb	3.5×10^4	292/day	$\chi_c \rightarrow J/\psi\gamma$	0.08%	28	7/month
χ_{c1}	0.32 pb	3.2×10^4	271/day	$J/\psi \rightarrow \mu^+\mu^-$	2%	628	162/month
χ_{c2}	1.3 pb	1.3×10^5	1105/day		1.1%	1450	366/month

TABLE III. The total production cross sections, production, and counting rates for different χ_c mesons, in ultraperipheral LHC kinematics at proton-proton energy $\sqrt{s_{pp}} = 13$ GeV. For estimates of the production and counting rates, we used the instantaneous luminosity $\mathcal{L} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 10^{-5} \text{ fb}^{-1} \text{ s}^{-1}$ and the integrated luminosity $\int dt \mathcal{L} = 100 \text{ fb}^{-1}$. The column “combined branching” corresponds to the product of branching fractions of $\chi_c \rightarrow J/\psi\gamma$ and $J/\psi \rightarrow \mu^+\mu^-$ decays.

	$\sigma_{\text{tot}}^{(pp)}$	Production rates		Decay channel	Combined branching	Counting rates	
		N	dN/dt			N_d	dN_d/dt
χ_{c0}	0.23 pb	2.3×10^4	195/day	$\chi_c \rightarrow J/\psi\gamma$	0.08%	18	4.7/month
χ_{c1}	0.19 pb	1.9×10^4	165/day	$J/\psi \rightarrow \mu^+\mu^-$	2%	384	99/month
χ_{c2}	0.9 pb	9×10^4	572/day		1.1%	750	189/month

the interest in this topic. However, the statistical significance of the observed signal still remains under discussion due to sizable uncertainties and possible background contributions. It could be extremely difficult to study in detail the odderon amplitude from the channels where odderons contribute as minor corrections. For this reason, the processes which proceed via the C -odd t -channel exchanges came into focus of odderon searches, and it is expected that such processes will be studied experimentally both at the HL-LHC and at the future EIC.

The exclusive photoproduction of C -even quarkonia, as for example $\gamma p \rightarrow \eta_c p$, historically attracted a lot of interest, since the heavy quark mass plays the role of the hard scale and justifies at least a partial description in perturbative QCD (see [66,67] for a short overview). However, that process obtains a sizable contribution from the photon-photon fusion (Primakoff mechanism), and for this reason, recently the exclusive χ_c photoproduction $\gamma p \rightarrow \chi_c p$ has been suggested in [68,69] as a potential cleaner alternative. The cross section of the latter process is comparable to that of η_c photoproduction; furthermore it may be easier to study experimentally due to the large branching fraction of radiative decays of χ_c to J/ψ . In this context, the photoproduction of $\chi_c\gamma$ pairs deserves attention as a potential background to odderon-mediated χ_c photoproduction: while the former is formally suppressed as $\mathcal{O}(\alpha_{\text{em}})$, the latter is suppressed by the smallness of the odderon amplitude, so numerically their cross sections are comparable. If the final-state photon is not detected,

potentially the $\chi_c\gamma$ photoproduction can be misinterpreted as a $\gamma p \rightarrow \chi_c p$ subprocess. Furthermore, if there are no constraints (cuts) on the invariant mass of the produced hadronic state, a sizable contribution could be obtained from the radiative decays of heavier charmonia. In order to estimate accurately these backgrounds, in general it is required to know in detail the geometry and acceptance of the detector. For simplicity, now we will consider that *all* the final photons are not detected, and we will consider the cross section $d\sigma/dt$ integrated over the phase space of the produced photon. This approach provides an upper estimate for the background. For the sake of definiteness, we will use for comparison only the result of [69] because the predictions of [68] are provided in the kinematics of relatively low energies, where the CGC approach might be not very reliable.

In Fig. 13 we show the cross sections of the χ_c and $\chi_c\gamma$ production via different mechanisms. The largest contribution comes from the $\psi(2S)$ photoproduction with subsequent radiative decay $\psi(2S) \rightarrow \chi_c\gamma$, since $\psi(2S)$ photoproduction does not require exchange of quantum numbers in the t -channel, and $\psi(2S)$ has remarkably large branching ratios of radiative decays, $\text{Br}(\psi(2S) \rightarrow \chi_{c0}\gamma) = 9.79 \pm 0.2\%$, $\text{Br}(\psi(2S) \rightarrow \chi_{c1}\gamma) = 9.75 \pm 0.24\%$, $\text{Br}(\psi(2S) \rightarrow \chi_{c2}\gamma) = 9.52 \pm 0.2\%$ [79]. We used for estimates the cross section of $\psi(2S)$ photoproduction found in the CGC framework in [102], and checked that it can reproduce the experimentally measured total (t -integrated) cross section [99] and the diffractive slope [103]. While the

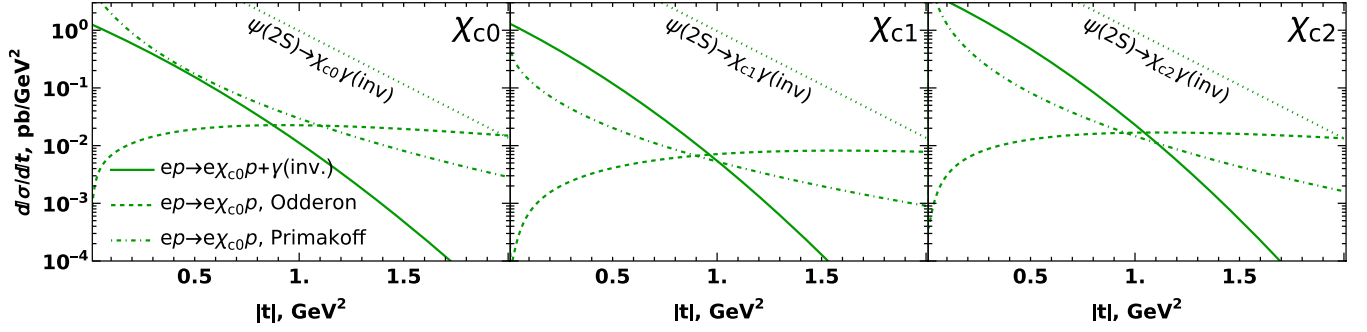


FIG. 13. Comparison of different mechanisms which contribute to electroproduction of mesons. The solid curve corresponds to the contribution of $ep \rightarrow e\gamma\chi_c p$ process discussed in this paper, with invisible (integrated out) final-state photon. The upper dotted curve corresponds to the contribution of $\chi_c\gamma$ pairs which stem from the radiative decays of $\psi(2S)$ charmonia (the $\chi_c\gamma$ pairs in this mechanism have invariant mass $M_{\gamma\chi_c} \sim M_{\psi(2S)}$). The dashed and dot-dashed lines correspond to exclusive electroproduction $ep \rightarrow e\chi_c p$ mediated by odderons and the Primakoff mechanism, as provided in [69] (see Fig. 5).

radiative decays of other excited quarkonia potentially could also contribute to the observed backgrounds, at present it is not possible to estimate accurately their contribution due to a sizable uncertainty in their wave functions.

The exclusive (nonresonant) production of $\chi_c\gamma$ pairs suggested in this paper exceeds significantly the odderon-mediated χ_c photoproduction at small- t ($|t| \lesssim 1 \text{ GeV}^2$), although it is less relevant in the large- t kinematics. The cross sections of both mechanisms of $\chi_c\gamma$ production [direct and via $\psi(2S)$ radiative decays] increase with energy, whereas the contribution of odderons should decrease due to different odderon intercepts.

For the sake of comparison we also have shown the contribution of the photon-photon fusion (so-called Primakoff mechanism) which exceeds the odderon contribution in the small- t kinematics, as discussed in [69]. While the Primakoff mechanism is less relevant for odderon searches on neutrons, namely for $\gamma n \rightarrow \chi_c n$ subprocess, the backgrounds due to the $\chi_c\gamma$ production with undetected photons are the same for proton and neutron targets.

The contributions of different mechanisms discussed in this section can be separated, imposing the kinematic cuts on the variable $(q - \Delta)^2 \approx t - Q^2 - 2q \cdot \Delta$. For exclusive χ_c production via odderon-mediated and Primakoff mechanisms, this variable corresponds to the square of charmonium mass, $M_{\chi_c}^2$. For production via radiative decays of $\psi(2S)$, this variable equals $M_{\psi(2S)}^2$. Finally, for the direct (nonresonant) production of $\chi_c\gamma$ pairs, this variable coincides with invariant mass $M_{\gamma\chi_c}^2$ of the $\chi_c\gamma$ pair. All the phenomenological estimates for the cross sections presented in this section are also available as tabulated plaintext data files as Supplemental Material [104].

IV. CONCLUSIONS

We analyzed the exclusive $\chi_c\gamma$ photoproduction, including its helicity dependence, for all χ_{cJ} states in the CGC

framework. We found that the amplitude of the process may be represented as a convolution of the process-dependent impact factors and forward dipole scattering amplitude. In the heavy quark mass limit, the kinematic distribution of produced $\chi_c\gamma$ pairs may be related to the dependence of the forward dipole scattering amplitude on dipole size, impact parameter, and rapidity dependence of the produced quarkonium-photon pair. The theoretical results of Sec. II can be used for estimates of the cross section with any parametrization of the forward dipole scattering amplitude at large energies $W \gtrsim 100 \text{ GeV}$. These results complement our previous study of $\chi_c\gamma$ in the collinear factorization framework [72], which is valid at moderate energies ($W \lesssim 100 \text{ GeV}$) but becomes less reliable at higher energies due to significant increase of uncertainties. Since the dominant contribution to the cross section comes from the quarks with relatively small transverse momenta, numerically the predictions of the CGC framework are significantly larger than those of the collinear factorization approach in this kinematics, in agreement with findings of [97,98].

Using a phenomenological parametrization of the forward color dipole amplitude available from the literature, we also estimated numerically the cross sections in the ultraperipheral kinematics at LHC and the future EIC. The cross sections of all χ_c mesons are of the same order of magnitude as the cross section of η_c production found earlier in [55]. The expected production rates of $\chi_c\gamma$ pairs constitute $\sim 10^4$ – 10^5 events per each 100 fb^{-1} of integrated luminosity, both at EIC and LHC. The expected detection (counting) rates in this kinematics constitute $\sim 10^2$ – 10^3 events for χ_{c1} and χ_{c2} mesons, if these charmonia are detected via their $\chi_c \rightarrow J/\psi\gamma$ decays. These numbers suggest that the cross section may be measured with reasonable precision. The χ_{c1} mesons are produced predominantly with helicity $H_{\chi_c} = 0$, whereas the polarization of χ_{c2} mesons depends on the kinematics. In the kinematics where χ_{c2} meson obtains the dominant contribution from

the helicity $H_{\chi_c} = 0$, the ratio of the cross sections of χ_{c1} and χ_{c2} mesons approaches unity due to HQSS, and the latter observable can be used to test the validity of the HQSS for the χ_c mesons.

We also found that the production of $\chi_c\gamma$ pairs with invisible photon (either direct or produced via radiative decay of $\psi(2S) \rightarrow \chi_c\gamma$) gives a sizable background to odderon-mediated exclusive photoproduction of χ_c mesons, which was suggested recently in [68,69] as an alternative tool for study of odderons. While the cross sections of $\chi_c\gamma$ production decrease rapidly as a function of the momentum transfer $|t|$, the radiative decay of $\psi(2S)$ remains the dominant mechanism up to $|t| \sim 2\text{GeV}^2$. These findings are in line with our previous analysis [55], which concluded that odderon-mediated quarkonia production can get sizable backgrounds from quarkonia-photon production.

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DATA AVAILABILITY

The data that support the findings of this article are openly available in the Supplemental Material [104].

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- [1] L. D. McLerran and R. Venugopalan, Computing quark and gluon distribution functions for very large nuclei, *Phys. Rev. D* **49**, 2233 (1994).
 - [2] L. D. McLerran and R. Venugopalan, Gluon distribution functions for very large nuclei at small transverse momentum, *Phys. Rev. D* **49**, 3352 (1994).
 - [3] L. D. McLerran and R. Venugopalan, Green's function in the color field of a large nucleus, *Phys. Rev. D* **50**, 2225 (1994).
 - [4] F. Gelis, E. Iancu, J. Jalilian-Marian, and R. Venugopalan, The color glass condensate, *Annu. Rev. Nucl. Part. Sci.* **60**, 463 (2010).
 - [5] E. Iancu and A. H. Mueller, From color glass to color dipoles in high-energy onium onium scattering, *Nucl. Phys.* **A730**, 460 (2004).
 - [6] C. A. Aidala *et al.*, Probing nucleons and nuclei in high energy collisions, *arXiv:2002.12333*.
 - [7] Y. Q. Ma and R. Venugopalan, Comprehensive description of J/ψ production in proton-proton collisions at collider energies, *Phys. Rev. Lett.* **113**, 192301 (2014).
 - [8] V. Cheung, Z. B. Kang, F. Salazar, and R. Vogt, Direct quarkonium production in DIS from a joint CGC and NRQCD framework, *Phys. Rev. D* **110**, 094039 (2024).
 - [9] H. Mäntysaari, K. Roy, F. Salazar, and B. Schenke, Gluon imaging using azimuthal correlations in diffractive scattering at the Electron-Ion Collider, *Phys. Rev. D* **103**, 094026 (2021).
 - [10] Z. B. Kang, E. Li, and F. Salazar, Direct quarkonium-plus-gluon production in DIS in the color glass condensate, *J. High Energy Phys.* **03** (2024) 027.
 - [11] K. Tuchin, Heavy quark production by a quasiclassical color field in proton nucleus collisions, *Phys. Lett. B* **593**, 66 (2004).
 - [12] J. P. Blaizot, F. Gelis, and R. Venugopalan, High-energy pA collisions in the color glass condensate approach. 2. Quark production, *Nucl. Phys.* **A743**, 57 (2004).
 - [13] B. Abelev *et al.* (ALICE Collaboration), Transverse momentum distribution and nuclear modification factor of charged particles in p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Rev. Lett.* **110**, 082302 (2013).
 - [14] G. Aad *et al.* (ATLAS Collaboration), Transverse momentum, rapidity, and centrality dependence of inclusive charged-particle production in $\sqrt{s_{NN}} = 5.02$ TeV p + Pb collisions measured by the ATLAS experiment, *Phys. Lett. B* **763**, 313 (2016).
 - [15] A. H. Rezaeian, M. Siddikov, M. Van de Klundert, and R. Venugopalan, Analysis of combined HERA data in the impact-parameter dependent saturation model, *Phys. Rev. D* **87**, 034002 (2013).
 - [16] A. H. Rezaeian and I. Schmidt, Impact-parameter dependent color glass condensate dipole model and new combined HERA data, *Phys. Rev. D* **88**, 074016 (2013).
 - [17] G. Watt and H. Kowalski, Impact parameter dependent color glass condensate dipole model, *Phys. Rev. D* **78**, 014016 (2008).
 - [18] G. G. Da Silveira and (ATLAS, LHCb and CMS Collaborations), QCD probes at LHC, *arXiv:1805.00772*.
 - [19] J. L. Albacete, Testing the CGC in proton-lead collisions at the LHC, *Nucl. Phys.* **A910-911**, 155 (2013).
 - [20] R. Aaij *et al.* (LHCb Collaboration), Study of exclusive photoproduction of charmonium in ultra-peripheral lead-lead collisions, *J. High Energy Phys.* **06** (2023) 146.
 - [21] H. Mäntysaari, F. Salazar, and B. Schenke, Energy dependent nuclear suppression from gluon saturation in exclusive vector meson production, *Phys. Rev. D* **109**, L071504 (2024).

- [22] Y. V. Kovchegov, Small xF_2 structure function of a nucleus including multiple pomeron exchanges, *Phys. Rev. D* **60**, 034008 (1999).
- [23] Y. V. Kovchegov and H. Weigert, Triumvirate of running couplings in small- x evolution, *Nucl. Phys. A* **784**, 188 (2007).
- [24] I. Balitsky and G. A. Chirilli, Next-to-leading order evolution of color dipoles, *Phys. Rev. D* **77**, 014019 (2008).
- [25] I. Balitsky, Operator expansion for high-energy scattering, *Nucl. Phys. B* **463**, 99 (1996).
- [26] I. Balitsky, Effective field theory for the small x evolution, *Phys. Lett. B* **518**, 235 (2001).
- [27] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, The BFKL equation from the Wilson renormalization group, *Nucl. Phys. B* **504**, 415 (1997).
- [28] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Wilson renormalization group for low x physics: Towards the high density regime, *Phys. Rev. D* **59**, 014014 (1998).
- [29] E. Iancu, A. Leonidov, and L. D. McLerran, The renormalization group equation for the color glass condensate, *Phys. Lett. B* **510**, 133 (2001).
- [30] E. Ferreiro, E. Iancu, A. Leonidov, and L. McLerran, Nonlinear gluon evolution in the color glass condensate: II, *Nucl. Phys. A* **703**, 489 (2002).
- [31] E. Iancu, A. Leonidov, and L. D. McLerran, Nonlinear gluon evolution in the color glass condensate: I, *Nucl. Phys. A* **692**, 583 (2001).
- [32] E. Iancu and D. N. Triantafyllopoulos, Higher-point correlations from the JIMWLK evolution, *J. High Energy Phys.* **11** (2011) 105.
- [33] G. Beuf, T. Lappi, and R. Paatelainen, Massive quarks in NLO dipole factorization for DIS: Transverse photon, *Phys. Rev. D* **106**, 034013 (2022).
- [34] G. Beuf, T. Lappi, and R. Paatelainen, Massive quarks in NLO dipole factorization for DIS: Longitudinal photon, *Phys. Rev. D* **104**, 056032 (2021).
- [35] B. Ducloué, H. Hänninen, T. Lappi, and Y. Zhu, Deep inelastic scattering in the dipole picture at next-to-leading order, *Phys. Rev. D* **96**, 094017 (2017).
- [36] H. Mäntysaari and J. Penttala, Complete calculation of exclusive heavy vector meson production at next-to-leading order in the dipole picture, *J. High Energy Phys.* **08** (2022) 247.
- [37] H. Mäntysaari and J. Penttala, Exclusive production of light vector mesons at next-to-leading order in the dipole picture, *Phys. Rev. D* **105**, 114038 (2022).
- [38] E. Iancu and Y. Mulian, Dihadron production in DIS at NLO: The real corrections, *J. High Energy Phys.* **07** (2023) 121.
- [39] K. Roy and R. Venugopalan, Extracting many-body correlators of saturated gluons with precision from inclusive photon + dijet final states in deeply inelastic scattering, *Phys. Rev. D* **101**, 071505 (2020).
- [40] K. Roy and R. Venugopalan, NLO impact factor for inclusive photon + dijet production in $e + A$ DIS at small x , *Phys. Rev. D* **101**, 034028 (2020).
- [41] J. W. Qiu and Z. Yu, Extracting transition generalized parton distributions from hard exclusive pion-nucleon scattering, *Phys. Rev. D* **109**, 074023 (2024).
- [42] J. W. Qiu and Z. Yu, Extraction of the parton momentum-fraction dependence of generalized parton distributions from exclusive photoproduction, *Phys. Rev. Lett.* **131**, 161902 (2023).
- [43] K. Deja, V. Martinez-Fernandez, B. Pire, P. Sznajder, and J. Wagner, Phenomenology of double deeply virtual Compton scattering in the era of new experiments, *Phys. Rev. D* **107**, 094035 (2023).
- [44] M. Siddikov and I. Schmidt, Exclusive production of quarkonia pairs in collinear factorization framework, *Phys. Rev. D* **107**, 034037 (2023).
- [45] M. Siddikov and I. Schmidt, Exclusive photoproduction of D -meson pairs with large invariant mass, *Phys. Rev. D* **108**, 096031 (2023).
- [46] J.-W. Qiu and Z. Yu, Exclusive production of a pair of high transverse momentum photons in pion-nucleon collisions for extracting generalized parton distributions, *J. High Energy Phys.* **08** (2022) 103.
- [47] J.-W. Qiu and Z. Yu, Single diffractive hard exclusive processes for the study of generalized parton distributions, *Phys. Rev. D* **107**, 014007 (2023).
- [48] M. Mangano, CERN Yellow Reports: Monographs, [arXiv:1710.06353](https://arxiv.org/abs/1710.06353).
- [49] R. Abdul Khalek *et al.*, Science requirements and detector concepts for the electron-ion collider: EIC Yellow Report, *Nucl. Phys. A* **1026**, 122447 (2022).
- [50] V. Burkert, L. Elouadrhiri, A. Afanasev, J. Arrington, M. Contalbrigo, W. Cosyn, A. Deshpande, D. Glazier, X. Ji and S. Liuti *et al.*, Precision studies of QCD in the low energy domain of the EIC, *Prog. Part. Nucl. Phys.* **131**, 104032 (2023).
- [51] G. T. Bodwin, E. Braaten, and G. P. Lepage, Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium, *Phys. Rev. D* **51**, 1125 (1995); **55**, 5853(E) (1997).
- [52] F. Maltoni, M. L. Mangano, and A. Petrelli, Quarkonium photoproduction at next-to-leading order, *Nucl. Phys. B* **519**, 361 (1998).
- [53] Y. Feng, J. P. Lansberg, and J. X. Wang, Energy dependence of direct-quarkonium production in pp collisions from fixed-target to LHC energies: Complete one-loop analysis, *Eur. Phys. J. C* **75**, 313 (2015).
- [54] N. Brambilla, S. Eidelman, B. Heltsley, R. Vogt, G. Bodwin *et al.*, Heavy quarkonium: Progress, puzzles, and opportunities, *Eur. Phys. J. C* **71**, 1534 (2011).
- [55] M. Siddikov, I. Zemlyakov, M. Roa, and S. Valdebenito, Photoproduction of pairs in the CGC framework, *Phys. Rev. D* **111**, 056024 (2025).
- [56] Y. Q. Ma, K. Wang, and K. T. Chao, QCD radiative corrections to χ_{cJ} production at hadron colliders, *Phys. Rev. D* **83**, 111503 (2011).
- [57] A. K. Likhoded, A. V. Luchinsky, and S. V. Poslavsky, Production of χ_{c-} and χ_{b-} mesons in high energy hadronic collisions, *Phys. Rev. D* **90**, 074021 (2014).
- [58] H. F. Zhang, L. Yu, S. X. Zhang, and L. Jia, Global analysis of the experimental data on χ_c meson hadroproduction, *Phys. Rev. D* **93**, 054033 (2016).
- [59] S. P. Baranov and A. V. Lipatov, Are there any challenges in the charmonia production and polarization at the LHC?, *Phys. Rev. D* **100**, 114021 (2019).

- [60] S. P. Baranov, A. V. Lipatov, A. A. Prokhorov, and X. Chen, Towards higher-order calculations of quarkonia production with k_T -factorization: P-wave charmonia, *Eur. Phys. J. C* **84**, 348 (2024).
- [61] A. Schäfer, L. Mankiewicz, and O. Nachtmann, Diffractive eta(c), eta-prime, J/psi and psi-prime production in electron-proton collisions at HERA energies, in *Proceedings of Physics at HERA*, edited by W. Buchmüller and G. Ingelman (DESY, Hamburg, Germany, 1992).
- [62] J. Bartels, High-energy behaviour in a non-abelian gauge theory (II). First corrections to $T_{n \rightarrow m}$ beyond the leading in s approximation, *Nucl. Phys.* **B175**, 365 (1980); T. Jaroszewicz, Infra-Red divergences and Regge behaviour in QCD, *Acta Phys. Pol. B* **11**, 965 (1980), <https://www.actaphys.uj.edu.pl/R/11/12/965>; J. Kwiecinski and M. Praszalowicz, Three gluon integral equation and odd C singlet Regge singularities in QCD, *Phys. Lett.* **94B**, 413 (1980).
- [63] J. Bartels, M. A. Braun, D. Colferai, and G. P. Vacca, Diffractive eta(c) photoproduction and electroproduction with the perturbative QCD odderon, *Eur. Phys. J. C* **20**, 323 (2001).
- [64] J. Czyzewski, J. Kwiecinski, L. Motyka, and M. Sadzikowski, Exclusive η_c photo- and electroproduction at HERA as a possible probe of the odderon singularity in QCD, *Phys. Lett. B* **398**, 400 (1997); **411**, 402 (1997).
- [65] R. Engel, D. Y. Ivanov, R. Kirschner, and L. Szymanowski, Diffractive meson production from virtual photons with odd charge-parity exchange, *Eur. Phys. J. C* **4**, 93 (1998).
- [66] A. Dumitru and T. Stebel, Multiquark matrix elements in the proton and three gluon exchange for exclusive η_c production in photon-proton diffractive scattering, *Phys. Rev. D* **99**, 094038 (2019).
- [67] S. Benic, D. Horvatic, A. Kaushik, and E. A. Vivoda, Exclusive η_c production from small- x evolved Odderon at a electron-ion collider, *Phys. Rev. D* **108**, 074005 (2023).
- [68] Y. Jia, Z. Mo, J. Pan, and J. Y. Zhang, Photoproduction of C-even quarkonia at the EIC and EicC, *Phys. Rev. D* **108**, 016015 (2023).
- [69] S. Benić, A. Dumitru, A. Kaushik, L. Motyka, and T. Stebel, Photon-odderon interference in exclusive X_c charmonium production at the Electron-Ion Collider, *Phys. Rev. D* **110**, 014025 (2024).
- [70] P. Agostini *et al.* (LHeC and FCC-he Study Group), The large hadron-electron collider at the HL-LHC, *J. Phys. G* **48**, 110501 (2021).
- [71] A. Abada *et al.* (FCC Collaboration), FCC physics opportunities, *Eur. Phys. J. C* **79**, 474 (2019).
- [72] M. Siddikov and I. Zemlyakov, Exclusive photoproduction of $\chi_{c\gamma}$ pairs, *Phys. Rev. D* **112**, 014021 (2025).
- [73] M. Diehl, Generalized parton distributions, *Phys. Rep.* **388**, 41 (2003).
- [74] M. Siddikov, Exclusive photoproduction of $\chi_{c\gamma}$ pairs with large invariant mass, *Phys. Rev. D* **110**, 056043 (2024).
- [75] S. J. Brodsky, H. C. Pauli, and S. S. Pinsky, Quantum chromodynamics and other field theories on the light cone, *Phys. Rep.* **301**, 299 (1998).
- [76] G. Duplančić, K. Passek-Passek-Kumerički, B. Pire, L. Szymanowski, and S. Wallon, Probing axial quark generalized parton distributions through exclusive photoproduction of a $\gamma\pi^\pm$ pair with a large invariant mass, *J. High Energy Phys.* **11** (2018) 179.
- [77] G. Duplančić, S. Nabeebaccus, K. Passek-Kumerički, B. Pire, L. Szymanowski, and S. Wallon, Accessing chiral-even quark generalised parton distributions in the exclusive photoproduction of a $\gamma\pi^\pm$ pair with large invariant mass in both fixed-target and collider experiments, *J. High Energy Phys.* **03** (2023) 241.
- [78] G. P. Lepage and S. J. Brodsky, Exclusive processes in perturbative quantum chromodynamics, *Phys. Rev. D* **22**, 2157 (1980).
- [79] S. Navas *et al.* (Particle Data Group), Review of Particle Physics, *Phys. Rev. D* **110**, 030001 (2024).
- [80] C. F. von Weizsacker, Radiation emitted in collisions of very fast electrons, *Z. Phys.* **88**, 612 (1934).
- [81] E. J. Williams, Correlation of certain collision problems with radiation theory, *Kong. Dan. Vid. Sel. Mat. Fys. Med.* **13N4**, 1 (1935), <https://inspirehep.net/literature/1377275>.
- [82] V. M. Budnev, I. F. Ginzburg, G. V. Meledin, and V. G. Serbo, The Two photon particle production mechanism. Physical problems. Applications. Equivalent photon approximation, *Phys. Rep.* **15**, 181 (1975).
- [83] J. P. Ma and Z. G. Si, NRQCD factorization for twist-2 light-cone wave-functions of Charmonia, *Phys. Lett. B* **647**, 419 (2007).
- [84] X. P. Wang and D. Yang, The leading twist light-cone distribution amplitudes for the S-wave and P-wave quarkonia and their applications in single quarkonium exclusive productions, *J. High Energy Phys.* **06** (2014) 121.
- [85] W. Wang, J. Xu, D. Yang, and S. Zhao, Relativistic corrections to light-cone distribution amplitudes of S-wave B_c mesons and heavy quarkonia, *J. High Energy Phys.* **12** (2017) 012.
- [86] I. Babiarz, R. Pasechnik, W. Schaefer, and A. Szczurek, Hadroproduction of scalar P-wave quarkonia in the light-front k_T -factorization approach, *J. High Energy Phys.* **06** (2020) 101.
- [87] A. Ayala, M. Hentschinski, J. Jalilian-Marian, and M. E. Tejeda-Yeomans, Spinor helicity methods in high-energy factorization: Efficient momentum-space calculations in the Color Glass Condensate formalism, *Nucl. Phys.* **B920**, 232 (2017).
- [88] P. Caucal, F. Salazar, and R. Venugopalan, Dijet impact factor in DIS at next-to-leading order in the color glass Condensate, *J. High Energy Phys.* **11** (2021) 222.
- [89] P. Caucal, F. Salazar, B. Schenke, and R. Venugopalan, Back-to-back inclusive dijets in DIS at small x : Sudakov suppression and gluon saturation at NLO, *J. High Energy Phys.* **11** (2022) 169.
- [90] J. B. Bronzan, G. L. Kane, and U. P. Sukhatme, Obtaining real parts of scattering amplitudes directly from cross section data using derivative analyticity relations, *Phys. Lett.* **49B**, 272 (1974).
- [91] E. Gotsman, E. M. Levin, and U. Maor, A quantitative investigation of the Pomeron, *Z. Phys. C* **57**, 677 (1993).
- [92] A. G. Shuvaev, K. J. Golec-Biernat, A. D. Martin, and M. G. Ryskin, Off-diagonal distributions fixed by diagonal partons at small x and ξ , *Phys. Rev. D* **60**, 014015 (1999).

- [93] T. Altinoluk, G. Beuf, A. Czajka, and A. Tymowska, DIS dijet production at next-to-eikonal accuracy in the CGC, *Phys. Rev. D* **107**, 074016 (2023).
- [94] Y. V. Kovchegov and M. Li, Weizscker-Williams gluon helicity distribution and inclusive dijet production in longitudinally polarized electron-proton collisions, *J. High Energy Phys.* **08** (2025) 206.
- [95] J. D. Bjorken, J. B. Kogut, and D. E. Soper, Quantum electrodynamics at infinite momentum: Scattering from an external field, *Phys. Rev. D* **3**, 1382 (1971).
- [96] H. G. Dosch, T. Gousset, G. Kulzinger, and H. J. Pirner, Vector meson leptonproduction and nonperturbative gluon fluctuations in QCD, *Phys. Rev. D* **55**, 2602 (1997).
- [97] F. Gelis and R. Venugopalan, Large mass $q\bar{q}$ production from the color glass condensate, *Phys. Rev. D* **69**, 014019 (2004).
- [98] H. Fujii, F. Gelis, and R. Venugopalan, Quark pair production in high energy pA collisions: General features, *Nucl. Phys.* **A780**, 146 (2006).
- [99] R. Aaij *et al.* (LHCb Collaboration), Central exclusive production of J/ψ and $\psi(2S)$ mesons in pp collisions at $\sqrt{s} = 13$ TeV, *J. High Energy Phys.* **10** (2018) 167.
- [100] G. Antchev *et al.* (TOTEM Collaboration), Elastic differential cross section $d\sigma/dt$ at $\sqrt{s} = 2.76$ TeV and implications on the existence of a colourless 3-gluon bound state, *Eur. Phys. J. C* **80**, 91 (2020).
- [101] V. M. Abazov *et al.* (D0 Collaboration), Measurement of the differential cross section $d\sigma/dt$ in elastic $p\bar{p}$ scattering at $\sqrt{s} = 1.96$ TeV, *Phys. Rev. D* **86**, 012009 (2012).
- [102] N. Armesto and A. H. Rezaeian, Exclusive vector meson production at high energies and gluon saturation, *Phys. Rev. D* **90**, 054003 (2014).
- [103] C. Adloff *et al.* (H1 Collaboration), Diffractive photo-production of $\psi(2S)$ mesons at HERA, *Phys. Lett. B* **541**, 251 (2002).
- [104] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/y73d-hzdd> for tabulated data and the file README.txt in that section for more details.