

Light-by-light contribution to the muon $g-2$ within the nonlocal chiral quark model with vector and axial-vector mesons

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The hadronic light-by-light contribution to the muon anomalous magnetic moment is calculated in the framework of a nonlocal quark model with scalar-pseudoscalar and vector-axial-vector channels. The effect of the dressing of internal photons by vector mesons is found to be tiny. As a result, the contribution from the nonstrange quark loop is $a_{\mu}^{\text{HLbL, Loop}} = (93 \pm 9.9) \times 10^{-11}$. The total value, including meson exchanges and the contribution of strange particles, is $a_{\mu}^{\text{HLbL}} = (154 \pm 12.4) \times 10^{-11}$.

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I. INTRODUCTION

The anomalous magnetic moments of the electron [1,2] and the muon [3] are rare examples in particle physics of quantities that can be precisely measured experimentally and calculated theoretically. To the current level of experimental precision, the anomalous magnetic moment (AMM) of the electron is mostly due to quantum electrodynamics (QED) corrections with a small strong contribution, while the anomalous magnetic moment of the muon is more sensitive to other interactions of the Standard Model. Theoretically, the electron is known with a precision of $1.4 \times 10^{-11} a_e$, while the experimental precision is $1.1 \times 10^{-10} a_e$ [4].

In Table I, the current experimental average from Brookhaven National Laboratory and Fermilab [3], as well as the theoretical predictions from the white paper (WP) 2020 [7], the SnowMass 2021 update [8] and latest white paper 2025 update [9], are presented for the different SM contributions (electromagnetic, strong, and weak).² While the 2020–2021 gap between experiment and theory spanned several standard deviations—a possible signature

of new physics—recent advances have reshaped the picture. The experimental precision has reached unprecedented levels, making Fermilab's results the dominant contribution to the current world average for the muon magnetic moment anomaly. The most significant theoretical advances in recent years concern strong interaction effects, especially hadronic vacuum polarization (HVP) and the fine-structure-constant-subleading light-by-light scattering (LbL) contribution. These contributions cannot be calculated theoretically using QCD perturbation theory. They represent the dominant sources of uncertainty in the theoretical prediction of the muon $g-2$ value. The lattice calculation from the BMW (Budapest-Marseille-Wuppertal) Collaboration [10] shows a subpercent level of uncertainty for the HVP contribution, which is comparable to the precision of estimations based on e^+e^- -hadronic annihilation. The uncertainties in the recent BMW result are reduced by 40% [11]. However, there is a significant difference between these estimates. Partial estimates of the HVP contribution from other lattice groups agree with the results of calculations from collaboration BMW. The light-quark connected contribution, estimated on the basis of e^+e^- hadronic annihilation data, differs significantly from lattice calculations [10–18]. On the other hand, the new measurement of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section, from the threshold to 1.2 GeV, using the CMD-3 detector, also differs from all previous measurements [19]. Consequently, the discrepancy between the experimental result [3] and the white paper 2025 update [9] has diminished, showing no remaining tension between experiment and theory.

Earlier estimates of the light-by-light contribution were based solely on effective models [20–25], but now estimates using other approaches have emerged. Recent advances include lattice QCD results from several collaborations [26–30], a dispersion-theory derivation [31–41], holographic approach in Anti-de Sitter space for calculations

¹It should be noted that there is a discrepancy between the recent measurement of α from atomic recoil measurements in caesium [5], rubidium [6], and the electron magnetic moment anomaly [1]. The three experimental results do not agree, and for the electron $g-2$ the difference between the direct measurement and the values calculated from the recoil measurements is $(-8.8 \pm 3.6) \times 10^{-13} (-2.4\sigma)$ for ^{133}Cs and $(4.8 \pm 3.0) \times 10^{-13} (+1.6\sigma)$ for ^{87}Rb [6].

²Through the paper, the LbL contribution is given in units of 10^{-11} .

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TABLE I. Muon anomaly. The experimental values are taken from Ref. [3]. The theoretical predictions are from the WP 2025 update (upper values) [9] and the WP 2020/Snowmass 2021 combination (lower values) [7,8]. The last column illustrates the difference between central values of WP 2025 and WP 2020.

Contribution	Value in 10^{-11}	WP25-WP20 difference
Exp	116 592 071.5 (14.5)	
Theory	116 592 033 (62)	223
	116 591 810 (43)	
QED	116 584 718.8 (2)	-0.1
	116 584 718.931 (104)	
EW	154.4 (4)	0.8
	153.6 (1.0)	
HVP	7045 (61)	200
	6845 (40)	
HLbL	115.5 (9.9)	23.5
	92 (18)	

of hadronic observables (AdS/QCD) [42–46], the Dyson-Schwinger approach [47–50], effective models [51–61], short-distance constraints from the operator product expansion [62–65], and a subleading contribution [66], among others.

In this paper, the estimation of the light-by-light contribution within the framework of the quark model with a scalar-pseudoscalar(S-PS) and a vector-axial-vector(V-AV) sector is presented. The paper extends the calculations of the quark models with only scalar-pseudoscalar particles [53] and the estimation of resonance exchanges by adding spin-1 particles [57]. A new effect that occurs in the model involving vector particles is the dressing of photons by intermediate vector meson exchanges. This effect was previously estimated within the extended Nambu-Jona-Lasinio (ENJL) model [20] and Dyson–Schwinger equations (DSE) [47]. In the ENJL case, this effect reduces the final answer for a quark box by a factor of three, while in the DSE case it results in only a few percent difference. The dressing of photons through the exchange of intermediate vector bosons naturally occurs in resonance chiral theory [56,60,61].

The paper is organized as follows. In Sec. II, the model is discussed. In Sec. III, the vertices with external currents and vector meson dressing are discussed. The diagrams of light-by-light processes and the calculation procedure are given in Sec. IV. Results are presented in Sec. V. In Sec. VI, a comparison with other approaches is presented. In Appendix A the nonlocal meson-quark-antiquark-photon(s) vertices are given. Local limits of calculations, which are important for cross-checking the calculation procedure, are considered Appendix B.

II. MODEL

The present calculation is performed within the framework of the extended nonlocal chiral quark model, as

described in Refs. [53,57,67]. The current-current interaction terms are taken into account in this model. After the Bosonization and Hubbard-Stratonovich transformations, the quark-meson effective Lagrangian of this nonlocal model takes the following form:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{q}(x)(i\hat{\partial}_x - M_c)q(x) + \sigma_0 J_S^0(x) - \frac{1}{2G_1} \left((P^a(x))^2 \right. \\ & + (\tilde{S}^a(x) + \sigma_0 \delta_0^a)^2 \Big) - \frac{1}{2G_2} \left((V^{a,\mu}(x))^2 + (A^{a,\mu}(x))^2 \right) \\ & + P^a(x) J_P^a(x) + \tilde{S}^a(x) J_S^a(x) + V_\mu^a(x) J_V^{a,\mu}(x) \\ & + A_\mu^a(x) J_A^{a,\mu}(x), \end{aligned} \quad (1)$$

where M_c is the current quark mass matrix with diagonal elements m_c , G_1 and G_2 are coupling constants in the pseudoscalar-scalar (P, S) and vector-axial-vector sectors (V, A), the mesonic fields denoted as P, S, V, A , J_i are the nonlocal quark currents, which are taken in the form motivated by the instanton liquid model (ILM) [67]

$$J_M^{a,\mu}(x) = \int d^4x_1 d^4x_2 f(x_1) f(x_2) \bar{q}(x - x_1) \Gamma_M^{a,\mu} q(x + x_2), \quad (2)$$

with $M = S, P, V, A$, and matrices are $\Gamma_S^a = \lambda^a$, $\Gamma_P^a = i\gamma^5 \lambda^a$, $\Gamma_V^{a,\mu} = \gamma^\mu \lambda^a$, $\Gamma_A^{a,\mu} = \gamma^5 \gamma^\mu \lambda^a$. For scalar-pseudoscalar currents, the vector index μ should be omitted. Since only light particles are considered the flavor matrices are: $\lambda^a \equiv \tau^a$, $a = 0, \dots, 3$ with $\tau^0 = 1$. In the ILM model, the form factor $f(x)$ is related to the profile function of the quark zero mode, but here it is taken as a Gaussian with a parameter fitted to mesonic observables. The scalar isoscalar field is shifted $S^0 = \tilde{S}^0 + \sigma^0$, to obtain a physical scalar mesonic field with a zero vacuum expectation value. σ_0 is the vacuum expectation value of the scalar isoscalar field, $\langle S^0 \rangle_0 = \sigma_0 \neq 0$. As a result, the term in the Lagrangian $\sigma_0 J_S^0(x)$ leads to the appearance of a momentum-dependent mass $m(p)$, where the momentum dependence is factored out as

$$m(p) = m_c + m_d f^2(p), \quad m_d = -\sigma_0, \quad (3)$$

and m_d (dynamical quark mass) can be self-consistently found from the “gap” equation

$$m_d = G_1 \frac{8N_c}{(2\pi)^4} \int d^4k \frac{f^2(k^2) m(k^2)}{k^2 + m^2(k^2)}. \quad (4)$$

The corresponding quark propagator is

$$S(p) = (\hat{p} - m(p))^{-1}. \quad (5)$$

To describe mesonic bound states, it is necessary to calculate quark polarization loops

$$\Pi_{M_1 M_2}(p^2) = i \frac{N_c}{(2\pi)^4} \int d^4 k f^2(k_+^2) f^2(k_-^2) \times \text{Tr}_{d,f} [S(k_-) \Gamma_{M_1}^a S(k_+) \Gamma_{M_2}^b],$$

where $k_{\pm} = k \pm p/2$ and the trace is taken over the Dirac and flavor matrices. For spin-1 bound states, the quark polarization loops and propagators should be split into transverse and longitudinal components

$$\Pi_{VV,AA}^{\alpha\beta}(p^2) = \Pi_{VV,AA}^T(p^2) P_p^{T;\alpha\beta} + \Pi_{VV,AA}^L(p^2) P_p^{L;\alpha\beta}, \quad (6)$$

using projectors

$$P_p^{T;\alpha\beta} = g^{\alpha\beta} - \frac{p^\alpha p^\beta}{p^2}, \quad P_p^{L;\alpha\beta} = \frac{p^\alpha p^\beta}{p^2}.$$

Then, the unrenormalized and renormalized meson propagators (without $\pi - a_1$ mixing) take the form

$$\begin{aligned} D_{S,P}(p^2) &= \frac{1}{-G_1^{-1} + \Pi_{SS,PP}(p^2)} = \frac{g_{S,P}^2(p^2)}{p^2 - M_{S,P}^2}, & D_{S,P}^R(p^2) &= \frac{D_{S,P}(p^2)}{g_{S,P}^2(p^2)} \equiv \frac{1}{p^2 - M_{S,P}^2}, \\ D_{V,A}^T(p^2) &= \frac{1}{-G_2^{-1} + \Pi_{VV,AA}^T(p^2)} = \frac{g_{V,A}^2(p^2)}{M_{V,A}^2 - p^2}, & D_{V,A}^{T;R}(p^2) &= \frac{D_{V,A}^T(p^2)}{g_{V,A}^2(p^2)} \equiv \frac{1}{M_{V,A}^2 - p^2}. \end{aligned} \quad (7)$$

The meson vertex functions (without $\pi - a_1$ mixing) in momentum space are

$$\Gamma_{p_1, p_2}^{M; \{\mu\}} = g_M(p^2) f(p_1) \Gamma_M^{a \{\mu\}} f(p_2) \quad (8)$$

where p_i, k are the quark and meson momenta, respectively ($p_1 = p_2 + k$). Longitudinal components are related to spin-0. In the case of a system of pseudoscalar-axial-vector states, mixing appears [68,69] due to a quark polarization loop with pseudoscalar and axial-vector [57]. The polarization loop between scalar and vector states is identically zero.

III. EXTERNAL CURRENTS

In the Lagrangian (1), exist three types of terms, which in the formalism of [70,71] generate terms corresponding to vertices of interaction with gauge electromagnetic

fields [53,72]

$$\bar{q}(x) i \hat{\partial}_x q(x), \quad \sigma_0 J_S^0(x), \quad M^{a \{\mu\}}(x) J_M^{a \{\mu\}}(x), \quad (9)$$

where M are all possible mesonic fields. Therefore, the interactions with the electromagnetic field should also be included in all these terms. In order to obtain the expressions for the vertices, the Schwinger phase factor is added to each quark field $Q(x, y) = E(x, y) q(y)$

$$E(x, y) = \exp \left\{ -ieQ \int_x^y du_\mu G^\mu(u) \right\}, \quad (10)$$

where G^μ is the electromagnetic field, e is the elementary charge, and Q is the charge matrix of the quark fields. Then, the following rules for the contour integral are used [72]

$$\frac{\partial}{\partial y^\mu} \int_x^y dz_\nu G^\nu(z) = G_\mu(y), \quad \delta^{(4)}(x - y) \int_x^y dz_\nu G^\nu(z) = 0, \quad (11)$$

which guarantee the absence of nonminimal terms. With these rules, the sum of the local and nonlocal one-photon quark-antiquark vertices leads to a result that coincides with the Ball-Chiu *Ansätze* [73]. The exponent inside the Schwinger factor, after expansion, generates vertices with an arbitrary number of external fields. In Fig. 1, the graphical representation of the vertices in (9) is shown: the first term corresponds to the local vertex $eQ\gamma^\mu$ generated by the quark kinetic term. The second term corresponds to nonlocal quark-antiquark-photon(s) vertices, and the third one corresponds to nonlocal meson-quark-antiquark-photon(s). The expression for quark-antiquark-photon(s) vertices $\Gamma_{p_2, p_1, q_1 \dots q_n}^{\gamma \dots \gamma; \mu_1 \dots \mu_n}$, with up to four photons, is given in [53]. For meson-quark-antiquark-photon(s) $\Gamma_{p_2, p_1, q_1 \dots q_n}^{M \gamma \dots \gamma; \mu_1 \dots \mu_n}$,

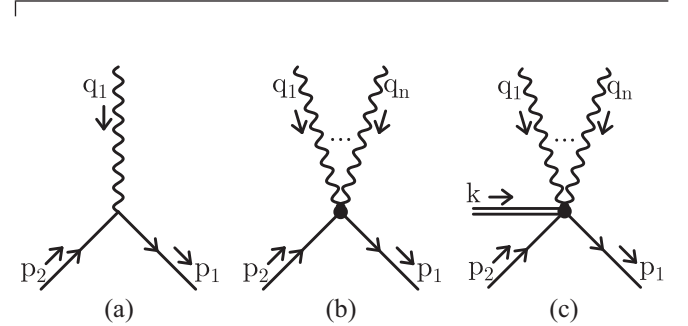


FIG. 1. Photon-quark vertices: the usual local one (a), the quark-antiquark vertex with n photons (b), and the meson-quark-antiquark vertex with n photons (c). Here p_1 and p_2 are the quark momenta, $q_1, \dots, q_n$ are the photon momenta, and k is the meson momentum.

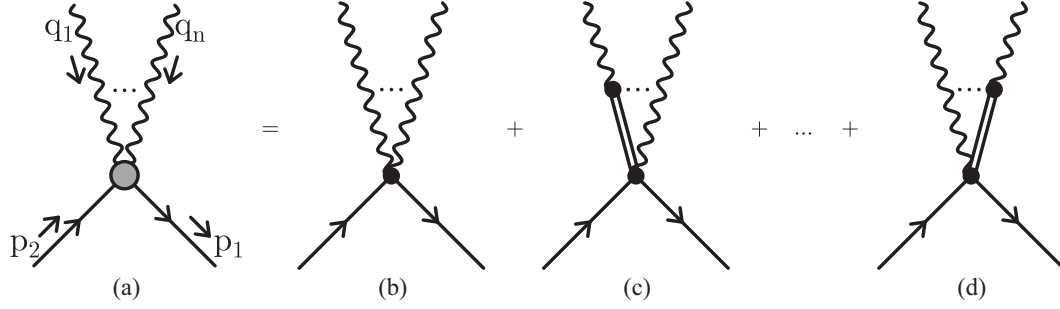


FIG. 2. Full quark-antiquark-(n) photon vertices: with nonlocal vertex (b) and with vector meson-photon transition (c), (d). Here, p_1 and p_2 are the quark momenta, $q_1, \dots, q_n$ are the photon momenta. The double line represents the intermediate vector meson.

with up to three photons, the expression is given in the Appendix A.

In the presence of a vector sector, the photon(s)-quark interaction vertices are additionally dressed by a $\rho(\omega) \rightarrow \gamma$ transition [57,74,75],

$$\begin{aligned}\Gamma_{p_2, p_1, q}^{\gamma; \mu_1} &= \Gamma_{p_2, p_1, q}^{\gamma; \mu} + \Delta \Gamma_{p_2, p_1, q_1}^{\gamma; \mu_1}, \\ \Gamma_{p_2, p_1, q_1, q_2}^{\gamma\gamma; \mu_1 \mu_2} &= \Gamma_{p_2, p_1, q_1, q_2}^{\gamma\gamma; \mu_1 \mu_2} + \Delta \Gamma_{p_2, p_1, q_1, q_2}^{\gamma\gamma; \mu_1 \mu_2}, \\ \Gamma_{p_2, p_1, q_1, \dots, q_n}^{\gamma \dots \gamma; \mu_1 \dots \mu_n} &= \Gamma_{p_2, p_1, q_1, \dots, q_n}^{\gamma \dots \gamma; \mu_1 \dots \mu_n} + \Delta \Gamma_{p_2, p_1, q_1, \dots, q_n}^{\gamma \dots \gamma; \mu_1 \dots \mu_n},\end{aligned}\quad (12)$$

where Δ represents the correction of the photon(s) vertex due to vector meson dressing. It is important to note that, from the third term in Eq. (9), only one vector meson can interact with a quark-antiquark pair and external electromagnetic fields at the point of interaction. This is illustrated in Figs. 2(c), 2(d) for the vertex of interaction with n photons. To understand the vertex corrections at the diagram level, one can join the dressing function with Fig. 1(c) to obtain the complete expression for the vertices. For the cases of one and two photons, these expressions are,

$$\begin{aligned}\Delta \Gamma_{p_2, p_1, q_1}^{\gamma; \mu_1} &= \sum_{V=\rho^0, \omega} \Gamma_{p_2, p_1}^{V; \alpha} P_{q_1}^{T; \alpha \mu_1} C_{\gamma V}(q_1), \\ \Delta \Gamma_{p_2, p_1, q_1, q_2}^{\gamma\gamma; \mu_1 \mu_2} &= \sum_{V=\rho^0, \omega} \left(\Gamma_{p_2, p_1, q_2}^{V\gamma; \alpha, \mu_2} P_{q_1}^{T; \alpha \mu_1} C_{\gamma V}(q_1) \right. \\ &\quad \left. + \Gamma_{p_2, p_1, q_1}^{V\gamma; \alpha, \mu_1} P_{q_2}^{T; \alpha \mu_2} C_{\gamma V}(q_2) \right)\end{aligned}\quad (13)$$

while general (n)photons vertex correction is the sum

$$\begin{aligned}\Delta \Gamma_{p_2, p_1, q_1, \dots, q_n}^{\gamma \dots \gamma; \mu_1 \dots \mu_n} \\ = \sum_{i=1}^n \sum_{V=\rho^0, \omega} P_{q_i}^{T; \alpha \mu_i} C_{\gamma V}(q_i) \Gamma_{p_2, p_1, q_1, \dots, q_n}^{V\gamma \dots \gamma; \alpha \mu_1 \dots \mu_n} \Big|_{\neq i}\end{aligned}$$

of quark-antiquark vertices with $(n-1)$ photons+vector meson. Here, $\Gamma_{p_2, p_1, q_1, \dots, q_n}^{V\gamma \dots \gamma; \alpha \mu_1 \dots \mu_n} \Big|_{\neq i}$ represents a vertex with $n-1$ photons + vector meson, where the photons are taken from 1 to $i-1$ and $i+1$ to n .

This dressing is transversal and can be written in the following form:

$$\begin{aligned}C_{\gamma V}(q^2) &= D_V^{T; R}(q^2) i N_c \frac{P_q^{T; \mu\nu}}{3} \int \frac{d^4 k}{(2\pi)^4} \left\{ \text{Tr} \left[S(k_+) \Gamma_{k_+, k_-, q}^{\gamma; \mu} \right. \right. \\ &\quad \left. \left. \times S(k_-) \Gamma_{k_-, k_+}^{V; \nu} \right] + \text{Tr} \left[\Gamma_{k, k, q}^{V\gamma; \mu\nu} S(k) \right] \right\},\end{aligned}\quad (14)$$

where V stands for ρ^0 or ω mesons. The transition has the property $C_{\gamma V}(0) = 0$ [74], and it does not lead to the renormalization of the photon mass or quark charge [75].

IV. DIAGRAMS

The hadronic LbL (HLbL) contribution to the anomalous magnetic moment of the muon can be calculated using the projection [76],

$$\begin{aligned}a_\mu^{\text{HLbL}} &= \frac{1}{48m_\mu} \text{Tr}((\hat{p} + m_\mu)[\gamma^\rho, \gamma^\sigma](\hat{p} + m_\mu)\Pi_{\rho\sigma}(p, p)), \\ \Pi_{\rho\sigma}(p', p) &= -ie^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1 + q_2 - k)^2} \\ &\quad \times \gamma^\mu \frac{\hat{p}' - \hat{q}_1 + m_\mu}{(p' - q_1)^2 - m_\mu^2} \gamma^\nu \frac{\hat{p} - \hat{q}_1 - \hat{q}_2 + m_\mu}{(p - q_1 - q_2)^2 - m_\mu^2} \\ &\quad \times \gamma^\lambda \frac{\partial}{\partial k^\rho} \Pi_{\mu\nu\lambda\sigma}(q_1, q_2, k - q_1 - q_2),\end{aligned}\quad (15)$$

where m_μ is the muon mass, and the static limit $k_\mu \equiv (p' - p)_\mu \rightarrow 0$ is implied. The calculation procedure is described in detail in [53]. By averaging over the direction of the muon momentum, the result for a_μ^{HLbL} becomes a three-dimensional integral with the radial integration variables $Q_1 = \sqrt{-q_1^2}$, $Q_2 = \sqrt{-q_2^2}$ and the angular variable [25,31,32,52,54].

The full set of diagrams for the four-rank polarization tensor $\Pi_{\mu\nu\lambda\sigma}$ is similar to those in the model without the vector sector [53], but now all the vertices are properly dressed with vector meson exchange, as shown in Fig. 3. As discussed in the previous section, since the model is based on the linear realization of chiral symmetry, only one $\rho(\omega)$ exchange can be present in any nonlocal photon-quark-antiquark vertex. Therefore, the exchange with one vector

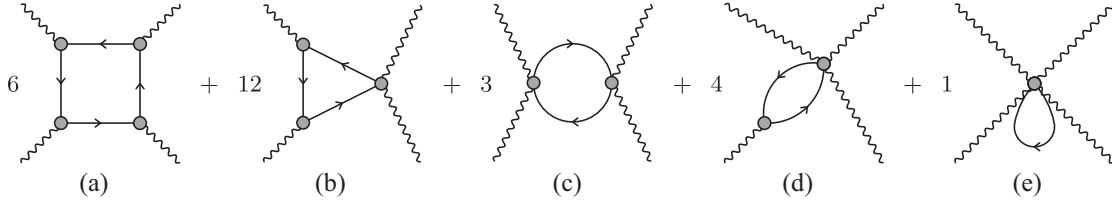


FIG. 3. Box diagram and the diagrams with nonlocal multiphoton interaction vertices represent the gauge invariant set of diagrams contributing to the polarization tensor $\Pi_{\mu\nu\lambda\rho}$. The numbers in front of the diagrams are the combinatorial factors for the crossed diagrams. In the LbL contribution, the three photons are connected to the muon line.

meson arises from diagrams Figs. 3(a)–3(e), two from Figs. 3(a)–3(d), three from Figs. 3(a), 3(b), and four from Fig. 3(a). Since at zero momentum the transition loop is zero, there is no contribution to $g - 2$ with exchanges of $\rho(\omega)$ on the external photon line. Therefore, the maximum number of vector meson exchanges contributing to LbL process is three. The vector meson exchange needs to be included in all types of diagrams presented in Figs. 3(a)–3(e). External gauge field lines are free and do not include vector meson exchanges.

V. RESULTS

The results in the model with spin-1 particles will differ from the result of the model without the vector meson sector for the following reasons:

- (i) the parameters of the model are different, as they are fitted to observables with the inclusion of a possible axial-vector component,
- (ii) there will be an intermediate axial-vector LbL contribution, and
- (iii) the internal photon line will be dressed with intermediate vector-meson exchange.

The parameters of the model are refitted in [57] and used to calculate the LbL contribution from axial-vector mesons. Due to the lack of confinement, the range of m_d , where the value of the G_2 constant can be unambiguously determined using the mass of the ρ meson, is limited to between 293 and 354 MeV. It is found that in this region the ratio of the four quark coupling constants, G_2/G_1 , ranges from -0.08 to -0.14 . Therefore, it is very useful to study the LbL contribution over a wider range of m_d using these constant ratios. The full quark loop contribution and the result of only diagrams without vector mesons are shown in Fig. 4 as a function of m_d . For comparison, the behavior of the quark loop contribution in a model without vector mesons is also shown [53]. One can see that, for small m_d , the contribution from diagrams without vector mesons gives contributions similar to those in [53]. On the other hand, for large m_d , the contribution of vector mesons is very small. The individual contributions from diagrams with one, two, and three vector meson exchanges are shown in Figs. 5–7. It can be seen that the contributions of diagrams involving vector mesons are small. For the set with $m_d = 310$ MeV, these contributions are -2.9 , 0.09 , and 0.001 , respectively.

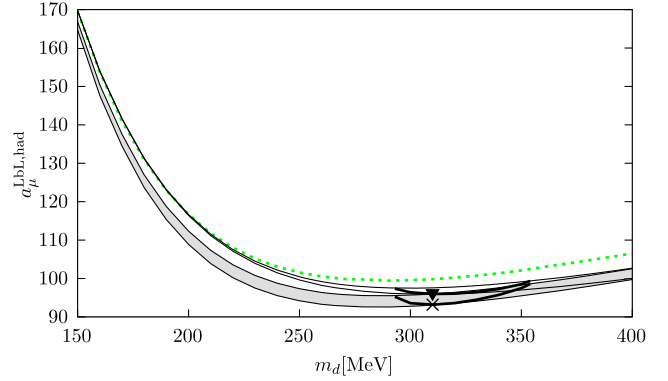


FIG. 4. (in units 10^{-11}) Left: The LbL contribution to the muon AMM from the quark loop diagram without $\rho(\omega)$ exchanges (thin upper solid lines) and with $\rho(\omega)$ exchanges (thin downer solid lines) for different parametrizations as a function of the dynamical quark mass. The shaded area corresponds to the region between the ratios $G_1/G_2 = -0.08$ and $G_1/G_2 = -0.14$. The thick black solid lines correspond to the parametrization where G_2 is fixed in order to reproduce the physical value of the ρ -meson mass. The triangle corresponds to the result with $m_d = 310$ MeV without $\rho(\omega)$ exchange and cross with $\rho(\omega)$. The dashed green line is the contribution in the model without the vector sector [53].

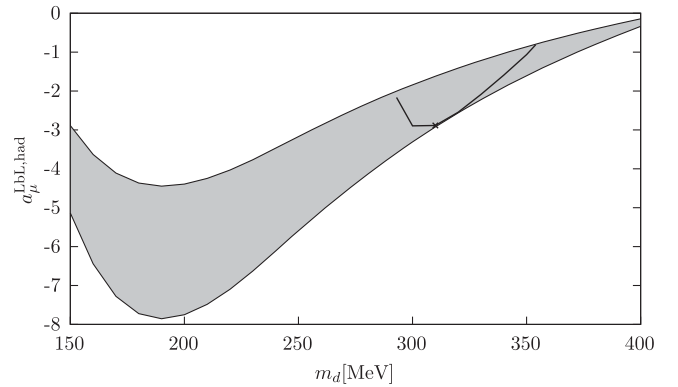


FIG. 5. (in units 10^{-11}) Left: The LbL contribution to the muon AMM from the quark loop diagram with one intermediate $\rho(\omega)$ exchange for different parametrizations as a function of the dynamical quark mass. The shaded area corresponds to the region between the ratios $G_1/G_2 = -0.08$ and $G_1/G_2 = -0.14$. The black solid lines correspond to the parametrization where G_2 is fixed in order to reproduce the physical value of the ρ -meson mass. The cross corresponds to the result with $m_d = 310$ MeV.

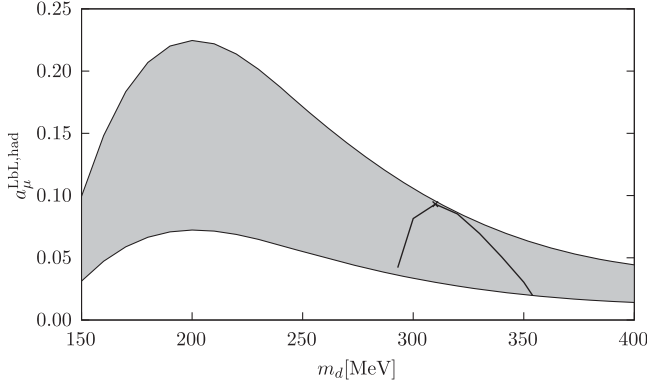


FIG. 6. (in units 10^{-11}) Left: The LbL contribution to the muon AMM from the quark loop diagram with two intermediate $\rho(\omega)$ exchanges for different parametrizations as a function of the dynamical quark mass. The shaded area corresponds to the region between the ratios $G_1/G_2 = -0.08$ and $G_1/G_2 = -0.14$. The black solid lines correspond to the parametrization where G_2 is fixed in order to reproduce the physical value of the ρ meson mass. The cross corresponds to the result with $m_d = 310$ MeV.

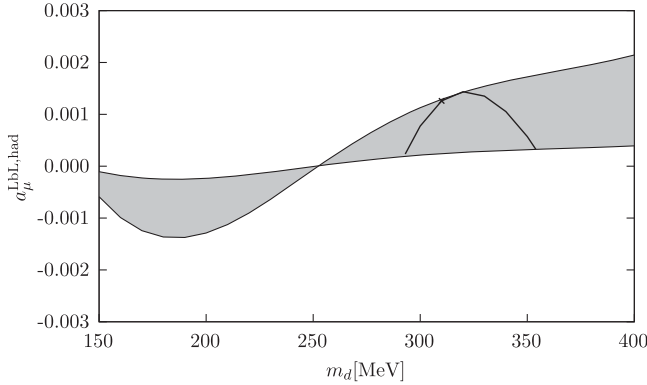


FIG. 7. (in units 10^{-11}) Left: The LbL contribution to the muon AMM from the quark loop diagram with three intermediate $\rho(\omega)$ exchanges for different parametrizations as a function of the dynamical quark mass. The shaded area corresponds to the region between the ratios $G_1/G_2 = -0.08$ and $G_1/G_2 = -0.14$. The black solid lines correspond to the parametrization where G_2 is fixed in order to reproduce the physical value of the ρ -meson mass. The cross corresponds to the result with $m_d = 310$ MeV.

The combination of quark loop contributions with mesonic exchanges is given in Fig. 8. It is interesting to note that, for a fixed value of m_d , the combined contribution from quark loops and mesons is less dependent on G_2 compared to the individual contributions. Namely, for a set with $m_d = 310$ MeV, the contributions to the ratio of -0.08 from quark loops and mesons are 96 and 51, respectively, while for -0.14 these values are 93 and 54. As a result, the total value of 147 coincides.

In the model without a vector sector [53], the central value is calculated as the average between the maximal and minimal values in the range m_d from 200 to 350 MeV,

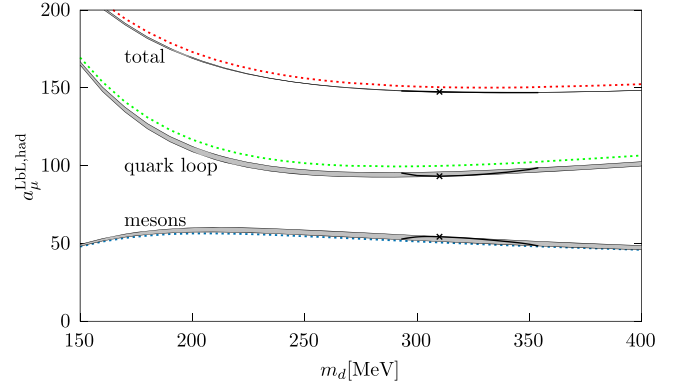


FIG. 8. (in units 10^{-11}) Left: The LbL contribution to the muon AMM from the π , σ , a_1 , f_1 mesons, the u and d quark loop, and the total $SU(2)$ contribution for different parametrizations as a function of the dynamical quark mass. The shaded area corresponds to the region between ratios $G_1/G_2 = -0.08$ and $G_1/G_2 = -0.14$. The thick black solid lines correspond to the parametrization where G_2 is fixed in order to reproduce the physical value of the ρ -meson mass. The cross corresponds to the result with $m_d = 310$ MeV. The dashed lines are the contributions in the model without the vector sector [51].

while the error bar is half the difference. Using this strategy, the total contribution from nonstrange quark loops in models with spin-1 mesons is 103 ± 9.9 , which is only slightly lower than the estimate of 108 ± 8.6 in the model without vector mesons.³ Adding the mesonic contributions, the $SU(2)$ result is 158 ± 11.4 with vector mesons, and 162 ± 14.3 without. Therefore, we can assume that the strange-particle contribution will not change much after the introduction of vector-axial-vector fields, and add to our estimate the strange-particle contribution in model without spin-1 mesons [53]. If we add naively the contributions of η , η' , $f_0(980)$, $a_0(980)$, and the strange quark loop (since we expect that the vector meson dressing will not drastically change them) from [53] on top of the $SU(2)$, the resulting value becomes 165 ± 12.4 , which is almost identical to the value of 168 ± 12.5 in [53].

However, in the model with vector mesons, it is proposed to change the method of calculating the central value. It seems more natural to take the central value at the point where it is possible to describe the vector meson and fix the G_2 constant using the physical value of the ρ -meson mass [57]. It is suggested to use the central value of $m_d = 310$ MeV, as at this point, it is also possible to describe the axial-vector meson. In this case nonstrange quark loop is 93 ± 9.9 , quark loop plus the $SU(2)$ mesonic contribution is 147 ± 11.4 . The final estimate of the HLbL contribution with strange particles included is

$$a_\mu^{\text{HLbL}} = (154 \pm 12.4) \times 10^{-11}. \quad (16)$$

³This value is rounded in [53].

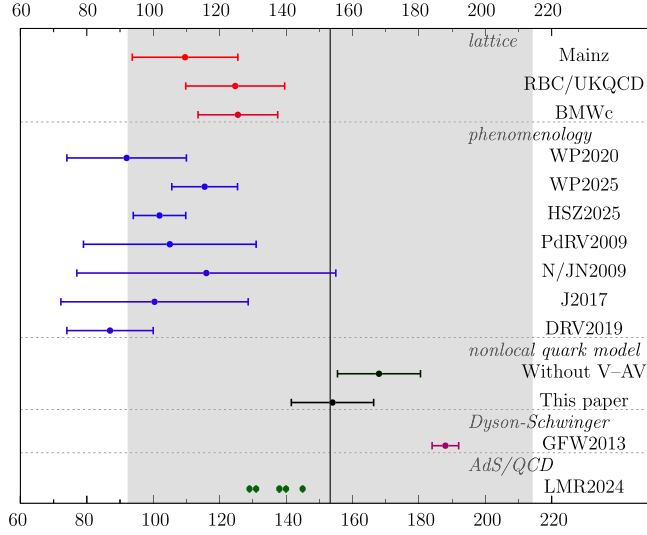


FIG. 9. (in units 10^{-11}) The HLbL contribution to the muon AMM: lattice QCD calculations (Mainz [28], (RBC/UKQCD) [29], and (BMWc)[30], phenomenological estimations “white paper” 2020 [7] and 2025 update [9], recent evaluation using a dispersive formalism (HSZ2025) [41], using combinations of different contributions (PdRV2009) [24], (N/JN2009) [25], (J2017) [54], (DRV2019) [58], in a nonlocal quark model without vector and axial-vector mesons [53] and present paper calculations with spin-1 particles, DSE and BSE studies (GFW2013) [47], AdS/QCD result for different models (LMR2024) [45]. The thin vertical line corresponds to difference between experimental value [3] and theoretical prediction minus HLbL contribution while the shaded region to the theoretical error without HLbL contribution from WP2025 [9].

VI. COMPARISON WITH OTHER APPROACHES AND SUMMARY

The comparison of our result (16) is given in Fig. 9 with recent lattice results by (Mainz [28], (RBC/UKQCD) [29], and (BMWc) [30], phenomenological estimations : “white paper” 2020 [7] and 2025 update [9], dispersive formalism (HSZ2025) [41], “Glasgow consensus” [24], DSE and BSE studies [47], and the AdS/QCD result for different models [45]. In the nonlocal quark model, the QCD asymptotics are automatically fulfilled by the diagrams with quark loops [53]. Furthermore, the inclusion of diagrams with vector meson dressing of photons and resonance exchanges does not alter this picture.⁴ This is similar to DSE and BSE studies [47] while in dispersive formalism [41] the matching to short-distance constraints⁵ is used. In the AdS/CFT correspondence [42–45], this matching occurs through the summation of a tower of resonances. One can see that our

⁴The suppression of quark loop contributions by soft gluon fields is discussed in [77].

⁵Short-distance constraints with gluonic corrections is given in [65].

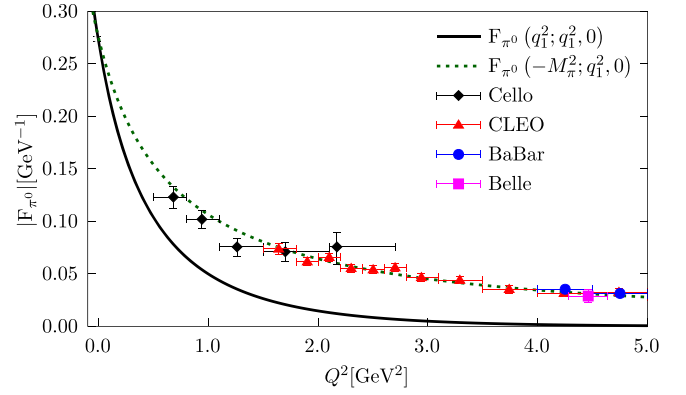


FIG. 10. $\gamma^*\gamma \rightarrow \pi^0$ transition form factor calculated in the nonlocal quark model with spin-1 particles, comparing two kinematic configurations: the LbL-relevant case $F_{\pi^0}(q_1^2; q_1^2, 0)$ (black solid line) and the experimentally measured case $F_{\pi^0}(-M_\pi^2; q_1^2, 0)$ (green dashed line). The experimental data from CELLO [80], CLEO [81], BABAR [82], and Belle [83] are also shown.

final result of is larger than the dispersive approach, the lattice QCD calculation and AdS/QCD. However, it is interesting to compare how the different results in Fig. 9 relate to the difference between theoretical calculations and experimental measurements. For comparison, the difference between the experimental value [3] and the theoretical prediction from the WP 2025 update [9] without HLbL contribution is also shown in Fig. 9. Even when incorporating our HLbL result (16) with the WP2020 [7,8] values in Table I, there is a significant theory-experiment gap of 200×10^{-11} . However, when using the WP2025 values from Table I [9] combined with our HLbL contribution (16), the discrepancy not only changes sign but reduces to approximately -1×10^{-11} . Therefore, incorporating our result substantially reduces the allowed parameter space for new physics scenarios while achieving excellent theory-experiment agreement for the muon’s anomalous magnetic moment.

The source of discrepancy between calculations in nonlocal model and dispersive formalism is partially known. Namely, the mean-field calculations in the quark model do not include the pion loop contribution, which is known to give a negative contribution [7,49]. The pion loop (or mesonic loop in general) corresponds to subleading terms in the $1/N_c$ expansion, which can be accounted for using the scheme presented in [78]. However, in the $1/N_c$ expansion, there will be additional terms that may also affect the contact term. On the other hand, within a non-local model, the two-photon form factors of mesons depend not only on the photon virtualities but also on the mesonic virtuality. To illustrate this, the transition pion form factor is shown in Fig. 10 for two different kinematic configurations: (1) the case where the meson virtuality matches that of the off-shell photon (relevant for the LbL process), and (2) the experimentally measured configuration with an on-mass-shell pion. This additional

dependence⁶ induces substantial suppression of massive resonance contributions to light-by-light scattering [53,57]. While [53] focuses on η and η' meson effects, [57] provides detailed examination of axial-vector form factors in the nonlocal model and their HLbL contributions.

In order to better understand the difference and as an independent cross-check, it would be interesting to repeat the calculations within the quark model framework in conditions similar to those used in lattice calculations, that is, in coordinate space with a semianalytical QED kernel [84]. This would allow us to compare not only the total number, but also the contributions from different distances, as shown in Fig. 3 of Ref. [27].

The results obtained within the framework of the non-local quark model for the LbL contribution to the muon magnetic-moment anomaly are shown to be stable when extending from the S-PS case to include V-AV currents. A similar effect is seen in the Nambu-Jona-Lasinio model describing $\pi\pi$ scattering [85].

Our mean-field analysis demonstrates good agreement with Fermilab's muon $g - 2$ measurements. The next step will incorporate pion loop contributions (a $1/N_c$ correction), which are expected to make significant contributions to the complete LbL scattering evaluation.

Diagrams are drawn with the help of the FEYN.GLE package [86].

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: MESON-QUARK-ANTIQUARK-PHOTON(S) VERTICES

Due to the nonlocal interaction between mesons and quarks, vertices with one, two, or three photons, using rules (11), take the form ($p_1 = p_2 + q_1 + \dots + q_i$)

$$\Gamma_{p_1, p_2, q_1}^{M\gamma; \{\alpha\}\mu_1} = -e g_M(p^2) \left(J_1^{\mu_1}(p_1, -q_1) Q \Gamma_M^{a\{\alpha\}} f(p_2) + f(p_1) \Gamma_M^{a\{\alpha\}} Q J_1^{\mu_1}(p_2, q_1) \right), \quad (\text{A1})$$

$$\begin{aligned} \Gamma_{p_1, p_2, q_1, q_2}^{M\gamma\gamma; \{\alpha\}\mu_1\mu_2} &= e^2 g_M(p^2) \left(J_2^{\mu_1\mu_2}(p_1, -q_1, -q_2) Q^2 \Gamma_M^{a\{\alpha\}} f(p_2) + J_1^{\mu_1}(p_1, -q_1) Q \Gamma_M^{a\{\alpha\}} Q J_1^{\mu_2}(p_2, q_2) \right. \\ &\quad \left. + J_1^{\mu_2}(p_1, -q_2) Q \Gamma_M^{a\{\alpha\}} Q J_1^{\mu_1}(p_2, q_1) + f(p_1) \Gamma_M^{a\{\alpha\}} Q^2 J_2^{\mu_1\mu_2}(p_2, q_1, q_2) \right), \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \Gamma_{p_1, p_2, q_1, q_2, q_3}^{M\gamma\gamma\gamma; \{\alpha\}\mu_1\mu_2\mu_3} &= -e^3 g_M(p^2) \left(J_3^{\mu_1\mu_2\mu_3}(p_1, -q_1, -q_2, -q_3) Q^3 \Gamma_M^{a\{\alpha\}} f(p_2) \right. \\ &\quad + J_2^{\mu_1\mu_2}(p_1, -q_1, -q_2) Q^2 \Gamma_M^{a\{\alpha\}} Q J_1^{\mu_3}(p_2, q_3) + J_2^{\mu_1\mu_3}(p_1, -q_1, -q_3) Q^2 \Gamma_M^{a\{\alpha\}} Q J_1^{\mu_2}(p_2, q_2) \\ &\quad + J_2^{\mu_2\mu_3}(p_1, -q_2, -q_3) Q^2 \Gamma_M^{a\{\alpha\}} Q J_1^{\mu_1}(p_2, q_1) + J_1^{\mu_3}(p_1, -q_3) Q \Gamma_M^{a\{\alpha\}} Q^2 J_2^{\mu_1\mu_2}(p_2, q_1, q_2) \\ &\quad + J_1^{\mu_1}(p_1, -q_1) Q \Gamma_M^{a\{\alpha\}} Q^2 J_2^{\mu_2\mu_3}(p_2, q_2, q_3) + J_1^{\mu_2}(p_1, -q_2) Q \Gamma_M^{a\{\alpha\}} Q^2 J_2^{\mu_1\mu_3}(p_2, q_1, q_3) \\ &\quad \left. + f(p_1) \Gamma_M^{a\{\alpha\}} Q^3 J_3^{\mu_1\mu_2\mu_3}(p_2, q_1, q_2, q_3) \right) \end{aligned} \quad (\text{A3})$$

where

$$J_1^{\mu_1}(k, q_1) = (k + k_1)^{\mu_1} f_{k, k_1}^{(1)}, \quad (\text{A4})$$

$$J_2^{\mu_1\mu_2}(k, q_1, q_2) = +(k + k_1)^{\mu_1} (k_1 + k_{12})^{\mu_2} f_{k, k_1, k_{12}}^{(2)} + (k + k_2)^{\mu_2} (k_2 + k_{12})^{\mu_1} f_{k, k_2, k_{12}}^{(2)} + 2g^{\mu_1\mu_2} f_{k, k_{12}}^{(1)}, \quad (\text{A5})$$

⁶The discussion of the dependence of the meson form factor on meson virtuality and dispersive result can be found in [79].

$$\begin{aligned}
J_3^{\mu_1\mu_2\mu_3}(k, q_1, q_2, q_3) = & +(k+k_1)^{\mu_1}(k_1+k_{12})^{\mu_2}(k_{12}+k_{123})^{\mu_3}f_{k,k_1,k_{12},k_{123}}^{(3)} + (k+k_1)^{\mu_1}(k_1+k_{13})^{\mu_3}(k_{13}+k_{123})^{\mu_2}f_{k,k_1,k_{13},k_{123}}^{(3)} \\
& + (k+k_2)^{\mu_2}(k_2+k_{12})^{\mu_1}(k_{12}+k_{123})^{\mu_3}f_{k,k_2,k_{12},k_{123}}^{(3)} + (k+k_2)^{\mu_2}(k_2+k_{23})^{\mu_3}(k_{23}+k_{123})^{\mu_1}f_{k,k_2,k_{23},k_{123}}^{(3)} \\
& + (k+k_3)^{\mu_3}(k_3+k_{13})^{\mu_1}(k_{13}+k_{123})^{\mu_2}f_{k,k_3,k_{13},k_{123}}^{(3)} + (k+k_3)^{\mu_3}(k_3+k_{23})^{\mu_2}(k_{23}+k_{123})^{\mu_1}f_{k,k_3,k_{23},k_{123}}^{(3)} \\
& + 2g^{\mu_1\mu_2}(k_{12}+k_{123})^{\mu_3}f_{k,k_{12},k_{123}}^{(2)} + 2g^{\mu_1\mu_3}(k_{13}+k_{123})^{\mu_2}f_{k,k_{13},k_{123}}^{(2)} + 2g^{\mu_2\mu_3}(k_{23}+k_{123})^{\mu_1}f_{k,k_{23},k_{123}}^{(2)} \\
& + 2g^{\mu_2\mu_3}(k+k_1)^{\mu_1}f_{k,k_1,k_{123}}^{(2)} + 2g^{\mu_1\mu_3}(k+k_2)^{\mu_2}f_{k,k_2,k_{123}}^{(2)} + 2g^{\mu_1\mu_2}(k+k_3)^{\mu_3}f_{k,k_3,k_{123}}^{(2)} \quad (A6)
\end{aligned}$$

additional momenta defined as $k_i = k + q_i$, $k_{ij} = k + q_i + q_j$, $k_{ijk} = k + q_i + q_j + q_k$ and the shorthand notations for first, second, and third order finite differences are introduced

$$f_{p,q}^{(1)} = \frac{f(p) - f(q)}{p^2 - q^2}, \quad f_{p,q,l}^{(2)} = \frac{f_{p,q}^{(1)} - f_{p,l}^{(1)}}{q^2 - l^2}, \quad f_{p,q,l,k}^{(3)} = \frac{f_{p,q,l}^{(2)} - f_{p,q,k}^{(2)}}{l^2 - k^2}.$$

Similar abbreviations are used for finite differences of the mass function $m \rightarrow m^{(i)}$.

APPENDIX B: CROSS-CHECK OF DRESSING AND LOCAL LIMITS

The influence of photon dressing by vector mesons on LbL contributions is studied in [21,47] within the framework of the ENJL model and the DSE approach. The effective vertex in the ENJL model [21] with vector meson exchange is

$$\Gamma_{\text{ENJL}}^{\gamma;\mu} = \Gamma_{\text{ENJL}}^{\gamma;\mu} + \Delta\Gamma_{\text{ENJL}}^{\gamma;\mu} = \gamma^\mu - \gamma_T^\mu \frac{Q^2}{Q^2 + M_V^2(Q^2)}, \quad (B1)$$

where γ_T^μ is the transversal projection of γ^μ . The transversal component of the full vertex can be rewritten identically as

$$\Gamma_{T,\text{ENJL}}^{\gamma;\mu} \equiv \gamma_T^\mu \frac{M_V^2(Q^2)}{Q^2 + M_V^2(Q^2)}. \quad (B2)$$

In ENJL model [21], the momentum-dependent mass of the vector meson is expressed via using the incomplete gamma function [87]

$$M_V^2(Q^2) = \frac{N_c \Lambda_\chi^2}{8\pi^2 G_V \bar{\Pi}^{(1)}(Q^2)}, \quad \bar{\Pi}^{(1)}(Q^2) = \frac{N_c}{2\pi^2} \int_0^1 dx x(1-x) \Gamma(0, x_{ij}), \quad x_{ij} = \frac{m_Q^2 + Q^2 x(1-x)}{\Lambda_\chi^2}, \quad (B3)$$

and model parameters [21] are $\Lambda_\chi = 1.16$ GeV, $G_V = 1.263$ GeV⁻² and the constituent quark mass is $m_Q = 275$ MeV. At zero momentum, the momentum-dependent mass of the vector meson is $M_V^2(0) = 0.678$ GeV² and for large Q^2 it is strongly increased, i.e., at $Q^2 = 4$ GeV² it is $M_V^2(4) \approx 3.55$ GeV².

In VMD approximation, which is discussed in [47], the vector meson mass is constant,⁷ i.e., $M_V^2(Q^2) \equiv M_V^2$. In the nonlocal model, the additional transition vertex with one photon (12), can be effectively represented by

$$\Delta\Gamma_{p_2,p_1,q_1}^{\gamma;\mu} = -\gamma_T^\mu \frac{Q^2}{Q^2 + M_V^2} f(p_1)f(p_2)F(Q^2), \quad (B5)$$

where $F(Q^2)$ is a smooth function. For symmetric splitting, $p_{1,2} = k \pm Q/2$, (k is the quark loop momentum) and a Gaussian form-factor, the combination of form-factors could be rewritten $f(p_1)f(p_2) = f(\sqrt{2}k)f(Q/\sqrt{2})$ in an identical way. In Fig. 11, the behavior of the additional dressed function related to the VMD result is shown for the ENJL model and $F(Q^2)$, $f(Q/\sqrt{2})F(Q^2)$ for the nonlocal model, for a set with $m_d = 310$ MeV. One can see that the VMD, ENJL, and nonlocal models produce the same sign of the correction, and the result of the nonlocal model is suppressed compared to the VMD or ENJL models. The constant for the $\rho \rightarrow e^+e^-$ process $g_{\rho\gamma} = 0.094$ GeV² deduced on a ρ -meson mass-shell is not far from experimental value $g_{\rho\gamma} = 0.121$ GeV².

In order to cross-check the calculations, a case similar to that of ENJL or VMD could be considered. In this case, the

⁷In the DSE approach, the transition vertex can be approximated as follows [47,88]:

$$\Gamma_{\text{DSE}}^{\gamma;\mu} = \Gamma_{\text{BC}}^{\gamma;\mu} - \gamma_T^\mu \frac{\omega^4 N_V}{\omega^4 + k^4} \frac{f_V}{M_V} \frac{Q^2}{Q^2 + M_V^2} e^{-\alpha(Q^2 + M_V^2)}, \quad (B4)$$

where $\Gamma_{\text{BC}}^{\gamma;\mu}$ is the vertex in Ball-Chiu *Ansätze* [73].

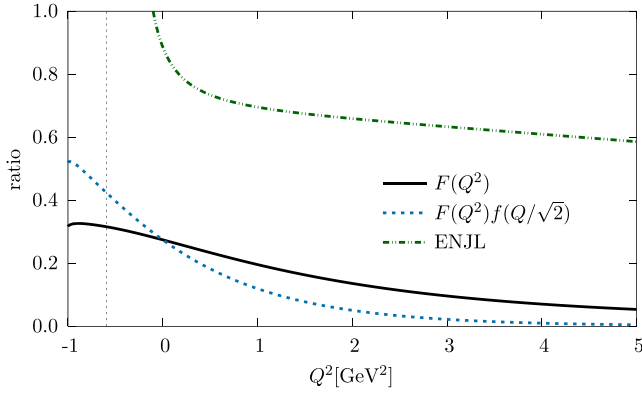


FIG. 11. Ratio of additional vertex with vector meson exchange with respect to VMD result $-\gamma_T^\mu Q^2/(Q^2 + M_V^2)$. The green double-dashed line is the ENJL result (B1), the black solid and blue dashed lines are the functions $F(Q^2)$ and $F(Q^2)f(Q/\sqrt{2})$ in the nonlocal model (B5). The VMD result corresponds to 1.

value of the current quark mass is taken to be the constituent quark mass, $m_c \equiv m_Q = 300$ MeV as in [20], the dynamical quark mass is set to zero $m_d = 0$, the function $F(Q^2)$ in the transition loop (B5) is set to 1. In this case, one can study the behavior of the vector meson correction to the LbL contribution as a function of the Λ parameter in the form factor of the vector meson vertex (B1). The pure local result will be $\Lambda \rightarrow \infty$. There are contributions with one, two, and three vector meson exchanges, as already discussed. The overall factor for exchanges involving an odd number of vector mesons is negative compared to the contribution without vector mesons, while for even exchanges it is positive. The behavior of the contributions

with different numbers of vector meson exchanges as a function of the Λ parameter is shown in Fig. 12 for VMD and ENJL vector meson dressings in linear scale (left) and logarithmic scale (right). In the ENJL model, the mass of the vector mesons depends on the momentum (B3). For infinitely large Λ , the total answers for VMD and ENJL are 9.4 and 15.4. It should be noted that in [20], the scheme for separating low and high energies is used with a scale parameter of $\mu \sim 0.7\text{--}4$ GeV. In this scheme, the low energy contribution is restricted such that all internal momentum is $|Q| \leq \mu$ in the integration procedure. For high energies, the contribution of a quark loop with mass $m_Q = \mu$ is taken. This separation is applied to all diagrams. One can see that the number 21, for the ENJL result quoted in [20] roughly corresponds to $\Lambda \sim 4$ GeV. On the other hand, in Fig. 12, one can see that even in the limit of constant mass, the value of the vector meson correction will be small if $\Lambda \sim 1$ GeV. Interestingly, the LbL contribution converges rather slowly, as seen in Fig. 11 (right).

On the other hand, in the local limit of $\Lambda \rightarrow \infty$, supplemented by the limit of zero vector-meson mass, the correction in Eq. (B1) becomes identical to $\Delta\Gamma^{\gamma\mu} = -\gamma_T^\mu$. Therefore, in this limit, the contribution of diagrams involving vector meson exchanges becomes proportional to that of constituent quark loops. Since there are three diagrams for one vector meson exchange, three for two vector meson exchanges, and only one for three vector meson exchanges, the corresponding coefficients of proportionality are $-3, 3$, and -1 . The sum of all vector meson exchanges should equal the contribution of the constituent quark loop, with a minus sign. The behavior of the vector meson contributions is shown in Fig. 13 as a function of its mass.

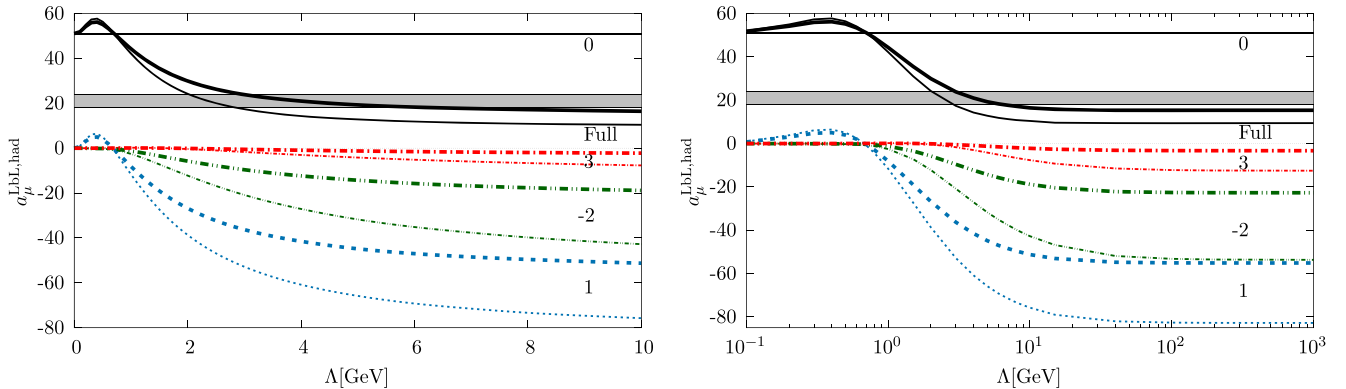


FIG. 12. (in units 10^{-11}) The LbL contribution to the muon AMM in the local limit for $m_Q = 300$ MeV: linear scale (left) and logarithmic scale (right). The thin horizontal line marked as 0 is the quark loop contribution without vector mesons. The contributions with vector mesons are: blue dashed line with 1 exchange, green dash-double-dotted line with 2 exchanges with minus sign, and red dash-dotted is with 3 exchanges. The black solid line is the sum of all contributions: pure quark loop “0” plus vector meson exchanges “1 + 2 + 3”. The thick lines correspond to the ENJL transition and the thin ones to the VMD one. The shaded area represents the result of the ENJL model for the quark loop contribution [20].

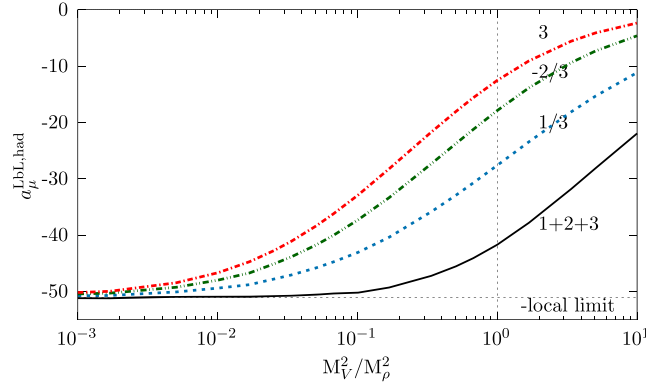


FIG. 13. (in units 10^{-11}) The LbL contribution from diagrams with additional vector meson vertices to the muon AMM in the local limit for $m_Q = 300$ MeV with VMD transition, as a function of the ratio of M_V^2 to the physical M_ρ^2 . The thin horizontal dashed line marked as the local limit is minus the quark loop contribution without vector mesons. Thin vertical dashed line corresponds to physical point $M_V^2/M_\rho^2 = 1$. The contributions with vector mesons are: blue dashed line with 1 exchange (divided by factor 3), green double dash-dotted line with 2 exchanges with minus sign (divided by factor 3) and the red dash-dotted line is with 3 exchanges. The black solid line is the sum of all vector-meson contributions.

- [1] X. Fan, T. G. Myers, B. A. D. Sukra, and G. Gabrielse, *Phys. Rev. Lett.* **130**, 071801 (2023).
- [2] D. Hanneke, S. Fogwell, and G. Gabrielse, *Phys. Rev. Lett.* **100**, 120801 (2008).
- [3] D. P. Aguillard *et al.* (Muon $g - 2$ Collaboration), *Phys. Rev. Lett.* **135**, 101802 (2025).
- [4] P. J. Mohr, D. B. Newell, B. N. Taylor, and E. Tiesinga, *Rev. Mod. Phys.* **97**, 025002 (2025).
- [5] R. H. Parker, C. Yu, W. Zhong, B. Estey, and H. Müller, *Science* **360**, 191 (2018).
- [6] L. Morel, Z. Yao, P. Cladé, and S. Guellati-Khélifa, *Nature (London)* **588**, 61 (2020).
- [7] T. Aoyama *et al.*, *Phys. Rep.* **887**, 1 (2020).
- [8] G. Colangelo *et al.*, *arXiv:2203.15810*.
- [9] R. Aliberti *et al.*, *Phys. Rep.* **1143**, 1 (2025).
- [10] S. Borsanyi *et al.*, *Nature (London)* **593**, 51 (2021).
- [11] A. Boccaletti *et al.* *arXiv:2407.10913*.
- [12] C. Aubin, T. Blum, M. Golterman, and S. Peris, in *39th International Symposium on Lattice Field Theory* (2022), *arXiv:2211.12057*.
- [13] T. Blum *et al.* (RBC and UKQCD Collaborations), *Phys. Rev. D* **108**, 054507 (2023).
- [14] G. Wang, T. Draper, K.-F. Liu, and Y.-B. Yang (chiQCD Collaboration), *Phys. Rev. D* **107**, 034513 (2022).
- [15] M. Cè *et al.*, *Phys. Rev. D* **106**, 114502 (2022).
- [16] C. Lehner and A. S. Meyer, *Phys. Rev. D* **101**, 074515 (2020).
- [17] C. Alexandrou *et al.* (Extended Twisted Mass Collaboration), *Phys. Rev. D* **107**, 074506 (2022).
- [18] A. Bazavov *et al.* (Fermilab Lattice, HPQCD, and MILC Collaborations), *Phys. Rev. D* **107**, 114514 (2023).
- [19] F. V. Ignatov *et al.* (CMD-3 Collaboration), *Phys. Rev. D* **109**, 112002 (2024).
- [20] J. Bijnens, E. Pallante, and J. Prades, *Nucl. Phys.* **B474**, 379 (1996).
- [21] J. Bijnens, E. Pallante, and J. Prades, *Phys. Rev. Lett.* **75**, 1447 (1995).
- [22] M. Hayakawa, T. Kinoshita, and A. I. Sanda, *Phys. Rev. D* **54**, 3137 (1996).
- [23] M. Knecht and A. Nyffeler, *Phys. Rev. D* **65**, 073034 (2002).
- [24] J. Prades, E. de Rafael, and A. Vainshtein, *Adv. Ser. Dir. High Energy Phys.* **20**, 303 (2009).
- [25] F. Jegerlehner and A. Nyffeler, *Phys. Rep.* **477**, 1 (2009).
- [26] T. Blum, N. Christ, M. Hayakawa, T. Izubuchi, L. Jin, C. Jung, and C. Lehner, *Phys. Rev. Lett.* **124**, 132002 (2020).
- [27] E.-H. Chao, R. J. Hudspith, A. Gérardin, J. R. Green, H. B. Meyer, and K. Ottnad, *Eur. Phys. J. C* **81**, 651 (2021).
- [28] E.-H. Chao, R. J. Hudspith, A. Gérardin, J. R. Green, and H. B. Meyer, *Eur. Phys. J. C* **82**, 664 (2022).
- [29] T. Blum, N. Christ, M. Hayakawa, T. Izubuchi, L. Jin, C. Jung, C. Lehner, and C. Tu (RBC and UKQCD Collaborations), *Phys. Rev. D* **111**, 014501 (2023).
- [30] Z. Fodor, A. Gerardin, L. Lellouch, K. K. Szabo, B. C. Toth, and C. Zimmermann, *Phys. Rev. D* **111**, 114509 (2025).
- [31] G. Colangelo, M. Hoferichter, M. Procura, and P. Stoffer, *J. High Energy Phys.* **09** (2015) 074.
- [32] G. Colangelo, M. Hoferichter, M. Procura, and P. Stoffer, *J. High Energy Phys.* **04** (2017) 161.
- [33] M. Hoferichter, B.-L. Hoid, B. Kubis, S. Leupold, and S. P. Schneider, *J. High Energy Phys.* **10** (2018) 141.
- [34] I. Danilkin, M. Hoferichter, and P. Stoffer, *Phys. Lett. B* **820**, 136502 (2021).
- [35] D. Stamen, D. Hariharan, M. Hoferichter, B. Kubis, and P. Stoffer, *Eur. Phys. J. C* **82**, 432 (2022).

- [36] M. Hoferichter, P. Stoffer, and M. Zillinger, *J. High Energy Phys.* **04** (2024) 092.
- [37] J. Lütke, M. Procura, and P. Stoffer, *J. High Energy Phys.* **04** (2025) 130.
- [38] O. Deineka, I. Danilkin, and M. Vanderhaeghen, *Phys. Rev. D* **111**, 034009 (2025).
- [39] M. Hoferichter, P. Stoffer, and M. Zillinger, *J. High Energy Phys.* **02** (2025) 121.
- [40] S. Holz, M. Hoferichter, B.-L. Hoid, and B. Kubis, *J. High Energy Phys.* **04** (2025) 147.
- [41] M. Hoferichter, P. Stoffer, and M. Zillinger, *Phys. Rev. Lett.* **134**, 061902 (2025).
- [42] J. Leutgeb and A. Rebhan, *Phys. Rev. D* **101**, 114015 (2020).
- [43] L. Cappiello, O. Catà, G. D'Ambrosio, D. Greynat, and A. Iyer, *Phys. Rev. D* **102**, 016009 (2020).
- [44] J. Leutgeb, J. Mager, and A. Rebhan, *Phys. Rev. D* **107**, 054021 (2022).
- [45] J. Leutgeb, J. Mager, and A. Rebhan, *Phys. Rev. D* **111**, 114001 (2025).
- [46] L. Cappiello, J. Leutgeb, J. Mager, and A. Rebhan, *J. High Energy Phys.* **07** (2025) 033.
- [47] T. Goecke, C. S. Fischer, and R. Williams, *Phys. Rev. D* **87**, 034013 (2013).
- [48] G. Eichmann, C. S. Fischer, E. Weil, and R. Williams, *Phys. Lett. B* **797**, 134855 (2019); **799**, 135029(E) (2019).
- [49] A. Miramontes, A. Bashir, K. Raya, and P. Roig, *Phys. Rev. D* **105**, 074013 (2022).
- [50] G. Eichmann, C. S. Fischer, T. Haeuser, and O. Regenfelder, *Eur. Phys. J. C* **85**, 445 (2025).
- [51] A. E. Dorokhov, A. E. Radzhabov, and A. S. Zhevlakov, *Eur. Phys. J. C* **71**, 1702 (2011).
- [52] A. E. Dorokhov, A. E. Radzhabov, and A. S. Zhevlakov, *Eur. Phys. J. C* **72**, 2227 (2012).
- [53] A. E. Dorokhov, A. E. Radzhabov, and A. S. Zhevlakov, *Eur. Phys. J. C* **75**, 417 (2015).
- [54] F. Jegerlehner, *The Anomalous Magnetic Moment of the Muon* (Springer, Cham, 2017), Vol. 274, pp. 1–693, 10.1007/978-3-319-63577-4.
- [55] P. Masjuan and P. Sanchez-Puertas, *Phys. Rev. D* **95**, 054026 (2017).
- [56] A. Guevara, P. Roig, and J. J. Sanz-Cillero, *J. High Energy Phys.* **06** (2018) 160.
- [57] A. E. Radzhabov, A. S. Zhevlakov, A. P. Martynenko, and F. A. Martynenko, *Phys. Rev. D* **108**, 014033 (2023).
- [58] I. Danilkin, C. F. Redmer, and M. Vanderhaeghen, *Prog. Part. Nucl. Phys.* **107**, 20 (2019).
- [59] M. Hoferichter, B. Kubis, and M. Zanke, *J. High Energy Phys.* **08** (2023) 209.
- [60] E. J. Estrada, S. González-Solís, A. Guevara, and P. Roig, *J. High Energy Phys.* **12** (2024) 203.
- [61] E. J. Estrada and P. Roig, *arXiv:2504.00448*.
- [62] P. Masjuan, P. Roig, and P. Sanchez-Puertas, *J. Phys. G* **49**, 015002 (2022).
- [63] J. Bijnens, N. Hermansson-Truedsson, L. Laub, and A. Rodríguez-Sánchez, *J. High Energy Phys.* **10** (2020) 203.
- [64] J. Bijnens, N. Hermansson-Truedsson, L. Laub, and A. Rodríguez-Sánchez, *J. High Energy Phys.* **04** (2021) 240.
- [65] J. Bijnens, N. Hermansson-Truedsson, and A. Rodríguez-Sánchez, *J. High Energy Phys.* **03** (2025) 094.
- [66] G. Colangelo, M. Hoferichter, A. Nyffeler, M. Passera, and P. Stoffer, *Phys. Lett. B* **735**, 90 (2014).
- [67] A. E. Dorokhov and L. Tomio, *Phys. Rev. D* **62**, 014016 (2000).
- [68] M. K. Volkov, *Sov. J. Part. Nucl.* **17**, 186 (1986).
- [69] U. G. Meissner, *Phys. Rep.* **161**, 213 (1988).
- [70] B. S. DeWitt, *Phys. Rev.* **125**, 2189 (1962).
- [71] S. Mandelstam, *Ann. Phys. (N.Y.)* **19**, 1 (1962).
- [72] J. Terning, *Phys. Rev. D* **44**, 887 (1991).
- [73] J. S. Ball and T.-W. Chiu, *Phys. Rev. D* **22**, 2542 (1980).
- [74] R. S. Plant and M. C. Birse, *Nucl. Phys. A* **628**, 607 (1998).
- [75] A. E. Dorokhov and W. Broniowski, *Eur. Phys. J. C* **32**, 79 (2003).
- [76] S. J. Brodsky and J. D. Sullivan, *Phys. Rev.* **156**, 1644 (1967).
- [77] S. Nedelko, A. Nikolskii, and V. Voronin, *J. Phys. G* **49**, 035003 (2022).
- [78] A. Radzhabov, D. Blaschke, M. Buballa, and M. Volkov, *Phys. Rev. D* **83**, 116004 (2011).
- [79] M. Hayakawa, *arXiv:2501.16547*.
- [80] H. J. Behrend *et al.* (CELLO Collaboration), *Z. Phys. C* **49**, 401 (1991).
- [81] J. Gronberg *et al.* (CLEO Collaboration), *Phys. Rev. D* **57**, 33 (1998).
- [82] B. Aubert *et al.* (The BABAR Collaboration), *Phys. Rev. D* **80**, 052002 (2009).
- [83] S. Uehara *et al.* (Belle Collaboration), *Phys. Rev. D* **86**, 092007 (2012).
- [84] N. Asmussen, E.-H. Chao, A. Gérardin, J. R. Green, R. J. Hudspith, H. B. Meyer, and A. Nyffeler, *J. High Energy Phys.* **04** (2023) 040.
- [85] V. Bernard, A. H. Blin, B. Hiller, Y. P. Ivanov, A. A. Osipov, and U.-G. Meissner, *Ann. Phys. (N.Y.)* **249**, 499 (1996).
- [86] A. Grozin, *Comput. Phys. Commun.* **283**, 108590 (2023).
- [87] J. Bijnens and J. Prades, *Z. Phys. C* **64**, 475 (1994).
- [88] P. Maris and P. C. Tandy, *Phys. Rev. C* **61**, 045202 (2000).