Erratum: Charged scalar field at future null infinity via nonlinear hyperboloidal evolution [Phys. Rev. D 112, 104053 (2025)]

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(Received 22 January 2026; published 24 February 2026)

DOI: 10.1103/y5w8-yvcl

This erratum to the paper takes into account the comments in [1]. The conclusions of the paper change only slightly, correcting the comparison to [2]. The corrections are numbered. Citation numbers are according to the original article, for ease of comparison.

I. ABSTRACT

I. Instead of "in a charged black hole background (…) of the scalar field prescribed in the literature," it should read "on a charged black hole have been considered in the literature, and here, we use the hyperboloidal formalism to present an independent comparison to prior nonlinear results. For the quasinormal modes, we show a fit of the spherical fundamental mode for the purely uncharged case and compare it to the charged one. We obtain good agreement with prior prescriptions for the oscillation frequency between real and imaginary parts of the scalar field, as well as for the exponent of the power-law decay."

II. INTRODUCTION

II. Equation (1) should read

$$\phi \propto t^{-(2l+3)}.$$

Right after this equation, instead of "Shortly afterwards, (…) dominated by slower decaying primary waves.," it should be read "Shortly afterwards, Ref. [8] analyzed the late-time power-laws of a neutral scalar field on an electrically charged star collapsing into a RN black hole. The same power-law as (1) was predicted, but with the signal dominated by slower decaying primary waves."

III. Equation (2) should read

$$\phi \propto e^{i\frac{qQ}{r_+}t} u^{-2\beta-2},$$

where $t = (u+v)/2$ with v the advanced time.

IV. After "The same goes for the tail decay.," we add "Note that (2) does not reduce to (1) for $qQ \to 0$, because this limit does not commute with $t \to \infty$, as understood in [9]. Finally, the numerical nonlinear results presented in [11] confirm predictions from the earlier papers."

III. TAIL REGIME

V. In Eq. (32) of the original article, where $p_{i^+} = \mathrm{Re}(-2\beta) + 2$ appears, it should read $p_{i^+} = \mathrm{Re}(-2\beta) - 2$.

VI. Instead of "To get precise values of p, we did a linear (…) decay that is partially affected by the gravitational curvature and by the electromagnetic coupling.," we explain in more detail, and it should be read "For $qQ \gtrsim 0.1$ performing the regression between $t = 200$ and $t = 500$ is valid, as the asymptotic regime is already reached at those times. However, in the limit $qQ \ll 1$, the electromagnetic contribution dominates asymptotically for $t \gg \frac{M}{qQ}$ [9,10], so that simulations need to be evolved for longer to reach the desired regime. In Fig. 7, we show the results of several simulations, where we have fixed $M = q = 1$, and we change the value of Q. This makes the overall factor qQ change exactly in the same way as Q. The asymptotic exponents (i.e., respecting the condition $t \gg \frac{M}{qQ}$) extracted from our simulation are plotted in triangles. Solid lines represent the analytical predictions as shown in Ref. [11], while dashed lines correspond to the predictions shown in Ref. [12]. We also show the values of p for the case when $Q = 0.001$ estimated between $t = 200$ and $t = 500$, represented

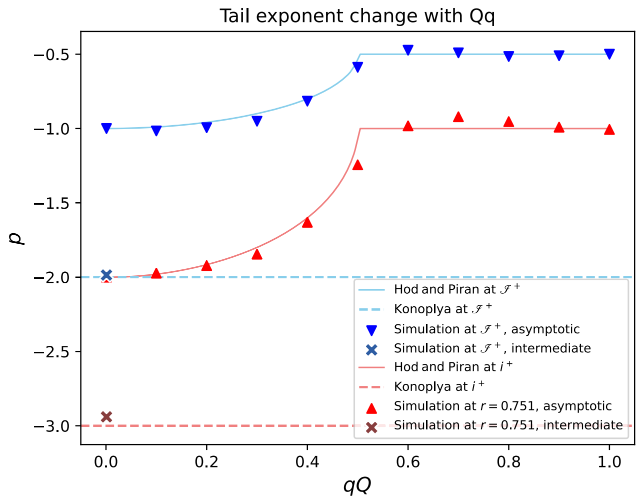

FIG. 7. Comparison of the results from our simulation (the triangles and crosses) with the analytical results coming from Konoplya and Zhidenko [12] and from Hod and Piran [9]. Blue lines and symbols correspond to values at $\mathcal{I}^+$. As mentioned in the main body of the text, the red lines correspond to the expected decay of the scalar field at i^+. The red triangles, however, correspond to the decay of the scalar field at a given radius $r \approx 0.75$ and at finite time, which is not i^+. Nonetheless, as the simulation time increases, the decay we expect to see at $r < 1$ should approach asymptotically the one at i^+.

through two crosses in the same color scheme. For this small value of qQ, these times correspond to the intermediate regime instead of the asymptotic one to obtain p for the latter we ran the simulation until $t = 6000$. It is interesting to note that initially, when the charged scalar field in the presence of a charged black hole enters the tail regime, it initially behaves as the neutral scalar field. This happens because the amplitude of the charged scalar field is mostly dominated initially by the real or complex part of the field, depending whether we perturb the former or the latter in the initial data. Only after some time, when both parts become comparable, does the asymptotic exponent corresponding to the charged scalar field appear. The main conclusion from this comparison is that our simulations coincide well with previous analytical results in the literature. Furthermore, in Fig. 7, we show not only the values of p at $\mathcal{I}^+$ (in blue), but also the values along a timelike hypersurface corresponding to the compactified value of $r = 0.751$ (in red). The main idea behind comparing the values of p along these hypersurfaces to the values of p as expected at i_+ is that as time progresses onwards, the observer moves ever closer to the position of i^+. It will never reach it, but it asymptotes to it. This is exactly what we see in the red quantities in Fig. 7."

VII. Figure 7 has a minor change.

VIII. After "We also wanted to see the effect of qQ in the oscillation frequency of the real and imaginary parts of the scalar field at $\mathcal{I}^+$.," we add "The oscillatory part of (4) is

$$\phi_{\text{oscil}} = e^{i\varphi_{\text{oscil}}} = e^{i\frac{qQ}{r_+}t} u^{i[qQ - \text{Im}(\beta)]} = e^{i\frac{qQ}{2r_+}u} e^{i[qQ-\text{Im}(\beta)]\log(u)} = e^{i\{\frac{qQ}{2r_+}u + [qQ - \text{Im}(\beta)]\log(u)\}},$$

so that its u-dependent angular frequency is

$$\omega = \frac{d\varphi_{\text{oscil}}}{du} = \frac{qQ}{2r_+} + [(qQ - \text{Im}(\beta)]\frac{1}{u}.$$

"

IX. Instead of "When plotting our results against the prediction (…) we see a clear divergence especially for $qQ > 1/2$.," it should be read "This analysis is shown by the red triangles in Fig. 8. The predicted frequency (33) (divided by 2π for ease of comparison) is evaluated at $u = 500$, i.e., the same value of the retarded time as that at which we are extracting the frequency. This is because our hyperboloidal time t at $\mathcal{I}^+$ and u are comparable in the asymptotic regime. As u increases, the shape of (33) becomes ever more dominated by the factor $qQ/(2r_+)$. We can therefore see that there is a good match between our results and the prediction by (33), except for large values of qQ close to extremality, where the divergence becomes quite stark and the data points from our simulation deviate. Understanding why this happens is left for future work."

X. The correct comparison changes Fig. 8 to.

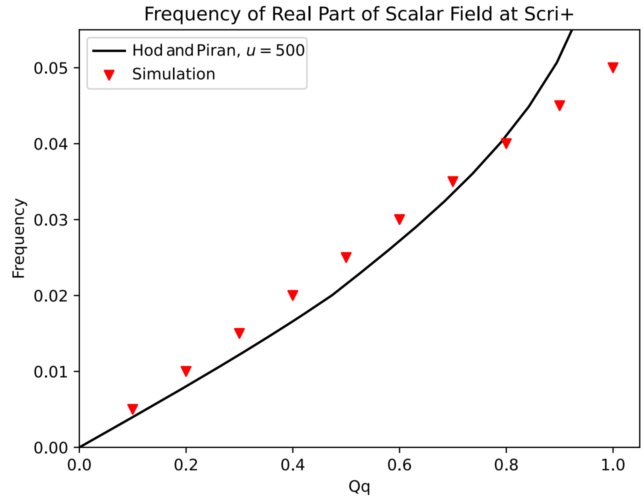

FIG. 8. Frequency comparison of the results from our simulation (the triangles) with the analytical results coming from Hod and Piran [9], (4), where the black line corresponds to the frequency given by (33) divided by 2π and evaluated at $u = 500$ (visually indistinguishable from the same expression evaluated at $u = 2000$).

IV. CONCLUSION

XI. Instead of “Regarding the tail regime, (…) occurs around $Qq \sim 10^{-3}$, means that, contrary (…),” it should be read: “our results agree very well with Ref. [11].”

XII. Instead of “We also analysed the frequency (…) the analytical predictions in Ref. [11].,” it should be read “We also analysed the frequency of oscillation of the real and imaginary parts during the tail regime and found good agreement with the analytical predictions in Ref. [11], although differences appear towards the extremal case.”

ACKNOWLEDGMENTS

We are indebted to Shahar Hod for pointing out several inaccuracies in an earlier version of our paper.

[1] S. Hod, arXiv:2511.21406.
[2] S. Hod and T. Piran, Phys. Rev. D **58**, 024019 (1998).