

Thermal corrections to Regge trajectories

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In this work, we investigate the behavior of Regge trajectories in the context of quantum field theory at finite temperature. For this purpose, we employ the $\lambda\phi^3$ model. We first compute the Regge trajectories at zero temperature, establishing a baseline for comparison. As a key novelty, we extend our analysis to finite temperature and derive an analytical expression for the Regge trajectories in this regime. We explore how temperature modifies the structure of the Regge trajectories and analyze the physical implications of these modifications, which arise from the resummation of thermal ladder diagrams. As an interesting consequence, we find the thermal evolution of masses belonging to different Regge trajectories.

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I. INTRODUCTION

The study of Regge trajectories, initially developed in the context of hadron scattering in the 1960s [1], continues to provide profound insight into the analytic structure of scattering amplitudes and the behavior of quantum field theories (QFTs) at high energies. These trajectories link the spin of particles to their mass squared in a nearly linear relationship, encapsulating fundamental dynamics of hadronic resonances and offering a geometrical interpretation of scattering processes in strong interactions. Traditionally, Regge theory has been applied successfully in the context of zero temperature QFT [2–21]. However, finite temperature systems are of great interest in the context of QFT. For example, different aspects concerning the behavior of hadronic parameters and the properties of the QCD phase diagram in the presence of thermal effects have been considered during the last years [22–37].

This paper explores the influence of thermal effects on Regge trajectories within the framework of a particular model in QFT.

This article is organized as follows. In Sec. II, we will present the topic of Regge trajectories. Then, in Sec. III, we will explore the Regge trajectories in the presence of temperature, and finally, in Sec. IV, we will present our conclusions.

II. REGGE TRAJECTORIES: GENERAL CONSIDERATIONS

In this section, we aim to get the power-law behavior of scattering amplitudes, characteristic of Regge theory, by considering a high-energy scattering process in the s channel, when the t channel has a fixed momentum. In particular, we are interested in the resummation of ladder diagrams, as in Fig. 1.

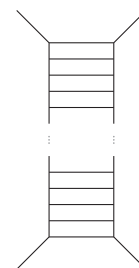


FIG. 1. Scattering process where “ladder” contributions are considered.

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Since we are interested in obtaining a thermal correction to the Regge trajectories, we begin our analysis by studying the zero temperature case, in the context of usual scalar field theory. In this sense, it is necessary to analyze the box diagram shown in Fig. 2, which has the following expression:

$$i\mathcal{M} = \lambda^4 \int \frac{d^4k}{(2\pi)^4} D(k) D(k + p_1) D(k + p_1 + p_3) \times D(k - p_2), \quad (1)$$

with $D(k)$ being the usual scalar propagator in Minkowski space, given by

$$D(k) = \frac{i}{k^2 - m^2 + i\epsilon}, \quad (2)$$

and where p_1 and p_2 are the incoming momenta and p_3 and p_4 are the outgoing momenta. In order to compute Eq. (1), we introduce four Feynman parameters, which allows us to write.

$$i\mathcal{M} = \lambda^4 \int \frac{d^4k}{(2\pi)^4} \left(\frac{i}{k^2 - m^2} \right) \left(\frac{i}{(k + p_1)^2 - m^2} \right) \left(\frac{i}{(k + p_1 + p_3)^2 - m^2} \right) \left(\frac{i}{(k - p_2)^2 - m^2} \right) \\ = \lambda^4 \int \frac{d^4k}{(2\pi)^4} \int \frac{3! \delta(x + y + z + w - 1) dx dy dz dw}{[x(k^2 - m^2) + y((k + p_1)^2 - m^2) + z((k + p_1 + p_3)^2 - m^2) + w((k - p_2)^2 - m^2)]^4}. \quad (3)$$

To continue the computation of this integral, we can manipulate the denominator to isolate the dependence k by the usual completion of squares. After the corresponding algebra, we can perform the change of variables $k \rightarrow q - yp_1 - z(p_1 - p_3) + wp_2$, to get

$$i\mathcal{M} = 3! \lambda^4 \int_0^1 dx dy dz dw \delta(x + y + z + w - 1) \cdot \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - \Delta)^4}, \quad (4)$$

where we have defined

$$\Delta = yws - zt(1 - z) + 2z(p_1 - p_3)(yp_1 - wp_2) - m^2[y + w - 1 - (y - w)^2], \quad (5)$$

with $s = (p_1 + p_2)^2$ and $t = (p_1 - p_3)^2$ being the usual Mandelstam variables. The next step is to integrate over q , where we obtain

$$\mathcal{M} = \frac{\lambda^4}{96\pi^2} \int \frac{\delta(x + y + z + w - 1) dx dy dz dw}{\Delta^2}. \quad (6)$$

In order to compute the remaining integral, we use Collins' technique [38], which exploits the fact that we are working in the regime when $s \rightarrow \infty$, which allows us to identify that the most relevant contribution in the denominator is given by the first term, while we can safely neglect

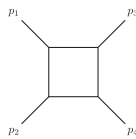


FIG. 2. Box diagram with circulating momentum k .

the other terms containing y and w . Therefore, and focusing only on the integrals over y and w ,

$$\int_0^1 \frac{1}{[yws - zt(1 - z) + m^2]^2} dy dw \\ = \int_0^1 \frac{1}{[m^2 - zt(1 - z)][sy - zt(1 - z) + m^2]} dy \\ = \frac{1}{s[m^2 - zt(1 - z)]} \ln \left(\frac{s - zt(1 - z) + m^2}{m^2 - zt(1 - z)} \right) \\ \approx \frac{1}{m^2 - zt(1 - z)} \cdot \frac{\ln s}{s} \quad (7)$$

and, therefore, the remaining integral looks like

$$\mathcal{M} = \frac{\lambda^4}{96\pi^2} \left(\frac{\ln s}{s} \right) \int_0^1 \int_0^1 \frac{\delta(x + z - 1)}{m^2 - zt(1 - z)} dx dz. \quad (8)$$

The logarithm term clearly emerges from the asymptotic region associated with the leading term when $s \rightarrow \infty$. Finally, by computing the integrals over x and y , we arrive at

$$\mathcal{M} = \lambda^2 K(t) \left(\frac{\ln s}{s} \right), \quad (9)$$

where we have defined

$$K(t) = \frac{\lambda^2}{24\pi^2 \sqrt{t(4m^2 - t)}} \arctan \left(\sqrt{\frac{t}{4m^2 - t}} \right). \quad (10)$$

Having determined the analytical behavior of the box diagram, we now proceed to compute the Regge trajectory. To obtain the trajectory, we must proceed with the resummation of diagrams by stacking box diagrams to form a “staircase diagram,” as shown in Fig. 1. Taking into

account each loop, this ladder diagram with n rungs in the $s \rightarrow \infty$ with t fixed can be expressed as

$$\mathcal{M}_n = \frac{\lambda^2}{s} \frac{[K(t) \ln s]^{n-1}}{(n-1)!}. \quad (11)$$

Thus, summing an infinite series of ladder diagrams yields the asymptotic behavior of the resulting amplitude as

$$\begin{aligned} A(s, t) &= \sum_{n=1}^{\infty} \mathcal{M}_n \\ &= \frac{\lambda^2}{s} \sum_{n=1}^{\infty} \frac{[K(t) \ln s]^{n-1}}{(n-1)!} \\ &= \frac{\lambda^2}{s} \cdot e^{K(t) \ln s} \\ &= \lambda^2 s^{\alpha(t)}, \end{aligned} \quad (12)$$

in which we can identify the Regge trajectory as

$$\alpha(t) = K(t) - 1. \quad (13)$$

Notice that we need $t < 4m^2$, i.e., t less than the s threshold for pair production, to have a real value for $K(t)$.

III. FINITE TEMPERATURE REGGE TRAJECTORIES

In order to calculate the finite temperature case, we start from Eq. (4), and incorporate temperature effects using the imaginary time formalism in the standard way [39], i.e.,

$$\int \frac{d^4 q}{(2\pi)^4} f(q) \rightarrow \frac{i}{\beta} \sum_n \int \frac{d^3 q}{(2\pi)^3} f(\omega_n, \mathbf{q}), \quad (14)$$

where we have introduced the bosonic Matsubara frequencies $\omega_n = 2\pi n/\beta$ with $\beta = 1/T$. Therefore, the expression for the box diagram at finite temperature is

$$\begin{aligned} \mathcal{M}_{0,T} &= 3! \lambda^4 \int_0^1 dx dy dz dw \delta(x+y+z+w-1) \\ &\quad \times \frac{1}{\beta} \sum_n \int \frac{d^3 q}{(2\pi)^3} \frac{1}{(\omega_n^2 + \mathbf{q}^2 + \Delta)^4}, \end{aligned} \quad (15)$$

where the subscripts 0 and T correspond to vacuum and finite temperature, respectively. Using the following identity:

$$\frac{1}{(\omega_n^2 + \mathbf{q}^2 + \Delta)^4} = -\frac{1}{3!} \left(\frac{\partial}{\partial \Delta} \right)^3 \frac{1}{(\omega_n^2 + \mathbf{q}^2 + \Delta)}, \quad (16)$$

we obtain

$$\begin{aligned} \mathcal{M}_{0,T} &= -\lambda^4 \int_0^1 dx dy dz dw \delta(x+y+z+w-1) \\ &\quad \times \left(\frac{\partial}{\partial \Delta} \right)^3 \frac{1}{\beta} \sum_n \int \frac{d^3 q}{(2\pi)^3} \frac{1}{(\omega_n^2 + \mathbf{q}^2 + \Delta)}, \end{aligned} \quad (17)$$

and performing the summation over Matsubara frequencies, we get

$$\begin{aligned} \mathcal{M}_{0,T} &= -\lambda^4 \int_0^1 dx dy dz dw \delta(x+y+z+w-1) \\ &\quad \times \left(\frac{\partial}{\partial \Delta} \right)^3 \int \frac{d^3 q}{(2\pi)^3} \left(\frac{1}{2(\mathbf{q}^2 + \Delta)^{1/2}} \right. \\ &\quad \left. + \frac{1}{(\mathbf{q}^2 + \Delta)^{1/2}} \frac{1}{e^{\beta\sqrt{\mathbf{q}^2 + \Delta}} - 1} \right) \\ &\equiv \mathcal{M}_0 + \mathcal{M}_T, \end{aligned} \quad (18)$$

where

$$\begin{aligned} \mathcal{M}_0 &= -\lambda^4 \int_0^1 dx dy dz dw \delta(x+y+z+w-1) \\ &\quad \times \left(\frac{\partial}{\partial \Delta} \right)^3 \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2(\mathbf{q}^2 + \Delta)^{1/2}}, \end{aligned} \quad (19)$$

and

$$\begin{aligned} \mathcal{M}_T &= -\lambda^4 \int_0^1 dx dy dz dw \delta(x+y+z+w-1) \\ &\quad \times \left(\frac{\partial}{\partial \Delta} \right)^3 \int \frac{d^3 q}{(2\pi)^3} \frac{1}{(\mathbf{q}^2 + \Delta)^{1/2}} \frac{1}{e^{\beta\sqrt{\mathbf{q}^2 + \Delta}} - 1}. \end{aligned} \quad (20)$$

We note that $\mathcal{M}_0$ is temperature independent and corresponds to the term calculated in Eq. (9). The temperature dependence is encoded in the term $\mathcal{M}_T$, with its calculation being not trivial. In order to handle this expression correctly, recalling that we are in the context of Regge trajectories, we take the limit $s \rightarrow \infty$ and, therefore, $\Delta \rightarrow \infty$. Using this limit and differentiating Eq. (20) with respect to Δ , we obtain

$$\begin{aligned} \mathcal{M}_T &= -\lambda^4 \int_0^1 dx dy dz dw \delta(x+y+z+w-1) \\ &\quad \times \left(-\int \frac{d^3 q}{(2\pi)^3} \left[\frac{\beta^3}{8(\mathbf{q}^2 + \Delta)^2} + \frac{3\beta^2}{4(\mathbf{q}^2 + \Delta)^{5/2}} + \frac{15\beta}{8(\mathbf{q}^2 + \Delta)^3} + \frac{15}{8(\mathbf{q}^2 + \Delta)^{7/2}} \right] e^{-\beta\sqrt{\mathbf{q}^2 + \Delta}} \right). \end{aligned} \quad (21)$$

Since we are interested in the low-temperature regime, we will focus on the first term of Eq. (21). Therefore, integrating over the variable q , we get

$$\mathcal{M}_T = \lambda^4 \int_0^1 dx dy dz dw \delta(x + y + z + w - 1) \times \frac{\beta^3}{64\pi\sqrt{\Delta}} e^{-\beta\sqrt{\Delta}}. \quad (22)$$

To perform the calculation in Feynman parameters, we will use the Collins method employed in the previous section. That is, we need to integrate the following expression:

$$\mathcal{M}_T = \lambda^4 \int dx dy dz dw \delta(x + y + z + w - 1) \times \frac{\beta^3}{64\pi\sqrt{ysw + tz(1-z) + m^2}} e^{-\beta\sqrt{ysw + tz(1-z) + m^2}}. \quad (23)$$

Integrating over the variables y , w , and x , we obtain

$$\mathcal{M}_T = -\lambda^4 \left(\frac{\ln s}{s} \right) \int_0^1 dz \frac{\beta^2 e^{-\beta\sqrt{tz(1-z) + m^2}}}{32\pi}. \quad (24)$$

Let us note that this last integral over dz cannot be performed analytically. Observe that the preceding equation has a form analogous to that found in Eq. (8). Therefore, we can express it as follows:

$$\mathcal{M}_T = \lambda^2 B(t, \beta) \left(\frac{\ln s}{s} \right), \quad (25)$$

where

$$B(t, \beta) = -\lambda^2 \int_0^1 dz \frac{\beta^2 e^{-\beta\sqrt{tz(1-z) + m^2}}}{32\pi}. \quad (26)$$

Finally, adding the zero temperature result, we obtain

$$\mathcal{M}_{0,T} = \lambda^2 (K(t) + B(t, \beta)) \left(\frac{\ln s}{s} \right). \quad (27)$$

Performing the analogous calculation to Sec. II, taking into account each loop, we obtain in this case

$$\mathcal{M}_{0,T,n} = \frac{\lambda^2 [(K(t) + B(t, \beta)) \ln s]^{n-1}}{s (n-1)!}. \quad (28)$$

Therefore, summing an infinite series of ladder diagrams, we get

$$\begin{aligned} A(s, t, \beta) &= \sum_{n=1}^{\infty} \mathcal{M}_{0,T,n} \\ &= \frac{\lambda^2}{s} \sum_{n=1}^{\infty} \frac{[(K(t) + B(t, \beta)) \ln s]^{n-1}}{(n-1)!} \\ &= \frac{\lambda^2}{s} \cdot e^{(K(t) + B(t, \beta)) \ln s} \\ &= \lambda^2 s^{\alpha(t, \beta)}. \end{aligned} \quad (29)$$

This result allows us to identify the Regge trajectory with a thermal contribution of the form

$$\alpha(t, \beta) = K(t) - 1 + B(t, \beta). \quad (30)$$

Regge trajectories can be expressed as a relation between the spin of hadronic states and their squared mass, of the form

$$\alpha(t) = \alpha_0 + \alpha' t, \quad (31)$$

where α_0 is the intercept with the vertical axis, α' is the Regge slope, and t is the Mandelstam variable, which is associated with the squared mass of the excited states that appear as resonances, expressed in units of GeV^2 . In order to obtain our analytic expressions in the form of Eq. (31), we perform a small t expansion. Let us note that with this expansion we will be able to obtain an analytical expression for Eq. (26) since we will be able to perform the integration over dz , yielding

$$\begin{aligned} \alpha(t, \beta) &= \frac{\lambda^2}{96\pi^2 m^2} - \lambda^2 \beta^2 \frac{e^{-m\beta}}{32\pi} \\ &+ \lambda^2 t \left(\frac{1}{576\pi^2 m^4} - \beta^3 \frac{e^{-m\beta}}{384\pi m} \right). \end{aligned} \quad (32)$$

We note that we have obtained an analytical expression for the Regge trajectories at finite temperature. In the literature [38,40,41], several trajectories are known, but only at zero temperature. These are shown in Table I.

In order to analyze how the Regge trajectories change with temperature, we first determine the fitted values of the parameters λ and m at $T = 0$ such that the trajectory reproduces the correct one, as shown in Table II.

TABLE I. Intercepts α_0 and slopes α' of the linear Regge trajectories $\alpha(t) \approx \alpha_0 + \alpha' t$ for different mesons at $T = 0$.

Meson	α_0	α' [GeV^{-2}]
ρ	0.500 ± 0.002	0.900 ± 0.004
K^*	0.300 ± 0.010	0.900 ± 0.010
ϕ	0.100 ± 0.009	0.900 ± 0.009
π	0.000 ± 0.001	0.800 ± 0.003

TABLE II. Values of the coupling constant λ and the mass parameter m extracted from the Regge parameters (α_0, α') at $T = 0$ for different mesons.

Meson	λ [GeV]	m [GeV]
ρ	19.869 ± 0.030	0.5270 ± 0.0009
K^*	17.219 ± 0.140	0.4906 ± 0.0042
ϕ	14.570 ± 0.451	0.1630 ± 0.0056
π	14.049 ± 0.140	0.4564 ± 0.0042

Using these fitted parameters, we then compute the temperature dependence of the trajectories. The results are displayed in Figs. 3–6 for the ρ , π , ϕ , and K^* mesons, respectively. In each case, the solid line corresponds to the $T = 0$ trajectory, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV. We observe in all cases a clear decrease in the Regge slope α' as the temperature increases.

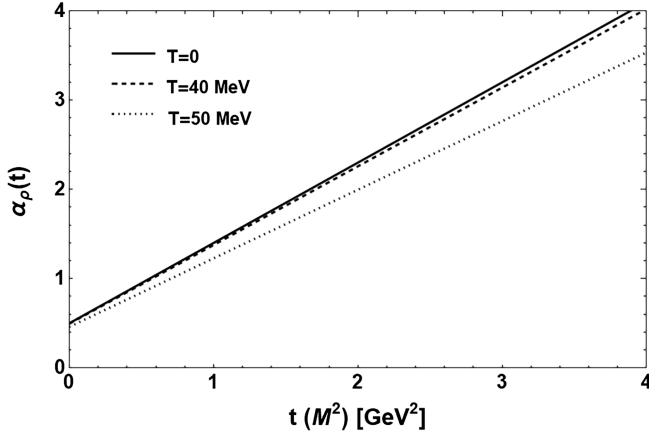


FIG. 3. $\alpha_\rho(t)$ vs t . The solid line corresponds to the Regge trajectory at $T = 0$, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV.

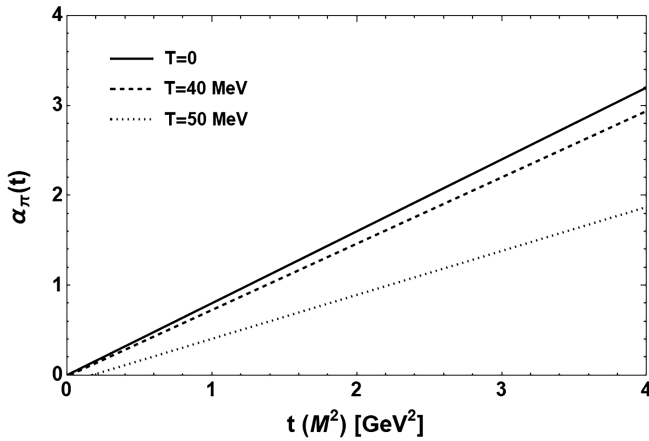


FIG. 4. $\alpha_\pi(t)$ vs t . The solid line corresponds to the Regge trajectory at $T = 0$, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV.

Another interesting piece of information that can be extracted from the plots is the particle mass. For example, if we consider the trajectory of the ρ meson, we can obtain the value of its mass at different temperatures, as shown in Fig. 7. From Fig. 7, we can extract the mass of the ρ meson in the zero temperature case as approximately 0.744 GeV, while at $T = 40$ MeV the ρ meson mass is 0.756 GeV and at $T = 50$ MeV the ρ meson mass is 0.839 GeV. We note that for small temperature values, there is no significant variation in the mass of the ρ meson. However, as the temperature increases, the mass clearly begins to rise, in agreement with the theoretical results of Ref. [42], where an analysis of the temperature evolution of the rho-meson mass was implemented. This was done in the context of unitarity restrictions, together with the virial expansion, applied to π - π scattering lengths in the chiral expansion. It was shown in this work that the growing behavior of the width $\Gamma(T)$, which turns out to be the crucial order

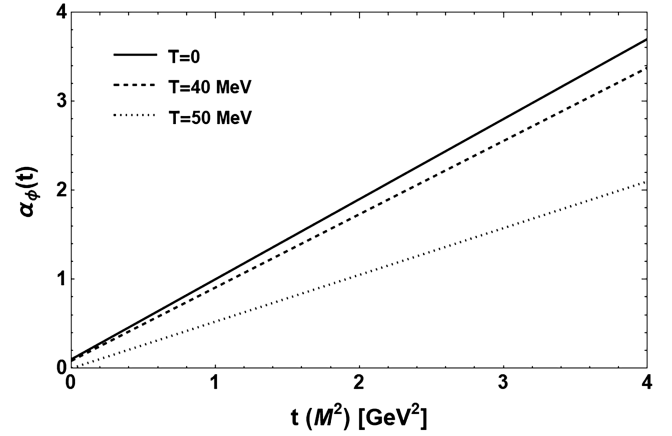


FIG. 5. $\alpha_\phi(t)$ vs t . The solid line corresponds to the Regge trajectory at $T = 0$, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV.

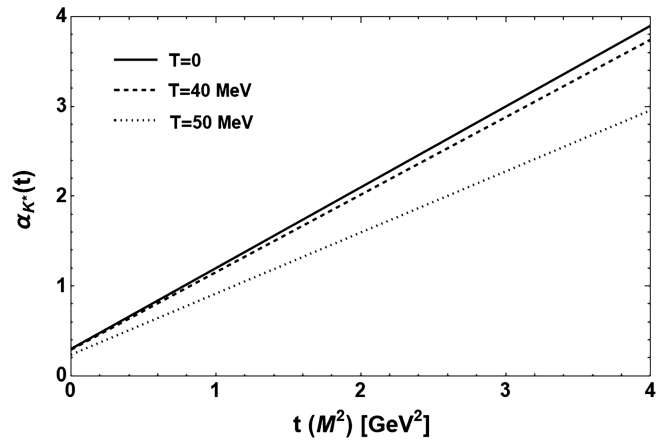


FIG. 6. $\alpha_{K^*}(t)$ vs t . The solid line corresponds to the Regge trajectory at $T = 0$, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV.

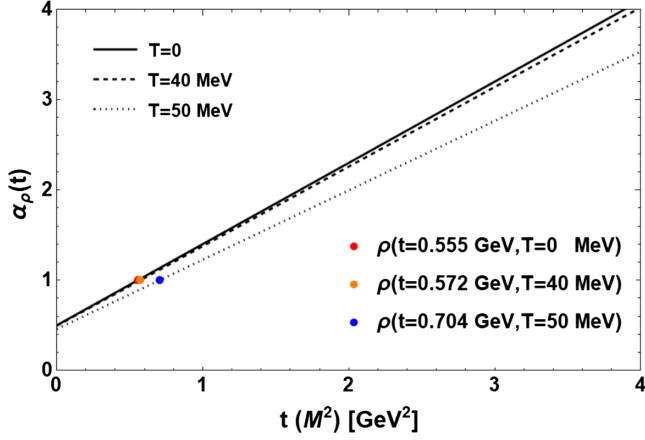


FIG. 7. $\alpha_\rho(t)$ vs t . The solid line corresponds to the Regge trajectory at $T = 0$, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV. The red point indicates the squared mass of the ρ meson in the vacuum case, the orange point corresponds to the ρ squared mass at $T = 40$ MeV, and the blue point corresponds to the ρ squared mass at $T = 50$ MeV.

parameter for the deconfinement phase transition, occurs together with a growing behavior of the mass $m_\rho(T)$. Here, we find a similar behavior for the rho mass evolution, in spite of the fact that our model is extremely simple. This is basically related to the first observation: the slope of the Regge trajectories decreases as a function of temperature. Moreover, we can carry out the same procedure for the ρ meson using several values of T , in order to obtain multiple Regge slopes and extract the mass of the ρ as a function of temperature, resulting in Fig. 8. We also performed the analysis for the pion mass, as shown in Fig. 9. The results indicate that $M_\pi(T)$ increases monotonically with temperature, starting from a value close to zero at $T = 0$ and then increasing with temperature. This behavior is consistent with previous studies based on lattice QCD [43] and effective models [44], where the pion mass also exhibits an increase with temperature.

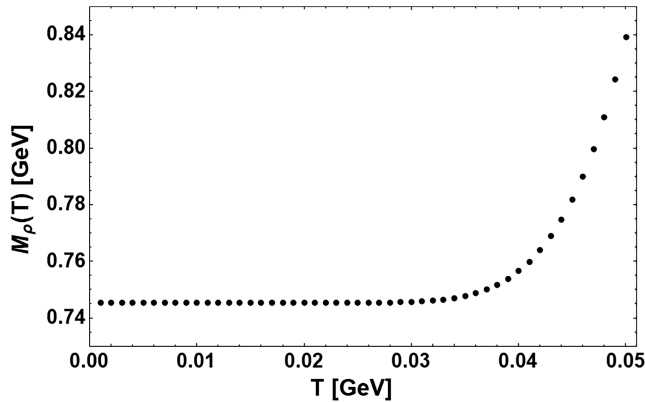


FIG. 8. $M_\rho(T)$ vs T .

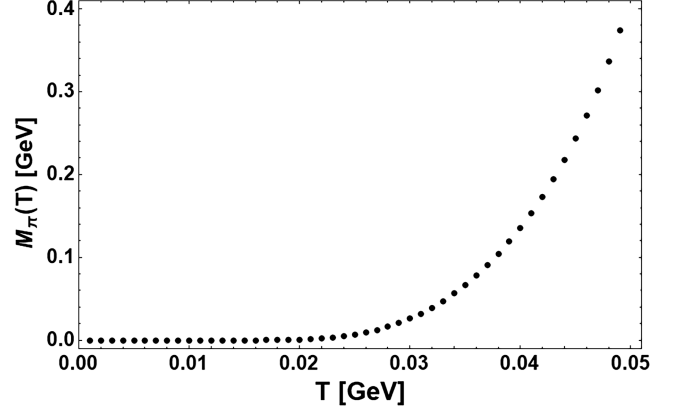


FIG. 9. $M_\pi(T)$ vs T .

Finally, it should be emphasized that our model, $\lambda\phi^3$, requires only two counterterms. In fact, in four space-time dimensions it is a super-renormalizable theory, since the coupling has a positive mass dimension. The first counterterm is related to the self-energy correction, and the second one to the tadpole diagram, as analyzed in Ref. [45]. Once these counterterms are fixed at a given scale, all other possible radiative corrections turn out to be finite. In our case, adopting a pragmatic point of view, we have fixed the two free parameters in our model, the coupling constant λ and the mass m , from the parameters associated with the Regge trajectories (intercept and slope). Once finite temperature effects are introduced, it is not necessary to include new temperature-dependent counterterms, since all thermal effects are finite. In principle, it would be possible to introduce thermal counterterms fixed at a certain temperature, but this is not required. We have followed this approach in our calculations, with renormalization performed at zero temperature.

IV. CONCLUSIONS

We have analytically derived the thermal corrections to Regge trajectories within a $\lambda\phi^3$ quantum field theory framework by resumming ladder diagrams using the imaginary time formalism. Our results show that finite temperature induces a reduction in the Regge slope, which leads to observable effects, such as the increase of meson masses with temperature. In particular, we analyze the behavior of the ρ meson and find that its mass exhibits a mild increase at low temperatures, followed by a more rapid growth as the temperature increases. This increase in mass aligns with theoretical predictions in thermal QFT [42,46] and supports the idea that the excitation spectrum of hadrons is modified in the presence of a thermal bath. A similar analysis was carried out for the pion, whose mass also increases with temperature, in agreement with previous studies based on lattice QCD [43] and effective models [44].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [38].

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