

Lepton-flavor violating decays induced by Lorentz violation in the Yukawa sector of the standard model extension

J. Montaña-Domínguez¹, F. Ramírez-Zavaleta², E. S. Tututi² and O. Vázquez-Hernández¹

¹*SECIHTI, Avenida Insurgentes Sur 1582, Colonia Crédito Constructor, Alcaldía Benito Juárez, 03940, CDMX, México*

²*Facultad de Ciencias Físico Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Avenida Francisco J. Múgica s/n, C. P. 58060, Morelia, Michoacán, México*



(Received 17 March 2026; accepted 8 June 2026; published 8 July 2026)

Tree-level lepton-flavor violating decays induced by Lorentz-violating effects within the Yukawa sector of the standard model extension are studied. These new-physics effects are parametrized by the $(Y_f)^{AB}_{\mu\nu}$ tensor, with μ and ν denoting Lorentz indices and A, B being indices in the flavor space. Since this tensor is antisymmetric under the interchange of Lorentz indices, in analogy with the electromagnetic tensor $F_{\mu\nu}$, $(Y_f)^{AB}_{\mu\nu}$ can be expressed in terms of six components associated with two complex three-vectors denoted by $\mathbf{e}_l^{AB}$ and $\mathbf{b}_l^{AB}$. On the assumption that these three-vectors are purely real or purely imaginary and mutually orthogonal, constraints on their magnitudes via experimental bounds on lepton-flavor violating processes $\text{Br}(l_B \rightarrow \gamma l_A)$ and $\text{Br}(l_B \rightarrow l_A l_C \bar{l}_C)$ are estimated. Thus, the $l_B \rightarrow \gamma l_A$ decay provides the following upper bounds: $|\mathbf{e}_l^{\mu\tau}| < 1.51 \times 10^{-11}$, $|\mathbf{e}_l^{e\tau}| < 1.34 \times 10^{-11}$, $|\mathbf{e}_l^{e\mu}| < 3.65 \times 10^{-18}$, $|\mathbf{b}_l^{\mu\tau}| < 1.95 \times 10^{-11}$, $|\mathbf{b}_l^{e\tau}| < 1.73 \times 10^{-11}$, $|\mathbf{b}_l^{e\mu}| < 4.71 \times 10^{-18}$. Conversely, by assuming that the Lorentz-violating coefficients are purely real, for the $l_B \rightarrow l_A l_C \bar{l}_C$ process it is found that $|\mathbf{e}_l^{\mu\tau}| < 2.87 \times 10^{-12}$ and $|\mathbf{b}_l^{\mu\tau}| < 4.05 \times 10^{-12}$. These results offer more restrictive bounds than those previously reported in the literature.

DOI: 10.1103/PhysRevD.114.015014

I. INTRODUCTION

The standard model (SM) of particle physics provides the most accurate description of the strong and electroweak interactions [1–8], where its Lagrangian is built upon the principles of quantum mechanics, special relativity and gauge invariance. Nevertheless, several investigations suggest that, at fundamental scales, such as the Planck scale, Lorentz symmetry could be explicitly or spontaneously broken [9–21]. The technological limitations in reaching such energy scales motivate the search for signals of Lorentz symmetry breaking in high-sensitivity experiments at low energies, such as the SM scale [22–27]. In this regard, low-energy effects of Lorentz violation (LV) can be parametrized in a model-independent framework known as the standard model extension (SME) [28,29]. The SME is an effective field theory that encompasses the SM along with general relativity and is structured by means of the sum of all independent

effective operators of the form

$$\mathcal{T}^{\mu\nu\lambda\dots}\mathcal{O}_{\mu\nu\lambda\dots}, \quad (1)$$

where $\mathcal{T}^{\mu\nu\lambda\dots}$ denotes constant Lorentz tensors and $\mathcal{O}_{\mu\nu\lambda\dots}$ represents *CPT*-even or *CPT*-odd gauge invariant operators built on SM fields [28,29]. The SME Lagrangian is manifestly invariant under observer Lorentz transformations, also known as passive transformations. Nevertheless, under particle Lorentz transformations, the components of the $\mathcal{T}^{\mu\nu\lambda\dots}$ tensor remain invariants, while the $\mathcal{O}_{\mu\nu\lambda\dots}$ ones transform covariantly, breaking Lorentz invariance. In its minimal version, the SME contains only renormalizable terms, i.e., those with mass dimension less than or equal to 4. However, at higher energies, nonrenormalizable terms must be considered in order to preserve causality and stability of the theory [30,31]. The Lorentz-violating tensors $\mathcal{T}^{\mu\nu\lambda\dots}$ can be interpreted as static background relic fields; in this sense, they do not represent new degrees of freedom, but they act on the SM fields as couplings that could lead to preferential spacetime directions. Accordingly, since Lorentz invariance is broken under particle transformations, their effects can be studied by varying the spatial orientation of the experimental setup. In this way, various experiments have been developed to search for LV sources, or for setting

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TABLE I. Experimental bounds on lepton-flavor violation for the two- and three-body decay processes $l_B \rightarrow l_A \gamma$ and $l_B \rightarrow l_A \bar{l}_C l_C$ or $l_B \rightarrow \bar{l}_A l_C l_C$, respectively.

Two-body	Three-body
$\text{BR}(\tau^- \rightarrow \mu^- \gamma) < 4.2 \times 10^{-8}$	$\text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 1.9 \times 10^{-8}$
$\text{BR}(\tau^- \rightarrow e^- \gamma) < 3.3 \times 10^{-8}$	$\text{BR}(\tau^- \rightarrow e^- e^+ e^-) < 2.5 \times 10^{-8}$
$\text{BR}(\mu^- \rightarrow e^- \gamma) < 3.1 \times 10^{-13}$	$\text{BR}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}$
	$\text{BR}(\tau^- \rightarrow e^- \mu^+ \mu^-) < 2.4 \times 10^{-8}$
	$\text{BR}(\tau^- \rightarrow \mu^- e^+ e^-) < 1.6 \times 10^{-8}$
	$\text{BR}(\tau^- \rightarrow \mu^+ e^- e^-) < 1.6 \times 10^{-8}$
	$\text{BR}(\tau^- \rightarrow e^+ \mu^- \mu^-) < 1.3 \times 10^{-8}$

bounds on the components of the $\mathcal{T}^{\mu\nu\lambda\dots}$ tensor [22–24,32,32–42]. A wide variety of studies have assessed the impact of LV, for instance, on QED interactions up to the one-loop level [31,43–45]. Moreover, studies on effects of LV in electroweak interactions [46–53] or in the extended QCD sector [54–58] have been addressed. Special attention has been given to the impact of LV on physical observables such as anomalous dipole moments [26,27,53,59–61], neutrino charge radius [62], LV running couplings [51,54,63], decay rates, and several scattering processes [43,46–50,52,55–58,64–66]. Also, a detailed summary of constraints on SME coefficients, which are periodically updated, can be consulted in Ref. [67].

One of the most interesting features provided by the SME is that it can generate charged flavor-violating transitions [28]. On this matter, the neutrino oscillations [68] reveals the presence of lepton-flavor violation (LFV), which is of fundamental relevance in particle physics. In contrast, the SM has been unable to explain this phenomenon since it was constructed under the assumption of massless neutrinos, in consequence, LFV interactions are forbidden at tree level. Nonetheless, experimental observations have established upper bounds on branching ratios (BRs) of various lepton-flavor violating processes such as $l_B \rightarrow l_A \gamma$, $l_B \rightarrow l_A \bar{l}_C l_C$ or $l_B \rightarrow \bar{l}_A l_C l_C$, with $l_{A,B,C} = e, \mu, \tau$; their numerical values are given in Table I [69,70]. Thus, the topic of flavor violation promoted by results on charged leptons is interesting in order to search for signals of new-physics effects as it occurs in the SME.

Several extensions of the SM contain interactions that can induce LFV processes [71–82]. For example, the minimally extended SM [77], in which three right-handed neutrinos are incorporated, predicts a value for $\text{BR}(\mu \rightarrow e \gamma)$ of the order of 10^{-54} [83], and also provides BRs for the three-body decays given in Table I of the order of $\sim 10^{-55} - 10^{-56}$. This implies that these processes are extremely suppressed and till now far from being detectable. The effective field theory addresses LFV via dimension-five and dimension-six operators, promisingly, at new-physics energy scales (Λ) of orders of magnitude of TeV up to 10^4 TeV [74,80].

On the other hand, the responsible part of the SME for LFV couplings is the minimal extended Yukawa (MEY) sector, where LV effects are parametrized by the complex two-tensor $(Y_f)_{\mu\nu}^{AB}$, with μ, ν denoting spacetime components and A, B labeling flavor indices; more details on these coefficients will be presented in the next section. To date, some bounds on the maxima of the values of such coefficients were derived in Refs. [46,53]. In the former [46], tree-level LV effects on the Higgs boson decay $h \rightarrow f^- f^+$, for $m_h = 100$ GeV, were studied, yielding, for any of the diagonal terms ($A = B$), $10^{-20} < |Y_f| < 10^{-17}$, being $|Y_f|^2 \equiv (Y_f)_{\alpha\beta}^{AA} (Y_f)^{\alpha\beta AA}$. In the latter, one-loop contributions of the MEY sector to the lepton magnetic and electric-dipole moments a_l and d_l , respectively, with $l = e, \mu, \tau$, were determined. In that work, only contributions that are independent of the four-momentum of the particles, i.e., those proportional to scalar-quadratic terms such as $(Y_l)_{\alpha\beta}^{AB} (Y_l)^{\alpha\beta AB}$, were considered. For the off-diagonal terms ($A \neq B$), the authors established bounds for the case $A = \mu, B = \tau$, where the most stringent bound corresponds to $|(V_l)_{\alpha\beta}^{\mu\tau} (V_l)^{\mu\tau\alpha\beta}| < 10^{-15}$, with $(V_l)_{\alpha\beta}^{AB} = 1/2[(Y_l)_{\alpha\beta}^{AB} + (Y_l)_{\alpha\beta}^{BA*}]$; meanwhile for the diagonal case $A = B = e$, their most restrictive bound is $\frac{1}{2}|\epsilon^{\alpha\beta\lambda\rho} (V_l)_{\alpha\beta}^{ee} (V_l)_{\lambda\rho}^{ee}| < 10^{-28}$.

In light of previous considerations, LFV decays offer an interesting scenario to study LV effects, since their stringent experimental bounds could lead to strong constraints on LV coefficients. In the present work, we calculate the impact of LV effects through LFV tree-level decays $l_B \rightarrow l_A \gamma$ and $l_B \rightarrow l_A \bar{l}_C l_C$ (with $l_{A,B,C} = e, \mu, \tau$), which are induced by the MEY sector. At tree level, the corresponding amplitudes are linear in the LV coefficients. On the contrary, processes such as $l_B \rightarrow \bar{l}_A l_C l_C$ are not considered, as they are induced through second-order terms of LV coefficients in the amplitudes, which results in negligible contributions.

Our manuscript is organized as follows: Sec. II presents the MEY Lagrangian. Section III is devoted to the calculation of the two- and three-body decay rates and the estimation for bounds on LV coefficients. Since the amplitudes involve several products of various Dirac matrices,

which could induce dipolar terms, we also calculate LV second-order corrections to the three-point vertex $l_A l_B \gamma$ function ($A \neq B$) in order to investigate if there exist tree-level contributions to transition magnetic or electric-dipole moments. Finally, in Sec. IV, the conclusions are presented.

II. THE YUKAWA EXTENDED SECTOR

The Yukawa sector of the SME is given by the Lagrangian [28]

$$\mathcal{L}_Y^{\text{mSME}} = \mathcal{L}_Y^{\text{SM}} - \frac{1}{2}(H_L)_{\mu\nu}^{AB} \bar{L}_A \Phi \sigma^{\mu\nu} R_B - \frac{1}{2}(H_U)_{\mu\nu}^{AB} \bar{Q}_A \Phi \sigma^{\mu\nu} U_B - \frac{1}{2}(H_D)_{\mu\nu}^{AB} \bar{Q}_A \tilde{\Phi} \sigma^{\mu\nu} D_B + \text{H.c.}, \quad (2)$$

where Q_A and L_A are the quark and lepton doublets of $SU_L(2)$, respectively, while R_B is a charged lepton singlet of $SU_L(2)$, U_B (D_B) is an up-type (down-type) quark singlet of $SU_L(2)$, and $\tilde{\Phi} = i\sigma^2 \Phi$, with Φ being the SM Higgs doublet. LV effects are induced by the $(H_F)_{\mu\nu}^{AB}$ components, with $F = L, U, D$; μ, ν represent spacetime components, whereas A, B symbolize either lepton or quark flavor indices. We use the metric $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and define $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. It should be noted that this two-rank tensor is antisymmetric under the interchange of Lorentz indices, i.e., $(H_F)_{\mu\nu}^{AB} = -(H_F)_{\nu\mu}^{AB}$. After spontaneous symmetry breaking in the unitary gauge, in the mass-eigenstates basis, Eq. (2) takes the form

$$\mathcal{L}_Y^{\text{mSME}} = - \sum_{f=l,u,d} \bar{f}_A \left\{ \left(m_{f_A} + g \frac{m_{f_A}}{2m_W} h \right) \delta^{AB} + \frac{1}{2}(v+h)[(Y_f)_{\mu\nu}^{AB} P_R + (Y_f)_{\mu\nu}^{BA*} P_L] \sigma^{\mu\nu} \right\} f_B. \quad (3)$$

Here, $(Y_f)_{\mu\nu}^{AB} = \sqrt{2} V_L^{f\dagger} (H_f)_{\mu\nu}^{AB} V_R^f$, being $V_{L,R}^f$ the components of a unitary matrix, which diagonalizes $\mathcal{L}_Y^{\text{mSME}}$ into the fermion-mass basis; f_A denotes the fermion field with mass m_{f_A} ; g is the $SU_L(2)$ gauge coupling, m_W the mass of the W boson, h is the physical Higgs boson, and $v = 246.22$ GeV is the vacuum expectation value. Notice that the antisymmetry property in the Lorentz indices for $(H_F)_{\mu\nu}^{AB}$ is preserved by $(Y_f)_{\mu\nu}^{AB}$. The $P_{L,R}$ correspond to the projectors $\frac{1}{2}(1 \mp \gamma_5)$. Since γ_5 will give rise to Levi-Civita tensors, when these are fully contracted with the Lorentz components of the form $(Y_f)_{\mu\nu}^{AB}$, then they will generate the dual tensor

$$(\tilde{Y}_f)_{\mu\nu}^{AB} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} (Y_f)^{AB\alpha\beta}. \quad (4)$$

On the other hand, in analogy with the relation between the electric and magnetic three-vectors, $\mathbf{E}$ and $\mathbf{B}$, respectively,

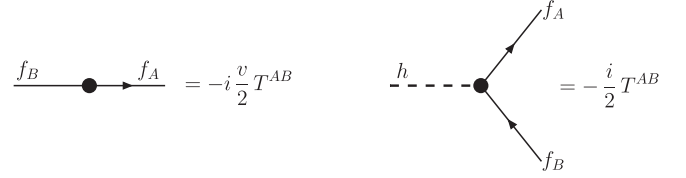


FIG. 1. Feynman rules derived from the MEY sector. Left: two-point $f_B f_A$ vertex function. Right: $h \bar{f}_A f_B$ three vertex.

derived from the electromagnetic tensor $F_{\mu\nu}$, two background three-vectors $\mathbf{e}_f^{AB}$ and $\mathbf{b}_f^{AB}$, can be introduced as [61]

$$(Y_f)_{0i}^{AB} = (\mathbf{e}_f^{AB})_i, \quad (Y_f)_{ij}^{AB} = +\epsilon^{ijk} (\mathbf{b}_f^{AB})^k, \quad (5)$$

$$(\tilde{Y}_f)_{0i}^{AB} = (\mathbf{b}_f^{AB})_i, \quad (\tilde{Y}_f)_{ij}^{AB} = -\epsilon^{ijk} (\mathbf{e}_f^{AB})^k. \quad (6)$$

For off-diagonal terms ($A \neq B$), the $\mathcal{L}_Y^{\text{mSME}}$ Lagrangian generates lepton and quark flavor-violating transitions through the two-point $f_A f_B$ vertex function and from the $h \bar{f}_A f_B$ coupling, at tree level. The respective Feynman rules are given in Fig. 1. The notation used in Fig. 1 is explicitly given as

$$T_f^{AB} = [(\mathcal{V}_f^{AB})_{\mu\nu} + (\mathcal{A}_f^{AB})_{\mu\nu} \gamma_5] \sigma^{\mu\nu}. \quad (7)$$

with

$$(\mathcal{V}_f^{AB})_{\mu\nu} \equiv \frac{1}{2} [(Y_f)_{\mu\nu}^{AB} + (Y_f)_{\mu\nu}^{BA*}], \quad (8)$$

$$(\mathcal{A}_f^{AB})_{\mu\nu} \equiv \frac{1}{2} [(Y_f)_{\mu\nu}^{AB} - (Y_f)_{\mu\nu}^{BA*}]. \quad (9)$$

However, under the assumption that LV coefficients are symmetric under the interchange of flavor indices, it is found that

$$(\mathcal{V}_f^{AB})_{\mu\nu} \equiv \text{Re}[(Y_f)_{\mu\nu}^{AB}], \quad (10)$$

$$(\mathcal{A}_f^{AB})_{\mu\nu} \equiv i \text{Im}[(Y_f)_{\mu\nu}^{AB}]. \quad (11)$$

Hereafter, we will work under this consideration. Besides, from Eqs. (5) and (6), it can be seen that

$$(\mathcal{V}_f^{AB})_{0i} = \text{Re}[\mathbf{e}_f^{AB}]_i, \quad (\mathcal{V}_f^{AB})_{ij} = \epsilon^{ijk} \text{Re}[\mathbf{b}_f^{AB}]^k, \quad (12)$$

$$(\mathcal{A}_f^{AB})_{0i} = i \text{Im}[\mathbf{e}_f^{AB}]_i, \quad (\mathcal{A}_f^{AB})_{ij} = i \epsilon^{ijk} \text{Im}[\mathbf{b}_f^{AB}]^k. \quad (13)$$

In the next section, for simplicity, we will only explore two scenarios: (a) purely real coefficients, when $(\mathcal{A}_f^{AB})_{\mu\nu} = 0$, and (b) purely imaginary coefficients, for $(\mathcal{V}_f^{AB})_{\mu\nu} = 0$.

III. THE CALCULATION

A. Two-body $f_B \rightarrow f_A \gamma$ decay

The first-order LV contributions of the MEY sector to the $f_B f_A \gamma$ vertex function are described by the Feynman diagrams in Fig. 2. The associated amplitude is given by

$$\Gamma_\mu^{O(1)f_B f_A \gamma} = ieQ_f \frac{v}{2} \bar{u}^{s'}(p', m_A) \left[T_f^{AB} \left(\frac{\not{p}' + m_B}{p'^2 - m_B^2} \right) \gamma_\mu + \gamma_\mu \left(\frac{\not{p} + m_A}{p^2 - m_A^2} \right) T_f^{AB} \right] u^s(p, m_B), \quad (14)$$

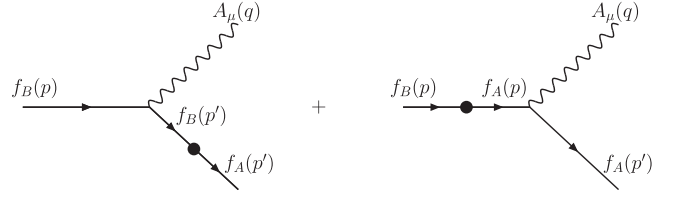


FIG. 2. First-order LV corrections to the $f_B f_A \gamma$ vertex.

where Q_f is the electric charge of the fermion in consideration. After some Dirac algebraic manipulations, this amplitude can be rewritten as follows:

$$\Gamma_\mu^{O(1)f_B f_A \gamma} = -\frac{2eQ_f v}{(m_B^2 - m_A^2)} \bar{u}^{s'}(p', m_A) \{ \gamma^\alpha [(m_A + m_B)(\mathcal{V}_f^{AB})_{\mu\alpha} - (m_B - m_A)(\mathcal{A}_f^{AB})_{\mu\alpha} \gamma_5] - P^\alpha [(\mathcal{V}_f^{AB})_{\mu\alpha} + (\mathcal{A}_f^{AB})_{\mu\alpha} \gamma_5] + i\sigma_\mu{}^\alpha q^\beta [(\mathcal{V}_f^{AB})_{\alpha\beta} + (\mathcal{A}_f^{AB})_{\alpha\beta} \gamma_5] \} u^s(p, m_B), \quad (15)$$

where $P = p + p'$ and $q = p - p'$. Since the amplitude satisfies the Ward identity $q^\mu \Gamma_\mu^{O(1)f_B f_A \gamma} = 0$, it is gauge invariant.

From Eq. (15), it can be observed that there are no contributions to vector or axial currents proportional to γ_μ or $\gamma_\mu \gamma_5$, respectively, nor contributions to the transition-magnetic or transition-electric dipole moments.

The invariant amplitude for the $f_B(p) \rightarrow f_A(p') \gamma(q)$ decay can be determined from Eq. (15) through

$$\mathcal{M}(f_B \rightarrow f_A \gamma) = \Gamma_\mu^{O(1)f_B f_A \gamma} \epsilon^{*\mu}(q). \quad (16)$$

Because we are not considering flavor conservation ($A = B$), it will not induce modifications to the energy-momentum dispersion relations of the fermionic fields. Consequently, there are no additional restrictions in the phase-space integration for the decay rates, and the average over initial spins by summing final spins remains unchanged; the photon polarization states also remain unmodified [43,46,47,49,50,52,55–58,64–66]. Accordingly, the mean square amplitude can be written as

$$|\bar{\mathcal{M}}(f_B \rightarrow f_A \gamma)|^2 = \frac{64\pi\alpha Q_f^2 v^2}{(m_B^2 - m_A^2)^2} \left\{ \left[\frac{(m_B^2 - m_A^2)^2}{4} (\mathcal{V}_f^{AB})^2 + \frac{m_B^2 + m_A^2}{2} (\mathcal{V}_f^{AB})_{\lambda q} (\mathcal{V}_f^{AB})^{\lambda q} - (m_B^2 - m_A^2) (\mathcal{V}_f^{AB})_{\lambda P} (\mathcal{V}_f^{AB})^{\lambda q} - (\mathcal{V} \rightarrow \mathcal{A}) \right] + i \left[(m_B - m_A)^2 (\mathcal{V}_f^{AB})_{\lambda q} (\tilde{\mathcal{A}}_f^{AB})^{\lambda q} - (m_B + m_A)^2 (\mathcal{A}_f^{AB})_{\lambda q} (\tilde{\mathcal{V}}_f^{AB})^{\lambda q} - (m_B^2 - m_A^2) ((\mathcal{V}_f^{AB})_{\lambda q} (\tilde{\mathcal{A}}_f^{AB})^{\lambda P} - (\mathcal{A}_f^{AB})_{\lambda q} (\tilde{\mathcal{V}}_f^{AB})^{\lambda P}) \right] + \frac{1}{2} \epsilon^{\rho\tau\alpha\beta} P_\alpha q_\beta ((m_B^2 - m_A^2) (\mathcal{A}_f^{AB})_{\lambda\rho} (\mathcal{V}_f^{AB})^\lambda{}_\tau + (\mathcal{A}_f^{AB})_{\rho q} (\mathcal{V}_f^{AB})_{\tau P} + (\mathcal{V}_f^{AB})_{\rho q} (\mathcal{A}_f^{AB})_{\tau P}) \right\}. \quad (17)$$

Here, $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$. In addition, we define $(\mathcal{K}_f^{AB})^2 = (\mathcal{K}_f^{AB})_{\lambda\rho} (\mathcal{K}_f^{AB})^{\lambda\rho}$, $(\mathcal{K}_f^{AB})_{\lambda r} = r^\rho (\mathcal{K}_f^{AB})_{\lambda\rho}$, and $(\tilde{\mathcal{K}}_f^{AB})_{\lambda r} = \frac{1}{2} r_\rho \epsilon_{\alpha\beta\lambda\rho} (\mathcal{K}_f^{AB})^{\alpha\beta}$, with $r = P, q$; $(\mathcal{K}_f^{AB})_{Pq} = P^\lambda q^\rho (\mathcal{K}_f^{AB})_{\lambda\rho}$, where $\mathcal{K} = \mathcal{A}, \mathcal{V}$. Notice that the tensor $(\mathcal{K}_f^{AB})_{\lambda r}$ and the scalar $(\mathcal{K}_f^{AB})_{Pq}$ have mass dimension 1 and 2, respectively. The contractions between LV tensors and four-momentum components, P and q , can be expressed in terms of dot and cross products between the three-vectors $\mathbf{P}, \mathbf{q}$ with the LV vectors $\mathbf{e}_f^{AB}$ and $\mathbf{b}_f^{AB}$; these expressions are detailed in Appendix A. Thereby, the decay rate of the $f_B \rightarrow f_A \gamma$ process can be written as

$$d\Gamma(f_B \rightarrow f_A \gamma) = \frac{1}{2m_B} \left(\frac{1}{2E_{\mathbf{p}'}} \frac{d^3 \mathbf{p}'}{(2\pi)^3} \right) \left(\frac{1}{2E_{\mathbf{q}}} \frac{d^3 \mathbf{q}}{(2\pi)^3} \right) (2\pi)^4 \delta^{(4)}(p - p' - q) |\bar{\mathcal{M}}(f_B \rightarrow f_A \gamma)|^2. \quad (18)$$

The integration over the volume element $d^3 \mathbf{p}'$ is carried out at the rest frame of the decaying particle, where $\mathbf{p}' = -\mathbf{q}$. Thus,

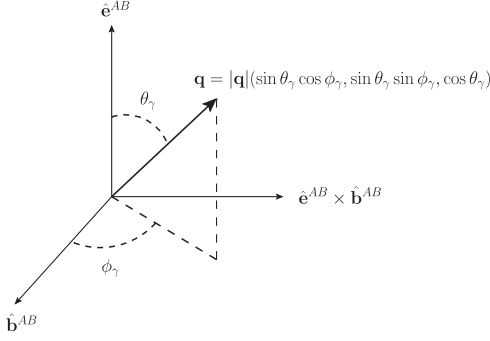


FIG. 3. Reference frame conformed by the unitary vectors along the real or imaginary parts of the $\mathbf{e}_f^{AB}$, $\mathbf{b}_f^{AB}$, and $\mathbf{e}_f^{AB} \times \mathbf{b}_f^{AB}$ vectors. The θ_γ and ϕ_γ correspond to the polar and azimuthal angles of the three-momentum of the final photon, respectively.

$$\Gamma(f_B \rightarrow f_A \gamma) = \frac{1}{32\pi^2 m_B} \int \frac{d^3 \mathbf{q}}{\bar{E}_{\mathbf{p}'} E_{\mathbf{q}}} \delta(m_B - \bar{E}_{\mathbf{p}'} - E_{\mathbf{q}}) \times \left| \bar{\mathcal{M}}(f_B \rightarrow f_A \gamma) \right|^2 \Big|_{\mathbf{p}' \rightarrow -\mathbf{q}}, \quad (19)$$

being $\bar{E}_{\mathbf{p}'} = \sqrt{m_A^2 + |\mathbf{q}|^2}$. Additionally, the $|\bar{\mathcal{M}}(f_B \rightarrow f_A \gamma)|^2$ quantity can be expressed in terms of masses m_A , m_B , and dot (cross) products between the three-vector $\mathbf{q}$ and the real or imaginary parts of $\mathbf{e}_f^{AB}$ and $\mathbf{b}_f^{AB}$. Notice that these vector products depend on the components of $\mathbf{q}$, which are integration variables. In this regard, the phase-space integration may become cumbersome due to the choice of the reference frame in which the volume element is integrated. For simplicity, we work the scenario where $\mathbf{e}_f^{AB}$ and $\mathbf{b}_f^{AB}$ are orthogonal in the rest frame of the decaying particle f_B . Moreover, since the phase-space volume is Lorentz invariant, we adopt the reference frame defined by the triad conformed by the unitary vectors along the $\mathbf{e}_f^{AB}$, $\mathbf{b}_f^{AB}$, and $\mathbf{e}_f^{AB} \times \mathbf{b}_f^{AB}$ vectors; this reference frame is illustrated in Fig. 3. On this basis, it follows that the decay rate is now

$$\Gamma(f_B \rightarrow f_A \gamma) = \frac{1}{64\pi^2} \frac{m_B^2 - m_A^2}{m_B^3} \int_0^{2\pi} d\phi_\gamma \times \int_0^\pi d\theta_\gamma \sin \theta_\gamma |\bar{\mathcal{M}}(f_B \rightarrow f_A \gamma)|^2. \quad (20)$$

Since the integration extends over the full angular distribution, the decay width $\Gamma(f_B \rightarrow f_A \gamma)$ will depend exclusively on the particle masses and the magnitudes of both real or imaginary parts of $\mathbf{e}_f^{AB}$ and $\mathbf{b}_f^{AB}$. The corresponding branching ratio is established as follows:

$$\text{BR}^{YSME}(f_B \rightarrow f_A \gamma) = \frac{\Gamma(f_B \rightarrow f_A \gamma)}{\Gamma_B}, \quad (21)$$

TABLE II. Numerical values of the $c_{R_{1,2}}^{AB}$ coefficients.

$\tau^- \rightarrow \mu^- \gamma$	$c_{R_1}^{\mu\tau} = 1.10 \times 10^{14}$	$c_{R_2}^{\mu\tau} = 1.83 \times 10^{14}$
$\tau^- \rightarrow e^- \gamma$	$c_{R_1}^{e\tau} = 1.10 \times 10^{14}$	$c_{R_2}^{e\tau} = 1.84 \times 10^{14}$
$\mu^- \rightarrow e^- \gamma$	$c_{R_1}^{e\mu} = 1.40 \times 10^{22}$	$c_{R_2}^{e\mu} = 2.33 \times 10^{22}$

where Γ_B is the total decay width of the f_B fermion. After several manipulations of tensor algebra, it is found that, for the cases of purely real coefficients ($\mathcal{A}_f^{AB} = 0$), or purely imaginary coefficients ($\mathcal{V}_f^{AB} = 0$), the respective BRs can be expressed as

$$\text{BR}_{\text{Re}}^{YSME}(f_B \rightarrow f_A \gamma) = c_{R_1}^{AB} |\text{Re}(\mathbf{b}_f^{AB})|^2 + c_{R_2}^{AB} |\text{Re}(\mathbf{e}_f^{AB})|^2, \quad (22)$$

$$\text{BR}_{\text{Im}}^{YSME}(f_B \rightarrow f_A \gamma) = c_{I_1}^{AB} |\text{Im}(\mathbf{b}_f^{AB})|^2 + c_{I_2}^{AB} |\text{Im}(\mathbf{e}_f^{AB})|^2, \quad (23)$$

where the coefficients $c_{I_{1,2}}^{AB}$, $c_{R_{1,2}}^{AB}$ are real and their explicit expressions are given by

$$c_{R_1}^{AB} = c_{I_1}^{AB} = \frac{\alpha v^2}{3\Gamma_B m_B} \left(3 - 2 \frac{m_A^2}{m_B^2} - \frac{m_A^4}{m_B^4} \right), \quad (24)$$

$$c_{R_2}^{AB} = c_{I_2}^{AB} = \frac{\alpha v^2}{3\Gamma_B m_B} \left(5 - 4 \frac{m_A^2}{m_B^2} - \frac{m_A^4}{m_B^4} \right), \quad (25)$$

and

$$|\text{Re}(\mathbf{c}_f^{AB})|^2 = \text{Re}(\mathbf{c}_f^{AB}) \cdot \text{Re}(\mathbf{c}_f^{AB}), \quad (26)$$

$$|\text{Im}(\mathbf{c}_f^{AB})|^2 = \text{Im}(\mathbf{c}_f^{AB}) \cdot \text{Im}(\mathbf{c}_f^{AB}), \quad (27)$$

with $\mathbf{c}_f^{AB} = \mathbf{e}_f^{AB}$, $\mathbf{b}_f^{AB}$, and the dot denoting the Euclidean scalar product. The numerical values for the $c_{R_{1,2}}^{AB}$ coefficients are given in Table II. The muon and τ decay widths are derived from their lifetimes; the corresponding masses and lifetimes were taken from Ref. [69].

In order to estimate LV effects, it is proposed that any new-physics effect must be smaller than current experimental bounds on LFV processes as seen through their BRs. Thus,

$$\text{BR}_{\text{Re,Im}}^{YSME}(f_B \rightarrow f_A \gamma) \leq \text{BR}^{\text{Exp}}(f_B \rightarrow f_A \gamma), \quad (28)$$

where $\text{BR}^{\text{Exp}}(f_B \rightarrow f_A \gamma)$ is expressly stated in the first column of Table I, accordingly for each decay mode. Thereafter, Eq. (28) restricts the maxima of the values allowed for $|\mathbf{b}_f^{AB}|$ and $|\mathbf{e}_f^{AB}|$. The pattern imposed by the method proposed for bounding the LV effects can be visualized in a two-dimensional plot, either in the $|\text{Re}(\mathbf{b}_f^{AB})|$ versus

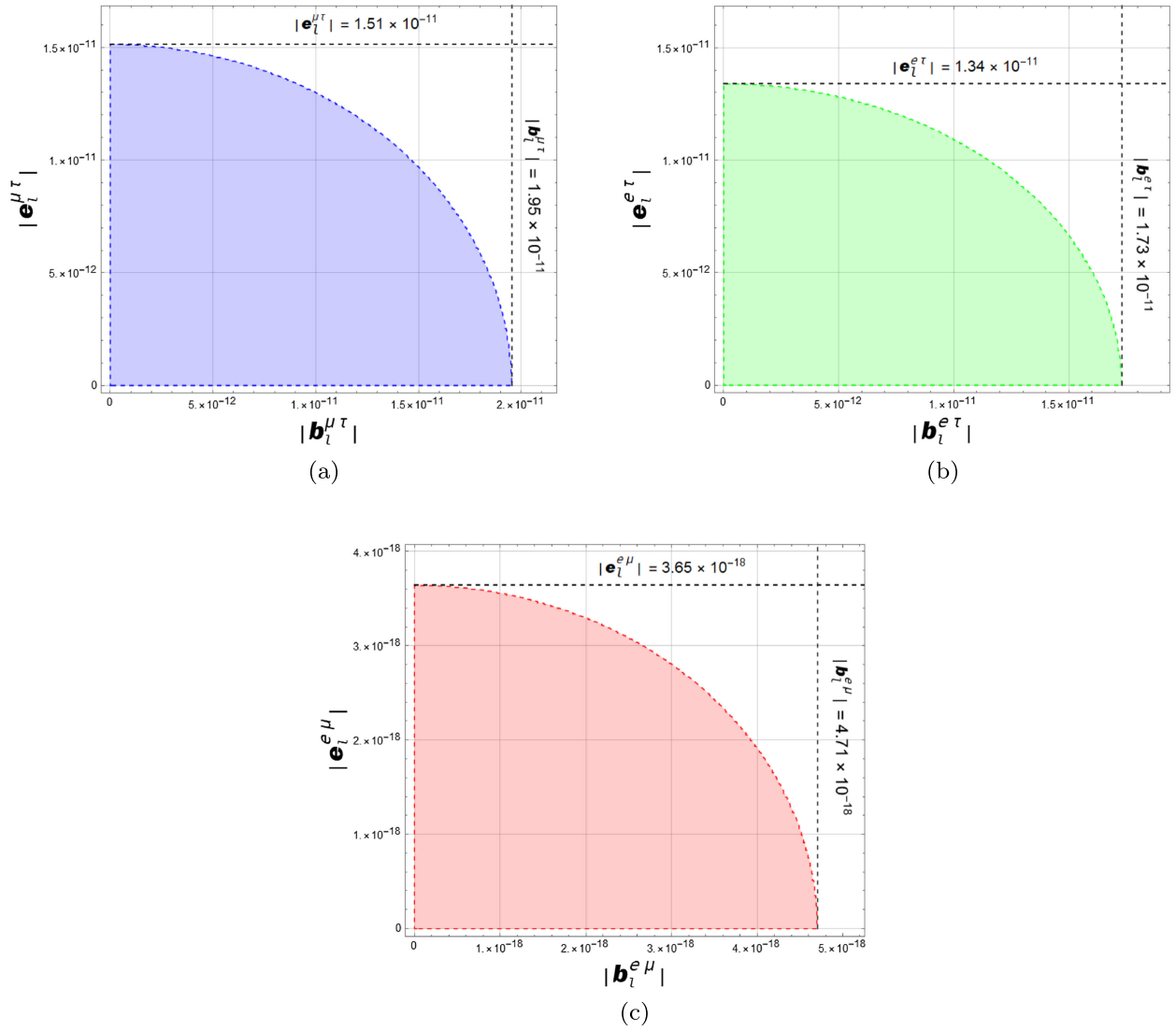


FIG. 4. Allowed regions for $|\mathbf{b}_l^{AB}|$, $|\mathbf{e}_l^{AB}|$ stem from current experimental bounds for: (a) $\tau \rightarrow \mu\gamma$, (b) $\tau \rightarrow e\gamma$, and (c) $\mu \rightarrow e\gamma$; the regions are identical for purely real and purely imaginary LV coefficients.

$|\text{Re}(\mathbf{e}_f^{AB})|$ plane or in the $|\text{Im}(\mathbf{b}_f^{AB})|$ versus $|\text{Im}(\mathbf{e}_f^{AB})|$ plane. However, since in Eqs. (22) and (23) the coefficients $c_{R,1,2}^{AB} = c_{I,1,2}^{AB}$ play the same role, both cases entail the same constraints for both purely real or purely imaginary LV coefficients via Eq. (28). For convenience, we express the bounds in terms of the magnitudes $|\mathbf{b}_f^{AB}| \equiv |\text{Re}(\mathbf{b}_f^{AB})|$ ($|\mathbf{b}_f^{AB}| \equiv |\text{Im}(\mathbf{b}_f^{AB})|$) and $|\mathbf{e}_f^{AB}| \equiv |\text{Re}(\mathbf{e}_f^{AB})|$ ($|\mathbf{e}_f^{AB}| \equiv |\text{Im}(\mathbf{e}_f^{AB})|$) in accordance with Eqs. (26) and (27), which correspond to LV coefficients in either the purely real or purely imaginary case, respectively. In this manner, the LV coefficients can be represented in a single plot $|\mathbf{b}_f^{AB}|$ versus $|\mathbf{e}_f^{AB}|$, as it is shown in Fig. 4. It should be noted that the resulting regions are delimited by ellipses in the $|\mathbf{b}_f^{AB}|$ versus $|\mathbf{e}_f^{AB}|$ plane.

In this context, the maxima of the permitted values for $|\mathbf{b}_l^{AB}|$, $|\mathbf{e}_l^{AB}|$ in the c.m. (center of mass) frame of the

decaying particle are presented in Table III. In the literature, it is reported a constraint for the LV coefficients $||\text{Re}(\mathbf{e}_l^{\mu\tau})|^2 - |\text{Re}(\mathbf{b}_l^{\mu\tau})|^2|$ [53], being explicitly $||\text{Re}(\mathbf{e}_l^{\mu\tau})|^2 - |\text{Re}(\mathbf{b}_l^{\mu\tau})|^2| \sim 10^{-15}$, whereas our estimation is of the order of 10^{-22} , which is 7 orders of magnitude more restrictive. Moreover, in contrast to Ref. [53], we explicitly consider Lorentz contractions with the momenta of the particles involved, which were neglected in that work, where the resulting contributions of this contractions stand out around 60% on all the partial decay widths.

Table III shows that the most restrictive bounds are for the LV coefficients involved in the $\mu \rightarrow e\gamma$ decay in comparison with those of the τ decay cases. This follows from the condition established in Eq. (28), where the allowed values of $|\mathbf{b}_l^{AB}|$ and $|\mathbf{e}_l^{AB}|$ depend on both the size of the $c_{R,I,1,2}^{AB}$ coefficients and the experimental bounds on

TABLE III. Upper bounds on $|\mathbf{b}_l^{AB}|$ and $|\mathbf{e}_l^{AB}|$ derived from the two-body $f_B \rightarrow f_A \gamma$ decay.

$ \mathbf{b}_l^{\mu\tau} \leq 1.95 \times 10^{-11}$	$ \mathbf{e}_l^{\mu\tau} \leq 1.51 \times 10^{-11}$
$ \mathbf{b}_l^{e\tau} \leq 1.73 \times 10^{-11}$	$ \mathbf{e}_l^{e\tau} \leq 1.34 \times 10^{-11}$
$ \mathbf{b}_l^{e\mu} \leq 4.71 \times 10^{-18}$	$ \mathbf{e}_l^{e\mu} \leq 3.65 \times 10^{-18}$

the BRs. Larger values of $c_{R,I,2}^{AB}$ therefore lead to a stronger suppression of the allowed parameter space. The $c_{R,I,2}^{AB}$ coefficients depend on the mass ratios m_A/m_B and on $(\Gamma_B m_B)^{-1}$, the inverse dependence on $\Gamma_B m_B$ leads to larger values of the coefficients in the muon decay relative to

the τ , as it can be seen in Table II. Additionally, the experimental bound $\text{BR}(\mu \rightarrow e\gamma) \lesssim 10^{-13}$ is several orders of magnitude more stringent than those for the $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ decays, which are below of 10^{-8} . Together, these effects result in more restrictive bounds on $|\mathbf{b}_l^{\mu e}|$ and $|\mathbf{e}_l^{\mu e}|$ than those obtained for the LFV τ transitions.

B. The three-body $f_B \rightarrow f_A f_C \bar{f}_C$ decay

The first-order contributions of the MEY sector to the $f_B \rightarrow f_A f_C \bar{f}_C$ process are calculated via the Feynman diagrams presented in Fig. 5.

The associated amplitude can be written as

$$\mathcal{M}(f_B \rightarrow f_A f_C \bar{f}_C) = \frac{2iv\pi\alpha Q_f^2}{(p_2 + p_3)^2} \bar{u}^{s'}(p_1, m_A) \left[T_f^{AB} \left(\frac{\not{p}_1 + m_B}{p_1^2 - m_B^2} \right) \gamma_\mu + \gamma_\mu \left(\frac{\not{p} + m_A}{p^2 - m_A^2} \right) T_f^{AB} \right] u^s(p, m_B) \times \bar{u}^{r'}(p_2, m_C) \gamma^\mu v^r(p_3, m_C). \quad (29)$$

The respective branching ratio is given by

$$\text{BR}(f_B \rightarrow f_A f_C \bar{f}_C) = \frac{1}{2m_B} \frac{1}{\Gamma_B} \int \left(\frac{1}{2E_{\mathbf{p}_1}} \frac{d^3\mathbf{p}_1}{(2\pi)^3} \right) \left(\frac{1}{2E_{\mathbf{p}_2}} \frac{d^3\mathbf{p}_2}{(2\pi)^3} \right) \left(\frac{1}{2E_{\mathbf{p}_3}} \frac{d^3\mathbf{p}_3}{(2\pi)^3} \right) \times (2\pi)^4 \delta^{(4)}(p - p_1 - p_2 - p_3) |\bar{\mathcal{M}}(f_B \rightarrow f_A f_C \bar{f}_C)|^2. \quad (30)$$

Just as in the case of the two-body decay, the volume element $d^3\mathbf{p}_3$ is integrated at the rest frame of the decaying particle f_B , whereas the phase-space volume elements $d^3\mathbf{p}_2$ and $d^3\mathbf{p}_1$ are integrated in the reference frame depicted in Fig. 3. Therefore, by using the same scenario for the LV coefficients as the one introduced for the case of the two-body decay, the BRs can be expressed as

$$\text{BR}_{\text{Re}}^{YSME}(f_B \rightarrow f_A f_C \bar{f}_C) = d_{R_1}^{AB} |\text{Re}(\mathbf{b}_f^{AB})|^2 + d_{R_2}^{AB} |\text{Re}(\mathbf{b}_f^{AB})| |\text{Re}(\mathbf{e}_f^{AB})| + d_{R_3}^{AB} |\text{Re}(\mathbf{e}_f^{AB})|^2, \quad (31)$$

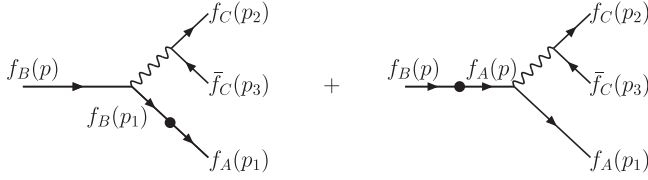
$$\text{BR}_{\text{Im}}^{YSME}(f_B \rightarrow f_A f_C \bar{f}_C) = d_{I_1}^{AB} |\text{Im}(\mathbf{b}_f^{AB})|^2 + d_{I_2}^{AB} |\text{Im}(\mathbf{b}_f^{AB})| |\text{Im}(\mathbf{e}_f^{AB})| + d_{I_3}^{AB} |\text{Im}(\mathbf{e}_f^{AB})|^2. \quad (32)$$

The explicit analytical expressions for the $d_{R_i}^{AB}$ and $d_{I_i}^{AB}$ coefficients are extremely lengthy and do not provide additional physical insight; however, they can be determined by using Eqs. (A2)–(A10). By comparing with the respective experimental BRs, in the same way considered for the procedure proposed in Eq. (28), we impose the condition

$$\text{BR}_{\text{Re,Im}}^{YSME}(f_B \rightarrow f_A f_C \bar{f}_C) \leq \text{BR}^{\text{Exp}}(f_B \rightarrow f_A f_C \bar{f}_C), \quad (33)$$

where $\text{BR}^{\text{Exp}}(f_B \rightarrow f_A f_C \bar{f}_C)$ denotes the first five processes listed in the second column of Table I. Similar to the case of two-body decays, the resulting constraints define regions in the parameter space determined by both the size of the coefficients d_{R_i, I_i}^{AB} (with $i = 1, 2, 3$) and the experimental bounds for the corresponding BRs. Among all the processes in question, only the $\tau^- \rightarrow \mu^- e^- e^+$ decay leads to more stringent constraints on $|\mathbf{b}_l^{\mu\tau}|$ and $|\mathbf{e}_l^{\mu\tau}|$ than those obtained from the two-body decay $\tau \rightarrow \mu\gamma$. The remaining three-body processes do not yield more restrictive constraints than those obtained from the two-body decays.

For the three-body decays that are the subject of our study, the analysis of the dependence of the different decay modes on their distribution in phase space is intricate. The reason lies in the fact that the mass dependence of the initial and final states interrelates with the dependence on the preferred direction of spacetime dictated by the violation of Lorentz symmetry, which, when integrated over phase space yields an overall contribution where the mass dependence is involved. Nonetheless, despite these difficulties, a mass dependence is observed in the decaying initial state and in the final state that changed flavor once

FIG. 5. First-order LV corrections to the $f_B \rightarrow f_A f_C \bar{f}_C$ decay.

the transition occurred due to the insertion of LV. This suggests that three-body decay processes are much more sensitive to LV effects as compared to the two-body decays studied. The only pattern observed in the mass dependence is that integration over phase space yields smaller contributions when the mass of the lepton that changed flavor is

small, which is appreciated for the τ decay modes $\mu^- e^- e^+$ and $e^- e^- e^+$. In contrast, it is worth mentioning that the $\tau^- \rightarrow \mu^- \mu^- \mu^+$ decay also constrains the $|\mathbf{e}_l^{\mu\tau}|$ and $|\mathbf{b}_l^{\mu\tau}|$ coefficients. However, in this case, the phase space is more restricted due to the heavier final-state masses, leading to smaller $d_{R_i, l_i}^{\mu\tau}$ coefficients and consequently to less-restrictive LV regions than those obtained from the $\tau^- \rightarrow \mu^- e^- e^+$ channel.

Furthermore, for the $e\mu$ sector, the experimental bound on $\text{BR}(\mu \rightarrow e\gamma)$ is more stringent than that for the $\mu^- \rightarrow e^- e^- e^+$ decay, favoring the LV constraints derived from the two-body decay case.

Specifically, by considering

$$\text{BR}_{\text{Re}}^{\text{YSME}}(\tau^- \rightarrow \mu^- e^- e^+) = (+9.70|\text{Re}(\mathbf{b}_l^{\mu\tau})|^2 - 1.28|\text{Re}(\mathbf{b}_l^{\mu\tau})||\text{Re}(\mathbf{e}_l^{\mu\tau})| + 19.4|\text{Re}(\mathbf{e}_l^{\mu\tau})|^2) \times 10^{14}, \quad (34)$$

$$\text{BR}_{\text{Im}}^{\text{YSME}}(\tau^- \rightarrow \mu^- e^- e^+) = (-4.46|\text{Im}(\mathbf{b}_l^{\mu\tau})|^2 + 29.6|\text{Im}(\mathbf{b}_l^{\mu\tau})||\text{Im}(\mathbf{e}_l^{\mu\tau})| - 2.17|\text{Im}(\mathbf{e}_l^{\mu\tau})|^2) \times 10^{13}, \quad (35)$$

and that

$$\text{BR}_{\text{Re,Im}}^{\text{YSME}}(\tau^- \rightarrow \mu^- e^- e^+) \leq 1.6 \times 10^{-8}, \quad (36)$$

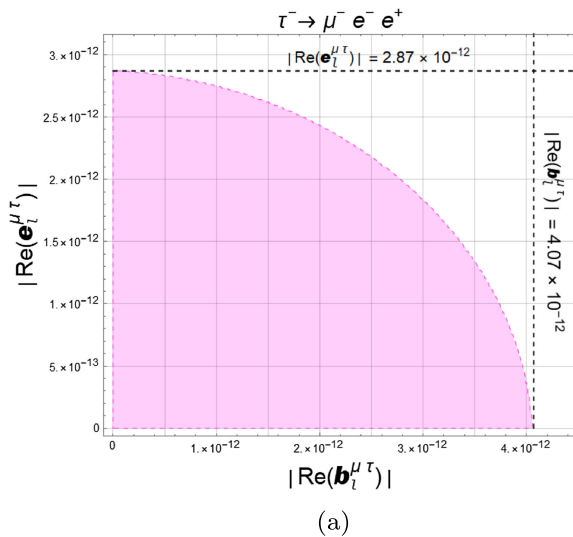
leads to regions delimited by an ellipse or a hyperbola in plots ($|\mathbf{b}_f^{AB}|$ versus $|\mathbf{e}_f^{AB}|$) (see Fig. 6).

Thus, for purely real LV coefficients, we obtain upper bounds for the $(Y_l)^{\mu\tau}$ components, as follows:

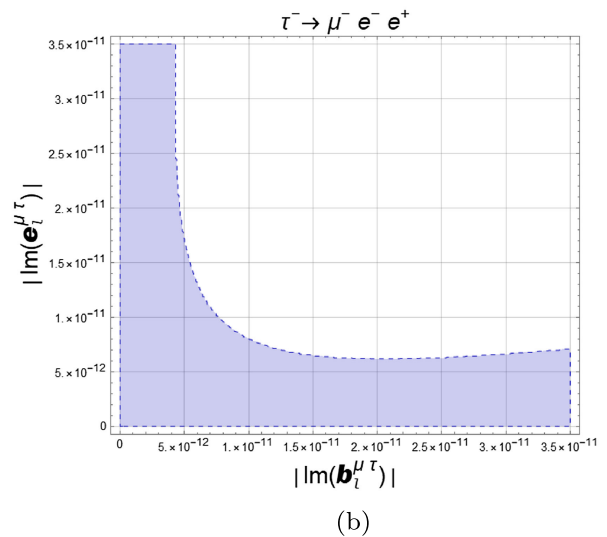
$$|\mathbf{b}_l^{\mu\tau}| \leq 4.07 \times 10^{-12}, \quad |\mathbf{e}_l^{\mu\tau}| \leq 2.87 \times 10^{-12}. \quad (37)$$

For purely imaginary LV coefficients, an unbounded region is obtained [see Fig. 6(b)] and no direct constraints can be deduced from this scenario. Nevertheless, by superimposing the plots $|\text{Im}(\mathbf{b}_l^{\mu\tau})|$ versus $|\text{Im}(\mathbf{e}_l^{\mu\tau})|$ for both two-body and three-body decays, we can discard the

unbounded region and to establish a bounded region for the LV coefficients in question, as it is shown in Fig. 7. Since there are no modifications to the upper limits in the resulting region, we will retain the upper bounds previously calculated from the two-body decay.



(a)



(b)

FIG. 6. Allowed regions for (a) $|\text{Re}(\mathbf{b}_l^{\mu\tau})|$ versus $|\text{Re}(\mathbf{e}_l^{\mu\tau})|$ and (b) $|\text{Im}(\mathbf{b}_l^{\mu\tau})|$ versus $|\text{Im}(\mathbf{e}_l^{\mu\tau})|$, derived from the experimental bound for the $\tau^- \rightarrow \mu^- e^- e^+$ decay.

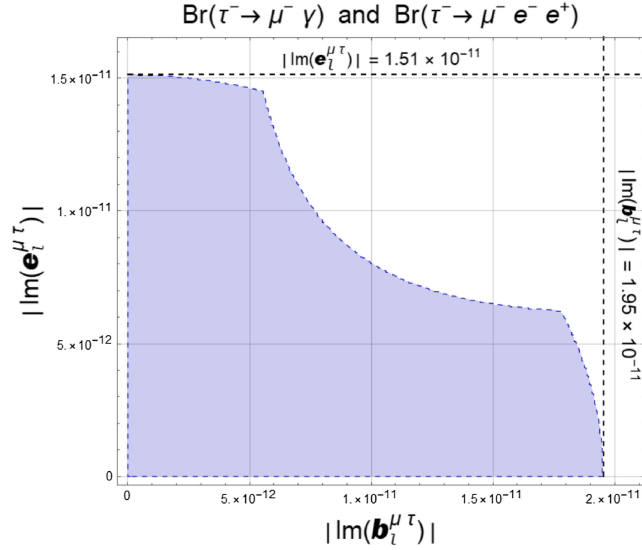


FIG. 7. Allowed region for the $|\text{Im}(\mathbf{b}_l^\mu \tau)|$ and $|\text{Im}(\mathbf{e}_l^\mu \tau)|$ coefficients derived from the combined results obtained with the $\tau \rightarrow \mu\gamma$ and $\tau^- \rightarrow \mu^- e^+ e^-$ decay processes.

Finally, the bounds given in both Table III and Eq. (37) can be translated into constraints on the $(Y_l)_{\alpha\beta}^{AB}$ components through the relations established in Eqs. (5) and (6). Thus, all the corresponding LV bounds in the rest frame of the decaying particles are presented in Table IV.

Since scalars of the form $(Y_f)_{\alpha\beta}^{AB}(Y_f)^{*BA\alpha\beta}$ are Lorentz invariants in the flavor space [see Eq. (A11)], we can use the bounds above obtained to establish limits on their values for any reference frame, which can be appreciated in Table V.

C. Second-order contributions

In order to search for dipolar-transition form factors, we compute second-order insertions in the $f_B f_A \gamma$ coupling at tree level, which is represented by the sum of the Feynman diagrams shown in Fig. 8. In this manner, the vertex function results in

$$\Gamma_\mu^{O(2)f_B f_A \gamma} = ieQ\bar{u}^{s'}(p', m_A)\Gamma_\mu^{O(2)AB}u^s(p, m_B), \quad (38)$$

where

$$\begin{aligned} \Gamma_\mu^{O(2)AB} = & (T_V^{AB})_\mu - (T_A^{AB})_\mu \gamma_5 + \gamma_\mu (g_V^{AB} - g_A^{AB} \gamma_5) + i\sigma_{\mu\lambda} q_\rho \left[\frac{(F_M^{AB})^{\lambda\rho}}{m_A + m_B} - \frac{(F_E^{AB})^{\lambda\rho}}{m_B - m_A} \gamma_5 \right] \\ & - i\epsilon_{\mu\lambda\rho\tau} \gamma^\lambda [(R_V^{AB})^{\rho\tau} - (R_A^{AB})^{\rho\tau} \gamma_5] + \gamma^\lambda [(S_V^{AB})_{\mu\lambda} - (S_A^{AB})_{\mu\lambda} \gamma_5] + i\sigma_{\lambda\rho} \left[\frac{(\hat{F}_M^{AB})^{\mu\lambda\rho}}{m_A + m_B} - \frac{(\hat{F}_E^{AB})^{\mu\lambda\rho}}{m_B - m_A} \gamma_5 \right]. \end{aligned} \quad (39)$$

In this case, $q = p' - p$. The explicit form for all the Lorentz-tensor structures present in Eq. (39) are given in Appendix B. This amplitude is gauge invariant since it satisfies the Ward identity $q^\mu \Gamma_\mu^{O(2)AB} = 0$.

Although in Eq. (39) there is no presence of dipolar-transition structures ($\sigma^{\mu\nu} q_\nu$), it is of worth to appreciate generalized vector and axial flavor-changing neutral currents, where g_V^{AB} and g_A^{AB} are

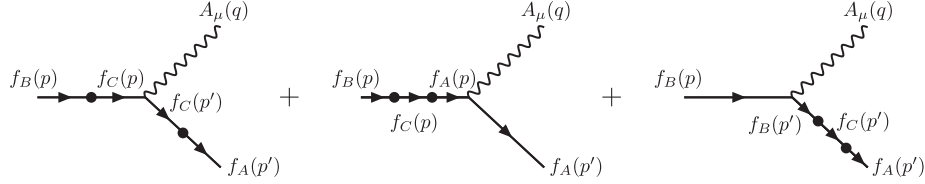
$$g_V^{AB} = \frac{2v^2}{m_{BA}^2 m_{CB}^2 m_{AC}^2} [m_{BA}^2 (\mathcal{V}_f^{AC})_{\lambda q} (\mathcal{V}_f^{CB})^{\lambda q} + m_{AC}^2 (\mathcal{V}_f^{AC})_{\lambda q} (\mathcal{V}_f^{CB})^{\lambda p} + m_{CB}^2 (\mathcal{V}_f^{AC})_{\lambda p} (\mathcal{V}_f^{CB})^{\lambda q} - (\mathcal{V} \rightarrow \mathcal{A})], \quad (40)$$

TABLE IV. Upper bounds for the $(Y_l)_{0i}^{AB}$ and $(Y_l)_{ij}^{AB}$ components; these bounds were computed in the rest frame of the decaying particles.

Purely real		Purely imaginary	
$(Y_l)_{0i}^{\mu\tau} (Y_l)_{0i}^{\mu\tau} < 8.24 \times 10^{-24}$	$(Y_l)_{ij}^{\mu\tau} (Y_l)_{ij}^{\mu\tau} < 3.31 \times 10^{-23}$	$(Y_l)_{0i}^{\mu\tau} (Y_l)_{0i}^{*\mu\tau} < 2.28 \times 10^{-22}$	$(Y_l)_{ij}^{\mu\tau} (Y_l)_{ij}^{*\mu\tau} < 7.60 \times 10^{-22}$
$(Y_l)_{0i}^{e\tau} (Y_l)_{0i}^{e\tau} < 1.80 \times 10^{-22}$	$(Y_l)_{ij}^{e\tau} (Y_l)_{ij}^{e\tau} < 5.99 \times 10^{-22}$	$(Y_l)_{0i}^{e\tau} (Y_l)_{0i}^{*e\tau} < 1.80 \times 10^{-22}$	$(Y_l)_{ij}^{e\tau} (Y_l)_{ij}^{*e\tau} < 5.99 \times 10^{-22}$
$(Y_l)_{0i}^{e\mu} (Y_l)_{0i}^{e\mu} < 1.33 \times 10^{-35}$	$(Y_l)_{ij}^{e\mu} (Y_l)_{ij}^{e\mu} < 4.44 \times 10^{-35}$	$(Y_l)_{0i}^{e\mu} (Y_l)_{0i}^{*e\mu} < 1.33 \times 10^{-35}$	$(Y_l)_{ij}^{e\mu} (Y_l)_{ij}^{*e\mu} < 4.44 \times 10^{-35}$

TABLE V. Upper bounds for the Lorentz-invariant scalars $(Y_l)_{\alpha\beta}^{AB}(Y_l)^{*BA\alpha\beta}$.

Purely real	Purely imaginary
$ (Y_l)_{\alpha\beta}^{\mu\tau} (Y_l)^{* \tau \mu \alpha \beta} < 4.96 \times 10^{-23}$	$ (Y_l)_{\alpha\beta}^{\mu\tau} (Y_l)^{* \tau \mu \alpha \beta} < 1.22 \times 10^{-21}$
$ (Y_l)_{\alpha\beta}^{e\tau} (Y_l)^{* \tau e \alpha \beta} < 9.58 \times 10^{-22}$	$ (Y_l)_{\alpha\beta}^{e\tau} (Y_l)^{* \tau e \alpha \beta} < 9.58 \times 10^{-22}$
$ (Y_l)_{\alpha\beta}^{e\mu} (Y_l)^{* \mu e \alpha \beta} < 7.10 \times 10^{-35}$	$ (Y_l)_{\alpha\beta}^{e\mu} (Y_l)^{* \mu e \alpha \beta} < 7.10 \times 10^{-35}$

FIG. 8. Second-order corrections to the LV coefficients Y_f^{AB} on the $f_B f_A \gamma$ vertex.

$$g_A^{AB} = \frac{2v^2}{m_{BA}^2 m_{CB}^2 m_{AC}^2} [m_{BA}^2 (\mathcal{A}_f^{AC})_{\lambda q} (\mathcal{V}_f^{CB})^{\lambda q} + m_{AC}^2 (\mathcal{A}_f^{AC})_{\lambda q} (\mathcal{V}_f^{CB})^{\lambda p} + m_{CB}^2 (\mathcal{A}_f^{AC})_{\lambda p} (\mathcal{V}_f^{CB})^{\lambda q} - (\mathcal{V} \leftrightarrow \mathcal{A})], \quad (41)$$

with $m_{XY}^2 = m_X^2 - m_Y^2$, for $X, Y = A, B, C$. Due to $(\mathcal{K}_f^{AC})_{\lambda r}$, with $\mathcal{K} = \mathcal{A}, \mathcal{V}$, and $r = P, q$, has mass dimension 1, the couplings g_V^{AB} and g_A^{AB} are dimensionless. An interesting property is that g_V^{AB} and g_A^{AB} are invariant under observer Lorentz transformations, but not under particle transformations.

IV. CONCLUSIONS

In this work, we calculated BRs for the LFV processes $f_B \rightarrow f_A \gamma$ and $f_B \rightarrow f_A f_C \bar{f}_C$ mediated by the Yukawa sector of the SME through the LV tensor $(Y_f)_{\mu\nu}^{AB}$. By comparing the theoretical branching ratios with their respective experimental bounds, parameter spaces for the LV couplings, $|\mathbf{e}_f^{AB}|$ and $|\mathbf{b}_f^{AB}|$, were determined. In order to perform the phase space integration for the different decays, some assumptions over the spatial orientation for the three-vectors $\mathbf{e}_f^{AB}$ and $\mathbf{b}_f^{AB}$ associated to $(Y_f)_{0i}^{AB}$ and $(Y_f)_{ij}^{AB}$, respectively, were considered. We took $\mathbf{e}_f^{AB}$ and $\mathbf{b}_f^{AB}$ as either purely real or purely imaginary, establishing them mutually orthogonal in the rest frame of the decaying particle. It was found that $|\mathbf{e}_f^{e\tau}|$ or $|\mathbf{b}_f^{e\tau}|$ are below of 10^{-11} , while $|\mathbf{e}_f^{\mu\tau}|$ or $|\mathbf{b}_f^{\mu\tau}|$ are below of 10^{-12} . The most stringent bounds corresponded to $|\mathbf{e}_f^{e\mu}|$ or $|\mathbf{b}_f^{e\mu}|$, being of the order of 10^{-18} . It should be recalled that this work introduces a methodology that addresses new energy-momentum dependent terms coupled to background LV-tensor fields that were ignored in previous research. Our upper bounds are more restrictive than one of those previously reported in the literature. In addition, we presented new constraints for Lorentz-invariant scalars $(Y_l)_{\alpha\beta}^{AB} (Y_l)^{*BA\alpha\beta}$ in the flavor space. On the other hand, we computed second-order LV corrections to the $f_B f_A \gamma$ vertex, finding that these only induce generalized vector and axial transition currents and a variety of new Lorentz structures, where there is no presence of dipolar-transition structures.

For future investigations, the upper bounds for the different LV coefficients that we estimated could be implemented in the study of LFV scattering processes, in which the spatial orientation of the momentum of the

initial particles with respect to the background fields $\mathbf{e}_f^{AB}$ and $\mathbf{b}_f^{AB}$ could be manifest through their cross sections.

ACKNOWLEDGMENTS

This work has been partially supported by SNII-SECIHTI and CIC-UMSNH. J. M.-D. thanks the SECIHTI program Investigadoras e Investigadores por México, Project No. 7009. O. V.-H. thanks SECIHTI for postdoctoral support.

DATA AVAILABILITY

All input data utilized in this study are available in the cited literature, including Ref. [69]. No new experimental datasets were generated. The results are derived from the theoretical framework, equations, and input parameters detailed in the manuscript.

APPENDIX A: LORENTZ CONTRACTIONS

In this Appendix, we present the scalar products that arise in this work. For convenience, we denote the LV tensors $(Y_f)_{\mu\nu}^{AB}$, $(Y_f)_{\mu\nu}^{*BA}$, $(\tilde{Y}_f)_{\mu\nu}^{AB}$, $(\tilde{Y}_f)_{\mu\nu}^{*BA}$ as $(\mathcal{X}^{AB})_{\mu\nu}$ or $(\mathcal{Z}^{AB})_{\mu\nu}$. As mentioned in the Introduction, the six independent components of $(Y_f)_{\mu\nu}^{AB}$ can be expressed in terms of $(\mathbf{e}_f^{AB})^i$ and $(\mathbf{b}_f^{AB})^i$ [see Eq. (5)]. Thus, the $(\mathcal{X}^{AB})_{\mu\nu}$ tensor can be associated with two three-vectors $(\mathbf{X}^{AB})^i$ and $(\mathbf{x}^{AB})^i$ through

$$(\mathcal{X}^{AB})_{0i} = (\mathbf{X}^{AB})_i, \quad (\mathcal{X}^{AB})_{ij} = \epsilon^{ijk} (\mathbf{x}^{AB})^k. \quad (A1)$$

TABLE VI. Associated $(\mathbf{e}_f^{AB})^i$ or $(\mathbf{b}_f^{AB})^i$ components for each $(\mathcal{X}^{AB})_{\mu\nu}$ tensor.

$(\mathcal{X}^{AB})_{\mu\nu}$	$(\mathbf{X}^{AB})^i$	$(\mathbf{x}^{AB})^i$
$(Y_f)_{\mu\nu}^{AB}$	$+(\mathbf{e}_f^{AB})^i$	$+(\mathbf{b}_f^{AB})^i$
$(Y_f)_{\mu\nu}^{*BA}$	$+(\mathbf{e}_f^{BA*})^i$	$+(\mathbf{b}_f^{BA*})^i$
$(\tilde{Y}_f)_{\mu\nu}^{AB}$	$+(\mathbf{b}_f^{AB})^i$	$-(\mathbf{e}_f^{AB})^i$
$(\tilde{Y}_f)_{\mu\nu}^{*BA}$	$+(\mathbf{b}_f^{BA*})^i$	$-(\mathbf{e}_f^{BA*})^i$

Where $\mathbf{X}^{AB}$ and $\mathbf{x}^{AB}$ correspond to $\mathbf{e}^{AB}$ or $\mathbf{b}^{AB}$, the corresponding relations are summarized in Table VI; see also Eqs. (5) and (6). In the $\mathbf{X}^{AB}$ and $\mathbf{x}^{AB}$ basis, the Lorentz scalar products that appear in this paper are given by

$$(\mathcal{X}^{AB})_{\mu\nu}(\mathcal{Z}^{AB})^{\mu\nu} = -2\mathbf{X}^{AB} \cdot \mathbf{Z}^{AB} + 2\mathbf{x}^{AB} \cdot \mathbf{z}^{AB}, \quad (\text{A2})$$

$$(\mathcal{X}^{AB})_{\alpha p_1}(\mathcal{Z}^{AB})^{\alpha p_2} = (\mathbf{X}^{AB} \cdot \mathbf{p}_1)(\mathbf{Z}^{AB} \cdot \mathbf{p}_2) - E_1 E_2 (\mathbf{X}^{AB} \cdot \mathbf{Z}^{AB}) + E_1 (\mathbf{X}^{AB} \times \mathbf{z}^{AB}) \cdot \mathbf{p}_2 \\ + E_2 (\mathbf{Z}^{AB} \times \mathbf{x}^{AB}) \cdot \mathbf{p}_1 - (\mathbf{x}^{AB} \times \mathbf{p}_1) \cdot (\mathbf{z}^{AB} \times \mathbf{p}_2), \quad (\text{A3})$$

$$(\mathcal{X}^{AB})_{p_1 p_2} = -E_1 (\mathbf{X}^{AB} \cdot \mathbf{p}_2) + E_2 (\mathbf{X}^{AB} \cdot \mathbf{p}_1) + (\mathbf{p}_1 \times \mathbf{p}_2) \cdot \mathbf{x}^{AB}, \quad (\text{A4})$$

$$(\mathcal{X}^{AB})_{\alpha i}(\mathcal{Z}^{AB})_{\beta}^{\lambda} \epsilon^{\alpha\beta p_1 p_2} = (\mathbf{X}^{AB} \times \mathbf{z}^{AB}) \cdot (\mathbf{p}_1 \times \mathbf{p}_2) - (\mathbf{Z}^{AB} \times \mathbf{x}^{AB}) \cdot (\mathbf{p}_1 \times \mathbf{p}_2) + E_2 (\mathbf{X}^{AB} \times \mathbf{Z}^{AB}) \cdot \mathbf{p}_1 \\ - E_1 (\mathbf{X}^{AB} \times \mathbf{Z}^{AB}) \cdot \mathbf{p}_2 - E_2 (\mathbf{x}^{AB} \times \mathbf{z}^{AB}) \cdot \mathbf{p}_1 + E_1 (\mathbf{x}^{AB} \times \mathbf{z}^{AB}) \cdot \mathbf{p}_2, \quad (\text{A5})$$

$$(\mathcal{X}^{AB})_{\alpha p_1}(\mathcal{Z}^{AB})_{\beta p_2} \epsilon^{\alpha\beta p_3 p_4} = -(\mathbf{X}^{AB} \cdot \mathbf{p}_1)[E_2 \mathbf{Z}^{AB} \cdot (\mathbf{p}_3 \times \mathbf{p}_4) - (\mathbf{z}^{AB} \times \mathbf{p}_2) \cdot (\mathbf{p}_3 \times \mathbf{p}_4)] \\ + (\mathbf{Z}^{AB} \cdot \mathbf{p}_2)[E_1 \mathbf{X}^{AB} \cdot (\mathbf{p}_3 \times \mathbf{p}_4) - (\mathbf{x}^{AB} \times \mathbf{p}_1) \cdot (\mathbf{p}_3 \times \mathbf{p}_4)] \\ - E_1 E_2 E_3 (\mathbf{X}^{AB} \times \mathbf{Z}^{AB}) \cdot \mathbf{p}_4 + E_1 E_3 [\mathbf{X}^{AB} \times (\mathbf{z}^{AB} \times \mathbf{p}_2)] \cdot \mathbf{p}_4 \\ + E_1 E_2 E_4 (\mathbf{X}^{AB} \times \mathbf{Z}^{AB}) \cdot \mathbf{p}_3 - E_1 E_4 [\mathbf{X}^{AB} \times (\mathbf{z}^{AB} \times \mathbf{p}_2)] \cdot \mathbf{p}_3 \\ - E_2 E_3 [\mathbf{Z}^{AB} \times (\mathbf{x}^{AB} \times \mathbf{p}_1)] \cdot \mathbf{p}_4 - E_3 [(\mathbf{x}^{AB} \times \mathbf{p}_1) \times (\mathbf{z}^{AB} \times \mathbf{p}_2)] \cdot \mathbf{p}_4 \\ + E_2 E_4 [\mathbf{Z}^{AB} \times (\mathbf{x}^{AB} \times \mathbf{p}_1)] \cdot \mathbf{p}_3 + E_4 [(\mathbf{x}^{AB} \times \mathbf{p}_1) \times (\mathbf{z}^{AB} \times \mathbf{p}_2)] \cdot \mathbf{p}_3, \quad (\text{A6})$$

$$(\mathcal{X}^{AB})_{\alpha p_1} \epsilon^{\alpha p_2 p_3 p_4} = (\mathbf{X}^{AB} \cdot \mathbf{p}_1)(\mathbf{p}_2 \times \mathbf{p}_3) \cdot \mathbf{p}_4 - E_1 E_2 \mathbf{X}^{AB} \cdot (\mathbf{p}_3 \times \mathbf{p}_4) + E_1 E_3 \mathbf{X}^{AB} \cdot (\mathbf{p}_2 \times \mathbf{p}_4) - E_1 E_4 \mathbf{X}^{AB} \cdot (\mathbf{p}_2 \times \mathbf{p}_3) \\ + E_2 (\mathbf{x}^{AB} \times \mathbf{p}_1) \cdot (\mathbf{p}_3 \times \mathbf{p}_4) - E_3 (\mathbf{x}^{AB} \times \mathbf{p}_1) \cdot (\mathbf{p}_2 \times \mathbf{p}_4) + E_4 (\mathbf{x}^{AB} \times \mathbf{p}_1) \cdot (\mathbf{p}_2 \times \mathbf{p}_3), \quad (\text{A7})$$

$$(\mathcal{X}^{AB})_{\alpha\beta}(\mathcal{Z}^{AB})^{\beta p_1} \epsilon^{\alpha p_2 p_3 p_4} = -E_1 (\mathbf{X}^{AB} \cdot \mathbf{Z}^{AB})(\mathbf{p}_2 \times \mathbf{p}_3) \cdot \mathbf{p}_4 + \mathbf{X}^{AB} \cdot (\mathbf{z}^{AB} \times \mathbf{p}_1)(\mathbf{p}_2 \times \mathbf{p}_3) \cdot \mathbf{p}_4 \\ + \mathbf{X}^{AB} \cdot [E_2 (\mathbf{p}_3 \times \mathbf{p}_4) - E_3 (\mathbf{p}_2 \times \mathbf{p}_4) + E_4 (\mathbf{p}_2 \times \mathbf{p}_3)](\mathbf{Z}^{AB} \cdot \mathbf{p}_1) \\ - (\mathbf{x}^{AB} \times \mathbf{Z}^{AB}) \cdot [E_1 E_2 (\mathbf{p}_3 \times \mathbf{p}_4) - E_1 E_3 (\mathbf{p}_2 \times \mathbf{p}_4) + E_1 E_4 (\mathbf{p}_2 \times \mathbf{p}_3)] \\ \times [\mathbf{x}^{AB} \times (\mathbf{z}^{AB} \times \mathbf{p}_1)] \cdot [E_2 (\mathbf{p}_3 \times \mathbf{p}_4) - E_3 (\mathbf{p}_2 \times \mathbf{p}_4) + E_4 (\mathbf{p}_2 \times \mathbf{p}_3)], \quad (\text{A8})$$

where p_r is a four-momentum with components $(p_r)^\mu = (E_r, \mathbf{p}_r)$, with $r = 1, 2, 3, 4$. The usual three-vector products are defined by

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a}^i \mathbf{b}^i, \quad (\text{A9})$$

$$(\mathbf{a} \times \mathbf{b})^i = \epsilon^{ijk} \mathbf{a}^j \mathbf{b}^k. \quad (\text{A10})$$

For example, taking $(\mathcal{X}^{AB})_{\mu\nu} = (Y_f)_{\mu\nu}^{AB}$ and $(\mathcal{Z}^{AB})_{\mu\nu} = (Y_f)_{\mu\nu}^{*BA}$, it follows that

$$(Y_f)_{\mu\nu}^{AB} (Y_f)^{*BA\mu\nu} = -2\mathbf{e}_f^{AB} \cdot \mathbf{e}_f^{BA*} + 2\mathbf{b}_f^{AB} \cdot \mathbf{b}_f^{BA*}. \quad (\text{A11})$$

APPENDIX B: LORENTZ TENSORS

In this Appendix, we present the various Lorentz tensors that arise from the second-order contributions to the $f_B f_A \gamma$ vertex given in Eq. (39). Let

$$c_\Delta = \frac{2v^2}{m_{BA}^2 m_{AC}^2 m_{BC}^2}, \quad (\text{B1})$$

thus,

$$\begin{aligned}
(T_V^{AB})_\mu = c_\Delta \{ & (m_C - m_A)m_{AC}^2(\mathcal{A}_f^{AC})_{\mu\lambda}(\mathcal{A}_f^{CB})^{\lambda P} + (m_B - m_C)m_{BC}^2(\mathcal{A}_f^{AC})^{\lambda P}(\mathcal{A}_f^{CB})_{\mu\lambda} \\
& + (m_A + m_C)m_{AC}^2(\mathcal{V}_f^{AC})_{\mu\lambda}(\mathcal{V}_f^{CB})^{\lambda P} - (m_B + m_C)m_{BC}^2(\mathcal{V}_f^{AC})^{\lambda P}(\mathcal{V}_f^{CB})_{\mu\lambda} \\
& + (m_C - m_A)m_{BA}^2(\mathcal{A}_f^{AC})_{\mu\lambda}(\mathcal{A}_f^{CB})^{\lambda q} + (m_B - m_C)m_{BA}^2(\mathcal{A}_f^{AC})^{\lambda q}(\mathcal{A}_f^{CB})_{\mu\lambda} \\
& + (m_A + m_C)m_{BA}^2(\mathcal{V}_f^{AC})_{\mu\lambda}(\mathcal{V}_f^{CB})^{\lambda q} - (m_B + m_C)m_{BA}^2(\mathcal{V}_f^{AC})^{\lambda q}(\mathcal{V}_f^{CB})_{\mu\lambda} \\
& + i[(m_B + m_C)m_{AC}^2(\mathcal{A}_f^{AC})^{\lambda q}(\tilde{\mathcal{V}}_f^{CB})_{\mu\lambda} + (m_A + m_C)m_{BC}^2(\tilde{\mathcal{V}}_f^{AC})_{\mu\lambda}(\mathcal{A}_f^{CB})^{\lambda q} \\
& + (m_C - m_B)m_{AC}^2(\mathcal{V}_f^{AC})^{\lambda q}(\tilde{\mathcal{A}}_f^{CB})_{\mu\lambda} + (m_C - m_A)m_{BC}^2(\tilde{\mathcal{A}}_f^{AC})_{\mu\lambda}(\mathcal{V}_f^{CB})^{\lambda q}] \}, \tag{B2}
\end{aligned}$$

$$\begin{aligned}
(T_A^{AB})_\mu = c_\Delta \{ & (m_A - m_C)m_{AC}^2(\mathcal{A}_f^{AC})_{\mu\lambda}(\mathcal{V}_f^{CB})^{\lambda P} - (m_A + m_C)m_{AC}^2(\mathcal{V}_f^{AC})_{\mu\lambda}(\mathcal{A}_f^{CB})^{\lambda P} \\
& \times (m_C - m_B)m_{BC}^2(\mathcal{V}_f^{AC})^{\lambda P}(\mathcal{A}_f^{CB})_{\mu\lambda} + (m_B + m_C)m_{BC}^2(\mathcal{A}_f^{AC})^{\lambda P}(\mathcal{V}_f^{CB})_{\mu\lambda} \\
& + (m_C - m_B)m_{BA}^2(\mathcal{V}_f^{AC})^{\lambda q}(\mathcal{A}_f^{CB})_{\mu\lambda} - (m_A + m_C)m_{BA}^2(\mathcal{V}_f^{AC})_{\mu\lambda}(\mathcal{A}_f^{CB})^{\lambda q} \\
& + (m_A - m_C)m_{BA}^2(\mathcal{A}_f^{AC})_{\mu\lambda}(\mathcal{V}_f^{CB})^{\lambda q} + (m_B + m_C)m_{BA}^2(\mathcal{A}_f^{AC})^{\lambda q}(\mathcal{V}_f^{CB})_{\mu\lambda} \\
& + i[(m_B - m_C)m_{AC}^2(\mathcal{A}_f^{AC})^{\lambda q}(\tilde{\mathcal{A}}_f^{CB})_{\mu\lambda} + (m_A - m_C)m_{BC}^2(\tilde{\mathcal{A}}_f^{AC})_{\mu\lambda}(\mathcal{A}_f^{CB})^{\lambda q} \\
& - (m_B + m_C)m_{AC}^2(\mathcal{V}_f^{AC})^{\lambda q}(\tilde{\mathcal{V}}_f^{CB})_{\mu\lambda} - (m_A + m_C)m_{BC}^2(\tilde{\mathcal{V}}_f^{AC})_{\mu\lambda}(\mathcal{V}_f^{CB})^{\lambda q}] \}, \tag{B3}
\end{aligned}$$

$$\begin{aligned}
(F_M^{AB})^{\lambda\rho} = c_\Delta(m_A + m_B) \{ & (m_A - m_C)m_{BC}^2(\mathcal{A}_f^{AC})^{\omega\lambda}(\mathcal{A}_f^{CB})_\omega^\rho - (m_B - m_C)m_{AC}^2(\mathcal{A}_f^{AC})^{\omega\rho}(\mathcal{A}_f^{CB})_\omega^\lambda \\
& - (m_A + m_C)m_{BC}^2(\mathcal{V}_f^{AC})^{\omega\lambda}(\mathcal{V}_f^{CB})_\omega^\rho + (m_B + m_C)m_{AC}^2(\mathcal{V}_f^{AC})^{\omega\rho}(\mathcal{V}_f^{CB})_\omega^\lambda \}, \tag{B4}
\end{aligned}$$

$$\begin{aligned}
(F_E^{AB})^{\lambda\rho} = c_\Delta(m_A - m_B) \{ & (m_A - m_C)m_{BC}^2(\mathcal{A}_f^{AC})^{\omega\lambda}(\mathcal{V}_f^{CB})_\omega^\rho - (m_B + m_C)m_{AC}^2(\mathcal{A}_f^{AC})^{\omega\rho}(\mathcal{V}_f^{CB})_\omega^\lambda \\
& + (m_A + m_C)m_{BC}^2(\mathcal{V}_f^{AC})^{\omega\lambda}(\mathcal{A}_f^{CB})_\omega^\rho + (m_B - m_C)m_{AC}^2(\mathcal{V}_f^{AC})^{\omega\rho}(\mathcal{A}_f^{CB})_\omega^\lambda \}, \tag{B5}
\end{aligned}$$

$$(R_V^{AB})^{\rho\tau} = c_\Delta \{ m_{BC}^2(\mathcal{A}_f^{AC})^{\rho P}(\mathcal{V}_f^{CB})^{\tau q} + m_{AC}^2(\mathcal{A}_f^{AC})^{\rho q}(\mathcal{V}_f^{CB})^{\tau P} + m_{BA}^2(\mathcal{A}_f^{AC})^{\rho q}(\mathcal{V}_f^{CB})^{\tau q} - (\mathcal{V}_f \leftrightarrow \mathcal{A}_f) \}, \tag{B6}$$

$$(R_A^{AB})^{\rho\tau} = c_\Delta \{ m_{BC}^2(\mathcal{V}_f^{AC})^{\rho P}(\mathcal{V}_f^{CB})^{\tau q} + m_{AC}^2(\mathcal{V}_f^{AC})^{\rho q}(\mathcal{V}_f^{CB})^{\tau P} + m_{BA}^2(\mathcal{V}_f^{AC})^{\rho q}(\mathcal{V}_f^{CB})^{\tau q} - (\mathcal{V}_f \rightarrow \mathcal{A}_f) \}, \tag{B7}$$

$$\begin{aligned}
(S_V^{AB})_{\mu\lambda} = c_\Delta \{ & m_{AC}^2[(\mathcal{V}_f^{AC})_{\lambda q}(\mathcal{V}_f^{BC})_{\mu q} - (\mathcal{V}_f^{AC})_{\mu P}(\mathcal{V}_f^{BC})_{\lambda P} + (\mathcal{A}_f^{AC})_{\mu P}(\mathcal{A}_f^{BC})_{\lambda P} - (\mathcal{A}_f^{AC})_{\lambda q}(\mathcal{A}_f^{BC})_{\mu q} \\
& - (m_A + m_B)(m_B + m_C)(\mathcal{V}_f^{AC})_{\mu\rho}(\mathcal{V}_f^{BC})_\lambda^\rho + (m_A + m_B)(m_B - m_C)(\mathcal{A}_f^{AC})_{\mu\rho}(\mathcal{A}_f^{BC})_\lambda^\rho] \\
& + m_{BC}^2[(\mathcal{V}_f^{AC})_{\lambda P}(\mathcal{V}_f^{BC})_{\mu P} - (\mathcal{V}_f^{AC})_{\mu q}(\mathcal{V}_f^{BC})_{\lambda q} + (\mathcal{A}_f^{AC})_{\mu q}(\mathcal{A}_f^{BC})_{\lambda q} - (\mathcal{A}_f^{AC})_{\lambda P}(\mathcal{A}_f^{BC})_{\mu P} \\
& + (m_A + m_B)(m_A + m_C)(\mathcal{V}_f^{AC})_\lambda^\rho(\mathcal{V}_f^{CB})_{\mu\rho} - (m_A + m_B)(m_A - m_C)(\mathcal{A}_f^{AC})_\lambda^\rho(\mathcal{A}_f^{CB})_{\mu\rho}] \\
& + m_{BA}^2[(\mathcal{V}_f^{AC})_{\lambda q}(\mathcal{V}_f^{CB})_{\mu P} - (\mathcal{V}_f^{AC})_{\mu P}(\mathcal{V}_f^{BC})_{\lambda q} + (\mathcal{A}_f^{AC})_{\mu P}(\mathcal{A}_f^{BC})_{\lambda q} - (\mathcal{A}_f^{AC})_{\lambda q}(\mathcal{A}_f^{BC})_{\mu P}] \\
& + i(m_B - m_A) \{ m_{AC}^2[(m_B + m_C)(\mathcal{A}_f^{AC})_{\mu\rho}(\tilde{\mathcal{V}}_f^{BC})_\lambda^\rho + (m_C - m_B)(\mathcal{V}_f^{AC})_{\mu\rho}(\tilde{\mathcal{A}}_f^{CB})_\lambda^\rho] \\
& + m_{BC}^2[(m_A + m_C)(\tilde{\mathcal{V}}_f^{AC})_\lambda^\rho(\mathcal{A}_f^{BC})_{\mu\rho} + (m_C - m_A)(\tilde{\mathcal{A}}_f^{AC})_\lambda^\rho(\mathcal{V}_f^{CB})_{\mu\rho}] \} \}, \tag{B8}
\end{aligned}$$

$$\begin{aligned}
(S_A^{AB})_{\mu\lambda} = c_\Delta \{ & m_{AC}^2[(\mathcal{V}_f^{AC})_{\mu P}(\mathcal{A}_f^{BC})_{\lambda P} - (\mathcal{A}_f^{AC})_{\mu P}(\mathcal{V}_f^{BC})_{\lambda P} - (\mathcal{V}_f^{AC})_{\lambda q}(\mathcal{A}_f^{BC})_{\mu q} + (\mathcal{A}_f^{AC})_{\lambda q}(\mathcal{V}_f^{BC})_{\mu q} \\
& + (m_B - m_A)(m_B - m_C)(\mathcal{V}_f^{AC})_{\mu\rho}(\mathcal{A}_f^{CB})_\lambda^\rho + (m_A - m_B)(m_B + m_C)(\mathcal{A}_f^{AC})_{\mu\rho}(\mathcal{V}_f^{BC})_\lambda^\rho] \\
& + m_{BC}^2[(\mathcal{A}_f^{AC})_{\lambda P}(\mathcal{V}_f^{CB})_{\mu P} - (\mathcal{V}_f^{AC})_{\lambda P}(\mathcal{A}_f^{BC})_{\mu P} + (\mathcal{V}_f^{AC})_{\mu q}(\mathcal{A}_f^{BC})_{\lambda q} - (\mathcal{A}_f^{AC})_{\mu q}(\mathcal{V}_f^{CB})_{\lambda q} \\
& + (m_B - m_A)(m_A + m_C)(\mathcal{V}_f^{AC})_\lambda^\rho(\mathcal{A}_f^{BC})_{\mu\rho} + (m_A - m_B)(m_A - m_C)(\mathcal{A}_f^{AC})_\lambda^\rho(\mathcal{V}_f^{CB})_{\mu\rho}] \\
& + m_{BA}^2[(\mathcal{V}_f^{AC})_{\mu P}(\mathcal{A}_f^{CB})_{\lambda q} - (\mathcal{V}_f^{AC})_{\lambda q}(\mathcal{A}_f^{BC})_{\mu P} + (\mathcal{A}_f^{AC})_{\lambda q}(\mathcal{V}_f^{BC})_{\mu P} - (\mathcal{A}_f^{AC})_{\mu P}(\mathcal{V}_f^{CB})_{\lambda q}] \\
& - i(m_A + m_B) \{ m_{AC}^2[(m_B - m_C)(\mathcal{A}_f^{AC})_{\mu\rho}(\tilde{\mathcal{A}}_f^{BC})_\lambda^\rho - (m_C + m_B)(\mathcal{V}_f^{AC})_{\mu\rho}(\tilde{\mathcal{V}}_f^{CB})_\lambda^\rho] \\
& + m_{BC}^2[(m_A - m_C)(\tilde{\mathcal{A}}_f^{AC})_\lambda^\rho(\mathcal{A}_f^{BC})_{\mu\rho} - (m_A + m_C)(\tilde{\mathcal{V}}_f^{AC})_\lambda^\rho(\mathcal{V}_f^{CB})_{\mu\rho}] \} \}, \tag{B9}
\end{aligned}$$

$$\begin{aligned}
(\hat{F}_M^{AB})^{\mu\lambda\rho} = & \frac{c_\Delta(m_A + m_B)}{2} \{ m_{AC}^2 [(m_B - m_C)(\mathcal{A}_f^{AC})^{\mu P}(\mathcal{A}_f^{CB})^{\lambda\rho} - (m_B + m_C)(\mathcal{V}_f^{AC})^{\mu P}(\mathcal{V}_f^{CB})^{\lambda\rho} \\
& + 2(m_A - m_C)(\mathcal{A}_f^{AC})^{\mu\lambda}(\mathcal{A}_f^{CB})^{\rho P} - 2(m_A + m_C)(\mathcal{V}_f^{AC})^{\mu\lambda}(\mathcal{V}_f^{CB})^{\rho P}] \\
& + m_{BC}^2 [(m_A - m_C)(\mathcal{A}_f^{AC})^{\lambda\rho}(\mathcal{A}_f^{CB})^{\mu P} - (m_A + m_C)(\mathcal{V}_f^{AC})^{\lambda\rho}(\mathcal{V}_f^{CB})^{\mu P} \\
& + 2(m_B - m_C)(\mathcal{A}_f^{AC})^{\rho P}(\mathcal{A}_f^{CB})^{\mu\lambda} - 2(m_B + m_C)(\mathcal{V}_f^{AC})^{\rho P}(\mathcal{V}_f^{CB})^{\mu\lambda}] \\
& + 2m_{BA}^2 [(m_A - m_C)(\mathcal{A}_f^{AC})^{\mu\lambda}(\mathcal{A}_f^{CB})^{\rho q} + (m_B - m_C)(\mathcal{A}_f^{AC})^{\rho q}(\mathcal{A}_f^{BC})^{\mu\lambda} \\
& - (m_A + m_C)(\mathcal{V}_f^{AC})^{\mu\lambda}(\mathcal{V}_f^{BC})^{\rho q} - (m_B + m_C)(\mathcal{V}_f^{AC})^{\rho q}(\mathcal{V}_f^{CB})^{\mu\lambda}] \}, \tag{B10}
\end{aligned}$$

$$\begin{aligned}
(\hat{F}_E^{AB})^{\mu\lambda\rho} = & \frac{c_\Delta(m_B - m_A)}{2} \{ m_{AC}^2 [(m_C - m_B)(\mathcal{V}_f^{AC})^{\mu P}(\mathcal{A}_f^{CB})^{\lambda\rho} + (m_B + m_C)(\mathcal{A}_f^{AC})^{\mu P}(\mathcal{V}_f^{CB})^{\lambda\rho} \\
& + 2(m_A + m_C)(\mathcal{V}_f^{AC})^{\mu\lambda}(\mathcal{A}_f^{CB})^{\rho P} + 2(m_C - m_A)(\mathcal{A}_f^{AC})^{\mu\lambda}(\mathcal{V}_f^{BC})^{\rho P}] \\
& + m_{BC}^2 [(m_C - m_A)(\mathcal{A}_f^{AC})^{\lambda\rho}(\mathcal{V}_f^{CB})^{\mu P} + (m_A + m_C)(\mathcal{V}_f^{AC})^{\lambda\rho}(\mathcal{A}_f^{BC})^{\mu P} \\
& + 2(m_B + m_C)(\mathcal{A}_f^{AC})^{\rho P}(\mathcal{V}_f^{BC})^{\mu\lambda} + 2(m_C - m_B)(\mathcal{V}_f^{AC})^{\rho P}(\mathcal{A}_f^{CB})^{\mu\lambda}] \\
& + 2m_{BA}^2 [(m_C - m_A)(\mathcal{A}_f^{AC})^{\mu\lambda}(\mathcal{V}_f^{CB})^{\rho q} + (m_B + m_C)(\mathcal{A}_f^{AC})^{\rho q}(\mathcal{V}_f^{BC})^{\mu\lambda} \\
& + (m_A + m_C)(\mathcal{V}_f^{AC})^{\mu\lambda}(\mathcal{A}_f^{BC})^{\rho q} + (m_C - m_B)(\mathcal{V}_f^{AC})^{\rho q}(\mathcal{A}_f^{CB})^{\mu\lambda}] \}. \tag{B11}
\end{aligned}$$

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