

Existence of the $DD^*\bar{K}^*$ and BB^*K^* three-body molecular states

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We investigate the existence of the three-body molecular state composed of $DD^*\bar{K}^*$ within the one-boson-exchange model. A major challenge is that while the pseudoscalar-meson couplings are well determined, the couplings for scalar- and vector-meson exchanges render significant model dependence. To ensure the reliability of our predictions and reduce model dependence, we recalibrate the coupling constants of the one-boson-exchange model. We treat the pole position of $Z_c(3900)$, or equivalently the scalar σ -exchange coupling constant, as the only unknown parameter. The coupling constants for the vector ρ and ω exchanges are determined by the pole positions of the well-established states $X(3872)$ and $T_{cc}(3875)$. We demonstrate that these parameter sets also successfully describe the $T_{cs0}(2870)$ without further tuning. For the three-body system, our results indicate that an $I(J^P) = 1/2(0^-)$ three-body molecular bound state exists when $Z_c(3900)$ is a virtual state located within approximately -10 MeV of the $D\bar{D}^*$ threshold. Furthermore, we extend our analysis to the complex energy plane using the complex scaling method to search for molecular resonances, though no evidence of resonances is found in considered channels. We also apply this formalism to the bottom analog BB^*K^* system. In this sector, the conditions for the existence of a three-body bound state are more relaxed, as a $Z_c(3900)$ virtual state located within -25 MeV below the threshold suffices, although three-body molecular resonances remain absent. We suggest that future experiments precisely measure the pole position of $Z_c(3900)$ or search for the three-body bound state in $DD\bar{K}\pi\pi$ and $DD\bar{K}$ channels, as these efforts would mutually illuminate the nature of the associated states.

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I. INTRODUCTION

Over the past two decades, the experimental discovery of numerous exotic hadrons that cannot be simply described as quark-antiquark mesons or three-quark baryons within the conventional quark model has revitalized the field. These discoveries have sparked significant advancements in hadron spectroscopy and provide an excellent platform for probing the nonperturbative dynamics of quantum chromodynamics (QCD), as reviewed in Refs. [1–12]. Several of these exotic states, such as the $X(3872)$ [also referred to as $\chi_{c1}(3872)$] [13], $Z_c(3900)$ [14,15], $T_{cs0}(2870)$ [16–18], and $T_{cc}(3875)$ [19,20] have attracted

particular attention due to their unique properties. As illustrated in Table I, these states are located close to S -wave meson-meson thresholds; therefore, it is natural to interpret them as hadronic molecular states of the corresponding meson pairs.

The concept of a hadronic molecular state composed of two color-singlet hadrons is not unique to the heavy-flavor sector. Indeed, the deuteron stands as the archetype of a two-body hadronic molecular state. Just as atomic nuclei with various mass numbers, such as ^3H and ^3He , exist in nature, anticipating the existence of heavy-flavor molecular states composed of three or more hadrons is logical. Theoretical studies of three-body hadronic molecules have been conducted on various systems, including three light mesons [22,23], $D^{(*)}D^{(*)}D^{(*)}$ [24–29], $D^{(*)}D^{(*)}\bar{D}^{(*)}$ [30,31], $D^{(*)}B^{(*)}\bar{B}^{(*)}$ [32], BBB^* [33], and systems where kaons replace one or more heavy mesons [34–43]. Furthermore, three-body universality in the charmed and bottom sectors was investigated in Refs. [44,45]. For a comprehensive review of three-body heavy-flavor systems, we refer the reader to Refs. [46,47].

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TABLE I. Examples for hadronic molecular states. The experimental data are taken from Ref. [21]. ΔE represents the mass difference between the molecular state and the threshold of the corresponding meson pairs. The symbol $\star$ indicates that the $I(J^P)$ quantum numbers have not been determined experimentally, and the listed values represent theoretically favored assignments.

State	$I^G(J^{PC})$	M [MeV]	Γ [MeV]	S -wave threshold	ΔE [MeV]
$X(3872)$	$0(1^{++})$	3871.64 ± 0.06	1.19 ± 0.21	$D^{*+}D^- + \text{c.c.}$ $D^{*0}\bar{D}^0 + \text{c.c.}$ $[D\bar{D}^*/\bar{D}D^*]^{C=+1}$	-8.1 ± 0.4 -0.05 ± 0.09 -4.09 ± 0.21
$Z_c(3900)$	$1^+(1^{+-})$	3887.1 ± 2.6	28.4 ± 2.6	$D^{*+}D^- - \text{c.c.}$ $D^{*0}\bar{D}^0 - \text{c.c.}$ $[D\bar{D}^*/\bar{D}D^*]^{C=-1}$	7.3 ± 2.6 15.4 ± 2.6 11.4 ± 2.6
$T_{cc}(3875)$	$0(1^+)\star$	3874.74 ± 0.10	$0.048^{+0.0020}_{-0.0141}$	$D^{*0}D^+$ $D^{*+}D^0$ D^*D	-1.6 ± 0.4 -0.36 ± 0.12 -0.99 ± 0.23
$T_{cs0}(2870)$	$0(0^+)\star$	2872 ± 16	67 ± 24	$D^*\bar{K}^*$	-30 ± 17

A system of particular interest that remains relatively unexplored is the $DD^*\bar{K}^*$. In this configuration, the DD^* subsystem can form the molecular state $T_{cc}(3875)$, and the $T_{cs0}(2870)$ was proposed as the $D^*\bar{K}^*$ molecular state [48–56]. Consequently, determining the existence of a three-body molecular state in this system is a compelling question.

In this work, we focus on the $DD^*\bar{K}^*$ three-body system and its bottom analog, the BB^*K^* system. We describe the interactions between hadrons using the one-boson-exchange (OBE) model, which incorporates the exchange of mesons such as π , η , ρ , ω , and σ . This model has achieved significant success in elucidating heavy-flavor hadronic molecules [57–81]. Notably, the existence of the DD^* bound state was predicted theoretically within the OBE model prior to the experimental discovery of the $T_{cc}(3875)$ [67,68]. Recently, the OBE model has been extended to investigate P -wave DD^* , $D\bar{D}^*/\bar{D}D^*$ [78,81], $D^*\bar{K}^*$ [79], and $B^{(*)}\bar{B}^{(*)}$ [80] systems.

A potential issue within the OBE model is that while the pseudoscalar-meson couplings are well determined, the couplings for scalar- and vector-meson exchanges introduce significant model dependence. For example, the σ -exchange potential in Refs. [24,70] is nearly 20 times stronger than that in Refs. [67,68]. To address the model dependence and make theoretical predictions more reliable, we adopt the approach from Ref. [82] to recalibrate the model parameters. The key strategy involves utilizing the $Z_c(3900)$ state, where the ρ - and ω -exchange interactions are expected to largely cancel, to constrain the strength of the scalar σ -exchange interaction. The pole position of the $Z_c(3900)$, which is equivalent to the scalar σ -exchange coupling constant, is treated as the only varying parameter. Then the coupling constants for the vector ρ and ω exchanges are fixed by the pole positions

of the well-established states $X(3872)$ and $T_{cc}(3875)$. Based on heavy quark symmetry, we apply the same model parameters from the $DD^*\bar{K}^*$ system to the BB^*K^* system.

Regarding the solution of the three-body problem, we employ the Gaussian expansion method (GEM) [83] to solve the Schrödinger equation. Furthermore, we utilize the complex scaling method (CSM) [84–87] to search for three-body molecular resonances, as this technique allows bound states and resonances to be treated on an equal footing via square-integrable basis expansion.

The paper is organized as follows. In Sec. II, we introduce the formalism. We construct the effective Lagrangians and derive the OBE effective potentials in Sec. II A. In Sec. II B, we present the methodology for studying the three-body systems, specifically the GEM and CSM. We discuss the numerical results in Sec. III. Finally, a brief summary is provided in Sec. IV.

II. FORMALISM

A. Effective Lagrangians and potentials

For the near-threshold hadronic molecular states considered in this work, the typical center-of-mass momenta are significantly smaller than the constituent masses of the charm and strange quarks. Therefore, it is reasonable to treat these heavy quarks as spectators and employ heavy meson effective Lagrangians to describe the interactions between hadrons. The Lagrangians in the OBE model are constructed based on heavy quark symmetry, chiral symmetry, and $SU(2)$ isospin symmetry [60,63,67,68,79,82,88–93]:

$$\mathcal{L}_{D^{(*)}D^{(*)}\sigma} = -2g_s D_b^\dagger D_b \sigma + 2g_s D_b^* \cdot D_b^{*\dagger} \sigma, \quad (1)$$

$$\mathcal{L}_{D^{(*)}D^{(*)}P} = +\frac{2g_a}{f_\pi}(D_b D_{a\lambda}^{*\dagger} + D_{b\lambda}^* D_a^\dagger)\partial^\lambda P_{ba} + i\frac{2g_a}{f_\pi}v^\alpha \varepsilon_{\alpha\mu\nu\lambda} D_b^{*\mu} D_a^{*\lambda\dagger} \partial^\nu P_{ba}, \quad (2)$$

$$\begin{aligned} \mathcal{L}_{D^{(*)}D^{(*)}V} = & +\sqrt{2}\beta g_V D_b D_a^\dagger v \cdot V_{ba} - 2\sqrt{2}\lambda g_V \\ & \times \varepsilon_{\lambda\mu\alpha\beta} v^\lambda (D_b D_a^{*\mu\dagger} + D_b^{*\mu} D_a^\dagger) (\partial^\alpha V_{ba}^\beta) \\ & - \sqrt{2}\beta g_V D_b^* \cdot D_a^{*\dagger} v \cdot V_{ba} \\ & - i2\sqrt{2}\lambda g_V D_b^{*\mu} D_a^{*\nu\dagger} (\partial_\mu V_\nu - \partial_\nu V_\mu)_{ba}, \end{aligned} \quad (3)$$

$$\mathcal{L}_{K^{(*)}K^{(*)}\sigma} = -2g'_s K_b^\dagger K_b \sigma + 2g'_s K_b^* \cdot K_b^{*\dagger} \sigma, \quad (4)$$

$$\mathcal{L}_{K^{(*)}K^{(*)}P} = +\frac{2g'_a}{f_\pi}(K_b K_{a\lambda}^{*\dagger} + K_{b\lambda}^* K_a^\dagger)\partial^\lambda P_{ba} + i\frac{2g'_a}{f_\pi}v'^\alpha \varepsilon_{\alpha\mu\nu\lambda} K_b^{*\mu} K_a^{*\lambda\dagger} \partial^\nu P_{ba}, \quad (5)$$

$$\begin{aligned} \mathcal{L}_{K^{(*)}K^{(*)}V} = & +\sqrt{2}\beta' g'_V K_b K_a^\dagger v' \cdot V_{ba} - 2\sqrt{2}\lambda' g'_V \\ & \times \varepsilon_{\lambda\mu\alpha\beta} v'^\lambda (K_b K_a^{*\mu\dagger} + K_b^{*\mu} K_a^\dagger) (\partial^\alpha V_{ba}^\beta) \\ & - \sqrt{2}\beta' g'_V K_b^* \cdot K_a^{*\dagger} v' \cdot V_{ba} \\ & - i2\sqrt{2}\lambda' g'_V K_b^{*\mu} K_a^{*\nu\dagger} (\partial_\mu V_\nu - \partial_\nu V_\mu)_{ba}, \end{aligned} \quad (6)$$

where $g_a^{(l)}$, $g_s^{(l)}$, $g_V^{(l)}$, $\beta^{(l)}$, and $\lambda^{(l)}$ are coupling constants. f_π is the pion decay constant. $v = (1, 0, 0, 0)$ is the velocity of the heavy meson. The matrices P and V represent the pseudoscalar and vector meson fields, respectively:

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \end{pmatrix}, \quad (7)$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \end{pmatrix}. \quad (8)$$

Here, we group the isotriplet π/ρ and the isosinglet η/ω into the same matrix according to SU(3) flavor symmetry to reduce the number of coupling constants.

Based on the OBE Lagrangians, the effective potentials are derived from the scattering Feynman diagrams shown in Fig. 1, including contributions from both direct and crossed diagrams. The effective potentials for the $D(p_1)D^*(p_2, \epsilon_2) \rightarrow D(p_3)D^*(p_4, \epsilon_4)$ are

$$V_\sigma^{DD^*} = -g_s^2 \frac{1}{q^2 + u_\sigma^2} \mathcal{O}_\sigma^D, \quad (9)$$

$$V_{\pi/\eta}^{DD^*} = +\frac{g_a^2}{f_\pi^2} \frac{(\mathbf{k} \cdot \boldsymbol{\epsilon}_4^\dagger)(\mathbf{k} \cdot \boldsymbol{\epsilon}_2)}{\mathbf{k}^2 + u_{\pi/\eta}^2} \mathcal{O}_{\pi/\eta}^C, \quad (10)$$

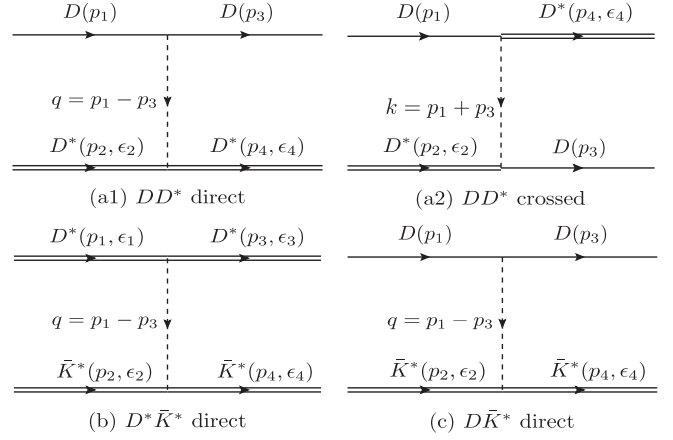


FIG. 1. Feynman diagrams to derive OBE potentials: (a1) and (a2) for DD^* , (b) for $D^*\bar{K}^*$, and (c) for $D\bar{K}^*$.

$$\begin{aligned} V_{\rho/\omega}^{DD^*} = & +\frac{\beta^2 g_v^2}{2} \frac{(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger)}{q^2 + u_{\rho/\omega}^2} \mathcal{O}_{\rho/\omega}^D \\ & + 2\lambda^2 g_v^2 \frac{\mathbf{k}^2 (\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) - (\mathbf{k} \cdot \boldsymbol{\epsilon}_4^\dagger)(\mathbf{k} \cdot \boldsymbol{\epsilon}_2)}{\mathbf{k}^2 + u_{\rho/\omega}^2} \mathcal{O}_{\rho/\omega}^C. \end{aligned} \quad (11)$$

The effective potentials for the $D^*(p_1, \epsilon_1)\bar{K}^*(p_2, \epsilon_2) \rightarrow D^*(p_3, \epsilon_3)\bar{K}^*(p_4, \epsilon_4)$ are

$$\mathcal{V}_\sigma^{D^*\bar{K}^*} = -g_s g'_s \frac{(\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger)}{q^2 + u_\sigma^2} \mathcal{O}_\sigma^D, \quad (12)$$

$$\mathcal{V}_{\pi/\eta}^{D^*\bar{K}^*} = +\frac{g_a g'_a}{f_\pi^2} \frac{((\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger) \cdot \mathbf{q})((\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \cdot \mathbf{q})}{q^2 + u_{\pi/\eta}^2} \mathcal{O}_{\pi/\eta}^D, \quad (13)$$

$$\begin{aligned} \mathcal{V}_{\rho/\omega}^{D^*\bar{K}^*} = & \left\{ +\frac{\beta\beta' g_V g'_V}{2} \frac{(\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger)}{q^2 + u_{\rho/\omega}^2} \right. \\ & + \frac{2\lambda\lambda' g_V g'_V}{q^2 + u_{\rho/\omega}^2} [(\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2)(\boldsymbol{\epsilon}_3^\dagger \cdot \mathbf{q})(\boldsymbol{\epsilon}_4^\dagger \cdot \mathbf{q}) \\ & - (\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_4^\dagger)(\boldsymbol{\epsilon}_3^\dagger \cdot \mathbf{q})(\boldsymbol{\epsilon}_2 \cdot \mathbf{q}) \\ & - (\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_4^\dagger \cdot \mathbf{q})(\boldsymbol{\epsilon}_1 \cdot \mathbf{q}) \\ & \left. + (\boldsymbol{\epsilon}_3^\dagger \cdot \boldsymbol{\epsilon}_4^\dagger)(\boldsymbol{\epsilon}_1 \cdot \mathbf{q})(\boldsymbol{\epsilon}_2 \cdot \mathbf{q}) \right] \mathcal{O}_{\rho/\omega}^D. \end{aligned} \quad (14)$$

The effective potentials for the $D(p_1)\bar{K}^*(p_2, \epsilon_2) \rightarrow D(p_3)\bar{K}^*(p_4, \epsilon_4)$ are

$$\mathcal{V}_\sigma^{D\bar{K}^*} = -g_s g'_s \frac{1}{q^2 + u_\sigma^2} \mathcal{O}_\sigma^D, \quad (15)$$

$$\mathcal{V}_{\rho/\omega}^{D\bar{K}^*} = +\frac{\beta\beta' g_V g'_V}{2} \frac{\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger}{q^2 + u_{\rho/\omega}^2} \mathcal{O}_{\rho/\omega}^D. \quad (16)$$

TABLE II. The isospin operators $\mathcal{O}_{\text{exc}}$ in Eqs. (9)–(16). $\mathcal{O}^D$ and $\mathcal{O}^C$ represent operators for the direct and crossed diagram, respectively. $\mathbb{I}$ is the 2×2 identity matrix and $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ denotes the SU(2) isospin Pauli matrix acting on the heavy meson fields.

Meson	σ	π	η	ρ	ω
$\mathcal{O}^D$	$\mathbb{I}$	$\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{2}$	$\frac{1}{6}\mathbb{I}$	$\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{2}$	$\frac{1}{2}\mathbb{I}$
$\mathcal{O}^C$		$-\frac{1}{4}(3\mathbb{I} - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$	$-\frac{1}{12}(\mathbb{I} + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$	$-\frac{1}{4}(3\mathbb{I} - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$	$-\frac{1}{4}(\mathbb{I} + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$

The effective potentials for the hidden-charm $D\bar{D}^*/\bar{D}D^*$ systems can be found in Ref. [82]. The isospin operators $\mathcal{O}$ are listed in Table II. In the direct diagram, the momentum of the propagator is $q = p_1 - p_3$, whereas in the crossed diagram, it is $k = p_1 + p_3$. As discussed in Refs. [78,82], this distinction significantly affects systems with odd partial waves. u_{exc} represents the effective mass of the exchanged meson, defined as

$$\frac{-1}{k_0^2 - \mathbf{k}^2 - m_{\text{exc}}^2} = \frac{1}{\mathbf{k}^2 + (m_{\text{exc}}^2 - k_0^2)} = \frac{1}{\mathbf{k}^2 + u_{\text{exc}}^2}, \quad (17)$$

$$\frac{-1}{q_0^2 - \mathbf{q}^2 - m_{\text{exc}}^2} = \frac{1}{\mathbf{q}^2 + (m_{\text{exc}}^2 - q_0^2)} = \frac{1}{\mathbf{q}^2 + u_{\text{exc}}^2}. \quad (18)$$

In the static approximation, the energy transfers k_0 and q_0 are the mass differences between the initial and final states. Notably, we have $q_0 = 0$ and $k_{D\bar{D}^*,0}^2 \approx m_\pi^2$. Therefore, we set $u = 0$ for the crossed pion exchange in the DD^* system and $u = m_{\text{exc}}$ for all other cases.

To regularize potential short-range singularities, we introduce a form factor to suppress high-momentum contributions:

$$V(\mathbf{p}, u) \rightarrow V(\mathbf{p}, u)F^2(u, \Lambda, \mathbf{p}^2),$$

$$F(u, \Lambda, \mathbf{p}^2) = \frac{\Lambda^2 - u^2}{\Lambda^2 + \mathbf{p}^2}, \quad (19)$$

where Λ denotes the cutoff parameter. In this work, we vary the cutoff from 1.10 to 1.35 GeV in steps of 0.05 GeV to address the cutoff dependence. The coordinate space potential can be readily obtained via Fourier transformation, as presented in the Appendix.

The values of the OBE parameters are listed in Table III. The pion decay constant f_π is a well-known quantity, while

the axial coupling constants g_a and g'_a are extracted from the widths of the $D^* \rightarrow D\pi$ and $K^* \rightarrow K\pi$, respectively. However, the remaining three vector coupling constants g_V , β , and λ , as well as the scalar coupling g_s , can only be determined through model-dependent methods. Three of these are independent, as g_V acts as an overall factor in the vector sector. The values in the table are adopted from Refs. [67,68] as a baseline, which are determined using vector meson dominance combined with results from lattice QCD and light cone sum rules [94]. Although the coupling constants in the charm and strange sectors may differ in principle, we set $g'_s = g_s$, $g'_V = g_V$, $\beta' = \beta$, and $\lambda' = \lambda$ to reduce the number of the parameters.

Following the approach in Ref. [82], we introduce three ratio factors:

$$\lambda \rightarrow \lambda R_\lambda, \quad \beta \rightarrow \beta R_\beta, \quad g_s \rightarrow g_s R_s, \quad (20)$$

to investigate the impact of variations in the scalar and vector coupling constants and to address model dependence. As demonstrated in Ref. [82] and illustrated in Fig. 2, for a special state $Z_c(3900)$, which is considered as a two-body $[D^*\bar{D}]_{I=1}^{C=-1}$ system, the exchange potentials of the ρ and ω mesons are nearly equal in magnitude and effectively cancel each other. This cancellation renders the pole position of the $Z_c(3900)$ primarily sensitive to σ exchange R_s . Considering the physical nature of the $Z_c(3900)$ remains a subject of debate [95–99], we assume $Z_c(3900)$ is a virtual state and treat its pole position, or equivalently R_s , as a free parameter. We vary the pole position of the $Z_c(3900)$ virtual state from -5 to -30 MeV in increments of 2.5 MeV. Subsequently, for each cutoff Λ and $Z_c(3900)$ pole position, we fix the R_λ and R_β by reproducing the isospin averaged binding energies of the $X(3872)$ ($\Delta E = -4.09$ MeV) and $T_{cc}(3875)$

TABLE III. Hadron masses and coupling constants in the OBE Lagrangians. The constants g_a and g'_a are extracted from the widths of the $D^* \rightarrow D\pi$ and $K^* \rightarrow K\pi$, respectively. The experimental data are taken from Ref. [21]. The values for g_s , λ , and β are adopted from Refs. [67,68,94] and serve as baseline parameters, where three rescaling factors in Eq. (20) are applied.

Mass	m_D [GeV]	m_{D^*} [GeV]	m_π [GeV]	m_η [GeV]	m_ρ [GeV]	m_ω [GeV]	m_σ [GeV]	m_{K^*} [GeV]
	1.867	2.009	0.137	0.548	0.775	0.783	0.600	0.894
Coupling	f_π [GeV]	g_a	g'_a	$g_V = g'_V$	$\beta = \beta'$	$\lambda = \lambda' [\text{GeV}^{-1}]$	$g_s = g'_s$	
	0.13025	0.57	0.88	5.8	0.9	0.56	0.76	

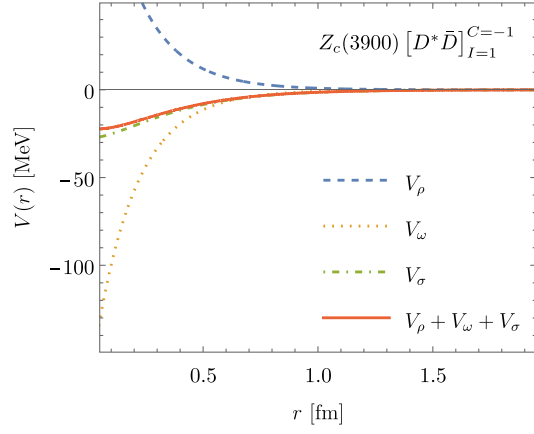


FIG. 2. Effective potentials of the scalar and vector meson exchange in the $Z_c(3900)[D^*\bar{D}]_{I=1}^{C=-1}$ system. The blue dashed, yellow dotted, and green dot-dashed lines represent the contributions from the ρ , ω , and σ exchanges, respectively. The red solid line denotes the total of them. The cutoff Λ is set to 1.10 GeV, and the parameters $(R_\beta, R_\lambda, R_s)$ are set to (1.0, 1.0, 2.0) as an example.

($\Delta E = -1.6$ MeV) as listed in Table I. The resulting coupling constants are presented in Fig. 3. It is evident that R_s correlates strongly with the pole position of the $Z_c(3900)$, whereas R_β and R_λ remain close to unity with minimal variation.

Three-body forces may also exist in the system. However, such interactions remain elusive even in nucleon systems [100]. In chiral effective field theory, the three-nucleon force first appears at next-to-next-to-leading order [101,102]. Power counting in this framework places the typical size of three-nucleon terms at only a few percent (roughly 5%) of the two-nucleon contribution [103]. Phenomenologically, light nuclei show a similar hierarchy between three- and two-nucleon binding-energy contributions [104]. Recently, Ref. [105] showed that, in hadronic systems with definite C parity, a C -parity-dependent transition term can act as an intrinsic three-body force. The present $DD^*\bar{K}^*$ system is different since it contains three isospin-1/2 hadrons whose light-flavor degrees of freedom are all antiquarks and has no definite charge parity. Its leading dynamics is therefore expected to be closer to

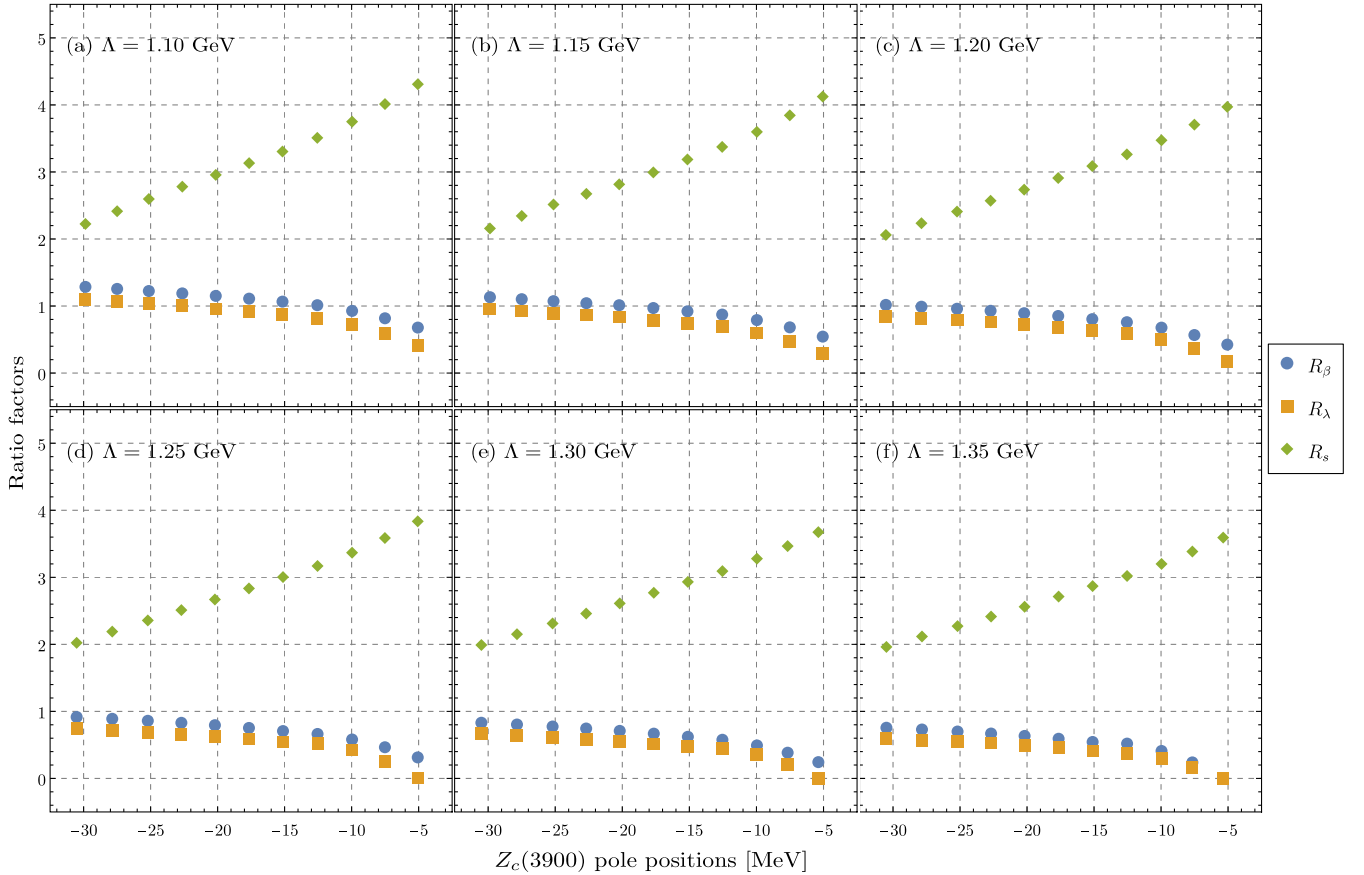


FIG. 3. The ratio factors for the scalar- and vector-meson-exchange interactions vary with the $Z_c(3900)$ virtual state pole position across different cutoffs. The blue circles, yellow squares, and green diamonds represent R_β , R_λ , and R_s , respectively. These ratios are defined relative to the baseline values as detailed in Eq. (20) and Table III.

that of the three-nucleon system, where pairwise interactions dominate. We expect the three-body-force effect in the present system to be of the same order as in the three-nucleon case and smaller than the present uncertainty associated with the two-body OBE interaction. Moreover, isolating three-body forces through physical observables is challenging. For these reasons, we neglect the three-body force in this work.

B. Methodology

The three-body system is investigated using a non-relativistic Hamiltonian

$$H = \sum_{i=1}^3 \left(m_i + \frac{p_i^2}{2m_i} \right) + \sum_{i<j} V_{ij}, \quad (21)$$

where V_{ij} is the two-body effective potential of the OBE model defined in Eqs. (9)–(16). The S -wave $DD^*\bar{K}^*$ three-body wave function can be expressed as

$$\Psi_{I,J} = \sum_{\alpha, n_i, n'_i, i} C_{\alpha, n_i, n'_i, i} \chi_{\alpha}^{I,J} \times \left[\phi_{n_i, l_m}^G(\mathbf{r}_i) \phi_{n'_i, l'_m}^G(\mathbf{R}_i) \right]_{L=0}, \quad (22)$$

where R_i and r_i represent the independent Jacobi coordinates shown in Fig. 4. The expansion coefficients $C_{\alpha, n_i, n'_i, i}$ are determined via the Rayleigh-Ritz variational method. $\chi_{\alpha}^{I,J}$ denotes the spin-isospin wave functions, given by

$$\begin{aligned} \chi_1^{\frac{1}{2}, J} &= \{D[D^*\bar{K}^*]^{0,J}\}^{\frac{1}{2}, J}, \\ \chi_2^{\frac{1}{2}, J} &= \{D[D^*\bar{K}^*]^{1,J}\}^{\frac{1}{2}, J}, \\ \chi_3^{\frac{3}{2}, J} &= \{D[D^*\bar{K}^*]^{1,J}\}^{\frac{3}{2}, J}, \end{aligned} \quad (23)$$

where $J = 0, 1, 2$.

We employ the GEM to solve the three-body Schrödinger equation. This approach is widely used in few-body calculations and has proven to be highly effective [83]. In GEM, the spatial wave functions take the form

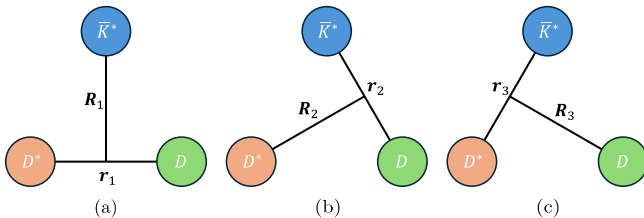


FIG. 4. The three sets of Jacobi coordinates for the $DD^*\bar{K}^*$ three-body system.

$$\phi_{nlm}(\mathbf{r}) = \sqrt{\frac{2^{l+5/2}}{\Gamma(l+\frac{3}{2})r_n^3}} \left(\frac{r}{r_n} \right)^l e^{-\frac{r}{r_n}} Y_{lm}(\hat{r}), \quad (24)$$

where Y_{lm} represents the spherical harmonic function. It is worth noting that any single set of Jacobi coordinates $(\mathbf{r}_i, \mathbf{R}_i)$ suffices when considering an infinite number of radially and orbitally excited basis functions. However, a more efficient approach involves restricting the calculation to the S -wave $\phi_{n00}(\mathbf{r}_i)\phi_{n'00}(\mathbf{R}_i)$, while utilizing all three sets of Jacobi coordinates ($i = 1$ to 3) [83,106,107]. The inclusion of S -wave functions from all Jacobi coordinates also partially compensates for contributions from higher partial wave angular momentum components within a single coordinate. The Gaussian size parameter r_n follows a geometric progression:

$$r_n = r_1 a^{n-1}, \quad a = \left(\frac{r_1}{r_{\max}} \right)^{(n_{\max}-1)}. \quad (25)$$

In this work, the size parameters are taken as

$$r_1 = 0.01 \text{ fm}, \quad r_{\max} = 10 \text{ fm}, \quad n_{\max} = 20, \quad (26)$$

which cover both long-range and short-range correlations and provide an efficient approximation for the radial component of the wave functions.

The approach described above is highly effective for solving bound states. However, the wave functions of resonances are not square integrable and cannot be obtained by directly solving the eigenequation of the Hermitian Hamiltonian. To identify three-body resonant solutions, we employ the CSM [84–87]. This technique introduces a complex rotation $U(\theta)$ to the radial coordinate r and the conjugate momentum p :

$$U(\theta)r = re^{i\theta}, \quad U(\theta)p = pe^{-i\theta}, \quad (27)$$

$$H(\theta) = \sum_{i=1}^3 \left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i} \right) + \sum_{i<j} V_{ij}(r_{ij}e^{i\theta}). \quad (28)$$

This transformation allows resonances to be treated similarly to bound states through square-integrable basis expansion. Solving the eigenvalue equation of the complex-scaled Hamiltonian yields the eigenenergies of bound, scattering, and resonant states simultaneously. Figure 5 illustrates a typical distribution of eigenenergies obtained by the CSM. The scattering states align along rays starting from threshold energies with $\text{Arg}(E) = -2\theta$, whereas bound states and resonances remain unchanged as θ varies. The resonance with energy $E_R = m_R - i\Gamma_R/2$ appears when $2\theta > |\text{Arg}(E_R)|$, where m_R and Γ_R denote the mass and width of the resonance, respectively. The combination of the GEM and the CSM has proven to be a powerful tool for investigating few-body resonances [87,108–117].

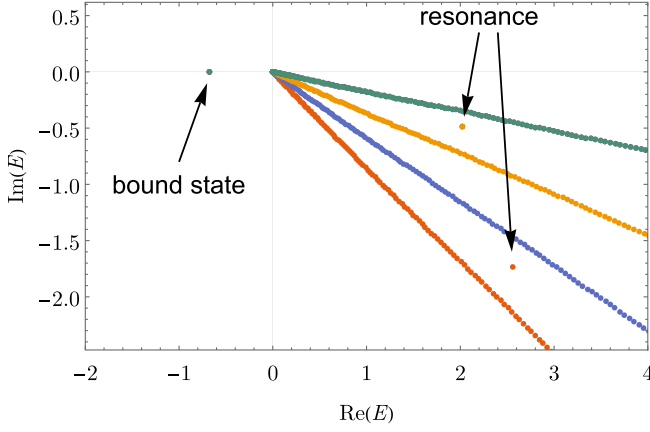


FIG. 5. A typical solution of the complex-scaled Schrödinger equation. The scattering states align along $\text{Arg}(E) = -2\theta$, whereas bound states and resonances do not shift as θ changes. The resonance with energy $E_R = m_R - i\frac{\Gamma_R}{2}$ can be obtained when $2\theta > |\text{Arg}(E_R)|$.

III. RESULTS AND DISCUSSIONS

A. $D^*\bar{K}^*$ system

Before investigating the three-body systems, we first calculate the binding energies of the $I(J^P) = 0(0^+)D^*\bar{K}^*$ two-body system using the parameters presented in Fig. 3, with the results illustrated in Fig. 6. For each parameter set, corresponding to various cutoff values and $Z_c(3900)$ virtual state pole positions, we consistently obtain a bound state with a binding energy of approximately 30 MeV. The slight cutoff dependence in the results arises from our assumption that the parameters Λ , β , λ , and g_s are identical for the charm and strange sectors, with they are calibrated solely by the charm sector. Nevertheless, each parameter set provides a good description of the experimentally observed $T_{cs0}(2870)$ without additional tuning, which lies 30 ± 17 MeV below the $D^*\bar{K}^*$ threshold [16–18,21]. This agreement indicates that treating the $\bar{K}^*$ as a heavy meson, combined with the OBE effective potential and the

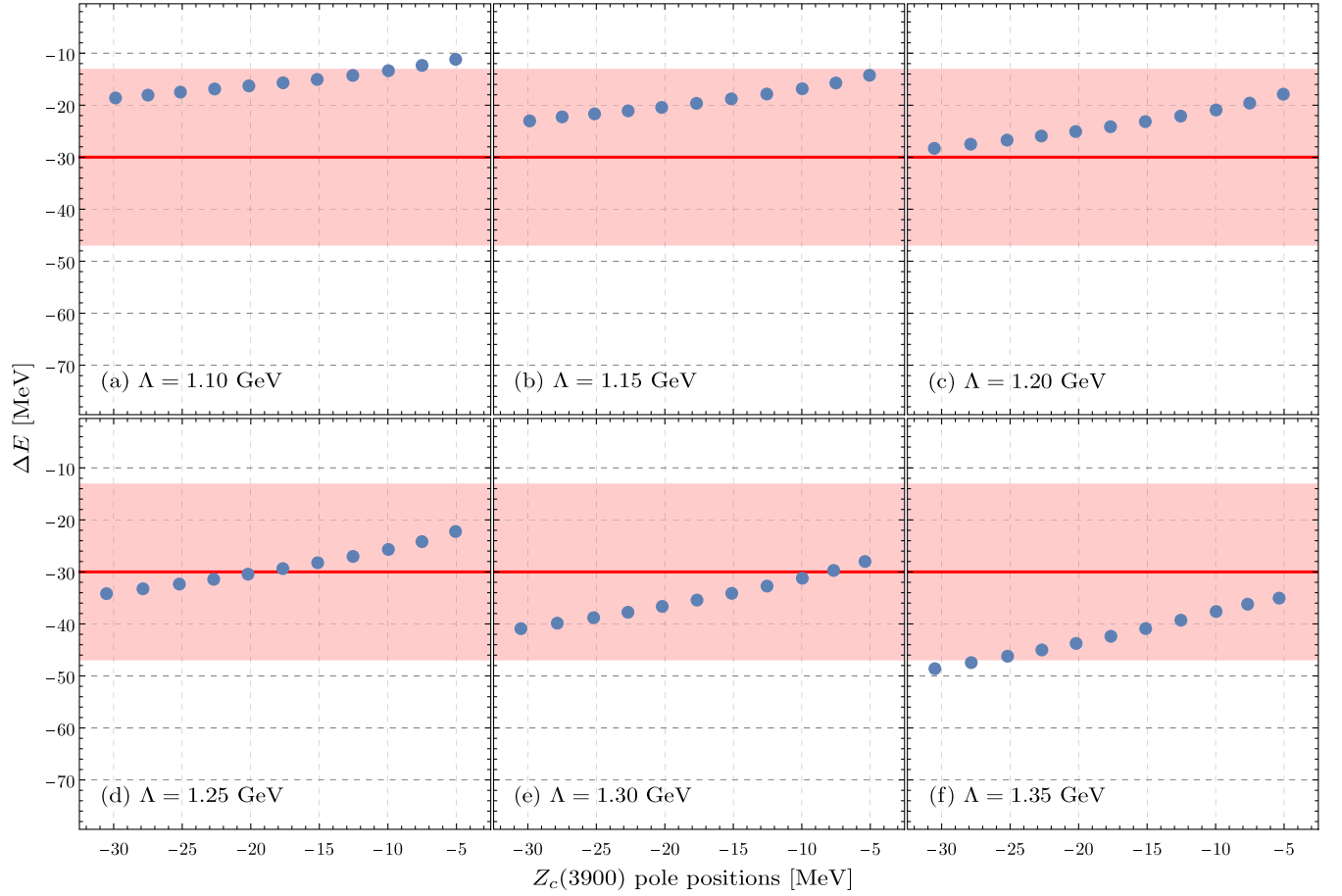


FIG. 6. The binding energies of the $I(J^P) = 0(0^+)D^*\bar{K}^*$ two-body system vary with the $Z_c(3900)$ virtual state pole position across different cutoffs. The red solid line indicates the experimental central value of the mass difference between the $T_{cs0}(2870)$ and the $D^*\bar{K}^*$ threshold, and the red shaded region represents the experimental uncertainty. The experimental data are taken from Ref. [21].

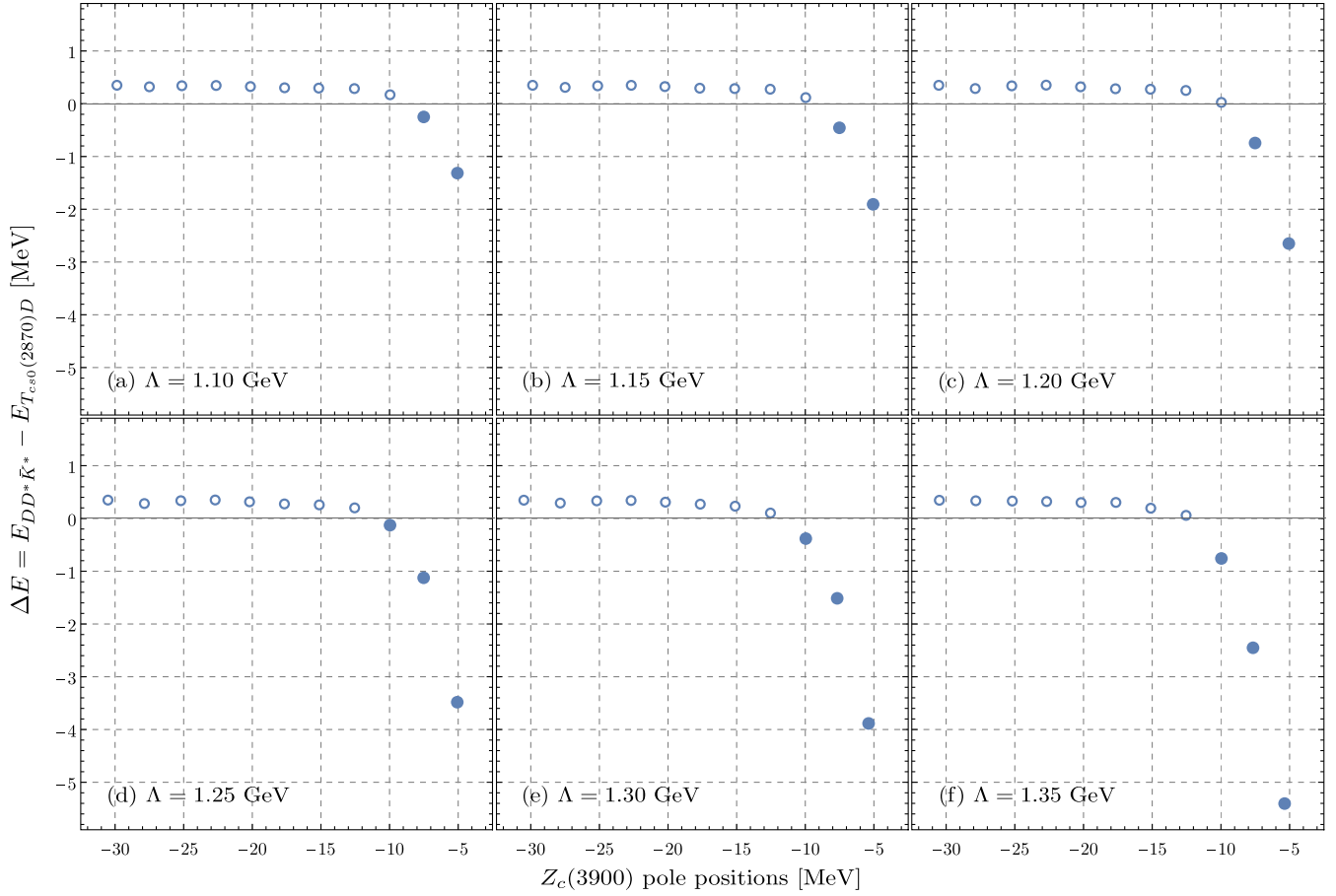


FIG. 7. The binding energies ΔE of the $I(J^P) = 1/2(0^-)$ $DD^*\bar{K}^*$ three-body system vary with the $Z_c(3900)$ virtual state pole position across different cutoffs. $\Delta E = E_{DD^*\bar{K}^*} - E_{T_{cs0}(2870)D}$ is defined as the energy difference between the three-body ground state and the theoretical $T_{cs0}(2870)D$ threshold, where the $T_{cs0}(2870)$ corresponds to the $I(J^P) = 0(0^+)D^*\bar{K}^*$ two-body bound state with binding energies shown in Fig. 6. Solid points indicate bound states ($\Delta E < 0$), while hollow points indicate unbound states ($\Delta E > 0$).

recalibrated parameters, provides a reliable description of the $D^*(\bar{K}^*)$ interactions. Furthermore, these results validate the application of this framework and these parameter sets to the investigation of other systems.

B. $DD^*\bar{K}^*$ system

We subsequently investigate the $DD^*\bar{K}^*$ three-body system. Using the parameter sets presented in Fig. 3, we calculate the ground state energy of the $DD^*\bar{K}^*$ system. The results for the $I(J^P) = 1/2(0^-)$ configuration are displayed in Fig. 7. The energy difference ΔE is defined relative to the theoretical $T_{cs0}(2870)D$ threshold, where the $T_{cs0}(2870)$ is treated as the $I(J^P) = 0(0^+)D^*\bar{K}^*$ two-body bound state, with binding energies shown in Fig. 6. Negative values of ΔE indicate the formation of a three-body molecular bound state, while positive values indicate unbound cases.

As shown in Fig. 7, the existence of a three-body bound state is strongly correlated with the pole position of the $Z_c(3900)$. As the $Z_c(3900)$ virtual state pole approaches

the threshold, corresponding to a stronger attractive σ exchange, the binding energy of the three-body system increases. Conversely, as the $Z_c(3900)$ pole moves away from the threshold, indicating a weaker σ -exchange attraction, the three-body binding energy decreases until the bound state disappears. Our results indicate that a three-body molecular bound state emerges when the $Z_c(3900)$ virtual state pole lies within approximately -10 MeV of the threshold. The binding energies are a few MeV. Notably, varying the cutoff does not qualitatively alter these results, implying that the connection between the existence of the $DD^*\bar{K}^*$ molecule and the $Z_c(3900)$ virtual state pole position is robust. At the quark level, this three-body bound state would correspond to a hexaquark state that has not yet been observed in experiment. Experimentally, this three-body bound state can be searched for in decay channels such as $DD\bar{K}\pi\pi$ and $DD\bar{K}$. Observation of such a state would constrain the position of the $Z_c(3900)$ pole. Meanwhile, a precise measurement of the $Z_c(3900)$ pole position would determine the existence or nonexistence of this three-body molecule.

We calculated the binding energies for all S -wave quantum numbers, specifically $I = (1/2, 3/2)$ with $J^P = (0, 1, 2)^-$. However, with the exception of the $1/2(0^-)$ channel, no three-body bound states were found in other configurations.

In our calculations, we neglect isospin breaking and coupled channels involving $D^*D^*\bar{K}^*$, $D^{(*)}D^{(*)}\bar{K}$, and baryon-antibaryon channels. Coupled-channel dynamics can be important for near-threshold systems, but its impact is expected to be limited for the present three-body bound state. For the lower mesonic channels, the nearest threshold is $D^*D^*\bar{K}$, which lies about $(m_{K^*} - m_K) - (m_{D^*} - m_D) \simeq 260$ MeV below the channel of interest. Such a large energy separation suppresses its contribution to the near-threshold eigenenergy. This expectation is also supported by the two-body coupled-channel calculation within the same OBE framework in Ref. [79], where both $T_{cs0}(2870)$ and $T_{cs1}(2900)$ were found to be almost pure $D^*\bar{K}^*$ molecular states. In addition, the K meson is a light pseudo-Goldstone boson, so treating the $D^{(*)}D^{(*)}\bar{K}$ channels consistently would require dynamics beyond the

heavy-meson formalism and nonrelativistic framework adopted for the present $D^{(*)}\bar{K}^*$ interactions. For the higher mesonic channel, the nearest $D^*D^*\bar{K}^*$ threshold lies about $m_{D^*} - m_D \simeq 140$ MeV above the states of interest. According to the variational principle, including this higher coupled channel would lower the ground-state energy and thus make the three-body bound state slightly more deeply bound. Owing to the sizable threshold gap, however, this correction should be moderate and would not change the qualitative conclusion on the existence of the $DD^*\bar{K}^*$ bound state. The baryon-antibaryon channels involve short-range dynamics beyond the molecular basis and are also not expected to dominate the near-threshold solution. For isospin breaking, Ref. [24] included effects from mass splittings and Coulomb interactions in the DDD^* three-body system. In Ref. [82], the isospin-breaking effects in the $X(3872)$ and $T_{cc}(3875)$ were examined in detail. The binding-energy shifts of the $X(3872)$ and $T_{cc}(3875)$ were found to mainly follow the mass differences between the physical $D^0\bar{D}^{*0}$ and $D^{*+}D^0$ thresholds and their corresponding isospin-averaged thresholds.

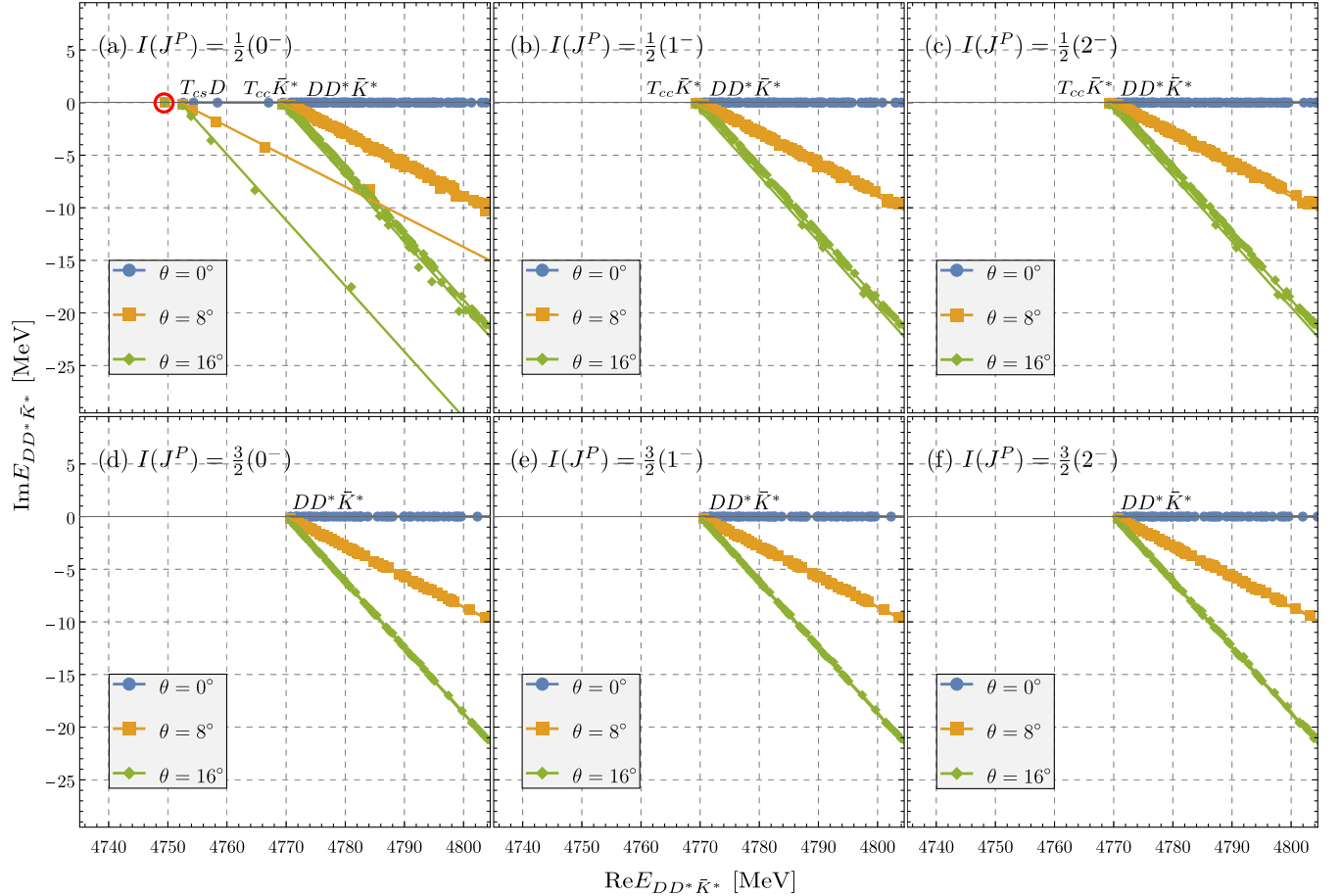


FIG. 8. Complex energy spectrum of the $DD^*\bar{K}^*$ three-body system calculated using the CSM with varying complex scaling angles θ . The parameter set corresponds to a cutoff $\Lambda = 1.20$ GeV and a $Z_c(3900)$ virtual state pole at -5 MeV. With the exception of the bound state marked by the red circle, all other states represent scattering states aligned along rays.

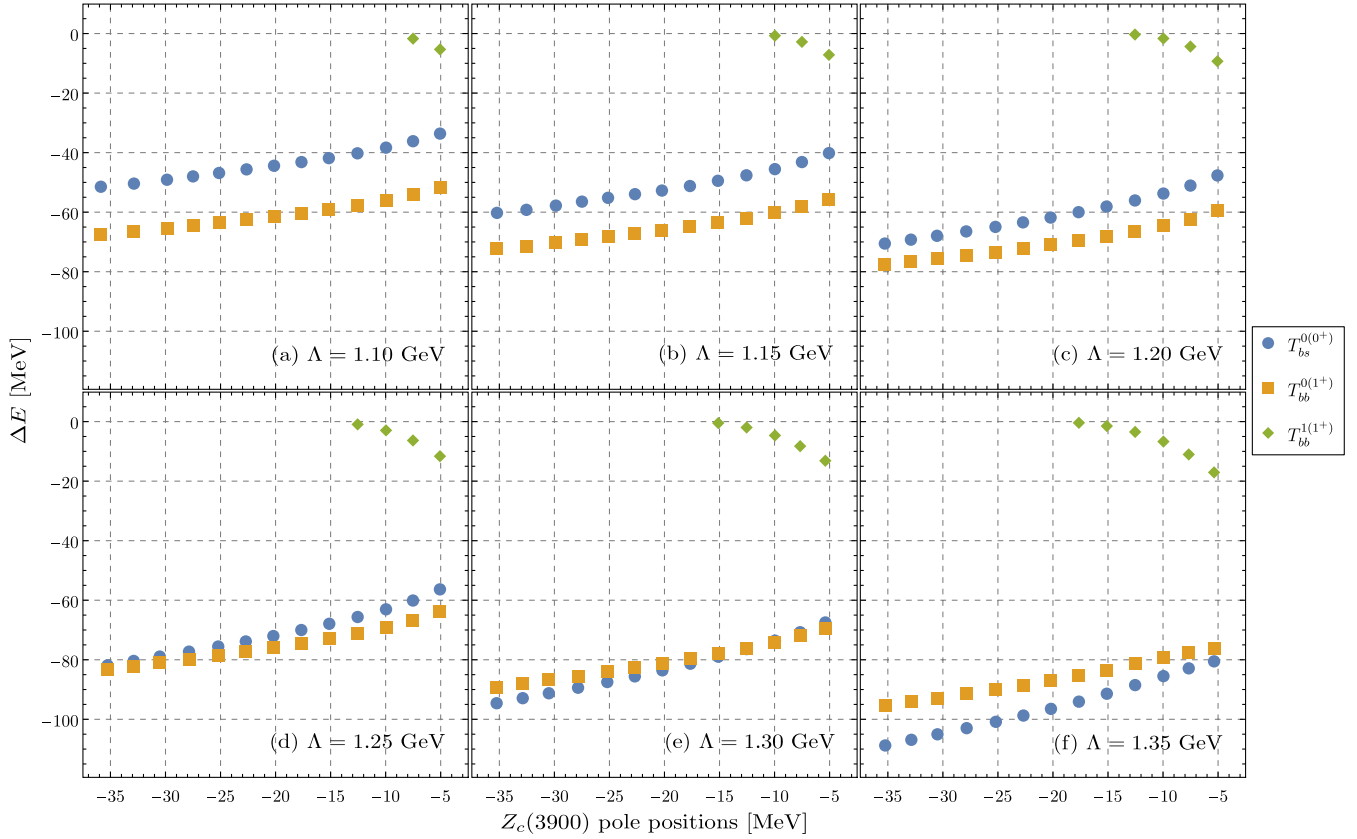


FIG. 9. The binding energies of the two-body subsystem bound states within the BB^*K^* three-body system vary with the $Z_c(3900)$ virtual state pole position across different cutoffs. The blue circles, yellow squares, and green diamonds represent the $0(0^+)B^*\bar{K}^*$ bound state $T_{bs}^{0(0+)}$, the $0(1^+)BB^*$ bound state $T_{bb}^{0(1+)}$, and the $1(1^+)BB^*$ bound state $T_{bb}^{1(1+)}$, respectively.

Applying this threshold estimate to the $DD^*\bar{K}^*$ system gives an effect of order 1 MeV. This is much smaller than the uncertainty from the strong interaction already present in the isospin-symmetric calculation. We therefore do not include the minor isospin-violation and Coulomb effects in this work.

In addition to bound states, we employ the CSM to search for resonances within the three-body system. Figure 8 illustrates a representative result. With the exception of a bound state pole, all other eigenenergies align along rays that rotate with the scaling angle θ , indicating that they correspond to scattering states. The results for other parameter sets are similar; we do not identify any three-body resonances in any of the considered channels.

C. BB^*K^* system

Regarding the BB^*K^* system, which serves as the bottom analog of the $DD^*\bar{K}^*$ system, the lack of experimental data prevents the extraction of the coupling constants. Consequently, based on heavy quark symmetry, we employ the same parameters as used for the $DD^*\bar{K}^*$

system, replacing only the charm meson masses (m_D, m_{D^*}) with the bottom meson masses (m_B, m_{B^*}). While this approach introduces theoretical uncertainties, it allows for a qualitative discussion.

The binding energies of the two-body subsystem bound states within the BB^*K^* three-body system are presented in Fig. 9. Evidently, the various two-body subsystems in the BB^*K^* sector exhibit deeper binding due to the larger mass of the bottom mesons. In particular, the BB^* system with $I(J^P) = 1(1^+)$ forms a shallow bound state, denoted as $T_{bb}^{1(1+)}$, when the σ -exchange interaction is strong, or equivalently, when the virtual state pole of the $Z_c(3900)$ lies close to the threshold. Furthermore, at a cutoff $\Lambda \approx 1.30$ GeV, the hierarchy of binding energies between the $0(1^+)BB^*$ bound state $T_{bb}^{0(1+)}$ and the $0(0^+)B^*\bar{K}^*$ bound state $T_{bs}^{0(0+)}$ is reversed. The lowest two-body threshold of the three-body system changes at this point.

The ground state energies of the $I(J^P) = 1/2(0^-)$ BB^*K^* system are displayed in Fig. 10. The energy difference ΔE is defined relative to the lowest two-body

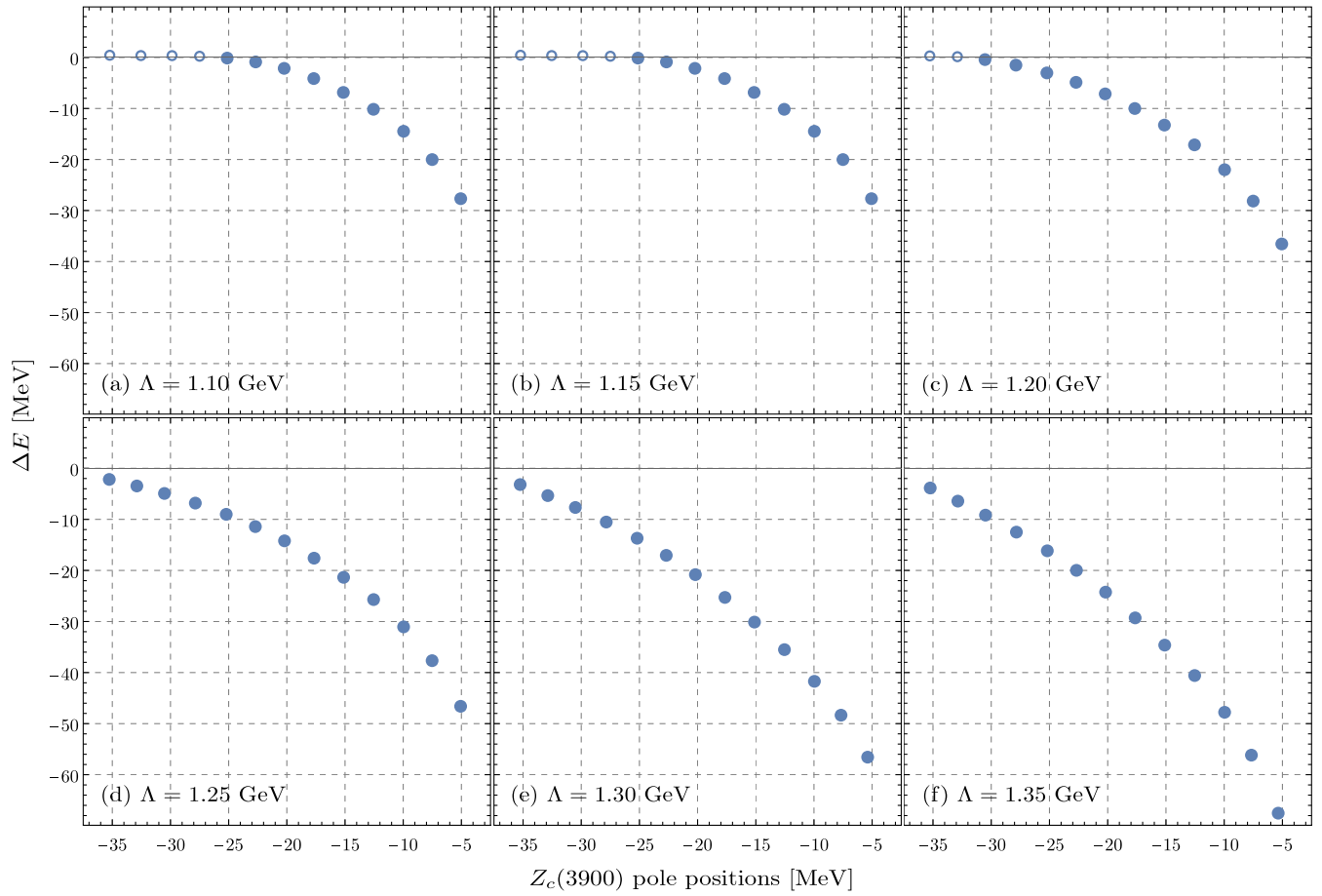


FIG. 10. The binding energies ΔE of the $I(J^P) = 1/2(0^-)$ BB^*K^* three-body system vary with the $Z_c(3900)$ virtual state pole position across different cutoffs. ΔE is defined as the energy difference between the three-body ground state and the lowest two-body threshold of the three-body system. Solid points indicate bound states ($\Delta E < 0$), while hollow points indicate unbound states ($\Delta E > 0$).

threshold of the three-body system. The results indicate that the conditions for forming a three-body bound state are significantly more relaxed in the BB^*K^* system. Depending on the specific cutoff, a $Z_c(3900)$ virtual state located between -25 and -35 MeV below the threshold suffices to form a three-body bound state. Moreover, the three-body state is more deeply bound, with binding energies reaching several tens of MeV. Apart from the

$1/2(0^-)$ channel, no three-body bound states are found in other configurations.

A typical complex energy spectrum of the BB^*K^* system calculated using the CSM is illustrated in Fig. 11. Similar to the $DD^*\bar{K}^*$ system, all solutions other than the bound state correspond to scattering states. We do not identify any three-body resonances in any of the considered channels.

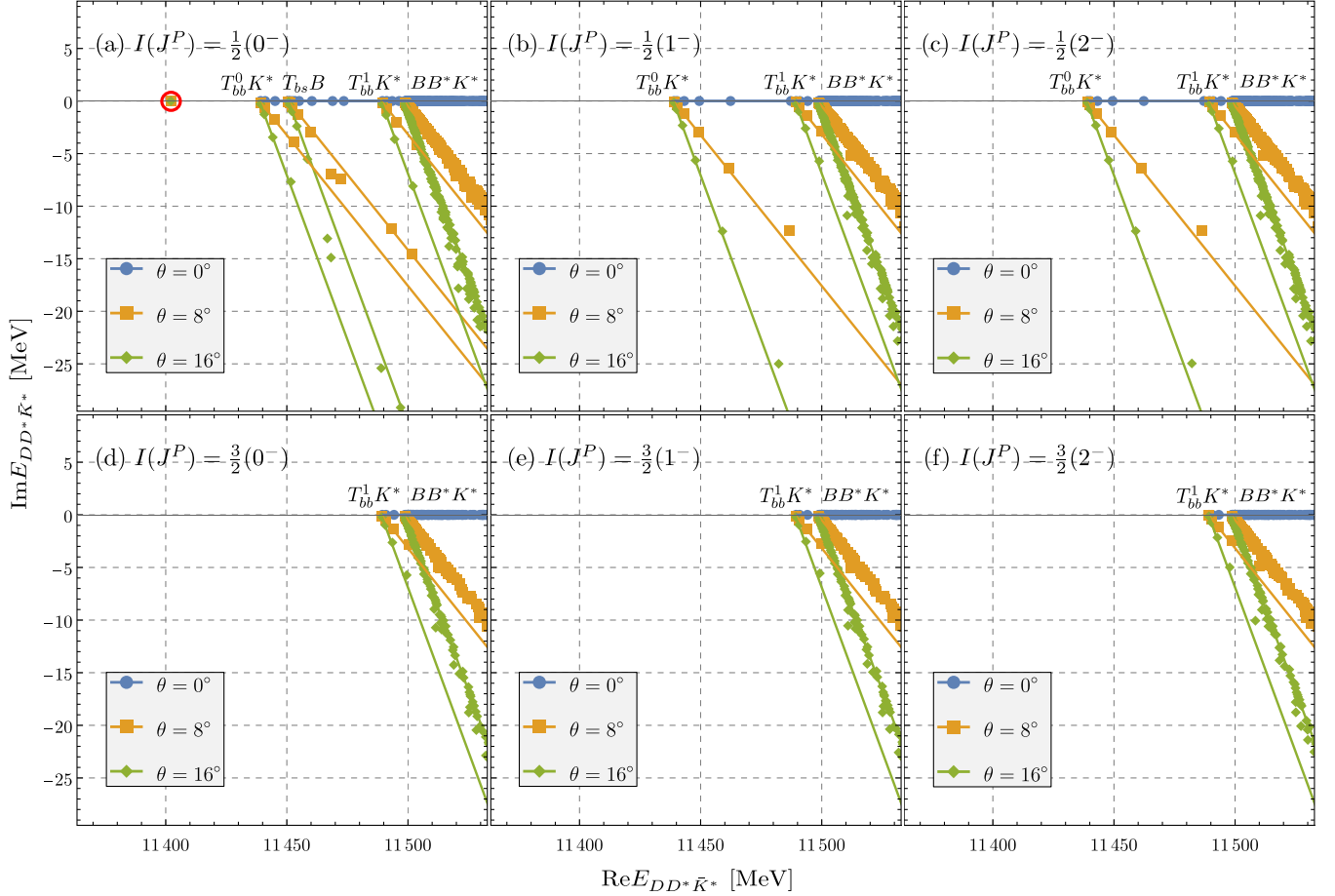


FIG. 11. Complex energy spectrum of the BB^*K^* three-body system calculated using the CSM with varying complex scaling angles θ . The parameter set corresponds to a cutoff $\Lambda = 1.20$ GeV and a $Z_c(3900)$ virtual state pole at -5 MeV. With the exception of the bound state marked by the red circle, all other states represent scattering states aligned along rays.

IV. SUMMARY

In this work, we investigated the existence of the $DD^*\bar{K}^*$ three-body molecular state and its bottom analog, the BB^*K^* system, within the OBE model. To address the inherent model dependence associated with scalar- and vector-meson-exchange interactions, we recalibrated their coupling constants. We assumed the $Z_c(3900)$ is a virtual state and treated its pole position as a free parameter, thereby constraining the strength of the scalar σ -exchange interaction. The vector couplings were determined using the well-established $X(3872)$ and $T_{cc}(3875)$ states. The resulting parameter sets successfully reproduce the mass of the $T_{cs0}(2870)$ without further tuning, validating the application of this framework and these parameter sets to the investigation of $D^{(*)}\bar{K}^*$ and other systems.

We employed the Gaussian expansion method to solve the three-body Schrödinger equation. Our results reveal a strong correlation between the existence of the three-body molecular bound state and the pole position of the $Z_c(3900)$ virtual state. Specifically,

an $I(J^P) = 1/2(0^-)$ $DD^*\bar{K}^*$ bound state exists when the $Z_c(3900)$ virtual state is located within approximately -10 MeV of the DD^* threshold, with a binding energy of a few MeV. For the BB^*K^* system, the conditions for forming a three-body bound state are more relaxed; a $Z_c(3900)$ virtual state located within -25 MeV of the threshold suffices. Furthermore, this state is more deeply bound, with a binding energy reaching several tens of MeV.

We also employ the complex scaling method to search for three-body resonances, but no such states are identified in either the charm or bottom sectors for any of the considered channels.

At the quark level, the predicted three-body bound states correspond to hexaquark states that have not yet been observed experimentally. We propose that future experiments search for the $DD^*\bar{K}^*$ three-body bound state in the $DD\bar{K}\pi\pi$ and $DD\bar{K}$ decay channels or precisely measure the pole position of the $Z_c(3900)$, as these efforts would mutually illuminate the nature of the associated states.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX: FOURIER TRANSFORMATION

Taking the direct diagrams as an example, the analytical expressions for the coordinate space potentials obtained via the Fourier transform are

$$\frac{1}{u^2 + q^2} F(u, \Lambda, q^2)^2 \rightarrow H_0(u, \Lambda, r), \quad (\text{A1})$$

$$\frac{q^2}{u^2 + q^2} F(u, \Lambda, q^2)^2 \rightarrow -H_1(u, \Lambda, r), \quad (\text{A2})$$

$$\begin{aligned} & \frac{q_i q_j}{u^2 + q^2} F(u, \Lambda, q^2)^2 \\ & \rightarrow - \left[H_3(u, \Lambda, r) T_{ij} + H_1(u, \Lambda, r) \frac{\delta_{ij}}{3} \right], \quad (\text{A3}) \end{aligned}$$

where $T_{ij} = \frac{3r_i r_j}{r^2} - \delta_{ij}$, and the expressions of $H_{0,1,3}$ are

$$H_0(u, \Lambda, r) = \frac{u}{4\pi} \left[\frac{e^{-ur} - e^{-\Lambda r}}{ur} - \frac{\Lambda^2 - u^2}{2u\Lambda} e^{-\Lambda r} \right], \quad (\text{A4})$$

$$H_1(u, \Lambda, r) = \frac{u^3}{4\pi} \left[\frac{e^{-ur} - e^{-\Lambda r}}{ur} - \frac{(\Lambda^2 - u^2)\Lambda^2}{2u^3\Lambda} e^{-\Lambda r} \right], \quad (\text{A5})$$

$$H_3(u, \Lambda, r) = \frac{u^3}{12\pi} \left[- \frac{e^{-\Lambda r} \Lambda^2 \left(\frac{3}{\Lambda^2 r^2} + \frac{3}{\Lambda r} + 1 \right)}{ru^3} \right. \quad (\text{A6})$$

$$\left. - \frac{e^{-\Lambda r} (\Lambda r + 1) (\Lambda^2 - u^2)}{2ru^3} + \frac{e^{-ur} \left(\frac{3}{r^2 u^2} + \frac{3}{ru} + 1 \right)}{ru} \right]. \quad (\text{A7})$$

The transformation for $V(\mathbf{k})$ follows a similar procedure. As discussed in Refs. [78,82], the coordinate space potentials derived from $V(\mathbf{k})$ and $V(\mathbf{q})$ are identical for even partial waves but differ by a sign for odd partial waves.

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