

Z_4 scotogenic model with a Higgs portal

Noel Jonathan Jobu^{*} and Kenji Nishiwaki[†]*Department of Physics, School of Natural Sciences, Shiv Nadar Institution of Eminence (Deemed to be University), Tehsil Dadri, Gautam Buddha Nagar, Uttar Pradesh, 201314, India*

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We propose a new scotogenic type model based on a global Z_4 symmetry involving dark matter candidates. After the symmetry breaking as Z_4 to Z_2 via the singlet scalar vacuum expectation value (VEV), the lightest Majorana fermion works as a viable thermal freeze-out dark matter (DM) candidate, and the mass terms for active neutrinos are generated as a finite quantum correction at the one-loop level. A key point of realizing our scotogenic structure is to introduce two types of Majorana fermions (heavy right-handed neutrinos) and inert Higgs doublets with opposite Z_4 parities. Since a large VEV for the singlet scalar is not so harmful in an appropriate realization of the Higgs mechanism for the SM gauge symmetry, we can naturally realize a TeV-scale fermionic DM candidate, where constraints via direct detection experiments are less than those for sub-TeV DM. Our scenario involves the Higgs-portal DM interactions, which help the realization of the correct DM relic abundance. Relying on the structure of the model, it is possible to find a natural partner for coannihilation. Our scenario can be investigated via the measurement of the Higgs trilinear self-coupling at the Large Hadron Collider. The simplest way to evade the domain-wall problem, by adding a tiny soft Z_2 breaking term, works, keeping a sufficient longevity of the decaying DM lifetime.

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I. INTRODUCTION

It is well known that the original form of the Standard Model (SM) should be upgraded, at least in the sector associated with neutrinos, due to the observations of neutrino oscillations, which originate from minuscule but nonzero masses of the neutrinos in the SM. Over the last two decades or more after the confirmation of the neutrino oscillations [1,2] (refer to review [3]), the experimental developments in neutrino oscillation have been significant, and now provide relevant constraints on the CP -violating phase of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [4–6]. The measurements of the three mixing angles and the CP phase will be significantly upgraded in the coming next-generation experiments, e.g., DUNE [7] and Hyper-Kamiokande [8]. Given this situation, it is becoming increasingly important to consider the structure of the neutrino sector in depth. Furthermore, it should be noted that the neutrino sector significantly impacts the possible scenarios in the early stages of the Universe in conjunction with dark matter and other matters.

A key question is: What is the origin of the tiny masses of the neutrinos? The simplest scenario relies on the Yukawa terms with the Higgs doublet, the doublet lepton fields, and the newly added right-handed singlet neutrinos, where the Dirac mass terms for the neutrinos are successfully generated via the Higgs mechanism, but we should allow to introduce tiny Yukawa couplings, at least $\mathcal{O}(10^{-12})$ or smaller. A relaxation in the tuning of Yukawa couplings is realized by adopting the see-saw mechanism, where the Yukawa structure of the sub-eV neutrino masses (and also mixings) involves connections with other heavy particles (see review [9]). Depending on how the left-handed active neutrinos couple with heavy particles, see-saw scenarios are classified as canonical/type-I [10–15], type-II [16–20], type-III [21], inverse [22,23], linear [23–25], and others. Note that a seminal application of the right-handed neutrinos, which can appear in the see-saw texture, is the baryogenesis via leptogenesis [26].

There are interesting variations of the see-saw scenario, suggested by the quantum nature of neutrinos. The above scenarios considered the case where neutrino mass terms exist at the lowest order of perturbation, but it is also possible that these are produced quantumly via loops of heavy new particles. As suggested by the results of the operator analysis [27,28], if active neutrino mass terms are represented as higher-dimensional operators of the SM fields in a gauge-invariant way (before the electroweak symmetry breaking), no divergences emerge in such loop-induced neutrino mass terms. We list early-stage concrete

^{*}Contact author: nj702@snu.edu.in[†]Contact author: kenji.nishiwaki@snu.edu.in

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TABLE I. Matter contents added to this model are summarized. $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ represent the degrees of freedom of the color, weak-isospin, and hypercharge, respectively. All Standard Model fields take $+1$ of the Z_4 parity. The index i takes one to three and discriminates three generations.

New particle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Z_4	Spin
S	1	1	0	-1	0 (real)
η_A	1	2	1	$+i$	0 (complex)
η_B	1	2	1	$-i$	0 (complex)
$N_{A,R} (i = 1, 2, 3)$	1	1	0	$+i$	1/2 (right handed)
$N_{B,R} (i = 1, 2, 3)$	1	1	0	$-i$	1/2 (right handed)

models based on this idea [29–34] (see also review on this kind of models [35]). A branch of such scenarios called the *scotogenic* model is a fascinating arena in particle physics, where additional symmetry/symmetries (imposed on such a model) ensure the loop origin of the active neutrino mass terms but also the stability of a new particle, which behaves as dark matter (DM) [36]. In this type of scenario, neutrino physics is mutually connected with dark matter physics, and the model predictability will be enhanced. Many studies have been done on such ideas in several contexts based on various kinds of symmetries; We can find various examples based on global Z_2 symmetry [37–101], based on other kinds of global discrete symmetries [102–124], and other embedding varieties, including those based on continuous-symmetry cases [125–190].¹

However, it should be noted here that the ordinary weakly interacting massive particle (WIMP) DM paradigm based on the thermal freeze-out scenario has recently reached a crossroads due to significant progress with null observations in the LUX-ZEPLIN (LZ) direct observation experiment [202,203]. Several ideas to evade the tight bounds in the freeze-out DM scenario include, e.g., fermionic DM with pseudoscalar mediator [204–209], and pseudo-Nambu-Goldstone boson (pNGB) DM [210–214] (see also [215–217] for embeddings to grand-unified scenarios). We can find other scenarios, for example, two-component dark matter (see e.g., [218]) and freeze-in production (see e.g., [219]).

Another simple scenario is to rely on the enhancement of the DM pair annihilation near a resonance of a mediator in an s -channel, where we can keep the coupling between DM and its mediator sizably small, and thereby, it is not difficult to suppress the (spin-independent) cross section in DM

direct detections. A favored region of the DM-mass range would be a TeV region since more severe bounds have been imposed in the sub-TeV domain [202,203]. We propose a new scotogenic scenario based on the global Z_4 symmetry, where the vacuum expectation value (VEV) of an additional singlet scalar breaks it to Z_2 . The residual Z_2 symmetry ensures the stability of DM, and a large VEV of the singlet scalar naturally realizes a TeV-scale Majorana fermionic DM.

This paper is organized as follows. In Sec. II, we introduce our scotogenic model based on a Z_4 global symmetry and derive the form of the one-loop induced neutrino mass terms. In Sec. III, we provide constraints on our scenario, namely, the lepton flavor violation, the oblique correction, and the stability of the scalar potential. In Sec. IV, we summarize our formulas for calculating the dark matter relic abundance and direct detection. In Sec. V, we provide our results of numerical calculations and show that we can reproduce the observed neutrino texture at one-loop level and the observed dark matter relic abundance consistently. In Sec. VI, we discuss the current constraint and future prospects of the constraint on our scenario via the measurement of the Higgs trilinear self-coupling at the Large Hadron Collider (LHC). In Sec. VII, we comment on possible prescriptions to evade the domain-wall problem in our scenario. In Sec. VIII, we summarize our results and provide some comments. In Appendix A, the decay width of the $SU(2)_L$ -singlet scalar is provided, and in Appendix B, the results of the numerical scans without the coannihilation effect are shown.

II. A Z_4 -BASED SCOTOGENIC MODEL

A. Model setup

Here, we introduce our model, which is a scotogenic-type model based on a discrete Z_4 symmetry, where no extension of the gauge groups from those of the Standard Model. Table I shows a summary of the matter contents we introduced, where we introduce one real $SU(2)_L$ -singlet scalar S , two complex $SU(2)_L$ -doublet scalars η_A and η_B , and six (two kinds of three-generation) $SU(2)_L$ -singlet right-handed neutrinos $N_{A,R}$ and $N_{B,R}$ ($i = 1, 2, 3$). Note that for the scalar doublets and right-handed fermions, type- A and type- B particles take opposite Z_4 parities as $+i$

¹Scenarios with bound-state dark matter are discussed in [191,192]. Another fascinating class (without DM candidates) is loop-induced neutrino masses under extra colored scalars; see e.g., [193–195] as early works. A recently related development is neutrino mass generations from generalized symmetry breaking and under noninvertible selection rules, see e.g., [196–198] (also refer to [199]). Another recent direction is to investigate precise calculations of how to probe a scotogenic model through flavors and electroweak precision measurements, see e.g., [200]. See also [201] as a scotogenic scenario in the so-called composite Higgs framework, alleviating the electroweak hierarchy problem.

and $-i$, respectively. The trivial Z_4 parity (+1) is assigned for all of the SM particles.

In our scenario, the possible leptonic Yukawa terms are as follows:

$$\begin{aligned} -\mathcal{L}_{\text{Yukawa}} = & Y_A^{ia} \overline{N'_{A,R}} \eta_A^T (i\sigma_2) L_{aL} + Y_B^{ia} \overline{N'_{B,R}} \eta_B^T (i\sigma_2) L_{aL} + \frac{m_{N_{ij}}}{2} \overline{(N'_{A,R})^c} N'_{B,R} \\ & + \frac{m_{N_{ij}}}{2} \overline{(N'_{B,R})^c} N'_{A,R} + \frac{y_{A_{ij}}}{2} S \overline{(N'_{A,R})^c} N'_{A,R} + \frac{y_{B_{ij}}}{2} S \overline{(N'_{B,R})^c} N'_{B,R} + \text{H.c.}, \end{aligned} \quad (2.1)$$

and the possible scalar potential terms are written down as

$$\begin{aligned} \mathcal{V} = & m_H^2 (H^\dagger H) + m_{\eta_A}^2 (\eta_A^\dagger \eta_A) + m_{\eta_B}^2 (\eta_B^\dagger \eta_B) + \frac{m_S^2}{2} S^2 + \frac{\lambda_{HS}}{2} S^2 (H^\dagger H) \\ & + \frac{\lambda_H}{2} (H^\dagger H)^2 + \frac{\lambda_S}{4} S^4 + \frac{\lambda_{2a}}{2} (\eta_A^\dagger \eta_A)^2 + \frac{\lambda_{2b}}{2} (\eta_B^\dagger \eta_B)^2 + \frac{\lambda_{2c}}{2} (\eta_A^\dagger \eta_A) (\eta_B^\dagger \eta_B) \\ & + \frac{\lambda_{2d}}{2} (\eta_A^\dagger \eta_B) (\eta_B^\dagger \eta_A) + \lambda_{3A} (H^\dagger H) (\eta_A^\dagger \eta_A) + \lambda_{3B} (H^\dagger H) (\eta_B^\dagger \eta_B) \\ & + \lambda_{4A} (H^\dagger \eta_A) (\eta_A^\dagger H) + \lambda_{4B} (H^\dagger \eta_B) (\eta_B^\dagger H) + \frac{1}{2} [\lambda_5 (H^\dagger \eta_A) (H^\dagger \eta_B) + \lambda_6 (\eta_A^\dagger \eta_B)^2 + \text{H.c.}] \\ & + \mu_S S (\eta_A^\dagger \eta_B) + \mu_S^* S (\eta_B^\dagger \eta_A) + \frac{\lambda_{S\eta_A}}{2} S^2 (\eta_A^\dagger \eta_A) + \frac{\lambda_{S\eta_B}}{2} S^2 (\eta_B^\dagger \eta_B), \end{aligned} \quad (2.2)$$

where H denotes the $SU(2)_L$ Higgs doublet to trigger the spontaneous electroweak symmetry breaking; i and α run from one to three, representing the three generations of the right-handed neutrinos and left-handed lepton doublets L_{aL} , respectively. σ_2 is the second Pauli matrix.

In the following analysis, we will assume that μ_S is a real parameter and $m_{N_{ij}}$, $y_{A_{ij}}$, and $y_{B_{ij}}$ are real and diagonal for simplicity. Also, we will take $\lambda_6 = 0$ to remove unnecessary complexity from the discussion without compromising the essence of the model, as the term does not contribute to mass terms under the inert condition for η_A and η_B . The fermion with the superscript $(N_R)^c$ denotes the charge conjugation of N_R .

The $SU(2)_L$ doublets are parametrized as²

$$L_{aL} = \begin{pmatrix} \nu'_a \\ l'^-_{aL} \end{pmatrix}, \quad H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v_H + h') \end{pmatrix}, \quad \eta_{A,B} = \begin{pmatrix} \eta'^+_{A,B} \\ \eta'^0_{A,B} \end{pmatrix} = \begin{pmatrix} \eta'^+_{A,B} \\ \frac{1}{\sqrt{2}}(\eta'_{R,A,B} + i\eta'_{I,A,B}) \end{pmatrix}, \quad (2.3)$$

and the singlet scalar,

$$S = (v_S + s'), \quad (2.4)$$

where we took the unitary gauge and the prime symbol for components, representing that they are gauge eigenstates. We focus on the following configuration of the VEVs of the scalars:

$$\langle H \rangle = \begin{pmatrix} 0 \\ v_H/\sqrt{2} \end{pmatrix}, \quad \langle S \rangle = v_S, \quad \langle \eta_A \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \eta_B \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (2.5)$$

where the Z_4 symmetry is spontaneously broken down to Z_2 due to the nonzero VEV of S (around the electroweak scale), and therefore, the lightest particle with $+i$ or $-i$ Z_4 parity is protected by a residual symmetry and becomes a dark matter candidate. The inert condition for η_A and η_B is necessary to keep the residual Z_2 symmetry. Note that the situation in Eq. (2.5) with nonzero VEVs v_H and v_S are valid after the spontaneous electroweak symmetry breaking.

²Note that

$$\begin{aligned} \overline{(N'_{A,R})^c} N'_{B,R} &= \overline{(N'_{B,R})^c} N'_{A,R}. \\ Y_A^{ia} \overline{N'_{A,R}} \eta_A^T (i\sigma_2) L_{aL} + \text{H.c.} &= Y_A^{ia} \overline{N'_{A,R}} [\nu'_{aL} \eta'^0_A - l'^-_{aL} \eta'^+_{A}] + (Y_A^{ia})^* [\overline{\nu'_{aL}} (\eta'^0_A)^* - \overline{l'^-_{aL}} \eta'^-_{A}] N'_{A,R}. \end{aligned}$$

The tadpole conditions,

$$\left. \frac{\partial \mathcal{V}}{\partial h} \right|_{h=s=0} = 0 = \left. \frac{\partial \mathcal{V}}{\partial s} \right|_{h=s=0}, \quad (2.6)$$

lead to the forms

$$\frac{\lambda_H}{2} v_H^2 + \frac{\lambda_{HS}}{2} v_S^2 = -m_H^2, \quad \frac{\lambda_{HS}}{2} v_H^2 + \lambda_S v_S^2 = -m_S^2, \quad (2.7)$$

which are solved as

$$v_H^2 = \frac{2[2(-m_H^2)\lambda_S - (-m_S^2)\lambda_{HS}]}{2\lambda_H\lambda_S - \lambda_{HS}^2},$$

$$v_S^2 = \frac{2[(-m_S^2)\lambda_H - (-m_H^2)\lambda_{HS}]}{2\lambda_H\lambda_S - \lambda_{HS}^2}, \quad (2.8)$$

where the Higgs VEV v_H should take $\simeq 246$ GeV. After the application of the tadpole conditions in Eq. (2.7), the mass matrix squared for h' and s' in this order reads

$$\mathcal{M}^2[h', s'] = \begin{bmatrix} \lambda_H v_H^2 & \lambda_{HS} v_H v_S \\ \lambda_{HS} v_H v_S & 2\lambda_S v_S^2 \end{bmatrix}$$

$$= \begin{bmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{bmatrix} \begin{bmatrix} M_h^2 & 0 \\ 0 & M_s^2 \end{bmatrix} \begin{bmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{bmatrix}, \quad (2.9)$$

where h is the SM-like Higgs boson with $M_h = 125$ GeV and s an additional CP -even scalar boson. The mixing angle α is determined as

$$\sin(2\alpha) = -\frac{2\lambda_{HS} v_H v_S}{M_h^2 - M_s^2}, \quad (2.10)$$

where the gauge eigenstates and the mass eigenstates (denoted without the prime symbol) are related as

$$\begin{bmatrix} h' \\ s' \end{bmatrix} = \begin{bmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{bmatrix} \begin{bmatrix} h \\ s \end{bmatrix}. \quad (2.11)$$

Next, we see the mass structure of the other scalars. Under our assumption that μ_S is real, the mass matrices squared for η'_{R_A} and η'_{R_B} , and η'_{I_A} and η'_{I_B} yield

$$\mathcal{M}^2[\eta'_{R_A}, \eta'_{R_B}] = \begin{bmatrix} m_{R_{AA}}^2 & m_{R_{AB}}^2 \\ m_{R_{AB}}^2 & m_{R_{BB}}^2 \end{bmatrix} = U_R \begin{bmatrix} M_{R_1}^2 & 0 \\ 0 & M_{R_2}^2 \end{bmatrix} U_R^\dagger, \quad (2.12)$$

$$\mathcal{M}^2[\eta'_{I_A}, \eta'_{I_B}] = \begin{bmatrix} m_{I_{AA}}^2 & m_{I_{AB}}^2 \\ m_{I_{AB}}^2 & m_{I_{BB}}^2 \end{bmatrix} = U_I \begin{bmatrix} M_{I_1}^2 & 0 \\ 0 & M_{I_2}^2 \end{bmatrix} U_I^\dagger, \quad (2.13)$$

where the components are defined as

$$m_{R_{AA}}^2 = m_{I_{AA}}^2 = m_{\eta_A}^2 + \frac{\lambda_{3A}}{2} v_H^2 + \frac{\lambda_{4A}}{2} v_H^2 + \frac{\lambda_{S\eta_A}}{2} v_S^2, \quad (2.14)$$

$$m_{R_{BB}}^2 = m_{I_{BB}}^2 = m_{\eta_B}^2 + \frac{\lambda_{3B}}{2} v_H^2 + \frac{\lambda_{4B}}{2} v_H^2 + \frac{\lambda_{S\eta_B}}{2} v_S^2, \quad (2.15)$$

$$m_{R_{AB}}^2 = \mu_S v_S + \frac{\lambda_5}{2} v_H^2, \quad (2.16)$$

$$m_{I_{AB}}^2 = \mu_S v_S - \frac{\lambda_5}{2} v_H^2. \quad (2.17)$$

The matrices diagonalize the mass matrices,

$$U_R = \begin{bmatrix} c_{\theta_R} & -s_{\theta_R} \\ s_{\theta_R} & c_{\theta_R} \end{bmatrix}, \quad U_I = \begin{bmatrix} c_{\theta_I} & -s_{\theta_I} \\ s_{\theta_I} & c_{\theta_I} \end{bmatrix};$$

$$\begin{bmatrix} \eta'_{R_A} \\ \eta'_{R_B} \end{bmatrix} = U_R \begin{bmatrix} \eta_{R_1} \\ \eta_{R_2} \end{bmatrix}, \quad \begin{bmatrix} \eta'_{I_A} \\ \eta'_{I_B} \end{bmatrix} = U_I \begin{bmatrix} \eta_{I_1} \\ \eta_{I_2} \end{bmatrix}, \quad (2.18)$$

$$\tan(2\theta_R) = \frac{2m_{R_{AB}}^2}{m_{R_{AA}}^2 - m_{R_{BB}}^2}, \quad \tan(2\theta_I) = \frac{2m_{I_{AB}}^2}{m_{I_{AA}}^2 - m_{I_{BB}}^2}, \quad (2.19)$$

where their mass eigenvalues are given as

$$M_{R_1}^2 = \frac{1}{2} \left[m_{R_{AA}}^2 + m_{R_{BB}}^2 + \sqrt{4m_{R_{AB}}^4 + (m_{R_{AA}}^2 - m_{R_{BB}}^2)^2} \right], \quad (2.20)$$

$$M_{R_2}^2 = \frac{1}{2} \left[m_{R_{AA}}^2 + m_{R_{BB}}^2 - \sqrt{4m_{R_{AB}}^4 + (m_{R_{AA}}^2 - m_{R_{BB}}^2)^2} \right], \quad (2.21)$$

$$M_{I_1}^2 = \frac{1}{2} \left[m_{I_{AA}}^2 + m_{I_{BB}}^2 + \sqrt{4m_{I_{AB}}^4 + (m_{I_{AA}}^2 - m_{I_{BB}}^2)^2} \right], \quad (2.22)$$

$$M_{I_2}^2 = \frac{1}{2} \left[m_{I_{AA}}^2 + m_{I_{BB}}^2 - \sqrt{4m_{I_{AB}}^4 + (m_{I_{AA}}^2 - m_{I_{BB}}^2)^2} \right]. \quad (2.23)$$

Note that the expressions from Eq. (2.20) to Eq. (2.23) hold at the tree level. This also applies to the following Eqs. (2.29), (2.30), (2.34), and (2.35). Similarly, the mass matrix for the charged scalars yields

$$\mathcal{M}^2[\eta'^+_A, \eta'^+_B] = \begin{bmatrix} m_{\eta'_A}^2 & m_{\eta'_B}^2 \\ m_{\eta'_B}^2 & m_{\eta'_A}^2 \end{bmatrix}, \quad (2.24)$$

with

$$m_{\eta_A^+}^2 = m_{\eta_A}^2 + \frac{\lambda_{3A}}{2} v_H^2 + \frac{\lambda_{S\eta_A}}{2} v_S^2, \quad (2.25)$$

$$m_{\eta_B^+}^2 = m_{\eta_B}^2 + \frac{\lambda_{3B}}{2} v_H^2 + \frac{\lambda_{S\eta_B}}{2} v_S^2, \quad (2.26)$$

$$m_{\eta_{AB}^+}^2 = \mu_S v_S, \quad (2.27)$$

$$\mathcal{M}^2[\eta_A^+, \eta_B^+] = U_{\text{cs}} \begin{bmatrix} M_{\eta_1^+}^2 & 0 \\ 0 & M_{\eta_2^+}^2 \end{bmatrix} U_{\text{cs}}^\dagger \begin{bmatrix} \eta_A^+ \\ \eta_B^+ \end{bmatrix} = U_{\text{cs}} \begin{bmatrix} \eta_1^+ \\ \eta_2^+ \end{bmatrix},$$

$$U_{\text{cs}} = \begin{bmatrix} c_{\theta_{\text{cs}}} & -s_{\theta_{\text{cs}}} \\ s_{\theta_{\text{cs}}} & c_{\theta_{\text{cs}}} \end{bmatrix}. \quad (2.28)$$

Deriving the formulas for mass eigenvalues and $\tan(2\theta_{\text{cs}})$ is straightforward as

$$M_{\eta_1^+}^2 = \frac{1}{2} \left[m_{\eta_A^+}^2 + m_{\eta_B^+}^2 + \sqrt{4m_{\eta_{AB}^+}^2 + (m_{\eta_A^+}^2 - m_{\eta_B^+}^2)^2} \right], \quad (2.29)$$

$$M_{\eta_2^+}^2 = \frac{1}{2} \left[m_{\eta_A^+}^2 + m_{\eta_B^+}^2 - \sqrt{4m_{\eta_{AB}^+}^2 + (m_{\eta_A^+}^2 - m_{\eta_B^+}^2)^2} \right], \quad (2.30)$$

$$\tan(2\theta_{\text{cs}}) = \frac{2m_{\eta_{AB}^+}^2}{m_{\eta_A^+}^2 - m_{\eta_B^+}^2}. \quad (2.31)$$

For the mass terms of the heavy fermions, to eliminate unnecessary complexity from the phenomenological discussion of neutrinos and dark matter—our subject of interest—and to make the essence of the model more transparently understandable, we manually constrain the degrees of freedom of the coupling constants under the following assumptions: all of $m_{N_{ij}}$, $y_{A_{ij}}$, and $y_{B_{ij}}$ ($i, j = 1, 2, 3$) are real and diagonal. Also, since physics involving CP violation (beyond the SM) is not the subject of this paper's discussion, physical complex coupling constants are not necessarily required. Note that, as we will see in Eq. (2.45), the Yukawa couplings with the two inert doublets ($Y_A^{i\alpha}$ and $Y_B^{i\alpha}$) should be complex to generate a nonzero CP -violating phase of the PMNS matrix.

The mass terms of the right-handed neutrinos decompose into each generation as follows:

$$-\frac{1}{2} \sum_{i=1}^3 \left[\overline{(N'_{A_i R})^c}, \overline{(N'_{B_i R})^c} \right] \mathcal{M}_{N_{ii}} \begin{bmatrix} N'_{A_i R} \\ N'_{B_i R} \end{bmatrix} + \text{H.c.}, \quad (2.32)$$

where a (complex) orthogonal matrix diagonalizes the symmetric Majorana mass terms as

$$\mathcal{M}_{N_{ii}} = \begin{bmatrix} y_{A_{ii}} v_S & m_{N_{ii}} \\ m_{N_{ii}} & y_{B_{ii}} v_S \end{bmatrix} = U_{N_i} \begin{bmatrix} M_{N_{A_i}} & 0 \\ 0 & M_{N_{B_i}} \end{bmatrix} (U_{N_i})^T, \quad U_{N_i} = \begin{bmatrix} c_{\theta_{N_i}} & -s_{\theta_{N_i}} \\ s_{\theta_{N_i}} & c_{\theta_{N_i}} \end{bmatrix}, \quad (2.33)$$

with

$$M_{N_{A_i}} = \frac{1}{2} \left[y_{A_{ii}} v_S + y_{B_{ii}} v_S + \sqrt{4m_{N_{ii}}^2 + (y_{A_{ii}} v_S - y_{B_{ii}} v_S)^2} \right], \quad (2.34)$$

$$M_{N_{B_i}} = \frac{1}{2} \left[y_{A_{ii}} v_S + y_{B_{ii}} v_S - \sqrt{4m_{N_{ii}}^2 + (y_{A_{ii}} v_S - y_{B_{ii}} v_S)^2} \right]. \quad (2.35)$$

The diagonalized basis (without the prime symbol) of the Majorana mass terms is defined as

$$\begin{bmatrix} N'_{A_i R} \\ N'_{B_i R} \end{bmatrix} = U_{N_i} \begin{bmatrix} N_{A_i R} \\ N_{B_i R} \end{bmatrix}. \quad (2.36)$$

We introduce the following compact notation for clarity in the mass eigenbases:

$$\mathcal{L}_{\text{Yukawa}} \supset - \sum_{s^0 = \eta_{R_1}, \eta_{R_2}, \eta_{I_1}, \eta_{I_2}} \left[\mathcal{Y}_{A_i \alpha}^{(s^0)} \overline{N_{A_i R}} \nu_{\alpha L} s^0 + \mathcal{Y}_{B_i \alpha}^{(s^0)} \overline{N_{B_i R}} \nu_{\alpha L} s^0 \right]$$

$$+ \sum_{s^+ = \eta_1^+, \eta_2^+} \left[\mathcal{Y}_{A_i \alpha}^{(s^+)} \overline{N_{A_i R}} \ell_{\alpha L}^- s^+ + \mathcal{Y}_{B_i \alpha}^{(s^+)} \overline{N_{B_i R}} \ell_{\alpha L}^- s^+ \right] + \text{H.c.}, \quad (2.37)$$

with

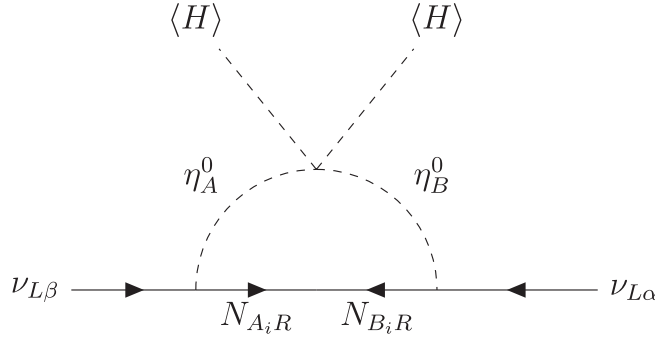


FIG. 1. The topology of loop-induced mass generation is depicted in gauge eigenstates. Note that all of the Feynman diagrams in this manuscript were drawn by TIKZ-FEYNMAN [220].

$$\begin{aligned}
 \mathcal{Y}_{A_i\alpha}^{(\eta_{R1})} &= +\frac{Y_A^{i\alpha}}{\sqrt{2}}c_{N_i}c_{\theta_R} + \frac{Y_B^{i\alpha}}{\sqrt{2}}s_{N_i}s_{\theta_R}, & \mathcal{Y}_{B_i\alpha}^{(\eta_{R1})} &= -\frac{Y_A^{i\alpha}}{\sqrt{2}}s_{N_i}c_{\theta_R} + \frac{Y_B^{i\alpha}}{\sqrt{2}}c_{N_i}s_{\theta_R}, \\
 \mathcal{Y}_{A_i\alpha}^{(\eta_{R2})} &= -\frac{Y_A^{i\alpha}}{\sqrt{2}}c_{N_i}s_{\theta_R} + \frac{Y_B^{i\alpha}}{\sqrt{2}}s_{N_i}c_{\theta_R}, & \mathcal{Y}_{B_i\alpha}^{(\eta_{R2})} &= +\frac{Y_A^{i\alpha}}{\sqrt{2}}s_{N_i}s_{\theta_R} + \frac{Y_B^{i\alpha}}{\sqrt{2}}c_{N_i}c_{\theta_R}, \\
 \mathcal{Y}_{A_i\alpha}^{(\eta_{I1})} &= +i\frac{Y_A^{i\alpha}}{\sqrt{2}}c_{N_i}c_{\theta_I} + i\frac{Y_B^{i\alpha}}{\sqrt{2}}s_{N_i}s_{\theta_I}, & \mathcal{Y}_{B_i\alpha}^{(\eta_{I1})} &= -i\frac{Y_A^{i\alpha}}{\sqrt{2}}s_{N_i}c_{\theta_I} + i\frac{Y_B^{i\alpha}}{\sqrt{2}}c_{N_i}s_{\theta_I}, \\
 \mathcal{Y}_{A_i\alpha}^{(\eta_{I2})} &= -i\frac{Y_A^{i\alpha}}{\sqrt{2}}c_{N_i}s_{\theta_I} + i\frac{Y_B^{i\alpha}}{\sqrt{2}}s_{N_i}c_{\theta_I}, & \mathcal{Y}_{B_i\alpha}^{(\eta_{I2})} &= +i\frac{Y_A^{i\alpha}}{\sqrt{2}}s_{N_i}s_{\theta_I} + i\frac{Y_B^{i\alpha}}{\sqrt{2}}c_{N_i}c_{\theta_I}, \\
 \mathcal{Y}_{A_i\alpha}^{(\eta_1^+)} &= +Y_A^{i\alpha}c_{N_i}c_{\theta_{cs}} + Y_B^{i\alpha}s_{N_i}s_{\theta_{cs}}, & \mathcal{Y}_{B_i\alpha}^{(\eta_1^+)} &= -Y_A^{i\alpha}s_{N_i}c_{\theta_{cs}} + Y_B^{i\alpha}c_{N_i}s_{\theta_{cs}}, \\
 \mathcal{Y}_{A_i\alpha}^{(\eta_2^+)} &= -Y_A^{i\alpha}c_{N_i}s_{\theta_{cs}} + Y_B^{i\alpha}s_{N_i}c_{\theta_{cs}}, & \mathcal{Y}_{B_i\alpha}^{(\eta_2^+)} &= +Y_A^{i\alpha}s_{N_i}s_{\theta_{cs}} + Y_B^{i\alpha}c_{N_i}c_{\theta_{cs}},
 \end{aligned} \tag{2.38}$$

where the relative sign originates from the $i\sigma_2$ matrix in Eq. (2.1).

B. Loop-induced neutrino mass terms

In our scenario, as in the original scotogenic model, the left-handed active neutrino mass terms are radiatively generated at a one-loop level, where we adopt the parametrization

$$\mathcal{L}_{\text{this model}} \supset -\frac{1}{2}\mathcal{M}_{\alpha\beta}^\nu \overline{(\nu'_{\alpha L})^c} \nu'_{\beta L} + \text{H.c.} \tag{2.39}$$

The calculation of the one-loop diagrams in Fig. 1 with the dimensional regularization brings us the result,

$$\begin{aligned}
 \mathcal{M}_{\alpha\beta}^\nu &= -\sum_{s^0=\eta_{R1}, \eta_{R2}, \eta_{I1}, \eta_{I2}} \sum_{i=1}^3 \frac{M_{N_{A_i}} \mathcal{Y}_{A_i\alpha}^{(s^0)} \mathcal{Y}_{A_i\beta}^{(s^0)}}{(4\pi)^2} \left\{ \frac{1}{\epsilon} - \gamma + \log(4\pi) - \int_0^1 dx \log \left[xM_{s^0}^2 + (1-x)M_{N_{A_i}}^2 - i\epsilon \right] \right\} \\
 &\quad - \sum_{s^0=\eta_{R1}, \eta_{R2}, \eta_{I1}, \eta_{I2}} \sum_{i=1}^3 \frac{M_{N_{B_i}} \mathcal{Y}_{B_i\alpha}^{(s^0)} \mathcal{Y}_{B_i\beta}^{(s^0)}}{(4\pi)^2} \left\{ \frac{1}{\epsilon} - \gamma + \log(4\pi) - \int_0^1 dx \log \left[xM_{s^0}^2 + (1-x)M_{N_{B_i}}^2 - i\epsilon \right] \right\} \\
 &= -\sum_{s^0=\eta_{R1}, \eta_{R2}, \eta_{I1}, \eta_{I2}} \sum_{i=1}^3 \frac{M_{N_{A_i}} \mathcal{Y}_{A_i\alpha}^{(s^0)} \mathcal{Y}_{A_i\beta}^{(s^0)}}{(4\pi)^2} \left\{ -\int_0^1 dx \log \left[xM_{s^0}^2 + (1-x)M_{N_{A_i}}^2 - i\epsilon \right] \right\} \\
 &\quad - \sum_{s^0=\eta_{R1}, \eta_{R2}, \eta_{I1}, \eta_{I2}} \sum_{i=1}^3 \frac{M_{N_{B_i}} \mathcal{Y}_{B_i\alpha}^{(s^0)} \mathcal{Y}_{B_i\beta}^{(s^0)}}{(4\pi)^2} \left\{ -\int_0^1 dx \log \left[xM_{s^0}^2 + (1-x)M_{N_{B_i}}^2 - i\epsilon \right] \right\},
 \end{aligned} \tag{2.40}$$

where $1/\varepsilon$ and γ represent a divergent pole and the Euler-Mascheroni constant. As expected from the nature of higher-dimensional operators, the divergences in the active-neutrino mass terms are canceled out in each of the A and B parts by use of Eq. (2.38). Note that this matrix is nondiagonal due to the degrees of freedom of α and β ($\alpha, \beta = 1, 2, 3$). Ignoring the $i\varepsilon$ term in the loop integral of heavy particles Eq. (2.40) we get,

$$\begin{aligned} & \int_0^1 dx \log \left[xM_{s^0}^2 + (1-x)M_{N_{A_i}}^2 \right] \\ &= -1 + \frac{M_{N_{A_i}}^2 \log(M_{N_{A_i}}^2) - M_{s^0}^2 \log(M_{s^0}^2)}{M_{N_{A_i}}^2 - M_{s^0}^2}. \end{aligned} \quad (2.41)$$

As is apparent from the summation of the neutral scalars in Eq. (2.40), $\mathcal{M}_{\alpha\beta}^\nu$ is dependent on the mass difference

between the real and imaginary scalars, since their contributions are opposite to each other. If $M_{R_1} = M_{I_1}$ and $M_{R_2} = M_{I_2}$ all mass contribution to $\mathcal{M}_{\alpha\beta}^\nu$ vanishes. The difference in their masses is parameterized by the coupling λ_5 as seen in Eqs. (2.16) and (2.17). v_S must also be nonzero for a finite mass contribution.

Hereafter, we restrict the Yukawa parameter under the condition for further simplicity,

$$Y_A^{i\alpha} \longrightarrow Y^{i\alpha} \longleftarrow Y_B^{i\alpha}, \quad (2.42)$$

where $\mathcal{M}_{\alpha\beta}^\nu$ becomes

$$\mathcal{M}_{\alpha\beta}^\nu \rightarrow \sum_{i=1}^3 Y^{i\alpha} \mathcal{X}_i Y^{i\beta}, \quad (2.43)$$

where the derivation of $\mathcal{X}_i$ is straightforward as

$$\begin{aligned} \mathcal{X}_i := & + \sum_{s^0=\eta_{R_1}, \eta_{R_2}, \eta_{I_1}, \eta_{I_2}} \frac{M_{N_{A_i}} (\mathcal{Y}_{A_i\alpha}^{(s^0)} / Y^{i\alpha}) (\mathcal{Y}_{A_i\beta}^{(s^0)} / Y^{i\beta})}{(4\pi)^2} \int_0^1 dx \log \left[xM_{s^0}^2 + (1-x)M_{N_{A_i}}^2 \right] \\ & + \sum_{s^0=\eta_{R_1}, \eta_{R_2}, \eta_{I_1}, \eta_{I_2}} \frac{M_{N_{B_i}} (\mathcal{Y}_{B_i\alpha}^{(s^0)} / Y^{i\alpha}) (\mathcal{Y}_{B_i\beta}^{(s^0)} / Y^{i\beta})}{(4\pi)^2} \int_0^1 dx \log \left[xM_{s^0}^2 + (1-x)M_{N_{B_i}}^2 \right], \end{aligned} \quad (2.44)$$

and this form follows the Casas-Ibarra parametrization [221],³

$$Y^{i\alpha} = \sqrt{\mathcal{X}^{-1}} R_\nu \sqrt{M_{\hat{m}_\nu}} U_{\text{PMNS}}^\dagger, \quad (2.45)$$

and where the following 3×3 matrices are introduced:

$$\mathcal{X} := \begin{bmatrix} \mathcal{X}_1 & 0 & 0 \\ 0 & \mathcal{X}_2 & 0 \\ 0 & 0 & \mathcal{X}_3 \end{bmatrix}, \quad \sqrt{\mathcal{X}^{-1}} = \begin{bmatrix} (\mathcal{X}_1)^{-1/2} & 0 & 0 \\ 0 & (\mathcal{X}_2)^{-1/2} & 0 \\ 0 & 0 & (\mathcal{X}_3)^{-1/2} \end{bmatrix}, \quad \sqrt{M_{\hat{m}_\nu}} = \begin{bmatrix} \sqrt{\hat{m}_{\nu_1}} & 0 & 0 \\ 0 & \sqrt{\hat{m}_{\nu_2}} & 0 \\ 0 & 0 & \sqrt{\hat{m}_{\nu_3}} \end{bmatrix}. \quad (2.46)$$

$\hat{m}_{\nu_i}$ ($i = 1, 2, 3$) are the mass eigenvalues of the three active neutrinos. Also, R_ν and $U_{\text{PMNS}}^\dagger$ are an arbitrary 3×3 orthogonal matrix and the PMNS matrix, respectively. As the only condition on R_ν is that it is orthogonal, it provides the physical degrees of freedom in the Yukawa couplings after realizing the observed masses and mixings of the active neutrinos.

While any orthogonal matrix R_ν may be used in the Casas-Ibarra parameterization, we scanned for an instance that produced a favorable Yukawa texture, i.e., high Yukawa for the third generation Majorana neutrinos (DM and coannihilating candidate). In the following analysis, we focus on the choice of R_ν ,⁴

$$R_\nu = \begin{bmatrix} 0.11641348 & 0.78579266 & 0.60743542 \\ -0.74517440 & -0.33523731 & 0.57648163 \\ 0.65663005 & -0.51975555 & 0.54652643 \end{bmatrix}. \quad (2.47)$$

³A systematic generalization of this parametrization is found in [222].

⁴The R_ν form in Eq. (2.47) was adopted in our analysis as a preferred example based on our empirical understanding from numerical scans, which showed that a form where all components are highly mixed more readily satisfies all phenomenological requirements in our model.

Also, we assume the normal ordering of the three neutrino masses. Based on the data [223], we determine $\hat{m}_{\nu 1} = 0.01$ eV, $\hat{m}_{\nu 2} = \hat{m}_{\nu 1} + \sqrt{\Delta m_{21}^2} = \hat{m}_{\nu 1} + 8.6 \times 10^{-3}$ eV = 0.0186 eV, and $\hat{m}_{\nu 3} = \hat{m}_{\nu 1} + \sqrt{\Delta m_{32}^2 + \Delta m_{21}^2} = \hat{m}_{\nu 1} + 0.05$ eV = 0.060 eV. For the mixing angles, we follow the parametrization

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix},$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, and δ_{CP} the CP -violating phase, and we will adopt the best-fit result for the normal mass ordering in Table 1 of [224]; $\theta_{12} = 33.68^\circ$, $\theta_{23} = 43.3^\circ$, $\theta_{13} = 8.56^\circ$, and $\delta_{CP} = 212^\circ$; we set the two possible Majorana phases as zeros because we do not address the physics related to CP violation in this manuscript.

III. NON-DM CONSTRAINTS ON THE MODEL

This section will discuss constraints other than DM physics on our Z_4 -based scotogenic scenario.

A. Lepton flavor violation

We present the formulas involved in calculating the branching ratios of $l_\alpha \rightarrow l_\beta + \gamma$ ($l_1 = e$, $l_2 = \mu$, $l_3 = \tau$), as shown in Fig. 2. The general form of the decay amplitudes squared is given by

$$|\mathcal{M}_{\alpha\beta}|^2 = \frac{e^2 m_{l_\alpha}^6}{4(4\pi)^4} \left| \sum_{i=1}^3 \sum_{s^+ = \eta_1^+, \eta_2^+} \left[\frac{(\mathcal{Y}_{A_i\beta}^{(s^+)})^* \mathcal{Y}_{A_i\alpha}^{(s^+)}}{M_{s^+}^2} F_2\left(\frac{M_{N_{A_i}}^2}{M_{s^+}^2}\right) + \frac{(\mathcal{Y}_{B_i\beta}^{(s^+)})^* \mathcal{Y}_{B_i\alpha}^{(s^+)}}{M_{s^+}^2} F_2\left(\frac{M_{N_{B_i}}^2}{M_{s^+}^2}\right) \right] \right|^2, \quad (3.1)$$

where α and β represent the initial and final lepton external states, assuming $m_{l_\alpha} > m_{l_\beta}$ and the function $F_2(x)$ is defined as

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1 - x^4)}. \quad (3.2)$$

The decay width can be calculated using [225],

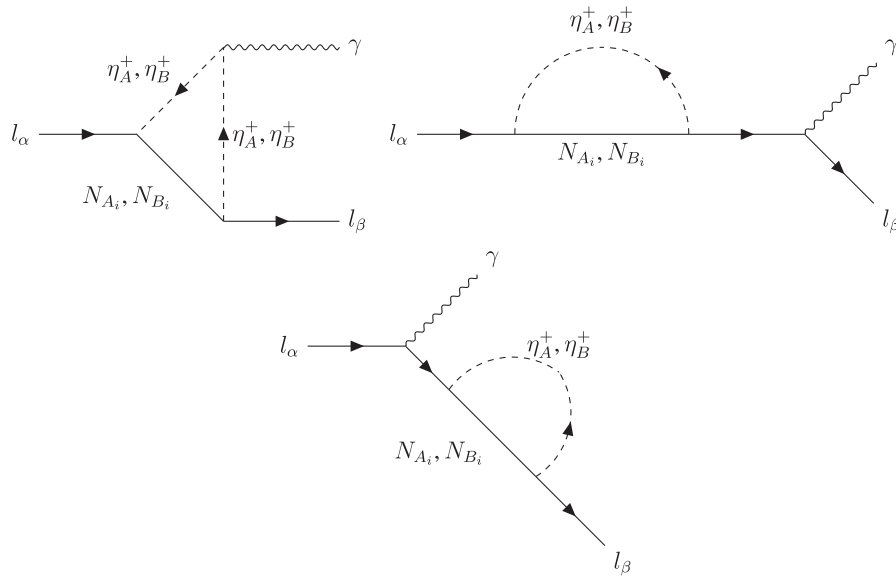


FIG. 2. The Feynman diagrams contributing to $l_\alpha \rightarrow l_\beta + \gamma$ are shown in mass eigenstates.

TABLE II. Lepton flavor violation observational bounds at 90% confidence level (CL).

Channel	Observational upper limits
$\text{Br}(\mu^- \rightarrow e^- + \gamma)$	$< 3.1 \times 10^{-13}$ [226]
$\text{Br}(\tau^- \rightarrow \mu^- + \gamma)$	$< 4.4 \times 10^{-8}$ [227]
$\text{Br}(\tau^- \rightarrow e^- + \gamma)$	$< 3.3 \times 10^{-8}$ [227]

$$\Gamma_{l_\alpha \rightarrow l_\beta + \gamma} = \frac{p^*}{8\pi m_{l_\alpha}^2} |\mathcal{M}_{\alpha\beta}|^2, \quad p^* = \frac{1}{2m_{l_\alpha}} (m_{l_\alpha}^2 - m_{l_\beta}^2). \quad (3.3)$$

Using the total decay width Γ_μ , the branching ratio of $l_\alpha \rightarrow l_\beta + \gamma$ is estimated as

$$\text{Br}(l_\alpha \rightarrow l_\beta + \gamma) = \frac{\Gamma_{l_\alpha \rightarrow l_\beta + \gamma}}{\Gamma_\alpha}. \quad (3.4)$$

We will use the lifetimes $\tau_\mu = 2.1969811 \times 10^{-6}$ s and $\tau_\tau = 2.903 \times 10^{-13}$ s [223] and the experimental constraints on the three processes, summarized in Table II.

B. Oblique constraint

Following the prescription in [228], we define two matrices U and V having dimensions $n_d \times n$ and $n_d \times m$. Here, $n_d (= 3)$ is the number of $SU(2)_L$ doublet scalars. In terms of the numbers of $n_c (= 0)$ and $n_n (= 1)$, which are the numbers of complex $SU(2)_L$ -singlet and real $SU(2)_L$ -singlet scalars with zero hypercharge, n and m are defined as $n := n_d + n_c (= 3)$ and $m := n_n + 2n_d (= 7)$, which represent the singly charged and neutral

components' numbers (including the would-be Nambu-Goldstone (NG) degrees of freedom), respectively. U and V must satisfy,

$$\begin{bmatrix} G^+ \\ \eta_A'^+ \\ \eta_B'^+ \end{bmatrix} = U \begin{bmatrix} G^+ \\ \eta_1^+ \\ \eta_2^+ \end{bmatrix}, \quad \begin{bmatrix} h' + iG^0 \\ \eta_{R_A}' + i\eta_{I_A}' \\ \eta_{R_B}' + i\eta_{I_B}' \end{bmatrix} = V \begin{bmatrix} G^0 \\ h \\ \eta_{R_1} \\ \eta_{I_1} \\ \eta_{R_2} \\ \eta_{I_2} \\ s \end{bmatrix}, \quad (3.5)$$

where G^+ and G^0 are the would-be NG bosons of the SM Higgs doublet, corresponding to the longitudinal degrees of freedom, respectively. The structures of U and V are identified as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\theta_{cs}} & -s_{\theta_{cs}} \\ 0 & s_{\theta_{cs}} & c_{\theta_{cs}} \end{pmatrix}, \quad (3.6)$$

$$V = \begin{pmatrix} i & c_\alpha & 0 & 0 & 0 & 0 & -s_\alpha \\ 0 & 0 & c_{\theta_R} & ic_{\theta_I} & -s_{\theta_R} & -is_{\theta_I} & 0 \\ 0 & 0 & s_{\theta_R} & is_{\theta_I} & c_{\theta_R} & ic_{\theta_I} & 0 \end{pmatrix}. \quad (3.7)$$

From this setup, we can calculate the deviation of the ρ parameter $\Delta\rho$ using the general formula in [228],

$$\begin{aligned} \Delta\rho = & \frac{g^2}{64\pi^2 M_W^2} \left[\sum_{a=2}^n \sum_{b=2}^m |(U^\dagger V)_{ab}|^2 F(m_a^2, m_b^2) - \sum_{b=2}^{m-1} \sum_{b'=b+1}^m [\text{Im}(V^\dagger V)_{bb'}]^2 F(m_b^2, m_{b'}^2) \right. \\ & - 2 \sum_{a=2}^{n-1} \sum_{a'=a+1}^n [(U^\dagger U)_{aa'}]^2 F(m_a^2, m_{a'}^2) + 3 \sum_{b=2}^m [\text{Im}(V^\dagger V)_{1b}]^2 [F(M_Z^2, m_b^2) - F(M_W^2, m_b^2)] \\ & \left. - 3[F(M_Z^2, M_h^2) - F(M_W^2, M_h^2)] \right], \end{aligned} \quad (3.8)$$

where g is the $SU(2)_L$ gauge coupling, and M_W and M_Z are the W -boson and Z -boson masses. m_a (also $m_{a'}$) and m_b (also $m_{b'}$) denote the masses of the charged scalars and neutral scalars in the mass eigenstates, respectively. The two states identified by $a = 1$ or $b = 1$ are unphysical; therefore, they are not considered in Eq. (3.8). Equation (3.8) is simplified in the models with no $SU(2)_L$ -singlet charged scalars, as in our model, where the third term proportional to $U^\dagger U$ vanishes as $U^\dagger U$ in unity, and since the summation is over off-diagonal elements, the term

has no contributions. The function $F(x, y)$ in Eq. (3.8) is defined as

$$F(x, y) = \begin{cases} 0 & \text{if } x = y, \\ \frac{x+y}{2} - \frac{xy \ln(x/y)}{x-y} & \text{otherwise.} \end{cases} \quad (3.9)$$

To compare with observational results [229], we use the parameter T ,

$$T = \frac{\Delta\rho}{\alpha(M_Z)}. \quad (3.10)$$

We consider the case when $U = 0$, and we use the central value of $\alpha(M_Z) = (127.951 \pm 0.009)^{-1}$, which is the quantum-electrodynamical fine structure constant defined in the $\overline{\text{MS}}$ scheme with five quark flavors [229]. We will target the following value via statistical analysis [223]:

$$T = 0.01 \pm 0.12. \quad (3.11)$$

C. Vacuum stability

1. Boundness from below

For the constraints via bounded from below, we need to look at the quartic part of the potential $\mathcal{V}^{(4)}$ defined as

$$\begin{aligned} \mathcal{V}^{(4)}(H, \eta_A, \eta_B, S) = & \frac{\lambda_{HS}}{2} S^2 (H^\dagger H) + \frac{\lambda_{S\eta_A}}{2} S^2 (\eta_A^\dagger \eta_A) + \frac{\lambda_{S\eta_B}}{2} S^2 (\eta_B^\dagger \eta_B) \\ & + \frac{\lambda_H}{2} (H^\dagger H)^2 + \frac{\lambda_S}{4} S^4 + \frac{\lambda_{2a}}{2} (\eta_A^\dagger \eta_A)^2 + \frac{\lambda_{2b}}{2} (\eta_B^\dagger \eta_B)^2 + \frac{\lambda_{2c}}{2} (\eta_A^\dagger \eta_A)(\eta_B^\dagger \eta_B) \\ & + \frac{\lambda_{2d}}{2} (\eta_A^\dagger \eta_B)(\eta_B^\dagger \eta_A) + \lambda_{3A} (H^\dagger H)(\eta_A^\dagger \eta_A) + \lambda_{3B} (H^\dagger H)(\eta_B^\dagger \eta_B) \\ & + \lambda_{4A} (H^\dagger \eta_A)(\eta_A^\dagger H) + \lambda_{4B} (H^\dagger \eta_B)(\eta_B^\dagger H) + \frac{1}{2} [\lambda_5 (H^\dagger \eta_A)(H^\dagger \eta_B) + \text{H.c.}], \end{aligned} \quad (3.12)$$

where we took $\lambda_6 = 0$ [refer to Eq. (2.2)]. Assuming fields are real, we can rewrite the potential $\mathcal{V}^{(4)}$ as described in [230],

$$\mathcal{V}^{(4)} = x^T \Lambda x, \quad (3.13)$$

with

$$x = \begin{pmatrix} H^2 \\ \eta_A^2 \\ \eta_A \eta_B \\ \eta_B^2 \\ S^2 \end{pmatrix}, \quad (3.14)$$

and

$$\Lambda = \begin{pmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{3A} + \lambda_{4A}}{2} & \frac{\lambda_5}{2} & \frac{\lambda_{3B} + \lambda_{4B}}{2} & \frac{\lambda_{HS}}{4} \\ \frac{\lambda_{3A} + \lambda_{4A}}{2} & \frac{\lambda_{2a}}{2} & 0 & \frac{(1-c)}{2} \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) & \frac{\lambda_{S\eta_A}}{4} \\ \frac{\lambda_5}{2} & 0 & c \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) & 0 & 0 \\ \frac{\lambda_{3B} + \lambda_{4B}}{2} & \frac{(1-c)}{2} \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) & 0 & \frac{\lambda_{2b}}{2} & \frac{\lambda_{S\eta_B}}{4} \\ \frac{\lambda_{HS}}{4} & \frac{\lambda_{S\eta_A}}{4} & 0 & \frac{\lambda_{S\eta_B}}{4} & \frac{\lambda_S}{4} \end{pmatrix}. \quad (3.15)$$

c is an arbitrary constant due to the ambiguity in $(\eta_A \eta_B)^2$ terms.

The requirement of stability means demanding that the quartic part of the potential $\mathcal{V}^{(4)} > 0$ as the fields $\phi_i \rightarrow \infty$. To find the conditions for $\mathcal{V}^{(4)} > 0$ we can impose the positivity condition on Λ [230]. We use the Sylvester criterion for the positivity of Λ , assuming $\phi_i \phi_j \in \mathcal{R}$. We must apply the Sylvester criterion to all permutations of the basis vector x .

Sylvester's criterion states that a $n \times n$ Hermitian matrix Λ is positive-definite if and only if all the following matrices have a positive determinant: the upper left 1×1 corner of Λ , the upper left 2×2 corner of Λ , and so on. This sequence continues till Λ itself.

We can then write down the general conditions,

$$\lambda_{ii} > 0, \quad (3.16)$$

$$\lambda_{ii}\lambda_{jj} - \lambda_{ij}^2 > 0 \quad \text{for } i \neq j, \quad (3.17)$$

$$\begin{vmatrix} \lambda_{ii} & \lambda_{ij} & \lambda_{ik} \\ \lambda_{ji} & \lambda_{jj} & \lambda_{jk} \\ \lambda_{ki} & \lambda_{kj} & \lambda_{kk} \end{vmatrix} > 0 \quad \text{for } i \neq j \neq k, \quad \begin{vmatrix} \lambda_{ii} & \lambda_{ij} & \lambda_{ik} & \lambda_{il} \\ \lambda_{ji} & \lambda_{jj} & \lambda_{jk} & \lambda_{jl} \\ \lambda_{ki} & \lambda_{kj} & \lambda_{kk} & \lambda_{kl} \\ \lambda_{li} & \lambda_{lj} & \lambda_{lk} & \lambda_{ll} \end{vmatrix} > 0 \quad \text{for } i \neq j \neq k \neq l, \quad (3.18)$$

$$\text{Det}[\Lambda] > 0. \quad (3.19)$$

λ_{ij} 's are the elements of matrix Λ where $i, j, k = 1, 2, 3, 4, 5$ represent the element position of the matrix.

We get five constraints from the 1×1 matrix,

$$\lambda_H > 0, \quad \lambda_{2a} > 0, \quad \lambda_{2b} > 0, \quad c(\lambda_{2c} + \lambda_{2d}) > 0, \quad \lambda_S > 0. \quad (3.20)$$

We get the following independent constraints from the 2×2 matrices:

$$\lambda_H\lambda_{2a} - (\lambda_{3A} + \lambda_{4A})^2 > 0, \quad \lambda_H\lambda_{2b} - (\lambda_{3B} + \lambda_{4B})^2 > 0, \quad (3.21)$$

$$2\lambda_{2a}\lambda_S - \lambda_{S\eta_A}^2 > 0, \quad 2\lambda_{2b}\lambda_S - \lambda_{S\eta_B}^2 > 0, \quad (3.22)$$

$$2\lambda_H\lambda_S - \lambda_{HS}^2 > 0, \quad c\lambda_H(\lambda_{2c} + \lambda_{2d}) - \lambda_S^2 > 0, \quad (3.23)$$

$$4\lambda_{2a}\lambda_{2b} - (1-c)^2(\lambda_{2c} + \lambda_{2d})^2 > 0, \quad (3.24)$$

where 10 constraints appear from them if we do not remove redundancies. We get the following independent constraints from the 3×3 matrices:

$$\begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{3A} + \lambda_{4A}}{2} & \frac{\lambda_S}{2} \\ \frac{\lambda_{3A} + \lambda_{4A}}{2} & \frac{\lambda_{2a}}{2} & 0 \\ \frac{\lambda_S}{2} & 0 & c\left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2}\right) \end{vmatrix} > 0, \quad \begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{3B} + \lambda_{4B}}{2} & \frac{\lambda_S}{2} \\ \frac{\lambda_{3B} + \lambda_{4B}}{2} & \frac{\lambda_{2b}}{2} & 0 \\ \frac{\lambda_S}{2} & 0 & c\left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2}\right) \end{vmatrix} > 0, \quad (3.25)$$

$$\begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{HS}}{4} & \frac{\lambda_{3A} + \lambda_{4A}}{2} \\ \frac{\lambda_{HS}}{4} & \frac{\lambda_S}{4} & \frac{\lambda_{S\eta_A}}{4} \\ \frac{\lambda_{3A} + \lambda_{4A}}{2} & \frac{\lambda_{S\eta_A}}{4} & \frac{\lambda_{2a}}{2} \end{vmatrix} > 0, \quad \begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{HS}}{4} & \frac{\lambda_{3B} + \lambda_{4B}}{2} \\ \frac{\lambda_{HS}}{4} & \frac{\lambda_S}{4} & \frac{\lambda_{S\eta_B}}{4} \\ \frac{\lambda_{3B} + \lambda_{4B}}{2} & \frac{\lambda_{S\eta_B}}{4} & \frac{\lambda_{2b}}{2} \end{vmatrix} > 0, \quad (3.26)$$

$$\begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{3A} + \lambda_{4A}}{2} & \frac{\lambda_{3B} + \lambda_{4B}}{2} \\ \frac{\lambda_{3A} + \lambda_{4A}}{2} & \frac{\lambda_{2a}}{2} & \frac{(1-c)}{2}\left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2}\right) \\ \frac{\lambda_{3B} + \lambda_{4B}}{2} & \frac{(1-c)}{2}\left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2}\right) & \frac{\lambda_{2b}}{2} \end{vmatrix} > 0, \quad \begin{vmatrix} \frac{\lambda_S}{4} & \frac{\lambda_{S\eta_A}}{4} & \frac{\lambda_{S\eta_B}}{4} \\ \frac{\lambda_{S\eta_A}}{4} & \frac{\lambda_{2a}}{2} & \frac{(1-c)}{2}\left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2}\right) \\ \frac{\lambda_{S\eta_B}}{4} & \frac{(1-c)}{2}\left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2}\right) & \frac{\lambda_{2b}}{2} \end{vmatrix} > 0, \quad (3.27)$$

$$\begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{HS}}{4} & \frac{\lambda_5}{2} \\ \frac{\lambda_{HS}}{4} & \frac{\lambda_S}{4} & 0 \\ \frac{\lambda_5}{2} & 0 & \frac{\lambda_{2b}}{2} \end{vmatrix} > 0, \quad (3.28)$$

where 10 constraints appear from them if we do not remove redundancies. We get the following independent constraints from the 4×4 matrices:

$$\begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{3A}+\lambda_{4A}}{2} & \frac{\lambda_5}{2} & \frac{\lambda_{3B}+\lambda_{4B}}{2} \\ \frac{\lambda_{3A}+\lambda_{4A}}{2} & \frac{\lambda_{2d}}{2} & 0 & \frac{(1-c)}{2} \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) \\ \frac{\lambda_5}{2} & 0 & c \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) & 0 \\ \frac{\lambda_{3B}+\lambda_{4B}}{2} & \frac{(1-c)}{2} \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) & 0 & \frac{\lambda_{2b}}{2} \end{vmatrix} > 0, \quad (3.29)$$

$$\begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{3A}+\lambda_{4A}}{2} & \frac{\lambda_5}{2} & \frac{\lambda_{HS}}{4} \\ \frac{\lambda_{3A}+\lambda_{4A}}{2} & \frac{\lambda_{2d}}{2} & 0 & \frac{\lambda_{S\eta A}}{4} \\ \frac{\lambda_5}{2} & 0 & c \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) & 0 \\ \frac{\lambda_{HS}}{4} & \frac{\lambda_{S\eta A}}{4} & 0 & \frac{\lambda_S}{4} \end{vmatrix} > 0, \quad (3.30)$$

$$\begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{3B}+\lambda_{4B}}{2} & \frac{\lambda_5}{2} & \frac{\lambda_{HS}}{4} \\ \frac{\lambda_{3B}+\lambda_{4B}}{2} & \frac{\lambda_{2b}}{2} & 0 & \frac{\lambda_{S\eta B}}{4} \\ \frac{\lambda_5}{2} & 0 & c \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) & 0 \\ \frac{\lambda_{HS}}{4} & \frac{\lambda_{S\eta B}}{4} & 0 & \frac{\lambda_S}{4} \end{vmatrix} > 0, \quad (3.31)$$

$$\begin{vmatrix} \frac{\lambda_H}{2} & \frac{\lambda_{3A}+\lambda_{4A}}{2} & \frac{\lambda_{3B}+\lambda_{4B}}{2} & \frac{\lambda_{HS}}{4} \\ \frac{\lambda_{3A}+\lambda_{4A}}{2} & \frac{\lambda_{2d}}{2} & \frac{(1-c)}{2} \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) & \frac{\lambda_{S\eta A}}{4} \\ \frac{\lambda_{3B}+\lambda_{4B}}{2} & \frac{(1-c)}{2} \left(\frac{\lambda_{2c}}{2} + \frac{\lambda_{2d}}{2} \right) & \frac{\lambda_{2b}}{2} & \frac{\lambda_{S\eta B}}{4} \\ \frac{\lambda_{HS}}{4} & \frac{\lambda_{S\eta A}}{4} & \frac{\lambda_{S\eta B}}{4} & \frac{\lambda_S}{4} \end{vmatrix} > 0, \quad (3.32)$$

where five constraints appear from them if we do not remove redundancies.

We must also constrain the value of c as described in [230], from the second constraint of Eq. (3.23) requires $|c| \geq |c_0|$,

$$c_0 = \frac{1}{(\lambda_{2c} + \lambda_{2d})} \left(\frac{\lambda_5^2}{\lambda_H} \right). \quad (3.33)$$

The extrema solutions of the left-hand side of the inequality Eq. (3.29) with respect to c are c_+ , c_- . We check which solution gives the maximum value for the inequality of Eq. (3.29) by substituting for c in its second derivative.

Finally, we have constraints,

$$c = \begin{cases} \max(c_0, c_+) & \text{if } (\lambda_{2c} + \lambda_{2d}) > 0, \\ \max(c_0, c_-) & \text{if } (\lambda_{2c} + \lambda_{2d}) < 0, \\ 0 & \text{if } (\lambda_{2c} + \lambda_{2d}) = 0. \end{cases} \quad (3.34)$$

We can compare and check our results with [231] for the Z_4 three doublet model if we turn off coupling with the S field and take its mass to zero. Our result is a match.

2. Inert condition via Hessian

The potential in Eq. (2.2) should not develop a VEV in the inert direction. We find these constraints by constructing the Hessian matrix H described in [231],

$$(H)_{ij} = \frac{\partial^2 \mathcal{V}}{\partial \phi_i \partial \phi_j} \Big|_{\phi_i = \langle \phi_i \rangle, \phi_j = \langle \phi_j \rangle}, \quad (3.35)$$

where $\phi_i, \phi_j = \{H, \eta_A, \eta_B, S\}$ assuming $\phi_i \in \mathcal{R}$. We consider the minimum case $\{v_H, 0, 0, v_S\}$,

$$H = \begin{pmatrix} 2m_H^2 + 3v_H^2\lambda_H + v_S^2\lambda_{HS} & 0 & 0 & \sqrt{2}v_H v_S \lambda_{HS} \\ 0 & 2m_{\eta_A}^2 + v_H^2(\lambda_{3A} + \lambda_{4A}) + v_S^2\lambda_{S\eta A} & \frac{\lambda_5 v_b^2}{4} & 0 \\ 0 & \frac{\lambda_5 v_b^2}{4} & m_{\eta_B}^2 + v_H^2(\lambda_{3B} + \lambda_{4B}) + v_S^2\lambda_{S\eta B} & 0 \\ \sqrt{2}v_H v_S \lambda_{HS} & 0 & 0 & m_S^2 + \frac{v_H^2}{2}\lambda_{HS} + 3v_S^2\lambda_S \end{pmatrix}. \quad (3.36)$$

The Hessian must be positive; therefore, we impose the Sylvester criterion to get the following constraints:

$$\begin{aligned}
2m_H^2 + 3v_H^2\lambda_H + v_S^2\lambda_{HS} &> 0, & 2m_{\eta_A}^2 + v_H^2(\lambda_{3A} + \lambda_{4A}) + v_S^2\lambda_{S\eta_A} &> 0, \\
2m_{\eta_B}^2 + v_H^2(\lambda_{3B} + \lambda_{4B}) + v_S^2\lambda_{S\eta_B} &> 0, & m_S^2 + \frac{v_H^2}{2}\lambda_{HS} + 3v_S^2\lambda_S &> 0.
\end{aligned} \tag{3.37}$$

$$\begin{aligned}
(2m_{\eta_A}^2 + v_H^2(\lambda_{3A} + \lambda_{4A}) + v_S^2\lambda_{S\eta_A})(m_{\eta_B}^2 + v_H^2(\lambda_{3B} + \lambda_{4B}) + v_S^2\lambda_{S\eta_B}) - \left(\frac{\lambda_5 v_h^2}{4}\right)^2 &> 0, \\
(2m_H^2 + 3v_H^2\lambda_H + \lambda_{HS}v_S^2)\left(m_S^2 + \frac{v_H^2}{2}\lambda_{HS} + 3v_S^2\lambda_S\right) - 2(v_H v_S \lambda_{HS})^2 &> 0.
\end{aligned} \tag{3.38}$$

Here, we have simplified and removed redundant constraints of the criteria.

IV. DARK MATTER PHENOMENOLOGY

As we discussed in Sec. II A, the lightest fermion that possesses the Z₄ parity $+i$ or $-i$ is stabilized by the residual Z₂ parity, and it can be a dark matter candidate χ . In this paper, we focus on the scenario that one of the heavy Majorana neutrinos N_{A_i} or N_{B_i} ($i = 1, 2, 3$) is a dark matter candidate. Which one is appropriate will be debated later.

We will consider the ordinary thermal freeze-out scenario in our scotogenic model. First, we will briefly summarize the scenario of dark matter. We follow the procedure in [232] in our implementation.

A. Relic density through thermal freeze-out

1. Thermal averaging

We first calculate the unpolarized annihilation rate,

$$\begin{aligned}
W_{ij \rightarrow kl} &= \frac{p_{kl}}{16\pi^2 g_i g_j S_{kl} \sqrt{s}} \int |\overline{\mathcal{M}_{ij \rightarrow kl}}|^2 d\Omega, \\
|\overline{\mathcal{M}_{ij \rightarrow kl}}|^2 &:= \sum_{\text{internal d.o.f.}} |\mathcal{M}_{ij \rightarrow kl}|^2,
\end{aligned} \tag{4.1}$$

where $\sqrt{s}$ is the total energy of the center-of-the-mass system and $d\Omega$ is the solid angle for the final state. $\sum_{\text{internal d.o.f.}} |\mathcal{M}_{ij \rightarrow kl}|^2$ is the square amplitude of the process $ij \rightarrow kl$ averaged over initial internal degrees of freedoms (d.o.f.) and summing over final d.o.f. [233]. The subscripts i, j in Eq. (4.1) represent different species of initial dark matter particles and k, l are final SM particle species. The indices k and l discriminate particles in final states, and p_{kl} is the final center-of-mass momentum, which is defined as

$$p_{kl} = \frac{\sqrt{s - (m_k + m_l)^2} \sqrt{s - (m_k - m_l)^2}}{2\sqrt{s}}, \tag{4.2}$$

where g_i is the degrees of freedom for the dark matter particles ($g_1 = 2$ for Majorana-fermion dark matter), and S_{kl} is the symmetry factor $S_{kl} = 2$ for identical final particles and $S_{kl} = 1$ otherwise. If we focus on a single DM candidate with no coannihilation, i and j take only one,

the effective unpolarized annihilation rate is

$$W_{\text{eff}} = \sum_{k,l} W_{11 \rightarrow kl}. \tag{4.3}$$

W_{eff} is the annihilation rate summed over all final state SM particles (k, l).

At equilibrium, we approximate the momentum distribution of the dark matter candidate to follow the Maxwell-Boltzmann distribution. We now integrate the annihilation rate over the momentum distribution of the dark matter population,

$$\mathcal{A} = \frac{g^2 T}{4\pi^4} \int_0^\infty dp_{\text{eff}} p_{\text{eff}}^2 W_{\text{eff}} K_1\left(\frac{\sqrt{s}}{T}\right), \tag{4.4}$$

$$p_{\text{eff}} = p_{11} = \frac{1}{2} \sqrt{s - 4M_\chi^2}, \tag{4.5}$$

where T describes the temperature, p_{eff} is the incoming momentum of the DM particle (in the center-of-mass frame) and $K_1(x)$ is a modified Bessel function of the second kind of order one. Equilibrium number density of the dark matter n_{eq} is

$$n_{\text{eq}} = \frac{T}{2\pi^2} g_\chi M_\chi^2 K_2\left(\frac{M_\chi}{T}\right), \tag{4.6}$$

where $K_2(x)$ is a modified Bessel function of the second kind of order two. The thermalized cross section can then be put together,

$$\langle \sigma_{\text{eff}} v \rangle = \frac{\mathcal{A}}{n_{\text{eq}}^2}. \tag{4.7}$$

In the case of coannihilating dark matter, there is an additional normalization for the case of coannihilation. We calculate the effective unpolarized annihilation rate,

$$W_{\text{eff}} = \sum_{i,j} \sum_{k,l} \frac{p_{ij}}{p_{11}} \frac{g_i g_j}{g_1^2} W_{ij \rightarrow kl}, \tag{4.8}$$

p_{11} is the initial center-of-mass momentum when the two initial-state particles are the dark matter ($i = j = 1$), while p_{ij} is that when the initial-state particles are the i th and j th dark matter candidates, defined as

$$p_{ij} = \frac{\sqrt{s - (M_{\chi_i} + M_{\chi_j})^2} \sqrt{s - (M_{\chi_i} - M_{\chi_j})^2}}{2\sqrt{s}}. \quad (4.9)$$

Also, the equilibrium number density in Eq. (4.6) is modified as

$$n_{\text{eq}} = \frac{T}{2\pi^2} \sum_i g_i M_{\chi_i}^2 K_2\left(\frac{M_{\chi_i}}{T}\right), \quad (4.10)$$

where the summation over i in Eq. (4.10) is taken over the dark-matter candidates relevant in the coannihilation.⁵

To calculate the thermal cross section in Eq. (4.7), we approximate $T = T_{f_i}$ and $x = x_f$ in the equilibrium number density as the different particles decouple at different temperatures,

$$T_{f_i} = \frac{M_{\chi_i}}{x_f}. \quad (4.11)$$

In the momentum integration of Eq. (4.4), we replace T with T_{f_1} as it is integrated over dark matter temperature.

2. Boltzmann equation

Let us consider the quantity Y ; it is the ratio of dark matter number density n to entropy density of the universe s ,

$$Y = \frac{n}{s}. \quad (4.12)$$

Y is the measure of dark matter number density; solving the following Boltzmann equation allows us to compare Y_∞ after the freeze-out mechanism as measured by Planck [234] $\Omega h^2 = 0.120 \pm 0.001$:

⁵Furthermore, we should modify the variable Y [defined in Eq. (4.12)] in the thermal equilibrium, appearing in the Boltzmann equation to describe the thermal freeze-out DM scenario in Eq. (4.13) for the coannihilation case as [232]

$$Y_{\text{eq}} = \frac{45x^2}{4\pi^4 h_{\text{eff}}} \sum_i g_i \left(\frac{M_{\chi_i}}{M_{\chi_1}}\right)^2 K_2\left(x \frac{M_{\chi_i}}{M_{\chi_1}}\right).$$

We make the same scaling modification to the rescaled variable [defined in Eq. (4.16)] as described earlier; the new yield at equilibrium is

$$y_{\text{eq}} = \frac{M_{\text{PL}} \sqrt{g^*} x^2}{\sqrt{90} h_{\text{eff}} \pi^3} \sum_i g_i \left(\frac{M_{\chi_i}}{M_{\chi_1}}\right)^2 K_2\left(x \frac{M_{\chi_i}}{M_{\chi_1}}\right).$$

$$\frac{dY}{dx} = -\sqrt{\frac{\pi}{45G}} \frac{\sqrt{g^*} M_\chi}{x^2} \langle \sigma_{\text{eff}} v \rangle (Y^2 - Y_{\text{eq}}^2), \quad (4.13)$$

and the distribution of Y in the thermal equilibrium is represented as

$$Y_{\text{eq}} = \frac{n_{\text{eq}}}{s} = \frac{45x^2}{4\pi^4 h_{\text{eff}}} g_1 K_2(x), \quad (4.14)$$

where we introduced the dimensionless inverse temperature x as

$$x := \frac{m_1}{T}. \quad (4.15)$$

g^* is the energy density degrees of freedom, h_{eff} is the entropy density degrees of freedom of the Universe, G is the gravitational constant. We use the scaling trick suggested by [235] to rescale the Boltzmann equation (4.13) by multiplying it with a constant $(M_{\text{PL}} (\frac{8\pi^2 g^*}{45})^{1/2})$, and we make the required changes to the Boltzmann equation,

$$\frac{dy}{dx} = -\langle \sigma_{\text{eff}} v \rangle M_\chi \frac{(y^2 - y_{\text{eq}}^2)}{x^2}, \quad y_{(\text{eq})} := M_{\text{PL}} \left(\frac{8\pi^2 g^*}{45}\right)^{1/2} Y_{(\text{eq})}, \quad (4.16)$$

$$y_{\text{eq}} = \frac{M_{\text{PL}} \sqrt{g^*} x^2}{\sqrt{90} h_{\text{eff}} \pi^3} g_1 K_2(x), \quad (4.17)$$

where $y_{(\text{eq})}$ is the new rescaled abundance and M_{PL} is the reduced Planck mass defined by $M_{\text{PL}} = (8\pi G)^{-1/2}$. This modification prevents numerical instabilities, which are common while solving the Boltzmann equation.

To solve Eq. (4.16), we approximate $T = T_f$ and $x = x_f$ when we calculate Eq. (4.7) which makes $\langle \sigma v \rangle_{\text{eff}}$ a constant within the region of integration, where T_f and x_f represent the temperature at which dark matter decouples or freezes out of the thermal bath ($x_f = M_\chi/T_f$). x_f typically ranges from 20 to 25, so we use $x_f = 20$ for our numerical analysis. Above 100 GeV, the degrees of freedom $g^* \simeq h_{\text{eff}} \approx 100$ and significantly decreases only at QCD phase transition (150–214 MeV) [236]. Since we are considering TeV-scale dark matter mass, we can use the typical value of $g^* \simeq h_{\text{eff}} \approx 100$ as a constant throughout the integration range of the Boltzmann equation because decoupling will occur before a significant change in g^* and h_{eff} .

To obtain relic density analytically, we need to integrate the Boltzmann equation from $x = 0$ to $x_\infty := M_\chi/T_0$ where T_0 is the present photon temperature of the Universe ($T_0 \sim 10^{-4}$ eV). We also have a boundary condition that is imposed $Y_{\text{eq}} = Y(x = 0)$, but practically, this is outside of the range of validity of the nonrelativistic expressions used. In our analysis we take $Y_{\text{eq}} = Y(x = 1)$ and integrate from $x = 1$ to $x = 10^3$. We have verified there is no strong

dependence on x for small x for the lower limit of the integration. We also check that $x = 10^3$ is sufficiently large, beyond which the changes in relic density are insignificant. At $x = 10^3$, we obtain the final abundance of dark matter, which can be plugged into the following formula to get the present dark matter relic density [232]:

$$\Omega h^2 = 2.755 \times 10^8 \times \frac{M_\chi}{\text{GeV}} \times Y_\infty, \quad (4.18)$$

where Y_∞ is defined as $Y(x = 10^3)$.

3. Lepton-portal contributions

In our scenario, where one of the heavy Majorana neutrinos is a thermal dark matter particle χ , through the Yukawa interaction in Eq. (2.37), the pair-annihilation channels $\chi\chi \rightarrow l_\alpha \bar{l}_\beta$ and $\chi\chi \rightarrow \nu_\alpha \bar{\nu}_\beta$ ($\alpha, \beta = 1, 2, 3$) contribute. We get the following scattering amplitudes contributing to the relic abundance in the freeze-out scenario; $\mathcal{M}_t$ and $\mathcal{M}_u$ are the total t -channel and u -channel amplitudes, respectively,

$$\begin{aligned} 4|\overline{\mathcal{M}_t^{(L)}}|^2 &= \sum_{\alpha, \beta=e, \mu, \tau} \left[(t - M_\chi^2 - m_{l_\alpha}^2)(t - M_\chi^2 - m_{l_\beta}^2) \right] \left| \sum_{s^+=\eta_1^+, \eta_2^+} \frac{(\mathcal{Y}_{\chi\alpha}^{(s^+)})^* \mathcal{Y}_{\chi\beta}^{(s^+)}}{t - M_{s^+}^2} \right|^2 \\ &+ \sum_{\alpha, \beta=e, \mu, \tau} \left[(t - M_\chi^2 - m_{\nu_\alpha}^2)(t - M_\chi^2 - m_{\nu_\beta}^2) \right] \left| \sum_{s^0=\eta_{R1}, \eta_{R2}, \eta_{I1}, \eta_{I2}} \frac{(\mathcal{Y}_{\chi\alpha}^{(s^0)})^* \mathcal{Y}_{\chi\beta}^{(s^0)}}{t - M_{s^0}^2} \right|^2, \end{aligned} \quad (4.19)$$

$$\begin{aligned} 4|\overline{\mathcal{M}_u^{(L)}}|^2 &= \sum_{\alpha, \beta=e, \mu, \tau} \left[(u - M_\chi^2 - m_{l_\alpha}^2)(u - M_\chi^2 - m_{l_\beta}^2) \right] \left| \sum_{s^+=\eta_1^+, \eta_2^+} \frac{(\mathcal{Y}_{\chi\alpha}^{(s^+)})^* \mathcal{Y}_{\chi\beta}^{(s^+)}}{u - M_{s^+}^2} \right|^2 \\ &+ \sum_{\alpha, \beta=e, \mu, \tau} \left[(u - M_\chi^2 - m_{\nu_\alpha}^2)(u - M_\chi^2 - m_{\nu_\beta}^2) \right] \left| \sum_{s^0=\eta_{R1}, \eta_{R2}, \eta_{I1}, \eta_{I2}} \frac{(\mathcal{Y}_{\chi\alpha}^{(s^0)})^* \mathcal{Y}_{\chi\beta}^{(s^0)}}{u - M_{s^0}^2} \right|^2, \end{aligned} \quad (4.20)$$

$$\begin{aligned} 4\left(\overline{\mathcal{M}_t^{(L)\dagger} \mathcal{M}_u^{(L)}} + \overline{\mathcal{M}_u^{(L)\dagger} \mathcal{M}_t^{(L)}}\right) &= - \sum_{\alpha, \beta=e, \mu, \tau} 2(s - m_{l_\alpha}^2 - m_{l_\beta}^2) M_\chi^2 \text{Re} \left[\sum_{s_1^+=\eta_1^+, \eta_2^+} \left(\frac{\mathcal{Y}_{\chi\alpha}^{(s_1^+)} (\mathcal{Y}_{\chi\beta}^{(s_1^+)})^*}{t - M_{s_1^+}^2} \right) \sum_{s_2^+=\eta_1^+, \eta_2^+} \left(\frac{(\mathcal{Y}_{\chi\alpha}^{(s_2^+)})^* \mathcal{Y}_{\chi\beta}^{(s_2^+)}}{u - M_{s_2^+}^2} \right) \right] \\ &- \sum_{\alpha, \beta=e, \mu, \tau} 2(s - m_{\nu_\alpha}^2 - m_{\nu_\beta}^2) M_\chi^2 \text{Re} \left[\sum_{s_1^0=\eta_{R1}, \eta_{R2}, \eta_{I1}, \eta_{I2}} \left(\frac{\mathcal{Y}_{\chi\alpha}^{(s_1^0)} (\mathcal{Y}_{\chi\beta}^{(s_1^0)})^*}{t - M_{s_1^0}^2} \right) \sum_{s_2^0=\eta_{R1}, \eta_{R2}, \eta_{I1}, \eta_{I2}} \left(\frac{(\mathcal{Y}_{\chi\alpha}^{(s_2^0)})^* \mathcal{Y}_{\chi\beta}^{(s_2^0)}}{u - M_{s_2^0}^2} \right) \right], \end{aligned} \quad (4.21)$$

where $\text{Re}[X]$ represents the real part of X , and we introduced the notation about the suffix for the Yukawa couplings in Eq. (2.37),

$$\mathcal{Y}_{\chi\alpha}^{(s)} := \text{the corresponding one from } \mathcal{Y}_{A_1\alpha}^{(s)}, \mathcal{Y}_{A_2\alpha}^{(s)}, \mathcal{Y}_{A_3\alpha}^{(s)}, \mathcal{Y}_{B_1\alpha}^{(s)}, \mathcal{Y}_{B_2\alpha}^{(s)}, \mathcal{Y}_{B_3\alpha}^{(s)}. \quad (4.22)$$

In the case of coannihilation, there are additional diagrams associated with the coannihilating particle; we use the convention χ_1 for the DM candidate and χ_2 for the coannihilating particle. In addition to the t -channel diagrams for $\chi_2\chi_2 \rightarrow l_\alpha \bar{l}_\beta$ that is added to dark matter annihilation, we will also have $\chi_1\chi_2 \rightarrow l_\alpha \bar{l}_\beta$ as described above. The u -channel and mixed interaction only contribute to $\chi_i\chi_j \rightarrow l_\alpha \bar{l}_\beta$ since χ_1 and χ_2 are not identical particles. The corresponding amplitudes squared for the coannihilation are written down as

$$\begin{aligned}
4(\overline{|\mathcal{M}_t^{(\text{L,co})}|^2})_{ij} = & \sum_{\alpha,\beta=e,\mu,\tau} \left[(t - M_{\chi_i}^2 - m_{l_\alpha}^2)(t - M_{\chi_j}^2 - m_{l_\beta}^2) \right] \left| \sum_{s^+=\eta_1^+, \eta_2^+} \frac{(\mathcal{Y}_{\chi_i \alpha}^{(s^+)})^* \mathcal{Y}_{\chi_j \beta}^{(s^+)}}{t - M_{s^+}^2} \right|^2 \\
& + \sum_{\alpha,\beta=e,\mu,\tau} \left[(t - M_{\chi_i}^2 - m_{\nu_\alpha}^2)(t - M_{\chi_j}^2 - m_{\nu_\beta}^2) \right] \left| \sum_{s^0=\eta_{R_1}, \eta_{R_2}, \eta_{I_1}, \eta_{I_2}} \frac{(\mathcal{Y}_{\chi_i \alpha}^{(s^0)})^* \mathcal{Y}_{\chi_j \beta}^{(s^0)}}{t - M_{s^0}^2} \right|^2, \quad (4.23)
\end{aligned}$$

$$\begin{aligned}
4(\overline{|\mathcal{M}_u^{(\text{L,co})}|^2})_{ii} = & \sum_{\alpha,\beta=e,\mu,\tau} \left[(u - M_{\chi_i}^2 - m_{l_\alpha}^2)(u - M_{\chi_i}^2 - m_{l_\beta}^2) \right] \left| \sum_{s^+=\eta_1^+, \eta_2^+} \frac{(\mathcal{Y}_{\chi_i \alpha}^{(s^+)})^* \mathcal{Y}_{\chi_i \beta}^{(s^+)}}{u - M_{s^+}^2} \right|^2 \\
& + \sum_{\alpha,\beta=e,\mu,\tau} \left[(u - M_{\chi_i}^2 - m_{\nu_\alpha}^2)(u - M_{\chi_i}^2 - m_{\nu_\beta}^2) \right] \left| \sum_{s^0=\eta_{R_1}, \eta_{R_2}, \eta_{I_1}, \eta_{I_2}} \frac{(\mathcal{Y}_{\chi_i \alpha}^{(s^0)})^* \mathcal{Y}_{\chi_i \beta}^{(s^0)}}{u - M_{s^0}^2} \right|^2, \quad (4.24)
\end{aligned}$$

$$\begin{aligned}
& 4 \left(\overline{\mathcal{M}_t^{(\text{L,co})\dagger} \mathcal{M}_u^{(\text{L,co})} + \mathcal{M}_u^{(\text{L,co})\dagger} \mathcal{M}_t^{(\text{L,co})}} \right)_{ii} \\
& = - \sum_{\alpha,\beta=e,\mu,\tau} 2(s - m_{l_\alpha}^2 - m_{l_\beta}^2) M_{\chi_i}^2 \text{Re} \left[\sum_{s_1^+=\eta_1^+, \eta_2^+} \left(\frac{\mathcal{Y}_{\chi_i \alpha}^{(s_1^+)} (\mathcal{Y}_{\chi_i \beta}^{(s_1^+)})^*}{t - M_{s_1^+}^2} \right) \sum_{s_2^+=\eta_1^+, \eta_2^+} \left(\frac{(\mathcal{Y}_{\chi_i \alpha}^{(s_2^+)})^* \mathcal{Y}_{\chi_i \beta}^{(s_2^+)}}{u - M_{s_2^+}^2} \right) \right] \\
& - \sum_{\alpha,\beta=e,\mu,\tau} 2(s - m_{\nu_\alpha}^2 - m_{\nu_\beta}^2) M_{\chi_i}^2 \text{Re} \left[\sum_{\substack{s_1^0=\eta_{R_1}, \eta_{R_2}, \\ \eta_{I_1}, \eta_{I_2}}} \left(\frac{\mathcal{Y}_{\chi_i \alpha}^{(s_1^0)} (\mathcal{Y}_{\chi_i \beta}^{(s_1^0)})^*}{t - M_{s_1^0}^2} \right) \sum_{\substack{s_2^0=\eta_{R_1}, \eta_{R_2}, \\ \eta_{I_1}, \eta_{I_2}}} \left(\frac{(\mathcal{Y}_{\chi_i \alpha}^{(s_2^0)})^* \mathcal{Y}_{\chi_i \beta}^{(s_2^0)}}{u - M_{s_2^0}^2} \right) \right]. \quad (4.25)
\end{aligned}$$

4. Higgs-portal contributions

In our scenario, the six heavy right-handed neutrinos N_{A_i} and N_{B_i} couple with the singlet scalar S , and after the spontaneous symmetry breaking with a nonzero VEV for S , the residual Z_2 symmetry remains, and the quantum fluctuation of S (s') mixes with the physical Higgs field h' [refer to Eqs. (2.3), (2.4), and (2.5)]. Therefore, $\chi\chi \rightarrow$ [a pair of SM particles] is also possible as a Higgs portal.

We introduce the following compact notation for clarity to the relevant part of our calculation:

$$\begin{aligned}
-\mathcal{L}_{\text{Higgs Yukawa}} \supset & \sum_{i=1}^3 \sum_{h^0=h,s} \left[\mathcal{Y}_{A_i}^{(h^0)} (\overline{N_{A_i R}})^c N_{A_i R} h^0 + \mathcal{Y}_{B_i}^{(h^0)} (\overline{N_{B_i R}})^c N_{B_i R} h^0 \right. \\
& \left. + \mathcal{Y}_{AB_i}^{(h^0)} (\overline{N_{B_i R}})^c N_{A_i R} h^0 + \mathcal{Y}_{AB_i}^{(h^0)} (\overline{N_{A_i R}})^c N_{B_i R} h^0 \right], \quad (4.26)
\end{aligned}$$

with

$$\begin{aligned}
\mathcal{Y}_{A_i}^{(h)} &= \frac{y_{A_{ii}}}{2} c_{N_i}^2 s_\alpha + \frac{y_{B_{ii}}}{2} s_{N_i}^2 s_\alpha, & \mathcal{Y}_{B_i}^{(h)} &= \frac{y_{A_{ii}}}{2} s_{N_i}^2 s_\alpha + \frac{y_{B_{ii}}}{2} c_{N_i}^2 s_\alpha, \\
\mathcal{Y}_{A_i}^{(s)} &= \frac{y_{A_{ii}}}{2} c_{N_i}^2 c_\alpha + \frac{y_{B_{ii}}}{2} s_{N_i}^2 c_\alpha, & \mathcal{Y}_{B_i}^{(s)} &= \frac{y_{A_{ii}}}{2} s_{N_i}^2 c_\alpha + \frac{y_{B_{ii}}}{2} c_{N_i}^2 c_\alpha, \\
\mathcal{Y}_{AB_i}^{(h)} &= \frac{y_{A_{ii}}}{2} s_{N_i} c_{N_i} s_\alpha - \frac{y_{B_{ii}}}{2} s_{N_i} c_{N_i} s_\alpha, & \mathcal{Y}_{AB_i}^{(s)} &= \frac{y_{A_{ii}}}{2} s_{N_i} c_{N_i} c_\alpha + \frac{y_{B_{ii}}}{2} s_{N_i} c_{N_i} c_\alpha, \quad (4.27)
\end{aligned}$$

where the six Yukawa couplings $y_{A_{ii}}$ and $y_{B_{ii}}$ ($i = 1, 2, 3$) do not appear in the active-neutrino mass formula and the lepton flavor violation, as shown in Eqs. (2.43), (2.44), and (3.1). So, these couplings are not constrained through these phenomena and play an important role in the Higgs-portal freeze-out DM scenario. Note that, in our current parametrization, the six effective couplings in Eq. (4.27) are real.

The following part of the SM Higgs couplings is relevant for our study:

$$\mathcal{L}_{\text{Higgs, SM}} = \sum_{h^0=h,s} h^0 \left[\mathcal{C}_W^{(h^0)} W_\mu^+ W^{-\mu} + \mathcal{C}_Z^{(h^0)} \frac{1}{2} Z_\mu Z^\mu \right] - \sum_{h^0=h,s} \mathcal{Y}_f^{(h^0)} h^0 \bar{f} f, \quad (4.28)$$

with

$$\begin{aligned} \mathcal{C}_W^{(h)} &= \frac{2M_W^2 c_\alpha}{v_H}, & \mathcal{C}_Z^{(h)} &= \frac{2M_Z^2 c_\alpha}{v_H}, & \mathcal{Y}_f^{(h)} &= \frac{M_f c_\alpha}{v_H}, \\ \mathcal{C}_W^{(s)} &= -\frac{2M_W^2 s_\alpha}{v_H}, & \mathcal{C}_Z^{(s)} &= -\frac{2M_Z^2 s_\alpha}{v_H}, & \mathcal{Y}_f^{(s)} &= -\frac{M_f s_\alpha}{v_H}, \end{aligned} \quad (4.29)$$

where M_f is the mass of a SM fermion f .⁶

Under the interaction in Eq. (4.26) and Eq. (4.28), we get the s -channel scattering amplitudes as follows:

$$\begin{aligned} 4|\overline{\mathcal{M}_s^{(H)}}|^2 &= \sum_f 16[(s - 4M_\chi^2)(s - 4M_f^2)] \left| \sum_{h^0=h,s} \frac{\mathcal{Y}_\chi^{(h^0)} \mathcal{Y}_f^{(h^0)}}{s - M_{h^0}^2 + iM_{h^0} \Gamma_{h^0}} \right|^2 \\ &+ 8 \left[(s - 4M_\chi^2) \left(2 - \frac{(s - 2M_W^2)^2}{4M_W^4} \right) \right] \left| \sum_{h^0=h,s} \frac{\mathcal{Y}_\chi^{(h^0)} \mathcal{C}_W^{(h^0)}}{s - M_{h^0}^2 + iM_{h^0} \Gamma_{h^0}} \right|^2 \\ &+ 8 \left[(s - 4M_\chi^2) \left(2 - \frac{(s - 2M_Z^2)^2}{4M_Z^4} \right) \right] \left| \sum_{h^0=h,s} \frac{\mathcal{Y}_\chi^{(h^0)} \mathcal{C}_Z^{(h^0)}}{s - M_{h^0}^2 + iM_{h^0} \Gamma_{h^0}} \right|^2, \end{aligned} \quad (4.30)$$

where f is summed over all SM fermions (coupled with the Higgs boson) and Γ_{h^0} represents the decay width of h^0 .⁷ The first term represents dark-matter pair annihilations into SM fermions, and the second and third terms represent those to SM massive gauge bosons W and Z , respectively.

In addition to the s -channel diagrams, we include t -channel and u -channel diagrams for $\chi\chi \rightarrow hh$, mediated via the Majorana fermions; this can be the dark matter particles and heavier ones. The following forms will be used for our numerical calculations:

$$\begin{aligned} 4|\overline{\mathcal{M}_t^{(H)}}|^2_{11} &= 4 \sum_{l,m=1,2} \left[\frac{1}{2} (M_{\chi_1}^2 + M_{\chi_1} M_{\chi_l} + M_{\chi_1} M_{\chi_m}) (s + u - t - 4M_{\chi_1}^2) + \frac{1}{2} M_{\chi_l} M_{\chi_m} (s - 4M_{\chi_1}^2) \right. \\ &\quad \left. + \frac{1}{2} M_{\chi_1}^2 (u - t) - \frac{M_h^4}{2} + \frac{1}{2} (M_{\chi_1}^2 - t)(M_{\chi_1}^2 - u) \right] \left(\frac{\mathcal{Y}_{\chi_l \chi_1}^{(h)} \mathcal{Y}_{\chi_l \chi_1}^{(h)}}{t - M_{\chi_l}^2} \right) \left(\frac{\mathcal{Y}_{\chi_l \chi_m}^{(h)} \mathcal{Y}_{\chi_l \chi_m}^{(h)}}{t - M_{\chi_m}^2} \right), \end{aligned} \quad (4.31)$$

$$\begin{aligned} 4|\overline{\mathcal{M}_u^{(H)}}|^2_{11} &= 4 \sum_{l,m=1,2} \left[\frac{1}{2} (M_{\chi_1}^2 + M_{\chi_1} M_{\chi_l} + M_{\chi_1} M_{\chi_m}) (s + t - u - 4M_{\chi_1}^2) + \frac{1}{2} M_{\chi_l} M_{\chi_m} (s - 4M_{\chi_1}^2) \right. \\ &\quad \left. + \frac{1}{2} M_{\chi_1}^2 (t - u) - \frac{M_h^4}{2} + \frac{1}{2} (M_{\chi_1}^2 - t)(M_{\chi_1}^2 - u) \right] \left(\frac{\mathcal{Y}_{\chi_l \chi_1}^{(h)} \mathcal{Y}_{\chi_l \chi_1}^{(h)}}{u - M_{\chi_l}^2} \right) \left(\frac{\mathcal{Y}_{\chi_l \chi_m}^{(h)} \mathcal{Y}_{\chi_l \chi_m}^{(h)}}{u - M_{\chi_m}^2} \right), \end{aligned} \quad (4.32)$$

$$\begin{aligned} 4 \left(\left(\mathcal{M}_t^{(H)} \right)^\dagger \mathcal{M}_u^{(H)} + \left(\mathcal{M}_u^{(H)} \right)^\dagger \mathcal{M}_t^{(H)} \right)_{11} &= -4 \sum_{l,m=1,2} \left[-M_h^4 + (M_{\chi_1}^2 - t)(M_{\chi_1}^2 - u) - M_{\chi_1} (M_{\chi_l} + M_{\chi_m})(u - t) \right. \\ &\quad \left. - (s - 4M_{\chi_1}^2)(M_{\chi_1}^2 - M_{\chi_1} M_{\chi_l} + M_{\chi_1} M_{\chi_m} - M_{\chi_l} M_{\chi_m}) \right] \\ &\quad \times \left[\left(\frac{\mathcal{Y}_{\chi_l \chi_1}^{(h)} \mathcal{Y}_{\chi_l \chi_1}^{(h)}}{t - M_{\chi_l}^2} \right) \left(\frac{\mathcal{Y}_{\chi_l \chi_m}^{(h)} \mathcal{Y}_{\chi_l \chi_m}^{(h)}}{u - M_{\chi_m}^2} \right) \right], \end{aligned} \quad (4.33)$$

⁶In the following analysis in Sec. V, we will consider the top, bottom quarks (if kinematically accepted) as the final state f in our numerical scans for the DM mass $\gtrsim 10^2$ GeV.

⁷We will provide the calculation of Γ_s in Appendix A.

where we put the condition that only the second-lightest Majorana fermion $\chi_{i=2}(=\chi_2)$ and the lightest as a dark matter $\chi_{i=1}(=\chi_1=\chi)$ are taken into account. Also, we introduced similar notations for the Yukawa couplings as for the lepton-portal channels in Eq. (4.22) [refer to Eqs. (4.27) and (4.29)],

$$\mathcal{Y}_{\chi_1}^{(h)} := \text{the corresponding one from } \mathcal{Y}_{A_1}^{(h)}, \mathcal{Y}_{A_2}^{(h)}, \mathcal{Y}_{A_3}^{(h)}, \mathcal{Y}_{B_1}^{(h)}, \mathcal{Y}_{B_2}^{(h)}, \mathcal{Y}_{B_3}^{(h)}, \quad (4.34)$$

$$\mathcal{Y}_{\chi_1\chi_2}^{(h)} = \mathcal{Y}_{\chi_2\chi_1}^{(h)} := \text{the corresponding one from } \mathcal{Y}_{AB_1}^{(h)}, \mathcal{Y}_{AB_2}^{(h)}, \mathcal{Y}_{AB_3}^{(h)}. \quad (4.35)$$

Similar to the lepton portal, we ought to consider additional diagrams involved in the coannihilation processes in the Higgs-portal scenario, too; $\chi_2\chi_2 \rightarrow f\bar{f}$, $\chi_1\chi_2 \rightarrow f\bar{f}$ as well as $\chi_2\chi_2 \rightarrow b\bar{b}$, $\chi_1\chi_2 \rightarrow b\bar{b}$ contribute to the relic in the coannihilation scenario. Here, f and b are the standard model fermions and bosons, respectively,

$$\begin{aligned} 4|\overline{\mathcal{M}_s^{(\text{H,co})}}|_{22}^2 &= \sum_f 16[(s-4M_{\chi_2}^2)(s-4m_f^2)] \left| \sum_{h^0=h,s} \frac{\mathcal{Y}_{\chi_2}^{(h^0)} \mathcal{Y}_f^{(h^0)}}{s-M_{h^0}^2+iM_{h^0}\Gamma_{h^0}} \right|^2 \\ &+ 8 \left[(s-4M_{\chi_2}^2) \left(2 - \frac{(s-2M_W^2)^2}{4M_W^4} \right) \right] \left| \sum_{h^0=h,s} \frac{\mathcal{Y}_{\chi_2}^{(h^0)} \mathcal{C}_W^{(h^0)}}{s-M_{h^0}^2+iM_{h^0}\Gamma_{h^0}} \right|^2 \\ &+ 8 \left[(s-4M_{\chi_2}^2) \left(2 - \frac{(s-2M_Z^2)^2}{4M_Z^4} \right) \right] \left| \sum_{h^0=h,s} \frac{\mathcal{Y}_{\chi_2}^{(h^0)} \mathcal{C}_Z^{(h^0)}}{s-M_{h^0}^2+iM_{h^0}\Gamma_{h^0}} \right|^2, \end{aligned} \quad (4.36)$$

$$\begin{aligned} 4|\overline{\mathcal{M}_s^{(\text{H,co})}}|_{12}^2 &= \sum_f 16[(s-(M_{\chi_1}+M_{\chi_2})^2)(s-4m_f^2)] \left| \sum_{h^0=h,s} \frac{\mathcal{Y}_{\chi_1\chi_2}^{(h^0)} \mathcal{Y}_f^{(h^0)}}{s-M_{h^0}^2+iM_{h^0}\Gamma_{h^0}} \right|^2 \\ &+ 8 \left[(s-(M_{\chi_1}+M_{\chi_2})^2) \left(2 - \frac{(s-2M_W^2)^2}{4M_W^4} \right) \right] \left| \sum_{h^0=h,s} \frac{\mathcal{Y}_{\chi_1\chi_2}^{(h^0)} \mathcal{C}_W^{(h^0)}}{s-M_{h^0}^2+iM_{h^0}\Gamma_{h^0}} \right|^2 \\ &+ 8 \left[(s-(M_{\chi_1}+M_{\chi_2})^2) \left(2 - \frac{(s-2M_Z^2)^2}{4M_Z^4} \right) \right] \left| \sum_{h^0=h,s} \frac{\mathcal{Y}_{\chi_1\chi_2}^{(h^0)} \mathcal{C}_Z^{(h^0)}}{s-M_{h^0}^2+iM_{h^0}\Gamma_{h^0}} \right|^2, \end{aligned} \quad (4.37)$$

where $\mathcal{Y}_{\chi_2}^{(h^0)}$ is defined as similar as $\mathcal{Y}_{\chi_1}^{(h^0)}$ in Eq. (4.34).

In the coannihilation scenario, we replace the mass difference between the two species with $\delta M_{ij} = |M_{\chi_j} - M_{\chi_i}|$, where $i = 1, 2$ and $j = 1, 2$, and $\delta M_{ii} = 0$:

$$\begin{aligned} 4|\overline{\mathcal{M}_t^{(\text{H,co})}}|_{ij}^2 &= 2 \sum_{l=1,2} \sum_{m=1,2} \left[-M_h^2(s-\delta M_{ij}^2 - (2M_{\chi_i} + M_{\chi_l} + M_{\chi_m})\delta M_{ij}) \right. \\ &+ (s-M_{\chi_i}^2-M_{\chi_j}^2)(M_{\chi_i}^2 + M_{\chi_i}M_{\chi_l} + M_{\chi_i}M_{\chi_m} + M_{\chi_l}M_{\chi_m}) \\ &+ (2M_{\chi_i} + M_{\chi_l} + M_{\chi_m})(M_{\chi_j}(M_{\chi_i}^2 - t) - M_{\chi_i}(M_{\chi_j}^2 - u)) \\ &\left. + \frac{1}{2}(M_h^2 + M_{\chi_i}^2 - t)(M_h^2 + M_{\chi_j}^2 - u) \right] \left(\frac{(\mathcal{Y}_{\chi_l\chi_i}^{(h)})\mathcal{Y}_{\chi_l\chi_j}^{(h)}}{t-M_{\chi_l}^2} \right) \left(\frac{(\mathcal{Y}_{\chi_i\chi_m}^{(h)})\mathcal{Y}_{\chi_j\chi_m}^{(h)}}{t-M_{\chi_m}^2} \right), \end{aligned} \quad (4.38)$$

$$\begin{aligned} 4|\overline{\mathcal{M}_u^{(\text{H,co})}}|_{22}^2 &= 4 \sum_{l=1,2} \sum_{m=1,2} \left[\frac{1}{2}(M_{\chi_2}^2 + M_{\chi_2}M_{\chi_l} + M_{\chi_2}M_{\chi_m})(s+t-u-4M_{\chi_2}^2) \right. \\ &+ \frac{1}{2}M_{\chi_l}M_{\chi_m}(s-4M_{\chi_2}^2) + \frac{1}{2}M_{\chi_2}^2(t-u) - \frac{M_h^4}{2} \\ &\left. + \frac{1}{2}(M_{\chi_2}^2 - t)(M_{\chi_2}^2 - u) \right] \left(\frac{(\mathcal{Y}_{\chi_l\chi_2}^{(h)})\mathcal{Y}_{\chi_l\chi_2}^{(h)}}{u-M_{\chi_l}^2} \right) \left(\frac{(\mathcal{Y}_{\chi_2\chi_m}^{(h)})\mathcal{Y}_{\chi_2\chi_m}^{(h)}}{u-M_{\chi_m}^2} \right), \end{aligned} \quad (4.39)$$

$$\begin{aligned}
4((\mathcal{M}_t^{(\text{H,co})})^\dagger \mathcal{M}_u^{(\text{H,co})} + (\mathcal{M}_u^{(\text{H,co})})^\dagger \mathcal{M}_t^{(\text{H,co})})_{22} = & -4 \sum_{l=1,2} \sum_{m=1,2} [-M_h^4 + (M_{\chi_2}^2 - t)(M_{\chi_2}^2 - u) - M_{\chi_2}(M_{\chi_l} + M_{\chi_m})(u - t) \\
& - (s - 4M_{\chi_2}^2)(M_{\chi_2}^2 - M_{\chi_2}M_{\chi_l} + M_{\chi_2}M_{\chi_m} - M_{\chi_l}M_{\chi_m})] \\
& \times \left(\frac{(\mathcal{Y}_{\chi_l \chi_2}^{(h)}) \mathcal{Y}_{\chi_l \chi_2}^{(h)}}{t - M_{\chi_l}^2} \right) \left(\frac{(\mathcal{Y}_{\chi_2 \chi_m}^{(h)}) \mathcal{Y}_{\chi_2 \chi_m}^{(h)}}{u - M_{\chi_m}^2} \right). \quad (4.40)
\end{aligned}$$

Note the other possible processes with s in the final state, $\chi + \chi \rightarrow h + s$ and $\chi + \chi \rightarrow s + s$. As we will discuss, we focus on the kinematic region in $2M_\chi \lesssim M_s$, where the two processes are kinematically suppressed and may not contribute significantly. Therefore, we ignore them.

B. Constraints via direct detection

1. Lepton-portal contributions

We define $\mathcal{A}_S$ as the anapole moment; it is the effective coupling between a photon and the dark matter candidate (Majorana fermion) [237] via a charged scalar-SM lepton loop,

$$\mathcal{A}_S = -\frac{e}{96\pi^2 M_\chi^2} \sum_{s^+ = \eta_1^+, \eta_2^+} \sum_{\alpha = \mu, \tau} |\mathcal{Y}_{\chi\alpha}^{(s^+)}|^2 \mathcal{F} \left(\frac{m_{l_\alpha}}{M_\chi}, \frac{M_{s^+}}{M_\chi} \right) \quad (4.41)$$

$$-\frac{e}{32\pi^2 M_\chi^2} \sum_{s^+ = \eta_1^+, \eta_2^+} |\mathcal{Y}_{\chi e}^{(s^+)}|^2 \mathcal{F}_e \left(\frac{\sqrt{|q|^2}}{M_\chi}, \frac{M_{s^+}}{M_\chi} \right). \quad (4.42)$$

This calculation assumes the mass of the electron to be negligible; hence α is only summed over μ and τ , the second line in Eq. (4.42) is the electron loop contribution. The formula holds only when momentum transfer from dark matter to the target nucleus q^2 is greater than the lepton mass squared ($q^2 > m_l^2$), as in our case. We use shorthand notations, $\mu := M_{s^+}/M_\chi$ and $\epsilon := m_{l_\alpha}/M_\chi$ below,

$$\mathcal{F}(\mu, \epsilon) = \frac{3}{2} \log \left(\frac{\mu^2}{\epsilon^2} \right) - \frac{1 + 3\mu^2 - 3\epsilon^2}{\sqrt{(\mu^2 - 1 - \epsilon^2)^2 - 4\epsilon^2}} \text{Arc tanh} \left(\frac{\sqrt{(\mu^2 - 1 - \epsilon^2)^2 - 4\epsilon^2}}{\mu^2 - 1 + \epsilon^2} \right), \quad (4.43)$$

$$\mathcal{F}_e \left(\frac{\sqrt{|q|^2}}{M_\chi}, \mu \right) = \frac{-10 + 12 \log \left(\frac{\sqrt{|q|^2}}{M_\chi} \right) - (3 + 9\mu^2) \log(\mu^2 - 1) - (3 - 9\mu^2) \log(\mu^2)}{9(\mu^2 - 1)}. \quad (4.44)$$

We quote (the relevant part of) the spin-independent differential cross section of dark matter scattering off a nucleus having atomic number Z formula from [237],⁸

$$\frac{d\sigma_\chi^{\text{I}}}{dE_R} = 4\mathcal{A}_S^2 \left[\alpha_{\text{EM}} Z^2 \left(2m_T - \left(1 + \frac{m_T}{M_\chi} \right)^2 \frac{E_R}{v^2} \right) F_Z^2(q^2) \right], \quad (4.45)$$

where E_R is the recoil energy, α_{EM} the fine-structure constant in electromagnetism, Z is the nuclear charge of the detector atom, m_T is the detector nucleus mass, and v is the velocity of the dark matter particle relative to that of the nucleus. $F_Z(q)$ is the nuclear charge form factor [237],

⁸Note that the full form of Eq. (4.45) is as follows:

$$\frac{d\sigma_\chi^{\text{I}}}{dE_R} = 4\mathcal{A}_S^2 \left[\alpha_{\text{EM}} Z^2 \left(2m_T - \left(1 + \frac{m_T}{M_\chi} \right)^2 \frac{E_R}{v^2} \right) F_Z^2(q^2) + d_A^2 \left(\frac{J+1}{3J} \right) \frac{2E_R m_T^2}{\pi v^2} F_S^2(q^2) \right],$$

where the second term in the cross section describes the scattering with the magnetic dipole moment of the nucleus. This is subdominant in the xenon detectors, which is relevant to our paper. Hence, it is dropped going forward. d_A , J , and $F_S(q^2)$ are the nuclear dipole moment, the spin form factor, and the spin of the target nucleus.

$$F_Z(q) = 3e^{-\frac{q^2 s^2}{2}} \frac{(\sin(qr) - qr \cos(qr))}{(qr)^3}, \quad (4.46)$$

where $s = 1$ fm, $r = \sqrt{R^2 - 5s^2}$, $R = 1.2A^{1/3}$ fm (with the nuclear mass number A). The (squared) momentum transfer from dark matter to the target nucleus q^2 takes

$$q^2 = 2m_T E_R. \quad (4.47)$$

To compare with the latest dark matter direct detection experiments [203], we must write down the total DM-nucleon spin-independent cross section. We can express the DM velocity relative to the nucleus in terms of nucleus recoil energy [238],

$$v = \left[\frac{m_T E_R}{\mu_{\chi T}^2 (1 - \cos \theta)} \right]^{1/2}, \quad (4.48)$$

where $\mu_{\chi T}$ is the reduced mass of the dark matter-nucleus system. As described in [238], we integrate Eq. (4.45) over the solid angle and the recoil energy the detector is sensitive to. In the case of the LZ detector, we integrate from 5.5 to 55 KeV recoil energies, in which region the detector efficiency is $> 50\%$ [203].

We can now compare the latest results of the LZ cross section per nucleon [203] with our calculated cross section

$$\sigma_{\chi p}^L = \frac{\sigma_{\chi}^L}{A}, \quad (4.49)$$

where $A = 129$ for Xenon. The detector uses a mixture of isotopes of which Xe^{129} has the largest contribution. In Eq. (4.49), we have used a conservative, simple way to convert our calculated total cross section into cross section per nucleon.

2. Higgs-portal contributions

We quote the spin-independent cross section of dark matter scattering off a nucleon, having the formula from [239],

$$y_{\chi H} = \begin{cases} \frac{y_{Aii}}{2} c_{N_i}^2 + \frac{y_{Bii}}{2} s_{N_i}^2 & \text{if } \chi \text{ is } N_{A_i} (i = 1, 2, 3), \\ \frac{y_{Aii}}{2} s_{N_i}^2 + \frac{y_{Bii}}{2} c_{N_i}^2 & \text{if } \chi \text{ is } N_{B_i} (i = 1, 2, 3), \end{cases} \quad (4.50)$$

$$\sigma_{\chi p}^H = \frac{\mu_{\chi p}^2 y_{\chi H}^2 s_a^2 c_a^2 m_p^2}{\pi v_H^2} \mathcal{F}_H(M_{\chi}, M_h, M_s, v) f_p^2. \quad (4.51)$$

We define coupling $y_{\chi H}$ between the neutral singlet S (gauge basis) and DM candidate χ (mass basis) as is described in [239],

$$\begin{aligned} \mathcal{F}_H(M_{\chi}, M_{H_1}, M_{H_2}, v) &= \frac{1}{4M_{\chi}^2 v^2} \left[\sum_{i=1,2} \left(\frac{1}{M_{H_i}^2} - \frac{1}{4M_{\chi}^2 v^2 + M_{H_i}^2} \right) \right. \\ &\quad \left. - \frac{2}{(M_{H_2}^2 - M_{H_1}^2)} \sum_{i=1}^2 (-1)^{i-1} \log \left(1 + \frac{4M_{\chi}^2 v^2}{M_{H_i}^2} \right) \right], \end{aligned} \quad (4.52)$$

$$\mu_{\chi p} = \frac{m_p M_{\chi}}{m_p + M_{\chi}}, \quad (4.53)$$

where we took $M_{H_1} = M_h$ and $M_{H_2} = M_s$. Here, proton mass m_p is assumed to be nucleon mass, $\mu_{\chi p}$ is dark matter-nucleon reduced mass. v is dark matter velocity in the laboratory frame and $f_p \approx 0.326$ is the nucleon form factor. To compare with the results of the LZ Collaboration [203], we input the dark matter velocity v in Eq. (4.51) for the spin-independent cross section per nucleon. The mean velocity of the dark matter halo is $v = 220$ km/s $= 73.5 \times 10^{-5} \approx 10^{-3}$ in natural units is used in our analysis.

V. NUMERICAL ANALYSIS

We will discuss the dark matter phenomenology of the current scenario, taking into account the observed active neutrino texture, and constraints via the lepton flavor violation ($l_{\alpha} \rightarrow l_{\beta} + \gamma$), T parameter oblique constraint, and the stability of the scalar potential. For simplicity, we focus on the normal ordering in the active neutrino mass matrix below.⁹

A key parameter of our dark matter scenario is the mixing angle α between the Higgs doublet and the singlet, which determines the magnitudes of the Higgs-portal channels in the relic abundance and constraints via the direct detection. Note that $\alpha = 0$ corresponds to the pure lepton-portal scenario. We choose the Majorana fermion $N_{B_3} (= \chi_1 = \chi)$ as a dark matter candidate, where under the texture in Eqs. (2.37) and (4.26). $N_{A_3} (= \chi_2)$ is an efficient partner of coannihilation. As shown in Eqs. (4.27) and (4.50), the relevant Yukawa couplings to the gauge singlet among them, namely $y_{A_{33}}$ and $y_{B_{33}}$, play a primary role in relic abundance and direct detection. We take the other $y_{A_{ii}}$ and $y_{B_{ii}}$ as zero ($y_{A_{11}} = y_{A_{22}} = y_{B_{11}} = y_{B_{22}} = 0$). Another important parameter is the mixing angle for the “third”-generation Majorana fermions θ_{N_3} , which modulates the interactions for relic abundance and direct detection. Finally, we should point out that the dark matter candidate can hit the pole of the (mostly singlet) scalar s if the mass is around half of it. The resonance enhancement in the

⁹All numerical calculations except for Fig. 5 in this paper were performed using Python3 [240] with the following packages: MATH [241], NumPy [242], PANDAS [243], RANDOM [241], and SciPy [244]; also MATPLOTLIB [245] for plotting. For Fig. 5, *Mathematica* was used.

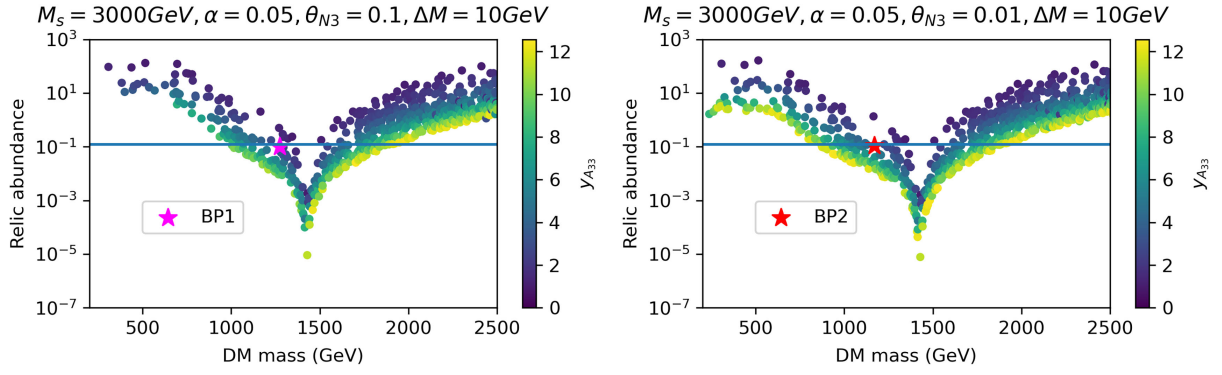


FIG. 3. Results of the relic abundance calculations under $M_s = 3$ TeV, $M_{\chi_2} - M_{\chi_1} (= \Delta M) = 10$ GeV, $\alpha = 0.05$; $\theta_{N_3} = 0.1$ (left panel) and $\theta_{N_3} = 0.01$ (right panel), taking account of coannihilation. The horizontal blue line depicts the experimental observation of the DM relic abundance ($\Omega h^2 = 0.120 \pm 0.001$) [234]. Note that each point of the panels is consistent with the observed neutrino profiles, the constraints via the lepton flavor violation ($l_\alpha \rightarrow l_\beta + \gamma$), and T parameter. The colors of the points indicate the magnitudes of $y_{A_{33}}$, while we take $y_{B_{33}} = 0$. The details of the two benchmark points are available in Table III.

annihilation (without large couplings) helps dark matter phenomenology.

As a benchmark, we choose $M_s = 3$ TeV, $\Delta M := M_{\chi_2} - M_{\chi_1} = 10$ GeV, $\alpha = 0.05$, where the Higgs-portal channel is switched on and the coannihilation mechanism may work. Figure 3 provides the results of our numerical scans

for $\theta_{N_3} = 0.1$ (left panel) and $\theta_{N_3} = 0.01$ (right panel), where other relevant parameters are scanned.

For each scan, we randomly selected the parameters $M_{\chi_1} (= M_{N_{B_3}})$, $M_{N_{A_1}}$, $M_{N_{B_1}}$, $M_{N_{A_2}}$, $M_{N_{B_2}}$, $M_{\eta_1^+}$, $M_{\eta_2^+}$, M_{R_1} , M_{R_2} , θ_R , λ_5 , $y_{A_{33}}$, while the following other independent relevant parameters are fixed; M_s , $M_{\chi_2} (= M_{N_{A_3}})$, θ_{N_1} , θ_{N_2} ,

TABLE III. The values of the relevant parameters for the seven benchmark points in Figs. 3 and 4 and realized relic abundances (Ωh^2) are shown. Note that they are consistent with the observed neutrino profiles, the lepton flavor violation ($l_\alpha \rightarrow l_\beta + \gamma$), T parameter, and DM direct detection.

Parameters	BP1	BP2	BP3	BP4	BP5	BP6	BP7
Ωh^2	0.1019	0.1119	0.1075	0.1042	0.1039	0.0830	0.1041
M_s (TeV)	3.0	3.0	2.0	2.0	2.0	2.0	2.0
$M_{\chi_1} (= M_{N_{B_3}})$ (GeV)	1274	1169	978	748	1230	965	778
$M_{\chi_2} (= M_{N_{A_3}})$ (GeV)	1284	1179	988	758	1240	975	788
$M_{\eta_1^+}$ (GeV)	2199.48	2795.66	2015.27	1299.27	2138.41	2157.74	1393.35
$M_{\eta_2^+}$ (GeV)	2006.75	2405.21	1173.00	897.00	1814.11	1706.14	933.00
M_{R_1} (GeV)	2235.39	2808.58	2056.27	1305.33	2182.50	2206.11	1408.59
M_{R_2} (GeV)	2019.67	2414.53	1190.37	857.16	1819.99	1716.45	962.39
$M_{N_{A_1}}$ (GeV)	1534	1408	1179	903	1482	1164	939
$M_{N_{A_2}}$ (GeV)	1533	1407	1178	902	1481	1163	938
$M_{N_{B_1}}$ (GeV)	1535	1409	1180	904	1483	1165	940
$M_{N_{B_2}}$ (GeV)	1532	1406	1177	901	1480	1162	937
θ_R	0.5859	0.5484	0.6322	0.6864	0.5482	0.6958	0.7500
$\theta_{N_1} (= \theta_{N_2}), \theta_{N_3}$	0.7, 0.1	0.7, 0.01	0.7, 0.1	0.7, 0.01	0.7, 0.01	0.7, 0.1	0.7, 0.01
θ_{cs}	0.6	0.6	0.6	0.6	0.6	0.6	0.6
α	0.05	0.05	0.1	0.1	0.1	0.05	0.05
$\log_{10}(\lambda_5)$	-6.0307	-6.1558	-6.8970	-6.7433	-6.1206	-6.4266	-6.6757
λ_6	0	0	0	0	0	0	0
$y_{A_{33}}$	2.8509	3.8996	1.5849	3.8393	4.9502	3.7451	5.2354
$y_{B_{33}}$	0	0	0	0	0	0	0
M_{I_1} (GeV)	2235.39	2808.58	2056.27	1305.33	2182.50	2206.11	1409.59
M_{I_2} (GeV)	2019.67	2414.53	1190.37	857.16	1819.99	1716.45	962.39

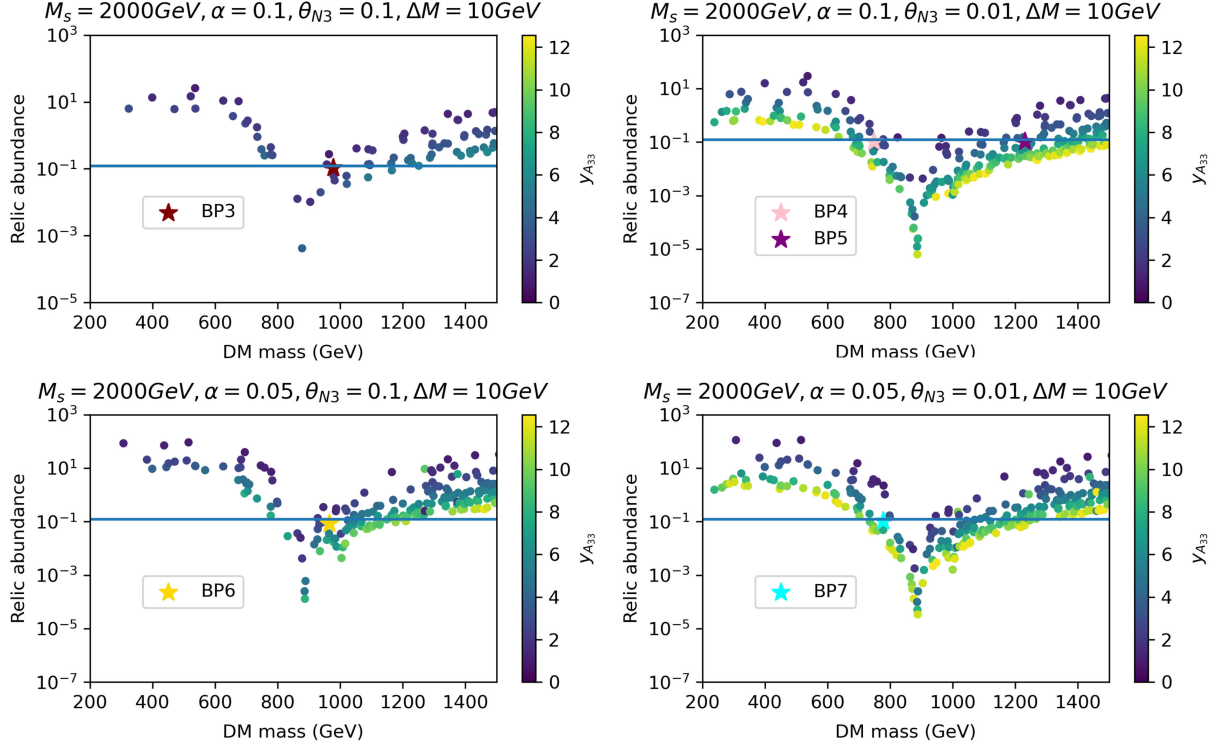


FIG. 4. Results of the relic abundance calculations under $M_s = 2$ TeV, $M_{\chi_2} - M_{\chi_1} (= \Delta M) = 10$ GeV; $(\alpha, \theta_{N_3}) = (0.1, 0.1)$ (left upper), $(0.1, 0.01)$ (right upper), $(0.05, 0.1)$ (left lower), $(0.05, 0.01)$ (right lower), respectively, taking account of coannihilation. The other parameter choices and conventions are the same as those of Fig. 3. The details of the five benchmark points are available in Table III.

θ_{N_3} , θ_{cs} , α , $\lambda_6 (= 0)$, and $y_{B_{33}} (= 0)$. Note that the values of M_{I_1} and M_{I_2} are determined by the parameters above, but we show the corresponding digits for convenience. Also, the two differences between M_{R_i} and M_{I_i} ($i = 1, 2$) are not observed within the shown digits in Table III, but they should be nonzero to produce the minuscule mass differences of the active neutrinos (see Sec. II B). Due to the same reason, θ_l (not shown in Table III) becomes extremely close to θ_R . See Table III for concrete information.¹⁰

Note that each point of the panels is consistent with the observed neutrino profiles, the constraints via the lepton flavor violation ($l_\alpha \rightarrow l_\beta + \gamma$), and the T parameter. The colors of the points indicate the magnitudes of $y_{A_{33}}$, while we take $y_{B_{33}} = 0$ to evade the direct-detection constraint. These results show that if we introduce an order-one Yukawa $y_{A_{33}}$ (keeping $y_{B_{33}} = 0$), our model can explain

¹⁰As mentioned above, the vital parameters for the residual amount of dark matter have been identified through phenomenological discussions, namely M_{χ_1} , M_{χ_2} , M_s , α , and θ_{N_3} , other than couplings such as $y_{A_{33}}$. We determined that fixing these parameters for comparison would clarify the characteristics of the models. In addition, several parameters that are not expected to affect the overall phenomenon significantly, such as θ_{N_1} and θ_{N_2} , have been fixed appropriately.

the TeV-scale dark matter remnant simultaneously with the preferred neutrino mass terms. The effective area extends over a wide range, not only in the vicinity of the resonance around 1.5 TeV. It can also be read that θ_{N_3} affects the valid range of the parameters; as shown in Fig. 3, a smaller θ_{N_3} is slightly more significant for a small value of α . We provide the full sets of model parameters for the two benchmarks (BP1, BP2) in Table III. Note that for a sizable choice of α , the Higgs-portal interaction increases rapidly, and the direct detection discards such a point.

We provide our investigations for a lighter s of $M_s = 2$ TeV, keeping $\Delta M (= M_{\chi_2} - M_{\chi_1}) = 10$ GeV, taking into account coannihilation. The results are available in Fig. 4, where we focus on the four choices of $(\alpha, \theta_{N_3}) = (0.1, 0.1)$ (left upper), $(0.1, 0.01)$ (right upper), $(0.05, 0.1)$ (left lower), and $(0.05, 0.01)$ (right lower), respectively.

Here, as a general trend, it becomes more difficult to avoid various restrictions because the masses of heavy particles tend to be smaller, compared with those in the scan for $M_s = 3$ TeV (refer to Fig. 3), in effective parameter points. As seen in the previous scans for $M_s = 3$ TeV (refer to Fig. 3), the smaller θ_{N_3} tends to yield more effective configurations. Furthermore, even with the same θ_{N_3} , a smaller nonzero α tended to result in more effective configurations. We provide the full sets of model parameters

TABLE IV. For each of the seven benchmark points (BP1 to BP7) of Figs. 3 and 4, we provide a set of valid parameters describing the potential (also not being relevant for the phenomena considered in the numerical scans). In any one of the sets, all of the conditions for the bounded from below [from Eq. (3.20) to Eq. (3.32)] and the inert condition [from Eqs. (3.37) and (3.38)] are satisfied.

Parameter	BP1	BP2	BP3	BP4	BP5	BP6	BP7
$\lambda_{S\eta_A}$	0.7105	0.9196	0.5929	0.4385	0.6829	0.6594	0.4845
$\lambda_{S\eta_B}$	0.6551	0.8175	0.4982	0.3963	0.6030	0.6052	0.3879
λ_{2a}	2.8	3.7	4	3.5	4	3.5	3.5
λ_{2b}	3	4	4	3.5	4	3.8	3.6
λ_{2c}	1	1	3	0.1	4	4.8	2.5
λ_{2d}	1	1	3	0.1	4.4	5	2.5
λ_{3A}	-3.5287	-4.3015	-0.3693	4.0333	-5.7771	1.7173	-8.6457
λ_{3B}	-1.4539	2.4240	-4.5045	-0.2452	0.7748	-7.8595	7.3934
λ_{4A}	4.5287	5.3014	1.3693	-3.0333	6.7771	-0.7173	9.6456
λ_{4B}	2.4539	-1.4240	5.5045	1.2452	0.2252	8.8595	-6.3934
m_{η_A} (GeV)	2048.17 <i>i</i>	2048.69 <i>i</i>	2048.30 <i>i</i>	2048.54 <i>i</i>	2048.72 <i>i</i>	2048.13 <i>i</i>	2048.18 <i>i</i>
m_{η_B} (GeV)	1964.41 <i>i</i>	1964.48 <i>i</i>	1964.47 <i>i</i>	1964.56 <i>i</i>	1965.08 <i>i</i>	1964.62 <i>i</i>	1964.71 <i>i</i>
λ_{HS}	0.3646	0.36461	0.3218	0.3218	0.3218	0.1617	0.1617
λ_H	0.6290	0.629044	0.9144	0.9144	0.9144	0.4227	0.4227
λ_S	0.1796	0.179551	0.0792	0.0792	0.0792	0.0798	0.0798

for the five benchmarks (BP3, BP4, BP5, BP6, BP7) in Table III.¹¹

The scan results in Figs. 3 and 4 clearly show that this scenario can still correctly reproduce the coordination of active neutrinos and the relic abundance of thermal WIMP dark matter while avoiding the latest direct search limitations on dark matter and other theoretical limitations, where active-neutrino masses and mixings are induced at one-loop level as shown in Fig. 1, and the Yukawa coupling $y_{A_{33}}$ tends to be $\mathcal{O}(1)$. We provide a few comments:

- (i) If we choose M_{χ_2} sizably above M_{χ_1} , coannihilation is disactivated and most of the relevant parameter points addressing the DM relic abundance are gone, see Appendix B for checking this point.
- (ii) If we switch off the Higgs portal coupling by $\alpha = 0$, only the lepton-portal coupling remains. As far as we checked, no valid parameter point for addressing the neutrino masses/mixings and the DM relic abundance (even with coannihilation), also evading the latest bound on the DM direct detection [203] and other constraints.¹²
- (iii) Since, as shown in Table III, also Figs. 3 and 4, the masses of the dark matter candidate and the other new fermions and scalars tend to be above (or around) 1 TeV, and thus to investigate the current scenario at the LHC by focusing on events with large missing E_T is uneasy. However, as we will debate in

Sec. VI, we can investigate the current scenario with the help of the LHC by focusing on the measurement of the trilinear Higgs self-coupling.

In the previous numerical scans in Figs. 3 and 4, also in Table IV, we did not include the constraints on the shape of the scalar potential: the bounded from below [from Eq. (3.20) to Eq. (3.32)] and the inert condition [from Eqs. (3.37) and (3.38)].¹³ To confirm this point, in Table IV, for each of the seven benchmark points (BP1 to BP7) of Figs. 3 and 4, we provide a set of valid parameters describing the potential (also not being relevant for the phenomena considered in the numerical scans), where, in any one of the sets, all of the conditions for the bounded from below [from Eq. (3.20) to Eq. (3.32)] and the inert condition [from Eqs. (3.37) and (3.38)] are satisfied. Here, at some benchmark points, some quartic couplings are quite large, but they still satisfy a naive perturbation limit of 4π . From the above discussion, it can be said that our model tends to maintain the stability of the potential perturbatively while keeping the stability of dark matter by appropriately selecting the parameters of the potential.

VI. BOUNDS ON HIGGS TRILINEAR SELF-COUPLING

The ATLAS experiment places stringent constraints on the Higgs trilinear self-coupling [247,248]. The parameter κ_3 quantifies the deviation from the

¹¹For the shift of the peak from the naive location $M_s/2 = 1$ TeV, see Chapter 6 of [246] for details.

¹²Note that we found parameter points with the correct amount of the DM relic abundance in the pure lepton-portal case ($\alpha = 0$) with coannihilation, being consistent with the previous LZ constraint on the direct detection [202], and the others.

¹³In our model, the potential has many parameters, making it structurally impossible to uniquely determine the potential parameters based solely on the information on the scalar's VEV, masses, and mixings. Due to this circumstance, we adopted the approach of separating the discussion of potential stability from other discussions.

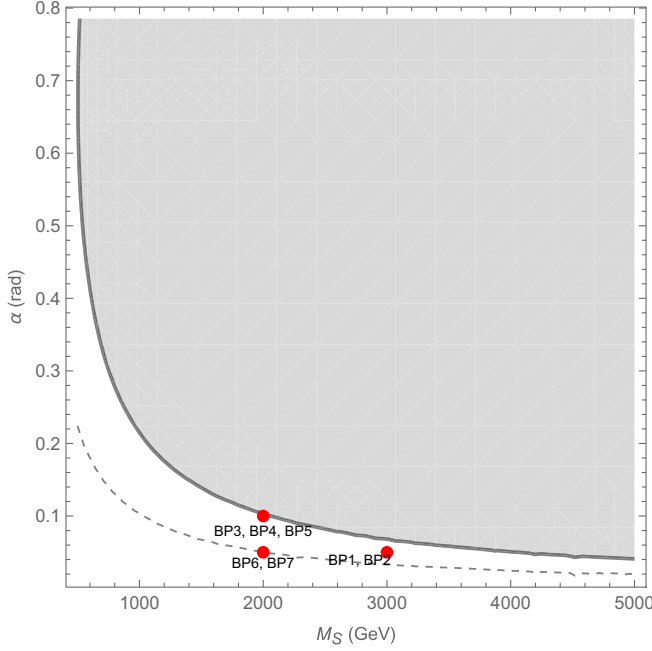


FIG. 5. The gray-shaded region shows the 95% CL disfavored region on the (M_s, α) plane (under the choice $v_s = 5$ TeV) by the ATLAS constraints on the trilinear Higgs self-coupling [247]. The dashed curve represents the 95% CL future prospect of the ATLAS and CMS HL-LHC projection under the SM hypothesis [249]. Red points represent the benchmark points discussed in Sec. V (see Figs. 3 and 4, and Table III for details).

Standard Model prediction for the trilinear Higgs self-coupling, defined as

$$\kappa_3 = \frac{\lambda_{hhh}}{\lambda_{hhh}^{\text{SM}}}. \quad (6.1)$$

The current bound by ATLAS on κ_3 [247] is

$$-0.4 \leq \kappa_3 \leq 6.3 \quad (\text{at 95\% confidence level}). \quad (6.2)$$

In the SM, the trilinear Higgs coupling arises from the scalar potential:

$$\begin{aligned} \mathcal{V}^{\text{SM}} &\supset \frac{\lambda_H}{2} (H^\dagger H)^2, \\ &\supset \frac{\lambda_H v_H}{2} h'^3 = \frac{1}{3!} 3\lambda_H v_H h'^3 = \frac{1}{3!} \lambda_{hhh}^{\text{SM}} h'^3, \end{aligned}$$

where the Higgs doublet is expanded around its VEV as shown in Eq. (2.3) [see also Eq. (2.4)]. Therefore, the SM trilinear coupling is fully determined by the electroweak symmetry-breaking scale $v_H = 246$ GeV and the Higgs mass $M_h = 125$ GeV,

$$\lambda_{hhh}^{\text{SM}} = 3\lambda_H v_H, \quad \lambda_H = \frac{M_h^2}{v_H^2} \simeq 0.26. \quad (6.3)$$

In our model, additional contributions to the trilinear coupling arise from the scalar singlet S , leading to the extended scalar potential:

$$\begin{aligned} \mathcal{V} &\supset \frac{\lambda_{HS}}{2} S^2 (H^\dagger H) + \frac{\lambda_H}{2} (H^\dagger H)^2 + \frac{\lambda_S}{4} S^4, \\ &\supset \frac{1}{2} \{v_H(\lambda_H c_\alpha^3 + \lambda_{HS} c_\alpha s_\alpha^3) + v_S(2\lambda_S s_\alpha^3 + \lambda_{HS} c_\alpha^2 s_\alpha)\} h^3, \\ &= \frac{1}{3!} \lambda_{hhh} h^3. \end{aligned} \quad (6.4)$$

After spontaneous symmetry breaking and mixing between the Higgs and singlet scalars, the physical trilinear coupling becomes

$$\lambda_{hhh} = 3\{v_H(\lambda_H c_\alpha^3 + \lambda_{HS} c_\alpha s_\alpha^3) + v_S(2\lambda_S s_\alpha^3 + \lambda_{HS} c_\alpha^2 s_\alpha)\}, \quad (6.5)$$

where we remind the reader of the mixing angle, $s_\alpha = \sin(\alpha)$, $c_\alpha = \cos(\alpha)$, namely, $s' = s_\alpha h + c_\alpha s$, $h' = c_\alpha h - s_\alpha s$. Here, λ_{hhh} can be completely determined if we specify the Z_4 symmetry breaking v_s , mass of the new singlet M_s , and Higgs mixing angle α via

$$\begin{aligned} \lambda_H &= \frac{M_h^2 c_\alpha^2 + M_s^2 s_\alpha^2}{v_H^2}, \quad \lambda_S = \frac{M_s^2 c_\alpha^2 + M_h^2 s_\alpha^2}{2v_s^2}, \\ \lambda_{HS} &= -\frac{(M_h^2 - M_s^2) \sin(2\alpha)}{2v_H v_S}. \end{aligned} \quad (6.6)$$

We pin down the corresponding points of the seven benchmark points discussed in Sec. V as a function of α and M_s under the choice $v_s = 5$ TeV, and the result is summarized in Fig. 5.¹⁴ Note that all of the benchmark points are consistent with the latest ATLAS result [247], and the five benchmark points (Nos. 1, 2, 3, 4, 5) are located near the current boundary. The 95% confidence level combined ATLAS and CMS HL-LHC projection under the SM hypothesis was estimated as $\kappa_3 \in [0.1, 2.3]$ [249], and the benchmark points will be indirectly investigated in the future.

VII. COMMENTS ON THE DOMAIN WALL PROBLEM

The authors of [250] make the observation that when a discrete symmetry is spontaneously broken, multiple degenerate ground states become available. As there is no requirement for the Universe to select a particular choice

¹⁴As recognized from Eq. (6.6), if v_s is notably greater than v_H , the disfavored region is not sensitive to the value of v_s .

of ground state, we will have causally disconnected spatial regions having different ground states. The different ground state regions will be separated by stable domain walls (DWs). The energy density of these domain walls scales as $\rho_{\text{DW}} \propto 1/R$, where R is the radius of the Universe, ρ_{DW} decreases more slowly than normal radiation or matter energy density dominating the energy density of the Universe, which can lead to disagreement with observation.

In our model, the Z_4 symmetry is spontaneously broken when the singlet field S acquires a VEV, to a Z_2 subgroup. We consider two possible mechanisms to address this issue in our model.¹⁵

A. Domain wall production before inflation

The simplest way is to consider the symmetry-breaking scale to be higher than the inflation scale. If the domain walls are produced before inflation, the resulting domain walls are inflated away. Low-scale inflation models with $V^{1/4} \simeq 10\text{--}10^4$ TeV [251–253] offer a viable setting in which domain walls produced by discrete symmetry breaking can be inflated away. Though our model benchmark points have $\text{VEV} \sim \mathcal{O}(10 \text{ TeV})$, we can increase the VEV above the inflation scale without altering the particle physics phenomenology discussed in this article to dilute away domain walls produced by discrete symmetry breaking.

B. Domain wall decay in a $U(1)_X$ -embedded scenario

Another way to address this issue is the use of the gauged discrete symmetry prescription described in [254,255]. We embed our Z_4 symmetry into a continuous gauge symmetry such as $U(1)_X$; in this new scenario the Universe first breaks the continuous gauge symmetry at some high scale v_1 to our Z_4 discrete symmetry forming strings and then subsequently Z_4 is broken to Z_2 at scale v_S forming domain walls bounded by strings,

$$U(1)_X \xrightarrow{v_1} Z_4 \xrightarrow{v_S} Z_2.$$

Inflation must occur before the high symmetry-breaking scale v_1 , so that the strings produced are not diluted away. These domain walls are metastable and collapse in time $t_c \sim \mu/\sigma \sim v_1^2/v_S^3$ (from dimension analysis in [256]), where μ and σ respectively denote the string mass per unit length and wall mass per unit area. If we assume scales of symmetry breaking $v_1 \sim 10^{16}$ GeV and $v_S \sim 10$ TeV or ~ 1 TeV, the domain-walls collapse in time $t_c \sim 10^{-4}$ s or $t_c \sim 10^{-2}$ s.¹⁶ The homogeneity and isotropy of our Universe requires that the domain walls disappear quite

early in the evolution of our Universe. The most stringent bound is from the big bang nucleosynthesis (BBN), the domain wall must disappear before BBN $T \sim 1$ MeV [223] if we assume there is not a domain wall dominated era in the Universe evolution the temperature corresponds to $t \sim 1$ s; therefore, the bound on the domain-wall collapse is $t_c \lesssim 1$ s [250]. This method was also used by the authors of [257] to analyze the gravitational signal resulting from the collapse of this metastable string-bounded domain-wall system. While a detailed discussion of this process is not in the scope of our article, we show that we are able to overcome the domain-wall issue within reasonable extensions of our model and its parameters while preserving our original result.¹⁷

C. Explicit soft Z_2 breaking

As a final mechanism to address the domain-wall problem, we consider introducing a soft, explicit breaking of the discrete symmetry, which avoids the domain-wall problem. The softly broken Z_4 symmetry lifts the degeneracy of the VEV, triggering domain-wall collapse. Most studies illustrating this mechanism typically use Z_2 as a starting point [258–261]. For Z_N with $N > 2$, the breaking is typically considered in the context of complex scalar fields [262,263], where the vacuum structure becomes elaborate. Nonetheless, heuristically, the arguments discussed may still be valid for our model. Let us begin with our potential for the S field,

$$V_0(S) = +\frac{m_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4, \quad (7.1)$$

with parameterization $S = (v_S + s')$. Though the Z_4 symmetry seems to allow four degenerate solutions for the VEV,

$$v_S = v_0 e^{\frac{N\pi}{2}}, \quad N = 0, 1, 2, 3. \quad (7.2)$$

These are only consistent if S were complex; if not, the imaginary solutions of the VEV lead to a complex potential which is unphysical. For a real scalar field S with Z_4 charge -1 , only the two real minima are allowed. Therefore, in our setup, S is effectively charged under a Z_2 symmetry and can only take VEV,

$$v_S = \pm v_0. \quad (7.3)$$

An explicit Z_4 symmetry-breaking term is added which introduces a bias in the vacuum structure and generates a

¹⁵We may mention that while some works [111,113,122] have proposed scotogenic scenarios based on Z_4 to a Z_2 , no comments on the domain-wall problem were found in them.

¹⁶Note that for a lower/higher v_1 , we get a looser/tighter bound.

¹⁷While a description of the formulas for μ and σ is given in the original paper [254], it contains a version of the formulas in Eqs. (12) and (13), which is a model-specific formula. A more general case is studied in [256], and the formulas can be found in Eqs. (4) and (11).

pressure on the domain walls,

$$\delta V = \epsilon S^3 \longrightarrow \epsilon v_0^3 e^{\frac{N_S}{2}}. \quad (7.4)$$

Here ϵ is a dimensionfull parameter controlling the strength of the explicit breaking and $N = 0, 2$ corresponding to the real field configuration of our setup. The energy difference between the two vacua reads

$$\Delta V = V(+v_S) - V(-v_S) = 2\epsilon v_S^3. \quad (7.5)$$

This energy difference between the two vacuum states provides a pressure leading to domain-wall collapse. We require that the domain walls disappear before nucleosynthesis [258]; therefore, the pressure must satisfy

$$\frac{\Delta V}{\sigma} > \frac{1}{10^{10} \text{ cm}} \sim 10^{-24} \text{ GeV}, \quad (7.6)$$

which leads to the lower bound on ϵ

$$\epsilon \gtrsim 5 \times 10^{-25} \text{ GeV}. \quad (7.7)$$

Here σ is the domain-wall tension $\sigma \sim v_S^3$.

On the other hand, the explicit soft-breaking term disables the absolute stability of the fermionic DM candidate, which becomes a very long-lived particle. A possible decay channel of χ is $\chi \rightarrow \nu_{\text{SM}} h h$ (ν_{SM} is a SM neutrino), and the magnitude of the corresponding total decay width of a decaying χ is very roughly estimated through naive dimensional analysis and the 3-body phase space factor (for massless final states) as

$$\Gamma_{\text{decaying-}\chi} \sim \frac{3}{192\pi^3} \times (\epsilon c_{\text{inert}})^2 \times \frac{M_\chi^3}{M_{\text{med}}^4}, \quad (7.8)$$

where the factor 3 comes from the three SM neutrinos, the violation of the inert condition in the terms $(H^\dagger H)(\eta_A^\dagger \eta_A)$ and $(H^\dagger H)(\eta_B^\dagger \eta_B)$ is parametrized as $\epsilon c_{\text{inert}}$ with a dimensionless constant c_{inert} , M_{med} represents a mass of the scalar mediator into a pair of the SM Higgs boson, and here we assumed that all of the dimensionless coupling constants are $\mathcal{O}(1)$. The corresponding mean lifetime is estimated as

$$\begin{aligned} \tau_{\text{decaying-}\chi} &= \Gamma_{\text{decaying-}\chi}^{-1} \\ &\sim (4 \times 10^{30} \text{ s}) \times \left(\frac{5 \times 10^{-25} \text{ GeV}}{\epsilon} \right)^2 \left(\frac{1}{c_{\text{inert}}} \right)^2 \\ &\quad \times \left(\frac{M_{\text{med}}}{1 \text{ TeV}} \right)^4 \left(\frac{1 \text{ TeV}}{M_\chi} \right)^3, \end{aligned} \quad (7.9)$$

where, if this value is longer than the age of the Universe $\sim 10^{17}$ s, this decaying DM can explain the relic abundance correctly. Via Eq. (7.9), we can conclude that,

for a very wide region of $\{\epsilon, c_{\text{inert}}\}$, the effect of the soft explicit breaking of the Z_2 symmetry is harmless for the existence of a suitable DM candidate. This means that the simplest option for evading the domain-wall problem works fine in our setup.

VIII. SUMMARY

We have constructed a new scotogenic type model based on a global Z_4 symmetry involving dark matter candidates. After the symmetry breaking of it as Z_4 to Z_2 via the singlet scalar VEV, the lightest Majorana fermion works as a viable WIMP DM candidate and the mass terms for active neutrinos are generated as a finite quantum correction. A key point of realizing our scotogenic structure is to introduce two types of Majorana fermions (heavy right-handed neutrinos) and inert Higgs doublets with opposite Z_4 parities. Since a large VEV for the singlet scalar is not so harmful in an appropriate realization of the Higgs mechanism for the SM gauge symmetry, we can naturally realize a TeV-scale fermionic DM candidate, where constraints via direct detection experiments are less than those for sub-TeV WIMPs. For the case that N_{B_3} is the DM candidate, we find a natural coannihilation partner N_{A_3} . By taking “asymmetric” profiles in the Yukawa couplings to the singlet scalar S as $y_{A_{33}} \sim \mathcal{O}(1)$ but $y_{B_{33}} = 0$ and small but nonzero α and θ_{N_3} , a “well-tempered” situation is realized, where the direct-detection cross section is relatively suppressed, and the decent-sized Higgs-portal interaction (with coannihilation) naturally leads to a correct amount of the relic abundance of the fermionic thermal WIMP DM. Our scenario has lots of parameter points where the observed DM relic abundance is explained under the correct realization of the active neutrino masses and mixing angles (refer to Figs. 3 and 4).

The current scenario, above-TeV DM and its mediators, are favored to evade the severe constraint via direct detection. At the LHC, such cases are hard to investigate via events with large missing transverse energy, but the development in the measurement of the Higgs trilinear self-coupling enables us to survey the validity of our scenario indirectly. As summarized in Table IV and Fig. 5, in our scenario, valid parameter points for the active neutrino profiles and the correct amount of WIMP DM tend to be located near or a bit away from the current boundary provided by the ATLAS experiment. The future ATLAS-CMS investigation at high-luminosity LHC will survey such benchmark points.

Since our scenario predicts the transition of the global discrete symmetry $Z_4 \rightarrow Z_2$, the formation of domain walls at an early stage of the Universe can lead to a mismatch with observations. We proposed three possible solutions, where the two require some extension of the model, while the last, simplest one, introducing an explicit soft Z_2 -symmetry-breaking term, can be addressed within the

present scenario. We checked that the simplest prescription works fine, where the lifetime of the decaying DM easily becomes longer than the age of the Universe.

We focus on a significant property of our scenario. As shown in Tables III and IV, the Yukawa coupling to the singlet scalar $y_{A_{33}}$ and some of the scalar quartic couplings tend to be $\mathcal{O}(1)$ at valid benchmarks. If they follow the running through the renormalization group equations, the running couplings may blow up or blow down at an energy scale that is not so far from the TeV scale. Therefore, if we attempt to explain the properties of neutrinos and dark matter in this scenario, the scale of inflation must be quite low. It looks like an interesting future study to investigate which low-energy inflation scenarios are consistent with our scenario. Furthermore, our scenario includes numerous right-handed neutrinos and numerous new scalar fields, including $SU(2)_L$ inert doublets. It is an interesting challenge to consider various TeV-scale leptogenesis scenarios under such circumstances. Also, the feature of Z_4 to Z_2 symmetry breaking with a real scalar field can also be studied in the future through gravitational wave production.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: DECAY WIDTH OF s

We begin with the scalar potential:

$$\mathcal{V} \supset \frac{\lambda_{HS}}{2} S^2 (H^\dagger H) + \frac{\lambda_H}{2} (H^\dagger H)^2 + \frac{\lambda_S}{4} S^4. \quad (\text{A1})$$

After acquiring VEV, the SM Higgs and singlet scalars are parametrized as

$$H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v_H + h') \end{pmatrix}, \quad S = (v_S + s'),$$

$$s' = s_\alpha h + c_\alpha s, \quad h' = c_\alpha h - s_\alpha s.$$

To compute the decay width Γ_s , we first isolate the interaction term in the potential that contributes to the decay of the singlet scalar s into two Higgs bosons hh :

$$\mathcal{V} \supset \frac{1}{2} \lambda_{shh} s h h.$$

The effective trilinear coupling coefficient c_{shh} is

$$\lambda_{shh} = -3\lambda_H v_H c_\alpha^2 s_\alpha + \frac{\lambda_{HS}}{4} \{v_S(c_\alpha + 3c_{3\alpha}) + v_H(-s_\alpha + 3s_{3\alpha})\} + 6\lambda_S v_S c_\alpha s_\alpha^2. \quad (\text{A2})$$

The square amplitude for this process is

$$|M_{shh}|^2 = \lambda_{shh}^2. \quad (\text{A3})$$

The decay width of a particle of mass M_s decaying into two identical Higgs particles is

$$\Gamma_{s \rightarrow hh} = \frac{1}{2!} \frac{p^*}{32\pi^2 M_s^2} \int |M_{shh}|^2 d\Omega, \quad (\text{A4})$$

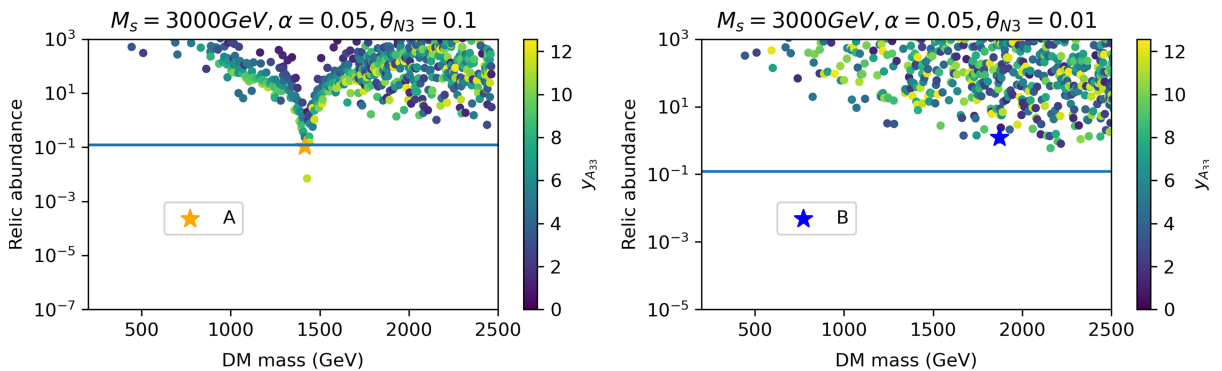


FIG. 6. Results of the relic abundance calculations under $M_s = 3$ TeV, $\alpha = 0.05$; $\theta_{N_3} = 0.1$ (left panel) and $\theta_{N_3} = 0.01$ (right), without coannihilation. The horizontal blue line depicts the experimental observation of the DM relic abundance ($\Omega h^2 = 0.120 \pm 0.001$) [234]. Note that each point of the panels is consistent with the observed neutrino profiles, the constraints via the lepton flavor violation ($l_\alpha \rightarrow l_\beta + \gamma$) and T parameter. The colors of the points indicate the magnitudes of $y_{A_{33}}$, while we take $y_{B_{33}} = 0$. The details of the two benchmark points are available in Table V.

TABLE V. [Top] The values of the relevant parameters for the two benchmark points without coannihilation in Fig. 6, and realized relic abundances (Ωh^2) are shown. Note that they are consistent with the observed neutrino profiles, the lepton flavor violation ($l_\alpha \rightarrow l_\beta + \gamma$), T parameter, and DM direct detection. [Bottom] For each of the two benchmark points (A and B) of Fig. 6, we provide a set of valid parameters describing the potential (also not being relevant for the phenomena considered in the numerical scans). In any one of the sets, all of the conditions for the bounded from below [from Eq. (3.20) to Eq. (3.32)] and the inert condition [from Eqs. (3.37) and (3.38)] are satisfied.

Parameters	A	B
Ωh^2	0.1107	1.2359
M_s (TeV)	3.0	3.0
$M_{X_1} (= M_{N_{B_3}})$ (GeV)	1415	1873
$M_{X_2} (= M_{N_{A_3}})$ (GeV)	1698	1883
$M_{\eta_1^+}$ (GeV)	3135.74	3645.31
$M_{\eta_2^+}$ (GeV)	3131.73	3478.74
M_{R_1} (GeV)	3171.44	3611.76
M_{R_2} (GeV)	3084.18	3469.54
$M_{N_{A_1}}$ (GeV)	1704	2253
$M_{N_{A_2}}$ (GeV)	1703	2252
$M_{N_{B_1}}$ (GeV)	1705	2254
$M_{N_{B_2}}$ (GeV)	1702	2251
θ_R	0.3529	0.3270
$\theta_{N_1} (= \theta_{N_2}), \theta_{N_3}$	0.7, 0.1	0.7, 0.01
θ_{cs}	0.6	0.6
α	0.05	0.05
$\log_{10}(\lambda_5)$	-6.7201	-7.2654
λ_6	0	0
$y_{A_{33}}$	8.0394	5.5746
$y_{B_{33}}$	0	0
M_{I_1} (GeV)	3171.44	3611.76
M_{I_2} (GeV)	3084.18	3469.54

Parameter	A	B
$\lambda_{S\eta A}$	1.1854	1.1480
$\lambda_{S\eta B}$	1.3617	1.1564
λ_{2_a}	5	5
λ_{2_b}	7	5
λ_{2_c}	7	5
λ_{2_d}	8	8.5
λ_{3A}	-4.5508	-0.0226
λ_{3B}	8.8775	12.1819
λ_{4A}	5.5508	1.0226
λ_{4B}	-7.8775	-11.1819
$m_{\eta A}$ (GeV)	2203.44 <i>i</i>	1199.68 <i>i</i>
$m_{\eta B}$ (GeV)	2733.92 <i>i</i>	1530.95 <i>i</i>
λ_{HS}	0.36461	0.36461
λ_H	0.629044	0.629044
λ_S	0.179551	0.179551

$$p^* = \frac{1}{2M_s} \sqrt{(M_s^2 - 4M_h^2)M_s^2} \simeq \frac{M_s}{2}, \quad (\text{A5})$$

where we used the approximation $M_h \ll M_s$ for our focused parameter region,

$$\Gamma_{s \rightarrow hh} = \frac{|M_{shh}|^2}{32\pi M_s}, \quad (\text{A6})$$

where the decay width of s is dominantly described by $s \rightarrow hh$ as $\Gamma_s \simeq \Gamma_{s \rightarrow hh}$.

APPENDIX B: DM RESULTS WITHOUT COANNIHILATION

Here, we provide the numerical scans similar to those in Sec. V, but without coannihilation by taking the mass of the coannihilation partner $M_{X_2} (= M_{N_{A_3}})$ as 20% larger than the DM mass $M_{X_1} (= M_{N_{B_3}})$, while the other schemes are the same. For each scan, as before, we randomly selected the parameters $M_{X_1} (= M_{N_{B_3}})$, $M_{N_{A_1}}$, $M_{N_{B_1}}$, $M_{N_{A_2}}$, $M_{N_{B_2}}$, $M_{\eta_1^+}$, $M_{\eta_2^+}$, M_{R_1} , M_{R_2} , θ_R , λ_5 , $y_{A_{33}}$, while the following other independent relevant parameters are fixed; M_s , $M_{X_2} (= M_{N_{A_3}})$, θ_{N_1} , θ_{N_2} , θ_{N_3} , θ_{cs} , α , $\lambda_6 (= 0)$, and $y_{B_{33}} (= 0)$. We chose the three significant parameters M_s , α , and θ_{N_3} as 3 TeV, 0.05, and 0.1 or 0.01, respectively. The results of the scans are available in Fig. 6, where left and right panels correspond to $\theta_{N_3} = 0.1$ and $\theta_{N_3} = 0.01$. It is apparent from the results, no valid points for addressing the DM relic and the active neutrino profiles are found, except for very near the narrow resonance of s .¹⁸ The details of the two focused points A and B are available in Table V.¹⁹ While the choice of asymmetric Yukawa couplings is not optimal in the noncoannihilation regime, the inferences remain unchanged, i.e. only the narrow resonance region can produce the correct relic abundance.

¹⁸In our scan for $\theta_{N_3} = 0.01$, due to a suppressed DM interaction owing to the smaller value of θ_{N_3} , we were not able to find a point near the resonance of s .

¹⁹Note that the values of M_{I_1} and M_{I_2} are determined by the parameters above, but we show the corresponding digits for convenience. Also, the two differences between M_{R_i} and M_{I_i} ($i = 1, 2$) are not observed within the shown digits in Table III, but they should be nonzero to produce the minuscule mass differences of the active neutrinos (see Sec. II B). Due to the same reason, θ_I (not shown in Table V) becomes extremely close to θ_R .

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