

Semileptonic decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$ within the covariant light-front approach

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(Received 18 June 2026; accepted 6 August 2026; published 8 September 2026)

We present a systematic analysis of the semileptonic decays $B_{(s)} \rightarrow D_{(s)}(1S, 2S)\ell\nu_\ell$ and $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)\ell\nu_\ell$ with $\ell = e, \mu, \tau$ within the covariant light-front quark model. Using the form factors of the transitions $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$ and $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)$, we calculate the branching ratios of the relevant semileptonic decays and find that $\text{Br}(B_{(s)} \rightarrow D_{(s)}\ell'\nu_{\ell'})$ and $\text{Br}(B_{(s)} \rightarrow D_{(s)}^*\ell'\nu_{\ell'})$ with $\ell' = e, \mu$ agree well with the data, while $\text{Br}(B_{(s)} \rightarrow D_{(s)}\tau\nu_\tau)$ and $\text{Br}(B_{(s)} \rightarrow D_{(s)}^*\tau\nu_\tau)$ are systematically smaller than the experimental measurements. This naturally gives rise to the so-called $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ anomalies. Our predictions $\mathcal{R}(D) = 0.261 \pm 0.013$ and $\mathcal{R}(D^*) = 0.228 \pm 0.026$ show 3.1σ and 2.1σ deviations from the current experimental world averages compiled by the Heavy Flavor Averaging Group, respectively, yet deviate only by 0.16σ and 1.5σ from the latest LHCb measurements. For the decays $B_{(s)} \rightarrow D_{(s)}(2S)\ell\nu_\ell$ and $B_{(s)} \rightarrow D_{(s)}^*(2S)\ell\nu_\ell$, their branching ratios lie in the range $10^{-4} - 10^{-3}$, which are much larger than the results from the Bethe Salpeter equation but agree with the relativistic quark model calculations. Furthermore, we also calculate the forward-backward asymmetries $\mathcal{A}_{\text{FB}}$ and longitudinal polarization fractions f_L for the corresponding decays. Our predictions are consistent with most other theoretical results and experimental data.

DOI: [10.1103/xwd5-ntgc](https://doi.org/10.1103/xwd5-ntgc)

I. INTRODUCTION

The semileptonic decays of heavy flavor mesons play an central role in constraining the Standard Model (SM) and searching for the signal of new physics (NP). These decays provide an ideal platform for precise extraction of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, the fundamental free parameters of the SM. The SM assumes that the electroweak couplings to all three generations of leptons are universal and the only difference arises due to their mass differences, a principle known as lepton flavor universality (LFU). Thus, the lepton flavor symmetry between semileptonic decay rates can be obtained after considering the charged lepton mass contributions to decay amplitudes and phase space. Any deviations from the CKM matrix unitarity constraints or the LFU will be a signal of NP.

Over the past several years, the $b \rightarrow c\ell\nu_\ell$ transition has served as a powerful precision probe for the search of NP,

especially experimental measurements of the semileptonic process $b \rightarrow c\tau\nu_\tau$ yield branching ratios systematically larger than SM expectations, which could be a hint of LFU violation [1]. The branching ratios of the decays $B \rightarrow D^{(*)}\ell\nu_\ell$ have been determined by the BABAR, Belle, and LHCb Collaborations. To minimize hadronic uncertainties, these experiments further extract the ratios of the branching fractions of the semileptonic decays $B \rightarrow D^{(*)}\ell\nu_\ell$ [2], which will be discussed in detail in the numerical results section.

Of particular interest are the LFU ratios

$$\begin{aligned}\mathcal{R}(D) &= \frac{\text{Br}(B \rightarrow D\tau\nu_\tau)}{\text{Br}(B \rightarrow D\ell'\nu_{\ell'})}, \\ \mathcal{R}(D^*) &= \frac{\text{Br}(B \rightarrow D^*\tau\nu_\tau)}{\text{Br}(B \rightarrow D^*\ell'\nu_{\ell'})},\end{aligned}\quad (1)$$

which have been measured by the different collaborations, and the latest world averages compiled by the Heavy Flavor Averaging Group (HFLAV) are listed as

$$\begin{aligned}\mathcal{R}(D)_{\text{exp}} &= 0.347 \pm 0.025, \\ \mathcal{R}(D^*)_{\text{exp}} &= 0.288 \pm 0.012,\end{aligned}\quad (2)$$

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where the results include both statistical and systematic uncertainties. On the theoretical side, the transition form factors, branching fractions, and related ratios are calculated in different approaches, such as the three-point QCD sum rules (QCDSR) [3,4], the light cone QCD sum rules (LCSR) [5,6], the relativistic quark model (RQM) [7], and so on. To achieve more precise determinations of the $B_{(s)} \rightarrow D_{(s)}$ and $B_{(s)} \rightarrow D_{(s)}^*$ transition form factors, the authors of Ref. [8] have calculated the next-to-leading-order QCD corrections at large hadronic recoil. Certainly, high-precision lattice QCD calculations for these transitions are also available in Refs. [9–13]. HFLAV provides the SM central values for the LFU ratios as

$$\begin{aligned}\mathcal{R}(D)_{\text{SM}} &= 0.296 \pm 0.004, \\ \mathcal{R}(D^*)_{\text{SM}} &= 0.254 \pm 0.005,\end{aligned}\quad (3)$$

which are obtained from global fits that combining lattice QCD inputs at high q^2 with experimental form factor determinations at low q^2 [14–16]. These results rely on parametrization schemes to extrapolate form factors from a restricted q^2 region to the full kinematic q^2 range. One can find that the measured $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ values exceed the SM predictions given above by 2.0σ and 2.7σ , respectively. Considering the $\mathcal{R}(D) - \mathcal{R}(D^*)$ correlation, the deviation between the data and the SM predictions remains at about 3.8σ . Though this is smaller than the earlier 4σ tension, it is still one of the most persistent hints of LFU violation in the B sector. Various heavy flavor anomalies associated with these transitions can be found in Refs. [17–19], and references therein.

The remainder of this paper is structured as follows. The formalism of the covariant light-front quark model (CLFQM), the hadronic matrix elements, and the helicity amplitudes combined via form factors for the¹ $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$ decays are given in Sec. II. Using the $B_{(s)} \rightarrow D_{(s)}, D_{(s)}^*$ form factor results derived in our previous work [20], we calculate the branching ratios, the forward-backward asymmetries, and the polarization fractions for the corresponding semileptonic decays, which are presented in Sec. III. Section IV provides a brief summary.

II. THEORETICAL FRAMEWORK

A. Covariant light-front approach

In the covariant quark model, the treatment of transition form factors is relatively covariant throughout the calculation process, where the light-front coordinates of a momentum p are used $p = (p^-, p^+, p_\perp)$ with $p^\pm = p^0 \pm p_z$, $p^2 = p^+ p^- - p_\perp^2$. The incoming (outgoing) meson has the mass M' (M'') with the momentum $P' = p'_1 + p_2$ ($P'' = p''_1 + p_2$), where p''_1 and p_2 are the momenta of the quark and antiquark inside the incoming (outgoing) meson with the mass $m_1^{(n)}$ and m_2 , respectively. Here, we follow the notation convention established in Refs. [21,22]. These momenta can be expressed in terms of the internal variables $(x_i, p'_\perp)$ as

$$p_{1,2}^{'+} = x_{1,2} P'^+, \quad p'_{1,2\perp} = x_{1,2} P'_\perp \pm p'_\perp \quad (4)$$

with $x_1 + x_2 = 1$. Using these internal variables, we can define some quantities for the incoming meson which are relevant to subsequent calculations:

$$\begin{aligned}M_0^2 &= (e'_1 + e_2)^2 = \frac{p_\perp'^2 + m_1'^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2}, \\ \tilde{M}'_0 &= \sqrt{M_0^2 - (m'_1 - m_2)^2}, \\ e_i^{(\prime)} &= \sqrt{m_i^{(\prime)2} + p_\perp^2 + p_z^2}, \\ p'_z &= \frac{x_2 M'_0}{2} - \frac{m_2^2 + p_\perp'^2}{2x_2 M'_0},\end{aligned}\quad (5)$$

where M'_0 is the kinetic invariant mass of the incoming meson and can be expressed as the energies of the quark and the antiquark $e_i^{(\prime)}$. Identical expressions hold for the outgoing meson, but with $'$ replaced by $''$ in superscripts.

B. Form factors

The Bauer-Stech-Wirble form factors for the $B \rightarrow D$ and $B \rightarrow D^*$ transitions are defined as follows:

$$\langle D(P'') | V_\mu | B(P') \rangle = \left(P_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right) F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2), \quad (6)$$

$$\begin{aligned}\langle D^*(P'', \epsilon''^*) | V_\mu - A_\mu | B(P') \rangle &= -\epsilon_{\mu\nu\alpha\beta} \epsilon''^{*\nu} P'^\alpha q^\beta \frac{V(q^2)}{m_B + m_{D^*}} - i \frac{2m_{D^*} \epsilon''^* \cdot P}{q^2} q_\mu A_0(q^2) \\ &\quad - i \epsilon''^{*\mu} (m_B + m_{D^*}) A_1(q^2) + i \frac{\epsilon''^* \cdot P}{m_B + m_{D^*}} P_\mu A_2(q^2) + i \frac{2m_{D^*} \epsilon''^* \cdot P}{q^2} q_\mu A_3(q^2),\end{aligned}\quad (7)$$

¹From now on, $D_{(s)}$ and $D_{(s)}^*$ refer to $D_{(s)}(1S, 2S)$ and $D_{(s)}^*(1S, 2S)$, respectively.

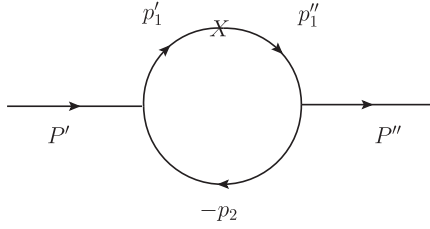


FIG. 1. Feynman diagram for the meson weak transition, where X denotes the vector or axial-vector transition vertex.

where $P = P' + P''$, $q = P' - P''$, and the convention $\epsilon_{0123} = 1$ is adopted. In order to calculate the amplitudes of the transition form factors, we need the following Feynman rules for the meson-quark-antiquark vertex $i\Gamma_M^{(\prime\prime)}$ with M representing a pseudoscalar (P) or vector (V) meson:

$$i\Gamma_P' = H_P'\gamma_5, \quad (8)$$

$$i\Gamma_P'' = \gamma_0 H_P''\gamma_5\gamma_0, \quad (9)$$

$$i\Gamma_V'' = i\gamma_0 H_V'' \left[\gamma_\mu - \frac{1}{W_V''} (p_1'' - p_2)_\mu \right] \gamma_0. \quad (10)$$

The form factors could be obtained by calculating the Feynman diagrams shown in Fig. 1. For the general $P \rightarrow P$ transition, the amplitude for the lowest order is

$$\mathcal{A}_\mu^{PP} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{H_P' H_P''}{N_1' N_1'' N_2} S_\mu^{PP}, \quad (11)$$

where $N_1'^{(\prime\prime)} = p_1'^{(\prime\prime)2} - m_1'^{(\prime\prime)2}$ and $N_2 = p_2^2 - m_2^2$ arise from the quark propagators and the trace S_μ^{PP} can be obtained directly by using the Lorentz contraction:

$$S_\mu^{PP} = \text{Tr}[\gamma_5 (\not{p}_1' + m_1') \gamma_\mu (\not{p}_1' + m_1') \gamma_5 (-\not{p}_2 + m_2)], \quad (12)$$

where the analytical expression for S_μ^{PP} can be found in Ref. [23]. It is similar for the $P \rightarrow V$ transition amplitude:

$$\mathcal{A}_\mu^{PV} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{H_P' (iH_V'')}{N_1' N_1'' N_2} S_{\mu\nu}^{PV} \epsilon^{*\nu}, \quad (13)$$

where

$$\begin{aligned} S_{\mu\nu}^{PV} &= (S_V^{PV} - S_A^{PV})_{\mu\nu} \\ &= \text{Tr} \left[\left(\gamma_\nu - \frac{1}{W_V''} (p_1'' - p_2)_\nu \right) (\not{p}_1'' + m_1'') (\gamma_\mu - \gamma_\mu \gamma_5) \right. \\ &\quad \left. \times (\not{p}_1' + m_1') \gamma_5 (-\not{p}_2 + m_2) \right]. \end{aligned} \quad (14)$$

In practice, we use the light-front decomposition of the Feynman loop momentum and integrate out the minus component through the contour method. If the covariant vertex functions are not singular when performing integration, the transition amplitudes will pick up the singularities in the antiquark propagators. The integration then leads to

$$\begin{aligned} N_1'^{(\prime\prime)} &\rightarrow \hat{N}_1'^{(\prime\prime)} = x_1 (M'^{(\prime\prime)2} - M_0'^{(\prime\prime)2}), \\ \int \frac{d^4 p_1'}{N_1' N_1'' N_2} H_P' H_M'' S^{PM} &\rightarrow -i\pi \int \frac{dx_2 d^2 p_\perp'}{x_2 \hat{N}_1' \hat{N}_1''} h_P' h_M'' \hat{S}^{PM}, \\ W_V'' &\rightarrow w_V'' = M_0'' + m_1'' + m_2, \end{aligned} \quad (15)$$

where

$$M_0''^2 = \frac{p_\perp'^2 + m_1''^2}{x_1} + \frac{p_\perp'^2 + m_2^2}{x_2}, \quad (16)$$

with $p_\perp'' = p_\perp' - x_2 q_\perp$. The explicit forms of h_M'' are given as [21]

$$h_P' = h_V'' = (M'^2 - M_0''^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2\tilde{M}_0}} \varphi'', \quad (17)$$

where φ'' is the light-front momentum distribution amplitude for S -wave mesons,

$$\varphi'' = \varphi''(x_2, p_\perp'') = 4 \left(\frac{\pi}{\beta^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp_z''}{dx_2}} \exp \left(-\frac{p_z''^2 + p_\perp'^2}{2\beta^2} \right). \quad (18)$$

Here, β is a phenomenological parameter and can be fixed by fitting the corresponding decay constant. For the first radially excited charmed mesons $D_{(s)}^*(2S)$ and $D_{(s)}(2S)$, the distribution amplitudes are given as

$$\begin{aligned} \varphi''(2S) &= 4 \left(\frac{\pi}{\beta^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp_z''}{dx_2}} \exp \left(-\frac{p_z''^2 + p_\perp'^2}{2\beta^2} \right) \\ &\quad \times \frac{1}{\sqrt{6}} \left(-3 + 2 \frac{p_z''^2 + p_\perp'^2}{\beta^2} \right). \end{aligned} \quad (19)$$

C. Helicity amplitudes and observables

The differential decay widths of the semileptonic decays $B_{(s)} \rightarrow D_{(s)} \ell \nu_\ell$ and $B_{(s)} \rightarrow D_{(s)}^* \ell \nu_\ell$ can be obtained by the combinations of helicity amplitudes, which are listed as follows:

$$\begin{aligned}
\frac{d\Gamma(B \rightarrow D\ell\nu)}{dq^2} &= (1 - \hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(m_B^2, m_D^2, q^2)} G_F^2 |V_{cb}|^2}{384 m_B^3 \pi^3} \{ (\hat{m}_\ell^2 + 2) \lambda(m_B^2, m_D^2, q^2) F_1^2(q^2) \\
&\quad + 3 \hat{m}_\ell^2 (m_B^2 - m_D^2)^2 F_0^2(q^2) \}, \\
\frac{d\Gamma_L(B \rightarrow D_{(s)}^* \ell \nu)}{dq^2} &= (1 - \hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(m_B^2, m_{D^*}^2, q^2)} G_F^2 |V_{cb}|^2}{384 m_B^3 \pi^3} \left\{ 3 \hat{m}_\ell^2 \lambda(m_B^2, m_{D^*}^2, q^2) A_0^2(q^2) + (\hat{m}_\ell^2 + 2) \right. \\
&\quad \times \left| \frac{1}{2m_{D^*}} \left[(m_B^2 - m_{D^*}^2 - q^2)(m_B + m_{D^*}) A_1(q^2) \right. \right. \\
&\quad \left. \left. - \frac{\lambda(m_B^2, m_{D^*}^2, q^2)}{m_B + m_{D^*}} A_2(q^2) \right] \right|^2 \Big\}, \\
\frac{d\Gamma^\pm(B \rightarrow D^* \ell \nu)}{dq^2} &= (1 - \hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(m_B^2, m_{D^*}^2, q^2)} G_F^2 |V_{cb}|^2}{384 m_B^3 \pi^3} \left\{ (m_\ell^2 + 2q^2) \lambda(m_B^2, m_{D^*}^2, q^2) \right. \\
&\quad \times \left| \frac{V(q^2)}{m_B + m_{D^*}} \mp \frac{(m_B + m_{D^*}) A_1(q^2)}{\sqrt{\lambda(m_B^2, m_{D^*}^2, q^2)}} \right|^2 \Big\},
\end{aligned}$$

where $\lambda(q^2) = \lambda(m_B^2, m_{D^*}^2, q^2) = (m_B^2 + m_{D^*}^2 - q^2)^2 - 4m_B^2 m_{D^*}^2$ and $\hat{m}_\ell^2 = m_\ell^2/q^2$ with m_ℓ being the lepton mass. Even though the electron and muon masses are very small relative to the tau mass, we retain full lepton mass dependence throughout numerical calculations to quantify the mass effects. The combined transverse and total differential decay widths are defined via

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2}, \quad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}. \quad (20)$$

For the $B \rightarrow D^* \ell \nu_\ell$ decays, one can define the longitudinal polarization fraction:

$$f_L = \frac{\Gamma_L}{\Gamma_L + \Gamma_+ + \Gamma_-}. \quad (21)$$

The analytical expression of the forward-backward asymmetry for the semileptonic decays considered in this work is defined as [24]

$$\mathcal{A}_{FB} = \frac{\int_0^1 \frac{d\Gamma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\Gamma}{d\cos\theta} d\cos\theta}{\int_{-1}^1 \frac{d\Gamma}{d\cos\theta} d\cos\theta} = \frac{\int b_\theta(q^2) dq^2}{\Gamma_{B \rightarrow D^{(*)} \ell \nu_\ell}}, \quad (22)$$

where θ is the angle between the three-momenta of the lepton ℓ and the initial meson in the $\ell \nu_\ell$ rest frame. The function $b_\theta(q^2)$ represents the angular coefficient and can be written as [24]

$$\begin{aligned}
b_\theta(q^2) &= \frac{G_F^2 |V_{cb}|^2}{128 \pi^3 m_B^3} \lambda(q^2) \left(1 - \frac{m_\ell^2}{q^2} \right)^2 \\
&\quad \times \frac{m_\ell^2}{q^2} (m_B^2 - m_D^2) F_1(q^2) F_0(q^2), \quad (23)
\end{aligned}$$

for the semileptonic decays $B \rightarrow D\ell\nu_\ell$, and that for the semileptonic decays $B \rightarrow D^* \ell \nu_\ell$ is given as

$$\begin{aligned}
b_\theta(q^2) &= \frac{G_F^2 |V_{cb}|^2}{128 \pi^3 m_{B(s)}^3} q^2 \sqrt{\lambda(q^2)} \left(1 - \frac{m_\ell^2}{q^2} \right)^2 \\
&\quad \times \left[\frac{1}{2} (H_{V,+}^2 - H_{V,-}^2) + \frac{m_\ell^2}{q^2} (H_{V,0} H_{V,t}) \right], \quad (24)
\end{aligned}$$

where the helicity amplitudes are defined as

$$H_{V,\pm}(q^2) = (m_B + m_{D^*}) A_1(q^2) \mp \frac{\sqrt{\lambda(q^2)}}{m_B + m_{D^*}} V(q^2), \quad (25)$$

$$\begin{aligned}
H_{V,0}(q^2) &= \frac{m_B + m_{D^*}}{2m_{D^*} \sqrt{q^2}} \left[-(m_B^2 - m_{D^*}^2 - q^2) A_1(q^2) \right. \\
&\quad \left. + \frac{\lambda(q^2) A_2(q^2)}{(m_B + m_{D^*})^2} \right], \quad (26)
\end{aligned}$$

$$H_{V,t}(q^2) = -\sqrt{\frac{\lambda(q^2)}{q^2}} A_0(q^2). \quad (27)$$

It is noticed that the subscript V in each helicity amplitude refers to the $\gamma_\mu(1 - \gamma_5)$ current.

III. NUMERICAL RESULTS OF SEMILEPTONIC DECAYS

Numerical inputs for meson and lepton masses, the CKM matrix elements, and $B_{(s)}$ meson lifetimes are taken from PDG [25]. In this section, we compare our predictions for the branching ratios of the semileptonic decays $B_{(s)} \rightarrow D_{(s)}(1S, 2S) \ell \nu_\ell$ and $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S) \ell \nu_\ell$ with those obtained from other theoretical methods and the available experimental data. The theoretical approaches employed in

TABLE I. The branching ratios of the semileptonic decays $B \rightarrow D(1S)\ell^+\nu_\ell$ (%) with the results from other theoretical approaches and data for comparison.

Decay modes	$B^0 \rightarrow D^-(1S)e^+\nu_e$	$B^0 \rightarrow D^-(1S)\mu^+\nu_\mu$	$B^0 \rightarrow D^-(1S)\tau^+\nu_\tau$
This work (CLFQM)	$1.96^{+0.01+0.00+0.06}_{-0.01-0.01-0.05}$	$1.96^{+0.01+0.00+0.06}_{-0.01-0.01-0.05}$	$0.51^{+0.00+0.01+0.02}_{-0.00-0.01-0.02}$
PQCD [34]	2.19	2.19	0.82
PQCD+Lattice [34]	1.95	1.95	0.62
CQM [32]	2.74	2.74	0.73
HQET [33]	...	...	0.64
PQCD [35]	2.03	2.03	0.87
PQCD [42]	...	1.65	0.554
PDG [25]	2.10 ± 0.07	2.10 ± 0.07	0.98 ± 0.21
Decay modes	$B^+ \rightarrow \bar{D}^0(1S)e^+\nu_e$	$B^+ \rightarrow \bar{D}^0(1S)\mu^+\nu_\mu$	$B^+ \rightarrow \bar{D}^0(1S)\tau^+\nu_\tau$
This work (CLFQM)	$2.11^{+0.01+0.00+0.06}_{-0.01-0.01-0.06}$	$2.11^{+0.01+0.00+0.06}_{-0.01-0.01-0.06}$	$0.55^{+0.00+0.01+0.02}_{-0.00-0.01-0.02}$
RQM [7]	2.53	2.53	0.68
PQCD [34]	2.29	2.29	0.86
PQCD+Lattice [34]	2.10	2.10	0.69
LQCD [11,39,40]	...	...	0.65
HQET [33]	...	...	0.66
PQCD [35]	2.19	2.19	0.95
PDG [25]	2.26 ± 0.07	2.26 ± 0.07	0.77 ± 0.25

related research include the RQM [7,26,27], the Bethe-Salpeter (BS) equation [28–30], the covariant quark model (CQM) [3,4,31,32], the heavy quark effective theory (HQET) [33], the perturbative QCD (PQCD) approach [34,35], the QCDSR [3,36], the lattice QCD (LQCD) [11,37–40], and the LCSR [41]. The uncertainties of our results arise from the lifetime of the $B_{(s)}$ meson and the decay constants of initial and final state mesons, respectively. The following are some comments: Our predictions for the branching ratios of the semileptonic decays $B \rightarrow D(1S)\ell^+\nu_\ell$ are listed in Table I. It is noted that the branching ratios of the decays $B \rightarrow D(1S)\ell^+\nu_\ell$ are in good agreement with the PQCD + Lattice results [34] and deviate from the experimental data by about 1.5σ . Meanwhile, for the decay $B^0 \rightarrow D^-(1S)\tau^+\nu_\tau$, the tension reaches 2.2σ . Although the deviation for the decay $B^+ \rightarrow \bar{D}^0(1S)\tau^+\nu_\tau$ is milder at only 0.9σ , the current experimental errors remain sizable. An analogous condition holds for the decays $B \rightarrow D^*(1S)\ell^+\nu_\ell$ is shown in Table II. The branching ratios of the decays $B \rightarrow D^*(1S)\ell^+\nu_\ell$ and $B_s \rightarrow D_s^*(1S)\mu^+\nu_\mu$ agree well with the data with only a 0.3σ deviation. However, the branching ratios of the decays $B \rightarrow D^*(1S)\tau^+\nu_\tau$ show a deviation exceeding 3σ from experimental data. In a word, our predictions for the decays $B \rightarrow D^{(*)}(1S)\ell^+\nu_\ell$ can explain the data well, while the branching ratios of the decays $B \rightarrow D^{(*)}(1S)\tau^+\nu_\tau$ are systematically smaller than the data. Similar situations also exist in other theoretical results. By the way, it is contrary for the nonleptonic B decays, such as

$B \rightarrow D(1S)\pi(K, \rho, K^*)$, where the theoretical results are larger than the data.

Tensions between the SM expectations and experimental measurements have driven extensive theoretical studies in the search for NP effects. Before claiming a new physics signal, it is crucial to substantially boost the precision of both theoretical calculations and experimental measurements. Current experiments still have relatively large uncertainties in the measured branching ratios of decays $B \rightarrow D(1S)\tau^+\nu_\tau$ shown in Table I.

For the branching ratios of the semileptonic decays $B_s \rightarrow D_s(1S)\mu^+\nu_\mu$ and $B_s \rightarrow D_s^*(1S)\mu^+\nu_\mu$, the discrepancies between our predictions and the data are approximately 1.1σ and 0.3σ , respectively. From Table III, one can find that the branching ratios of the decays $B_s \rightarrow D_s\ell^+\nu_\ell$ given by the LCSR [6,41], BS equation [28], and PQCD [34] approaches are smaller than 2×10^{-2} . In contrast, those from the QCDSR [3], CQM [4,31,43], and RQM [26] are larger than 2.5×10^{-2} . Meanwhile, the results obtained by the same methods as listed above, such as the QCDSR [36], PQCD [44], and RQM [27], fall within the range $(2.013\text{--}2.469) \times 10^{-2}$ given by the LQCD [37] and well account for the data. In Table IV, apart for the results given by the PQCD + Lattice [34] and CQM [31,43], many other theoretical predictions are consistent with the available data within errors. Further precise experimental data are needed, which are useful not only for clarifying different theoretical approaches, but also for constraining parameters in relevant models.

TABLE II. The branching ratios of the semileptonic decays $B \rightarrow D^*(1S)l^+\nu_l(\%)$ with the results from other theoretical approaches and data for comparison.

Decay modes	$B^0 \rightarrow D^{*-}(1S)e^+\nu_e$	$B^0 \rightarrow D^{*-}(1S)\mu^+\nu_\mu$	$B^0 \rightarrow D^{*-}(1S)\tau^+\nu_\tau$
This work	$4.97^{+0.01+0.03+0.36}_{-0.01-0.01-0.16}$	$4.95^{+0.01+0.03+0.36}_{-0.01-0.01-0.16}$	$1.13^{+0.00+0.01+0.07}_{-0.00-0.01-0.04}$
RQM [7]	6.28	6.28	1.45
PQCD [34]	5.32	5.32	1.53
PQCD+Lattice [34]	4.63	4.63	1.25
PQCD [35]	4.52	4.52	1.36
HQET [33]	...	...	1.29
CQM [32]	6.64	6.64	1.57
PQCD [42]	...	4.33	1.175
PDG [25]	4.87 ± 0.09	4.87 ± 0.09	1.48 ± 0.09
Decay modes	$B^+ \rightarrow \bar{D}^{*0}(1S)e^+\nu_e$	$B^+ \rightarrow \bar{D}^{*0}(1S)\mu^+\nu_\mu$	$B^+ \rightarrow \bar{D}^{*0}(1S)\tau^+\nu_\tau$
This work	$5.37^{+0.01+0.03+0.39}_{-0.01-0.01-0.17}$	$5.35^{+0.01+0.03+0.38}_{-0.01-0.01-0.17}$	$1.22^{+0.00+0.01+0.08}_{-0.00-0.01-0.04}$
RQM [7]	6.81	6.77	1.52
PQCD [34]	5.53	5.53	1.60
PQCD+Lattice [34]	4.89	4.89	1.34
PQCD [35]	4.87	4.87	1.47
HQET [33]	...	...	1.34
PDG [25]	5.26	5.26	1.88

TABLE III. The branching ratios of the semileptonic decays $B_s^0 \rightarrow D_s^-(1S)l^+\nu_l(\%)$ with the results from other theoretical approaches and data for comparison.

Decay modes	$B_s^0 \rightarrow D_s^-(1S)e^+\nu_e$	$B_s^0 \rightarrow D_s^-(1S)\mu^+\nu_\mu$	$B_s^0 \rightarrow D_s^-(1S)\tau^+\nu_\tau$
This work (CLFQM)	$2.05^{+0.00+0.11+0.05}_{-0.01-0.11-0.06}$	$2.04^{+0.01+0.11+0.01}_{-0.01-0.11-0.01}$	$0.53^{+0.00+0.03+0.03}_{-0.00-0.07-0.02}$
NCQM [45]	2.32	...	0.67
LCSR [6]	1.817	1.817	0.606
QCDSR [3]	2.8–3.5	2.8–3.5	...
QCDSR [36]	2.03	2.03	...
LCSR [41]	1.0	1.0	0.33
PQCD [44]	2.13	2.13	0.84
RQM [27]	2.1	2.1	0.62
RQM [26]	2.54	2.54	0.70
CQM [31]	2.73–3.00	2.73–3.00	...
PQCD [34]	1.97	1.97	0.72
PQCD+Lattice [34]	1.84	1.84	0.63
LQCD [37]	2.013–2.469	2.013–2.469	0.619–0.724
BS [28]	1.4–1.7	1.4–1.7	0.47–0.55
LQCD [38]	2.31	2.31	0.69
CQM [3,4]	2.8–3.8	2.8–3.8	...
CQM [43]	2.89	2.88	0.78
PDG [25]	...	2.29 ± 0.21	...

TABLE IV. The branching ratios of the semileptonic decays $B_s^0 \rightarrow D_s^{*-}(1S)l^+\nu_l$ (%) with the results from other theoretical approaches and data for comparison.

Decay modes	$B_s^0 \rightarrow D_s^{*-}(1S)e^+\nu_e$	$B_s^0 \rightarrow D_s^{*-}(1S)\mu^+\nu_\mu$	$B_s^0 \rightarrow D_s^{*-}(1S)\tau^+\nu_\tau$
This work (CLFQM)	$5.02^{+0.02+0.25+0.41}_{-0.02-0.27-0.47}$	$5.00^{+0.02+0.25+0.41}_{-0.02-0.27-0.46}$	$1.13^{+0.00+0.05+0.08}_{-0.00-0.06-0.09}$
NCQM [45]	6.26	...	1.53
PQCD [44]	4.76	4.76	1.44
RQM [27]	5.3	5.3	1.3
RQM [26]	2.54	2.54	0.70
CQM [31]	7.49–7.66	7.49–7.66	...
PQCD [34]	5.04	5.04	1.45
PQCD + Lattice [34]	4.42	4.42	1.20
BS [28]	5.1–5.8	5.1–5.8	1.2–1.3
LQCD [38]	5.25	5.25	1.31
CQM [3,4]	1.89–6.61	1.89–6.61	...
CQM [43]	6.42	6.39	1.53
PDG [25]	...	5.2 ± 0.5	...

Compared to the semileptonic decays $B \rightarrow D^{(*)}(1S)\ell\nu_\ell$, the theoretical studies on the semileptonic $B \rightarrow D^{(*)}(2S)\ell\nu_\ell$ decays remain relatively scarce. Currently, only the BS equation method [29,30] and the RQM [27] have been applied to in this class of decays. In Tables V and VI, we present the branching ratios of the semileptonic decays $B \rightarrow D^{(*)}(2S)\ell\nu_\ell$ and $B_s \rightarrow D_s^{(*)}(2S)\ell\nu_\ell$, respectively, which are at least one order of magnitude smaller than those of the decays $B \rightarrow D^{(*)}(1S)\ell\nu_\ell$ and $B_s \rightarrow D_s^{(*)}(1S)\ell\nu_\ell$. These semileptonic $B_{(s)}$ decays to the first radially excited charmed mesons have branching ratios up to $10^{-4} - 10^{-3}$, which are within the detection accuracy of current experiments. It is noted that our predictions are much larger than those from the improved BS equation approach [29], where the authors considered that the contribution of the relativistic correction cannot be ignored when radially excited mesons were included. For example, the branching ratios of the decays $B_s^0 \rightarrow D_s^{*-}(2S)e^+\nu_e$ were given as 3.14×10^{-4} [29] and 9.9×10^{-4} [30] with and without including the relativistic correction, respectively. That is to say, relativistic effects

significantly suppress the branching ratio by a factor of 1/3. Our prediction $\text{Br}(B_s^0 \rightarrow D_s^{*-}(2S)\ell^+\nu_\ell) = 1.35 \times 10^{-3}$ is consistent with the previous BS equation result. Certainly, all our results for the decays $B_s \rightarrow D_s^{(*)-}(2S)\ell\nu_\ell$ agree with the RQM calculations [27] as shown in Table VI. Thus, it is interesting to test whether significant relativistic effects exist in these considered decays in future experiments.

Both semileptonic decays $B \rightarrow D^{(*)}(1S)\ell\nu_\ell$ and $B \rightarrow D^{(*)}(2S)\ell\nu_\ell$ are induced by the same $b \rightarrow c\ell\nu_\ell$ transition, so the decays $B \rightarrow D^{(*)}(2S)\ell\nu_\ell$ can also serve as a probe for NP searches. The tensions between experimental data and SM predictions in the $B \rightarrow D^{(*)}(1S)\ell\nu_\ell$ decays have attracted widespread attention. Whether a similar anomaly also occurs in the semileptonic $B \rightarrow D^{(*)}(2S)\ell\nu_\ell$ decays is worthy of in-depth investigation. There is a long-standing puzzle in flavor physics about determining the V_{cb} , that is, 3σ deviations between the inclusive and exclusive determinations. To resolve this inconsistency, the LHCb Collaboration has determined the following branching fraction ratios [2]:

TABLE V. The branching ratios of the semileptonic decays $B \rightarrow D^{(*)}(2S)\ell\nu_\ell$ (10^{-3}) with the results from the BS equation for comparison.

Decay modes	This work	BS [29]	Decay modes	This work	BS [29]
$B^0 \rightarrow D^{*-}(2S)e^+\nu_e$	$1.44^{+0.00+0.18+0.15}_{-0.00-0.19-0.19}$	0.139	$B^+ \rightarrow \bar{D}^0(2S)e^+\nu_e$	$1.55^{+0.00+0.19+0.17}_{-0.00-0.21-0.20}$	0.15
$B^0 \rightarrow D^{*-}(2S)\mu^+\nu_\mu$	$1.43^{+0.00+0.18+0.15}_{-0.00-0.19-0.19}$	0.138	$B^+ \rightarrow \bar{D}^0(2S)\mu^+\nu_\mu$	$1.54^{+0.00+0.19+0.17}_{-0.00-0.21-0.20}$	0.149
$B^0 \rightarrow D^{*-}(2S)\tau^+\nu_\tau$	$0.18^{+0.00+0.03+0.03}_{-0.00-0.03-0.03}$	0.0135	$B^+ \rightarrow \bar{D}^0(2S)\tau^+\nu_\tau$	$0.19^{+0.00+0.03+0.03}_{-0.00-0.04-0.04}$	0.0149
$B^0 \rightarrow D^{*-}(2S)e^+\nu_e$	$2.67^{+0.00+0.20+0.11}_{-0.00-0.24-0.14}$	0.1942	$B^+ \rightarrow \bar{D}^{*0}(2S)e^+\nu_e$	$2.93^{+0.00+0.21+0.13}_{-0.00-0.26-0.16}$	0.2123
$B^0 \rightarrow D^{*-}(2S)\mu^+\nu_\mu$	$2.65^{+0.00+0.20+0.11}_{-0.00-0.24-0.14}$	0.1932	$B^+ \rightarrow \bar{D}^{*0}(2S)\mu^+\nu_\mu$	$2.91^{+0.00+0.21+0.13}_{-0.00-0.26-0.16}$	0.2113
$B^0 \rightarrow D^{*-}(2S)\tau^+\nu_\tau$	$0.19^{+0.00+0.02+0.10}_{-0.00-0.02-0.11}$	0.0137	$B^+ \rightarrow \bar{D}^{*0}(2S)\tau^+\nu_\tau$	$0.21^{+0.00+0.02+0.11}_{-0.00-0.03-0.11}$	0.0155

TABLE VI. The branching ratios of the semileptonic decays $B_s^0 \rightarrow D_s^{(*)-}(2S)l^+\nu_l$ (10^{-3}) with the results from the BS equation and RQM for comparison.

Decay modes	This work	BS [29]	BS [30]	RQM [27]	Decay modes	This work	[29]	RQM [27]
$B_s^0 \rightarrow D_s^{*-}(2S)e^+\nu_e$	$1.35^{+0.00+0.25+0.14}_{-0.00-0.23-0.22}$	0.314	0.99	2.7	$B_s^0 \rightarrow D_s^{*-}(2S)e^+\nu_e$	$2.25^{+0.01+1.09+0.64}_{-0.01-0.70-0.80}$	0.5873	3.8
$B_s^0 \rightarrow D_s^{*-}(2S)\mu^+\nu_\mu$	$1.34^{+0.00+0.25+0.14}_{-0.00-0.23-0.22}$	0.312	...	...	$B_s^0 \rightarrow D_s^{*-}(2S)\mu^+\nu_\mu$	$2.23^{+0.01+1.09+0.64}_{-0.01-0.70-0.80}$	0.5842	...
$B_s^0 \rightarrow D_s^{*-}(2S)\tau^+\nu_\tau$	$0.14^{+0.00+0.03+0.02}_{-0.00-0.03-0.03}$	0.0244	...	0.11	$B_s^0 \rightarrow D_s^{*-}(2S)\tau^+\nu_\tau$	$0.15^{+0.00+0.05+0.06}_{-0.00-0.07-0.06}$	0.0405	0.15

$$\frac{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}\mu^+\nu_\mu)}{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}\mu^+\nu_\mu)} = 0.464 \pm 0.013_{\text{stat}} \pm 0.043_{\text{syst}} \pm 0.05_{\text{ext}}, \quad (28)$$

$$\frac{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}\mu^+\nu_\mu)}{\mathcal{B}r(B^0 \rightarrow D^{*-}\mu^+\nu_\mu)} = 1.09 \pm 0.05_{\text{stat}} \pm 0.06_{\text{syst}} \pm 0.05_{\text{ext}}, \quad (29)$$

$$\frac{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}\mu^+\nu_\mu)}{\mathcal{B}r(B^0 \rightarrow D^{*-}\mu^+\nu_\mu)} = 1.06 \pm 0.05_{\text{stat}} \pm 0.07_{\text{syst}} \pm 0.05_{\text{ext}}. \quad (30)$$

For comparison, we present our predictions for the corresponding branching fraction ratios, as well as those for the case where $D_{(s)}(1S)$ and $D_{(s)}^*(1S)$ are replaced by $D_{(s)}(2S)$ and $D_{(s)}^*(2S)$, respectively:

$$\frac{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}\mu^+\nu_\mu)}{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}\mu^+\nu_\mu)} = 0.408 \pm 0.029, \quad (31)$$

$$\frac{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}(2S)\mu^+\nu_\mu)}{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}(2S)\mu^+\nu_\mu)} = 0.601 \pm 0.342,$$

$$\frac{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}\mu^+\nu_\mu)}{\mathcal{B}r(B^0 \rightarrow D^{*-}\mu^+\nu_\mu)} = 1.041 \pm 0.074, \quad (32)$$

$$\frac{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}(2S)\mu^+\nu_\mu)}{\mathcal{B}r(B^0 \rightarrow D^{*-}(2S)\mu^+\nu_\mu)} = 0.937 \pm 0.269,$$

$$\frac{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}\mu^+\nu_\mu)}{\mathcal{B}r(B^0 \rightarrow D^{*-}\mu^+\nu_\mu)} = 1.010 \pm 0.216, \quad (33)$$

$$\frac{\mathcal{B}r(B_s^0 \rightarrow D_s^{*-}(2S)\mu^+\nu_\mu)}{\mathcal{B}r(B^0 \rightarrow D^{*-}(2S)\mu^+\nu_\mu)} = 0.842 \pm 0.446,$$

where the errors stem from the $B_{(s)}$ meson lifetime and the decay constants of initial and final state mesons.

As one of the pillars of the SM, the ratios $\mathcal{R}(D^{(*)})$ and $\mathcal{R}(D_{(s)}^{(*)})$ are a powerful test of the LFU. Their values have attracted widespread attention from both experimentalists and theorists. Our predictions are presented in Table VII, along with experimental measurements and other

theoretical results for comparison. From the initial value $0.44 \pm 0.058 \pm 0.042$ reported by Belle [46] to the recent one $0.249 \pm 0.043 \pm 0.047$ given by LHCb, the $\mathcal{R}_D$ measurements exhibit a decreasing trend over time. Accordingly, relative to the experimental measurements, our prediction $\mathcal{R}(D) = 0.260 \pm 0.014$ yields a shrinking tension, which changes from 2.4σ to 0.16σ , while the deviation compared to the HFAG value still amounts to 3.1σ . It is similar for the $\mathcal{R}(D^*)$ values between our predictions and the data. For example, our value $\mathcal{R}(D^{*-}) = 0.228 \pm 0.026$ compares with the measurement $0.260 \pm 0.015 \pm 0.016 \pm 0.012$ given by LHCb [47], corresponding to a 1.3σ deviation, while it sits at a 3.7σ below the HFAG average $0.295 \pm 0.010 \pm 0.010$. If we adopt the latest LHCb measurement $\mathcal{R}(D^*) = 0.402 \pm 0.081 \pm 0.085$, the tension reduces to 1.5σ , which falls within the range allowed by statistical fluctuations again. More precise experimental data for $\mathcal{R}(D_{(s)})$ and $\mathcal{R}(D_{(s)}^*)$ are urgently needed. Certainly, our predictions are consistent with the results obtained from the RQM [7,26,27] and the PQCD [35] calculations within errors.

Next, we calculate other two physical observables, namely, the forward-backward asymmetry $\mathcal{A}_{\text{FB}}$ and the longitudinal polarization fraction f_L , using Eqs. (21) and (22). Their values are listed in Tables VIII–X. These physical quantities are also sensitive to some kinds of new physics and can be measured in the present experiments. So they have attracted widespread attention from various theoretical approaches. From our calculations, we arrive at the following conclusions.

- (i) The forward-backward asymmetries $\mathcal{A}_{\text{FB}}$ for the decays $B_{(s)} \rightarrow D_{(s)}\ell\nu_\ell$ exhibit a clear hierarchical relationship, because they are proportional to the square of the lepton mass as shown in Eq. (23), while the difference among $\mathcal{A}_{\text{FB}}$ values of the decays $B_{(s)} \rightarrow D_{(s)}^*\ell\nu_\ell$ is not significant.
- (ii) The $\mathcal{A}_{\text{FB}}$ values for the decays $B_{(s)} \rightarrow D_{(s)}\ell\nu_\ell$ are positive, while those of the decays $B_{(s)} \rightarrow D_{(s)}^*\ell\nu_\ell$ are negative. Our predictions are consistent with the results given by the PQCD and PQCD + Lattice approaches [34] and HQET [50] but have opposite signs relative to the CQM [32] and RQM [7] calculations except for those of the decays $B_{(s)} \rightarrow D_{(s)}^*\tau\nu_\tau$.

TABLE VII. The values of ratios $\mathcal{R}(D^{(*)})$ and $\mathcal{R}(D_s^{(*)})$, where the individual errors, as those in branching ratios, have been added in quadrature.

	$\mathcal{R}(D^-(1S))$	$\mathcal{R}(D^0(1S))$	$\mathcal{R}(D^{*-}(1S))$	$\mathcal{R}(D^{*0}(1S))$
This work	0.260 ± 0.014	0.261 ± 0.013	0.228 ± 0.026	0.228 ± 0.026
RQM [7]	...	0.269	0.231	0.224
PQCD+Lattice [34]	0.318	0.329	0.270	0.274
PQCD [35]	0.429	0.434	0.301	0.302
Belle-II [48]	0.418	...	0.306	...
LHCb [49]	0.249	...	0.402	...
	$\mathcal{R}(D^-(2S))$	$\mathcal{R}(D^0(2S))$	$\mathcal{R}(D^{*-}(2S))$	$\mathcal{R}(D^{*0}(2S))$
This work	0.125 ± 0.037	0.123 ± 0.039	0.072 ± 0.0431	0.072 ± 0.043
BS [29]	0.097	0.099	0.071	0.073
	$\mathcal{R}(D_s^-(1S))$	$\mathcal{R}(D_s^{*-}(1S))$	$\mathcal{R}(D_s^-(2S))$	$\mathcal{R}(D_s^{*-}(2S))$
This work	0.259 ± 0.033	0.226 ± 0.031	0.104 ± 0.038	0.067 ± 0.052
RQM [26]	0.276	0.276	...	...
RQM [27]	0.295	0.245	...	...
PQCD+Lattice [34]	0.342	0.271	...	...
BS [29]	...	...	0.078	0.069

- (iii) It is worth noting that the magnitudes of the $\mathcal{A}_{\text{FB}}$ values for the decays $B_{(s)} \rightarrow D_{(s)}\tau\nu_\tau$ given by these theoretical approaches are in excellent agreement with each other. However, the discrepancies among the results for the decays $B_{(s)} \rightarrow D_{(s)}^*\tau\nu_\tau$ are significant.
- (iv) From Table IX, one can find that the forward-backward asymmetries of the decays $B_{(s)} \rightarrow D_{(s)}(2S)\ell\nu_\ell, D_{(s)}^*(2S)\ell\nu_\ell$ are similar to those of corresponding decays $B_{(s)} \rightarrow D_{(s)}(1S)\ell\nu_\ell, D_{(s)}^*(1S)\ell\nu_\ell$. These results can be verified by future LHCb and Belle II experiments.

In order to investigate the dependence of the polarization on q^2 , we calculate the longitudinal polarization fractions by dividing the full energy region into two regions for each decay as listed in Table X, where region 1 is defined as $m_\ell^2 < q^2 < \frac{(m_B - m_{D^*(nS)})^2 + m_\ell^2}{2}$ and region 2 is $\frac{(m_B - m_{D^*(nS)})^2 + m_\ell^2}{2} < q^2 < (m_B - m_{D^*(nS)})^2$ with $n = 1, 2$. Some comments are in order. For the decays $B_{(s)} \rightarrow D_{(s)}^*(nS)\ell'\nu_{\ell'}$, the longitudinal polarization fractions from region 1 are larger than those from region 2, while it is contrary to the decays $B_{(s)} \rightarrow D_{(s)}^*(nS)\tau\nu_\tau$. For the decays $B_{(s)} \rightarrow D_{(s)}^*(nS)\ell'\nu_{\ell'}$, the longitudinal polarizations are larger than the transverse ones, while for the decays $B_{(s)} \rightarrow D_{(s)}^*(nS)\tau\nu_\tau$, the longitudinal polarization fractions are almost equal to, or even smaller than, the

transverse ones. In other words, the longitudinal polarization fractions of the decays $B_{(s)} \rightarrow D_{(s)}^*(nS)\ell'\nu_{\ell'}$ are always larger than those of the decays $B_{(s)} \rightarrow D_{(s)}^*(nS)\tau\nu_\tau$. Furthermore, the longitudinal polarization fractions of the decays $B_{(s)} \rightarrow D_{(s)}^*(2S)\ell\nu_\ell$ are larger than those of the corresponding decays $B_{(s)} \rightarrow D_{(s)}^*(1S)\ell\nu_\ell$. These relations can be expressed using the following formulas:

$$f_L(B_{(s)} \rightarrow D_{(s)}^*(nS)\ell'\nu_{\ell'}) > f_L(B_{(s)} \rightarrow D_{(s)}^*(nS)\tau\nu_\tau),$$

$$n = 1, 2, \quad (34)$$

$$f_L(B_{(s)} \rightarrow D_{(s)}^*(2S)\ell\nu_\ell) > f_L(B_{(s)} \rightarrow D_{(s)}^*(1S)\ell\nu_\ell). \quad (35)$$

All the results for our considered decays are consistent with other theoretical calculations, such as the RQM [7], PQCD + Lattice [34], HQET [50], and model-independent analysis (MIA) [51]. It is worth noting that our prediction for the value of $f_L(B^0 \rightarrow D^{*-}(1S)\tau^+\nu_\tau)$ is in good agreement with the experimental data given by LHCb [53] but smaller than the Belle measurement [52]. We look forward to more experimental data to verify our predictions. The q^2 dependence of the forward-backward asymmetries $\mathcal{A}_{\text{FB}}$ (Figs. 2–4) and the differential decay rates $d\Gamma^{(L)}/dq^2$ (Figs. 5 and 6) is listed in the Appendix.

TABLE VIII. Forward-backward asymmetries $\mathcal{A}_{\text{FB}}$ for the decays $B_{(s)} \rightarrow D_{(s)}(1S)\ell\nu_\ell$ and $B_{(s)} \rightarrow D_{(s)}^*(1S)\ell\nu_\ell$ with the results from other theoretical approaches for comparison.

Channels	$B^0 \rightarrow D^-(1S)e^+\nu_e$	$B^0 \rightarrow D^-(1S)\mu^+\nu_\mu$	$B^0 \rightarrow D^-(1S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$4.36^{+0.01+0.01+0.13}_{-0.01-0.01-0.13} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.36^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$
CQM [32]	-11.7×10^{-7}	...	-0.36
PQCD [34]	...	...	0.35
PQCD+Lattice [34]	...	...	0.36
Channels	$B^+ \rightarrow \bar{D}^0(1S)e^+\nu_e$	$B^+ \rightarrow \bar{D}^0(1S)\mu^+\nu_\mu$	$B^+ \rightarrow \bar{D}^0(1S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$4.36^{+0.01+0.01+0.13}_{-0.01-0.01-0.13} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.36^{+0.00+0.04+0.01}_{-0.00-0.05-0.01}$
RQM [7]	-9.8×10^{-7}	-0.013	-0.37
Channels	$B_s^0 \rightarrow D_s^-(1S)e^+\nu_e$	$B_s^0 \rightarrow D_s^-(1S)\mu^+\nu_\mu$	$B_s^0 \rightarrow D_s^-(1S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$4.39^{+0.06+0.26+0.14}_{-0.06-0.19-0.13} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.36^{+0.00+0.04+0.02}_{-0.00-0.02-0.01}$
RQM [7]	-9.7×10^{-7}	-0.013	-0.36
PQCD [34]	...	...	0.36
PQCD+Lattice [34]	...	...	0.36
Channels	$B^0 \rightarrow D^{*-}(1S)e^+\nu_e$	$B^0 \rightarrow D^{*-}(1S)\mu^+\nu_\mu$	$B^0 \rightarrow D^{*-}(1S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$-0.20^{+0.00+0.01+0.01}_{-0.00-0.00-0.01}$	$-0.20^{+0.00+0.01+0.01}_{-0.00-0.00-0.01}$	$-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$
CQM [32]	0.19	...	0.027
PQCD [34]	...	...	-0.085
PQCD+Lattice [34]	...	...	-0.054
HQET [50]	...	...	-0.084
Channels	$B^+ \rightarrow \bar{D}^{*0}(1S)e^+\nu_e$	$B^+ \rightarrow \bar{D}^{*0}(1S)\mu^+\nu_\mu$	$B^+ \rightarrow \bar{D}^{*0}(1S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$-0.20^{+0.00+0.01+0.01}_{-0.00-0.00-0.01}$	$-0.20^{+0.00+0.01+0.01}_{-0.00-0.00-0.01}$	$-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$
RQM [7]	-0.22	-0.23	-0.32
Channels	$B_s^0 \rightarrow D_s^{*-}(1S)e^+\nu_e$	$B_s^0 \rightarrow D_s^{*-}(1S)\mu^+\nu_\mu$	$B_s^0 \rightarrow D_s^{*-}(1S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$-0.20^{+0.00+0.01+0.00}_{-0.00-0.00-0.01}$	$-0.20^{+0.00+0.01+0.02}_{-0.00-0.00-0.01}$	$-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$
RQM [7]	-0.26	-0.27	-0.32
PQCD [34]	...	...	-0.083
PQCD+Lattice [34]	...	...	-0.050

TABLE IX. Forward-backward asymmetries $\mathcal{A}_{\text{FB}}$ for the decays $B_{(s)} \rightarrow D_{(s)}(2S)\ell\nu_\ell$ and $B_{(s)} \rightarrow D_{(s)}^*(2S)\ell\nu_\ell$.

Channels	$B^0 \rightarrow D^-(2S)e^+\nu_e$	$B^0 \rightarrow D^-(2S)\mu^+\nu_\mu$	$B^0 \rightarrow D^-(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$6.60^{+0.02+0.08+0.08}_{-0.02-0.09-0.07} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.35^{+0.00+0.05+0.05}_{-0.00-0.06-0.06}$
Channels	$B^+ \rightarrow \bar{D}^0(2S)e^+\nu_e$	$B^+ \rightarrow \bar{D}^0(2S)\mu^+\nu_\mu$	$B^+ \rightarrow \bar{D}^0(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$6.60^{+0.02+0.09+0.07}_{-0.02-0.08-0.08} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.35^{+0.00+0.05+0.05}_{-0.00-0.06-0.06}$
Channels	$B_s^0 \rightarrow D_s^-(2S)e^+\nu_e$	$B_s^0 \rightarrow D_s^-(2S)\mu^+\nu_\mu$	$B_s^0 \rightarrow D_s^-(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$6.63^{+0.02+1.09+0.58}_{-0.02-1.04-1.03} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.37^{+0.00+0.07+0.06}_{-0.00-0.08-0.08}$
Channels	$B^0 \rightarrow D^{*-}(2S)e^+\nu_e$	$B^0 \rightarrow D^{*-}(2S)\mu^+\nu_\mu$	$B^0 \rightarrow D^{*-}(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.11^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.05^{+0.00+0.00+0.03}_{-0.00-0.02-0.03}$

(Table continued)

TABLE IX. (Continued)

Channels	$B^0 \rightarrow D^-(2S)e^+\nu_e$	$B^0 \rightarrow D^-(2S)\mu^+\nu_\mu$	$B^0 \rightarrow D^-(2S)\tau^+\nu_\tau$
Channels	$B^+ \rightarrow \bar{D}^{*0}(2S)e^+\nu_e$	$B^+ \rightarrow \bar{D}^{*0}(2S)\mu^+\nu_\mu$	$B^+ \rightarrow \bar{D}^{*0}(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.05^{+0.00+0.00+0.03}_{-0.00-0.02-0.03}$
Channels	$B_s^0 \rightarrow D_s^{*-}(2S)e^+\nu_e$	$B_s^0 \rightarrow D_s^{*-}(2S)\mu^+\nu_\mu$	$B_s^0 \rightarrow D_s^{*-}(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\text{FB}}$	$-0.13^{+0.00+0.04+0.06}_{-0.00-0.06-0.05}$	$-0.13^{+0.00+0.04+0.06}_{-0.00-0.06-0.05}$	$-0.07^{+0.00+0.02+0.04}_{-0.00-0.06-0.03}$

TABLE X. The longitudinal polarization fractions f_L for the decays $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)\ell\nu_\ell$ in region 1, region 2, and total physical region, respectively.

Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B^0 \rightarrow D^{*-}(1S)l'^+\nu_{l'})$	0.35	0.20	$0.55^{+0.00+0.00+0.03}_{-0.00-0.04-0.02}$	$f_L(B^0 \rightarrow D^{*-}(2S)l'^+\nu_{l'})$	0.50	0.18	$0.68^{+0.00+0.06+0.03}_{-0.00-0.05-0.03}$
$f_L(B^0 \rightarrow D^{*-}(1S)\tau^+\nu_\tau)$	0.20	0.25	$0.45^{+0.00+0.00+0.01}_{-0.00-0.03-0.02}$	$f_L(B^0 \rightarrow D^{*-}(2S)\tau^+\nu_\tau)$	0.22	0.29	$0.51^{+0.00+0.03+0.01}_{-0.00-0.06-0.02}$
PQCD [34]	...	...	0.42	...	...	...	...
PQCD+Lattice [34]	...	...	0.43	...	...	...	...
MIA [51]	...	...	0.457	...	...	...	...
HQET [50]	...	...	0.441	...	...	...	...
Belle [52]	...	...	$0.60 \pm 0.08 \pm 0.04$	...	...	...	...
LHCb [53]	...	...	$0.41 \pm 0.06 \pm 0.03$	...	...	...	...
Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B^+ \rightarrow \bar{D}^{*0}(1S)l'^+\nu_{l'})$	0.35	0.20	$0.55^{+0.00+0.00+0.03}_{-0.00-0.03-0.02}$	$f_L(B^+ \rightarrow \bar{D}^{*0}(2S)l'^+\nu_{l'})$	0.49	0.18	$0.68^{+0.00+0.05+0.03}_{-0.00-0.06-0.03}$
RQM [7]	...	...	0.55	...	...	...	...
$f_L(B^+ \rightarrow \bar{D}^{*0}(1S)\tau^+\nu_\tau)$	0.20	0.25	$0.45^{+0.00+0.00+0.01}_{-0.00-0.03-0.06}$	$f_L(B^+ \rightarrow \bar{D}^{*0}(2S)\tau^+\nu_\tau)$	0.22	0.29	$0.51^{+0.00+0.04+0.03}_{-0.00-0.06-0.03}$
RQM [7]	...	...	0.47	...	...	...	...
Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B_s^0 \rightarrow D_s^{*-}(1S)l'^+\nu_{l'})$	0.34	0.20	$0.54^{+0.00+0.04+0.05}_{-0.00-0.03-0.07}$	$f_L(B_s^0 \rightarrow D_s^{*-}(2S)l'^+\nu_{l'})$	0.51	0.17	$0.68^{+0.00+0.31+0.15}_{-0.00-0.20-0.22}$
RQM [7]	...	...	0.49	...	...	...	...
$f_L(B_s^0 \rightarrow D_s^{*-}(1S)\tau^+\nu_\tau)$	0.20	0.25	$0.45^{+0.00+0.00+0.05}_{-0.00-0.02-0.03}$	$f_L(B_s^0 \rightarrow D_s^{*-}(2S)\tau^+\nu_\tau)$	0.23	0.28	$0.51^{+0.01+0.12+0.11}_{-0.01-0.18-0.11}$
PQCD [34]	...	...	0.42	...	...	...	...
PQCD+Lattice [34]	...	...	0.43	...	...	...	...
RQM [7]	...	...	0.42	...	...	...	...

IV. SUMMARY

Inspired by the recent experimental anomalies in B meson decays, especially the improved precision of LFU measurements, we investigate the semileptonic decays $B_{(s)} \rightarrow D_{(s)}(nS)\ell\nu_\ell$ and $B_{(s)} \rightarrow D_{(s)}^*(nS)\ell\nu_\ell$ with $n = 1, 2$ using the CLFQM and calculate several physical observables, including the branching ratio $\mathcal{R}(D^{(*)})$, the forward-backward asymmetry $\mathcal{A}_{\text{FB}}$, and longitudinal polarization fraction f_L . Our predictions for the branching ratios of the

decays $B_{(s)} \rightarrow D_{(s)}(1S)\ell'\nu_{\ell'}$, $D_{(s)}^*(1S)\ell'\nu_{\ell'}$ agree well with data, and the deviations are no more than 1.5σ , reaching as low as 0.3σ for the decays $B_{(s)} \rightarrow D_{(s)}^*(1S)\ell'\nu_{\ell'}$. Meanwhile, the branching ratios of the decays $B_{(s)} \rightarrow D_{(s)}(1S)\tau\nu_\tau$, $D_{(s)}^*(1S)\tau\nu_\tau$ are systematically smaller than the experimental data. The deviations between our predictions and the data for the decays $B^0 \rightarrow D^{(*)-}\tau^+\nu_\tau$, $B^+ \rightarrow \bar{D}^{*0}\tau^+\nu_\tau$ exceed 2σ , or even 3σ . Accordingly, the extracted values of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are naturally lower than the

experimental measurements. Nevertheless, such tensions display a gradual downward trend. The deviations between our results and the latest LHCb data for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are 0.16σ and 1.5σ , respectively. In contrast, the tensions compared with the current HFLAV experimental world averages still amount to 3.1σ and 2.1σ , respectively. Meanwhile, the branching ratios of the decays $B \rightarrow D^{(*)}(2S)\ell\nu_\ell$ are also calculated, which lie in the range $10^{-4} - 10^{-3}$ and are about one order of magnitude larger than those obtained from the BS equation. For decays $B_s \rightarrow D_s^{(*)}(2S)\ell\nu_\ell$, our predictions are 3–5 times those given by the BS equation and are consistent with the RQM calculations. Furthermore, we present the forward-backward asymmetries $\mathcal{A}_{\text{FB}}$ and the longitudinal polarization

fractions f_L for relevant decays, which can be tested in future LHCb and Belle II experiments.

ACKNOWLEDGMENTS

This work is partly supported by the National Natural Science Foundation of China under Grant No. 11347030 and the Natural Science Foundation of Henan Province under Grants No. 232300420116, No. 242300421679, and No. 252300421302.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

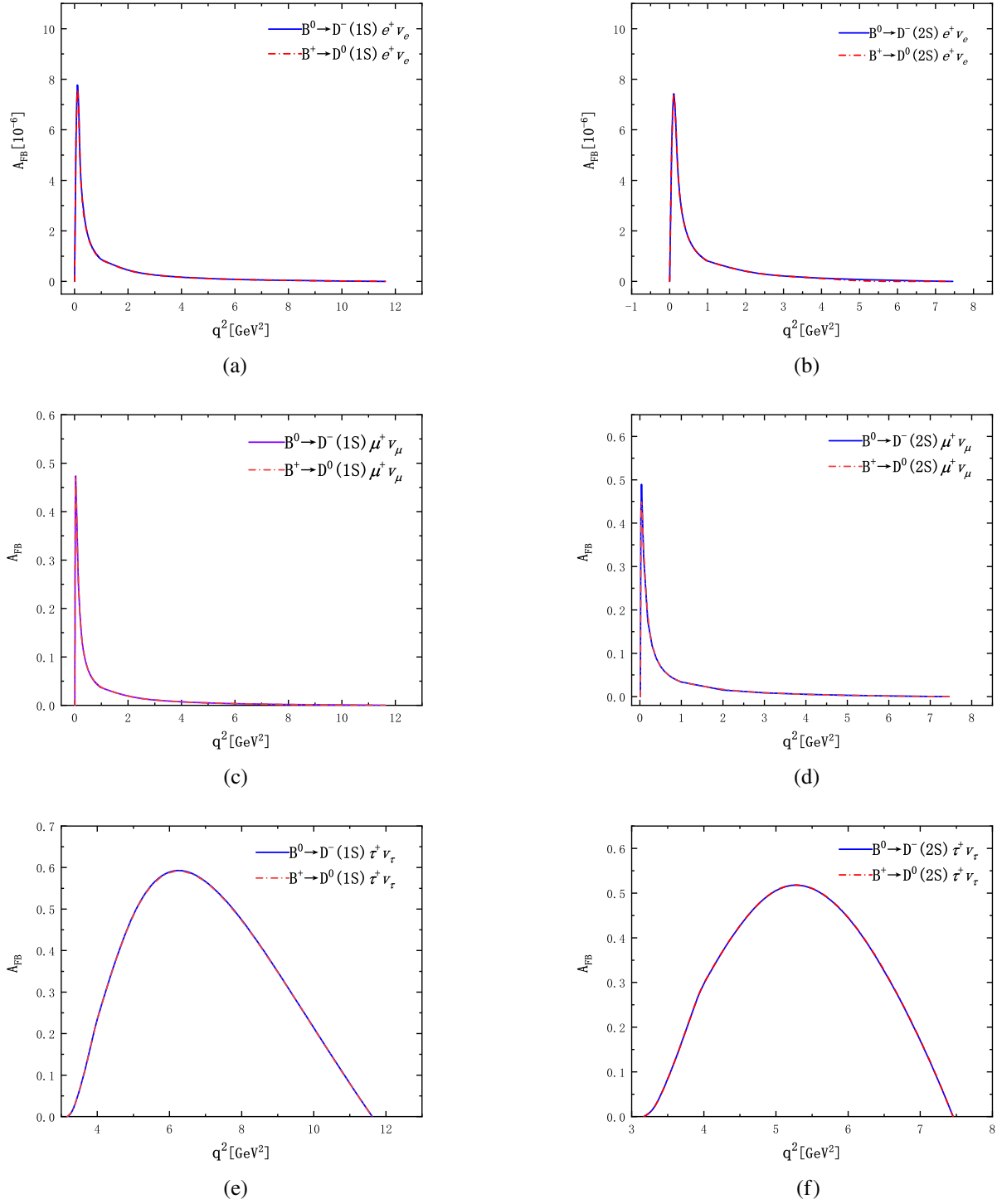
APPENDIX: q^2 DEPENDENCE OF THE $\mathcal{A}_{\text{FB}}$ AND $d\Gamma^{(L)}/dq^2$ 

FIG. 2. The q^2 dependence of the forward-backward asymmetries $\mathcal{A}_{\text{FB}}$ for the decays $B \rightarrow D(1S)e\nu_e$ (a), $B \rightarrow D(2S)e\nu_e$ (b), $B \rightarrow D(1S)\mu\nu_\mu$ (c), $B \rightarrow D(2S)\mu\nu_\mu$ (d), $B \rightarrow D(1S)\tau\nu_\tau$ (e), and $B \rightarrow D(2S)\tau\nu_\tau$ (f).

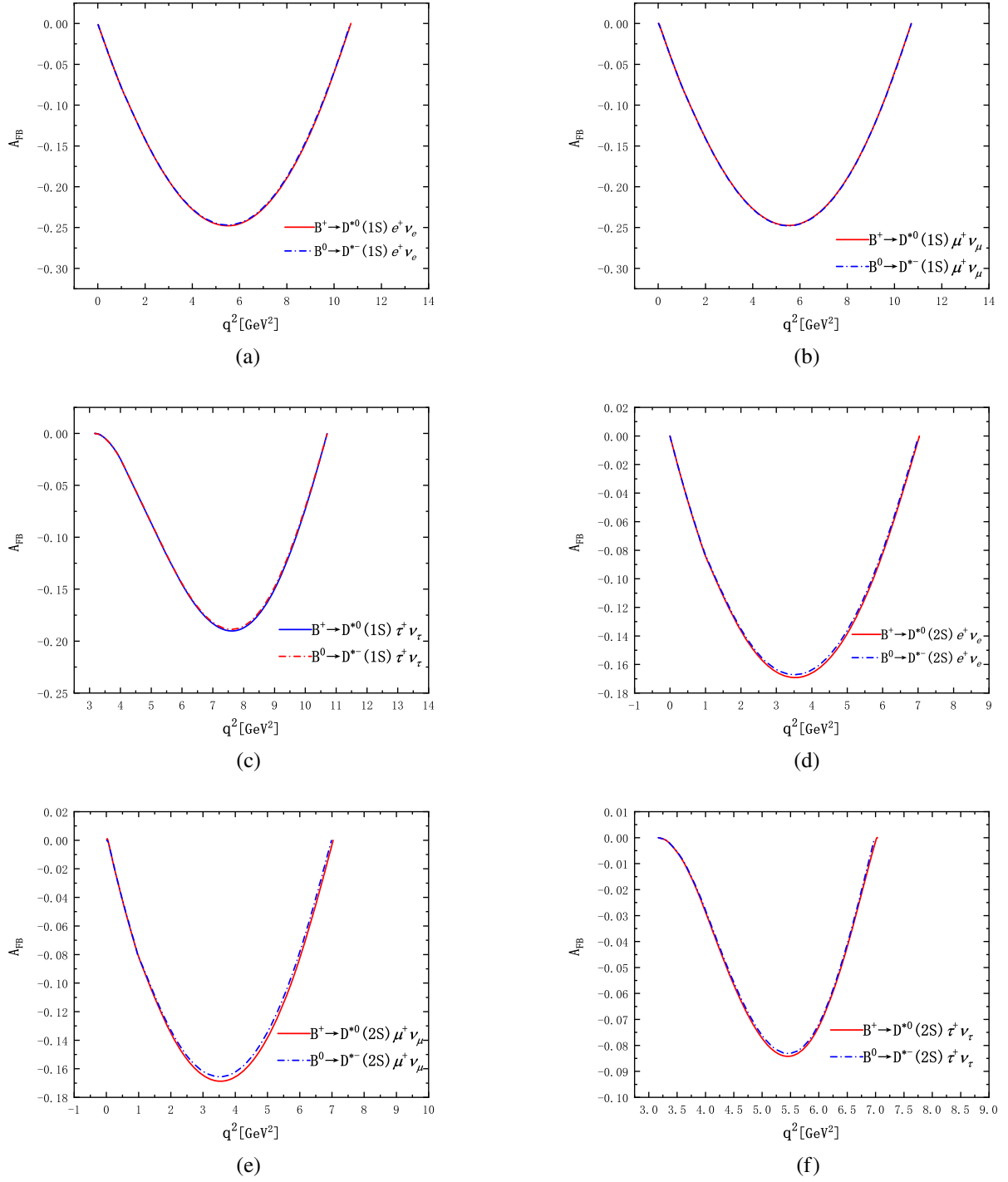


FIG. 3. The q^2 dependence of the forward-backward asymmetries A_{FB} for the decays $B \rightarrow D^*(1S)e\nu_e$ (a), $B \rightarrow D^*(1S)\mu\nu_\mu$ (b), $B \rightarrow D^*(1S)\tau\nu_\tau$ (c), $B \rightarrow D^*(2S)e\nu_e$ (d), $B \rightarrow D^*(2S)\mu\nu_\mu$ (e), and $B \rightarrow D^*(2S)\tau\nu_\tau$ (f).

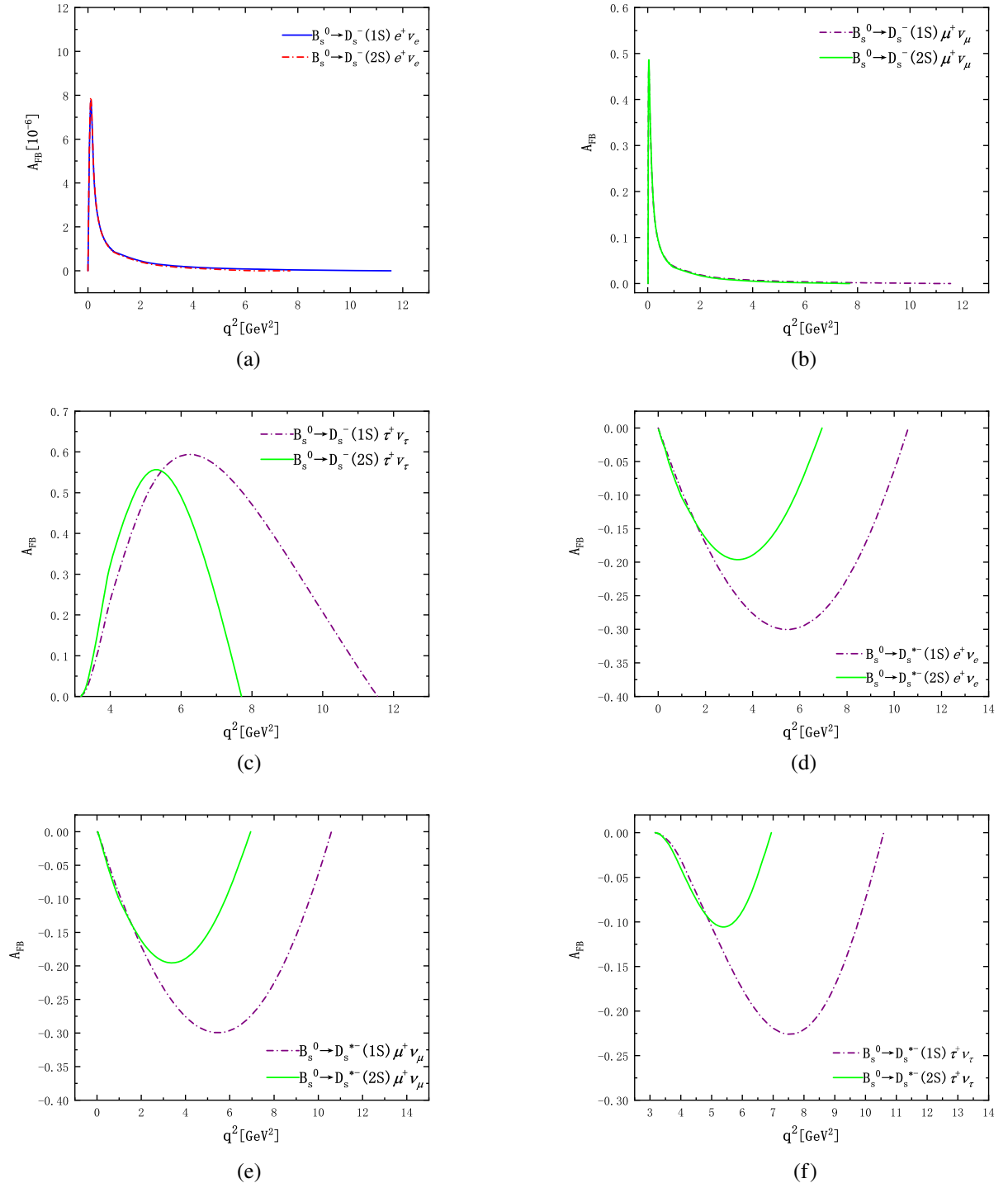


FIG. 4. The q^2 dependence of the forward-backward asymmetries A_{FB} for the decays $B_s \rightarrow D_s(1S, 2S)e\nu_e$ (a), $B_s \rightarrow D_s(1S, 2S)\mu\nu_\mu$ (b), $B_s \rightarrow D_s(1S, 2S)\tau\nu_\tau$ (c), $B_s \rightarrow D_s^*(1S, 2S)e\nu_e$ (d), $B_s \rightarrow D_s^*(1S, 2S)\mu\nu_\mu$ (e), and $B_s \rightarrow D_s^*(1S, 2S)\tau\nu_\tau$ (f).

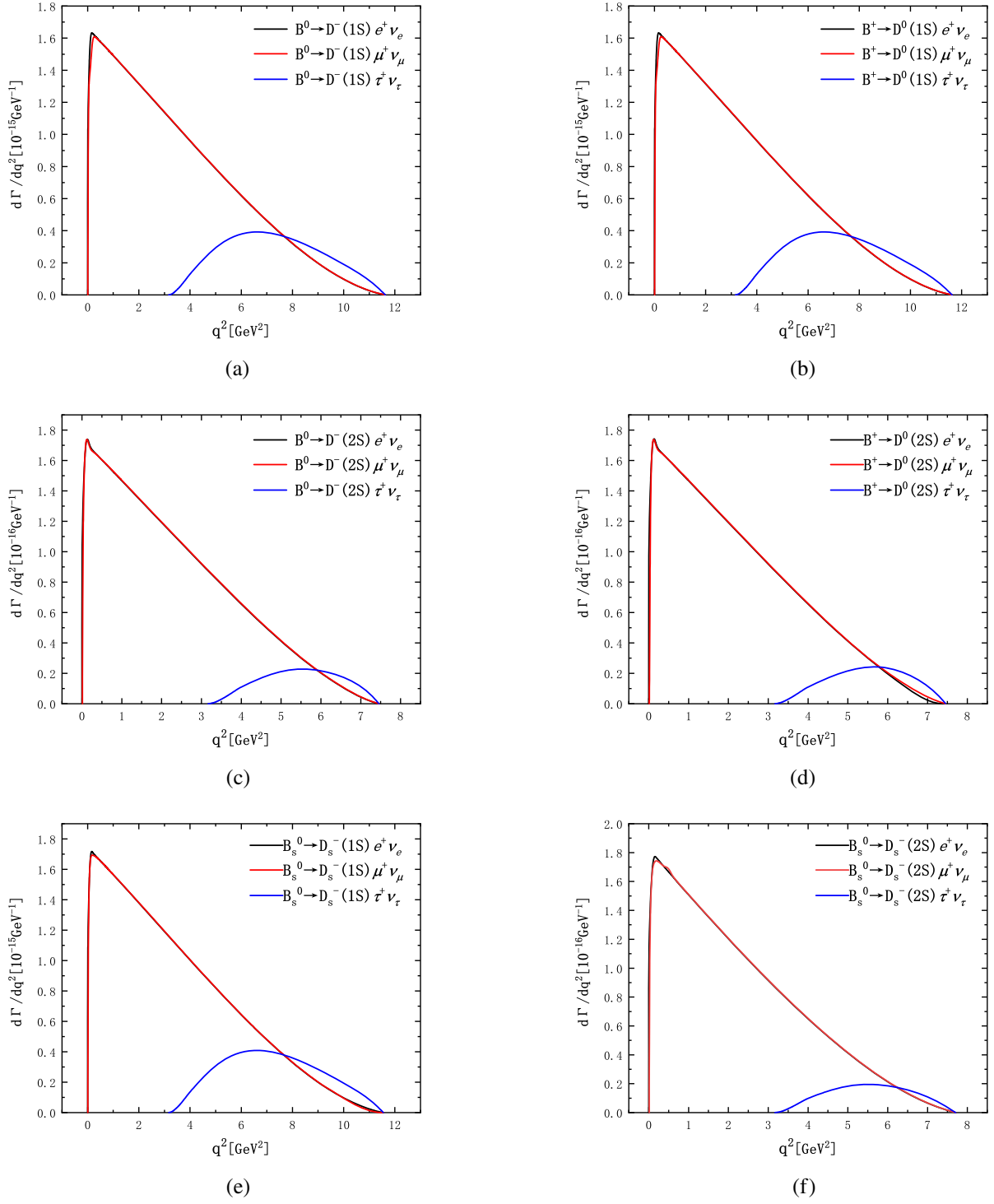


FIG. 5. The q^2 dependence of the differential decay rates $d\Gamma/dq^2$ for the decay $B^0 \rightarrow D^-(1S)\ell^+\nu_\ell$ (a), $B^+ \rightarrow \bar{D}^0(1S)\ell^+\nu_\ell$ (b), $B^0 \rightarrow D^-(2S)\ell^+\nu_\ell$ (c), $B^+ \rightarrow \bar{D}^0(2S)\ell^+\nu_\ell$ (d), $B_s^0 \rightarrow D_s^-(1S)\ell^+\nu_\ell$ (e), and $B_s^0 \rightarrow D_s^-(2S)\ell^+\nu_\ell$ (f).

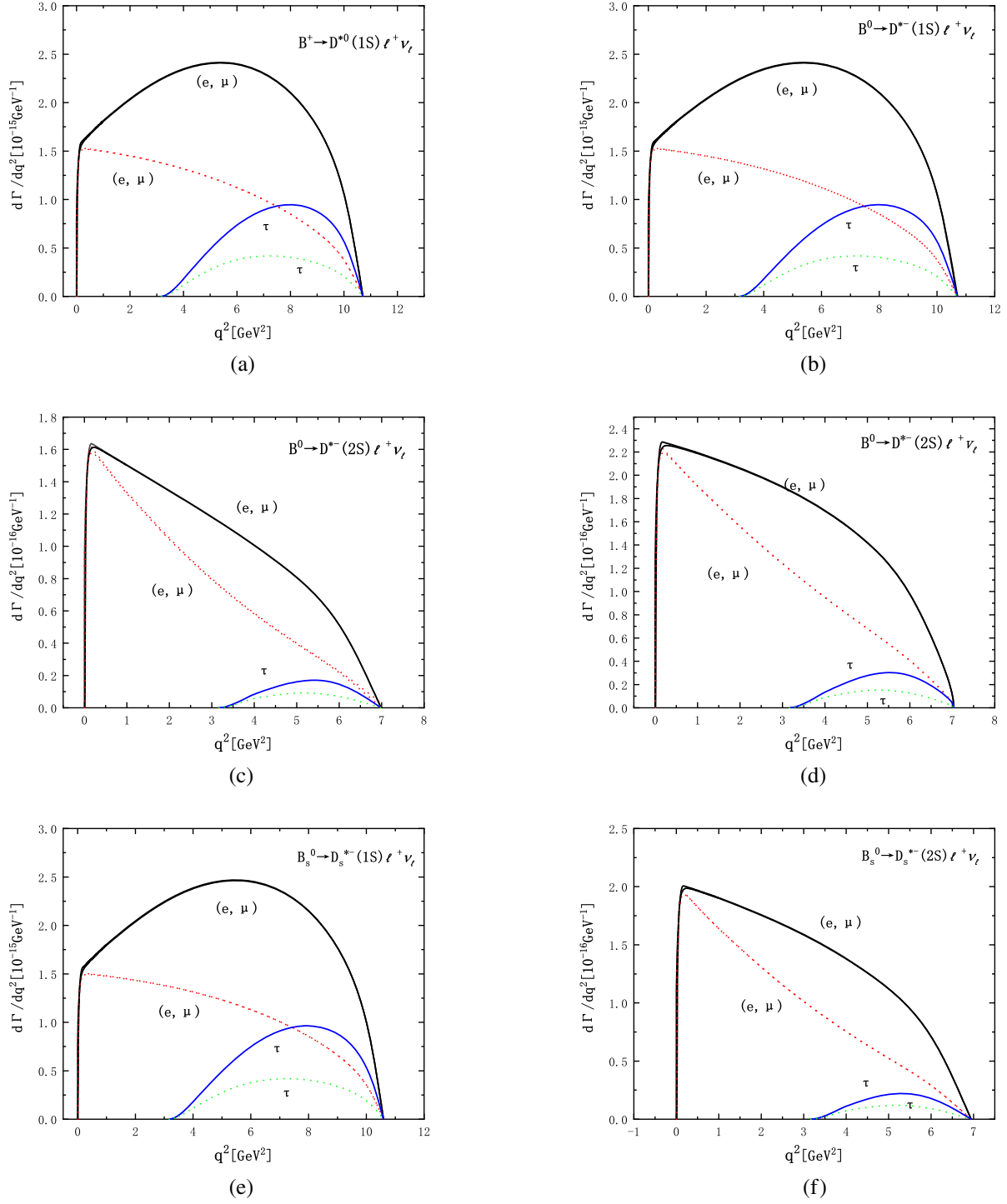


FIG. 6. The q^2 dependence of the differential decay rates $d\Gamma/dq^2$ (solid lines) and $d\Gamma^L/dq^2$ (the dashed lines refer to $\ell^i\nu_{\ell^i}$ mode, and the dotted lines represent for $\tau\nu_\tau$ mode) for the decays $B^+ \rightarrow \bar{D}^{*0}(1S)\ell^+\nu_\ell$ (a), $B^0 \rightarrow D^{*-}(1S)\ell^+\nu_\ell$ (b), $B^+ \rightarrow \bar{D}^{*0}(2S)\ell^+\nu_\ell$ (c), $B^0 \rightarrow D^{*-}(2S)\ell^+\nu_\ell$ (d), $B_s^0 \rightarrow D_s^{*-}(1S)\ell^+\nu_\ell$ (e), and $B_s^0 \rightarrow D_s^{*-}(2S)\ell^+\nu_\ell$ (f).

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