

Searching for leptophilic composite asymmetric dark sector at e^+e^- colliders

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Composite asymmetric dark matter (ADM) models provide a well-motivated paradigm that simultaneously explains dark matter (DM) relic density and matter-antimatter asymmetry. In these models, the mass of the DM candidate (the lightest dark baryon) is generated through the dark confinement scale dynamics. Although the leptophilic composite ADM model offers a viable framework, comprehensive studies of its collider phenomenology are absent. This work systematically explores novel signatures from the leptophilic composite asymmetric dark sector at both low-energy and high-energy e^+e^- colliders as well as other existing collider constraints. We demonstrate detectability of TeV-scale mediators along with sub-GeV to GeV-scale lightest dark mesons at Belle II and its proposed far detector, GAZELLE, as well as Circular Electron Positron Collider/Future Circular Collider-electron-positron experiments. Moreover, these experiments exhibit complementary coverage of the model parameter space.

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I. INTRODUCTION

The particle nature of dark matter (DM) and the origin of matter-antimatter asymmetry remain outstanding challenges in particle physics, astrophysics, and cosmology. The proposed DM models span mass scales from ultralight to tens of solar masses [1–4], with well-motivated candidates including weakly interacting massive particles (WIMPs) [5,6], strongly interacting massive particles [7], hidden sector DM [11], WIMPLESS DM [8], asymmetric DM (ADM) [9,10], freeze-in DM [11], QCD axions [12], and primordial black holes [13]. Despite extensive searches, no conclusive evidence supports any candidate to date. Consequently, diverse experimental strategies have been developed to probe their properties.

Dynamically generating the observed matter-antimatter asymmetry requires satisfying Sakharov's three conditions [14]: (1) baryon number violation, (2) C/CP violation, and (3) departure from thermal equilibrium. Successful mechanisms include electroweak baryogenesis [15], leptogenesis [16], and the Affleck-Dine mechanism [17]. The observed DM-to-baryon density ratio $\Omega_{\text{DM}}/\Omega_{\text{B}} \approx 5$ suggests a common origin for both abundances. The ADM paradigm

[9,10] achieves this by positing that DM abundance, like baryonic abundance, arises from particle-antiparticle asymmetry generated by shared physics connecting dark and standard model (SM) sectors.

Composite ADM [18–31] represents a prominent scenario in which the dark sector features a hidden $SU(N)$ gauge symmetry analogous to SM QCD. In such models, accidental dark baryon number conservation stabilizes the lightest dark baryon, making it a natural DM candidate. Crucially, the DM mass is directly related to the dark confinement scale Λ_d , thereby being dynamically determined rather than a free parameter. Mediators facilitate dark sector and SM sector coupling to generate shared particle-antiparticle asymmetry. We adopt the framework in Refs. [23,28], where bi-charged scalar mediator decays produce asymmetries in both sectors.

Two bi-charged scalar mediators exist in the literature: baryophilic [23] and leptophilic [28] types, both capable of simultaneously explaining DM relic density and matter-antimatter asymmetry. The baryophilic mediator carries charges under both hidden $SU(N)$ and SM QCD symmetries, coupling to a dark quark and an SM quark. This enables copious dark quark production at the LHC where, given that the dark quark mass is much smaller than the collision energy, the produced dark quarks become energetic and highly boosted. When the scale of Λ_d is also much smaller than the collision energy, subsequent QCD-like showering and hadronization generate distinctive dark jets that facilitate novel search

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strategies at the LHC [32–42]. In contrast, the leptophilic mediator is coupled with a dark quark and an SM lepton, suppressing LHC production rates, as noted in [28]. Therefore, lepton colliders consequently provide ideal environments for probing such models. Although Ref. [28] proposed preliminary studies, comprehensive analyses remain lacking. This work, therefore, systematically explores leptophilic composite ADM signatures on low-energy and high-energy e^+e^- colliders.

Collider searches for leptophilic scalar mediators resemble searches of slepton in supersymmetry models when dark hadrons inside dark jets escape detection at the LHC and Large Electron-Positron Collider (LEP) [43–48]. We, therefore, rescale existing slepton constraints to these mediators. While collision energies exceeding the dark confinement scale Λ_d can produce dark showers, the intensity of showering depends critically on energy: higher energies generate denser parton showers. At low-energy e^+e^- colliders such as B factories, sparse dark showers enable tracking of individual displaced dark pions and kinematic reconstruction. Similar approaches have been studied for the dark photon mediator [49], though signatures differ fundamentally from this work. Conversely, at high-energy e^+e^- colliders [Circular Electron Positron Collider (CEPC), Future Circular Collider-electron-positron (FCC-ee), and International Linear Collider (ILC)], dense dark showers necessitate identification of displaced, collimated lepton jets rather than individual dark pions [28]. When dark jets become undetectable, monophoton signatures [50] become relevant. This work establishes complementary coverage of parameter space through these distinct search strategies.

This paper is structured as follows: Sec. II reviews simplified leptophilic composite ADM models. Section III examines existing collider constraints. Novel dark shower signatures at B factories and Higgs factories are explored in Secs. IV and V, respectively. Section VI presents our conclusions and future research directions.

II. A BRIEF REVIEW OF QCD-LIKE DARK SECTOR WITH BI-CHARGED MEDIATORS

In the leptophilic composite ADM models proposed in Ref. [28], new heavy particles, including Majorana fermions and $SU(3)_D$ charged Dirac fermions, generate out-of-equilibrium processes and explicitly break hidden baryon number conservation. However, these particles lie significantly above current collider energy scales and are consequently decoupled from our analysis. We, therefore, restrict the Lagrangian to the relevant scalar mediator X and dark quark q_d interactions as follows¹:

¹The possible Higgs portal coupling between the SM Higgs doublet field and the X field is irrelevant to this study. Therefore, we assume that its contributions are negligible and can be neglected here.

$$\mathcal{L} \supset \overline{q_d}(\not{D} - m_{q_d})q_d + (D_\mu X)^\dagger(D^\mu X) - m_X^2 X^\dagger X - \frac{1}{4}G^{d,\mu\nu}G_{\mu\nu}^d - (\kappa X \overline{q_{dL}} l_R + \text{H.c.}), \quad (1)$$

where the covariant derivative $D_\mu = \partial_\mu - ig_d G_\mu^d$ is defined with g_d and G_μ^d , representing the $SU(3)_D$ gauge coupling and dark gluon field, respectively. The field strength tensor $G_{\mu\nu}^d$ corresponds to the $SU(3)_D$ gauge field G_μ^d , while m_{q_d} and m_X denote the dark quark and mediator masses. The coupling matrix κ (a $n \times 3$ matrix) corresponds to interactions between n -flavor dark quarks and three-flavor charged leptons. Note that the mediator X plays a crucial role in generating asymmetries in both SM and dark sectors through its decays, as it carries both SM lepton number and dark baryon number.

At lepton and hadron colliders, dark quarks are produced through t -channel processes and/or mediator decays, as shown in Fig. 1. Postproduction, they acquire significant kinetic energy ($E \gg \Lambda_d$), where Λ_d denotes the dark confinement scale. Assuming the dark QCD sector mirrors SM QCD dynamics, these dark quarks undergo dark parton shower evolution and hadronization, forming a collinear bunch of dark mesons.

In subsequent sections, we adopt the parameter settings for the QCD-like dark sector from Refs. [32,37,39,40] to enable concrete simulation predictions. We assume three dark colors ($N_{C_d} = 3$) and seven dark flavors ($N_{f_d} = 7$), with the following mass relations: $\Lambda_d = m_{q_d} = 2m_{\pi_d}$, and $m_{\rho_d} = 4m_{\pi_d}$. Note that the dark confinement scale Λ_d is closely related to the mass of the lightest dark baryon, which serves as the DM candidate [23,28,32,36,37]. All dark quarks are taken to be mass-degenerate, while dark pions and dark rho mesons are similarly degenerate in both flavor diagonal and off-diagonal states. Other dark baryons and mesons, assumed substantially heavier than dark pions and rho mesons, exhibit negligible production rates from dark showers and hadronization due to mass suppression. We, therefore, exclude them from this analysis.

Integrating out the heavy mediator [final terms in Eq. (1)] yields a dimension-six operator,

$$\mathcal{L}_{\text{dim-6}} \supset \frac{\kappa^2}{m_X^2} (\overline{q_{dL}} \ell_R) (\overline{\ell_R} q_{dL}), \quad (2)$$

which enables dark pion (π_d) decays to charged lepton pairs. The decay width is [32]

$$\Gamma(\pi_d \rightarrow \overline{\ell} \ell) = \frac{\kappa^4 f_{\pi_d}^2 m_\ell^2 m_{\pi_d}}{32\pi m_X^4} \sqrt{1 - \frac{4m_\ell^2}{m_{\pi_d}^2}}, \quad \text{with} \quad \ell = e, \mu, \tau. \quad (3)$$

Here, f_{π_d} is the dark pion decay constant and m_ℓ is the lepton mass. Parametrizing the proper lifetime as

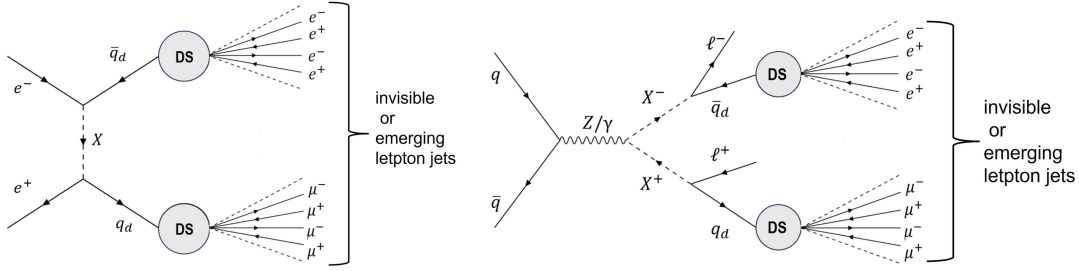


FIG. 1. Diagrams illustrating dark shower production evolving into invisible or emerging lepton jets. Left panel: e^+e^- annihilation via t -channel mediator X producing a dark quark pair ($q_d\bar{q}_d$). Right panel: $q\bar{q}$ annihilation yielding mediator pairs, where each mediator decays to a dark quark and a SM lepton.

$$c\tau_0 = \frac{c\hbar}{\Gamma_{\pi_d}} \approx 1.92 \text{ m} \times \left(\frac{1}{\kappa^4}\right) \left(\frac{1 \text{ GeV}}{f_{\pi_d}}\right)^2 \left(\frac{0.1 \text{ GeV}}{m_\ell}\right)^2 \times \left(\frac{1 \text{ GeV}}{m_{\pi_d}}\right) \left(\frac{m_X}{1 \text{ TeV}}\right)^4 \left(1 - \frac{4m_\ell^2}{m_{\pi_d}^2}\right)^{-\frac{1}{2}} \quad (4)$$

reveals that dark pions propagate measurable distances from the primary vertex before decaying into charged lepton pairs.

To illustrate the typical proper decay length of π_d versus m_{π_d} , we fix $\kappa = 1.0$, $f_{\pi_d} = m_{\pi_d}$, and $m_X = 1 \text{ TeV}$ as benchmark values. In order to simplify our discussions here, the lepton flavor universality (LFU) is assumed such that the common coupling κ applies to the interactions of a mediator with a dark quark and a SM charged lepton for each lepton flavor. Figure 2 shows that for $2m_e < m_{\pi_d} < 2m_\mu$, π_d decays to electron-positron pairs. The proper decay length exceeds 10^7 m , allowing electrons and positrons to escape detection. When $2m_\mu < m_{\pi_d} < 2m_\tau$, the muon decay channel opens, with π_d predominantly decaying into muon pairs. The resulting decay length $\sim 1 \text{ m}$ makes these long-lived π_d detectable. Finally, for $m_{\pi_d} > 2m_\tau$, the tau channel becomes

accessible, and π_d primarily decays to tau pairs, potentially exhibiting prompt decays.

III. EXISTING COLLIDER CONSTRAINTS ON MODEL PARAMETERS

Figure 2 reveals a dramatic change in the proper decay length of π_d at the $m_{\pi_d} \geq 2m_\mu$ threshold. We, therefore, classify two dark pion mass ranges: $10 \text{ MeV} \leq m_{\pi_d} < 2m_\mu$ and $2m_\mu \leq m_{\pi_d} < 2m_\tau$, and discuss their respective existing collider constraints.

First, we consider the production of scalar mediator pairs through $pp \rightarrow X^+X^-$, with decays $X^+ \rightarrow \ell^+q_d$ and $X^- \rightarrow \ell^- \bar{q}_d$ ($\ell = e, \mu, \tau$). Here, κ in Eq. (1) affects only the $X^\pm$ lifetime, not the production cross section. When $X^\pm$ decays promptly and dark parton showers are invisible at ATLAS and CMS detectors, signatures resemble slepton and stau searches in supersymmetry models [43,47]. Under the LFU assumption, we recast these constraints onto this model. We generate a Universal FeynRules Output (UFO) model file for Eq. (1) via FeynRules [51] and simulate signal processes with MadGraph5_aMC@NLO [52] (version 3.5.8). For each m_X , we scan κ and define the mediator prompt decay region by $\gamma\beta c\tau_X < 0.1 \text{ mm}$ in the lab frame. Figures 3 and 4 show regions above the red dashed line [$\kappa \gtrsim \mathcal{O}(10^{-7}-10^{-6})$] for the mediator prompt decays recast from slepton and stau constraints.² Additionally, because there are plenty of dark pions produced after dark showers and hadronization, we require the π_d proper decay length (without any boost) larger than 10 m to conservatively assume all dark pions escape ATLAS and CMS detectors as missing energy. Specifically, we set $m_{\pi_d} = f_{\pi_d} = 0.2 \text{ GeV}$ in Fig. 3 and $m_{\pi_d} = f_{\pi_d} = 3 \text{ GeV}$ in Fig. 4 to estimate the upper bounds of κ . Regions below blue dashed lines [$\kappa \lesssim \mathcal{O}(1-10)$ for $10 \text{ MeV} \leq m_{\pi_d} < 2m_\mu$; $\kappa \lesssim \mathcal{O}(0.1-1)$ for $2m_\mu \leq m_{\pi_d} < 2m_\tau$] correspond to invisible dark showers, which are adapted for recasting.

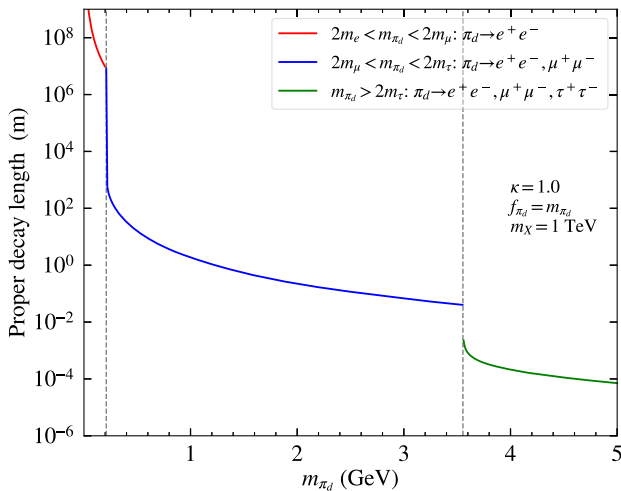


FIG. 2. The dark pion proper decay length with respect to m_{π_d} . Here, we fixed the parameters $\kappa = 1.0$, $f_{\pi_d} = m_{\pi_d}$, and $m_X = 1 \text{ TeV}$.

²The signal production cross sections rescaled to match Fig. 3 of Ref. [28] with QCD next-to-leading order + next-to-leading logarithmic corrections at the 13 TeV LHC [53].

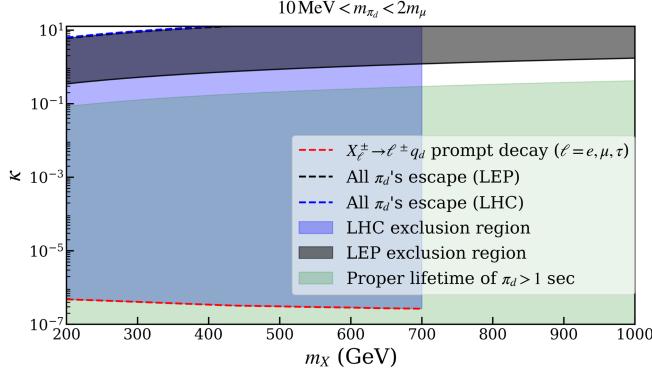


FIG. 3. Exclusion region for dark pion masses $10 \text{ MeV} \leq m_{\pi_d} < 2m_\mu$. The red dashed line separates long-lived (below) and prompt (above) $X^\pm$ decays. The blue and black dashed lines indicate where all π_d particles escape detection at the LHC and LEP, respectively. The blue and black shaded regions correspond to slepton search constraints (LHC) [43] and monophoton search constraints (LEP) [48,50]. The green region represents the parameter space where the proper lifetime of π_d exceeds one second, conservatively evaluated at $m_{\pi_d} = 0.2 \text{ GeV}$.

Under the LFU assumption, the mediator $X^\pm$ has approximately equal decay branching ratios of $1/3$ to each flavor channel ($e^\pm q_d$, $\mu^\pm q_d$, and $\tau^\pm q_d$). Consequently, we include an extra factor of $1/9$ when applying stau pair production constraints. Comparison with ATLAS results from Table 1 of auxiliary material in Ref. [47] shows that for $m_X > 200 \text{ GeV}$ and the dark quark mass $m_{q_d} = 1 \text{ GeV}$, the effective cross section is at least a factor of 2 below the experimental bounds. Thus, no exclusion region exists for $m_X > 200 \text{ GeV}$, though stau constraints exclude $m_X \lesssim 160 \text{ GeV}$.

For slepton pair production constraints in Fig. 3(c) of Ref. [43], the analysis involves all combinations of a pair of leptons from electrons and muons in the final states. Therefore, we apply a factor of $4/9$ in our process. Notably, while the constraints include contributions from both $\tilde{\ell}_L^+ \tilde{\ell}_L^-$ and $\tilde{\ell}_R^+ \tilde{\ell}_R^-$ production, our signal process specifically corresponds to the $\tilde{\ell}_R^+ \tilde{\ell}_R^-$ channel. We, therefore, isolate the right-handed slepton constraints using the fraction of $pp \rightarrow \tilde{\ell}_L^+ \tilde{\ell}_L^-$ and $pp \rightarrow \tilde{\ell}_R^+ \tilde{\ell}_R^-$ production cross sections from Refs. [53,54]. The resulting effective cross section in our process exceeds ATLAS exclusion limits for $\tilde{\ell}_R^+ \tilde{\ell}_R^-$ in the mass range $m_X \in [200, 700] \text{ GeV}$. Additionally, after accounting for the average Lorentz boost of the mediator ($\gamma\beta \simeq 1.5$), we estimate the prompt decay boundaries within this mass range.

Conversely, for π_d with moderate decay lengths, the novel signatures of displaced lepton jets are expected to be observable at the LHC. However, Ref. [28] notes detection challenges for the $pp \rightarrow X^+ X^-$ production channel due to (1) insufficient production cross sections and (2) detection efficiencies after considering the whole cut-flow below ~ 0.1 . Moreover, significant irreducible cosmic-ray backgrounds

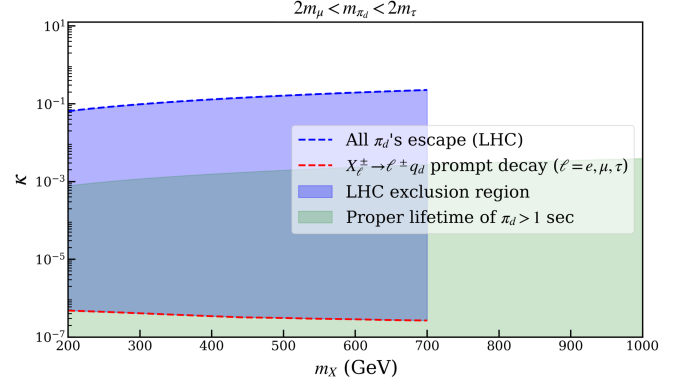


FIG. 4. Exclusion region for dark pion masses $2m_\mu < m_{\pi_d} < 2m_\tau$. The blue dashed line indicates where all π_d particles escape detection at the LHC, making this region subject to slepton search constraints. The red dashed line retains the same interpretation as in Fig. 3. The green region represents the parameter space where the proper lifetime of π_d exceeds one second, conservatively evaluated at $m_{\pi_d} = 3 \text{ GeV}$.

further complicate the observation. While LHC Run 3 and HL-LHC may improve sensitivity, current ATLAS displaced lepton jet searches [55] exclusively target Higgs exotic decays, distinct from this model's signature of two opposite-sign leptons plus two displaced lepton jets. Consequently, recasting strategies from Ref. [55] are inapplicable. A dedicated analysis (beyond the scope of this work) would be required, though we establish that the LHC constraints remain weaker than those derived for e^+e^- colliders in Secs. IV and V.

Dark quark pairs can also be produced via t -channel mediator exchange at the LEP. Following dark showers and hadronization, if all dark mesons escape the detector, they contribute to missing energy. With initial-state radiation, this also yields a monophoton signature. For $10 \text{ MeV} \leq m_{\pi_d} < 2m_\mu$, we conservatively set $m_{\pi_d} = f_{\pi_d} = 0.2 \text{ GeV}$ and require π_d proper decay length greater than 12 m to ensure that all dark mesons escape the detector volume of LEP. The black dashed line in Fig. 3 gives upper bounds $\kappa \sim \mathcal{O}(1-10)$. We further recast monophoton constraints at the LEP from the t -channel scalar mediator and Dirac fermionic DM model with the cutoff scale Λ [Eq. (4)] and the purple line in Fig. 4 ($\bar{\chi} e \bar{e} \chi$) of Ref. [48] to the leptophilic composite ADM via

$$\frac{1}{\Lambda^2} \simeq \frac{1}{(333.86 \text{ GeV})^2} = 3 \frac{\kappa^2}{m_X^2}, \quad (5)$$

where the factor of 3 originates from $SU(3)_D$ charges. Assuming light dark quarks and determining lower bounds of Λ , the black regions in Fig. 3 indicate where predicted cross sections exceed experimental bounds, consistent with the $1/\Lambda^2$ scaling.

Finally, a dark pion lifetime exceeding one second would violate observed light-element abundances and the cosmic

microwave background (CMB) spectrum, as constrained by big bang nucleosynthesis and CMB measurements [56,57]. We, therefore, incorporate these constraints by excluding regions where the proper lifetime of π_d is greater than one second, indicated by the green shaded areas in Figs. 3 and 4. Notably, only a narrow region of parameter space remains in Fig. 3 for $m_X \lesssim 1$ TeV.

Although portions of the parameter space are excluded in Figs. 3 and 4, significant regions remain accessible for exploration at both low-energy and high-energy e^+e^- colliders. In subsequent sections, we derive exclusion limits for three benchmark points (BPs): $m_{\pi_d} = 0.32, 0.7$, and 1 GeV.

IV. SPARSE DARK SHOWERS AT *BABAR*, *BELLE II*, AND *GAZELLE*

As previously discussed, the intensity of dark showers depends critically on the energy of the produced dark quarks. This section focuses on low-energy e^+e^- colliders capable of generating sparse dark showers, where individual displaced vertices from long-lived dark pion decays can be resolved. We specifically examine the *BABAR* and Belle II experiments, along with the proposed GAZELLE far detector for Belle II.

In this and the following section, we employ FeynRules [51] to generate a UFO model file for Eq. (1), and then parton-level Monte Carlo events are generated using MadGraph5_aMC@NLO [52]. Subsequently, the Hidden Valley (HV) module within PYTHIA8.313 [58] simulates dark parton showering and hadronization processes. Parameter settings closely follow Refs. [32,37,39,40], specifically, $N_{C_d} = 3$, $N_{f_d} = 7$, $\Lambda_d = m_{q_d} = 2m_{\pi_d}$, and $m_{\rho_d} = 4m_{\pi_d}$. In our simulation, diagonal and off-diagonal HV-mesons share identical decay patterns. We implement the running coupling α_d and apply a p_T cutoff of $1.1\Lambda_d$ to dark showers. The fraction of vector and pseudoscalar HV-mesons is calculated by following Appendix A of Ref. [59]. Finally, the ratio of dark pion mass over dark pion decay constant, m_{π_d}/f_{π_d} , can range from $\mathcal{O}(1-10)$ [60–63]. For this analysis, we fix $f_{\pi_d} = m_{\pi_d}$. This choice only affects the dark pion decay width, which can be easily rescaled using Eq. (3).

A. Recasting LLP searches at the *BABAR*

The *BABAR* experiment utilized the PEP-II collider, colliding 9 GeV electrons with 3.1 GeV positrons at a center-of-mass energy $\sqrt{s} = 10.58$ GeV. Dark quark pairs are produced via the t -channel process (Fig. 1, left panel).

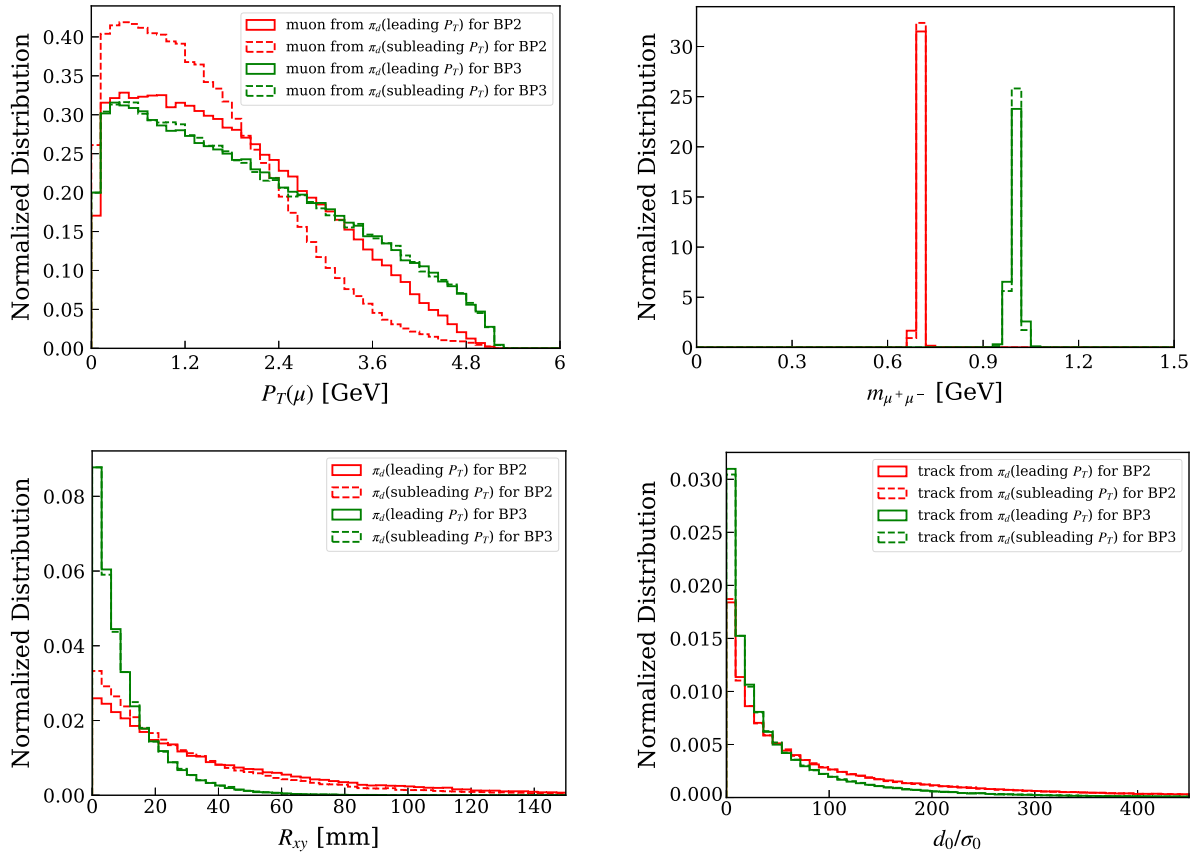


FIG. 5. Kinematic distributions for the signal benchmark points BP2 and BP3 at the *BABAR*. The distributions of the leading p_T and subleading p_T dark pions and their daughter muons are represented by solid and dashed lines, respectively. Top left: transverse momentum p_T of muons from dark pion decays. Top right: invariant mass $m_{\mu^+\mu^-}$ of muon pairs. Bottom left: transverse decay distance R_{xy} of dark pions. Bottom right: track impact parameter significance d_0/σ_0 .

Following dark showering and hadronization, long-lived dark pions decay to muon pairs, creating multiple displaced vertices. Such novel signature is an icon of dark showers.

To recast the previous *BABAR* long-lived particle (LLP) searches to this model, we implement the event selection criteria from Refs. [49,64]:

- (i) The transverse displacement of π_d is required to be within the range $10 \text{ mm} < R_{xy} < 500 \text{ mm}$ from the interaction point (IP).
- (ii) The final state must include at least three charged tracks with $p_T > 0.12 \text{ GeV}$, among which at least two of them must have $p_T > 0.18 \text{ GeV}$.
- (iii) The invariant mass of the decay products must satisfy either $m_{\mu^+\mu^-} < 0.37 \text{ GeV}$ or $m_{\mu^+\mu^-} > 0.5 \text{ GeV}$.
- (iv) Each muon is required to have $d_0/\sigma_{d_0} > 3$, where d_0 is the transverse impact parameter and σ_{d_0} is its uncertainty.

The detector resolution is performed by the smearing effect of the relevant uncertainties of observables. Since recasting efficiencies are only available for LLPs with $m > 0.5 \text{ GeV}$ and $c\tau > 5 \text{ mm}$, as noted in Refs. [49,64], we restrict the analysis to two of our BPs: $m_{\pi_d} = 0.7 \text{ GeV}$ and 1 GeV , requiring $c\tau > 5 \text{ mm}$ for dark pions.

Specifically, to calculate the impact parameter d_0 , we first select the long-lived dark pion candidate and its daughter muons. We record their respective three-momenta and compute the sine of the angle α between them. The

transverse displacement is defined as $R_{xy} = \sqrt{R_x^2 + R_y^2}$, where R is the flight distance of the dark pion in the lab frame, and R_x (R_y) is the x (y) component of R . The impact parameter is then given by $d_0 = R_{xy} \sin \alpha$. The position resolution of displaced vertices and the momentum resolution of the final-state particles are given by [65]

$$\sigma_{xy} = \sigma_z = \left[\frac{50 \text{ GeV}}{p_T} + 15 \right] \mu\text{m}, \quad (6)$$

$$\frac{\sigma_{p_T}}{p_T} = \left[0.21 + 0.14 \frac{p_T}{\text{GeV}} \right] \%. \quad (7)$$

Here, p_T denotes the transverse momentum of the charged track. In the calculation of d_0 , we use σ_{xy} to account for the detector resolution in the transverse plane.

Figure 5 shows the relevant kinematic distributions of these two signal BPs. First, the transverse momentum distribution of the muon, $P_T(\mu)$ (top left), is generally larger for heavier π_d masses. This is because, under a lower total π_d yield, each π_d carries higher average energy, leading to muons from its decay with larger p_T . Second, in the $\mu^+\mu^-$ invariant mass distribution, $m_{\mu^+\mu^-}$ (top right), the position of the reconstructed mass peak directly reflects the mass of π_d . Regarding the transverse decay length, R_{xy} (bottom left), since π_d is long-lived, its decay vertex is

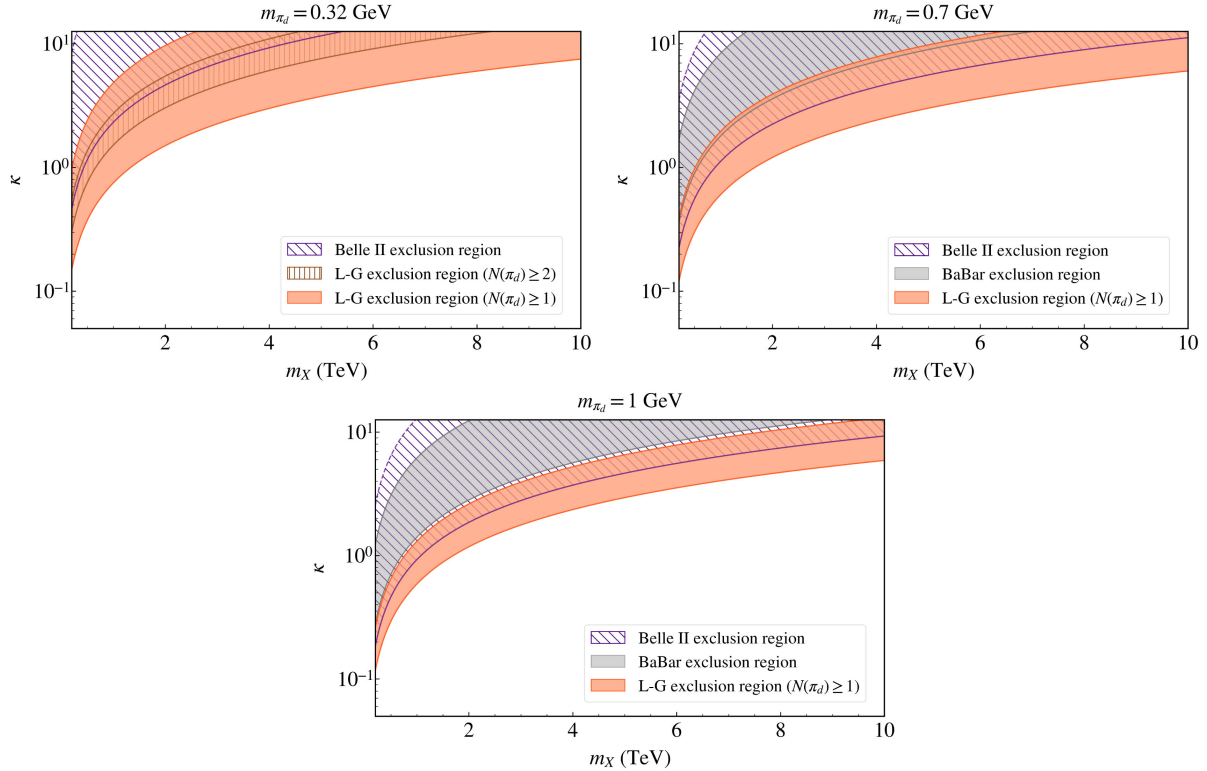


FIG. 6. Exclusion limits for three benchmark points, $m_{\pi_d} = 0.32, 0.7$, and 1 GeV , for three experiments: *BABAR*, Belle II, and L-GAZELLE.

significantly displaced from the IP in the transverse plane. Moreover, due to the proper decay length $c\tau_0 \propto 1/m_{\pi_d}^3$, a lighter π_d has a longer decay length, resulting in an R_{xy} distribution that extends to larger values and exhibits a more pronounced long tail for BP2 compared to BP3. Finally, the transverse impact parameter d_0 (bottom right) further characterizes the long-lived nature of π_d . The value of d_0 is significantly larger than the uncertainty σ_0 , which clearly distinguishes π_d decays from prompt decays.

The production cross section for $e^+e^- \rightarrow \bar{q}_d q_d$ scales as $(\kappa/m_X)^4$, while the proper lifetime of π_d is inversely proportional to this factor. We, therefore, recast *BABAR* LLP search constraints onto the (m_X, κ) parameter plane. Specifically, we extract exclusion limits from the $\mu^+\mu^-$ channel (red curve in Fig. 2 of Ref. [64]), plotting LLP mass versus $\sigma(e^+e^- \rightarrow LX)\mathcal{B}(L \rightarrow f)\epsilon(f)$, where $\sigma(e^+e^- \rightarrow LX)$ is the LLP production cross section (L is the LLP candidate, and X are associated particles), $\mathcal{B}(L \rightarrow f)$ is the branching fraction to final state of a pair of f particles (a muon pair in our case), and $\epsilon(f)$ is the detection efficiency to such final state. For $m_{\pi_d} = 0.7$ GeV and 1 GeV, exclusion limits appear in gray regions of Fig. 6. The upper boundaries correspond to short dark pion lifetimes that preclude LLP classification, while lower boundaries indicate long-lived pions escaping the detector's sensitive volume. The inverse relationship of constraints between m_X and κ is clearly evident in Fig. 6.

B. Detecting dark showers at the Belle II

The Belle II experiment operates at the SuperKEKB collider, which collides 7 GeV electrons with 4 GeV positrons at a center-of-mass energy of $\sqrt{s} = 10.58$ GeV. Dark quark pairs are produced via the process $e^+e^- \rightarrow \bar{q}_d q_d$, followed by dark showers and hadronization, as

TABLE I. The average dark pion π_d multiplicity, the average energy of each π_d , and the average transverse momentum of each muon in the final state for three BPs with $m_{\pi_d} = 0.32, 0.7$, and 1 GeV at the Belle II.

m_{π_d} (GeV)	$\langle N_{\pi_d} \rangle$	$\langle E(\pi_d) \rangle$ (GeV)	$\langle P_T(\mu) \rangle$ (GeV)
0.32	6.20	1.78	0.63
0.7	3.15	3.46	1.26
1	2.03	5.43	1.96

illustrated in the left panel of Fig. 1. This leads to the production of several long-lived dark pions, which predominantly decay into muon pairs. As a result, multiple displaced vertices can be identified as the novel signal signature. Three BPs with $m_{\pi_d} = 0.32, 0.7$, and 1 GeV are chosen for illustration, corresponding to $\kappa = 1.0$ and $m_X = 200$ GeV.

In all BPs, the average number of π_d per event exceeds two, as shown in Table I. Thus, we require at least two π_d 's per event for the signal process in subsequent analyses. All events must satisfy the Belle II trigger selections outlined in Table II and then pass the event selection criteria outlined in Table III [49]. Moreover, key kinematic properties for each BP are summarized in Table I, including the average π_d multiplicity, the average energy of each π_d , and the average transverse momentum of each muon in the final state.

In this study, the trigger strategy at the Belle II relies on three categories, as follows:

- (i) Two tracks trigger: an event passes this trigger if it contains two muon tracks, each with $p_T > 0.3$ GeV, and the azimuthal opening angle between them at the IP in the lab frame satisfies $\Delta\phi > 90^\circ$. In addition, the angle between the position vector of decay vertex and the beam axis in the lab frame must satisfy $38^\circ < \theta_{\text{lab}} < 127^\circ$.
- (ii) One muon trigger: an event passes this trigger if it contains at least one muon with $p_T > 0.9$ GeV, and $38^\circ < \theta_{\text{lab}} < 127^\circ$.
- (iii) Displaced vertex trigger: this trigger is satisfied when the event contains at least one LLP (dark pion) with a transverse decay length $9 \text{ mm} < R_{xy} < 600 \text{ mm}$, which decays into two muons, each with $p_T > 0.1$ GeV, and $38^\circ < \theta_{\text{lab}} < 127^\circ$.

We further apply the event selection criteria summarized in Table III to identify the signal events and suppress the background ones. An event is selected if at least two leading p_T π_d particles satisfy the following conditions: $0.2 \text{ cm} < R_{xy} < 60 \text{ cm}$, $-55 \text{ cm} \leq z \leq 140 \text{ cm}$, and $17^\circ \leq \theta_{\text{lab}} \leq 150^\circ$. Additionally, their daughter muons must satisfy $p_T > 0.05$ GeV, and the reconstructed invariant mass of the muon pair must fulfill $m_{\mu^+\mu^-} < 0.48$ GeV or $m_{\mu^+\mu^-} > 0.52$ GeV. In this analysis, we require at least two muon pairs with significantly displaced vertices, which effectively suppresses backgrounds from punch-through pions. For decay vertices with $R_{xy} > 0.2 \text{ cm}$, the significant displacement from the IP eliminates photon conversion

TABLE II. Three Belle II triggers are considered in this analysis: two tracks, one muon, and a displaced vertex, adapted from Ref. [49].

Trigger	Conditions
Two tracks	Two tracks with $p_T > 0.3$ GeV for each and an azimuthal opening angle $\Delta\phi > 90^\circ$ at the interaction point, as well as the angle between the position vector of decay vertex and the beam axis with $38^\circ < \theta_{\text{lab}} < 127^\circ$ in the lab frame
One muon	One muon with $p_T > 0.9$ GeV and $38^\circ < \theta_{\text{lab}} < 127^\circ$
Displaced vertex	At least one displaced vertex in the event with $9 \text{ mm} < R < 600 \text{ mm}$, formed from two tracks with $p_T > 0.1$ GeV and $38^\circ < \theta_{\text{lab}} < 127^\circ$

TABLE III. The event selection criteria for muons and displaced vertices in LLP searches at the Belle II [49].

Object	Selection criteria
Muons	$p_T(\mu^+), p_T(\mu^-) > 0.05$ GeV $m_{\mu^+\mu^-} < 0.48$ GeV or $m_{\mu^+\mu^-} > 0.52$ GeV
Displaced vertices	$0.2 \text{ cm} < R_{xy} < 60 \text{ cm}$ $-55 \text{ cm} \leq z \leq 140 \text{ cm}$ $17^\circ \leq \theta_{\text{lab}} \leq 150^\circ$

TABLE IV. Trigger efficiencies for three different dark pion masses with fixed $\kappa = 1.0, m_X = 200$ GeV.

Trigger type	$m_{\pi_d} = 0.32$ GeV	$m_{\pi_d} = 0.70$ GeV	$m_{\pi_d} = 1.00$ GeV
Two tracks	75.82%	58.72%	34.16%
After cuts	17.78%	47.48%	25.31%
One muon	83.76%	89.22%	84.17%
After cuts	9.93%	65.01%	51.62%
Displaced vertex	91.24%	79.41%	46.24%
After cuts	11.39%	60.12%	35.16%

backgrounds. Furthermore, backgrounds originating from detector structures can be effectively rejected for nonelectron final states. All signal events are required to satisfy at least one of the triggers listed in Table II and the selection criteria outlined in Table III. Under these comprehensive selection criteria, the signal region can be treated as effectively background-free [49,66].

Table IV summarizes the efficiencies of three trigger types and after applying event selection criteria for three BPs. Overall, the efficiencies of all three triggers exhibit significant variations with m_{π_d} , and the relative ordering of efficiencies among the BPs changes considerably after the same selection is applied. Specifically, the efficiency of the two tracks trigger decreases as m_{π_d} increases, dropping from 75.82% for BP1 to 34.16% for BP3. This behavior can be understood from the distribution of θ_{lab} . For $m_{\pi_d} = 1$ GeV, the tracks from the leading and subleading $p_T \pi_d$'s are more dispersed in θ_{lab} distribution, with many falling outside the trigger acceptance region of $38^\circ \leq \theta_{\text{lab}} \leq 127^\circ$. In this case, the two selected tracks are very likely to originate from the same dark pion, and their opening angle $\Delta\phi$ rarely exceeds $\pi/2$, leading to the lowest efficiency. In contrast, for $m_{\pi_d} = 0.32$ and 0.7 GeV, the θ_{lab} distribution is more concentrated, and the higher π_d multiplicity results in higher efficiency. The one muon trigger maintains high efficiency across all three BPs, peaking at 89.22% for BP2. The efficiency of the displaced vertex trigger decreases with increasing m_{π_d} . This condition strongly depends on the decay length of π_d . The proper decay lengths for the three BPs are $c\tau_0 = 116.40$ mm (BP1), 8.80 mm (BP2), and 2.94 mm (BP3), respectively. The proper decay length of BP1 is an order of magnitude larger than that of BP2, and after multiplying by $\beta\gamma$ in the

lab frame, it often exceeds the upper limit of the trigger condition. However, since the proper decay length follows an exponential distribution in the Monte Carlo simulation, the high multiplicity of BP1 enhances the probability that at least one displaced vertex satisfies the trigger condition. In contrast, for $m_{\pi_d} = 1$ GeV, the R_{xy} distribution of the two leading $p_T \pi_d$'s is concentrated in a very small region, resulting in the fewest events satisfying the trigger condition. After applying the cuts in Table III, the order of efficiencies in these three cases may change. From the R_{xy} distribution of π_d 's, for $m_{\pi_d} = 0.32$ GeV, many events have decay lengths for both π_d 's exceeding 60 cm, leading to a significant reduction in efficiency. In comparison, for $m_{\pi_d} = 0.7$ GeV, most events fall within the required range, resulting in the highest efficiency.

We then analyze relevant signal kinematic distributions for π_d 's and muons at the Belle II in Figs. 7 and 8. First, as the m_{π_d} increases from BP1 to BP3, the N_{π_d} distribution (top left) decreases from approximately 6 to 2. Regarding the decay length, R_{xy} and R_z represent the transverse and longitudinal decay lengths of the dark pions in the lab frame, respectively. According to Eq. (4), under fixed parameters and with $f_{\pi_d} = m_{\pi_d}$, the proper decay length satisfies $c\tau_0 \propto 1/m_{\pi_d}^3$. Therefore, lighter π_d 's have longer decay lengths, and their R_{xy} (top right) and R_z (bottom left) distributions exhibit larger values and longer tails. Finally, the $\theta_{\text{lab}}(\text{track})$ distribution (bottom right) is significantly dispersed for the BP1, while the other two BPs exhibit relatively concentrated distributions. By comparing E and p_T of daughter particles, it can be seen that for BP1 and BP2, the leading and subleading $p_T \pi_d$'s have similar p_z values, leading to small differences in their $\Delta\theta_{\text{lab}}$ distributions. In contrast, for the BP3, although its p_T distribution is similar, the leading π_d carries higher energy, resulting in a larger p_z difference. The greater difference in boost further leads to a significant separation in their θ_{lab} distributions.

As shown in Fig. 8, the $p_T(\mu)$ distribution (top left) is similar to that in the BABAR analysis. In the $m_{\mu^+\mu^-}$ distribution (top right), the reconstructed mass peaks clearly appear near the respective m_{π_d} , 0.32 GeV, 0.7 GeV, and 1 GeV for BP1, BP2, and BP3, respectively, but exhibit different levels of broadening. The width of these distributions is influenced by momentum resolutions of muons [67,68],

$$\frac{\sigma_p}{p_\mu} = 0.0011 p_\mu [\text{GeV}] \oplus \frac{0.0025}{\beta}, \quad (8)$$

and the width of the $m_{\mu^+\mu^-}$ distribution increases from BP1 to BP3 as the momentum resolution grows with the $p_T(\mu)$. The $E_{\mu^+\mu^-}$ distribution (bottom left) shows a clear m_{π_d} and $p_T(\mu)$ dependence—muon pairs from the decay of heavier and higher $p_T \pi_d$'s carry more energy. Finally, in the $\Delta R_{\mu^+\mu^-}$ distribution (bottom right), the average angular separation

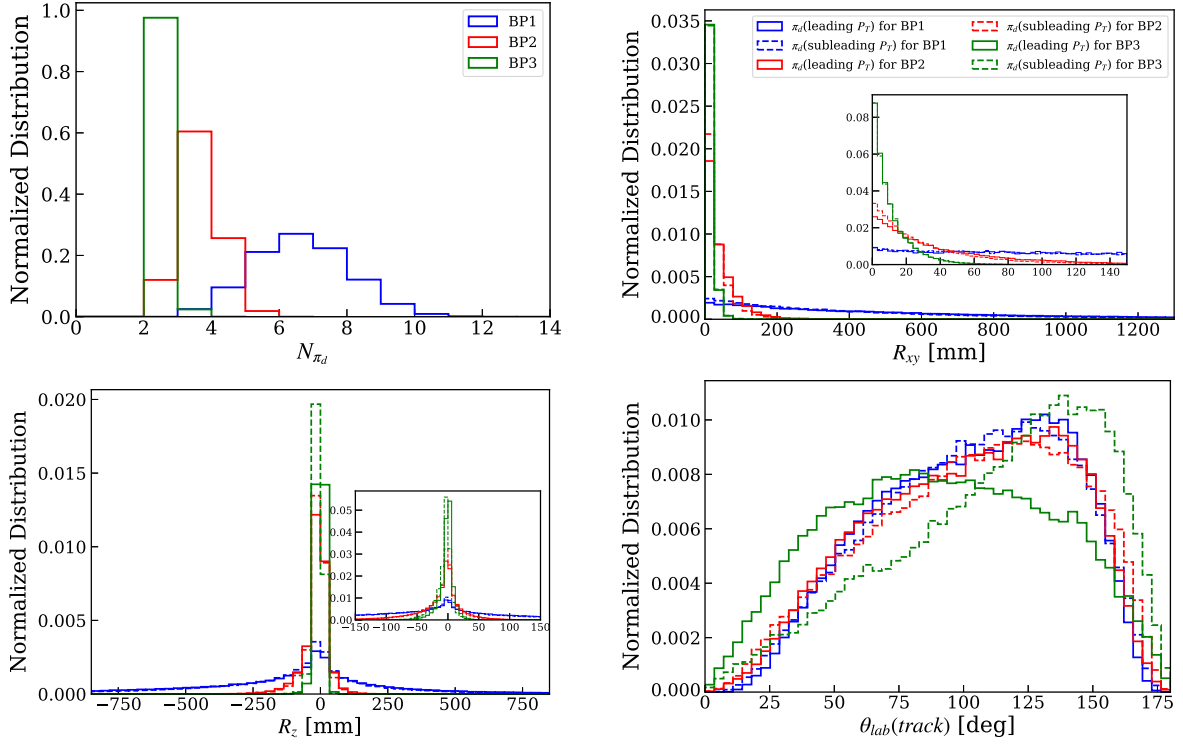


FIG. 7. Distributions for leading (solid line) and subleading (dashed line) p_T dark pions (π_d) in BP1, BP2, and BP3 at the Belle II. Top left: number of dark pions per event, N_{π_d} . Top right: transverse decay length, R_{xy} , of leading and subleading π_d 's. Bottom left: longitudinal flight distance, R_z . Bottom right: angle between the decay vertex vector and beam axis, θ_{lab} .

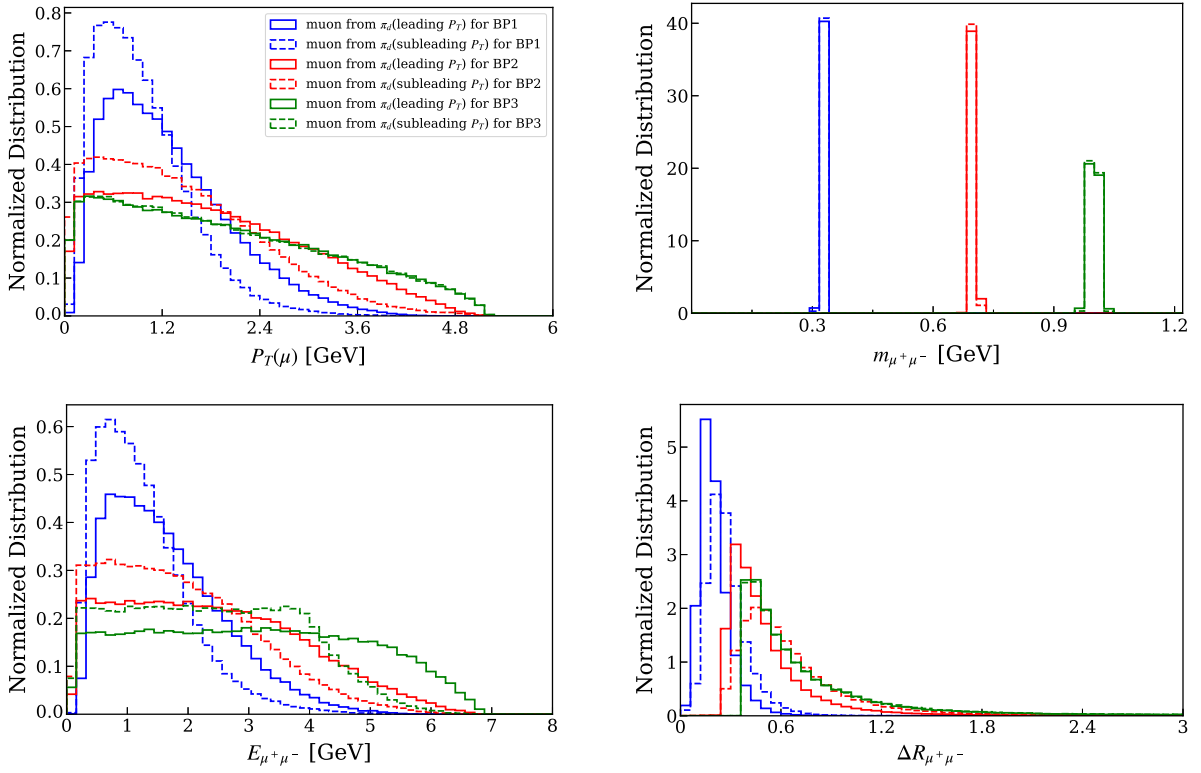


FIG. 8. The kinematic distributions for muons from leading p_T (solid line) and subleading p_T (dashed line) dark pions in BP1, BP2, and BP3 at the Belle II. Top left: transverse momentum of muons, $p_T(\mu)$. Top right: invariant mass of muon pairs, $m_{\mu^+\mu^-}$. Bottom left: energy of muon pairs, $E_{\mu^+\mu^-}$. Bottom right: angular separation between opposite-sign muons, $\Delta R_{\mu^+\mu^-}$.

between the muons is smaller for higher π_d multiplicity, while in BP3, which has fewer π_d 's, the ΔR distribution is generally larger.

Since the coupling κ and mediator mass m_X affect both the $e^+e^- \rightarrow \bar{q}_d q_d$ production cross section and π_d lifetime, we map exclusion limits in the parameter space of the (m_X, κ) plane. In particular, $\sigma(e^+e^- \rightarrow \bar{q}_d q_d)$ and Γ_{π_d} both scale as $(\kappa/m_X)^4$. Applying the trigger conditions and event selection criteria (Tables II and III), we obtain the signal efficiency ϵ_S . Here, an event is considered to pass the trigger selection if at least one of the three trigger conditions is satisfied. The expected signal yield is $N_S = \mathcal{L} \times \sigma(e^+e^- \rightarrow \bar{q}_d q_d) \times \epsilon_S$, assuming optimistic integrated luminosity $\mathcal{L} = 50 \text{ ab}^{-1}$ for the Belle II experiment. With a background-free assumption, $N_S = 2.3$ events establish the exclusion contours of the 90% confidence level (C.L.), which correspond to the lower boundaries with purple solid lines in Fig. 6.

As expected, small κ/m_X yields a tiny cross section and excessive π_d decay length, reducing N_S below the detection ability of the Belle II experiment. Conversely, large κ/m_X produces prompt π_d decays without the signature of displaced vertices. In this situation, the efficiency drops sharply, necessitating enormous Monte Carlo samples. Therefore, we conservatively set $N_S = 10$ to estimate upper boundaries with purple dashed lines in Fig. 6.³ We can observe that the projected Belle II constraints also indicate inverse proportionality between m_X and κ for these three BPs. Notably, blank regions in the upper-left corner of the (m_X, κ) plane could be covered by searching for multi-muons from the prompt decay of dark pions. However, detailed signal-background analysis is beyond the scope of this work.

Unlike most light DM searches at the Belle II, the monophoton search strategy lacks sufficient discriminating power between signal and background events in this model. Although undetectable dark pions produce missing energy signatures, their kinematic distributions remain similar to SM backgrounds. Because the mediator mass exceeds the center-of-mass energy at the Belle II, this absence of light mediator production results in smooth, continuous photon energy distributions that resemble SM backgrounds [49]. We, therefore, exclude monophoton constraints from Belle II in this analysis.

C. Long-lived dark pion searches in proposed GAZELLE detector

Positioned downstream of the Belle II detector at a considerable distance from the IP, the proposed GAZELLE detector facilitates long-distance tracking and displaced vertex reconstruction, which enhances the overall sensitivity for probing LLPs. The GAZELLE detector has three proposed configurations: Baby-GAZELLE, L-GAZELLE,

and GODZILLA [69]. L-GAZELLE comprises two components, LG-B1 (Region I) and LG-B2 (Region II), which select signal events, as detailed later. We chose L-GAZELLE for this analysis due to its superior angular coverage (LG-B1: 0.34 sr [2.7%]; LG-B2: 0.76 sr [6.0%]), exceeding Baby-GAZELLE (0.12 sr, 0.95%) and GODZILLA (0.74 sr, 5.9%) and thereby improving the detection efficiency. Its moderate detection distances (LG-B1: $x = 35 \text{ m}$, $y = 2.3 \text{ m}$, $z = 0 \text{ m}$; LG-B2: $x = 19 \text{ m}$, $y = 2.3 \text{ m}$, $z = 10.5 \text{ m}$) optimally capture long-lived dark pion decays. This contrasts with Baby-GAZELLE's proximity ($x = 10 \text{ m}$, $y = -3.7 \text{ m}$, $z = 10 \text{ m}$), which limits the far decay sensitivity, and the GODZILLA distance ($x = -27 \text{ m}$, $y = 18 \text{ m}$, $z = 20 \text{ m}$), which reduces the near decay sensitivity. Thus, L-GAZELLE offers the optimal balance for our study.

We apply a DDC [70] for the L-GAZELLE detector simulation.⁴ The decay probability of LLPs within a certain range is calculated as $P(\ell_1, \ell_2) = \exp(-\frac{\ell_1}{\gamma|\vec{v}|\tau_0}) - \exp(-\frac{\ell_2}{\gamma|\vec{v}|\tau_0})$, where $\gamma \equiv (1 - |\vec{v}|^2)^{-1/2}$, and $|\vec{v}|$ and τ_0 are the velocity and proper decay length of a LLP with the natural units. The weight for each layer is $\Delta\phi/(2\pi)$, where $\Delta\phi$ is the angular aperture of each layer. The weighted sum of probabilities from all detector layers gives the acceptance A_i for each LLP_{*i*}, which represents the detection probability of each LLP. For event selections, we calculate the probabilities of detecting at least one LLP and at least two LLPs:

- (i) At least one LLP detected: $\langle P(\geq 1) \rangle = 1 - \langle \prod (1 - A_i) \rangle$.
- (ii) At least two LLPs detected: $\langle P(\geq 2) \rangle = 1 - \langle \prod (1 - A_i) \rangle - \langle \sum A_i \prod_{j \neq i} (1 - A_j) \rangle$.

For the signal efficiency ϵ_S obtained for both scenarios, the expected signal yield is given by $N_S = \mathcal{L} \times \sigma(e^+e^- \rightarrow \bar{q}_d q_d) \times \epsilon_S$, assuming an optimistic integrated luminosity $\mathcal{L} = 50 \text{ ab}^{-1}$. The 90% C.L. exclusion contours correspond to the lower boundaries, shown as orange-red (at least one LLP detected) and brown (at least two LLPs detected) solid lines in Fig. 6. Compared to recast BABAR and projected Belle II constraints, the far detector L-GAZELLE coverage extends to smaller κ and larger m_X values.

We further evaluated detection efficiencies for different m_{π_d} and τ_0 values, as shown in Table V. The detection efficiency for at least two LLPs [$\langle P(\geq 2) \rangle$] is significantly lower than for at least one LLP [$\langle P(\geq 1) \rangle$]. For BP1, $\langle P(\geq 2) \rangle$ is about 2 orders of magnitude smaller than $\langle P(\geq 1) \rangle$. As the m_{π_d} increases, the efficiency gap widens, particularly at $m_{\pi_d} = 1 \text{ GeV}$, where events typically produce two LLPs that travel nearly back-to-back, making it

³Even with this relaxed requirement, more than 10^8 simulated events are required for this estimation.

⁴The L-GAZELLE detector is approximated with LG-B1 (single-layer approximation within 8.5% error) and LG-B2 (four-layer approximation within 5.0% error).

TABLE V. Detection efficiency $\langle P(\geq 1) \rangle$ and $\langle P(\geq 2) \rangle$ for various dark pion proper decay length τ_0 and dark pion mass m_{π_d} values. Corresponding production cross sections are 5.40×10^{-2} fb for $m_{\pi_d} = 0.32$ GeV at $\tau_0 = 54.91$ m, 6.59×10^{-2} fb for $m_{\pi_d} = 0.7$ GeV at $\tau_0 = 5.25$ m, and 6.32×10^{-2} fb for $m_{\pi_d} = 1$ GeV at $\tau_0 = 1.80$ m.

m_{π_d} (GeV)	τ_0 (m)	$\langle P(\geq 1) \rangle$	$\langle P(\geq 2) \rangle$
0.32	54.91	9.02×10^{-3}	3.71×10^{-5}
	10.98	2.16×10^{-2}	1.96×10^{-4}
	5.49	2.27×10^{-2}	1.99×10^{-4}
0.70	5.25	1.22×10^{-2}	2.91×10^{-5}
	1.05	2.68×10^{-3}	8.37×10^{-8}
	0.53	3.17×10^{-4}	3.33×10^{-11}
1.00	1.80	4.90×10^{-3}	7.36×10^{-8}
	0.36	7.32×10^{-6}	9.96×10^{-19}
	0.18	2.02×10^{-9}	0

highly unlikely for both to be detected within L-GAZELLE. Since τ_0 and the production cross section are anticorrelated, a decrease in τ_0 enhances the cross section but causes a more rapid decline in detection efficiency. This efficiency loss prevents meaningful constraints for BP2 and BP3 in the scenario requiring at least two detected LLPs. Consequently,

the brown solid line corresponding to BP2 and BP3 is absent in Fig. 6.

V. DENSE DARK SHOWERS AT THE CEPC/FCC-EE

This section further investigates the detection prospects at high-energy e^+e^- colliders, specifically the proposed CEPC and the FCC-ee. Both are considered to operate at a center-of-mass energy of $\sqrt{s} = 240$ GeV, with integrated luminosities of 5.6 ab^{-1} and 5.0 ab^{-1} , respectively. Given the expected comparable sensitivity, the analysis in the main text focuses on CEPC, while a direct comparison of the exclusion regions for FCC-ee and CEPC is provided in Appendix. We also present exclusion limits for two distinct dark pion mass ranges: $10 \text{ MeV} \leq m_{\pi_d} < 2m_\mu$ and $2m_\mu \leq m_{\pi_d} < 2m_\tau$, which correspond to the monophoton and displaced muon jet (DMJ) signatures, respectively.

A. The monophoton signature

For the dark pion mass range $10 \text{ MeV} \leq m_{\pi_d} < 2m_\mu$, we maintain the conservative setting $m_{\pi_d} = f_{\pi_d} = 0.2$ GeV and impose the requirement that the π_d proper decay length exceeds 6.08 m to guarantee that all dark mesons decay outside the CEPC detector volume. The resulting

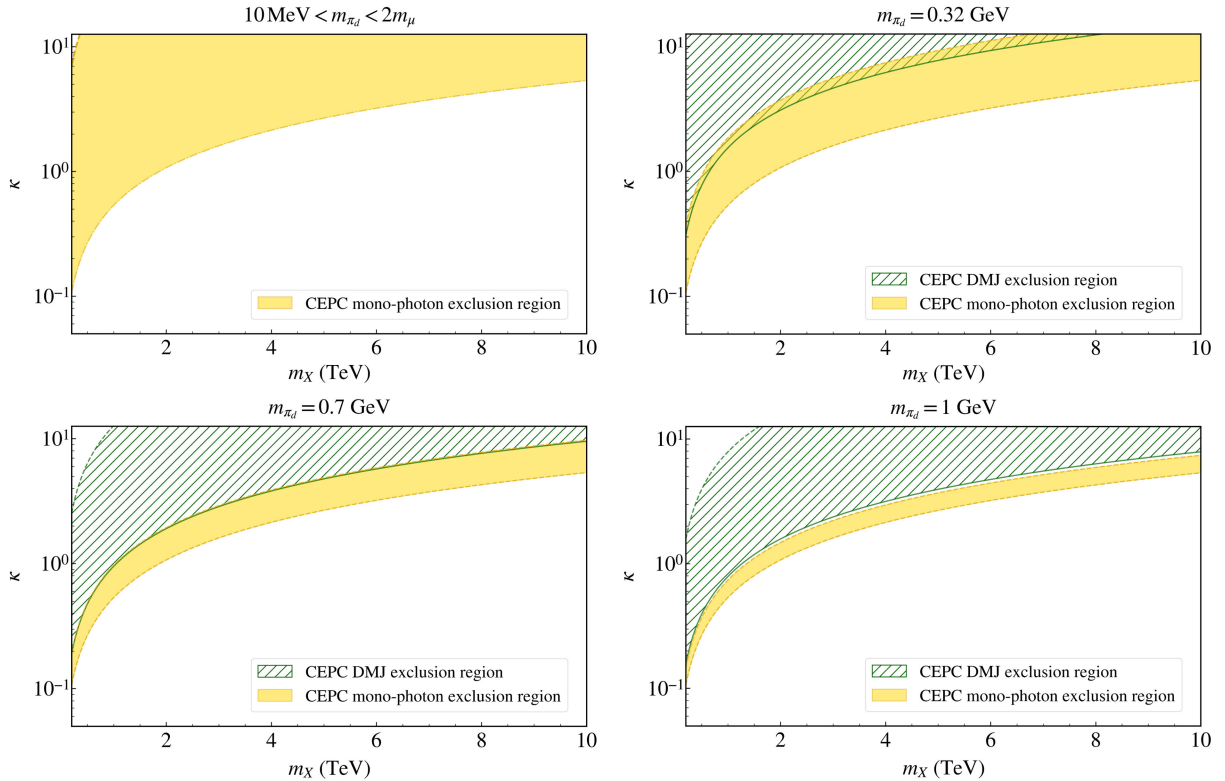


FIG. 9. Exclusion regions for different dark pion mass ranges. The first panel shows the monophoton signature for $10 \text{ MeV} < m_{\pi_d} < 2m_\mu$, while the remaining three panels show benchmark points $m_{\pi_d} = 0.32, 0.7$, and 1 GeV for both CEPC monophoton and DMJ signatures, covering $2m_\mu < m_{\pi_d} < 2m_\tau$.

upper bounds on κ , shown as the yellow dashed curve in Fig. 9, reach values of $\mathcal{O}(1-10)$. We recast the previous CEPC monophoton analysis, derived for the t -channel scalar mediator effective operator with cutoff scale Λ_t [corresponding to the t -channel case in Eq. (2.3)] and the upper-right panel (H-mode) of Fig. 16 in Ref. [50], to our leptophilic composite ADM framework through the relation

$$\frac{1}{\Lambda^2} \simeq \frac{1}{(1078 \text{ GeV})^2} = 3 \frac{\kappa^2}{m_\chi^2}. \quad (9)$$

Assuming light dark quarks and determining the lower limits on Λ , the yellow region in Fig. 9 indicates parameter space where the predicted cross sections exceed the experimental bounds.

TABLE VI. The CEPC detector geometry and its relevant transverse spatial resolution [28].

Detector component	R_{in}	R_{out}	σ_{xy}
Vertex detector	16 mm	60 mm	$2.80 \mu\text{m} - 6.00 \mu\text{m}$
Silicon tracker	0.15 m	1.81 m	$7.20 \mu\text{m}$
Hadron calorimeter	2.30 m	3.34 m	30 mm
Muon system	4.40 m	6.08 m	20 mm

B. The displaced muon jet signature

For the dark pion mass range $2m_\mu \leq m_{\pi_d} < 2m_\tau$, unlike low-energy e^+e^- colliders, due to the high center-of-mass energy, the initially produced dark quarks undergo dark showering and hadronization processes, yielding a large number of dark pions with the same model parameter settings. These dark pions subsequently decay to muons, resulting in a high multiplicity of displaced muons that collectively form DMJs.

The detailed definition of DMJs in this analysis is as follows. All candidate muons inside the DMJ are selected by requiring $p_T(\mu) > 1 \text{ GeV}$ and $|\eta_\mu| < 3.0$, parent particles (π_d 's) with R_{xy} within one of the four regions defined in Table VI, and muon pairs with $\Delta\theta > 0.02$ radians. Muons satisfying these criteria are identified as displaced muons. We employ the anti- k_t algorithm [71], as implemented in FASTJET [72], with a jet radius parameter of $R = 0.5$ to cluster muons into jets, and subsequently identify those containing at least six muons as DMJs.

We then select the leading p_T DMJ in each event and present in Fig. 10 the main kinematic distributions of its constituent muons and their mother dark pions, using three BPs with $m_{\pi_d} = 0.32 \text{ GeV}$, 0.7 GeV , and 1 GeV , corresponding to $\kappa = 5$ and $m_\chi = 1 \text{ TeV}$. The R_{xy} (top left) and $p_T(\mu)$ (top right) distributions follow trends similar to those

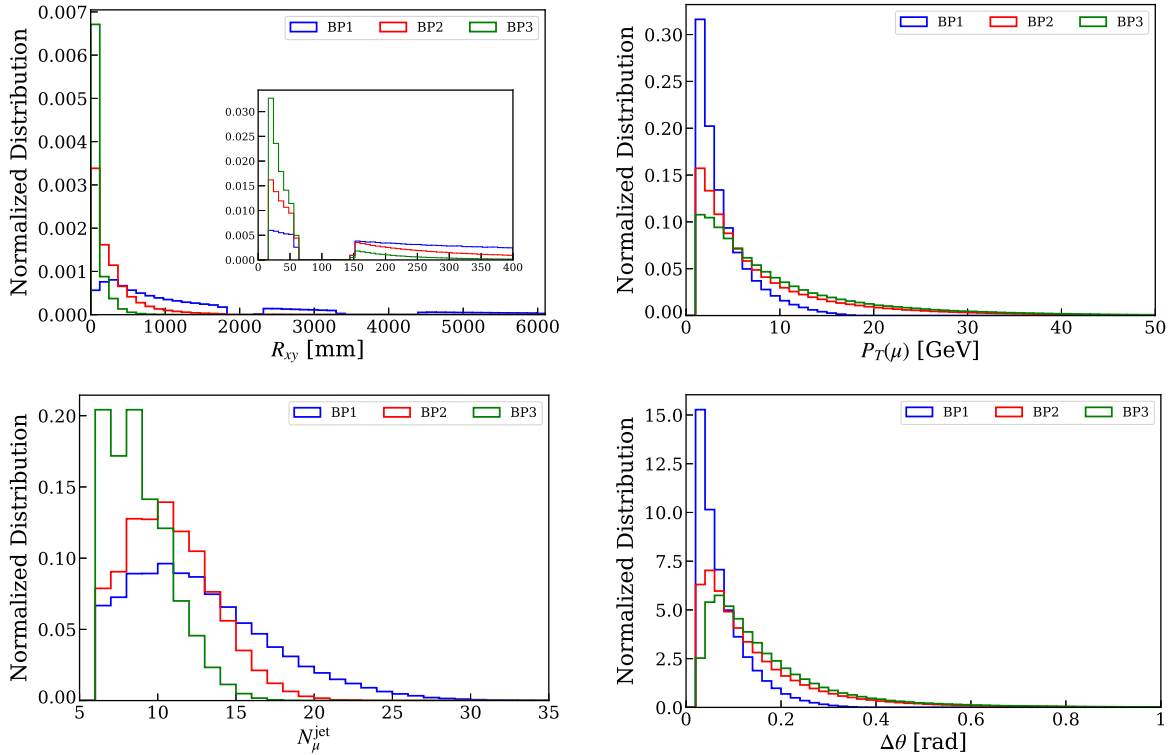


FIG. 10. The main kinematic distributions for the DMJ analysis at the CEPC with three benchmark points, $m_{\pi_d} = 0.32, 0.7$, and 1 GeV , corresponding to $\kappa = 5$ and $m_\chi = 1 \text{ TeV}$. Top left: the dark pion transverse decay length, R_{xy} . Top right: muon transverse momentum, $p_T(\mu)$. Bottom left: the number of muons in each muon jet. Bottom right: the opening angle $\Delta\theta$ between two muons originating from the same dark pion.

observed at low-energy e^+e^- colliders, but at the CEPC they extend to significantly higher values due to its much larger collision energy. Since the CEPC detector is segmented into four noncontiguous radial regions, as shown in Table VI, the R_{xy} distribution reflects this structure. For three BPs considered here, most dark pions are concentrated in the innermost region, namely, the vertex detector. The calculation of R_{xy} also includes Gaussian smearing effects, with the transverse position resolution for displaced vertices given by [73]

$$\sigma_{xy} = \left(a \oplus \frac{b}{p_T \cdot \sin^{3/2}\theta} \right) \mu\text{m}, \quad (10)$$

where $a = 5$, $b = 10$ GeV, p_T is the average transverse momentum of the track, and θ is its polar angle. The distribution of the number of muons inside the leading p_T DMJ (bottom left) illustrates the multiplicity of displaced muons. Across all three BPs, DMJs exhibit high-multiplicity features, with lighter π_d 's showing a multiplicity advantage. Such high-multiplicity signatures are distinctive and very difficult for SM backgrounds to mimic [28]. The opening angle $\Delta\theta$ between two muons originating from the same dark pion (bottom right) increases from BP1 to BP3. This wider separation for heavier π_d results from the larger p_T

carried by the muons. However, due to the finite spatial resolution of the detector, muon pairs with excessively small $\Delta\theta$ cannot be resolved and are, therefore, excluded from the π_d reconstruction.

To further identify the signal signature, all events containing at least two DMJs are selected. The CEPC muon system comprises eight sensitive layers alternating with iron absorber plates. With a total iron thickness equivalent to 6.7 interaction lengths, this configuration offers sufficient protection against punch-through backgrounds. The detector achieves excellent muon identification performance while maintaining exceptionally low pion misidentification rates [73]. When combined with the signal selection strategy detailed earlier, this methodology guarantees a background-free analysis [28]. The signal efficiency ϵ_S is then evaluated from the above event selection criteria. The expected signal yield is given by $N_S = \mathcal{L} \times \sigma(e^+e^- \rightarrow q_d \bar{q}_d) \times \epsilon_S$. Under the background-free hypothesis, the 2σ exclusion limit is set at $N_S = 3$, from which we derive the exclusion contour in the (m_X, κ) plane, as shown in Fig. 9.

When determining the upper exclusion boundary, the same challenge as in the Belle II analysis is encountered, and thus we conservatively set $N_S = 10$ for the CEPC upper limit estimation, while the lower boundary remains determined by the condition $N_S = 3$. However, for $m_{\pi_d} = 0.32$ GeV, it

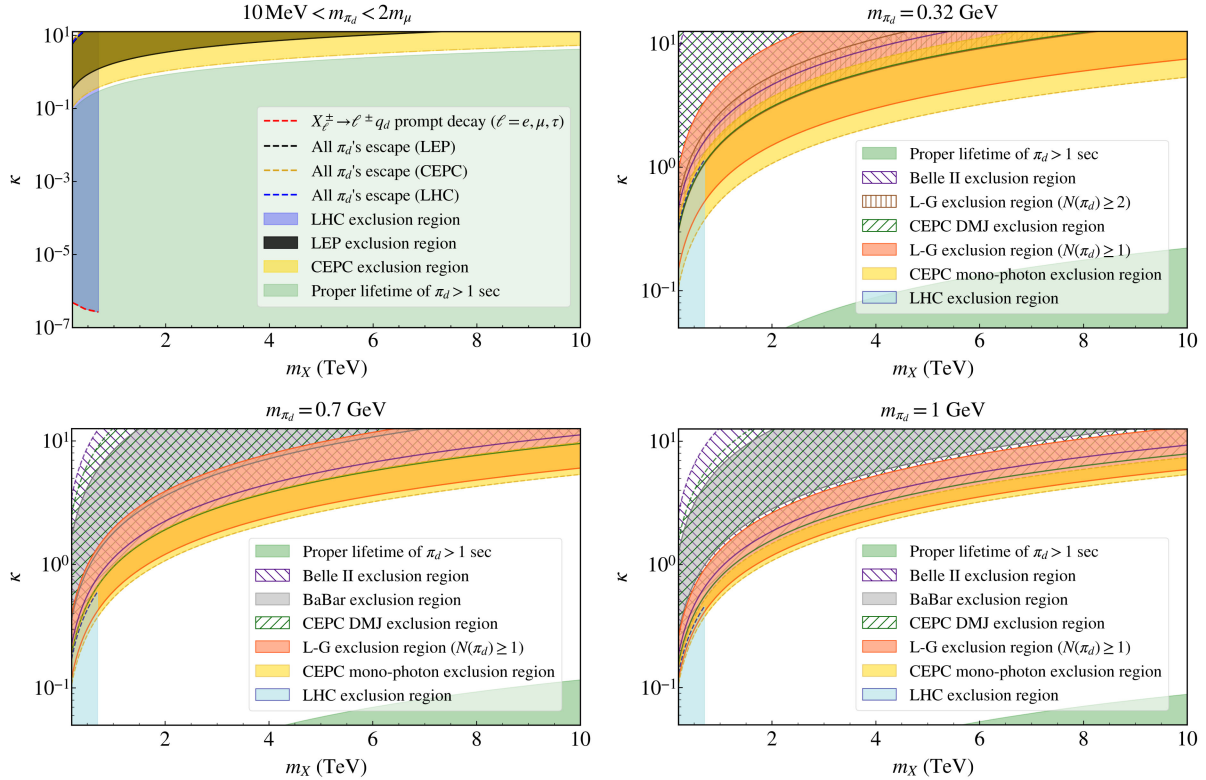


FIG. 11. Combined exclusion limits for different dark pion mass ranges. The first panel shows the exclusion region for $10 \text{ MeV} < m_{\pi_d} < 2m_{\mu}$, while the remaining three panels show benchmark points $m_{\pi_d} = 0.32, 0.7$, and 1 GeV from the LHC, monophoton searches at the LEP and CEPC, displaced vertex searches (*BABAR*, Belle II, and L-GAZELLE), and a DMJ search at the CEPC. All four panels include the parameter space where the proper lifetime of π_d exceeds one second, conservatively evaluated at $m_{\pi_d} = 0.2 \text{ GeV}$ for the light mass range.

remains hopeless to establish an upper boundary under this conservative assumption. We have estimated that more than 10^9 Monte Carlo events are required for this situation, beyond the scope of this work. Therefore, no upper exclusion boundary is presented for $m_{\pi_d} = 0.32$ GeV.

VI. SUMMARY AND DISCUSSION

We explored novel signatures of a leptophilic QCD-like composite asymmetric dark sector at e^+e^- colliders by simulating dark parton showering, hadronization, and detector responses to map the experimental reach. The study reveals a sharp transition in the dark pion proper decay length at the muon threshold (Fig. 2). Recast constraints from monophoton searches and from slepton/stau reinterpretations limit the low-mass bi-charged scalar mediator region (Figs. 3 and 4), while combined projections for *BABAR*, Belle II, L-GAZELLE, and CEPC produce the characteristic band-shaped sensitivity in the (m_X, κ) plane, as summarized in Fig. 11. After applying cosmological constraints on the π_d lifetime, nearly all parameter space for $10 \text{ MeV} \leq m_{\pi_d} < 2m_\mu$ and $m_X \lesssim 10 \text{ TeV}$ is excluded. Additionally, for $2m_\mu \leq m_{\pi_d} < 2m_\tau$, a significant fraction of parameter space with $\kappa \gtrsim 1$ and $m_X \lesssim 10 \text{ TeV}$ is also excluded.

The primary experimental targets we identify are (i) sparse displaced dimuon vertices at B factories (kinematics in Figs. 5, 7, and 8, while trigger and event selection conditions are summarized in Tables II and III), which Belle II can probe with realistic triggers and event selections, (ii) far detectors (L-GAZELLE) that extend sensitivity to substantially longer lifetimes, and (iii) DMJ searches at high-energy e^+e^- colliders (main kinematics shown in Fig. 10 and detector assumptions in Table VI). Although our projections adopt simplifying assumptions, LFU of the mediator coupling, the benchmark relation $f_{\pi_d} = m_{\pi_d}$, and optimistic/background-free hypotheses in some regions, they nevertheless provide concrete, experimentally accessible targets and strongly motivate a coordinated program across B factories, far detectors, and Higgs factories.

Finally, we outline promising directions for future work. Advanced data-scouting or trigger-level analyses for low- p_T displaced dimuon pairs at CMS and LHCb could significantly strengthen LHC constraints [74–76]. High-energy muon colliders could directly produce the bi-charged scalar mediator via Drell-Yan processes and thereby provide an independent probe of the parameter space. Furthermore, when the dark pion is heavier than $2m_\tau$, dark jets become τ -rich. This regime will require dedicated strategies (e.g., displaced and boosted τ -tagging, and optimized reconstruction of hadronic τ decays) to retain sensitivity.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: FCC-ee AND CEPC EXCLUSION REGION COMPARISON

This appendix compares the FCC-ee and CEPC experiments in terms of their exclusion regions from monophoton and DMJ searches. We adopt the same parameter settings and analysis strategies for the monophoton and DMJ signatures as in the CEPC study. The observed differences are primarily attributed to the distinct detector geometries and the modest variation in integrated luminosity between FCC-ee and CEPC experiments. The geometry and relevant transverse spatial resolutions of the FCC-ee detector are summarized in Table VII, following Refs. [77,78]. The transverse spatial resolutions are taken with reference to the CEPC. Here, we take $m_{\pi_d} = 1.0$ GeV as an example to demonstrate this similarity—other benchmark points exhibit nearly identical behavior.

As shown in Fig. 12, we compare the parameter space exclusion capabilities between the FCC-ee and CEPC for $m_{\pi_d} = 1.0$ GeV. Considering the overall coverage of the four detector subsystems, the effective detector length of the FCC-ee is slightly larger than that of the CEPC, therefore, the FCC-ee demonstrates a mild advantage. However, this improvement is very small, and the exclusion capabilities of the two experiments are quite comparable. Therefore, for high-energy e^+e^- colliders, we take the CEPC as a representative example and perform detailed studies on the three BPs in the main text.

TABLE VII. The FCC-ee detector geometry and its relevant transverse spatial resolution.

Detector component	R_{in}	R_{out}	σ_{xy}
Vertex detector (inner)	13.70 mm	35.60 mm	2.80 μm –6.00 μm
Vertex detector (outer)	0.13 m	0.32 m	2.80 μm –6.00 μm
Drift chamber	0.35 m	2.00 m	7.20 μm
Hadron calorimeter	2.80 m	4.60 m	30 mm
Muon system	4.60 m	5.24 m	20 mm

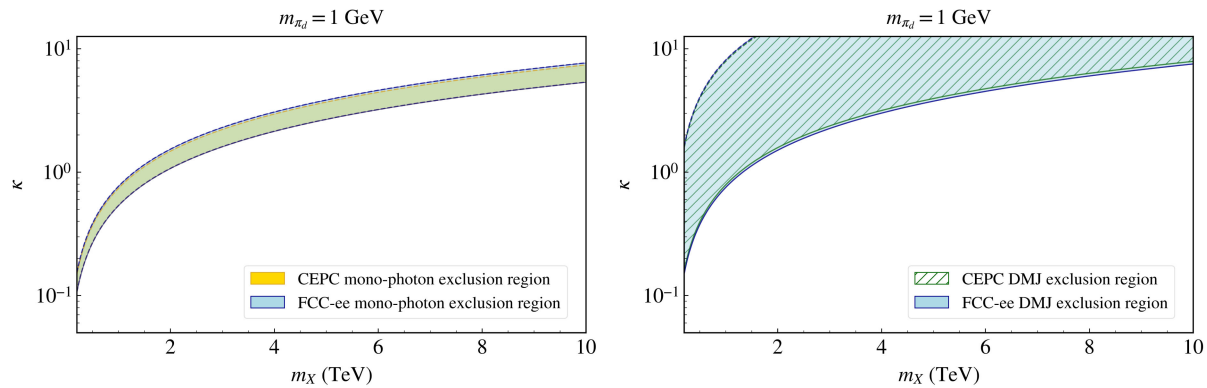


FIG. 12. The comparison of exclusion limits from monophoton (left) and DMJ (right) searches between the CEPC and FCC-ee experiments, for a dark pion mass of $m_{\pi_d} = 1$ GeV.

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