

Bound of Casimir effect by holography

Rong-Xin Miao 

School of Physics and Astronomy, Sun Yat-Sen University, Zhuhai, 519082, China



(Received 19 December 2024; accepted 10 September 2025; published 2 October 2025)

Inspired by the Kovtun-Son-Starinets bound, we propose that holography imposes a lower bound on the Casimir effect. For simplicity, we focus on the Casimir effect between parallel planes for three-dimensional conformal field theories and briefly comment on the generalizations to other boundary shapes and higher dimensions. Remarkably, the ghost-free holographic models impose a universal lower bound on the Casimir effect. We verify the holographic bound by free theories, the Ising model, and $O(N)$ models with $N = 2, 3$ at critical points and prove it for the two-dimensional case. Remarkably, a general class of quantum field theories without conformal symmetries also obeys the holographic bound.

DOI: [10.1103/xs4m-qjsb](https://doi.org/10.1103/xs4m-qjsb)

Introduction. Casimir effect [1] is an interesting quantum effect that originates from the changes of vacuum fluctuations due to the boundary [2–5]. It is a candidate of dark energy [6] and plays a vital role in QCD [7] and wormhole physics [8,9]. It has been measured in experiments [10–12] and has significant potential applications in nanotechnology. As the vacuum energy, Casimir energy is defined as the lowest energy of a quantum system with a boundary. Since energy should be bounded from below, it is expected that there is a fundamental lower bound of the Casimir effect. This letter explores this vital problem. Holography (AdS/CFT) [13] provides a powerful tool to study the strongly coupled conformal field theory (CFT) and usually imposes bounds on various physical phenomena. The most famous example is the Kovtun-Son-Starinets (KSS) bound [14] for hydromechanics, which conjectures that the ratio of shear viscosity η to entropy density s has a lower bound,

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}, \quad (1)$$

where $\hbar$ is the reduced Planck constant and k_B is the Boltzmann constant. The lower bound is obtained from the strongly coupled CFT dual to Einstein gravity [15] and can be modified slightly by higher derivative gravity [16–18].

Now, let us explore whether AdS/CFT sets a similar lower bound for the Casimir effect. For simplicity, we focus on the Casimir effect for three-dimensional (3D) CFTs between parallel planes (strip) and comment on various

generalizations at the end of the Letter. We consider flat space and impose the same boundary conditions on the two planes. Then, the Casimir effect takes a universal form [19],

$$\langle T^i_j \rangle_{\text{strip}} = \kappa_1 \frac{\hbar c}{L^3} \text{diag}(1, -2, 1), \quad (2)$$

where κ_1 is the dimensionless Casimir amplitude, c is the speed of light, and L is the strip width. For the fixed L , $-\kappa_1$ should be bounded from below to avoid negative infinity energy density $T_{tt} = -\kappa_1 \hbar c / L^3$ and pressure $T_{nn} = -2\kappa_1 \hbar c / L^3$. Note that the Casimir amplitude κ_1 is an extensive parameter in the sense that if one CFT has a Casimir amplitude κ_1 then N -copies of the same CFT have a Casimir amplitude $N\kappa_1$. It implies that only $-\kappa_1$ over another extensive parameter could have a lower bound. Natural candidates of these extensive parameters are boundary central charges in Weyl anomaly [20] $\mathcal{A} = \int_{\partial M} d^2x \sqrt{|\sigma|} (aR_{\partial M} + b\text{tr}\bar{k}^2)$, where $R_{\partial M}$ and $\bar{k}$ are the intrinsic Ricci scalar and traceless parts of extrinsic curvatures on the boundary ∂M , respectively. a is the A-type boundary central charge, which decreases under the boundary renormalization group (RG) flow [20]. We remark that a can be positive, zero, or negative. For example, $a = \pm 1/(384\pi)$ for conformally coupled free scalar with Robin boundary condition (RBC) and Dirichlet boundary condition (DBC) [20], respectively; $a \sim \sinh(\rho)$ with $-\infty < \rho < \infty$ for holographic CFT₃ [21]. Since a can be zero and negative, there is no lower bound for $-\kappa_1/a$. On the other hand, the B-type boundary central charge b is always positive since it is related to the Zamolodchikov norm of the displacement operator, i.e., $b = C_D \pi / 32 > 0$. The displacement operator $D(y)$ describes the violation of space translation invariance normal to the boundary [22], i.e., $\nabla_i T^{ij} = \delta(x) D(y) n^j$. The two-point function defines its Zamolodchikov norm, i.e., $\langle D(y) D(0) \rangle = C_D c^2 \hbar^2 / |y|^6$.

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

By definition, C_D is dimensionless and positive. The above arguments suggest $-\kappa_1/C_D$ should have a physical lower bound. Usually, a bound is set either by free theories or strongly coupled CFTs dual to gravity. We have checked all the known examples and find that, similar to the KSS bound, holography gives the smallest $-\kappa_1/C_D$. Thus, we propose that the ratio of Casimir amplitude to the norm of the displacement operator has a lower bound set by holography,

$$-\frac{\kappa_1}{C_D} \geq -\frac{\pi^{5/2}\Gamma(\frac{1}{3})^3}{108\Gamma(\frac{5}{6})^3} \approx -2.17, \quad \text{for 3D CFTs.} \quad (3)$$

It means the energy density $-\kappa_1/L^3$ per degree of freedom C_D is bounded from below for a physical system with fixed size L . Remarkably, unlike the KSS bound (1), different holographic models give the same universal lower bound (3) for the Casimir effect. We verify the lower bound (3) by free CFTs, the Ising model, and the $O(N)$ model with $N = 2, 3$ at critical points. Furthermore, we prove the corresponding bound for two-dimensional (2D) CFTs, which strongly supports our general proposal. Below, we discuss the test of our proposal, the derivation of the holographic bound, and its various generalizations. For simplicity, we focus on natural units with $c = \hbar = k_B = 1$ in the following.

Tests. Let us test the holographic bound (3) by various 3D CFTs, which is summarized in Table I. We have $-\kappa_1/C_D = -3\pi\zeta(3)/8 \approx -1.42$ for Dirac fermions with bag boundary condition (BC) [23,24], and $-\kappa_1/C_D = -\pi\zeta(3)/2 \approx -1.89$ for free scalars with RBC and DBC [25,26]. For Ising model with ϕ^4 interaction at the critical point, we have $2\kappa_1 \approx 0.820$ and $C_D \approx 0.193$ [27–29], which yields $-\kappa_1/C_D \approx -2.12$. Here, we impose with $(++)$ BC for the Ising model, where the spins on the two boundaries have the same signs. Near the strip boundary, the scalar operator behaves as [30]

$$\langle \sigma(z) \rangle = \frac{a_\sigma}{(2z)^{\Delta_\sigma}} \left(1 + B_\sigma \left(\frac{z}{L} \right)^d + \dots \right), \quad (4)$$

where Δ_σ is the conformal dimension and B_σ obeys the universal relation

$$\frac{B_\sigma}{\Delta_\sigma} = 2^{d-1}(d-1)\pi^{-\frac{d}{2}}\Gamma\left(\frac{d}{2}\right) \frac{\kappa_1}{C_D}. \quad (5)$$

Here, we use the notation of Ref. [27] with $d = 3$ for 3D CFTs. For $O(N)$ model with $N = 2, 3$ and open BCs, Ref. [27] obtains $B_\sigma \approx 1.21, 1.07$ and $\Delta_\sigma \approx 0.519088$,

TABLE I. $(-\kappa_1/C_D)$ for various 3D CFTs.

Dirac fermion	Scalar	Ising	O(2)	O(3)	AdS/BCFT
-1.42	-1.89	-2.12	-1.83	-1.62	-2.17

0.518920 by applying Monte Carlo simulations. Then, Eq. (5) gives $-\kappa_1/C_D \approx -1.83, -1.62$ for $N = 2, 3$, respectively. We summarize the results of free theories, the Ising and $O(N)$ models in Table I, which all obey the holographic bound (3) will be derived below. It strongly supports our proposal. Interestingly, Eq. (5) suggests the bound of the Casimir effect imposes an upper bound of B_σ/Δ_σ .

Holographic bound. Let us study the holographic bound of the Casimir effect of a strip in anti-deSitter/boundary conformal field theory(AdS/BCFT) [31]. We are interested in the general ghost-free gravity in AdS_4 , which turns out to be Dvali-Gabadadze-Porrati (DGP) gravity [32],

$$I = \int_N d^4x \sqrt{|g|} (R + 6) + 2 \int_Q d^3y \sqrt{|h|} (K - T + \lambda \mathcal{R}), \quad (6)$$

where K is the extrinsic curvature, T is the brane tension, λ is the DGP parameter, and R and $\mathcal{R}$ are Ricci scalars in bulk N and on the brane Q , respectively. We have set Newton's constant $16\pi G_N = 1$ and AdS radius $l = 1$ for simplicity. Note that higher derivative gravity and negative DGP gravity generally suffer the ghost problems [33], while Gauss-Bonnet/Lovelock gravity appears only in dimensions higher than 4. Thus, positive DGP gravity with $\lambda \geq 0$ is the general ghost-free gravity theory in AdS_4 . The case $\lambda = 0$ corresponds to Einstein gravity. Following Ref. [31], we impose the Neumann boundary condition (NBC) on the brane Q ,

$$K^{ij} - (K - T + \lambda \mathcal{R})h^{ij} + 2\lambda \mathcal{R}^{ij} = 0, \quad (7)$$

where $T = 2 \tanh(\rho) - 2\lambda \text{sech}^2(\rho)$ is a useful parametrization [33]. The holographic norm of the displacement operator is given by [25,33]

$$C_D = \frac{32}{\pi \left(\frac{2\lambda}{2\lambda \sinh(\rho) + \cosh(\rho)} + 2 \tan^{-1} \left(\tanh\left(\frac{\rho}{2}\right) \right) + \frac{\pi}{2} \right)}. \quad (8)$$

$C_D \geq 0$ together with $\lambda \geq 0$ yields the constraints: $0 \leq \lambda$ for $\rho \geq 0$; $0 \leq \lambda \leq -\frac{\coth(\rho)}{2} \geq 1/2$ for $\rho \leq 0$. Below, we will show that $\lambda = 1/2$ is a phase-transition point and there are other physical constraints for λ . In the following, we show only key points and leave the detailed derivations to the Supplemental Material [34].

The vacuum of a strip is dual to the AdS soliton [21]

$$ds^2 = \frac{\frac{dz^2}{h(z)} + h(z)d\theta^2 - dt^2 + dy^2}{z^2}, \quad (9)$$

where $h(z) = 1 - z^3/z_h^3$. The absence of conical singularities fixes the period of angle θ in bulk

$\beta = 4\pi/|h'(z_h)| = 4z_h\pi/3$. The strip is defined on the AdS boundary $z = 0$ with $0 \leq \theta \leq L$, where $L \geq 0$ is the strip width. We derive the energy density $T_{tt} = -1/z_h^3$ from (9) by applying the holographic renormalization [35]. Compared with the Casimir effect (2), we get the holographic Casimir amplitude

$$\kappa_1 = L^3/z_h^3, \quad (10)$$

where L can be fixed by NBC (7). By using the rescale invariance of AdS, we set $z_h = 1$ below. For negative brane tension $T \leq 0$, we derive the strip width

$$L_1 = \int_0^{z_{\max}} \frac{2(\sqrt{h(z_{\max})} - \lambda h(z))dz}{h(z)\sqrt{H(z, z_{\max}) - h(z_{\max})}}, \quad (11)$$

where $H = \frac{1}{2}h(z)(1 + 2\lambda\sqrt{h(z_{\max})} + X)$, $X = \sqrt{1 + 4\lambda^2 h(z) - 4\lambda\sqrt{h(z_{\max})}}$, and $z_{\max}$ is the turning point obeying $T = -2\sqrt{h(z_{\max})}$. The strip width with $T > 0$ can be obtained from the complement of that with $T < 0$ by $L_{\text{II}} = \beta - L_1(T \rightarrow -T, \lambda \rightarrow -\lambda)$. It is the typical trick used in Ref. [21], based on extrinsic curvature flip signs when crossing the brane. In the viewpoint of NBC (7), T , λ change signs equivalently. Some comments are in order. First, L_1 (11) is negative for sufficiently large λ . The physical constraint $L \geq 0$ yields $\lambda \leq 1$. Second, the case $0 \leq \lambda < 1/2$ can continuously transform into that of Einstein gravity, while the case $1/2 < \lambda \leq 1$ cannot. See Fig. 1 for instance. We name them normal and singular phases, respectively. Third, the strip width L decreases with brane tension T [21], and $L \geq 0$ sets an upper bound of T [36]. For $T \geq 0$, L should be smaller than the angle period β to avoid conical singularity. Luckily, $L \leq \beta$ is satisfied automatically for $T \geq 0$ [36]. On the other hand, there is no upper bound of L for $T < 0$, since the conical singularity is hidden behind the brane [37]. Thus, it is irrelevant to the bulk dual of the strip. As a result, there is no extra constraint on the lower bound of T , and it can take the minimal value $T = -2$ [36,37]. We remark that $T \geq -2$ sets no constraint on ρ in the normal phase, while it sets a lower bound of ρ in the singular phase. See Fig. 1 (bottom).

We can derive the ratio $-\kappa_1/C_D$ from (8), (10), (11). Refer to Fig. 1 for details. Figure 1 (top) denotes the normal phase with $0 \leq \lambda < 1/2$, which shows that the smaller ρ , the smaller the $-\kappa_1/C_D$. Besides, all curves approach the same lower bound (red line) from above in the limit $T \rightarrow -2$. On the other hand, Fig. 1 (bottom) labels the singular phase with $1/2 \leq \lambda \leq 1$. It shows that $-\kappa_1/C_D$ with $\lambda > 1/2$ is always larger than that of Einstein gravity (blue curve). Besides, the minimum of $-\kappa_1/C_D$ increases with λ and approaches zero when $\lambda \rightarrow 1$. Since we are interested in the lower bound of $(-\kappa_1/C_D)$, we focus on the normal phase below.

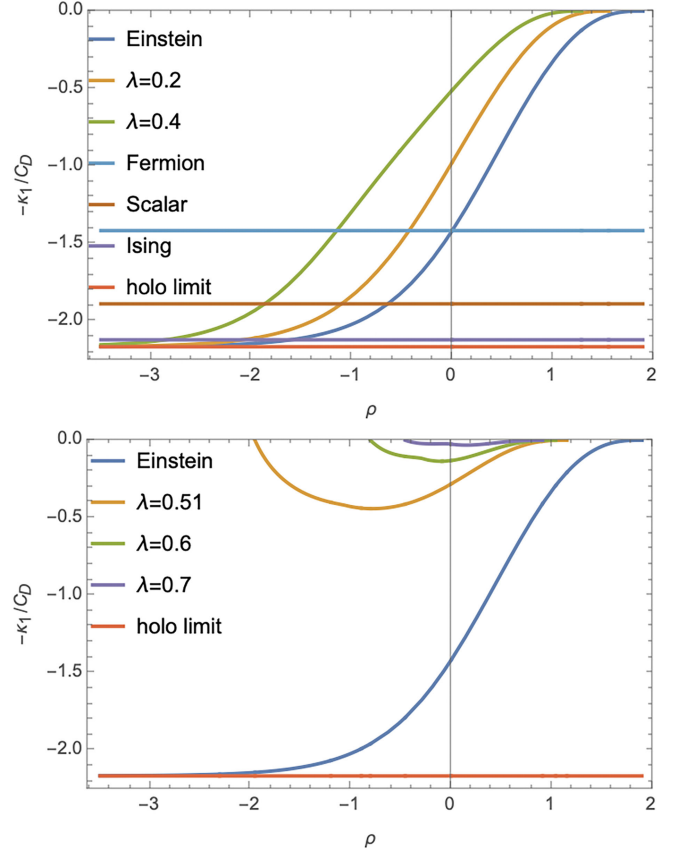


FIG. 1. $-\kappa_1/C_D$ in normal phase with $0 \leq \lambda < 1/2$ (up) and singular phase with $1/2 < \lambda \leq 1$ (down) for DGP gravity. Einstein gravity corresponds to $\lambda = 0$. The top figure shows all holographic curves approach the same limit (red line) from above for $\rho \rightarrow -\infty$ ($T \rightarrow -2$) in the normal phase. The bottom figure shows $-\kappa_1/C_D$ increases with λ and approaches zero for $\lambda \rightarrow 1$ in the singular phase. Together, it shows all examples obey the holographic bound (3) (red line).

Let us explain the physical reasons why the minimal brane tension yields the lower bound of the Casimir effect in the normal phase. First, the Casimir energy equals the free energy at zero temperature, which can be calculated holographically by the Euclidean gravitational action. Naturally, the smaller the brane tension (energy), the smaller the Euclidean action, and, thus, the smaller the Casimir energy. Second, the brane tension decreases under boundary RG flow, i.e., $T_{\text{IR}} < T_{\text{UV}}$ [21]. At IR, many modes become inactive, resulting in reduced energy. Third, the Casimir effect is driven by long-range quantum correlations [38]. As the tension (energy) decreases, these quantum correlations become stronger, which in turn increases the Casimir amplitude and reduces the Casimir energy. In summary, the minimal brane tension corresponds to the minimal Casimir energy.

Now, we give some analytical discussions. Performing coordinate transformation $z = z_{\max}y$ and expanding (11) around $x = \text{sech}^2(\rho) \rightarrow 0$, we derive perturbatively for

normal phase

$$L_1 = \frac{2\sqrt{\pi}\Gamma(\frac{4}{3})}{\Gamma(\frac{5}{6})} \left(\frac{(1-2\lambda)^2}{x} \right)^{\frac{1}{6}} + O\left(\frac{x}{(1-2\lambda)^2} \right)^{\frac{5}{6}}. \quad (12)$$

Taking limit $x \rightarrow 0$ ($T \rightarrow -2$) with $\lambda < 1/2$, we obtain a universal limit independent of λ ,

$$\lim_{T \rightarrow -2} \frac{-\kappa_1}{C_D} = \lim_{x \rightarrow 0} \frac{-L_1^3}{C_D} = -\frac{\pi^{5/2}\Gamma(\frac{1}{3})^3}{108\Gamma(\frac{5}{6})^3}. \quad (13)$$

It verifies our claim that different holographic models give the same lower bound (3) in the normal phase. The lower bound of $(-\kappa_1/C_D)$ is saturated by negative brane tension. We remark that the negative brane tension is well defined in AdS/BCFT [21,31,36,37]. First, the brane tension is a cosmological constant rather than a kinetic-energy term on the end-of-the-world brane. Of course, the negative cosmological constant is well defined in AdS/CFT. Second, negative brane tension generally yields a negative A-type boundary central charge. But nothing goes wrong. Recall that the free scalar with the DBC has a negative A-type boundary central charge [20].

General boundaries. Holography is expected to set a bound on the Casimir effect for general boundary shapes, not limited to strips. Let us provide some evidence. We first consider the wedge space, which is the simplest generalization of a strip. The Casimir effect takes the form

$$\langle T^i_j \rangle_{\text{wedge}} = \frac{f(\Omega)}{r^3} \text{diag}(1, -2, 1), \quad (14)$$

where $f(\Omega)$ is the Casimir amplitude and Ω is the opening angle of wedge. In the limit $\Omega \rightarrow 0$, $r \rightarrow \infty$ with $r\Omega = L$ fixed, the large r region of the wedge approaches a strip. It results in $f(\Omega) \rightarrow \kappa_1/\Omega^3$ for $\Omega \rightarrow 0$ [39]. On the other hand, we have $f(\Omega) \rightarrow (C_D\pi/64)(\pi - \Omega)$ in the smooth limit $\Omega \rightarrow \pi$ [19]. These two limits suggest AdS/BCFT with minimal brane tension sets the lower bound of the wedge Casimir effect,

$$\frac{-f(\Omega)}{C_D} \geq \lim_{T \rightarrow -2} \left(\frac{-f(\Omega)}{C_D} \right)_{\text{holo}}, \quad \text{for } 0 < \Omega \leq \pi. \quad (15)$$

See the following long paper [40] for the derivation and test of (15). Let us go on to discuss the general boundary. Near any smooth boundary, the normal-normal component of the stress tensor takes a universal form in flat space [39,41],

$$\langle T_{nn}(z) \rangle = \alpha \frac{\text{Tr} \bar{k}^2}{z} - \frac{\alpha}{2} k \text{Tr} \bar{k}^2 \log\left(\frac{z}{L}\right) + t_{nn} + O(z), \quad (16)$$

where z is the distance to boundary, L is the system size, and t_{nn} denotes the finite term. The divergent terms appear since we consider the ideal boundary. There is a natural cutoff $z \geq \epsilon$ in real material with ϵ the lattice length. Remarkably, the displacement operator universally determines the divergent terms, i.e., $\alpha = -C_D\pi/16$ [41]. It strongly suggests $\langle T_{nn}(\epsilon) \rangle / C_D$, or equivalently, t_{nn}/C_D has a lower bound for fixed system size and boundary shape. A weaker proposal is for the average pressure

$$\left(\frac{\bar{t}_{nn}}{C_D} \right) \geq \lim_{T \rightarrow -2} \left(\frac{\bar{t}_{nn}}{C_D} \right)_{\text{holo}}, \quad (17)$$

where $\bar{t}_{nn} = (\int_{\partial M} t_{nn} dS) / (\int_{\partial M} dS)$ is related to the variation of the total energy concerning the system size, i.e., $-dE/dL$.

General dimensions. Let us comment on the holographic bound of the Casimir effect for a strip in general dimensions. The corresponding Casimir amplitude κ_1 and norm of displacement operator C_D are given by Ref. [19] for Einstein gravity. By taking the limit $T \rightarrow -(d-1)$, we derive a universal lower bound,

$$\frac{-\kappa_1}{C_D} \geq \frac{-2^{d-2} d \pi^{d-\frac{1}{2}} \Gamma(\frac{d-1}{2}) \Gamma(\frac{1}{d})^d}{\Gamma(d+2) \left(d \Gamma(\frac{1}{2} + \frac{1}{d}) \right)^d}. \quad (18)$$

We verify that the free scalar obeys the bound, i.e., $(-\kappa_1/C_D)_{\text{scalar}} = -2^{1-d} \pi^{d/2} \zeta(d) / \Gamma(\frac{d}{2}) \geq (18)$ [25,26]. For $d = 2$, we can prove the bound (18)

$$\left(\frac{-\kappa_1}{C_D} \right) \Big|_{d=2} \geq -\frac{\pi^3}{12}. \quad (19)$$

According to Ref. [42], the inequality is saturated for general 2D CFTs with the same boundary conditions on the two planes. On the other hand, if we impose different boundary conditions on the two planes, we get larger Casimir energy [43,44],

$$(-\kappa_1)_{\text{differ BC}} \geq (-\kappa_1)_{\text{same BC}}. \quad (20)$$

Note that $C_D = c/(2\pi^2)$ is independent of the choices of boundary conditions, where c is the central charge of 2D CFTs. Considering all the above statements, we get solid proof of the bound (19) for 2D CFTs, which is strong support for our proposal (18).

Non-CFTs. Interestingly, the lower bound of (3) also works for a general class of non-CFTs. Take an example of a 3D free scalar with mass m and coupling ξ with the Ricci scalar. It is a CFT only if $\xi = 1/8$ and $m = 0$. For non-CFT parameters, the energy density between parallel plates is no longer a constant [5]. What is worse, it is divergent near the

boundary. Fortunately, the total energy per area is finite [5]. On the other hand, the pressure is still a finite constant. Thus, we define $\kappa_1 = \langle T_{nn}(0) \rangle L^3 / (-2)$ generally. By applying the textbook method, we obtain for both RBC and DBC, i.e., $16\pi\kappa_1 = -2M^2 \log(1 - e^{-2M}) + 2MLi_2(e^{-2M}) + Li_3(e^{-2M})$, where $M = mL > 0$ is a dimensionless parameter, and Li denotes the polylogarithmic function. Remarkably, κ_1 is independent of ξ and decreases with mass,

$$\frac{d\kappa_1}{dM} = \frac{M^2(1 - \coth(M))}{8\pi} < 0. \quad (21)$$

For free non-CFTs, we can define C_D via $\langle D(y)D(0) \rangle = C_D/|y|^6 + O(1/|y|^5)$ in half-space, where $D = T_{nn}(0)$ [22]. At a small distance $m|y| \ll 1$, the mass effect can be ignored. As a result, C_D is independent of mass. The above discussions show $-\kappa_1/C_D$ is larger than that of a massless scalar and thus obeys the holographic lower bound (3). The above arguments apply to general massive free theories. The Casimir effect arises from long-range quantum correlations [38]. When mass is introduced, these correlations become short range, which generally reduces the Casimir amplitude κ_1 [5]. Recall that C_D is mass independent. Consequently, $-\kappa_1/C_D$ reaches its minimum value in the massless limit for general free theories.

Conclusion. In conclusion, we propose that holography sets a fundamental bound on the ratio of the Casimir amplitude to the displacement operator norm. The Casimir amplitude is defined as the average pressure $\bar{t}_{nn}$ on the boundary. We take 3D CFTs in a strip as an example and then briefly discuss the generalization to other boundary shapes, higher dimensions, and free non-CFTs. It is interesting to test our proposal in experiments of critical systems [45]. Finding a field-theoretical proof or counterexample for our proposal is also enjoyable. Our findings help estimate how much the Casimir effect contributes to dark energy and whether

sufficient Casimir energy can be generated to support a traversable wormhole macroscopically. Additionally, it helps evaluate the maximum Casimir force between nano-devices, which can enhance nanotechnology designs. Usually, a bound is set by either strongly coupled or free theories [14,46,47]. For instance, the KSS bound [14] for fluid is set by the strongly coupled theory dual to gravity, while the Hofman-Maldacena bound [46,47] for bulk central charges is set by free theories. For entanglement entropy, there is a similar bound set by free theories, i.e., $(-F_0/C_T) \geq (-F_0/C_T)_{\text{free scalar}}$ [48], where F_0 is the universal coefficient of entanglement entropy and C_T is a bulk central charge. We remark that the dual brane for entanglement entropy is tensionless while the one for the Casimir effect is tensile. With a tensile brane, we have more parameters to get a smaller lower bound of the Casimir effect in holography than in free theory. The Ising model of Table I also shows that a free scalar does not set the lower bound of the Casimir effect. In AdS_3 , a conformal bootstrap restricts brane tension $|T| \leq 0.99$ slightly stronger than $|T| \leq 1$ [49]. If this is also the case for both T and DGP coupling λ in higher dimensions, it may give a slightly stronger lower bound of the Casimir effect. Anyway, this Letter takes a significant step toward exploring the lower bound of the Casimir effect. Many open problems remain to be explored, particularly the generalization to non-CFTs with interactions.

Acknowledgments. We thank T. Takayanagi and J. X. Lu for valuable comments and discussions. We are grateful to the conference “Gauge Gravity Duality 2024,” where the work was completed. This work is supported by the National Natural Science Foundation of China (Grant No. 12275366).

Data availability. The data that support the findings of this article are openly available [23–29,40].

-
- [1] H. B. G. Casimir, *Indagat. Math.* **10**, 261 (1948).
 - [2] G. Plunien, B. Muller, and W. Greiner, *Phys. Rep.* **134**, 87 (1986).
 - [3] M. Bordag, U. Mohideen, and V. M. Mostepanenko, *Phys. Rep.* **353**, 1 (2001).
 - [4] K. A. Milton, *J. Phys. A* **37**, R209 (2004).
 - [5] M. Bordag, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, *International Series of Monographs on Physics* (Oxford University Press, Oxford, 2009), Vol. 145, pp. 1–768.
 - [6] S. Wang, Y. Wang, and M. Li, *Phys. Rep.* **696**, 1 (2017).
 - [7] L. Vepstas, A. D. Jackson, and A. S. Goldhaber, *Phys. Lett.* **140B**, 280 (1984).
 - [8] M. S. Morris, K. S. Thorne, and U. Yurtsever, *Phys. Rev. Lett.* **61**, 1446 (1988).
 - [9] J. Maldacena and A. Milekhin, *Phys. Rev. D* **103**, 066007 (2021).
 - [10] U. Mohideen and A. Roy, *Phys. Rev. Lett.* **81**, 4549 (1998).
 - [11] G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, *Phys. Rev. Lett.* **88**, 041804 (2002).
 - [12] G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, *Rev. Mod. Phys.* **81**, 1827 (2009).

- [13] J. M. Maldacena, *Int. J. Theor. Phys.* **38**, 1113 (1999) [*Adv. Theor. Math. Phys.* **2**, 231 (1998)].
- [14] P. Kovtun, D. T. Son, and A. O. Starinets, *Phys. Rev. Lett.* **94**, 111601 (2005).
- [15] G. Policastro, D. T. Son, and A. O. Starinets, *Phys. Rev. Lett.* **87**, 081601 (2001).
- [16] M. Brigante, H. Liu, R. C. Myers, S. Shenker, and S. Yaida, *Phys. Rev. D* **77**, 126006 (2008).
- [17] Y. Kats and P. Petrov, *J. High Energy Phys.* 01 (2009) 044.
- [18] M. Brigante, H. Liu, R. C. Myers, S. Shenker, and S. Yaida, *Phys. Rev. Lett.* **100**, 191601 (2008).
- [19] R. X. Miao, *J. High Energy Phys.* 06 (2024) 084.
- [20] K. Jensen and A. O'Bannon, *Phys. Rev. Lett.* **116**, 091601 (2016).
- [21] M. Fujita, T. Takayanagi, and E. Tonni, *J. High Energy Phys.* 11 (2011) 043.
- [22] M. Billò, V. Goncalves, E. Lauria, and M. Meineri, *J. High Energy Phys.* 04 (2016) 091.
- [23] S. Bellucci and A. A. Saharian, *Phys. Rev. D* **80**, 105003 (2009).
- [24] D. M. McAvity and H. Osborn, *Nucl. Phys.* **B406**, 655 (1993).
- [25] R. X. Miao, *J. High Energy Phys.* 07 (2019) 098.
- [26] A. Romeo and A. A. Saharian, *J. Phys. A* **35**, 1297 (2002).
- [27] F. P. Toldin and M. A. Metlitski, *Phys. Rev. Lett.* **128**, 215701 (2022).
- [28] F. Parisen Toldin and S. Dietrich, *J. Stat. Mech.* 11 (2010) P11003.
- [29] M. Hasenbusch, *Phys. Rev. B* **82**, 104425 (2010).
- [30] J. L. Cardy, *Phys. Rev. Lett.* **65**, 1443 (1990).
- [31] T. Takayanagi, *Phys. Rev. Lett.* **107**, 101602 (2011).
- [32] G. R. Dvali, G. Gabadadze, and M. Porrati, *Phys. Lett. B* **485**, 208 (2000).
- [33] R. X. Miao, *J. High Energy Phys.* 06 (2024) 043.
- [34] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/xs4m-qjsb> for holographic bound of Casimir effect for DGP gravity.
- [35] S. de Haro, S. N. Solodukhin, and K. Skenderis, *Commun. Math. Phys.* **217**, 595 (2001).
- [36] M. Miyaji, T. Takayanagi, and T. Ugajin, *J. High Energy Phys.* 06 (2021) 023.
- [37] We thank Takayanagi for valuable comments on negative brane tension and its irrelevance to conical singularity, See also Ref. [36].
- [38] D. M. Dantchev and S. Dietrich, *Phys. Rep.* **1005**, 1 (2023).
- [39] D. Deutsch and P. Candelas, *Phys. Rev. D* **20**, 3063 (1979).
- [40] R. X. Miao, *J. High Energy Phys.* 04 (2025) 023.
- [41] R. X. Miao and C. S. Chu, *J. High Energy Phys.* 03 (2018) 046.
- [42] H. W. J. Bloete, J. L. Cardy, and M. P. Nightingale, *Phys. Rev. Lett.* **56**, 742 (1986).
- [43] C. P. Bachas, *J. Phys. A* **40**, 9089 (2007).
- [44] O. Diatlyk, H. Khanchandani, F. K. Popov, and Y. Wang, *Phys. Rev. Lett.* **133**, 261601 (2024).
- [45] M. Krech, *The Casimir Effect in Critical Systems* (World Scientific, London, 1994).
- [46] D. M. Hofman and J. Maldacena, *J. High Energy Phys.* 05 (2008) 012.
- [47] D. M. Hofman, D. Li, D. Meltzer, D. Poland, and F. Rejon-Barrera, *J. High Energy Phys.* 06 (2016) 111.
- [48] P. Bueno, H. Casini, O. L. Andino, and J. Moreno, *Phys. Rev. Lett.* **131**, 171601 (2023).
- [49] S. Collier, D. Mazac, and Y. Wang, *J. High Energy Phys.* 02 (2023) 019.