

Bottom baryon decays into pseudoscalar meson and dark baryon within perturbative QCD approach

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In the work, we study the production of dark baryon ψ with the bottom-baryon decays $\Lambda_b \rightarrow P\psi$ under the perturbative quantum chromodynamics (QCD) approach. For the type-I/II models in B -mesogenesis scenarios, we calculate the form factors for bottom-baryon decays into meson. Using these results, we obtain branching ratios for $\Lambda_b \rightarrow P\psi$ decays. The PQCD approach calculations show these branching ratios reach $\mathcal{O}(10^{-5})$ in both models, with $\mathcal{B}(\Lambda_b \rightarrow K\psi)$ specifically reaching $\mathcal{O}(10^{-4})$ in type-II. These accessible magnitudes make the predictions testable at the LHCb and B -factories with improved precision.

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I. INTRODUCTION

The Standard Model (SM) of particle physics and the standard cosmological model (Λ CDM) constitute two remarkably successful theoretical frameworks, describing the fundamental interactions of elementary particles and the large-scale evolution of the Universe, respectively. However, their mutual incompatibility raises profound questions, including the origin of dark matter and the observed matter-antimatter asymmetry. Since Sakharov first established the necessary conditions for baryogenesis [1], extensive efforts have been devoted to identifying viable mechanisms capable of explaining these phenomena. Conventional approaches typically rely on high-energy scales and the existence of beyond-SM particles with extreme masses, rendering them experimentally inaccessible in the foreseeable future.

A recently proposed B -mesogenesis scenario [2–5] presents a compelling alternative, offering a unified resolution to both the dark matter relic abundance and baryon asymmetry. In the B -mesogenesis scenario, decays of a heavy scalar field Φ in the early Universe generate $b - \bar{b}$ quark pairs that subsequently hadronize into B -mesons. The neutral $B^0/\bar{B}^0$ mesons undergo CP -violating oscillations before decaying asymmetrically into: (i) dark sector baryons ($\mathcal{B} = -1$) and (ii) visible hadrons ($\mathcal{B} = +1$), thereby producing the observed baryon asymmetry while preserving total baryon number conservation. This framework naturally connects baryogenesis to DM production through experimentally accessible B -meson phenomenology. A distinctive feature of this framework is its testability: predictions can be directly probed at hadron colliders and B -factories [3,6], as well as indirectly constrained through precision measurements at kaon and hyperon facilities [7,8]. Currently, dedicated experimental searches for B -mesogenesis signatures—particularly B meson decays into baryons accompanied by missing energy—are being conducted by the Belle-II [9] and LHCb [10] Collaborations.

Recent theoretical studies have extensively investigated B -meson decays within the B -mesogenesis framework, encompassing both exclusive and semi-inclusive processes. The exclusive decay $B \rightarrow P\psi$ was initially analyzed via leading-twist light-cone sum rules (LCSR) [11], with

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higher-twist corrections subsequently computed [12]. A comprehensive treatment of B -meson decays to octet baryons or charmed antitriplet baryons plus ψ was presented in [13,14], while analogous exclusive decays to baryons with missing energy were explored in [15] for neutralino detection. Additionally, Λ_b decays to light mesons and dark baryons have been studied using both QCD factorization [16] and LCSR methods [17]. Beyond exclusive channels, semi-inclusive decays $B \rightarrow X_{u/c,d/s}\psi$ were investigated via heavy quark expansion (HQE) in [18], yielding constraints on the B -mesogenesis coupling parameters. Previous LCSR studies of B -meson decays in B -mesogenesis have uncertainties from threshold and Borel parameters. In this work, we will use perturbative QCD (pQCD) with k_T -factorization to improve precision in predicting the exclusive decay channel $B \rightarrow P\psi$. This approach which has been successfully applied to the meson and baryon system [19–25], systematically accounts for transverse momentum effects and provides a more reliable theoretical framework for evaluating the decay amplitudes.

This paper is organized as follow: In Sec. II, we introduce the wave functions of Λ_b , π , K , and D in turn, while Sec. III discuss the B -mesogenesis scenario. Section IV contains our perturbative calculation within the PQCD framework. In Sec. V, we study the numerical results, and a conclusion is presented in the last section.

II. WAVE FUNCTIONS

In general, the Lorentz structure of Λ_b baryon wave function can be simplified using the Bargmann-Wigner equation [26] in the heavy quark limit, where the spin and orbital degrees of freedom of the light quark system are decoupled. In coordinate space, the wave function of the Λ_b baryon is defined as [27,28],

$$\begin{aligned} & \langle 0 | u_a^{iT}(x) d_\beta^j(y) b_\gamma^k(0) | \Lambda_b(p_{\Lambda_b}) \rangle \\ &= \frac{1}{6} \epsilon^{ijk} \left\{ f_{\Lambda_b}^{(1)} \left[\left(\frac{\not{n}\not{\bar{n}}}{8} \phi_3^{+-}(x_2, x_3) + \frac{\not{n}\not{\bar{n}}}{8} \phi_3^{-+}(x_2, x_3) \right) \gamma_5 C^T \right] \right. \\ & \quad + f_{\Lambda_b}^{(2)} \left[\left(\frac{\not{n}}{4} \phi_2(x_2, x_3) \right. \right. \\ & \quad \left. \left. + \frac{\not{\bar{n}}}{4} \phi_4(x_2, x_3) \right) \gamma_5 C^T \right] \right\}_{\alpha\beta} \{ u_{\Lambda_b}(k_i, \mu) \}_\gamma. \end{aligned} \quad (1)$$

where the light-cone vectors $\mathbf{n} = 1/\sqrt{2}(1, 0, 0, 1)$ and $\bar{\mathbf{n}} = 1/\sqrt{2}(1, 0, 0, -1)$, satisfying $\mathbf{n} \cdot \bar{\mathbf{n}} = 1$. Here b , u , and d denote quark fields, i , j , and k are color indices, α , β , and γ are spinor indices, C is the charge conjugation matrix, and u_{Λ_b} represents the Λ_b baryon spinor. The normalization constant corresponds to the wave function at the origin in the configuration space. The numerical value $f_{\Lambda_b}^{(1)} \approx f_{\Lambda_b}^{(2)} \equiv f_{\Lambda_b} = 0.021 \pm 0.004 \text{ GeV}^3$ is adopted from experimental data on the semileptonic decay

$\Lambda_b \rightarrow \Lambda_c l \bar{\nu}_l$. The light-cone distribution amplitudes (LCDAs) ϕ_2 , ϕ_3^{+-} , ϕ_3^{-+} and ϕ_4 are expanded in Gegenbauer polynomials. This expansion is derived from QCD sum rules including only the leading-order perturbative contribution,

$$\begin{aligned} \phi_2(x_2, x_3) &= m_{\Lambda_b}^4 x_2 x_3 \left[\frac{1}{\epsilon_0^4} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_0} \right. \\ & \quad \left. + a_2 C_2^{3/2} \left(\frac{x_2 - x_3}{x_2 + x_3} \right) \frac{1}{\epsilon_1^4} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_1} \right], \\ \phi_3^{+-}(x_2, x_3) &= \frac{2m_{\Lambda_b}^3 x_2}{\epsilon_3^3} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_3}, \\ \phi_3^{-+}(x_2, x_3) &= \frac{2m_{\Lambda_b}^3 x_3}{\epsilon_3^3} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_3}, \\ \phi_4(x_2, x_3) &= \frac{5}{\mathcal{N}} m_{\Lambda_b}^2 \int_{m_{\Lambda_b}}^{s_0} ds e^{-s/\tau} (s - m_{\Lambda_b}(x_2 + x_3)/2)^3, \end{aligned} \quad (2)$$

where the relevant parameters, Gegenbauer moment a_2 , dimensionful parameters ϵ_i ($i = 0, 1, 3$), Borel mass τ , continuum threshold s_0 and normalization constant $\mathcal{N}$ are used as

$$\begin{aligned} a_2 &= 0.333_{-0.333}^{+0.250}, & \epsilon_0 &= 200_{-60}^{+130} \text{ MeV}, \\ \epsilon_1 &= 650_{-300}^{+650} \text{ MeV}, & \epsilon_3 &= 230_{-60}^{+60} \text{ MeV}, \\ \tau &\in (0.4, 0.8) \text{ GeV}, & s_0 &= 1.2 \text{ GeV}, \\ \mathcal{N} &= \int_0^{s_0} ds s^5 e^{-s/\tau}. \end{aligned} \quad (3)$$

In this work, the Λ_b baryon decays may proceed to final states containing either light pseudoscalar mesons or charmed meson. For a light pseudoscalar meson moving along the light-cone direction n , its light-cone wave function is defined as [29],

$$\begin{aligned} & \langle P(p_2) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle \\ &= -\frac{i}{\sqrt{2N_c}} \int_0^1 dx e^{ixp_2 \cdot z} \left\{ \gamma_5 \not{p}_2 / \phi^A(x) + m_0^P \gamma_5 \phi^P(x) \right. \\ & \quad \left. - m_0^P \sigma_{\mu\nu} \gamma_5 p_2^\mu z^\nu \frac{\phi^\sigma(x)}{6} \right\}_{\alpha\beta}, \end{aligned} \quad (4)$$

$$\begin{aligned} &= -\frac{i}{\sqrt{2N_c}} \int_0^1 dx e^{ixp_2 \cdot z} \left\{ \gamma_5 \not{p}_2 \phi^A(x) + m_0^P \gamma_5 \phi^P(x) \right. \\ & \quad \left. + m_0^P \gamma_5 (\not{n}\not{\bar{n}} - 1) \phi^T(x) \right\}_{\alpha\beta}, \end{aligned} \quad (5)$$

while the wave function of charmed D meson is defined by the light cone matrix element [30]

$$\int_0^1 \frac{d^4 z}{(2\pi)^4} e^{ik_2 \cdot z} \langle 0 | \bar{c}_\alpha(0) q_\beta(z) | \bar{D}^0(P_D) \rangle \rangle = -\frac{i}{\sqrt{2N_c}} \{ (\not{P}_D + m_0^D) \gamma_5 \phi_D(k_2) \}_{\beta\alpha}. \quad (6)$$

For the numerical calculation, we adopt the parametrization of distribution amplitude [31],

$$\phi_\pi^A(x) = \frac{3f_\pi}{\sqrt{6}} x(1-x)(1 + 0.44C_2^{3/2}(t)), \quad (7)$$

$$\phi_\pi^P(x) = \frac{f_\pi}{2\sqrt{6}} (1 + 0.43C_2^{1/2}(t)), \quad (8)$$

$$\phi_\pi^T(x) = -\frac{f_\pi}{2\sqrt{6}} (C_1^{1/2}(t) + 0.55C_3^{1/2}(t)), \quad (9)$$

$$\phi_K^A(x) = \frac{3f_K}{\sqrt{6}} x(1-x)(1 + 0.17C_1^{3/2}(t) + 0.115C_2^{3/2}(t)), \quad (10)$$

$$\phi_K^P(x) = \frac{f_K}{2\sqrt{6}} (1 + 0.24C_2^{1/2}(t)), \quad (11)$$

$$\phi_K^T(x) = -\frac{f_K}{2\sqrt{6}} (C_1^{1/2}(t) + 0.35C_3^{1/2}(t)), \quad (12)$$

$$\phi_D(x) = \frac{f_D}{2\sqrt{2N_c}} 6x(1-x)[1 + C_D(1-2x)], \quad (13)$$

here $t = 2x - 1$, and the Gegenbauer polynomials are shown as,

$$\begin{aligned} C_1^{1/2}(t) &= t, & C_1^{3/2}(t) &= 3t, & C_2^{1/2}(t) &= \frac{1}{2}(3t^2 - 1), \\ C_2^{3/2}(t) &= \frac{3}{2}(5t^2 - 1), & C_3^{1/2}(t) &= \frac{1}{2}t(5t^2 - 3), \\ C_4^{1/2}(t) &= \frac{1}{8}(35t^4 - 30t^2 + 3), \\ C_4^{3/2}(t) &= \frac{15}{8}(21t^4 - 14t^2 + 1). \end{aligned} \quad (14)$$

with the free shape parameters C_D , ω_D [32], chiral enhancement parameter $m_0^P = m_P^2/(m_{q_1} + m_{q_2})$, decay constants f_π , f_K , f_D [33] in our numerical calculations are adopted as,

$$\begin{aligned} C_D &= 0.5 \pm 0.1, & \omega_D &= 0.1, \\ m_0^\pi &= 1.4 \pm 0.1 \text{ GeV}, & m_0^K &= 1.6 \pm 0.1 \text{ GeV}, \\ m_0^D &= 1.87 \pm 0.1 \text{ GeV}, & f_\pi &= 130 \text{ MeV}, \\ f_K &= 160 \text{ MeV}, & f_D &= 0.209 \pm 0.002 \text{ GeV}, \end{aligned} \quad (15)$$

III. THE B-MESOGENESIS SCENARIO

In B-mesogenesis scenarios, the b -quark can decay into two light quarks and a dark baryon ψ . While the total baryon number is conserved in this process, the invisibility of dark baryon ψ creates an apparent baryon number violation in the visible sector. This signature of baryon nonconservation emerges from the following two types effective Hamiltonian [3]

$$\begin{aligned} \mathcal{H}_{\text{eff}}^I &= -G_{uq}^I \bar{\mathcal{O}}_{uq}^I \psi^c + \text{H.c.} \quad G_{uq}^I = \frac{y_{ub} y_{\phi q}}{M_Y^2}, \\ \mathcal{O}_{uq}^I &= -i\epsilon_{ijk} (u^{iT} C P_R b^j) P_R q^k, \\ \mathcal{H}_{\text{eff}}^{II} &= -G_{uq}^{II} \bar{\mathcal{O}}_{uq}^{II} \psi^c + \text{H.c.} \quad G_{uq}^{II} = \frac{y_{\phi b} y_{uq}}{M_Y^2}, \\ \mathcal{O}_{uq}^{II} &= -i\epsilon_{ijk} (u^{iT} C P_R q^j) P_R b^k, \end{aligned} \quad (16)$$

with $P_R = (1 + \gamma_5)/2$ and C is the charge conjugate matrix, y_{ij} being the product of the two relevant dimensionless couplings, M_Y is the mass of dark baryon. Using the effective Hamiltonians, the decay amplitude for $\Lambda_b \rightarrow M\psi$ can be expressed as,

$$iM = -G_{uq} \bar{u}_\psi(q, s_\psi) \langle M(p_2) | \mathcal{O}_{uq}(0) | \Lambda_b(p_1, s_{\Lambda_b}) \rangle, \quad (17)$$

where u_ψ is the spinor of dark baryon, and momentum $q = p_1 - p_2$. The transition matrix element on the right hand side above can be parametrized by two possible form factors $F_1(q^2)$ and $F_2(q^2)$,

$$\begin{aligned} \langle M(p_2) | \mathcal{O}_{uq}(0) | \Lambda_b(p_1, s_{\Lambda_b}) \rangle \\ = P_R \left[F_1(q^2) + \frac{\not{q}}{m_{\Lambda_b}} F_2(q^2) \right] u_{\Lambda_b}(p_1, s_{\Lambda_b}). \end{aligned} \quad (18)$$

Using the preceding definitions, the decay width for the characteristic baryon-number-violating decays $\Lambda_b \rightarrow \pi\phi$, $\Lambda_b \rightarrow K\psi$, and $\Lambda_b \rightarrow D\psi$ are derived as follows,

$$\begin{aligned} \Gamma(\Lambda_b \rightarrow M\psi) &= \frac{G_{uq}^2 |\vec{q}|}{8m_{\Lambda_b}^2 (2\pi)^5} \left[(m_{\Lambda_b}^2 + m_\psi^2 - m_M^2) \right. \\ &\quad \times \left(F_1^2(q^2) + \frac{m_\psi^2}{m_{\Lambda_b}^2} F_2^2(q^2) \right) \\ &\quad \left. + 4m_\psi^2 F_1(q^2) F_2(q^2) \right]. \end{aligned} \quad (19)$$

The kinematic variables $|\vec{q}|$ denote the magnitudes of the dark baryon ψ momenta in the center-of-mass frame,

$$|\vec{q}| = \frac{1}{2m_{\Lambda_b}} \sqrt{[m_{\Lambda_b}^2 - (m_\psi + m_M)^2][m_{\Lambda_b}^2 - (m_\psi - m_M)^2]}. \quad (20)$$

The coupling constants G_{uq} can be constrained through B -meson semi-inclusive decays. Available theoretical

determinations using heavy quark operator product expansion (OPE) yield upper limits consistent with current experimental results [17],

$$\text{Type I: } G_{ud}^2 < (1.8 \pm 0.35) \times 10^{-14} \text{ GeV}^{-4}, \quad (21)$$

$$G_{us}^2 < (3.75 \pm 0.74) \times 10^{-14} \text{ GeV}^{-4}, \quad (22)$$

$$G_{cd}^2 < (1.06 \pm 0.21) \times 10^{-12} \text{ GeV}^{-4}, \quad (23)$$

$$G_{cs}^2 < (1.63 \pm 0.33) \times 10^{-12} \text{ GeV}^{-4}, \quad (24)$$

$$\text{Type II: } G_{us}^2 < (1.07 \pm 0.21) \times 10^{-11} \text{ GeV}^{-4}, \quad (25)$$

$$G_{cs}^2 < (3.62 \pm 0.72) \times 10^{-10} \text{ GeV}^{-4}, \quad (26)$$

In the numerical analysis, the dark baryon mass is restricted to the kinematic range $0 < m_\psi < m_{\Lambda_b} - m_M$, while the couplings G_{ij} are fixed at their center values of upper limit. For the type-II scheme, an additional four-fermion coupling is required in this work, parametrized as $|G_{ud}|^2 = 10^{-13} \text{ GeV}^{-4}$ [11].

IV. PERTURBATIVE CALCULATIONS

The process $\Lambda_b \rightarrow P\psi$ can be handled by factorization theorems. In general, the factorization theorem states that the transition form factor can be expressed as the convolution of hadronic wave functions $\phi_i(x_i, b_i)$ s and the hard scattering amplitude $H(x_i, b_i)$ by both the longitudinal and the transverse momenta [19]. Radiative corrections to the hadronic wave function can generate double logarithms, which can be resummed into the Sudakov factor. Furthermore, loop corrections to the b -quark decay vertex also give rise to another double logarithms, usually factorized from the hard amplitude and resummed into the threshold Sudakov function [34–38]. Combining all the elements together, the typical factorization formula reads as,

$$\begin{aligned} \mathcal{M} &\sim \int_0^1 dx_2 dx_3 dx_4 \\ &\times \int d^2\mathbf{b}_2 d^2\mathbf{b}_3 d^2\mathbf{b}_4 G_{ij}(t) \phi_{\Lambda_b}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3, t) \\ &\times H(x_2, x_3, x_4, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4, t) \phi_M(x_4, \mathbf{b}_4, t), \end{aligned} \quad (27)$$

where the coefficients $C_{ij}(t)$, organizing the QCD corrections for the effective operator in Eq. (16). The perturbative hard kernel $H(x_i, \mathbf{b}_i, t)$, representing b -quark decay sub-amplitude, and the nonperturbative meson wave function $\phi_i(x_i, \mathbf{b}_i, t)$, describes the evolution from scale t to the lower hadronic scale Λ . For a review of this approach, see Ref. [39].

We will work in the light-cone coordinates. The initial Λ_b baryon is assumed to be at rest, with the final meson recoiling along the light-cone direction $\mathbf{n}$. Then the momentums of hadrons and their valence quarks are parametrized as follows,

$$p_1(\Lambda_b) = \frac{m_{\Lambda_b}}{\sqrt{2}}(1, 1, \mathbf{0}_\perp), \quad p_2(M) = \frac{m_{\Lambda_b}}{\sqrt{2}}(\eta, 0, \mathbf{0}_\perp), \quad (28)$$

$$\begin{aligned} k_1 &= \frac{m_{\Lambda_b}}{\sqrt{2}}(1, x_1, \mathbf{k}_{1\perp}), & k_2 &= \frac{m_{\Lambda_b}}{\sqrt{2}}(0, x_2, \mathbf{k}_{2\perp}), \\ k_3 &= \frac{m_{\Lambda_b}}{\sqrt{2}}(0, x_3, \mathbf{k}_{3\perp}), \end{aligned} \quad (29)$$

$$k_4 = \frac{m_{\Lambda_b}}{\sqrt{2}}(x_4\eta, 0, \mathbf{k}_{4\perp}), \quad k_5 = \frac{m_{\Lambda_b}}{\sqrt{2}}(\bar{x}_4\eta, 0, -\mathbf{k}_{4\perp}). \quad (30)$$

where p_1 and p_2 denote the momentum of the Λ_b baryon and final meson, k_1 is the b -quark momentum, k_2 and k_3 are the two light quark momentum in Λ_b , k_4 and k_5 are quark momentums in final meson. x_i are their longitudinal momentum fractions, and $\mathbf{k}_{i\perp}$ are the corresponding transverse momenta, satisfying $\sum_{i=1,2,3} \mathbf{k}_{i\perp} = 0$. Accordingly, the transfer momentum and light-cone components can be achieved as $q^2 = (p_1 - p_2)^2 = (1 - \eta)m_{\Lambda_b}^2$. In the heavy quark limit, $m_{\Lambda_b} = m_b + \bar{\Lambda}$ ($\bar{\Lambda}$ is the order of QCD scale). Since $m_{\Lambda_b} \gg \bar{\Lambda}$, we expand the amplitudes in terms of $\frac{\bar{\Lambda}}{m_{\Lambda_b}}$.

At the leading order of expansion, $\eta \sim 1$, $q^2 \sim 0$.

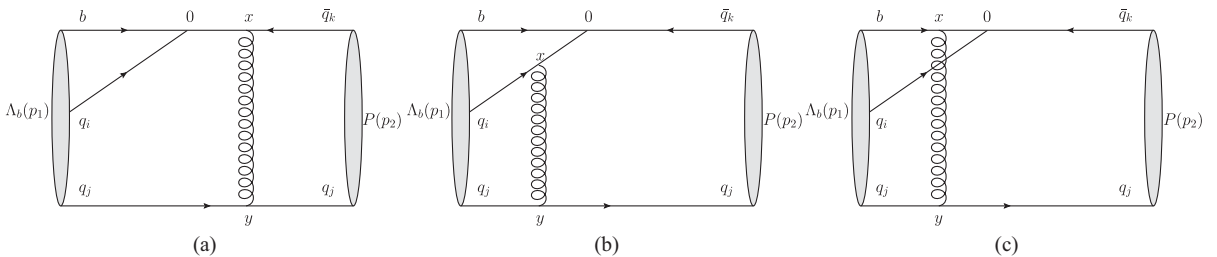


FIG. 1. Typical leading order Feynman diagrams for the production of dark baryon with the bottom-baryon decays $\Lambda_b \rightarrow P\psi$. q_i, q_j are the quarks in initial Λ_b baryon. The final meson P can be chosen as π, K, D .

In the k_T factorization, the perturbative hard-scattering amplitude $H(x_i, b_i)$ is the amplitude for a quark and an antiquark to scatter collinearly via a hard-gluon exchange with wavelengths limited by the impact parameter b_i . In particular, the hard gluon exchange is essential to ensure the applicability of the twist expansion, while soft contributions are expected to be less important owing to the suppression by the Sudakov factor [40]. At the leading order, there are only three possible hard gluon exchange diagrams for $\Lambda_b \rightarrow M(\psi)$ decays, which are shown in Fig. 1. The decay amplitudes are then obtained by convolving the perturbative hard-scattering kernel with the nonperturbative wave functions. The corresponding decay factorization amplitudes take two distinct contraction forms $\mathcal{M}_x(q_i, q_j, \bar{q}_k) = \mathcal{M}_x^{ud}(u, d, \bar{d}/\bar{s}) + \mathcal{M}_x^{du}(d, u, \bar{u}/\bar{c})$. For the type-I effective Hamiltonian, these amplitudes are explicitly given by,

$$\begin{aligned} \mathcal{M}_a = & \frac{2\pi C_F m_{\Lambda_b}}{9\sqrt{3}} E(t_a) \int_0^1 dx_3 dx_4 \int_0^{1/\Lambda} \mathbf{b}_3 d\mathbf{b}_3 \mathbf{b}_4 d\mathbf{b}_4 h_1(x_3, x_4, \mathbf{b}_3, \mathbf{b}_4) P_R \left\{ \left[-\sqrt{2} m_0 m_{\Lambda_b} \eta f_{\Lambda_b}^{(2)} \phi^P(x_4, \mathbf{b}_4) \phi_4(x_3, \mathbf{b}_3) \right. \right. \\ & + \not{n} \left(m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \phi^P(x_4, \mathbf{b}_4) (x_3 \phi_2(x_3, \mathbf{b}_3) + \eta \phi_4(x_3, \mathbf{b}_3)) - \frac{\sqrt{2}}{4} m_{\Lambda_b}^2 x_3 \eta f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) \phi_3^{+-}(x_3, \mathbf{b}_3) \right) \Big]^{ud} \\ & + \left[\sqrt{2} m_0 m_{\Lambda_b} x_3 f_{\Lambda_b}^{(2)} \phi^P(x_4, \mathbf{b}_4) \phi_2(x_3, \mathbf{b}_3) + \frac{1}{2} m_{\Lambda_b}^2 \eta x_3 f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) \phi_3^{-+}(x_3, \mathbf{b}_3) \right. \\ & \left. \left. + \not{n} \left(-\frac{\sqrt{2}}{4} m_{\Lambda_b}^2 x_3 \eta f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) \phi_3^{-+}(x_3, \mathbf{b}_3) - m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \Phi^P(x_4, \mathbf{b}_4) (x_3 \phi_2(x_3, \mathbf{b}_3) + \eta \phi_4(x_3, \mathbf{b}_3)) \right) \right]^{du} \right\} u_{\Lambda_b}, \quad (31) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_b = & \frac{\pi C_F m_{\Lambda_b}^2}{9\sqrt{6}} E(t_b) \int_0^1 dx_2 dx_3 dx_4 \int_0^{1/\Lambda} \mathbf{b}_2 d\mathbf{b}_2 \mathbf{b}_3 d\mathbf{b}_3 \mathbf{b}_4 d\mathbf{b}_4 h_2(x_2, x_3, x_4, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4) P_R \left\{ \left[\frac{\sqrt{2}}{2} m_0 m_{\Lambda_b} (x_2 + x_3) f_{\Lambda_b}^{(2)} \right. \right. \\ & \times \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^T(x_4, \mathbf{b}_4) - \phi^P(x_4, \mathbf{b}_4)) - \frac{1}{2} m_{\Lambda_b}^2 \eta (x_2 + x_3) f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) (\phi_3^{+-}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) \\ & + \phi_3^{-+}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3)) + \not{n} \left(\frac{1}{2} m_0 m_{\Lambda_b} \eta x_4 f_{\Lambda_b}^{(2)} \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) + \phi^T(x_4, \mathbf{b}_4)) \right. \\ & + \frac{1}{2} m_0 m_{\Lambda_b} (x_2 + x_3) f_{\Lambda_b}^{(2)} \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) - \phi^T(x_4, \mathbf{b}_4)) + \frac{\sqrt{2}}{4} m_{\Lambda_b}^2 \eta (x_2 + x_3) f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) \\ & \left. \left. \times (\phi_3^{+-}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + \phi_3^{-+}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3)) \right]^{ud} + \left[\frac{\sqrt{2}}{2} m_0 m_{\Lambda_b} \eta x_4 f_{\Lambda_b}^{(2)} (\phi^P(x_4, \mathbf{b}_4) + \phi^T(x_4, \mathbf{b}_4)) \right. \right. \\ & \times \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + \not{n} \left(-\frac{1}{2} x_4 \eta m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) + \phi^T(x_4, \mathbf{b}_4)) - \frac{1}{2} m_0 m_{\Lambda_b} (x_2 + x_3) f_{\Lambda_b}^{(2)} \right. \\ & \times \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) - \phi^T(x_4, \mathbf{b}_4)) - \frac{1}{2} (\phi_3^{+-}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + \phi_3^{-+}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3)) \\ & \left. \left. \times (x_2 + x_3) \eta m_{\Lambda_b}^2 f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) \right]^{du} \right\} u_{\Lambda_b}, \quad (32) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_c = & \frac{\pi C_F m_{\Lambda_b}^2}{9\sqrt{6}} E(t_c) \int_0^1 dx_2 dx_3 dx_4 \int_0^{1/\Lambda} \mathbf{b}_2 d\mathbf{b}_2 \mathbf{b}_3 d\mathbf{b}_3 \mathbf{b}_4 d\mathbf{b}_4 h_3(x_2, x_3, x_4, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4) P_R \left\{ \left[\frac{1}{2} \bar{x}_2 \eta m_{\Lambda_b}^2 f_{\Lambda_b}^{(1)} \right. \right. \\ & \times \phi^A(x_4, \mathbf{b}_4) \phi_3^{+-}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + \frac{\sqrt{2}}{2} m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} (\bar{x}_2 \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + (1 - x_4 \eta) \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3)) \\ & \times (\phi^P(x_4, \mathbf{b}_4) - \phi^T(x_4, \mathbf{b}_4)) + \not{n} \left(m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \phi^T(x_4, \mathbf{b}_4) (\bar{x}_2 \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + (1 - x_4 \eta) \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3)) \right. \\ & \left. \left. - \frac{\sqrt{2}}{4} \bar{x}_2 \eta m_{\Lambda_b}^2 f_{\Lambda_b}^{(1)} \phi_3^{+-}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) \phi^A(x_4, \mathbf{b}_4) \right]^{ud} + \left[\frac{1}{2} m_{\Lambda_b}^2 \eta \bar{x}_3 f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) \phi_3^{-+}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) \right. \right. \end{aligned}$$

$$\begin{aligned}
& + \frac{\sqrt{2}}{2} m_0 m_{\Lambda_b} \bar{x}_3 f_{\Lambda_b}^{(2)} \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) - \phi^T(x_4, \mathbf{b}_4)) + \not{p} \left(\frac{1}{2} m_0 m_{\Lambda_b} \bar{x}_3 f_{\Lambda_b}^{(2)} \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) \right. \\
& \times (\phi^T(x_4, \mathbf{b}_4) - \phi^P(x_4, \mathbf{b}_4)) + \frac{1}{2} m_0 m_{\Lambda_b} (1 - \eta x_4) f_{\Lambda_b}^{(2)} \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) + \phi^T(x_4, \mathbf{b}_4)) \\
& \left. + \frac{\sqrt{2}}{4} m_{\Lambda_b}^2 \eta \bar{x}_3 f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) (\phi_3^{+-}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) - \phi_3^{-+}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3)) \right) \Big] \Big\} u_{\Lambda_b}, \quad (33)
\end{aligned}$$

where C_F is the color factor, $h_i(x_i, b_i)$ s are the perturbatively calculable hard-scattering kernel mediated by hard-gluon exchange. $E(t_x)$ s are the nonperturbative evolution factors of wave functions. The indices ud and du denote the contraction sequences of the valence quarks $q_i q_j$ within the initial Λ_b baryon in Fig. 1. For the $\pi^0(\psi)$ final state, the decay amplitudes combines contributions from $-\frac{1}{\sqrt{2}} \mathcal{M}_x^{ud}(u, d, \bar{d})$ and $\frac{1}{\sqrt{2}} \mathcal{M}_x^{du}(d, u, \bar{u})$. In contract, the $K^0(\psi)$ and $\bar{D}^0(\psi)$ final states receive single-contribution amplitudes $\mathcal{M}_x^{ud}(u, d, \bar{s})$ or $\mathcal{M}_x^{du}(d, u, \bar{c})$ respectively. The corresponding amplitudes derived from the type-II effective Hamiltonian are

$$\begin{aligned}
\mathcal{M}'_a = & -\frac{2\pi C_F m_{\Lambda_b}}{9\sqrt{3}} E(t_a) \int_0^1 dx_3 dx_4 \int_0^{1/\Lambda} \mathbf{b}_3 d\mathbf{b}_3 \mathbf{b}_4 d\mathbf{b}_4 h_1(x_3, x_4, \mathbf{b}_3, \mathbf{b}_4) P_R \left\{ \left[\sqrt{2} m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \phi^P(x_4, \mathbf{b}_4) (x_3 \phi_2(x_3, \mathbf{b}_3) \right. \right. \\
& - \eta \phi_4(x_3, \mathbf{b}_3)) + \frac{1}{2} m_{\Lambda_b}^2 x_3 \eta f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) \phi_3^{+-}(x_3, \mathbf{b}_3) \Big]^{ud} + \left[\sqrt{2} m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \phi^P(x_4, \mathbf{b}_4) (x_3 \phi_2(x_3, \mathbf{b}_3) \right. \\
& - \eta \phi_4(x_3, \mathbf{b}_3)) + \frac{1}{2} m_{\Lambda_b}^2 x_3 \eta f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) \phi_3^{-+}(x_3, \mathbf{b}_3) \Big]^{du} \Big\} u_{\Lambda_b}, \quad (34)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}'_b = & -\frac{\pi C_F m_{\Lambda_b}^2}{9\sqrt{6}} E(t_b) \int_0^1 dx_2 dx_3 dx_4 \int_0^{1/\Lambda} \mathbf{b}_2 d\mathbf{b}_2 \mathbf{b}_3 d\mathbf{b}_3 \mathbf{b}_4 d\mathbf{b}_4 h_1(x_2, x_3, x_4, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4) P_R \left\{ \left[\frac{\sqrt{2}}{2} (x_2 + x_3) m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \right. \right. \\
& \times \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^T(x_4, \mathbf{b}_4) - \phi^P(x_4, \mathbf{b}_4)) - \frac{\sqrt{2}}{2} \eta x_4 m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) + \phi^T(x_4, \mathbf{b}_4)) \\
& - \frac{1}{2} (x_2 + x_3) \eta m_{\Lambda_b}^2 f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) (\phi_3^{+-}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + \phi_3^{-+}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3)) \Big]^{ud} + \left[-\frac{\sqrt{2}}{2} (x_2 + x_3) m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \right. \\
& \times \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) - \phi^T(x_4, \mathbf{b}_4)) - \frac{\sqrt{2}}{2} \eta x_4 m_0 m_{\Lambda_b} f_{\Lambda_b}^{(2)} \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) + \phi^T(x_4, \mathbf{b}_4)) \\
& - \frac{1}{2} (x_2 + x_3) \eta m_{\Lambda_b}^2 f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) (\phi_3^{+-}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + \phi_3^{-+}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3)) \Big]^{du} \Big\} u_{\Lambda_b}, \quad (35)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}'_c = & -\frac{\pi C_F m_{\Lambda_b}^2}{9\sqrt{6}} E(t_c) \int_0^1 dx_2 dx_3 dx_4 \int_0^{1/\Lambda} \mathbf{b}_2 d\mathbf{b}_2 \mathbf{b}_3 d\mathbf{b}_3 \mathbf{b}_4 d\mathbf{b}_4 h_1(x_2, x_3, x_4, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4) P_R \left\{ \left[\frac{\sqrt{2}}{2} m_0 m_{\Lambda_b} (1 - \eta x_4) f_{\Lambda_b}^{(2)} \right. \right. \\
& \times (\phi^P(x_4, \mathbf{b}_4) - \phi^T(x_4, \mathbf{b}_4)) \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + \not{p} \left(\frac{1}{2} m_0 m_{\Lambda_b} (1 - \eta x_4) f_{\Lambda_b}^{(2)} (\phi^T(x_4, \mathbf{b}_4) - \phi^P(x_4, \mathbf{b}_4)) \right. \\
& \times \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + \frac{1}{2} m_0 m_{\Lambda_b} \bar{x}_2 f_{\Lambda_b}^{(2)} \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) + \phi^T(x_4, \mathbf{b}_4)) - \frac{\sqrt{2}}{4} m_{\Lambda_b}^2 \eta \bar{x}_2 f_{\Lambda_b}^{(1)} \phi^A(x_4, \mathbf{b}_4) \\
& \times \phi_3^{-+}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) \Big]^{ud} + \left[\frac{\sqrt{2}}{2} m_0 m_{\Lambda_b} (1 - \eta x_4) f_{\Lambda_b}^{(2)} (\phi^P(x_4, \mathbf{b}_4) - \phi^T(x_4, \mathbf{b}_4)) \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) \right. \\
& + \not{p} \left(\frac{1}{2} m_0 m_{\Lambda_b} (1 - \eta x_4) f_{\Lambda_b}^{(2)} (\phi^T(x_4, \mathbf{b}_4) - \phi^P(x_4, \mathbf{b}_4)) \phi_4(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) + \frac{1}{2} m_0 m_{\Lambda_b} \bar{x}_3 f_{\Lambda_b}^{(2)} \right. \\
& \times \phi_2(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) (\phi^P(x_4, \mathbf{b}_4) + \phi^T(x_4, \mathbf{b}_4)) - \frac{\sqrt{2}}{4} m_{\Lambda_b}^2 \eta \bar{x}_3 \phi^A(x_4, \mathbf{b}_4) \phi_3^{+-}(x_2, x_3, \mathbf{b}_2, \mathbf{b}_3) \Big]^{du} \Big\} u_{\Lambda_b}. \quad (36)
\end{aligned}$$

The forms for the offshellness of the intermediate gluon β_{e_x} and quarks α_{e_x} in the hard-scattering kernel functions $h_i(x_i, b_i)$ s are derived as [24],

$$\begin{aligned}
 \alpha_1^{ud/du} &= x_3 \eta m_{\Lambda_b}^2, & \alpha_2^{ud} &= x_4(x_2 + x_3) \eta m_{\Lambda_b}^2, & \alpha_2^{du} &= x_4(x_2 + x_3) \eta m_{\Lambda_b}^2, & \alpha_3^{ud} &= \bar{x}_2(x_4 \eta - 1) m_{\Lambda_b}^2, \\
 \alpha_3^{du} &= \bar{x}_3(x_4 \eta - 1) m_{\Lambda_b}^2, & \beta_1^{ud/du} &= x_3 x_4 \eta m_{\Lambda_b}^2, & \beta_2^{ud} &= x_3 x_4 \eta m_{\Lambda_b}^2, & \beta_2^{du} &= x_2 x_4 \eta m_{\Lambda_b}^2, \\
 \beta_3^{ud} &= x_3 \eta x_4 m_{\Lambda_b}^2, & \beta_3^{du} &= x_2 \eta x_4 m_{\Lambda_b}^2,
 \end{aligned} \tag{37}$$

The hard kernel functions $h_i(h'_i)$ [31] consist of the threshold resummation factor $S_t(x_i)$ and the propagator of virtual quark and gluon. The integration of the quark and gluon propagators yields specific Bessel functions, with the hard-kernel functions for the type-I effective Hamiltonian respectively given by

$$\begin{aligned}
 h_1(x_3, x_4, \mathbf{b}_3, \mathbf{b}_4) &= S_t(x_3) K_0(\sqrt{\alpha_1} |\mathbf{b}_4 - b_3|) K_0(\sqrt{\beta_1} |\mathbf{b}_4|), \\
 h_2(x_2, x_3, x_4, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4) &= S_t(x_2 x_4) K_0(\sqrt{\alpha_2} |\mathbf{b}_2|) K_0(\sqrt{\beta_2} |\mathbf{b}_2 - b_3|) |_{b_4 = -b_2}, \\
 h_3(x_2, x_3, x_4, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4) &= S_t(x_4) \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\sqrt{|\alpha_3|} |\mathbf{b}_2|) H_0^{(1)}(\sqrt{|\beta_3|} |\mathbf{b}_3|) |_{b_4 = b_2 - b_3}.
 \end{aligned} \tag{38}$$

where K_0 and $H_0 = J_0 + iY_0$ are Bessel functions [27]. For the type-II model, h'_1 is identical to h_1 , while h'_2 and h'_3 are obtained through the symmetry operation $\mathbf{b}_2 \leftrightarrow \mathbf{b}_3$. The derivation of the relevant Fourier transformation proceeds as,

$$\int d^2k \frac{e^{ik \cdot b}}{\alpha + k^2} = 2\pi \left[K_0(\sqrt{\alpha} b) \theta(\alpha) + \frac{i\pi}{2} H_0(\sqrt{\alpha} b) \theta(-\alpha) \right]. \tag{39}$$

Loop corrections to the b-quark decay vertex give rise to double logarithms. The resummation of this type of double logarithms leads to the Sudakov factor $S_t(x)$. The factor is universal, independent of flavors of internal quarks, twists, or the specific process, and follows the standard parametrization [41]

$$S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c, \tag{40}$$

the parameter $c = 0.4$ is determined from the best fit to the next-to-leading-logarithm threshold resummation in moment space, diminishing as $x \rightarrow 0, 1$, modifying the end-point behavior of baryon/meson distribution amplitudes effectively [42,43]. The evolution factors $E(t_x)$ in the factorization formulas are given by

$$E(t_a) = \alpha_s(t) \exp(-S_{\Lambda_b}(t) - S_P(t)), \tag{41}$$

where the Sudakov exponential stems from the resummation of double logarithms generated by radiative corrections to the hadronic wave function. It decreases fast with increasing b and vanishes at $b = 1/\Lambda$ [44].

$$\begin{aligned}
 S_{\Lambda_b}(t) &= s(x_2 m_{\Lambda_b}, \omega) + s(x_3 m_{\Lambda_b}, \omega) \\
 &\quad + \frac{8}{3} \int_{\omega}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \\
 S_M(t) &= s(x_4 \eta m_{\Lambda_b}, \mathbf{b}_4) + s(\bar{x}_4 \eta m_{\Lambda_b}, \mathbf{b}_4) \\
 &\quad + 2 \int_{1/b_3}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})),
 \end{aligned} \tag{42}$$

$$S_D(t) = s(x_4 \eta m_{\Lambda_b}, \mathbf{b}_4) + 2 \int_{1/b_4}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})). \tag{43}$$

The factorization scales ω represent the inverse of a typical transverse distance among the three valence quarks of the initial baryon [27]. The choices of ω is

$$\begin{aligned}
 \omega &= \min\left(\frac{1}{b_1}, \frac{1}{b_2}, \frac{1}{b_3}\right), & b_1 &= |\mathbf{b}_2 - \mathbf{b}_3|, \\
 b_2 &= |\mathbf{b}_1 - \mathbf{b}_3|, & b_3 &= |\mathbf{b}_1 - \mathbf{b}_2|,
 \end{aligned} \tag{44}$$

with the quark mass anomalous dimension $\gamma_q = -\alpha_s/\pi$ [45]. The explicit expression of $s(Q, b)$ can be found, for example, in Appendix A of Ref [46]. The hard scales t are chosen as the maxima of the virtualities of internal particles involved in the hard amplitudes $H(x_i, b_i)$ [42]. It reflects the fact that energy releases and evolution effects involved in different B meson decay modes are different.

$$t_a = \max(\sqrt{\alpha_1}, \sqrt{\beta_1}, \omega, 1/b_4), \tag{45}$$

$$t_b = \max(\sqrt{\alpha_2}, \sqrt{\beta_2}, \omega, 1/b_3, 1/|\mathbf{b}_2 - \mathbf{b}_3|), \tag{46}$$

$$t_c = \max(\sqrt{\alpha_3}, \sqrt{\beta_3}, \omega, 1/b_2, 1/b_3). \tag{47}$$

It is straightforward to extract the form factor F_1 and F_2 by comparing with Eq. (18), where $\mathcal{M}_i(\not{p})$ and

$\mathcal{M}_i(I)$ correspond to the Lorentz structures $\not{p}$ and I in the amplitudes.

$$F_1^\pi(q^2=0) = \frac{1}{\sqrt{2}} \sum_i^{a,b,c} (\mathcal{M}_i^{du}(I) - \mathcal{M}_i^{ud}(I)) + \frac{1-\eta}{\eta} (\mathcal{M}_i^{du}(\not{p}) - \mathcal{M}_i^{ud}(\not{p})), \quad (48)$$

$$F_2^\pi(q^2=0) = \sum_i^{a,b,c} \frac{1}{\eta} \mathcal{M}_i^{du}(\not{p}) - \frac{1}{\eta} \mathcal{M}_i^{ud}(\not{p}), \quad (49)$$

$$F_1^K(q^2=0) = \sum_i^{a,b,c} \mathcal{M}_i^{ud}(I) + \frac{\sqrt{2}(1-\eta)}{\eta} \mathcal{M}_i^{ud}(\not{p}), \quad (50)$$

$$F_2^K(q^2=0) = \sum_i^{a,b,c} \frac{\sqrt{2}}{\eta} \mathcal{M}_i^{ud}(\not{p}), \quad (51)$$

$$F_1^D(q^2=0) = \sum_i^{a,b,c} \mathcal{M}_i^{du}(I) + \frac{\sqrt{2}(1-\eta)}{\eta} \mathcal{M}_i^{du}(\not{p}), \quad (52)$$

$$F_2^D(q^2=0) = \sum_i^{a,b,c} \frac{\sqrt{2}}{\eta} \mathcal{M}_i^{du}(\not{p}). \quad (53)$$

V. NUMERICAL RESULTS

We take the QCD scale $\Lambda = 0.25 \pm 0.05$ GeV, and the wave function parameters of in Eqs. (3) and (15). In addition, the transverse momentum dependent parton distributions of the final meson are also included. In the

work, we use the Gaussian form to factorize the wave functions [47],

$$\phi(x, \mathbf{b}) = \phi(x) \exp\left(-\frac{\mathbf{b}^2}{4\beta^2}\right), \quad (54)$$

where $\phi(x)$ is the longitudinal momentum distribution amplitude, the exponential factor describes the transverse momentum distribution of final meson. The parameters β characterize the shape of the transverse momentum distributions, which is expected at the order of QCD scale Λ and related with the root of the averaged transverse momentum square $\langle k_T^2 \rangle$. Following Ref. [48] we choose $\langle k_T^2 \rangle^{1/2} = 0.35$ GeV, $\beta^2 = 4$ GeV⁻².

Accordingly, the form factor $F_{1,2}(0)$ for bottom-baryon decays into meson can be obtained, the corresponding numerical results for the type-I/II models are listed in Table I. The tabulated uncertainties primarily arise from the parameters of wave function and QCD scale Λ . Then the branching ratio can be obtained by substituting the results for the form factors in to Eq. (19). It is shown that these branching ratios are larger than those obtained via HQE-LCSR [17], yet consistent with prediction in Ref [3]. Under the type-I model, the branching ratios calculated using the k_T -factorization approach are consistently about two to three orders of magnitude larger than those obtained via LCSR with $m_\psi = 2$ GeV. However, as the value of m_ψ increases, the discrepancy between them diminishes significantly. At m_ψ around 5 GeV, the results from the two methods become consistent within their respective uncertainties.

$$\begin{aligned} \mathcal{B}^{\text{PQCD}}(\Lambda_b^0 \rightarrow \pi^0 \psi) &= \begin{cases} 0.14_{-0.10}^{+0.12} \times 10^{-5}, m_\psi = 2.0 \text{ GeV} \\ 0.40_{-0.20}^{+0.20} \times 10^{-7}, m_\psi = 5.0 \text{ GeV} \end{cases}, & \mathcal{B}^{\text{LCSR}}(\Lambda_b^0 \rightarrow \pi^0 \psi) &= \begin{cases} 0.15_{-0.10}^{+0.05} \times 10^{-8}, m_\psi = 2.0 \text{ GeV} \\ 0.85_{-0.40}^{+0.35} \times 10^{-8}, m_\psi = 5.0 \text{ GeV} \end{cases}, \\ \mathcal{B}^{\text{PQCD}}(\Lambda_b^0 \rightarrow K^0 \psi) &= \begin{cases} 0.85_{-0.55}^{+1.00} \times 10^{-5}, m_\psi = 2.0 \text{ GeV} \\ 0.10_{-0.08}^{+0.10} \times 10^{-6}, m_\psi = 5.0 \text{ GeV} \end{cases}, & \mathcal{B}^{\text{LCSR}}(\Lambda_b^0 \rightarrow K^0 \psi) &= \begin{cases} 0.45_{-0.25}^{+0.25} \times 10^{-8}, m_\psi = 2.0 \text{ GeV} \\ 0.15_{-0.05}^{+0.10} \times 10^{-8}, m_\psi = 5.0 \text{ GeV} \end{cases}, \\ \mathcal{B}^{\text{PQCD}}(\Lambda_b^0 \rightarrow \bar{D}^0 \psi) &= \begin{cases} 2.0_{-1.2}^{+2.2} \times 10^{-5}, m_\psi = 2.0 \text{ GeV} \\ 0.80_{-0.75}^{+2.20} \times 10^{-5}, m_\psi = 3.7 \text{ GeV} \end{cases}, & \mathcal{B}^{\text{LCSR}}(\Lambda_b^0 \rightarrow \bar{D}^0 \psi) &= \begin{cases} 0.65_{-0.25}^{+0.20} \times 10^{-7}, m_\psi = 2.0 \text{ GeV} \\ 0.50_{-0.15}^{+0.35} \times 10^{-7}, m_\psi = 3.7 \text{ GeV} \end{cases}. \end{aligned} \quad (55)$$

TABLE I. The form factors for the transition $\Lambda_b \rightarrow \pi(\psi), K(\psi), D(\psi)$ in the type-I and II models.

	Type-I			Type-II	
	$F_1(0)$	$F_2(0)$		$F_1(0)$	$F_2(0)$
$\Lambda_b^0 \rightarrow \pi^0$	$1.189_{-0.492}^{+0.474}$	$-1.400_{-0.591}^{+0.440}$	$\Lambda_b^0 \rightarrow \pi^0$	$0.059_{-0.006}^{+0.003}$	$0.055_{-0.031}^{+0.082}$
$\Lambda_b^0 \rightarrow K^0$	$-1.852_{-0.839}^{+0.786}$	$0.768_{-0.305}^{+0.325}$	$\Lambda_b^0 \rightarrow K^0$	$1.846_{-0.672}^{+0.837}$	$-0.146_{-0.068}^{+0.071}$
$\Lambda_b^0 \rightarrow \bar{D}^0$	$0.569_{-0.174}^{+0.256}$	$-2.227_{-0.948}^{+1.017}$	$\Lambda_b^0 \rightarrow \bar{D}^0$	$1.264_{-0.354}^{+0.246}$	$0.107_{-0.079}^{+0.051}$

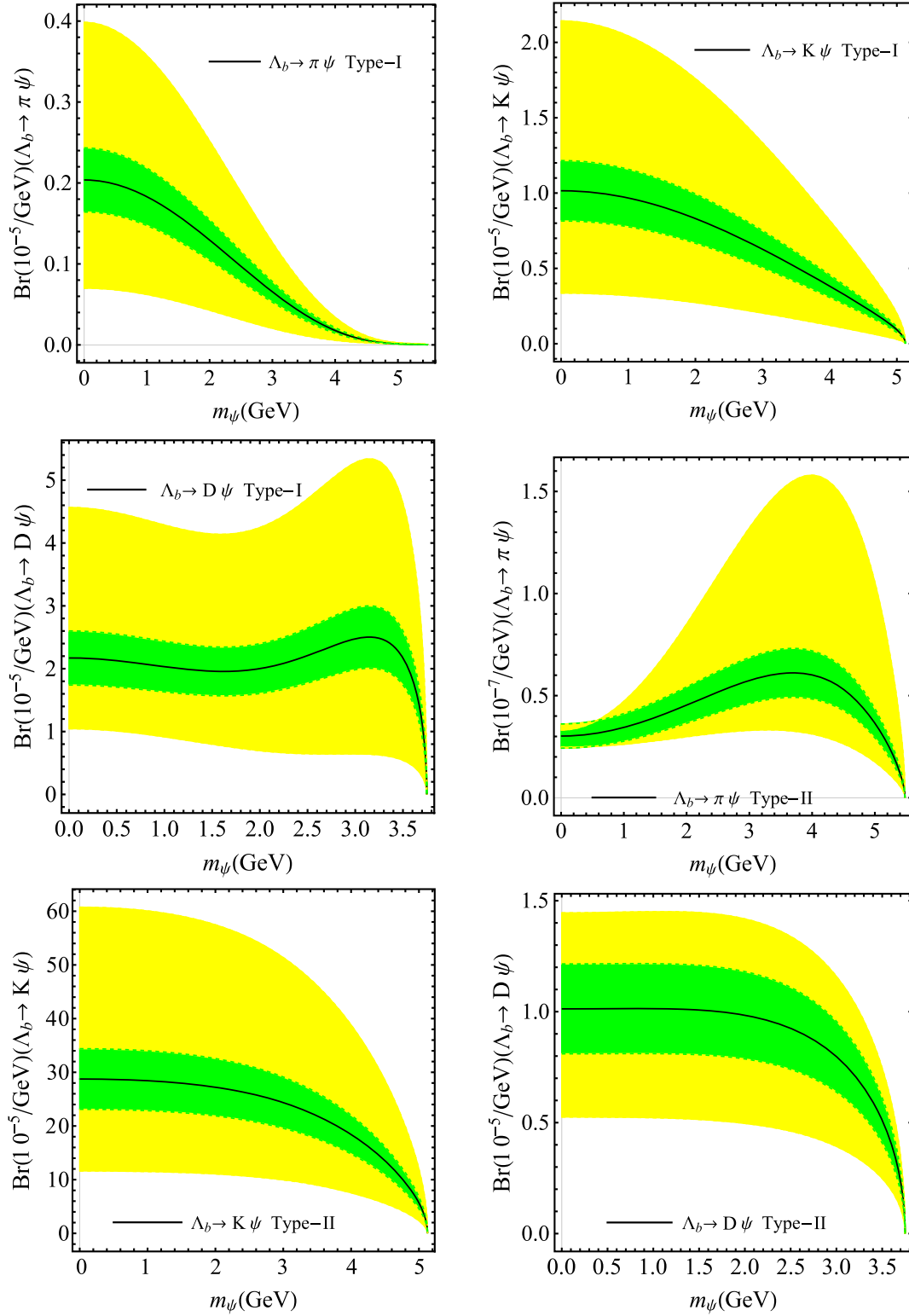


FIG. 2. The upper limits of branching fractions for $\Lambda_b \rightarrow M\psi$ as function of dark baryon mass m_ψ in the type-I and II models. The yellow and green bands denote the uncertainties from couplings G_{ij} and form factor F_i .

In the type-II model, the LCSR calculation yields a branching ratio for $\Lambda_b \rightarrow K\psi$ of less than 10^{-10} , while that for $\Lambda_b \rightarrow \pi\psi$ is even smaller. In contrast, the

k_T -factorization calculation indicates that the branching ratio for $\Lambda_b \rightarrow K\psi$ can reach up to 10^{-4} at $m_\psi = 2.0$ GeV.

$$\begin{aligned}
\mathcal{B}^{\text{PQCD}}(\Lambda_b^0 \rightarrow \pi^0 \psi) &= \begin{cases} 0.40_{-0.15}^{+0.40} \times 10^{-5}, m_\psi = 2.0 \text{ GeV} \\ 0.30_{-0.15}^{+0.70} \times 10^{-5}, m_\psi = 5.0 \text{ GeV} \end{cases}, & \mathcal{B}^{\text{PQCD}}(\Lambda_b^0 \rightarrow K^0 \psi) &= \begin{cases} 2.8_{-1.7}^{+3.0} \times 10^{-4}, m_\psi = 2.0 \text{ GeV} \\ 0.8_{-0.6}^{+0.4} \times 10^{-4}, m_\psi = 5.0 \text{ GeV} \end{cases}, \\
\mathcal{B}^{\text{PQCD}}(\Lambda_b^0 \rightarrow \bar{D}^0 \psi) &= \begin{cases} 1.0_{-0.5}^{+0.45} \times 10^{-5}, m_\psi = 2.0 \text{ GeV} \\ 0.20_{-0.10}^{+0.20} \times 10^{-5}, m_\psi = 3.7 \text{ GeV} \end{cases}, & \mathcal{B}^{\text{LCSR}}(\Lambda_b^0 \rightarrow \bar{D}^0 \psi) &= \begin{cases} 0.35_{-0.10}^{+0.05} \times 10^{-6}, m_\psi = 2.0 \text{ GeV} \\ 0.15_{-0.05}^{+0.05} \times 10^{-6}, m_\psi = 3.7 \text{ GeV} \end{cases}. \quad (56)
\end{aligned}$$

Given the discrepancies between the two approaches, hard dynamics is assumed to dominate in the B meson transition form factors in the k_T factorization, while soft contribution is less important because of suppression from the Sudakov mechanism. This stands in contrast to the light cone sum rules formalism. Numerically, in the LCSR approach, the final results exhibit a strong dependence on both the threshold parameter s_0 and the Borel parameter T^2 , whose values are often not trivially determined. As demonstrated in Ref. [17], the lower limits of T^2 cannot be well determined at the leading order. This may therefore introduce significant differences in the branching ratios derived from the two methods.

We present the full m_ψ -dependence of the predicted branching ratios in Fig. 2, showing upper limits for $\Lambda_b \rightarrow P\psi$ decays in type I/II models. The yellow and green bands quantify uncertainties from couplings G_{ij} and form factors F_i respectively. Branching ratios reach $\mathcal{O}(10^{-5})$ in both models, with $\mathcal{B}(\Lambda_b \rightarrow K\psi)$ in type II attaining $\mathcal{O}(10^{-4})$. These accessible magnitudes make our predictions testable at the LHCb and B-factories with improved precision. The pronounced sensitivity of branching ratios to the couplings G_{ij} , evident from the Fig. 2, motivates defining new ratios of different decay channels. For the type I model, we define ratios R_1^I and R_2^I ,

$$\begin{aligned}
R_1^I &= \frac{\mathcal{B}(\Lambda_b \rightarrow K^0 \psi)}{\mathcal{B}(\Lambda_b \rightarrow \pi^0 \psi)} \sim 8.4, \\
R_2^I &= \frac{\mathcal{B}(\Lambda_b \rightarrow \bar{D}^0 \psi)}{\mathcal{B}(\Lambda_b \rightarrow \pi^0 \psi)} \sim 28.8, \quad (57)
\end{aligned}$$

with branching ratios at the benchmark dark baryon mass $m_\psi = \frac{1}{2}(m_{\Lambda_b} - m_\pi)$. These ratios significantly deviates from the ratio of G_{ij} factors

$$R_1^I = \left| \frac{y_{\psi s}}{y_{\psi d}} \right|^2 \sim 2.1, \quad R_2^I = \left| \frac{y_{cb}}{y_{ub}} \right|^2 \sim 58.9, \quad (58)$$

which indicates large contribution at the amplitude level. For completeness, we obtain the upper limits of the $\Lambda_b \rightarrow P\psi$ branching fractions as functions of m_ψ for type I and II models, shown in Fig. 2. The yellow and green bands represent uncertainties from the couplings G_{ij} and form factors F_i . Given the strong dependence of these parameters G_{ij} and F_i , future measurements will provide essential constraints for exploring ψ -related physics.

VI. CONCLUSIONS

In this work, we studied the production of dark baryon ψ by the decays of Λ_b within the PQCD framework for the first time. In the type-I/II models, form factors and branching ratios of $\Lambda_b \rightarrow \pi\psi$, $\Lambda_b \rightarrow D\psi$ and $\Lambda_b \rightarrow K\psi$ are calculated respectively. The numerical results show that those branching ratios can reach the order of 10^{-5} . The accessible magnitudes make our predictions testable at the LHCb and B-factories experiments. It should be pointed out that the coupling constants of the two types effective operators G_{uq} may alter the results by a factor of two.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [49].

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