

Holographic open-closed exchange in double deeply virtual Compton scattering: Fixed- j structural matching to the $\pm$ -basis Wilson kernels

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We show that fixed- j holographic double deeply virtual Compton scattering (DDVCS) and its DVCS limit give a fixed-scale structural match to the singlet conformal-operator product expansion Wilson-kernel family of QCD in the leading-twist unpolarized singlet vector channel. The open-string $C_1(\delta, \vartheta)$ hypergeometric kernel was derived by Nishio-Watari; the new closed-string result is that the Brower-Polchinski-Strassler-Tan (BPST) upper Witten vertex gives the same family with a different near-boundary power count, because $\Delta_c(j)$ is fixed by the BPST trajectory rather than by open-Regge data. The Mellin exponent is therefore derived as $\delta_c(j) = j + \Delta_c(j) - 2 = 2j + \gamma_c(j)$ rather than inserted by hand. The even open branch gives the parallel unprotected counterpart in the projected singlet vector amplitude. At $Q = \mu = \mu_0 = \mu_*$, the point is not merely that equal-endpoint evolution is trivial; it is that the ultraviolet Witten vertex has already produced the fixed- j conformal kernel before the lower conformal moment is matched. In the conformal partial-wave/conformal subtraction representation, beta-proportional conformal-anomaly terms and scheme transformations are coefficient/evolution bookkeeping away from the matching point, not replacements for the projected fixed-scale hypergeometric η/ξ Wilson-kernel family. The protected/unprotected $j = 2$ split identifies the protected closed branch with the $(-)$ conformal partial wave and the even open branch with the unprotected $(+)$ counterpart; these are mixed singlet eigenchannels, not literal unmixed quark/gluon operators at finite N_c . The comparison therefore gives a direct conformal-basis interpretation of the fixed- j holographic t -channel Witten diagram: the ultraviolet vertex generates the universal Wilson-kernel family, while the lower vertex supplies the channel-dependent nonperturbative conformal moment.

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I. INTRODUCTION

Deeply virtual Compton scattering (DVCS), timelike Compton scattering (TCS), and double deeply virtual Compton scattering (DDVCS) are among the cleanest probes of generalized parton distributions (GPDs), which were introduced in the original momentum fraction x -space/nonforward-distribution formulations by Müller *et al.*, by Ji and by Radyushkin, and subsequently developed into a broad QCD framework [1–8]. At leading power, the DVCS amplitude admits a collinear factorization theorem: the short-distance current-current correlation function is separated from the long-distance hadronic matrix element in the generalized Bjorken regime, to all orders in perturbation theory [9,10]. The same amplitude can be organized not only in the original momentum-fraction language but also in the conformal-basis/conformal-moment language, where

complex conformal spin diagonalizes the leading-order singlet evolution and makes the operator product expansion of the two currents particularly transparent [11–17]. In the present work this conformal representation supplies the perturbative-QCD lens through which the fixed- j holographic t -channel Witten diagram is interpreted. Recent two-loop calculations make the relevant conformal-basis kernels and moments increasingly explicit [18–23].

The conformal/Gegenbauer basis is used here because its moments are matrix elements of local twist-two conformal operators, the skewness dependence of the physical even-spin moments is constrained by Lorentz covariance and polynomiality, and the leading-order singlet evolution is diagonal in the $\pm$ basis. Holographic QCD supplies a geometric representation of the corresponding fixed- j hadronic matrix elements through the lower Witten vertex. The conformal operator product expansion (OPE) therefore provides a precise structural language for identifying the upper Witten vertex with the hard kernel and the lower Witten vertex with the nonperturbative moment when the holographic diagram is viewed through the perturbative-QCD lens.

From the holographic side, a classic ultraviolet anchor is the vacuum vector-current two-point function. In bottom-up

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anti-de Sitter (AdS)/QCD, the on-shell bulk action reproduces the logarithmic Q^2 behavior of the QCD current-current correlator and fixes the bulk gauge coupling [24]. In the soft-wall model, the same current is sourced by the bulk-to-boundary propagator of Grigoryan and Radyushkin [25]. This vacuum matching fixes the non-normalizable current source; it does not by itself fix spin- j source normalizations, trajectory data, or hadronic conformal moments. The claim below is that the same current insertion, embedded in the fixed- j hadronic Witten diagram, yields the QCD conformal Wilson-kernel family, while the exchanged-operator normalization and lower conformal moment remain matched or modeled inputs. At high energy and small x , the closed-string channel is anchored by the Brower-Polchinski-Strassler-Tan (BPST) graviton trajectory [26], while holographic DVCS amplitudes have been analyzed in conformal Regge theory by Costa and Djurić, in related work by Brower *et al.* [27,28], and on s -channel/large- x scalar target in [29]. The connection between holographic DVCS/GPD amplitudes and conformal-OPE language, including the open-string sector, was made especially transparent by Nishio and Watari [30,31]. Appendix B, especially Eqs. (285)–(289) of [30,31], defines the $C_1(\delta, \vartheta)$ AdS integral and rewrites it as the same Gauss-hypergeometric conformal-OPE kernel used below. More broadly, gravitonlike effective descriptions of 2^{++} tensor mesons have long appeared in AdS/QCD [32], and tensor-meson exchange remains phenomenologically relevant in current hadronic analyses [33].

The novelty relative to Nishio-Watari is therefore sharply delimited. Their Appendix B supplies the open-string C_1 hypergeometric kernel. Here the same kernel is derived for the closed BPST exchange for the first time in this fixed- j Compton setting, with the exponent $\delta_c(j) = j + \Delta_c(j) - 2$ fixed by the explicit BPST upper-vertex z -power counting. This closed result is not an immediate corollary of the open one: the closed exponent follows from the BPST boundary mode and trajectory data and therefore requires an independent near-boundary derivation with no open-string input. The paper then places the closed BPST and even open Nishio-Watari branches in the modern QCD $\pm$ eigenbasis, keeps the finite forward normalizations $c_j^\pm$, interprets the lower Witten vertex as the nonperturbative conformal moment, and uses the protected/unprotected $j = 2$ split to identify the protected branch and supply the analytic-continuation prescription for the structural closed/open $\leftrightarrow (-)/(+)$ assignment. In one sentence: the novelty is not the open C_1 identity itself, but the independent closed BPST fixed- j upper-Witten-vertex derivation of the same conformal Wilson-kernel family, with the Mellin label fixed by z -power counting, and the resulting structural open/closed $\leftrightarrow (+)/(-)$ operator-basis mapping that identifies what the fixed- j holographic t -channel Witten diagrams compute in the perturbative-QCD conformal basis. This is a structural matching result, not a rediscovery of the open-string hypergeometric identity.

The central subject of this paper is the fixed- j holographic description of DDVCS and its DVCS limit. We derive the amplitude directly from the t -channel Witten diagram, separating the universal ultraviolet photon vertex from the infrared hadronic matrix element. The conformal-basis perturbative-QCD representation is then used as a structural diagnostic: it identifies the upper Witten vertex with the fixed- j Wilson-kernel family and the lower Witten vertex with the associated nonperturbative conformal moment, thereby clarifying the operator content of the holographic construction.

The cleanest route is to begin with the fixed- j t -channel Witten diagram and factorize it before performing the Sommerfeld-Watson resummation. In that order of limits the physics is transparent. The upper vertex depends only on the two virtual photons and the near-boundary behavior of the exchanged spin- j mode. The lower vertex is the nucleon matrix element of the same spin- j operator. The spin projector supplies the kinematical polynomial in the skewness variables. What remains after this separation is the holographic analogue of a fixed- j Compton form factor.

The crucial result is that the upper holographic impact factor is universal. In the conformal limit it is determined entirely by the pure-AdS photon wave functions and yields a closed-form Gauss hypergeometric function of η^2/ξ^2 . The lower hadronic vertex carries all infrared model dependence and may be evaluated in the soft-wall model, or in any other infrared completion, without altering the hard kernel. This already gives a clean holographic factorization statement, see Figs. 1 and 2. The structural comparison made here is that the same hypergeometric family is precisely the one that appears in the conformal-basis Wilson coefficients of the singlet vector Compton form factor in perturbative QCD. Evidence that the requisite near-boundary factorization of the bulk-to-bulk propagator is not an artifact of a particular infrared model can be traced through the pure-AdS/hard-wall analysis of Nishio and Watari, the coordinate-space graviton analysis of Hatta and Yang, and the

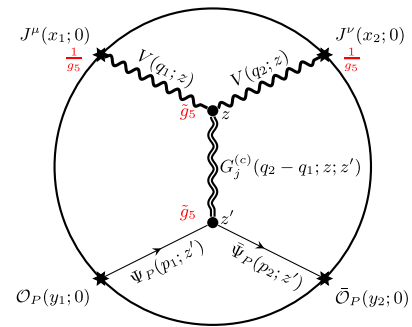


FIG. 1. Standard closed-channel Witten diagram before holographic collinear factorization.

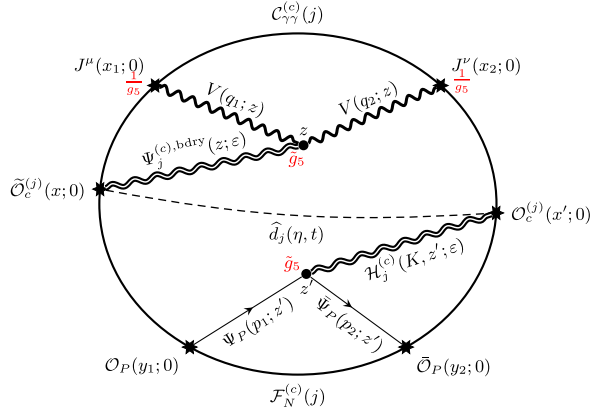


FIG. 2. Factorized closed-channel Witten diagram for holographic collinear factorization of the DDVCS Compton amplitude.

hard-/soft-wall Witten-diagram treatment of heavy quarkonium photoproduction by Mamo and Zahed [30,31,34,35].

In concrete terms, the UV part of the holographic Witten diagram produces the same fixed- j conformal Wilson-kernel family that appears in the conformal-basis collinear-factorization formula, the IR part provides the corresponding matched nonperturbative conformal moment, and the first physical even moment $j = 2$ anchors the closed/(-) and open/(+) channel assignment dynamically rather than conventionally. At $Q = \mu = \mu_0 = \mu_*$, the evolution factors are identities, but the fixed-scale coefficient is the hypergeometric Wilson kernel multiplied by the finite normalization $c_j^\pm$, and the lower conformal moment remains a matched input. The precise fixed- j invariant and matching scale are specified in Sec. II A.

More precisely, in the kinematic window

$$Q_1^2 \sim Q_2^2 \sim \tilde{Q}^2 \equiv Q^2, \quad -t, M_N^2 \ll Q_i^2, \\ \xi, \eta \ll 1, \quad \vartheta \equiv \frac{\eta}{\xi} = \mathcal{O}(1), \quad i = 1, 2. \quad (1)$$

The projection used throughout is the transverse-transverse scalar component of the current correlator. In this kinematic window the matching is applied after this transverse-transverse (TT) scalar projection: $F_{z\mu}$ -induced structures involving K_0 or $z\partial_z K_1$ project onto longitudinal or distinct invariant amplitudes and are not part of the scalar kernel matched below. At the matching point

$$Q = \mu = \mu_0 = \mu_*, \quad (2)$$

we will show that the even open-string channel matches the + conformal-basis Wilson coefficient and conformal moment, while the closed-string channel matches the corresponding - objects. The natural arena for stating this result is DDVCS, where both photons are off shell and the conformal structure is fully visible; the DVCS limit may then be taken only after the fixed- j analysis. This ordering

matters physically: when both photons remain off shell, the ratio η^2/ξ^2 appears in the hard kernel in its fully visible conformal form, whereas taking one photon on shell too early hides part of that structure even though it does not change the underlying fixed- j operator statement [3,22].

The most decisive diagnostic is the first physical even moment, $j = 2$. On the perturbative side, the (-) singlet anomalous dimension vanishes at $j = 2$ as the leading-order manifestation of the momentum-sum rule, while the (+) eigenvalue remains finite; the explicit formulas are collected in Appendix B, especially Eqs. (B15) and (B16). On the holographic side, the closed branch passes through the bulk graviton and therefore through the conserved total energy-momentum tensor, so the corresponding anomalous dimension vanishes at the same point. The open branch has no such symmetry protection at $j = 2$; in the large- λ regime its value is finite and positive, while any accidental zero at a special finite value of λ would only produce a temporary degeneracy at that point, not a Ward identity. The open/+ and closed/- assignment is therefore not a fit-driven analogy but a structural prescription anchored by symmetry and by the analytic behavior of the two branches.

It is equally important to state the role of conformality precisely. The conformal fixed-point formulas provide the cleanest analytic representation of the hard kernel because they display the scale dependence as a pure power law. No claim is made that real QCD is near a conformal fixed point; the conformal expressions are used as the conformal partial-wave/conformal subtraction (CS) representation, where the hard kernel is most transparent. The matching itself is imposed at the single scale (2). At that point, the matched object is the fixed- j conformal Wilson-kernel family. Beta-proportional conformal-anomaly terms, finite scheme transformations, and logarithmic running are part of the coefficient/evolution bookkeeping away from the matching point. They refine the anomalous dimensions, finite coefficient normalizations, and evolution factors, but they are not an alternative fixed-scale η/ξ kernel for the projected invariant. Recent next-to-next-to-leading order (NNLO) coefficient functions for DVCS and DDVCS sharpen the perturbative input for $\gamma_j^\pm(\alpha_s)$, $c_j^\pm(\alpha_s)$, and evolution effects [18–23]. They refine the perturbative data entering the same conformal partial wave rather than replacing the fixed-scale hypergeometric kernel.

The paper is conceptual rather than numerical. It determines the operator meaning of the holographic fixed- j amplitude, isolates the universal part from the infrared-sensitive part, and places the resulting nonperturbative moments in the same language used by collinear factorization and, ultimately, by lattice-QCD determinations of moments and form factors. In this sense it is a structural paper about what the holographic amplitude is computing. The phenomenological application of the holographic DDVCS/DVCS amplitude to experimental observables, Compton form-factor extraction, and uncertainty analysis is reserved for a separate study.

The main body is organized around this theoretical narrative. Section II states the fixed- j holographic amplitude and the precise structural dictionary to the conformal-basis perturbative-QCD result. Section III derives the universal holographic hard kernel from the Witten diagram and isolates the model-dependent lower conformal moments. Section IV presents the singlet conformal-OPE amplitude in the $\pm$ basis and formulates the comparison as a single-scale structural statement. Appendix A contains the detailed holographic derivation, including near-boundary factorization, the explicit open-channel vertex, the reconstruction of the holographic Compton form factor, and the open/closed replacement rule. Appendix B collects only the closed-form perturbative-QCD results needed for the matching; the detailed conformal-OPE derivations and basis conventions are delegated to Ref. [17].

II. CENTRAL FIXED- J STATEMENT AND MATCHING DICTIONARY

We now state the object that is being matched before turning to its holographic and perturbative derivations. Throughout this section the object is the leading-twist unpolarized singlet vector invariant amplitude in the kinematic window (1). The upper current insertion is the TT-projected scalar component defined below. Full Compton-tensor components, helicity structures, axial-vector or transversity channels, contact terms, target-mass effects, and higher-twist terms define distinct invariant amplitudes. In particular, terms in a fully gauge-invariant $VV\mathcal{O}_j$ coupling that contain $F_{z\mu}$, K_0 , or $z\partial_z K_1$ feed longitudinal or tensor-distinct projections. The matched invariant and scale are summarized in Sec. II A. The schematic projected upper-vertex statement is

$$-\frac{1}{2}g_{\mu\nu}^T\mathcal{V}_{VV\mathcal{O}_j}^{\mu\nu}\Big|_{\text{lead}} = \mathcal{N}_j\mathcal{V}(Q_1, z)\mathcal{V}(Q_2, z) + \mathcal{O}\left(\frac{M_N^2}{Q^2}, \frac{-t}{Q^2}\right), \quad (3)$$

while the complementary $F_{z\mu}$ -induced terms define longitudinal-longitudinal (LL), transverse-longitudinal (TL), or helicity-distinct kernels. This is the same TT/LL separation used in the Nishio-Watari decomposition and in the holographic electroproduction projection of Sec. II.D, especially Eq. (II.17), of Ref. [36].

We use the average hadron and photon momenta

$$P \equiv \frac{p+p'}{2}, \quad \tilde{q} \equiv \frac{q_1+q_2}{2}, \quad \Delta \equiv p-p' = q_2-q_1, \quad (4)$$

with invariants

$$p^2 = p'^2 = M_N^2, \quad q_1^2 = -Q_1^2, \quad q_2^2 = -Q_2^2, \quad \tilde{q}^2 = -\tilde{Q}^2, \quad t \equiv \Delta^2 < 0, \quad s \equiv (p+q_1)^2, \quad (5)$$

and the standard off-forward variables

$$\eta \equiv \frac{\Delta \cdot \tilde{q}}{2P \cdot \tilde{q}}, \quad \xi \equiv \frac{\tilde{Q}^2}{2P \cdot \tilde{q}}, \quad \vartheta \equiv \frac{\eta}{\xi}, \quad \chi \equiv \frac{4M_N^2\eta^2}{-t}. \quad (6)$$

For definiteness, introduce lightlike collinear vectors p_c^μ and n_c^μ with $p_c^2 = n_c^2 = 0$ and $p_c \cdot n_c = 1$, and define the transverse metric

$$g_T^{\mu\nu} = g^{\mu\nu} - p_c^\mu n_c^\nu - p_c^\nu n_c^\mu. \quad (7)$$

The scalar invariant matched in this paper is the TT projection of the singlet vector Compton tensor,

$$\hat{\mathcal{H}} \equiv -\frac{1}{2}g_{T\mu\nu}T_{\text{sing},V}^{\mu\nu} + \mathcal{O}\left(\frac{M_N^2}{Q^2}, \frac{-t}{Q^2}\right), \quad (8)$$

with the omitted terms belonging to longitudinal, helicity-flip, target-mass, or higher-twist invariant amplitudes. All occurrences of the upper photon vertex below are understood after this projection. Throughout, $X = o, c$ labels the even open and closed channels, respectively. We also use the convention

$$\mathcal{K}_j^{(\gamma)}(\vartheta) \equiv {}_2F_1\left(\frac{j}{2} + \frac{\gamma}{4}, \frac{j+1}{2} + \frac{\gamma}{4}; j + \frac{3}{2} + \frac{\gamma}{2}; \vartheta^2\right), \quad (9)$$

and reserve the term “Wilson kernel” for this hypergeometric functional dependence. The corresponding “Wilson coefficient” is the full normalized coefficient, including $c_j^\pm$, scale/evolution factors, and basis-normalization conventions.

In the twist-two unpolarized sector, the fixed- j holographic amplitudes may be written in the compact channel-unified form

$$\begin{aligned} \hat{\mathcal{H}}_{\text{holo}}^{(X)}(j) &= \xi^{-j} \left(\frac{\mu}{Q}\right)^{\gamma_X(j)} \mathcal{K}_j^{(\gamma_X(j))}(\vartheta) \left(\frac{\mu_0}{\mu}\right)^{\gamma_X(j)} \\ &\quad \times \phi_0^X(j, \tilde{\delta}_X, \tilde{\delta}_0) \hat{d}_j(\eta, t) \mathbf{g}_X \hat{\mathcal{F}}_N^{(X)}(j; t), \\ X &= o, c, \end{aligned} \quad (10)$$

where

$$\mathbf{g}_o \equiv 1, \quad \mathbf{g}_c \equiv \frac{\tilde{g}_5^2}{g_5^2}, \quad (11)$$

and

$$\hat{d}_j(\eta, t) \equiv {}_2F_1\left(-\frac{j}{2}, \frac{1-j}{2}; \frac{1}{2} - j; -\frac{4M_N^2\eta^2}{-t}\right) \quad (12)$$

is the kinematical polynomial produced by the spin- j projector. For physical even j , it is the finite polynomial in $4M_N^2\eta^2/(-t)$ obtained by contracting the

symmetric-traceless spin- j projector between the two hadron momenta in the off-forward t channel; Eq. (12) is the analytic continuation of that polynomial used in the fixed- j Regge representation. It is part of the lower matrix-element kinematics and does not modify the upper Q^2 and η/ξ Wilson kernel. Equation (10) is the precise fixed- j object whose universal kernel is being matched. The quantity ϕ_0^X is the residual UV source normalization, while $\hat{\mathcal{F}}_N^{(X)}(j; t)$ is the bottom conformal moment supplied by the hadronic vertex.

The point of Eq. (10) is that the full dependence on Q^2 and η/ξ factorizes into the universal hypergeometric hard kernel. All infrared model dependence is pushed into $\hat{\mathcal{F}}_N^{(X)}$. The holographic derivation in Sec. III and Appendix A shows that the Mellin exponent is fixed to be

$$\delta_X(j) \equiv j + \Delta_X(j) - 2 = 2j + \gamma_X(j), \quad X = o, c, \quad (13)$$

where $\Delta_X(j)$ is the dimension of the exchanged spin- j mode and $\gamma_X(j)$ its anomalous contribution.

To compare directly with the perturbative literature, we organize the singlet QCD amplitude in the $\pm$ basis of Ref. [17], especially Sec. IV.1, and make only one convention change:

$$j_{\text{ref}} \rightarrow j - 1, \quad (14)$$

so that our label j coincides with the physical spin- j exchanged in the fixed- j channel. Recent conformal-moment calculations for DVCS and DDVCS can be interfaced naturally with the same organization [22,23].

The fixed- j singlet vector Compton form factor in the conformal basis that diagonalizes the LO singlet evolution takes the form

$$\hat{\mathcal{H}}_{\text{pQCD}}^{\text{sing}}(j; \xi, \eta, t, Q^2; \mu) = \sum_{a=\pm} \xi^{-j} C_j^a \left(\frac{\eta}{\xi}, \frac{Q^2}{\mu^2}; \alpha_s \right) H_j^a(\eta, t; \mu), \quad (15)$$

and at a conformal fixed point $\alpha_s = \alpha_s^*$ the Wilson coefficients become

$$C_j^a \left(\frac{\eta}{\xi}, \frac{Q^2}{\mu^2}; \alpha_s^* \right) = c_j^a(\alpha_s^*) \left(\frac{\mu}{Q} \right)^{\gamma_j^a(\alpha_s^*)} \mathcal{K}_j^{(\gamma_j^a(\alpha_s^*))}(\vartheta), \quad (16)$$

so that

$$\begin{aligned} \hat{\mathcal{H}}_{\text{pQCD}}^{\text{sing}}(j; \xi, \eta, t, Q^2; \mu) &= \sum_{a=\pm} \xi^{-j} c_j^a(\alpha_s^*) \left(\frac{\mu}{Q} \right)^{\gamma_j^a(\alpha_s^*)} \mathcal{K}_j^{(\gamma_j^a(\alpha_s^*))}(\vartheta) \left(\frac{\mu_0}{\mu} \right)^{\gamma_j^a(\alpha_s^*)} \\ &\times H_j^a(\eta, t; \mu_0). \end{aligned} \quad (17)$$

The historical antecedents of this representation are the conformal-OPE constructions of Refs. [11–13], while the practically useful DVCS synthesis is Ref. [17]; recent NNLO

extensions may be found in Refs. [18–23]. Appendix B quotes the closed-form pQCD results needed for the comparison, while the detailed derivations are delegated to Ref. [17].

The matching statement is now immediate. In the kinematic window (1) and at the matching point (2), the fixed- j holographic amplitudes (10) and the conformal-basis pQCD amplitudes (17) have the same fixed- j kernel-and-moment structure under the dictionary

$$\gamma_o(j) \leftrightarrow \gamma_j^+(\alpha_s^*), \gamma_c(j) \leftrightarrow \gamma_j^-(\alpha_s^*), \quad (18)$$

with the nonperturbative factors identified as

$$\phi_0^o(j, \tilde{\delta}_o, \tilde{\delta}_0) \hat{d}_j(\eta, t) \hat{\mathcal{F}}_N^{(o)}(j; t) \leftrightarrow c_j^+(\alpha_s^*) H_j^+(\eta, t; \mu_*), \quad (19a)$$

$$\phi_0^c(j, \tilde{\delta}_c, \tilde{\delta}_0) \hat{d}_j(\eta, t) \left(\frac{\tilde{g}_5^2}{g_5^2} \right) \hat{\mathcal{F}}_N^{(c)}(j; t) \leftrightarrow c_j^-(\alpha_s^*) H_j^-(\eta, t; \mu_*). \quad (19b)$$

This is the precise matching statement: a channel-by-channel identification of the fixed- j hard kernel and its associated conformal moment in the stated window and at the stated scale. The arrows in Eq. (18) identify the conformal-dimension label inside the fixed- j kernel; they do not equate weak-coupling QCD anomalous-dimension functions with strong-coupling holographic trajectory functions as functions of j . The lower factor on the holographic side, including $\hat{d}_j(\eta, t) \hat{\mathcal{F}}_N^{(X)}(j; t)$, is the holographic model representation of the conformal moment. It is not a general theorem that QCD conformal moments factorize into this particular kinematical polynomial times a t -form factor. The UV kernel is universal; the lower moment is matched or modeled. The remaining normalization freedom is purely conventional: changing the normalization of the UV source rescales ϕ_0^X , while changing the normalization of the $\pm$ eigenvectors rescales $c_j^\pm$ and $H_j^\pm$ inversely. In the explicit UV convention used in Appendix A, the cutoff factor in the boundary mode cancels against the source rescaling, so ϕ_0^X is a finite residual source normalization to be matched together with the forward coefficient normalization. This is the intended level of the matching.

Fixed- j claim. In the kinematic window (1), after the TT scalar projection (8), and at the single matching scale (2), the pure-AdS upper Witten vertex produces the same hypergeometric conformal Wilson-kernel family (9) as the singlet conformal OPE. The protected closed BPST branch is structurally identified with the $(-)$ conformal partial wave, the even Nishio-Watari open branch gives the unprotected $(+)$ counterpart in the projected singlet vector amplitude, and all finite source/forward normalizations and lower conformal moments are matched through (19) rather than predicted by the UV kernel.

TABLE I. Fixed- j matching dictionary. The table summarizes the structural assignments used below; the derivation and normalization conventions are given in the text and Appendixes.

Holographic object	QCD Conformal-basis object	Diagnostic	Structural role
Closed BPST branch	(−) singlet eigenchannel	$\gamma_c(2) = 0 \leftrightarrow \gamma_2^{-(0)} = 0$	Protected energy–momentum branch
Even Nishio-Watari open branch	(+) singlet eigenchannel	No Ward-identity zero at $j = 2$	Unprotected even open Reggeon branch
Upper Witten vertex	Conformal Wilson kernel	Same $\mathcal{K}_j^{(r)}(\eta/\xi)$	Universal projected hard factor
Lower Witten vertex	Conformal moment	Matched nonperturbative input	IR-sensitive operator matrix element

Only the products $c_j^\pm H_j^\pm$ have normalization-independent meaning: a rescaling of the $\pm$ eigenvectors changes the two factors inversely and leaves the fixed- j Compton amplitude unchanged. The structural dictionary is therefore formulated at the level of these products. See Table I for a summary.

Two immediate comments are essential. First, the matching is naturally formulated for DDVCS, where both photons are off shell and the conformal structure is fully visible. The DVCS limit may then be taken after the fixed- j matching. Second, beyond the conformal fixed point the pure power-law scaling in μ is replaced by the familiar logarithmic LO evolution, but the same factorized logic survives. In that sense the fixed-point formula is the cleanest place to state the open/+ and closed/− correspondence, while the running-coupling generalization tells us how the same kernel is embedded in real-QCD scale evolution.

A. Matched invariant and scale

The fixed- j comparison concerns the TT-projected leading-twist unpolarized singlet vector invariant in the kinematic window (1). At the scale $Q = \mu = \mu_0 = \mu_*$, the ultraviolet Witten vertex produces the same hypergeometric conformal Wilson-kernel family as the QCD conformal OPE, while the finite source/forward normalizations and lower conformal moments enter through (19). The dictionary for $\gamma_X(j)$ identifies the conformal-dimension label carried by the fixed- j kernel in the two organizations.

B. The $j = 2$ structural anchor

This is the point where the matching stops being only a kernel comparison and becomes a structural channel assignment. The first nontrivial even moment is the sharpest place to test the dictionary in Eq. (18). On the perturbative side, Appendix B gives the LO eigenanomalous dimensions in Eq. (B5). At $j = 2$ they reduce to

$$\gamma_2^{-(0)} = 0, \quad \gamma_2^{+(0)} = \frac{4}{3}(2C_F + T_F n_f), \quad (20)$$

as shown explicitly in Eq. (B15). The vanishing of the (−) eigenvalue is the leading-order manifestation of the singlet momentum-sum rule, while the (+) channel remains finite. Near $j = 2$, the (−) eigenvalue starts linearly from zero

whereas the (+) eigenvalue receives a regular finite correction, cf. Eq. (B16). This is the perturbative side of the protected/unprotected split already emphasized in the conformal-OPE literature [17,23].

On the holographic side, the same protected/unprotected pattern follows from the BPST assignments for the closed and open trajectories displayed in Appendix A immediately below Eq. (A10). Using Eq. (13), one finds for the closed channel

$$\begin{aligned} \Delta_c(2) &= 4, & \delta_c(2) &= 2 + \Delta_c(2) - 2 = 4, \\ \gamma_c(2) &= 0. \end{aligned} \quad (21)$$

This is the holographic signature of the bulk graviton and hence of the conserved total energy-momentum tensor. This protected branch should not be read as a literal unmixed gluon operator at finite N_c ; the perturbative $\pm$ eigenchannels are quark-gluon mixtures, and open/closed here denotes the effective diagonal branches of the projected large- N_c /holographic description. By contrast, the open branch is unprotected at the same point:

$$\begin{aligned} \Delta_o(2) &= 2 + \sqrt{1 + \sqrt{\lambda}}, \\ \gamma_o(2) &= \Delta_o(2) - 4 = \sqrt{1 + \sqrt{\lambda}} - 2. \end{aligned} \quad (22)$$

The invariant statement is not the literal nonzero value for every possible finite λ , since the expression above has an accidental zero at $\lambda = 9$. At that accidental zero the open and closed values are temporarily degenerate at $j = 2$, but the open anomalous dimension remains unprotected for $j \neq 2$, and the full trajectory structure in Eq. (24), not the isolated value at one coupling, carries the unprotected assignment. The invariant statement is therefore that no Ward identity or conservation law enforces $\gamma_o(2) = 0$ in the open branch. At fixed j , both for the physical even-spin sequence and for the complex- j continuation used in Regge reconstruction, the $j = 2$ match identifies the protected branch and supplies the analytic-continuation prescription used for the structural assignment:

$$\text{closed} \leftrightarrow (-), \quad \text{open} \leftrightarrow (+). \quad (23)$$

The same conclusion is visible directly in the analytic structure of the two holographic branches. At strong coupling,

$$\gamma_c(j) = -j + \sqrt{4 + 2\sqrt{\lambda}(j-2)} \simeq \sqrt{2\sqrt{\lambda}(j-2)}, \quad (24a)$$

$$\gamma_o(j) = -j + \sqrt{1 + \sqrt{\lambda}(j-1)} \simeq \sqrt{\sqrt{\lambda}(j-1)}. \quad (24b)$$

The closed trajectory is anchored by the protected spin-2 graviton point, while the open trajectory is anchored by an unprotected Reggeon branch. In the strong-coupling/intercept shorthand this appears as the familiar $\sqrt{j-2}$ versus $\sqrt{j-1}$ diagnostic. At finite λ , however, the branch points are displaced to $j_{0c} = 2 - 2/\sqrt{\lambda}$ and $j_{0o} = 1 - 1/\sqrt{\lambda}$; the shorthand should not be read as a statement that the finite- λ branch points sit exactly at 2 and 1. The point is not that the full amplitude is saturated by $j = 2$; rather, $j = 2$ is the first physical even moment and therefore the cleanest diagnostic identifying the protected branch and fixing the analytic-continuation prescription used for the fixed- j amplitude. Because this diagnostic rests on conservation laws and on the analytic structure of the trajectories, it is not sensitive to the particular soft-wall realization of the lower moments.

III. HOLOGRAPHIC FACTORIZATION FROM THE WITTEN DIAGRAM

The holographic side is now derived explicitly. The strategy is simple: derive the closed channel directly from the Witten diagram, isolate the universal upper vertex, display the parallel Nishio-Watari open vertex, and use the open/closed replacement rule only as a notation-saving summary after both vertices have been written.

The closed channel is the directly derived building block because it follows from the standard t -channel Witten diagram and its BPST continuation [26]. A key point of this paper is that the BPST closed exchange, not only the Nishio-Watari open exchange, produces the same C_1 hypergeometric kernel once the near-boundary z powers are counted at fixed j . The even open channel is then written as the parallel Nishio-Watari construction of Refs. [30,31]; their Appendix B gives the relevant open-string C_1 integral and hypergeometric form explicitly in Eqs. (285)–(289). The full derivation is collected in Appendix A; here we isolate the steps that make the matching to pQCD manifest.

One endpoint of the exchanged spin- j propagator may be pushed to the boundary, so that the original Witten diagram factorizes into an upper three-point function and a lower hadronic matrix element. Schematically,

$$G_c(j, z' \rightarrow 0, z; t) \simeq \Psi_j^{(c),\text{bdry}}(z'; \varepsilon) \mathcal{H}_j^{(c)}(K, z; \varepsilon), \quad (25)$$

where $\Psi_j^{(c),\text{bdry}}$ is the near-boundary mode and $\mathcal{H}_j^{(c)}$ the bulk-to-boundary kernel associated with the lower hadronic

vertex. In pure AdS, hard-wall, and soft-wall realizations, this near-boundary factorization is the holographic counterpart of collinear factorization [30,34,35]. The fixed- j closed-channel amplitude then takes the factorized form

$$\begin{aligned} \hat{\mathcal{H}}_{\text{holo}}^{(c)}(j; s, t, \chi; Q_1^2, Q_2^2) \\ = \bar{s}_c^j \mathcal{C}_{\gamma\gamma}^{(c)}(j; Q_1^2, Q_2^2, \bar{\varepsilon}_c) \hat{d}_j(\eta, t) \hat{\mathcal{F}}_N^{(c)}(j; t), \end{aligned} \quad (26)$$

with the hadronic information isolated in the bottom moment $\hat{\mathcal{F}}_N^{(c)}(j; t)$.

The universal upper vertex is obtained by taking the conformal limit of the photon bulk wave functions,

$$\mathcal{V}^{\text{conf}}(Q_i, z) = Q_i z K_1(Q_i z), \quad i = 1, 2. \quad (27)$$

These are the same pure-AdS wave functions that underlie the vacuum current-current matching of Refs. [24,25]. The scalar kernel written below is the TT-projected upper vertex defined by Eq. (8). In the full gauge-invariant VVh or $VV\mathcal{O}_j$ coupling one also generates longitudinal terms from $F_{z\mu}$ and derivative structures, which in the conformal limit may be represented through K_0 or $z\partial_z K_1$. After applying the scalar TT projector, these components belong to longitudinal or distinct helicity/tensor invariants rather than to the kernel matched here; the complementary LL/TL projections are the natural place where those derivative structures appear, as in Nishio-Watari and in the projection formula of Sec. II.D, Eq. (II.17), of Ref. [36]. The leading scalar invariant retained here is therefore represented by $\mathcal{V}(Q_1, z)\mathcal{V}(Q_2, z)$. The top integral reduces to

$$\begin{aligned} C_1(\delta, \eta/\xi) \equiv & \left(1 - \frac{\eta^2}{\xi^2}\right)^{1/2} \int_0^\infty dy y^{1+\delta} K_1\left(y \sqrt{1 + \frac{\eta}{\xi}}\right) K_1 \\ & \times \left(y \sqrt{1 - \frac{\eta}{\xi}}\right), \end{aligned} \quad (28)$$

with $y = Qz \simeq \sqrt{(Q_1^2 + Q_2^2)/2z}$. The displayed square-root form is the spacelike DDVCS domain $|\eta/\xi| < 1$ in which both virtualities are kept off shell; endpoint limits such as DVCS and complex physical continuations are taken after the fixed- j kernel has been identified. The integral is elementary:¹

$$C_1(\delta, \eta/\xi) = 2^{\delta-1} \frac{\delta + 2\Gamma(\frac{\delta}{2} + 1)^4}{\delta \Gamma(\delta + 2)} {}_2F_1\left(\frac{\delta}{4}, \frac{\delta}{4} + \frac{1}{2}; \frac{\delta}{2} + \frac{3}{2}; \frac{\eta^2}{\xi^2}\right). \quad (29)$$

The hard kernel is therefore universal. It is not a model artifact of the soft-wall; it is fixed completely by the

¹Equivalently, Eq. (29) is the standard product-Bessel integral for $K_1(ay)K_1(by)$ after setting $a^2 = 1 + \eta/\xi$ and $b^2 = 1 - \eta/\xi$. Nishio-Watari use the same identity for the open-string C_1 integral in Appendix B of Ref. [31].

pure-AdS photon wave functions and by the power of z carried by the exchanged spin- j field. For the closed BPST branch this is the new part of the construction: the same hypergeometric family arises from a closed-string Witten diagram with its Mellin exponent fixed by BPST power counting. This is the precise mathematical sense in which the hard kernel is a consequence of the AdS geometry rather than of a phenomenological ansatz.

The exponent entering Eq. (28) is not assumed. It follows directly from the Witten diagram. In the upper vertex the measure contributes z^{-5} , the tensor vertex contributes $z^{4+2(j-2)}$, the bulk-to-boundary propagator contributes $z^{-(j-2)}$, the boundary mode contributes $z^{\Delta_X(j)}$, and the two photon wave functions contribute z^2 . Hence the integrand scales as

$$\begin{aligned} & z^{-5} \times z^{4+2(j-2)} \times z^{-(j-2)} \times z^{\Delta_X(j)} \times z^2 \\ & = z^{j+\Delta_X(j)-1} = z^{1+\delta_X(j)}, \end{aligned} \quad (30)$$

which fixes the Mellin exponent to Eq. (13). This z -power counting is the structural anchor on the holographic side: it is what turns the Witten diagram into the same anomalous-dimension combination that appears in the conformal-basis Wilson coefficients of pQCD.

All infrared model dependence is confined to the lower vertex. In the soft-wall model the right- and left-handed nucleon contributions reduce to gamma-function expressions, and the bottom conformal moment is

$$\hat{\mathcal{F}}_N^{(X)}(j; t) = \frac{1}{2} \left[\hat{\mathcal{F}}_R^{(X)}(j; t) + \hat{\mathcal{F}}_L^{(X)}(j; t) \right], \quad X = o, c, \quad (31)$$

with the explicit formulas collected in Appendix A. The important point is not the particular soft-wall expression, but the separation itself: the hard upper factor is universal, while the hadronic vertex carries the entire infrared completion. In this sense the soft-wall model is used here as an input for the lower conformal moments, not as the source of the Wilson-kernel-like factor.

Before using the compact map, it is helpful to display the corresponding open upper Witten vertex explicitly. In the even Nishio-Watari open channel one has

$$\begin{aligned} \tilde{\mathcal{C}}_{\gamma\gamma}^{(o)}(j; Q_1^2, Q_2^2, \bar{\epsilon}_o) &= \frac{g_5}{2g_5^2} \int_0^\infty dz \sqrt{g} e^{-\Phi_o} z^{4+2(j-2)} \\ &\times \mathcal{K}_{\gamma\gamma}^{(o)}(Q_1, Q_2; z) z^{-(j-2)} \Psi_j^{(o), \text{bdry}}(z; \epsilon), \end{aligned} \quad (32)$$

with

$$\begin{aligned} \mathcal{K}_{\gamma\gamma}^{(o)}(Q_1, Q_2; z) &= \mathcal{V}(Q_1, z) \mathcal{V}(Q_2, z), \\ \Psi_j^{(o), \text{bdry}}(z; \epsilon) &= - \frac{(\sqrt{2}\kappa_o z)^{\Delta_o(j)} (\sqrt{2}\bar{\epsilon}_o)^{4-\Delta_o(j)}}{\Delta_o(j)}. \end{aligned} \quad (33)$$

Taking $\mathcal{V}(Q_i, z) \rightarrow Q_i z K_1(Q_i z)$, the same z -power counting gives $z^{j+\Delta_o(j)-1} = z^{1+\delta_o(j)}$, and hence the same $C_1(\delta_o, \eta/\xi)$ integral and the same kernel $\mathcal{K}_j^{(\gamma_o)}(\vartheta)$. Thus the open result is not asserted from the closed result; it is the parallel Nishio-Watari open-string Witten vertex written in the same fixed- j factorized form. Appendix B of Ref. [31] displays this C_1 integral and its hypergeometric form in Eqs. (285)–(289).

Once the open vertex has been displayed, the even open channel can be summarized by the replacement rule

$$\begin{aligned} \tilde{g}_5 &\rightarrow g_5, & \kappa_c &\rightarrow \kappa_o, & h_{\mu\nu}^c &\rightarrow h_{\mu\nu}^o, \\ \Delta_c &\rightarrow \Delta_o, & \gamma_c &\rightarrow \gamma_o. \end{aligned} \quad (34)$$

This rule is a shorthand after the two projected vertices have been independently specified; it is not a claim about the full open-string tensor sector. It is used here at the level of the projected upper scalar kernel and the resulting fixed- j invariant amplitude. In the Nishio-Watari construction, the same current insertions and the same spin- j kinematical projector couple to an open-string Regge trajectory with different open-string boundary data, couplings, and intercept [30,31]. Moreover, graviton-like effective descriptions of 2^{++} tensor mesons have long been used in AdS/QCD [32], so employing a parallel spin-2 action on the open side is not foreign to holographic hadron phenomenology. The continued phenomenological relevance of tensor-meson exchange in modern hadronic amplitudes provides an additional supporting perspective [33]. Once the upper vertex is fixed by the pure-AdS photon wave functions, the only channel dependence compatible with that construction enters through the substitutions in Eq. (34). The open and closed channels therefore have the same functional dependence in the universal kernel, differing only in the trajectory data, the overall coupling factor, and the lower hadronic moments. This is precisely the structure needed for the open/+ and closed/− correspondence.

IV. PQCD CONFORMAL OPE AND FIXED- j STRUCTURAL MATCHING

With the holographic side in hand, the remaining task is to write the perturbative amplitude in the same fixed- j language and match the two structures term by term.

The perturbative side of the comparison is most transparent in the conformal basis that diagonalizes the LO singlet evolution. Historically, this representation grows out of the conformal-OPE program of Refs. [11,12], the

complex conformal-spin construction of Ref. [13], and the explicit LO evolution solutions of Refs. [14,15]. For DVCS conventions and the explicit conformal representation, however, the practically most useful anchor is Ref. [17], especially Sec. IV.1, which is the basis followed here. Denote the corresponding conformal moments by $H_j^\pm(\eta, t; \mu)$. At a conformal fixed point the fixed- j Wilson coefficients are given by Eq. (16), and the full singlet fixed- j amplitude is Eq. (17). The key fact is that the same Gauss hypergeometric function appears on both sides, with the same combination of spin and anomalous dimension entering its parameters.

This agreement is not automatic merely because both descriptions know about the conformal group. Conformal symmetry fixes the possible three-point functional form once the spin and dimension of the exchanged operator are specified, but the nontrivial content here is the following three-part statement:

- (i) the Witten diagram factorizes at fixed j into an upper current kernel and a lower conformal moment;
- (ii) the upper vertex fixes the specific Mellin label $\delta_X(j) = j + \Delta_X(j) - 2 = 2j + \gamma_X(j)$ through the z -power count in Eq. (30), rather than by assumption;
- (iii) the protected/unprotected $j = 2$ diagnostic identifies the closed protected branch and supplies the analytic-continuation prescription for the closed/open $\leftrightarrow (-)/(+)$ channel assignment used in this projected amplitude.

The shared conformal group explains why a hypergeometric kernel can appear; these three Witten-diagram and channel facts explain why the particular kernel and dictionary are the ones matched here.

The matching can therefore be stated without ambiguity. In the kinematic window and at the matching point $Q = \mu = \mu_0 = \mu_*$,

$$\hat{\mathcal{H}}_{\text{holo}}^{(o)}(j) \Big|_{\mu=Q=\mu_0=\mu_*} \leftrightarrow \xi^{-j} c_j^+(\alpha_s^*) \mathcal{K}_j^{(\gamma_j^+(\alpha_s^*))}(\vartheta) H_j^+(\eta, t; \mu_*), \quad (35a)$$

$$\hat{\mathcal{H}}_{\text{holo}}^{(c)}(j) \Big|_{\mu=Q=\mu_0=\mu_*} \leftrightarrow \xi^{-j} c_j^-(\alpha_s^*) \mathcal{K}_j^{(\gamma_j^-(\alpha_s^*))}(\vartheta) H_j^-(\eta, t; \mu_*). \quad (35b)$$

In this sense the open-string contribution provides the fixed- j holographic realization of the (+) channel, while the closed-string contribution provides the corresponding realization of the (−) channel in the projected singlet vector amplitude. These equations are the core result. They state that at fixed j , in the window (1) and at the single scale (2), the two descriptions have the same kernel-and-moment structure under the stated dictionary.

The $j = 2$ anchor makes this statement sharper. Equation (20) shows that the perturbative (−) eigenvalue

is the protected one, while Eqs. (21) and (22) show that the protected holographic branch is the closed channel and that the open branch has no symmetry-enforced zero at the same point. The analytic behavior in Eq. (24) then extends this protected/unprotected distinction away from $j = 2$. The channel assignment is therefore anchored before any resummation or numerical implementation is considered.

The finite constants $c_j^\pm$ are fixed by matching the nonforward coefficients to the forward deep-inelastic scattering (DIS) normalization. In the unit-diagonal eigenvector convention used here, the final LO result is quoted in Appendix B, Eq. (B12), while the detailed basis-rotation derivation is given in Ref. [17]. A rescaling of the $\pm$ eigenvectors changes $c_j^\pm$ and $H_j^\pm$ inversely, so the structural correspondence is formulated at the level of the normalization-independent products $c_j^\pm H_j^\pm$.

It is equally important to state what the conformal fixed-point formulas do and do not assume. They provide the cleanest analytic representation of the hard kernel, but the matching itself is local in scale. At any chosen scale μ_* , one may evaluate both descriptions at

$$Q = \mu = \mu_0 = \mu_*, \quad (36)$$

so that the fixed- j pQCD amplitude takes the schematic form

$$\hat{\mathcal{H}}_{\text{pQCD}}^{\text{sing}}(j) \Big|_{\mu=Q=\mu_0=\mu_*} = \sum_{a=\pm} \xi^{-j} c_j^a(\alpha_s(\mu_*)) \mathcal{K}_j^{(\gamma_j^a(\alpha_s(\mu_*)))} \times (\vartheta) H_j^a(\eta, t; \mu_*), \quad (37)$$

where the displayed $\mathcal{K}_j^{(\gamma_j^a)}$ is the same hypergeometric kernel with the anomalous dimension evaluated at the chosen scale. In this form the matching is an ordinary effective-theory matching at one scale. In the conformal partial-wave/CS representation of Ref. [17], the object matched here is the fixed- j conformal kernel of the projected invariant. Conformal-breaking, beta-proportional terms, and finite scheme transformations are assigned to coefficient/evolution bookkeeping away from $Q = \mu = \mu_0 = \mu_*$; they do not define a second fixed-scale η/ξ kernel for this projection. Higher-order perturbative information enters through $\gamma_j^a(\alpha_s(\mu_*))$ and $c_j^a(\alpha_s(\mu_*))$; the remaining conformal-anomaly and running-coupling bookkeeping governs the evolution away from the matching point.

Away from a conformal fixed point the coupling runs. The pure power laws in μ are then replaced by the usual logarithmic evolution factors,

$$\mathcal{E}_j^\pm(\mu, \mu_0) = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\gamma_j^{\pm(0)}/\beta_0}, \quad (38)$$

and the Wilson coefficients admit the usual LO RG improvement. The explicit formulas are given in

Appendix B, especially Eqs. (B13) and (B14). The main point is that the same factorized logic survives: logarithmic running changes the scale dependence and the bookkeeping of moments and coefficients, not the fixed-scale hypergeometric functional form of the hard kernel or the open/ $+$, closed/ $-$ assignment. This is why the fixed-point formulas should be read as a presentational convenience, not as a hidden dynamical assumption.

Recent NNLO results for DVCS and DDVCS coefficient functions sharpen the perturbative side of the story [18–23]. They refine the anomalous dimensions $\gamma_j^\pm(\alpha_s)$, the finite coefficient normalizations $c_j^\pm(\alpha_s)$, and the evolution bookkeeping entering the same conformal partial wave, whose fixed-scale hypergeometric dependence on η/ξ is the kernel identified by the holographic upper vertex.

V. CONCLUSION

We have reformulated the holographic description of DDVCS as a fixed- j structural matching problem. The result of the paper has three parts. First, the holographic fixed- j amplitude factorizes into a universal upper kernel and a lower hadronic conformal moment. Second, the universal upper kernel is the same Gauss hypergeometric function that appears in the conformal-basis Wilson coefficients of the singlet vector Compton form factor in QCD. Third, the first physical even moment $j = 2$ identifies the protected closed branch and supplies the analytic-continuation prescription for the closed/open $\leftrightarrow (-)/(+)$ channel assignment: the closed trajectory is anchored by the protected spin-2 graviton point, while the open trajectory is anchored by an unprotected Reggeon branch. In the strong-coupling/intercept shorthand this appears as the familiar $\sqrt{j-2}$ versus $\sqrt{j-1}$ diagnostic, with the finite- λ branch points displaced as stated above.

The distinction from Nishio-Watari is central. Their open-string analysis already contains the C_1 integral and its hypergeometric form. The closed channel here is the new rigorous starting point: its BPST near-boundary factorization isolates a universal upper photon vertex and derives the same hypergeometric family with the precise Mellin exponent $\delta_c(j) = j + \Delta_c(j) - 2$ fixed by z -power counting. Put tersely, the novelty is not the open C_1 identity itself; it is that the closed BPST fixed- j upper Witten vertex independently produces the same conformal Wilson-kernel family, fixes the Mellin label by z -power counting, and thereby completes the structural open/closed $\leftrightarrow (+)/(-)$ operator-basis mapping that identifies the fixed- j holographic amplitude in the perturbative-QCD conformal basis. The even open channel is displayed through its

corresponding Nishio-Watari open Witten vertex and then summarized by a parallel replacement rule. The new use of the open channel here is not the rediscovery of its C_1 integral; it is its placement in the modern $\pm$ eigenbasis with explicit $c_j^\pm$ normalization bookkeeping and with the $j = 2$ diagnostic supplying the $(+)$ branch prescription. In both channels the upper impact factor is fixed by the pure-AdS photon wave functions and gives a closed-form Gauss hypergeometric function of η^2/ξ^2 , with Mellin exponent $\delta_X(j) = j + \Delta_X(j) - 2 = 2j + \gamma_X(j)$. This is the point at which the holographic construction acquires a precise fixed- j interpretation in the conformal-OPE language.

The conformal fixed-point formulas provide the cleanest analytic representation of the hard kernel, with the comparison imposed at the single scale (2) in the kinematic window (1). Away from the fixed point, LO running replaces the pure power laws by the familiar logarithmic evolution factors while preserving the channel-by-channel fixed- j organization of the kernel and conformal moment. Recent NNLO DVCS/DDVCS coefficient functions refine the anomalous dimensions $\gamma_j^\pm(\alpha_s)$, the finite normalizations $c_j^\pm(\alpha_s)$, and the evolution terms entering the same conformal partial wave [18–23].

The main implication is that the holographic DDVCS/DVCS amplitude has a precise fixed- j operator interpretation. Its universal upper vertex generates the conformal Wilson-kernel family, its lower Witten vertex supplies the channel-dependent nonperturbative conformal moment, and the protected/unprotected $j = 2$ structure fixes the closed/open $\leftrightarrow (-)/(+)$ assignment. Viewed through the perturbative-QCD conformal representation, the t -channel Witten diagrams therefore compute the same kernel-and-moment organization that appears in the singlet conformal OPE.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: DETAILED HOLOGRAPHIC DERIVATION

1. Kinematics, soft-wall setup, and canonical normalization

We work in the Poincaré patch of AdS₅,

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1), \quad (\text{A1})$$

with sector-dependent soft-wall dilaton profiles

$$\Phi_o(z) = \kappa_o^2 z^2, \quad \Phi_c(z) = \kappa_c^2 z^2, \quad (\text{A2})$$

and we set $R = 1$ from now on. The external transverse photon kernel satisfies

$$\partial_z \left(\frac{e^{-\Phi_o}}{z} \partial_z \mathcal{V}(Q_1, z) \right) - \frac{Q_1^2 e^{-\Phi_o}}{z} \mathcal{V}(Q_1, z) = 0, \quad \mathcal{V}(Q_1, \varepsilon) = 1, \quad (\text{A3})$$

with solution

$$\mathcal{V}(Q_1, z) = \kappa_o^2 z^2 \Gamma \left(1 + \frac{Q_1^2}{4\kappa_o^2} \right) U \left(1 + \frac{Q_1^2}{4\kappa_o^2}, 2; \kappa_o^2 z^2 \right). \quad (\text{A4})$$

For the upper vertex we will later take the conformal limit,

$$\mathcal{V}(Q_1, z) \longrightarrow \mathcal{V}^{\text{conf}}(Q_1, z) = Q_1 z K_1(Q_1 z), \quad (\text{A5})$$

because the hard upper impact factor is insensitive to the IR completion. That is precisely why the upper vertex is universal.

For the twist- τ nucleon ground state we use the soft-wall wave functions

$$\psi_R(z; 0) = \frac{\tilde{n}_R}{\kappa_o^{\tau-2}} z^{5/2} (\kappa_o^2 z^2)^{(\tau-2)/2}, \quad (\text{A6a})$$

$$\psi_L(z; 0) = \frac{\tilde{n}_L}{\kappa_o^{\tau-1}} z^{5/2} (\kappa_o^2 z^2)^{(\tau-1)/2}, \quad (\text{A6b})$$

$$\tilde{n}_L = \kappa_o^\tau \sqrt{\frac{2}{\Gamma(\tau)}}, \quad \tilde{n}_R = \tilde{n}_L \kappa_o^{-1} \sqrt{\tau - 1}. \quad (\text{A6c})$$

The symmetric closed sector is described by the gravitonlike fluctuation $h_{\mu\nu}^c$ with quadratic action

$$S_c^{(2)} = \frac{1}{R} \int d^4x dz \sqrt{g} e^{-2\Phi_c} \left[-\frac{1}{4\tilde{g}_5^2} g^{MN} \eta^{\lambda\rho} \eta^{\sigma\tau} \partial_M h_{\lambda\sigma}^c \partial_N h_{\rho\tau}^c + \frac{1}{8\tilde{g}_5^2} g^{MN} \eta^{\alpha\beta} \eta^{\gamma\sigma} \partial_M h_{\alpha\beta}^c \partial_N h_{\gamma\sigma}^c \right], \quad (\text{A7})$$

where $\tilde{g}_5^2 = 2\kappa_5^2$ and axial gauge $h_{\mu z}^c = h_{zz}^c = 0$ is understood. The even-spin open sector is described by the parallel gravitonlike field $h_{\mu\nu}^o$,

$$S_o^{(2)} = \frac{1}{R} \int d^4x dz \sqrt{g} e^{-2\Phi_o} \left[-\frac{1}{4\tilde{g}_5^2} g^{MN} \eta^{\lambda\rho} \eta^{\sigma\tau} \partial_M h_{\lambda\sigma}^o \partial_N h_{\rho\tau}^o + \frac{1}{8\tilde{g}_5^2} g^{MN} \eta^{\alpha\beta} \eta^{\gamma\sigma} \partial_M h_{\alpha\beta}^o \partial_N h_{\gamma\sigma}^o \right], \quad (\text{A8})$$

which is the closed action with $\tilde{g}_5 \rightarrow g_5$, $\Phi_c \rightarrow \Phi_o$, and $h_{\mu\nu}^c \rightarrow h_{\mu\nu}^o$. The field $h_{\mu\nu}^o$ is used here as an effective projected even-spin open Reggeon/tensor exchange in the scalar invariant, not as a complete open-string tensor action.

The canonically normalized bulk fluctuations are therefore

$$\Psi \rightarrow g_5 \Psi, \quad V_M \rightarrow g_5 V_M, \quad h_{\mu\nu}^o \rightarrow g_5 h_{\mu\nu}^o, \quad h_{\mu\nu}^c \rightarrow \tilde{g}_5 h_{\mu\nu}^c. \quad (\text{A9})$$

Each external non-normalizable photon source contributes $1/g_5$, each open cubic vertex contributes g_5 , and each closed cubic vertex contributes $\tilde{g}_5$. Those are the coupling factors shown in red in Figs. 1 and 2.

The Regge trajectories are written as

$$\Delta_X(j) = 2 + j + \gamma_X(j), \quad \delta_X(j) = j + \Delta_X(j) - 2 = 2j + \gamma_X(j), \quad X = o, c. \quad (\text{A10})$$

For the closed branch we use the BPST form

$$\Delta_c(j) = 2 + \sqrt{2\sqrt{\lambda}(j - j_{0c})}, \quad j_{0c} = 2 - \frac{2}{\sqrt{\lambda}}, \quad (\text{A11})$$

while the even open branch is taken to be

$$\Delta_o(j) = 2 + \sqrt{\sqrt{\lambda}(j - j_{0o})}, \quad j_{0o} = 1 - \frac{1}{\sqrt{\lambda}}. \quad (\text{A12})$$

The rest of the derivation is organized so that the closed channel is derived directly and the open channel is introduced through the parallel Nishio-Watari vertex before the replacement rule (34) is used as shorthand. Also note that at strong 't Hooft coupling $\lambda \gg 1$ and fixed- j , the closed BPST branch can be rewritten as

$$\begin{aligned} \gamma_c(j) &= \Delta_c(j) - 2 - j = -j + \sqrt{4 + 2\sqrt{\lambda}(j - 2)} \\ &\simeq \sqrt{2\sqrt{\lambda}(j - 2)}, \end{aligned} \quad (\text{A13})$$

which vanishes at $j = 2$. Similarly, the open BPST branch can be rewritten as

$$\begin{aligned} \gamma_o(j) &= \Delta_o(j) - 2 - j = -j + \sqrt{1 + \sqrt{\lambda}(j - 1)} \\ &\simeq \sqrt{\sqrt{\lambda}(j - 1)}, \end{aligned} \quad (\text{A14})$$

which is finite and positive at $j = 2$ in the large- λ regime. At finite λ , the important structural point is that this open-branch value is not protected by a conservation law; an accidental zero at a special finite coupling would only create a local degeneracy at $j = 2$ and would not change the open/closed branch prescription away from that point.

2. Closed channel: from the standard Witten diagram to holographic collinear factorization

The closed channel is the directly derived building block because it follows from the standard soft-wall graviton Witten diagram. The first step is the spectral decomposition of the spin- j propagator.

3. Spin- j modes, propagator, and near-boundary factorization

The normalizable closed modes are

$$\psi_n^{(c)}(j, z) = c_n^{(c)}(j) z^{\Delta_c(j)} L_n^{\Delta_c(j)-2}(2\kappa_c^2 z^2), \quad (\text{A15})$$

with normalization constant

$$c_n^{(c)}(j) = \left[\frac{2^{\Delta_c(j)+1} \kappa_c^{2\Delta_c(j)} \Gamma(n+1)}{\Gamma(n + \Delta_c(j) - 1)} \right]^{1/2}. \quad (\text{A16})$$

They satisfy

$$\frac{1}{2\kappa_c^2} \int_0^\infty dz \sqrt{g} e^{-2\Phi_c} |g^{xx}| \psi_n^{(c)}(j, z) \psi_m^{(c)}(j, z) = \delta_{nm}, \quad (\text{A17})$$

and the bulk-to-bulk propagator has the spectral representation

$$G_c(j, z, z'; t) = 2\kappa_c^2 \sum_{n=0}^\infty \frac{\psi_n^{(c)}(j, z) \psi_n^{(c)}(j, z')}{-t + m_{c,n}^2(j)}, \quad (\text{A18})$$

with the mass spectrum

$$M_{c,n}^2(j) = 8\kappa_c^2 \left(n + \frac{\Delta_c(j)}{2} \right) = 8\kappa_c^2 \left(n + \frac{j + \gamma_c(j) + 2}{2} \right). \quad (\text{A19})$$

The generalized decay constants are defined by the ultra-violet derivative

$$\mathcal{F}_n^{(c)}(j, \varepsilon) = \frac{1}{2\kappa_c^2} \frac{1}{\tilde{g}_5} \left[-\sqrt{g} e^{-2\Phi_c} |g^{xx}| \partial_z \psi_n^{(c)}(j, z) \right]_{z=\varepsilon}, \quad (\text{A20})$$

so that one endpoint of the propagator may be pushed to the boundary as

$$\begin{aligned} G_c(j, z' \rightarrow 0, z; t) &\simeq \frac{\psi_0^{(c)}(j, z' \rightarrow 0)}{\tilde{g}_5 \mathcal{F}_0^{(c)}(j, \varepsilon)} \\ &\times \sum_{n=0}^\infty \frac{\tilde{g}_5 \mathcal{F}_n^{(c)}(j, \varepsilon) \psi_n^{(c)}(j, z)}{-t + m_{c,n}^2(j)}. \end{aligned} \quad (\text{A21})$$

The required n independence follows directly from the universal near-boundary expansion. Since

$$L_n^{\Delta_c(j)-2}(0) = \frac{\Gamma(n + \Delta_c(j) - 1)}{\Gamma(\Delta_c(j) - 1) \Gamma(n + 1)}, \quad (\text{A22})$$

one has

$$\begin{aligned} \psi_n^{(c)}(j, z' \rightarrow 0) &= c_n^{(c)}(j) L_n^{\Delta_c(j)-2}(0) z'^{\Delta_c(j)} \\ &[1 + \mathcal{O}(z'^2)], \end{aligned} \quad (\text{A23})$$

while the UV derivative defining $\mathcal{F}_n^{(c)}$ gives the same factor $c_n^{(c)}(j) L_n^{\Delta_c(j)-2}(0)$ multiplying $\varepsilon^{\Delta_c(j)-4} [1 + \mathcal{O}(\varepsilon^2)]$. Hence this factor cancels in

$$\frac{\psi_n^{(c)}(j, z' \rightarrow 0)}{\tilde{g}_5 \mathcal{F}_n^{(c)}(j, \varepsilon)} \propto z'^{\Delta_c(j)} \varepsilon^{4-\Delta_c(j)} [1 + \mathcal{O}(z'^2, \varepsilon^2)], \quad (\text{A24})$$

and the ratio is independent of n to leading near-boundary order; Eq. (A25) fixes the proportionality constant in the present normalization. The same cancellation is the hard-wall statement written with the small-argument Bessel expansion. This is the holographic origin of collinear factorization: the first factor becomes the boundary mode entering the upper three-point function, whereas the second factor becomes the bulk-to-boundary kernel entering the lower hadronic matrix element.

It is therefore natural to define

$$\Psi_j^{(c),\text{bdry}}(z'; \varepsilon) = \frac{\psi_0^{(c)}(j, z' \rightarrow 0)}{\tilde{g}_5 \mathcal{F}_0^{(c)}(j, \varepsilon)} = -\frac{(\sqrt{2}\kappa_c z')^{\Delta_c(j)} (\sqrt{2}\bar{\varepsilon}_c)^{4-\Delta_c(j)}}{\Delta_c(j)}, \quad \bar{\varepsilon}_c \equiv \kappa_c \varepsilon, \quad (\text{A25})$$

and

$$\begin{aligned} \mathcal{H}_j^{(c)}(K, z; \varepsilon) &= \sum_{n=0}^{\infty} \frac{\tilde{g}_5 \mathcal{F}_n^{(c)}(j, \varepsilon) \psi_n^{(c)}(j, z)}{-t + m_{c,n}^2(j)} \\ &= \mathcal{N}_c(j, \bar{K}_c, \bar{\varepsilon}_c) (\sqrt{2}\kappa_c z)^{\Delta_c(j)} U\left(\frac{a_{t,c}}{2} + \frac{\Delta_c(j)}{2}, \Delta_c(j) - 1; 2\kappa_c^2 z^2\right), \end{aligned} \quad (\text{A26})$$

with

$$\mathcal{N}_c(j, \bar{K}_c, \bar{\varepsilon}_c) = (\sqrt{2}\bar{\varepsilon}_c)^{\Delta_c(j)-4} \frac{\Gamma(\frac{a_{t,c}}{2} + \frac{\Delta_c(j)}{2})}{\Gamma(\Delta_c(j) - 2)}, \quad \bar{K}_c \equiv \frac{K}{\kappa_c}, \quad a_{t,c} \equiv -\frac{t}{4\kappa_c^2}. \quad (\text{A27})$$

Equation (A21) is precisely the smooth transition from Figs. 1 and 2. The normalization factor $\mathcal{N}_c(j, \bar{K}_c, \bar{\varepsilon}_c)$ is fixed by the standard normalization condition of bulk-to-boundary propagators in AdS namely

$$\mathcal{H}_j^{(c)}(K, z = \varepsilon; \varepsilon) = 1.$$

4. Why the upper integral depends on $\delta_c = j + \Delta_c - 2$

The fixed- j closed amplitude before evaluating the bulk integrals can be written schematically as

$$\hat{\mathcal{H}}^{(c)}(j; s, t, \chi; Q_1^2, Q_2^2) = \bar{s}_c \tilde{\mathcal{C}}_{\gamma\gamma}^{(c)}(j; Q_1^2, Q_2^2, \bar{\varepsilon}_c) \hat{d}_j(\eta, t) \mathcal{F}_N^{(c)}(j; t, \bar{\varepsilon}_c), \quad \bar{s}_c \equiv \frac{P \cdot \tilde{q}}{\kappa_c^2} \simeq \frac{Q^2}{2\kappa_c^2 \xi}. \quad (\text{A28})$$

The upper vertex is

$$\tilde{\mathcal{C}}_{\gamma\gamma}^{(c)}(j; Q_1^2, Q_2^2, \bar{\varepsilon}_c) = \frac{\tilde{g}_5}{2g_5^2} \int_0^\infty dz \sqrt{g} e^{-\Phi_c} z^{4+2(j-2)} \mathcal{K}_{\gamma\gamma}^{(c)}(Q_1, Q_2; z) z^{-(j-2)} \Psi_j^{(c),\text{bdry}}(z; \varepsilon), \quad (\text{A29})$$

where

$$\mathcal{K}_{\gamma\gamma}^{(c)}(Q_1, Q_2; z) = \mathcal{V}(Q_1, z) \mathcal{V}(Q_2, z). \quad (\text{A30})$$

Equation (A30) is the TT-projected scalar part of the gauge-invariant photon-photon-spin- j upper vertex, with the scalar projection defined in Eq. (8). Explicitly, at leading power the projected tensor structure is summarized by

$$-\frac{1}{2} g_{\mu\nu}^T \mathcal{V}_{VV\mathcal{O}_j}^{\mu\nu} \Big|_{\text{lead}} = \mathcal{N}_j \mathcal{V}(Q_1, z) \mathcal{V}(Q_2, z) + \mathcal{O}\left(\frac{M_N^2}{Q^2}, \frac{-t}{Q^2}\right), \quad (\text{A31})$$

whereas the complementary projections retain the $F_{z\mu}$, K_0 , and $z\partial_z K_1$ structures. The full Maxwell stress-tensor coupling also contains terms with $F_{z\mu}$ and derivatives of the bulk-to-boundary propagator; in the conformal limit these generate structures proportional to K_0 or $z\partial_z K_1$. After the scalar TT projection, such terms feed longitudinal or distinct tensor amplitudes outside the leading-twist unpolarized singlet scalar invariant considered here, as in the TT/LL decompositions of Refs. [30,31,36]. The matching claim therefore uses the TT-projected upper vertex, not the full helicity decomposition. The sector label on the dilaton in Eq. (A29) is a normalization convention for the exchanged closed-channel cubic vertex. In the hard upper-vertex region $z \sim 1/Q$,

$$e^{-\Phi_c(z)} - e^{-\Phi_o(z)} = -(\kappa_c^2 - \kappa_o^2)z^2 + \mathcal{O}(\kappa_X^4 z^4), \quad (\text{A32})$$

so replacing one by the other changes the conformal upper integral only by relative corrections of order $(\kappa_c^2 - \kappa_o^2)/Q^2$ plus IR-sensitive normalization effects. It therefore does not affect the leading conformal C_1 kernel or the exponent derived below. In the conformal limit,

$$\mathcal{K}_{\gamma\gamma}^{(c)}(Q_1, Q_2; z) \longrightarrow Q_1 Q_2 z^2 K_1(Q_1 z) K_1(Q_2 z), \quad (\text{A33})$$

and the complete z power multiplying the two Bessel functions is

$$\begin{aligned} &(-5) + (4 + 2(j-2)) - (j-2) + \Delta_c(j) + 2 \\ &= j + \Delta_c(j) - 1. \end{aligned} \quad (\text{A34})$$

Thus, after $y = Qz$ with

$$Q_1^2 \simeq \left(1 + \frac{\eta}{\xi}\right) Q^2, \quad Q_2^2 \simeq \left(1 - \frac{\eta}{\xi}\right) Q^2, \quad (\text{A35})$$

the universal upper integral necessarily takes the form

$$\left(1 - \frac{\eta^2}{\xi^2}\right)^{1/2} \int_0^\infty dy y^{1+\delta_c(j)} K_1\left(y \sqrt{1 + \frac{\eta}{\xi}}\right) K_1\left(y \sqrt{1 - \frac{\eta}{\xi}}\right), \quad (\text{A36})$$

with

$$\delta_c(j) = j + \Delta_c(j) - 2 = 2j + \gamma_c(j), \quad (\text{A37})$$

and the overall kinematic factor coming from the relation

$$Q_1 Q_2 \simeq Q \left(1 + \frac{\eta}{\xi}\right)^{1/2} \times Q \left(1 - \frac{\eta}{\xi}\right)^{1/2} = Q^2 \left(1 - \frac{\eta^2}{\xi^2}\right)^{1/2}.$$

The same reasoning goes through verbatim in the even open channel, so the exponent is universally $\delta_X(j) = j + \Delta_X(j) - 2$.

5. Universal upper vertex and the explicit source normalization

Evaluating (A29) with the conformal photon wave functions gives

$$\mathcal{C}_{\gamma\gamma}^{(c)}(j; Q_1^2, Q_2^2, \bar{\epsilon}_c) = \frac{\tilde{g}_5}{(\bar{\epsilon}_c)^{4-\Delta_c(j)}} \times \left(\frac{Q^2}{2\kappa_c^2}\right)^j \times \tilde{\mathcal{C}}_{\gamma\gamma}^{(c)}(j; Q_1^2, Q_2^2, \bar{\epsilon}_c) = -\frac{\tilde{g}_5^2}{2g_5^2} \frac{(\sqrt{2})^{4-\Delta_c(j)}}{\Delta_c(j)} 2j \left(\frac{Q}{2\kappa_c}\right)^{-\delta_c(j)+2j} C_1\left(\delta_c(j), \frac{\eta}{\xi}\right), \quad (\text{A38})$$

where C_1 is defined in (28). Using (29), one obtains

$$\mathcal{C}_{\gamma\gamma}^{(c)} = -\frac{\tilde{g}_5^2}{2g_5^2} \frac{(\sqrt{2})^{4-\Delta_c(j)}}{\Delta_c(j)} \left(\frac{Q}{2\kappa_c}\right)^{-\delta_c(j)+2j} 2^{\delta_c(j)+j-1} \frac{\delta_c(j) + 2}{\delta_c(j)} \frac{\Gamma(\frac{\delta_c(j)}{2} + 1)^4}{\Gamma(\delta_c(j) + 2)} \times {}_2F_1\left(\frac{j}{2} + \frac{\gamma_c}{4}, \frac{j+1}{2} + \frac{\gamma_c}{4}; j + \frac{3}{2} + \frac{\gamma_c}{2}; \frac{\eta^2}{\xi^2}\right). \quad (\text{A39})$$

Writing

$$\kappa_X = \tilde{\delta}_X \Lambda_{\text{QCD}}(X = o, c), \quad \mu_0 = \tilde{\delta}_0 \Lambda_{\text{QCD}}, \quad \epsilon = \frac{\tilde{\delta}_\epsilon}{\mu}, \quad (\text{A40})$$

one isolates the physical QCD-like scale dependence as

$$\left(\frac{Q}{2\kappa_c}\right)^{-\delta_c(j)+2j} = \left(\frac{Q}{2\kappa_c}\right)^{-\gamma_c(j)} = \left(\frac{\mu}{Q}\right)^{\gamma_c(j)} \left(\frac{\mu_0}{\mu}\right)^{\gamma_c(j)} \left(\frac{2\tilde{\delta}_c}{\tilde{\delta}_0}\right)^{\gamma_c(j)}. \quad (\text{A41})$$

The remaining t - and η -independent normalization can be written as a source factor

$$\phi_0^c(j, \tilde{\delta}_c, \tilde{\delta}_0) \equiv -2^{j+\delta_c(j)-2} \frac{\delta_c(j) + 2}{\delta_c(j) \Delta_c(j)} \frac{\Gamma(\frac{\delta_c(j)}{2} + 1)^4}{\Gamma(\delta_c(j) + 2)} (\sqrt{2})^{4-\Delta_c(j)} \left(\frac{2\tilde{\delta}_c}{\tilde{\delta}_0}\right)^{\gamma_c(j)}. \quad (\text{A42})$$

This is the explicit leftover source normalization produced by the closed upper vertex in the UV source convention used here. In this convention the cutoff parameter $\tilde{\delta}_\epsilon$ has cancelled between the boundary-mode normalization and the source rescaling, so it is not kept as an argument of ϕ_0^X . At the protected spin-2 point this normalization is finite:

$$\begin{aligned} \gamma_c(2) &= 0, & \delta_c(2) &= \Delta_c(2) = 4, \\ \phi_0^c(2, \tilde{\delta}_c, \tilde{\delta}_0) &= -\frac{4}{5}, \end{aligned} \quad (\text{A43})$$

before the overall factor $\tilde{g}_5^2/g_5^2$ is included. On the QCD side, in the LO forward normalization convention of Appendix B,

$$c_2^{-(0)} = -\frac{\gamma_2^{\Sigma g, (0)}}{\gamma_2^{+, (0)} - \gamma_2^{-(0)}} = \frac{T_F n_f}{2C_F + T_F n_f}, \quad (\text{A44})$$

so the source-factor/forward-normalization matching is finite at the structural anchor. The minus sign in Eq. (A43) is not an invariant obstruction: the phase of

the spin- j source/eigenoperator and the sign convention for the matched moment can be reversed, while the physical product in Eq. (19) is unchanged. The lower soft-wall moment is also finite there for the twist values used in this model.

The crucial point is that none of the soft-wall input used for the nucleon enters (A39). The entire upper vertex is fixed by pure AdS photon wave functions and by the quantum numbers of the exchanged spin- j field. This is the precise sense in which the upper vertex is universal and model independent.

6. Model-dependent lower vertex in the soft-wall model

All model dependence is isolated in the lower impact factor

$$\mathcal{F}_N^{(c)}(j; t, \bar{\epsilon}_c) = \frac{\tilde{g}_5}{2} \int_0^\infty dz \sqrt{g} e^{-\Phi_c} z^{1+2(j-2)} (\psi_R^2 + \psi_L^2) z^{-(j-2)} \mathcal{H}_j^{(c)}(K, z; \epsilon). \quad (\text{A45})$$

With $u = 2\kappa_c^2 z^2$ and the standard integral

$$\int_0^\infty du u^{\lambda-1} e^{-u} U(a, b; u) = \frac{\Gamma(\lambda)\Gamma(\lambda-b+1)}{\Gamma(a+\lambda-b+1)}, \quad (\text{A46})$$

the lower vertex reduces to pure gamma functions. Separating the right- and left-handed nucleon contributions gives

$$\hat{\mathcal{F}}_R^{(c)}(j; t) = 2 \times \frac{(\bar{\epsilon}_c)^{4-\Delta_c(j)}}{\tilde{g}_5} \times \mathcal{F}_N^{(c)}(j; t, \bar{\epsilon}_c) = (\tau-1) \frac{\Gamma(\tau-1-\frac{\gamma_c}{2})\Gamma(j+\tau-1+\frac{\gamma_c}{2})\Gamma(a_{t,c}+j+\gamma_c)}{\Gamma(j+\gamma_c)\Gamma(\tau)\Gamma(a_{t,c}+j+\tau-1+\frac{\gamma_c}{2})}, \quad (\text{A47a})$$

$$\hat{\mathcal{F}}_L^{(c)}(j; t) = 2 \times \frac{(\bar{\epsilon}_c)^{4-\Delta_c(j)}}{\tilde{g}_5} \times \mathcal{F}_N^{(c)}(j; t, \bar{\epsilon}_c) = \frac{\Gamma(\tau-\frac{\gamma_c}{2})\Gamma(j+\tau+\frac{\gamma_c}{2})\Gamma(a_{t,c}+j+\gamma_c)}{\Gamma(j+\gamma_c)\Gamma(\tau)\Gamma(a_{t,c}+j+\tau+\frac{\gamma_c}{2})}, \quad (\text{A47b})$$

$$\hat{\mathcal{F}}_N^{(c)}(j; t) = \frac{1}{2} [\hat{\mathcal{F}}_R^{(c)}(j; t) + \hat{\mathcal{F}}_L^{(c)}(j; t)], \quad (\text{A47c})$$

with $a_{t,c} = -t/(4\kappa_c^2)$. Equation (A47) makes the factorization transparent: the upper vertex is universal, the lower vertex carries all model information, and in the present paper that information is taken from the soft-wall model.

Combining the upper and lower pieces yields the closed fixed- j amplitude in the form quoted in Eq. (10), namely

$$\hat{\mathcal{H}}^{(c)}(j) = \xi^{-j} \left(\frac{\mu}{Q} \right)^{\gamma_c} {}_2F_1 \left(\frac{j}{2} + \frac{\gamma_c}{4}, \frac{j+1}{2} + \frac{\gamma_c}{4}; j + \frac{3}{2} + \frac{\gamma_c}{2}; \frac{\eta^2}{\xi^2} \right) \left(\frac{\mu_0}{\mu} \right)^{\gamma_c} \phi_0^c(j, \tilde{\delta}_c, \tilde{\delta}_0) \hat{d}_j(\eta, t) \left(\frac{\tilde{g}_5^2}{g_5^2} \right) \hat{\mathcal{F}}_N^{(c)}(j; t). \quad (\text{A48})$$

7. Even-spin open channel as the parallel Nishio-Watari construction

The even open channel is not derived from the closed channel by fiat. The Nishio-Watari open-string construction supplies the parallel fixed- j upper vertex, whose projected scalar part is the object relevant for the unpolarized singlet vector invariant amplitude. In the conventions used here this upper vertex is

$$\tilde{\mathcal{C}}_{\gamma\gamma}^{(o)}(j; Q_1^2, Q_2^2, \bar{\epsilon}_o) = \frac{g_5}{2g_5^2} \int_0^\infty dz \sqrt{g} e^{-\Phi_o} z^{4+2(j-2)} \mathcal{K}_{\gamma\gamma}^{(o)}(Q_1, Q_2; z) z^{-(j-2)} \Psi_j^{(o), \text{bdry}}(z; \epsilon), \quad (\text{A49})$$

with

$$\mathcal{K}_{\gamma\gamma}^{(o)}(Q_1, Q_2; z) = \mathcal{V}(Q_1, z)\mathcal{V}(Q_2, z), \quad (\text{A50a})$$

$$\Psi_j^{(o), \text{bdry}}(z; \varepsilon) = -\frac{(\sqrt{2}\kappa_o z)^{\Delta_o(j)}(\sqrt{2}\bar{\varepsilon}_o)^{4-\Delta_o(j)}}{\Delta_o(j)}. \quad (\text{A50b})$$

Taking the same conformal photon limit $\mathcal{V}(Q_i, z) \rightarrow Q_i z K_1(Q_i z)$ gives the open-channel power count

$$z^{-5} z^{4+2(j-2)} z^{-(j-2)} z^{\Delta_o(j)} z^2 = z^{j+\Delta_o(j)-1} = z^{1+\delta_o(j)}, \quad (\text{A51})$$

so the open upper vertex contains the same Bessel product integral

$$C_1\left(\delta_o(j), \frac{\eta}{\xi}\right) = \left(1 - \frac{\eta^2}{\xi^2}\right)^{1/2} \int_0^\infty dy y^{1+\delta_o(j)} \times K_1\left(y\sqrt{1+\frac{\eta}{\xi}}\right) K_1\left(y\sqrt{1-\frac{\eta}{\xi}}\right), \quad (\text{A52})$$

and hence the same Gauss-hypergeometric kernel with $\gamma_c \rightarrow \gamma_o$. This is precisely the C_1 integral and hypergeometric transformation displayed by Nishio-Watari in Appendix B of Ref. [31]. The open field $h_{\mu\nu}^o$ should therefore be read here as the effective even-spin open Reggeon/tensor exchange after projection onto the same scalar invariant, not as a claim about the full tensor structure of the open-string sector.

Only after the open and closed projected vertices have both been specified do we summarize their relation by the direct replacement rule (34). The rule is therefore a shorthand between two fixed- j Witten vertices, not a derivation of the open sector from the closed one. It acts on the projected upper scalar kernel and the resulting fixed- j invariant amplitude, not on the full open-string tensor sector. The open trajectory is

$$\Delta_o(j) = 2 + j + \gamma_o(j) = 2 + \sqrt{\lambda}(j - j_{0o}), \quad j_{0o} = 1 - \frac{1}{\sqrt{\lambda}}, \quad (\text{A53})$$

so that

$$\delta_o(j) = j + \Delta_o(j) - 2 = 2j + \gamma_o(j). \quad (\text{A54})$$

Because the upper vertex depends only on the conformal photon wave functions and on the spin and dimension of the exchanged field, it has the same functional form as in the closed channel, with the substitutions $\Delta_c \rightarrow \Delta_o$ and $\gamma_c \rightarrow \gamma_o$. Thus the open source factor is obtained directly from (A42) by replacement,

$$\phi_0^o(j, \tilde{\delta}_o, \tilde{\delta}_0) = \phi_0^c(j, \tilde{\delta}_c, \tilde{\delta}_0) \Big|_{\delta_c \rightarrow \delta_o, \Delta_c \rightarrow \Delta_o, \gamma_c \rightarrow \gamma_o}. \quad (\text{A55})$$

The lower soft-wall moments are likewise obtained by the channel replacement

$$\hat{\mathcal{F}}_R^{(o)}(j; t) = (\tau - 1) \frac{\Gamma(\tau - 1 - \frac{\gamma_o}{2}) \Gamma(j + \tau - 1 + \frac{\gamma_o}{2}) \Gamma(a_{t,o} + j + \gamma_o)}{\Gamma(j + \gamma_o) \Gamma(\tau) \Gamma(a_{t,o} + j + \tau - 1 + \frac{\gamma_o}{2})}, \quad (\text{A56a})$$

$$\hat{\mathcal{F}}_L^{(o)}(j; t) = \frac{\Gamma(\tau - \frac{\gamma_o}{2}) \Gamma(j + \tau + \frac{\gamma_o}{2}) \Gamma(a_{t,o} + j + \gamma_o)}{\Gamma(j + \gamma_o) \Gamma(\tau) \Gamma(a_{t,o} + j + \tau + \frac{\gamma_o}{2})}, \quad (\text{A56b})$$

$$\hat{\mathcal{F}}_N^{(o)}(j; t) = \frac{1}{2} \left[\hat{\mathcal{F}}_R^{(o)}(j; t) + \hat{\mathcal{F}}_L^{(o)}(j; t) \right], \quad (\text{A56c})$$

with $a_{t,o} = -t/(4\kappa_o^2)$. For the Sommerfeld-Watson reconstruction the gamma-function expressions in Eqs. (A47) and (A56) are understood by analytic continuation from the physical even-spin sequence. The contours $\mathcal{C}_X$ are chosen to avoid isolated poles generated by the soft-wall lower vertex; such poles are model-dependent lower-moment singularities, not additional ultraviolet Wilson kernels. In applications they are treated by the usual contour-deformation or residue prescription. The final open fixed- j amplitude is therefore

$$\hat{\mathcal{H}}^{(o)}(j) = \xi^{-j} \left(\frac{\mu}{Q}\right)^{\gamma_o} F_1\left(\frac{j}{2} + \frac{\gamma_o}{4}, \frac{j+1}{2} + \frac{\gamma_o}{4}; j + \frac{3}{2} + \frac{\gamma_o}{2}; \frac{\eta^2}{\xi^2}\right) \left(\frac{\mu}{\mu}\right)^{\gamma_o} \phi_0^o(j, \tilde{\delta}_o, \tilde{\delta}_0) \hat{d}_j(\eta, t) \hat{\mathcal{F}}_N^{(o)}(j; t). \quad (\text{A57})$$

This is a notation-saving summary after the parallel Nishio-Watari open Witten vertex has been specified; it is not a derivation of the open sector from the closed replacement rule.

8. Reconstruction of the physical Compton form factor and operator interpretation

The fixed- j amplitudes (A48) and (A57) are the building blocks of the physical DDVCS Compton form factor. The Reggeized amplitude is obtained by the even-signature Sommerfeld-Watson transform,

$$\hat{\mathcal{H}}(\bar{s}, \bar{t}, \chi; Q_1^2, Q_2^2) = \sum_{X=o,c} \int_{C_X} \frac{dj}{2\pi i} \xi_+(j) \hat{\mathcal{H}}^{(X)}(j),$$

$$\xi_+(j) = \frac{1 + e^{-i\pi j}}{\sin \pi j}, \quad (\text{A58})$$

with only even spins contributing in the present sector.

After holographic collinear factorization the operator interpretation is immediate. The upper vertex is the universal three-point function of two electromagnetic currents and a normalized spin- j operator. The lower vertex is the matrix element of that same operator between nucleon states. In symbolic form,

$$\text{upper vertex} \sim \langle J^\mu J^\nu \tilde{\mathcal{O}}_X^{(j)} \rangle, \quad \text{lower vertex} \sim \langle P' | \mathcal{O}_X^{(j)} | P \rangle. \quad (\text{A59})$$

The universal hypergeometric function is therefore the holographic analogue of the perturbative Wilson coefficient, whereas the bottom moments $\hat{\mathcal{F}}_N^{(X)}$ are the holographic analogue of the nonperturbative conformal moments.

This factorized structure also makes the model dependence completely explicit. The universal upper vertex does not know about the soft-wall. The soft-wall model enters only through the lower hadronic integral, and hence only through the bottom moments $\hat{\mathcal{F}}_N^{(X)}(j; t)$. Any other IR

model would modify those moments while leaving the hypergeometric hard kernel unchanged.

9. Useful algebra for the lower soft-wall moments

For completeness we record the lower-vertex integral in a form that makes the gamma-function reduction completely transparent. Inserting (A26) and the soft-wall nucleon wave functions into the lower vertex gives integrals of the schematic form

$$\int_0^\infty dz z^{\alpha-1} e^{-2\kappa_X^2 z^2} U\left(\frac{a_{t,X}}{2} + \frac{\Delta_X}{2}, \Delta_X - 1; 2\kappa_X^2 z^2\right), \quad (\text{A60})$$

which, after $u = 2\kappa_X^2 z^2$, reduce to

$$\int_0^\infty du u^{\lambda-1} e^{-u} U(a, b; u) = \frac{\Gamma(\lambda)\Gamma(\lambda - b + 1)}{\Gamma(a + \lambda - b + 1)}. \quad (\text{A61})$$

This is the elementary identity behind the gamma-function expressions (A47) and (A56).

10. Unified fixed- j formula

It is sometimes useful to write the open and closed results in a single line. Defining

$$\mathfrak{g}_o \equiv 1, \quad \mathfrak{g}_c \equiv \frac{\tilde{g}_5^2}{g_5^2}, \quad (\text{A62})$$

we may combine the two channels as

$$\hat{\mathcal{H}}^{(X)}(j) = \xi^{-j} \left(\frac{\mu}{Q}\right)^{\gamma_X} F_1\left(\frac{j}{2} + \frac{\gamma_X}{4}, \frac{j+1}{2} + \frac{\gamma_X}{4}; j + \frac{3}{2} + \frac{\gamma_X}{2}; \frac{\eta^2}{\xi^2}\right) \left(\frac{\mu_0}{\mu}\right)^{\gamma_X} \times \phi_0^X(j, \tilde{\delta}_X, \tilde{\delta}_0) \hat{d}_j(\eta, t) \mathfrak{g}_X \hat{\mathcal{F}}_N^{(X)}(j; t),$$

$$X = o, c. \quad (\text{A63})$$

This compact form makes the universal structure obvious: channel dependence enters only through the trajectory data (Δ_X, γ_X) , the overall coupling factor $\mathfrak{g}_X$, and the lower moments $\hat{\mathcal{F}}_N^{(X)}$.

APPENDIX B: PQCD CONFORMAL-OPE RESULTS USED IN THE MATCHING

For completeness, this Appendix collects only the closed-form pQCD results used in the main text. The conformal partial-wave construction, the LO singlet diagonalization, the forward DIS normalization, and the renormalization-group derivations are given in detail in Ref. [17], especially Sec. IV.1, and are not repeated here. We use the physical-spin convention $j_{\text{ref}} \rightarrow j - 1$, for which the vector projector is $[1 + (-1)^j]$ and the fixed- j factor is ξ^{-j} .

Singlet eigenchannels.

Our anomalous dimensions are defined by

$$\mu \frac{d}{d\mu} H_j^a(\mu) = -\gamma_j^a(\alpha_s) H_j^a(\mu), \quad a = \pm. \quad (\text{B1})$$

In the unpolarized singlet vector sector, the LO anomalous-dimension matrix in the quark-gluon basis is

$$\gamma_j^{(0)} = \begin{pmatrix} \gamma_j^{\Sigma\Sigma, (0)} & \gamma_j^{\Sigma g, (0)} \\ \gamma_j^{g\Sigma, (0)} & \gamma_j^{gg, (0)} \end{pmatrix}, \quad (\text{B2})$$

with

$$\gamma_j^{\Sigma\Sigma, (0)} = -C_F \left[3 + \frac{2}{j(j+1)} - 4S_1(j) \right], \quad (\text{B3a})$$

$$\gamma_j^{\Sigma g, (0)} = -4n_f T_F \frac{j^2 + j + 2}{j(j+1)(j+2)}, \quad (\text{B3b})$$

$$\gamma_j^{\pm, (0)} = \frac{1}{2} \left[\gamma_j^{\Sigma \Sigma, (0)} + \gamma_j^{gg, (0)} \pm \sqrt{(\gamma_j^{\Sigma \Sigma, (0)} - \gamma_j^{gg, (0)})^2 + 4\gamma_j^{\Sigma g, (0)} \gamma_j^{g \Sigma, (0)}} \right]. \quad (\text{B5})$$

$$\gamma_j^{g \Sigma, (0)} = -2C_F \frac{j^2 + j + 2}{(j-1)j(j+1)}, \quad (\text{B3c})$$

We use the unit-diagonal eigenvector convention of Ref. [17],

$$\gamma_j^{gg, (0)} = C_A \left[4S_1(j) + \frac{4}{j(j+1)} - \frac{12}{(j-1)(j+2)} \right] - \beta_0, \quad (\text{B3d})$$

$$\begin{pmatrix} H_j^+ \\ H_j^- \end{pmatrix} = U_j \begin{pmatrix} H_j^\Sigma \\ H_j^g \end{pmatrix}, \quad U_j = \begin{pmatrix} 1 & a_j \\ b_j & 1 \end{pmatrix}, \quad (\text{B6})$$

where

where

$$a_j = \frac{\gamma_j^{gg, (0)} - \gamma_j^{-, (0)}}{\gamma_j^{g \Sigma, (0)}}, \quad b_j = \frac{\gamma_j^{\Sigma \Sigma, (0)} - \gamma_j^{+, (0)}}{\gamma_j^{\Sigma g, (0)}}. \quad (\text{B7})$$

$$S_1(j) = \psi(j+1) + \gamma_E, \quad C_F = \frac{N_c^2 - 1}{2N_c},$$

$$C_A = N_c, \quad T_F = \frac{1}{2}, \quad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f. \quad (\text{B4})$$

The eigenvector normalization remains conventional:

$$H_j^\pm \longrightarrow N_j^\pm H_j^\pm, \quad C_j^\pm \longrightarrow (N_j^\pm)^{-1} C_j^\pm, \quad (\text{B8})$$

so only the products $C_j^\pm H_j^\pm$ are invariant.

Conformal fixed-point coefficient.

The two eigenanomalous dimensions are

At a conformal fixed point,

$$H_j^\pm(\eta, t; \mu) = \left(\frac{\mu_0}{\mu} \right)^{\gamma_j^\pm(\alpha_s^*)} H_j^\pm(\eta, t; \mu_0), \quad (\text{B9})$$

and the normalized fixed- j Wilson coefficients are

$$C_j^\pm \left(\frac{\eta}{\xi}, \frac{Q^2}{\mu^2}; \alpha_s^* \right) = c_j^\pm(\alpha_s^*) \left(\frac{\mu}{Q} \right)^{\gamma_j^\pm(\alpha_s^*)} {}_2F_1 \left(\frac{j}{2} + \frac{\gamma_j^\pm(\alpha_s^*)}{4}, \frac{j+1}{2} + \frac{\gamma_j^\pm(\alpha_s^*)}{4}; j + \frac{3}{2} + \frac{\gamma_j^\pm(\alpha_s^*)}{2}; \frac{\eta^2}{\xi^2} \right). \quad (\text{B10})$$

Hence

$$\hat{\mathcal{H}}_{\text{pQCD}}^{\text{sing}}(j) = \sum_{a=\pm} \xi^{-j} c_j^a(\alpha_s^*) \left(\frac{\mu}{Q} \right)^{\gamma_j^a(\alpha_s^*)} \mathcal{K}_j^{(\gamma_j^a(\alpha_s^*))}(\vartheta) \left(\frac{\mu_0}{\mu} \right)^{\gamma_j^a(\alpha_s^*)} H_j^a(\eta, t; \mu_0). \quad (\text{B11})$$

The finite constants $c_j^\pm$ are fixed by requiring the nonforward coefficients to reduce in the forward limit to the standard DIS Wilson coefficients. The detailed row-vector basis rotation is given in Ref. [17]. At LO the forward quark-gluon coefficient is (1, 0), and in the convention (B6) the final result is

$$c_j^{+, (0)} = \frac{\gamma_j^{\Sigma \Sigma, (0)} - \gamma_j^{-, (0)}}{\gamma_j^{+, (0)} - \gamma_j^{-, (0)}} = \frac{\gamma_j^{+, (0)} - \gamma_j^{gg, (0)}}{\gamma_j^{+, (0)} - \gamma_j^{-, (0)}}, \quad (\text{B12a})$$

$$c_j^{-,(0)} = -\frac{\gamma_j^{\Sigma g,(0)}}{\gamma_j^{+,(0)} - \gamma_j^{-,(0)}}. \quad (\text{B12b})$$

LO running and the spin-2 anchor.

With a running coupling, the LO eigenmoments evolve as

$$H_j^\pm(\eta, t; \mu) = \mathcal{E}_j^\pm(\mu, \mu_0) H_j^\pm(\eta, t; \mu_0), \quad \mathcal{E}_j^\pm(\mu, \mu_0) = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\gamma_j^{\pm,(0)}/\beta_0}. \quad (\text{B13})$$

The corresponding LO fixed- j singlet amplitude may be written directly as

$$\hat{\mathcal{H}}_{\text{pQCD}}^{\text{sing}}(j; \xi, \eta, t, Q^2; \mu) = \sum_{a=\pm} \xi^{-j} c_j^a(\alpha_s(Q)) \mathcal{K}_j^{(\gamma_j^a(\alpha_s(Q)))}(\vartheta) \left[\frac{\alpha_s(Q)}{\alpha_s(\mu_0)} \right]^{\gamma_j^{a,(0)}/\beta_0} H_j^a(\eta, t; \mu_0). \quad (\text{B14})$$

At the first physical even spin, Eq. (B5) gives

$$\gamma_2^{-,(0)} = 0, \quad \gamma_2^{+,(0)} = \frac{4}{3}(2C_F + T_F n_f), \quad (\text{B15})$$

which is the protected/unprotected split used in the main text. Expanding the same closed-form eigenvalues about $j = 2$ gives

$$\gamma_j^{-,(0)} = \frac{C_F[C_A(24\pi^2 - 65) + T_F n_f(12\pi^2 - 5)]}{18(2C_F + T_F n_f)}(j - 2) + \mathcal{O}((j - 2)^2), \quad (\text{B16a})$$

$$\gamma_j^{+,(0)} = \frac{4}{3}(2C_F + T_F n_f) + \frac{4C_F^2(12\pi^2 - 85) + T_F n_f[C_A(24\pi^2 - 65) - 160C_F]}{36(2C_F + T_F n_f)}(j - 2) + \mathcal{O}((j - 2)^2). \quad (\text{B16b})$$

Equations (B5)–(B16) are the complete pQCD input required for the fixed- j structural comparison in the main text.

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