

Three family supersymmetric Pati-Salam model from rigid intersecting D6-branes

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We construct, for the first time, a three family $\mathcal{N} = 1$ supersymmetric Pati-Salam model from rigid intersecting D6-branes on a factorizable $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orientifold with discrete torsion. The factorizable geometry allows for explicit control over rigid cycles and moduli stabilization. We can break the Pati-Salam gauge symmetry down to the Standard Model (SM) gauge symmetry via the supersymmetry preserving Higgs mechanism, generate the SM fermion masses and mixings, and break the supersymmetry via gaugino condensations in the hidden sector.

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Introduction. A great challenge in string phenomenology has been to construct the realistic string vacua, which can give the low-energy supersymmetric Standard Models (SMs) without exotic particles, and stabilize the moduli fields. We can connect such string models to the low-energy realistic particle physics via renormalization group equation running and thus probe these models at the Large Hadron Collider and the future colliders, etc. We would like to emphasize that the semirealistic supersymmetric standardlike models and grand unified theories (GUTs) have been constructed extensively in type IIA theory on the $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold [1–22].

The key point is that only Pati-Salam-like models can explain all the Yukawa couplings [4]. Such kind of models can also be constructed in the heterotic string framework [23,24]. In heterotic models, the gauge group arises from the breaking of $E_8 \times E_8$ via internal bundles and orbifold twists, with matter spectra determined by the bundle structure; thus, there exists gauge coupling unification naturally. In contrast, the intersecting D6-brane models generate gauge groups from stacks of branes

wrapping factorizable three-cycles, with chiral matter localized at brane intersections. This approach provides a clear geometric understanding of gauge symmetries, chirality, and coupling hierarchies and allows for the systematic construction of rigid cycles without adjoint matter. Furthermore, couplings such as μ -terms and non-perturbative neutrino masses can arise from D-brane instantons. Global consistency conditions such as RR tadpole and K-theory constraints are manifest, ensuring tight control over the resulting low-energy spectrum.

The three family supersymmetric Pati-Salam models have been constructed systematically first in [4]. The SM fermion masses and mixings in some models have been explained explicitly [9,10,14,20]. In particular, a systematic method was proposed to construct all the three family $\mathcal{N} = 1$ supersymmetric Pati-Salam models, where the Pati-Salam gauge symmetry can be broken down to the SM gauge symmetry via the D-brane splitting and supersymmetry preserving Higgs mechanism [11,12]. However, in these semirealistic models, there exist three adjoint multiplets for $U(N)$ gauge symmetries since the D6-branes are not rigid, and the models from the rigid intersecting D6-branes are still very far from realistic [25–32]. Therefore, how to construct the three family $\mathcal{N} = 1$ supersymmetric Pati-Salam models from rigid intersecting D6-branes is a big challenge.

In Ref. [25], it was first shown that the global $\mathcal{N} = 1$ chiral models can be constructed from the rigid intersecting D6-branes on a factorizable $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orientifold with discrete torsion. However, the consistent three-generation models in this setup with factorizable tori were very

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difficult to engineer, and the examples in [25] generically had four generations. In the case of nonrigid branes, we need at least one torus to be tilted to generate three-generation models on the mirror orbifold $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [1–18,20]. However, in the present setup with rigid three-cycles, no consistent three-generation models employing tilted tori are known. In Ref. [26], a first example of a three-generation model was indeed presented; however, it was constructed on nonfactorizable tori. The equivalent model with factorizable tori does not satisfy $\mathcal{N} = 1$ supersymmetry conditions.

The main advantage of using factorizable toroidal orbifolds is the enhanced control over D6-brane configurations and the systematic construction of rigid three-cycles. On factorizable spaces, D-brane cycles decompose into products of one-cycles on each two-torus, simplifying the computation of intersection numbers, tadpole conditions, and chiral spectra. In contrast, nonfactorizable geometries often obscure the localization of exceptional cycles and complicate the classification of rigid three-cycles, as discussed in [33]. Furthermore, the intersection number formulas and tadpole cancellation conditions depend on the choice of lattice (e.g., A or B type), which is more transparent in factorizable settings. In this Letter, we for the first time construct a three family $\mathcal{N} = 1$ supersymmetric Pati-Salam model with factorizable tori from rigid intersecting D6-branes on a factorizable $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orientifold, utilizing only the rectangular tori (A-type lattice), and briefly study its phenomenological consequences.

Model building from rigid intersecting D6-branes. Discrete torsion leads to rigid D6-branes wrapping fractional and exceptional three-cycle collapsed to a lower-dimensional fixed cycle in the orbifold limit of the Calabi-Yau space [27]. As the D6-branes are rigid, adjoint moduli disappear from the open string spectra: The irregularity of gauge symmetry breaking is removed along the flat direction, and one-loop β functions are improved. Rigid branes allow instanton contributions to the Kähler potential, superpotential generating μ -terms [28], nonperturbative neutrino masses [29,30], and forbidden Yukawa couplings [31,32] or might even trigger supersymmetry breaking [34].

In setup [25], the six-torus is factorizable as $\mathbb{T}^6 = \mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$, and the orbifold group is $\mathbb{Z}_2 \times \mathbb{Z}_2'$, where each $\mathbb{Z}_2$ factor inverts two two-tori. In addition to the untwisted cycles, there are 48 independent collapsed three-cycles from the twisted sectors. Thus, in this setup, there are, in total, 104 three-cycles, i.e., $b_3(\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')) = 2 + 2h_{unt}^{21} + 2h_{tw}^{21}$, with $(h_{unt}^{21}, h_{tw}^{21}) = (3, 48)$. For each sector, we denote the 16 fixed points on $(\mathbb{T}_1^2 \times \mathbb{T}_2^2)/\mathbb{Z}_2$ by $[e_{i,j}^g]$, with $g \in \{\Theta, \Theta', \Theta\Theta'\}$, and $i, j \in \{1, 2, 3, 4\}$ as depicted in Fig. 1. After blowing up the orbifold singularities, these become two-cycles with the topology of $\mathbb{S}^2$. Each such four-dimensional $\mathbb{T}^4/\mathbb{Z}_2$ is an orbifold of $\mathbb{K}3$ before taking

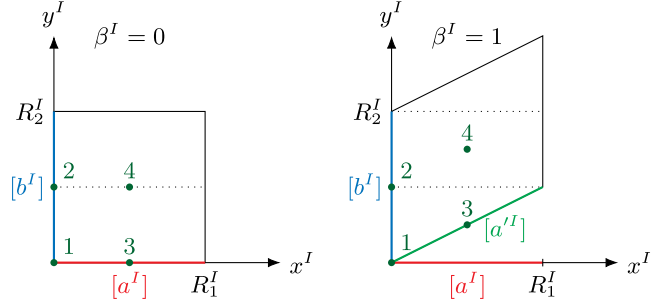


FIG. 1. The $\mathbb{Z}_2$ invariant a-type (left) and b-type (right) lattices. $\mathbb{Z}_2$ fixed points are depicted in dots. The $\mathcal{R}$ invariant x^I axis is along the one-cycle $[a^I] - \frac{\beta^I}{2}[b^I]$ with $\beta^I = 0$ for the a-type lattice and $\beta^I = 1$ for the b-type lattice. $\mathcal{R}$ acts as reflection along the y^I axis, which is spanned by the one-cycle $[b^I]$. For the a-type lattice, all $\mathbb{Z}_2$ fixed points are invariant under $\mathcal{R}$, whereas for the b-type lattice, only 1 and 2 are invariant, while 3 and 4 exchange as $3 \xleftrightarrow{\mathcal{R}} 4$.

the other elements of the orientifold group into account. There are three $\mathbb{Z}_2$ -twisted sectors with 16 fixed points each. The three two-tori have radii $R_{1,2}^{(i)}$ along the x^i and y^i axes, $i \in \{1, 2, 3\}$. The tori may $(\beta^i = 1)^1$ or may not $(\beta^i = 0)$ be tilted. There are four orbifold fixed points on each torus at $(0,0)$, $(0, R_2^{(i)}/2)$, $(R_1^{(i)}/2, \beta^i R_2^{(i)}/4)$, and $(R_1^{(i)}/2, (2 + \beta^i)R_2^{(i)}/4)$. They will be labeled fixed points 1, 2, 3, and 4. All these geometrical data are shown in Fig. 1 for an untilted and a tilted torus. The orientifold group is $\Omega\mathcal{R}(-1)^{F_L}$, where F_L is the left-moving spacetime fermion number, Ω is world sheet parity, and $\mathcal{R}$ is an isometric antiholomorphic involution of internal space acting on the Kähler class J and three-form Ω_3 as

$$\mathcal{R}J = -J, \mathcal{R}\Omega_3 = e^{2i\phi}\bar{\Omega}_3, \quad (1)$$

with ϕ being real. The $\mathcal{R}$ can be thought of as complex conjugation if ϕ is vanishing. This gives rise to the orientifold plane at the fixed locus of $\mathcal{R}$. The branes (split into fractional branes) from the hidden sector [35] are utilized in this Letter to cancel the untwisted tadpoles.

The (stacks of) D6-branes on this background are described by the wrapping numbers (n^i, m^i) , the charges under the twisted RR fields $\epsilon^i \in \{-1, 1\}$, the position $\delta^i \in \{0, 1\}$, and the discrete Wilson lines $\lambda^i \in \{0, 1\}$. The D6-brane wraps the one-cycle $n^i[a^i] + m^i[b^i]$ on the i th torus, and the fundamental one-cycles $[a^i] = [a^i] + \frac{\beta^i}{2}[b^i]$ and $[b^i]$ are shown in Fig. 1. The ϵ^i satisfy $\epsilon^1 = \epsilon^2\epsilon^3$ and are related to the discrete Wilson lines [36]. The position is described by the three parameters δ^i , where $\delta^i = 0$ if the D6-brane goes through fixed point 1 on the i th torus, and $\delta^i = 1$ otherwise. An alternative way to

¹Unlike [25], we take $\beta = 1$ and not $1/2$ for the tilted torus.

characterize a D6-brane is to use $\epsilon_{kl}^i \in \{-1, 0, 1\}$, $i \in \{1, 2, 3\}$, $k, l \in \{1, 2, 3, 4\}$ [25], instead of ϵ^i , δ^i , and λ^i . ϵ_{kl}^i is the charge of the D6-brane under the fixed point labeled kl in the i th twisted sector. The ϵ_{kl}^i can be determined from ϵ^i , δ^i , and λ^i . Note that for each i only 4 out of the 16 ϵ_{kl}^i are nonzero. In both these descriptions, there is some redundancy [25]. Rather than fixing some of the ϵ_{kl}^i charges to be 1 [25], it will here be more convenient to choose $n^{1,2} > 0$ (or m^i positive if n^i vanishes).

It is useful to define $\tilde{m}^i = m^i + \frac{\beta^i}{2} n^i$, such that a D6-brane wraps the one-cycle $n^i[a^i] + \tilde{m}^i[b^i]$ on the i th torus. The volume of this one-cycle is given by

$$V^i = \sqrt{(n^i)^2 (R_1^{(i)})^2 + (\tilde{m}^i)^2 (R_2^{(i)})^2}, \quad (2)$$

and the tree-level gauge coupling is

$$\begin{aligned} \frac{1}{g_{\text{tree}}^2} &= e^{-\phi_{10}} \prod_i V^i = \frac{e^{-\phi_4}}{(T_1 T_2 T_3)^{1/2}} \prod_i V^i \\ &= \frac{(SU_1 U_2 U_3)^{1/4}}{(T_1 T_2 T_3)^{1/2}} \prod_i V^i, \end{aligned} \quad (3)$$

where $\phi_{10}(\phi_4)$ is the 10(4)-dimensional dilaton in string frame, S is the dilaton in Einstein frame, U_i are the (real parts of the) complex structure moduli in Einstein frame, and $T_i = R_1^{(i)} R_2^{(i)}$ are the (real parts of the) Kähler moduli. From (3) one can, using the supersymmetry condition (see below), derive the dependence of the tree-level gauge kinetic function on the untwisted moduli [37]

$$\hat{f}_{\text{tree}} = S^c n^1 n^2 n^3 - \sum_{i \neq j \neq k=1}^3 U_i^c n^i \tilde{m}^j \tilde{m}^k, \quad (4)$$

where S^c and U_i^c are the complexified dilaton and complex structure moduli, the axions being RR fields. Similarly, T_i^c are complexified Kähler moduli with the axions stemming from the Neveu-Schwarz Neveu-Schwarz two-form field.

The D6-brane is rotated by the angles θ^i , defined via $\tan \theta^i = \tilde{m}^i R_2^{(i)} / n^i R_1^{(i)}$, with respect to the x axes of the three tori. Only supersymmetric configurations will be considered in this Letter as

$$\sum_{i=1}^3 \theta^i = 0 \mod 2\pi. \quad (5)$$

The charges of the four orientifold planes are denoted as $\eta_{\Omega\mathcal{R}}$ and $\eta_{\Omega\mathcal{R}i}$ with $i \in \{1, 2, 3\}$ and have to satisfy

$$\eta_{\Omega\mathcal{R}} \prod_{i=1}^3 \eta_{\Omega\mathcal{R}i} = -1 \quad (6)$$

in the present case of the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold with $h_{21} = 51$ [25,38]. The tadpole cancellation conditions are given by

TABLE I. Chiral spectrum for intersecting D6-branes at angles. Positive intersection numbers refer to the left-handed fermions.

Sector	Representation
aa	$U(N_a/2)$ vector multiplet
$ab + ba$	$I_{ab}(\square_a, \square_b) = \Pi_a \circ \Pi_b$
$ab' + b'a$	$I_{ab'}(\square_a, \square_b) = \Pi_a \circ \Pi'_b$
$aa' + a'a$	$n_{\square\square} = \frac{1}{2}(\Pi_a \circ \Pi'_a - \Pi_a \circ \Pi_{O6})$
	$n_{\square} = \frac{1}{2}(\Pi_a \circ \Pi'_a + \Pi_a \circ \Pi_{O6})$

$$\begin{aligned} \sum_a N_a n_a^1 n_a^2 n_a^3 &= 16 \eta_{\Omega\mathcal{R}}, \\ \sum_a N_a n_a^i \tilde{m}_a^j \tilde{m}_a^k &= -2^{4-\beta^j-\beta^k} \eta_{\Omega\mathcal{R}i}, \quad i \neq j \neq k \in \{1, 2, 3\}, \\ \sum_a N_a n_a^i (\epsilon_{a,kl}^i - \eta_{\Omega\mathcal{R}} \eta_{\Omega\mathcal{R}i} \epsilon_{a,\mathcal{R}(k)\mathcal{R}(l)}^i) &= 0, \\ \sum_a N_a \tilde{m}_a^i (\epsilon_{a,kl}^i + \eta_{\Omega\mathcal{R}} \eta_{\Omega\mathcal{R}i} \epsilon_{a,\mathcal{R}(k)\mathcal{R}(l)}^i) &= 0, \end{aligned} \quad (7)$$

where $\mathcal{R}(k) = k$ in case of an untwisted torus and $\mathcal{R}(\{1, 2, 3, 4\}) = \{1, 2, 4, 3\}$ in the other case [25], and the sum is a sum over all stacks of D6-branes. N_a denotes the number of D6-branes on stack a . The wrapping numbers and twisted charges carry an index a denoting the D6-brane stack which they describe. The orientifold projection acts on the wrapping numbers and twisted charges as follows:

$$\begin{aligned} \tilde{m}^I &\rightarrow -\tilde{m}^I, \\ \epsilon_{kl}^i &\rightarrow -\eta_{\Omega\mathcal{R}} \eta_{\Omega\mathcal{R}i} \epsilon_{\mathcal{R}(k)\mathcal{R}(l)}^i. \end{aligned} \quad (8)$$

A three family $\mathcal{N} = 1$ supersymmetric Pati-Salam model from rigid cycles. To construct the three family Pati-Salam model using rigid, semirigid, and nonrigid branes, we follow the strategy outlined in [26]. In the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold, the fractional branes invariant under $\Omega\mathcal{R}$ are those placed on top of an exotic $O6^{(+,+)}$ plane that is taken as $O_{\Omega\mathcal{R}}$ in our choice (6). The adjoint fields from aa sector do not arise due to the rigid D6-branes.

The generic massless particle spectrum for intersecting D6-branes at general angles can be expressed via the intersection numbers as listed in Table I. We present the model in Table II with the corresponding fixed points of the brane stacks given in Table III. We deform the hidden sector D6-branes to move away from the origin in order to get rid of some of the associated exotic particles. The complete perturbative particle spectrum is shown in Table IV. The several exotic particles X_L^i and X_R^i can be decoupled from low-energy spectrum via strong dynamics as discussed in the companion paper [35].

TABLE II. Model with gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_1 \times SU(2)_2 \times SU(2)^3 \times SU(4)^4 \times USp(4)^4$. The torus moduli are $\chi_1 = 8$, $\chi_2 = 4$, $\chi_3 = 3$, and the tree-level gauge coupling relation is $g_C^2 = \frac{5}{4}g_L^2 = \frac{25}{44}g_R^2 = \frac{125}{182}g_Y^2 = 2\sqrt{2}\pi e^{\phi_4}$.

Stack	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a_1	4	$(0, 1) \times (1, 0) \times (0, -1)$
a_2	2	$(-4, -1) \times (-2, 1) \times (1, 0)$
a_3	2	$(4, 1) \times (2, 1) \times (-3, -1)$
a_4	2	$(-1, 0) \times (-2, 1) \times (2, 1)$
a_5	2	$(1, 0) \times (0, 1) \times (0, -1)$
b_1	2	$(1, 0) \times (0, -1) \times (0, 1)$
b_2	2	$(-1, 0) \times (0, 1) \times (0, 1)$
c_1	4	$(0, -1) \times (1, 0) \times (0, 1)$
c_2	4	$(0, 1) \times (-1, 0) \times (0, 1)$
d_1	4	$(0, -1) \times (0, 1) \times (1, 0)$
d_2	4	$(0, -1) \times (0, -1) \times (-1, 0)$
e_1	2	$(1, 0) \times (1, 0) \times (1, 0)$
e_2	2	$(-1, 0) \times (-1, 0) \times (1, 0)$
e_3	2	$(1, 0) \times (-1, 0) \times (-1, 0)$
e_4	2	$(-1, 0) \times (1, 0) \times (-1, 0)$

The additional constraint from (7) makes it very difficult to construct the consistent vacua. Indeed, the model reported in Table II is of phenomenological interest due to the breaking of the Pati-Salam gauge symmetry down to the SM gauge symmetry. The purpose of the present section is to give an example of these more involved constructions that also admit the low-energy theories, very much closer to realistic particle physics. Models presented in [35] usually do not have the Higgs fields to break the Pati-Salam gauge symmetry down to the SM. We address this important feature of model building for the model in Table II. With the choice of the fixed points listed in Table III, one can calculate the chiral spectrum presented in Table IV. Table II consists of five sets D6-branes, i.e., a, b, c, d, e . The set a consists of five stacks of fractional D6-branes, but each with different bulk wrapping numbers. The set a , most importantly, gives rise to the gauge symmetries and chiral spectrum. In contrast to the set a , the sets b, c, d , and e are the bulk D6-branes which are split into fractional constituents. It can be checked that Table II satisfies all the constraints presented in (7) for untwisted and twisted tadpoles and in (5) for supersymmetry condition.

The gauge symmetries derived from the set a are $SU(4) \times SU(2)_L \times SU(2)_1 \times SU(2)_2 \times SU(2)$, which give rise to the Pati-Salam-like theory from the subsets a_1, a_2, a_3 , and a_4 . In contrast to set a , the sets b, c, d , and e yield the gauge symmetries $U(2)^2, U(4)^2, U(4)^2$, and $USp(4)^4$, respectively. The latter gauge groups from the fractional D6-branes can be deformed into bulk D6-branes upon Higgsings as follows:

TABLE III. The hidden sector D6-branes, shown with asterisks, are deformed to move away from the origin in order to remove the associated exotic particles.

Cycles	Θ' sector	$\Theta\Theta'$ sector	Θ sector
Π_{a_1}	$((1, 3), (1, 2))$	$((1, 2), (1, 2))$	$((1, 2), (1, 3))$
Π_{a_2}	$((1, 2), (1, 3))$	$((1, 2), (1, 3))$	$((1, 2), (1, 2))$
Π_{a_3}	$((1, 2), (1, 2))$	$((1, 3), (1, 2))$	$((1, 3), (1, 2))$
Π_{a_4}	$((1, 2), (1, 4))$	$((1, 2), (1, 4))$	$((1, 2), (1, 2))$
$\Pi_{a_5}^*$	$((3, 4), (3, 4))$	$((2, 4), (3, 4))$	$((2, 4), (3, 4))$
$\Pi_{b_1}^*$	$((3, 4), (3, 4))$	$((2, 4), (3, 4))$	$((2, 4), (3, 4))$
$\Pi_{b_2}^*$	$((3, 4), (3, 4))$	$((2, 4), (3, 4))$	$((2, 4), (3, 4))$
$\Pi_{c_1}^*$	$((2, 4), (3, 4))$	$((3, 4), (3, 4))$	$((3, 4), (2, 4))$
$\Pi_{c_2}^*$	$((2, 4), (3, 4))$	$((3, 4), (3, 4))$	$((3, 4), (2, 4))$
$\Pi_{d_1}^*$	$((3, 4), (2, 4))$	$((3, 4), (2, 4))$	$((3, 4), (3, 4))$
$\Pi_{d_2}^*$	$((3, 4), (2, 4))$	$((3, 4), (2, 4))$	$((3, 4), (3, 4))$
$\Pi_{e_1}^*$	$((2, 4), (2, 4))$	$((2, 4), (2, 4))$	$((2, 4), (2, 4))$
$\Pi_{e_2}^*$	$((2, 4), (2, 4))$	$((2, 4), (2, 4))$	$((2, 4), (2, 4))$
$\Pi_{e_3}^*$	$((2, 4), (2, 4))$	$((2, 4), (2, 4))$	$((2, 4), (2, 4))$
$\Pi_{e_4}^*$	$((2, 4), (2, 4))$	$((2, 4), (2, 4))$	$((2, 4), (2, 4))$

$$\{b_1, b_2\}, \{c_1, c_2\}, \{d_1, d_2\}, \{e_1, e_2, e_3, e_4\} \rightarrow b, c, d, e, \quad (9)$$

$$U(2)^2 \times U(4)^2 \times U(4)^2 \times USp(4)^4 \\ \longrightarrow U(1) \times U(2) \times U(2) \times USp(4). \quad (10)$$

Some of the $U(1)$ s are not really gauge symmetries, but only global ones because their gauge bosons would receive Stueckelberg mass through the Green-Schwarz mechanism [2]. Thus, Pati-Salam gauge symmetries $SU(4) \times SU(2) \times SU(2)$ are obtained from D-branes a_1, a_2, a_3 , and a_4 , while the extra symmetries $U(2)^2 \times U(4)^2 \times U(4)^2 \times USp(4)^4$ arise from sets b, c, d , and e , which turns out to be, after Higgsing (10), the gauge symmetries $U(1) \times U(2) \times U(2) \times USp(4)$.

The rigid brane a_4 is moved into the visible sector for the sake of symmetry breaking, thus modifying the condition for three generations,

$$I_{a_1 a_2} + I_{a_1 a'_2} = \pm 3, \quad (11)$$

$$I_{a_1 a_3} + I_{a_1 a'_3} + I_{a_1 a_4} + I_{a_1 a'_4} = \mp 3. \quad (12)$$

We present the chiral spectrum of this theory in Table IV. In particular, we present the conventions of some particles concretely, for example, the SM fermions, Higgs fields, etc. Moreover, the visible sector will suffer from the uncanceled twisted tadpoles. Therefore, an additional D6-brane a_5 is added to cancel the latter. In order to get the consistent model, untwisted tadpoles are canceled by adding more semirigid branes from the set b, c, d , and e such that they do not make any contribution to the twisted tadpoles.

TABLE IV. The chiral particle spectrum for the model in Table II, and the hidden part of the symmetry is $SU(2)^3 \times SU(4)^4 \times USp(4)^4$. The chiral spectrum is singlet under this symmetry.

Sector	$SU(4) \times SU(2)_L \times SU(2)_1 \times SU(2)_2 \times \text{Hidden}$	
$a_1 a_2$	$3 \times (4, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
$a_1 a_3'$	$6 \times (\bar{4}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
$a_1 a_3$	$1 \times (4, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^c(Q_R, L_R)$
$a_1 a_4$	$2 \times (4, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^{ic}(Q_R, L_R)$
$a_2 \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$4 \times (1, \bar{1}, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	S_L^i
$a_2 a_3$	$3 \times (1, \bar{2}, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi_i(H_u', H_d')$
$a_2 a_3'$	$5 \times (1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi_i(H_u', H_d')$
$a_2 a_4'$	$1 \times (1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	Ξ_i
$a_3 \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$60 \times (1, 1, \bar{1}, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	S_R^i
$a_3 \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$6 \times (1, 1, \bar{3}, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	T_R^i
$a_3 a_4'$	$3 \times (1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	Δ_i
$a_2 a_5'$	$1 \times (1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	X_L^i
$a_2 b_1$	$1 \times (1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	X_L^i
$a_2 b_2'$	$1 \times (1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	X_L^i
$a_2 c_1'$	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1)$	X_L^i
$a_2 c_2$	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1, 1)$	X_L^i
$a_3 a_5'$	$2 \times (1, 1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
$a_3 a_5$	$1 \times (1, 1, \bar{2}, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
$a_3 b_1'$	$1 \times (1, 1, \bar{2}, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
$a_3 b_1$	$2 \times (1, 1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
$a_3 b_2'$	$2 \times (1, 1, \bar{2}, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
$a_3 b_2$	$1 \times (1, 1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
$a_3 c_1'$	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1)$	X_R^i
$a_3 c_1$	$4 \times (1, 1, \bar{2}, 1, 1, 1, 1, 4, 1, 1, 1, 1, 1, 1)$	X_R^i
$a_3 c_2'$	$4 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1)$	X_R^i
$a_3 c_2$	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1, 1)$	X_R^i
$a_4 c_1'$	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1)$	X_R^i
$a_4 c_2'$	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1)$	X_R^i
$a_3 d_1'$	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1)$	X_R^i
$a_3 d_1$	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1)$	X_R^i
$a_3 d_2'$	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1)$	X_R^i
$a_3 d_2$	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1)$	X_R^i
$a_4 d_1'$	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1)$	X_R^i
$a_4 d_2$	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1)$	X_R^i
$a_3 e_2'$	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 4, 1)$	X_R^i
$a_3 e_2$	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1)$	X_R^i

Phenomenological study. The gauge symmetries from a_1 , a_2 , a_3 , and a_4 are $U(4)_C$, $U(2)_L$, $U(2)_1$, and $U(2)_2$, respectively. The anomalies from four global $U(1)$'s of $U(4)_C$, $U(2)_L$, $U(2)_1$, and $U(2)_2$ are canceled by the Green-Schwarz mechanism, and the gauge fields of these $U(1)$'s obtain masses via the linear $B \wedge F$ couplings. So, the effective gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_1 \times SU(2)_2$. When Δ_i obtain vacuum expectation values (VEVs), the $SU(2)_1 \times SU(2)_2$ gauge symmetry is broken down to the diagonal $SU(2)_R$ gauge symmetry. Thus, we obtain the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge

symmetry. We have three families of left-handed fermions F^i and three families of right-handed fermions from F_R^i , F_R^c , F_R^{ci} , i.e., $6 - 1 - 2 = 3$.

Moreover, by giving VEVs to F_R^c and a linear combination of F_R^i , we can break the Pati-Salam gauge symmetry down to the SM gauge symmetry and preserve the four-dimensional $\mathcal{N} = 1$ supersymmetry by keeping the D and F flatness. In addition, we can generate the SM fermion masses and mixing, as well as the vectorlike particle masses for F_R^{ci} and two other linear combinations of F_R^i via the following superpotential:

$$W \supset y_{ij}^k F_L^i F_R^j \Phi_k + y_{ij}^k F_R^{ci} F_R^j \Delta_k. \quad (13)$$

Furthermore, the $SU(4)$ gauge symmetries in the hidden sector have negative β functions, and then the supersymmetry can be broken via gaugino condensations.

In intersecting D-brane models, proton decay mediating operators are absent at all orders in perturbation theory due to world sheet consistency conditions. All interactions originate from open string diagrams, which require an even number of string end points and conservation of brane charges. As a result, processes involving an odd number of $SU(3)$ triplets, such as $U + D + D \rightarrow$ leptons, cannot be realized perturbatively [39]. To be concrete, the gauge symmetry of our Pati-Salam models is $U(4)_C \times U(2)_L \times U(2)_R$; thus, we have three anomalous $U(1)$ gauge symmetries $U(1)_C \times U(1)_L \times U(1)_R$ whose gauge anomalies are canceled by the generalized Green-Schwarz mechanism. The proton decay operators such as $QQQL$ and $U^c D^c U^c E^c$ are forbidden by these anomalous $U(1)$ gauge symmetries.

The tree-level gauge coupling relations for the model in Table II are

$$g_C^2 = \frac{5}{4} g_L^2 = \frac{25}{44} g_R^2 = \frac{125}{182} \cdot \frac{5}{3} g_Y^2, \quad (14)$$

where the weak hypercharge arises from a canonical Pati-Salam embedding involving the $SU(4)_C$ and $SU(2)_{1,2}$ generators. This implies

$$g_L^2 = \frac{250}{273} g_Y^2 \Rightarrow \sin^2 \theta_W = \frac{g_Y^2}{g_Y^2 + g_L^2} = \frac{273}{523} \approx 0.521,$$

valid at the string/unification scale. While this differs from the observed low-energy value $\sin^2 \theta_W(M_Z) \approx 0.231$, the renormalization group equation running by considering the vectorlike particles and adjoint fields [13] as well as string threshold corrections [40,41] can bridge the gap. In fact, the hypercharge normalization k_Y , defined via $g_Y^2 = k_Y g^2$, is not fixed universally in D-brane models, as $U(1)_Y$ typically arises from a linear combination of $U(1)_a$ factors with model-dependent coefficients and wrapping numbers [5,42]. By choosing alternative, anomaly-free hypercharge embeddings, it is possible to adjust k_Y to bring

the tree-level prediction of $\sin^2 \theta_W$ closer to the minimal supersymmetric Standard Model or SU(5) GUT value $\sin^2 \theta_W(M_{\text{GUT}}) = 3/8$. Additionally, kinetic mixing and generalized Chern-Simons terms can lead to effective shifts in the gauge couplings and k_Y at low energies [43].

Finally, the string scale M_S , which is determined by the dilaton and internal volumes, can be tuned to stay in the range of $M_S \sim 10^{14} - 10^{17}$ GeV to remain compatible with phenomenological constraints on gauge coupling unification and electroweak observables.

Conclusions. We have presented a three-generation $\mathcal{N} = 1$ supersymmetric Pati-Salam model from rigid intersecting D6-branes on $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold with discrete torsion. The model is the first of its kind constructed in the factorizable compactification lattice with moduli stabilization. The construction is particularly simple as it only involves rectangular two-tori.

We showed that the Pati-Salam gauge symmetry can be broken down to the SM gauge symmetry via the supersymmetry preserving Higgs mechanism, the SM fermion masses and mixings can be generated, and the supersymmetry can be broken via gaugino condensations in the hidden sector. We have presented the complete perturbative particle spectrum, tree-level gauge coupling relations. Further detailed phenomenology of the model needs a thorough analysis of the supersymmetry breaking soft

terms and the allowed masses of the SM fermions with moduli stabilization.

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- [1] M. Cvetič, G. Shiu, and A. M. Uranga, Three family supersymmetric standard-like models from intersecting brane worlds, *Phys. Rev. Lett.* **87**, 201801 (2001).
 - [2] M. Cvetič, G. Shiu, and A. M. Uranga, Chiral four-dimensional $N = 1$ supersymmetric type 2A orientifolds from intersecting D6 branes, *Nucl. Phys.* **B615**, 3 (2001).
 - [3] M. Cvetič, I. Papadimitriou, and G. Shiu, Supersymmetric three family SU(5) grand unified models from type IIA orientifolds with intersecting D6-branes, *Nucl. Phys.* **B659**, 193 (2003); **B696**, 298(E) (2004).
 - [4] M. Cvetič, T. Li, and T. Liu, Supersymmetric Pati-Salam models from intersecting D6-branes: A road to the standard model, *Nucl. Phys.* **B698**, 163 (2004).
 - [5] R. Blumenhagen, M. Cvetič, P. Langacker, and G. Shiu, Toward realistic intersecting D-brane models, *Annu. Rev. Nucl. Part. Sci.* **55**, 71 (2005).
 - [6] C.-M. Chen, T. Li, and D. V. Nanopoulos, Standard-like model building on Type II orientifolds, *Nucl. Phys.* **B732**, 224 (2006).
 - [7] C.-M. Chen, T. Li, and D. V. Nanopoulos, Type IIA Pati-Salam flux vacua, *Nucl. Phys.* **B740**, 79 (2006).
 - [8] C.-M. Chen, T. Li, and D. V. Nanopoulos, Flipped and unflipped SU(5) as type IIA flux vacua, *Nucl. Phys.* **B751**, 260 (2006).
 - [9] C.-M. Chen, T. Li, V. E. Mayes, and D. V. Nanopoulos, A realistic world from intersecting D6-branes, *Phys. Lett. B* **665**, 267 (2008).
 - [10] C.-M. Chen, T. Li, V. E. Mayes, and D. V. Nanopoulos, Towards realistic supersymmetric spectra and Yukawa textures from intersecting branes, *Phys. Rev. D* **77**, 125023 (2008).
 - [11] W. He, T. Li, R. Sun, and L. Wu, The final model building for the supersymmetric Pati-Salam models from intersecting D6-branes, *Eur. Phys. J. C* **82**, 710 (2022).
 - [12] W. He, T. Li, and R. Sun, The complete search for the supersymmetric Pati-Salam models from intersecting D6-branes, *J. High Energy Phys.* **08** (2022) 044.
 - [13] T. Li, R. Sun, and L. Wu, String-scale gauge coupling relations in the supersymmetric Pati-Salam models from intersecting D6-branes, *J. High Energy Phys.* **03** (2023) 210.
 - [14] M. Sabir, T. Li, A. Mansha, and X.-C. Wang, The supersymmetry breaking soft terms, and fermion masses and mixings in the supersymmetric Pati-Salam model from intersecting D6-branes, *J. High Energy Phys.* **04** (2022) 089.
 - [15] T. Li, A. Mansha, and R. Sun, Revisiting the supersymmetric Pati-Salam models from intersecting D6-branes, *Eur. Phys. J. C* **81**, 82 (2021).

- [16] T. Li, A. Mansha, R. Sun, L. Wu, and W. He, $N = 1$ supersymmetric $SU(12)_C \times SU(2)_L \times SU(2)_R$ models, $SU(4)_C \times SU(6)_L \times SU(2)_R$ models, and $SU(4)_C \times SU(2)_L \times SU(6)_R$ models from intersecting D6-branes, *Phys. Rev. D* **104**, 046018 (2021).
- [17] A. Mansha, T. Li, M. Sabir, and L. Wu, Three-family supersymmetric Pati–Salam models with symplectic groups from intersecting D6-branes, *Eur. Phys. J. C* **84**, 151 (2024).
- [18] A. Mansha, T. Li, and L. Wu, The hidden sector variations in the $\mathcal{N} = 1$ supersymmetric three-family Pati–Salam models from intersecting D6-branes, *Eur. Phys. J. C* **83**, 1067 (2023).
- [19] M. Sabir, A. Mansha, T. Li, and Z.-W. Wang, Fermion masses and mixings in string theory with Dirac neutrinos, *Phys. Rev. D* **111**, L071702 (2025).
- [20] M. Sabir, A. Mansha, T. Li, and Z.-W. Wang, Fermion masses and mixings in the supersymmetric Pati–Salam landscape from Intersecting D6-Branes, *J. High Energy Phys.* **10** (2024) 252.
- [21] M. Sabir, A. Mansha, T. Li, and Z.-W. Wang, Susy breaking soft terms in the supersymmetric Pati–Salam landscape from intersecting D6-branes, *J. High Energy Phys.* **01** (2025) 201.
- [22] A. Mansha, T. Li, and M. Sabir, Revisiting the supersymmetric trinification models from intersecting D6-branes, *Commun. Theor. Phys.* **76**, 095201 (2024).
- [23] B. Assel, K. Christodoulides, A. E. Faraggi, C. Kounnas, and J. Rizos, Classification of heterotic Pati–Salam models, *Nucl. Phys.* **B844**, 365 (2011).
- [24] A. E. Faraggi, V. G. Matyas, and B. Percival, Classification of nonsupersymmetric Pati–Salam heterotic string models, *Phys. Rev. D* **104**, 046002 (2021).
- [25] R. Blumenhagen, M. Cvetič, F. Marchesano, and G. Shiu, Chiral D-brane models with frozen open string moduli, *J. High Energy Phys.* **03** (2005) 050.
- [26] S. Forste and I. Zavala, Oddness from rigidity, *J. High Energy Phys.* **07** (2008) 086.
- [27] R. Blumenhagen, V. Braun, B. Kors, and D. Lust, Orientifolds of K3 and Calabi–Yau manifolds with intersecting D-branes, *J. High Energy Phys.* **07** (2002) 026.
- [28] L. E. Ibanez and R. Richter, Stringy instantons and Yukawa couplings in MSSM-like orientifold models, *J. High Energy Phys.* **03** (2009) 090.
- [29] R. Blumenhagen, M. Cvetič, and T. Weigand, Spacetime instanton corrections in 4D string vacua: The Seesaw mechanism for D-Brane models, *Nucl. Phys.* **B771**, 113 (2007).
- [30] L. E. Ibanez and A. M. Uranga, Neutrino Majorana masses from string theory instanton effects, *J. High Energy Phys.* **03** (2007) 052.
- [31] S. A. Abel and M. D. Goodsell, Realistic Yukawa couplings through instantons in intersecting brane worlds, *J. High Energy Phys.* **10** (2007) 034.
- [32] R. Blumenhagen, M. Cvetič, D. Lust, R. Richter, and T. Weigand, Non-perturbative Yukawa couplings from string instantons, *Phys. Rev. Lett.* **100**, 061602 (2008).
- [33] S. Forste, C. Timirgaziu, and I. Zavala, Orientifold’s landscape: Non-factorisable six-tori, *J. High Energy Phys.* **10** (2007) 025.
- [34] M. Cvetič and T. Weigand, A string theoretic model of gauge mediated supersymmetry breaking, [arXiv:0807.3953](https://arxiv.org/abs/0807.3953).
- [35] A. Mansha, M. Sabir, T. Li, and L. Wang, Three-family supersymmetric Pati–Salam models from intersecting D6-branes on rigid cycles, [arXiv:2505.03664](https://arxiv.org/abs/2505.03664).
- [36] A. Sen, Stable nonBPS bound states of BPS D-branes, *J. High Energy Phys.* **08** (1998) 010.
- [37] D. Lust, P. Mayr, R. Richter, and S. Stieberger, Scattering of gauge, matter, and moduli fields from intersecting branes, *Nucl. Phys.* **B696**, 205 (2004).
- [38] R. Blumenhagen and M. Schmidt-Sommerfeld, Gauge thresholds and Kaehler metrics for rigid intersecting D-brane models, *J. High Energy Phys.* **12** (2007) 072.
- [39] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan, and A. M. Uranga, $D = 4$ chiral string compactifications from intersecting branes, *J. Math. Phys. (N.Y.)* **42**, 3103 (2001).
- [40] D. Lust and S. Stieberger, Gauge threshold corrections in intersecting brane world models, *Fortschr. Phys.* **55**, 427 (2007).
- [41] D. M. Ghilencea and G. G. Ross, On gauge unification in type I / I-prime models, *Phys. Lett. B* **480**, 355 (2000).
- [42] L. E. Ibanez, F. Marchesano, and R. Rabadan, Getting just the standard model at intersecting branes, *J. High Energy Phys.* **11** (2001) 002.
- [43] P. Anastasopoulos, M. Bianchi, E. Dudas, and E. Kiritsis, Anomalies, anomalous $U(1)$ ’s and generalized Chern–Simons terms, *J. High Energy Phys.* **11** (2006) 057.