

Exploring bottom-charmed molecular tetraquarks with the complex scaling method

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In this work, we adopt the one-boson exchange model to analyze the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ interactions from a coupled-channel perspective. For the $I(J^P) = 1(1^+) DB^*/D^*B/D^*B^*$ system, employing the complex scaling method, we obtain a loosely bound state and two resonant states located below the DB^* , D^*B , and D^*B^* thresholds, respectively. Assuming that the charmoniumlike states $Z_c(3900)$ and $Z_c(4020)$, as well as the bottomoniumlike states $Z_b(10610)$ and $Z_b(10650)$, are molecular states near their respective thresholds, we suggest that the predicted states in the $I(J^P) = 1(1^+)D^{(*)}B^{(*)}$ system could be their bottom-charmed analogs. In addition, we extend our analysis to other $D_{(s)}^{(*)}B_{(s)}^{(*)}$ combinations with various quantum numbers. Our results indicate that, besides the $I(J^P) = 1(1^+)$ channel, more molecular states with quantum numbers $I(J^P) = 0(0^+), 0(1^+), 0(2^+)$, and $1(0^+)$ may also exist in the bottom-charmed sector. Compared with our earlier study of hidden-bottom systems [Phys. Rev. D **110**, 074038 (2024)], where two bound states were obtained in the $I(J^P) = 1(1^{+-})$ channel, the present work instead finds resonant states in the $I(J^P) = 1(1^+)$ sector. Moreover, while no molecular state with strangeness was identified in the previous hidden bottom systems, we now obtain molecular states near the $D_s^{(*)}B_s^*$ threshold. Finally, we provide possible decay channels for future experimental searches, recommending the B_c/B_c^* plus light mesons for bound states, and the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ channels for resonances.

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I. INTRODUCTION

The discovery of the hidden-charm state $X(3872)$ with $J^{PC} = 1^{++}$ in the process $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$ by the Belle Collaboration in 2003 caused a series of interesting consequences for the establishment of the hadronic spectrum [1]. Since $X(3872)$ lies very close to the $D\bar{D}^* + \text{H.c.}$ threshold, it is naturally interpreted as a $D\bar{D}^*$ hadronic molecule. Nevertheless, other explanations such as compact tetraquarks and hybrid configurations have also been proposed [2–6]. Distinguishing among these scenarios remains a central challenge in hadron spectroscopy. In this

work, we focus exclusively on the study of loosely bound molecular states. This discovery has sparked broad interest in exotic hadrons, especially in the charm sector. In 2013, the BESIII [7] and Belle [8] Collaborations observed a charged charmoniumlike structure, $Z_c(3900)$, located slightly above the $D\bar{D}^* + \text{H.c.}$ threshold, which further stimulated discussions on the dynamics of near-threshold states. According to heavy-quark spin symmetry (HQSS), the molecular states of $D\bar{D}$, $D\bar{D}^* + \text{H.c.}$, and $D^*\bar{D}^*$ are expected to form a charmoniumlike multiplet with quantum numbers $J^P = 0^{++}, 1^{++}, 1^{+-}$, and 2^{++} . Experimentally, the $X(3872)$ is established as an isoscalar with $I(J^{PC}) = 0(1^{++})$ [9], lying extremely close to the $D\bar{D}^*$ threshold, which strongly supports its molecular interpretation. While the $Z_c(3900)$ is observed in the $J/\psi \pi^+ \pi^-$ decay channel and has quantum numbers of $I(J^{PC}) = 1(1^{+-})$ [9]. In the $D^*\bar{D}^*$ channel, the BESIII Collaboration reported an isovector resonance, $Z_c(4020)$ [10], while the Belle Collaboration observed a possible isoscalar candidate, $X(4013)$ [11]. Although the internal structures of these states and their possible connections to the well-known

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$X(3872)$ remain unclear, they can naturally be interpreted as the heavy quark spin partners of $Z_c(3900)$ and $X(3872)$ within the molecular framework. Taken together, these near threshold structures $X(3872)$ and $X(4013)$ in the isoscalar sector, and $Z_c(3900)$ and $Z_c(4020)$ in the isovector sector may be regarded as two HQSS related doublets, reflecting common dynamical origins governed by heavy-quark spin symmetry rather than a strict one to one correspondence. Meanwhile, in the bottom sector, two analogous bottomoniumlike resonances, $Z_b(10610)$ and $Z_b(10650)$, were observed by the Belle Collaboration in 2011 [12]. These states, both with $I(J^{PC}) = 1(1^{+-})$, provide complementary evidence for the existence of heavy-quark spin partner structures across both the charm and bottom sectors. A variety of phenomenological investigations on the XYZ and P_c/T_{cc} families can be found in Refs. [2–6,13–24].

The bottom-charmed molecular tetraquarks, also known as B_c -like states, have not yet been observed experimentally. Nevertheless, the fundamental principles of quantum chromodynamics (QCD) do not forbid their existence. Many theoretical studies have explored their properties. In Ref. [25], the author calculated the mass spectra of the $cq\bar{b}\bar{q}$, $cs\bar{b}\bar{q}$, and $cs\bar{b}\bar{s}$ configurations using the chromomagnetic interaction model. Many references have extensively investigated the properties of B_c -like states within the QCD sum rules method and QCD light-cone sum rules [26–34]. For instance, Ref. [34] examined the masses of $Q\bar{q}\bar{Q}^{(\prime)}q$ molecules and found some bound states in the bottom-charmed sector within the QCD sum rules framework. The chiral unitary approach and hidden gauge symmetry Lagrangians have also been used to study B_c -like molecular states [35,36]. Additionally, in Ref. [37], the interaction between $B_{(s)}^{(*)}$ and $D_{(s)}^{(*)}$ mesons was studied using the one-boson exchange (OBE) model, which suggested the possible existence of molecular states. In Ref. [38], the authors also investigated radiative decays and magnetic moments of bottom-charmed molecular tetraquarks. Moreover, several studies have also focused on bottom-charmed tetraquark systems [39–43].

However, we are still far from establishing the complete spectroscopy for B_c -like molecular tetraquark states, and our understanding of them remains limited. Bottom-charmed molecular tetraquark states may provide insights into the dynamics of exotic hadrons in other sectors. As we know that the heavy quark spin symmetry (HQSS) implies the existences of spin multiplets in different sectors. Also, the heavy quark flavor symmetry (HQFS) indicates there may be similar patterns among hidden-charm, hidden-bottom, and bottom-charm systems. That is, the $b \leftrightarrow c$ correspondence can help us to build bridges between several energy sectors. Therefore, studying B_c -like molecular tetraquarks can help disentangle heavy-quark and light-quark dynamics and offer complementary information to the hidden-charm and hidden-bottom systems.

Charmed Sector	Bottom Charmed Sector	Bottom Sector
$D^* \bar{D}^*$ $Z_c(4020), X(4014)$	$D^* B^*$ $Z(?), X(?)$	$B^* \bar{B}^*$ $Z_b(10650), X(?)$
$D \bar{D}^*$ $Z_c(3900), X(3872)$	$DB^* \text{ or } D^* B$ $Z(?), X(?)$	$B \bar{B}^*$ $Z_b(10610), X(?)$

FIG. 1. Comparison of states near the corresponding thresholds for the charmed, bottom-charmed, and bottom sectors.

Taken together, as illustrated in Fig. 1, heavy quark symmetry (HQS) allows for the existence of molecular states in the bottom-charmed sector, which are expected to be counterparts of those already observed in the charm and bottom sectors. Furthermore, coupled-channel effects and $S - D$ wave mixing effects are expected to play an important role in the production of these molecular states. Therefore, it is important to systematically study the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems using a coupled-channel approach, exploring possible excited resonances as well as bound states.

Recently, we have systematically explored the properties of the hidden-bottom molecular tetraquarks and the bottom-strange molecular pentaquarks by adopting the one-boson exchange model [44,45]. We employ the Gaussian expansion method (GEM) [46,47] and the complex scaling method (CSM) [48,49] to solve the coupled-channel Schrödinger equation and extract the pole positions. In the present work, we apply the same procedure to systematically investigate bottom-charmed molecular tetraquark states. First, we investigate the $D^{(*)}B^{(*)}$ systems, starting with the $I(J^P) = 1(1^+)$ sector to constrain the model's free cutoff parameter Λ . We then extend our study to a variety of $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems with different quantum numbers. According to heavy-quark symmetry, our numerical results suggest the possible existence of several molecular states near the corresponding $D^{(*)}B^{(*)}$ thresholds, which may be related to the structures reported by experiments in both the charm and bottom sectors. We hope that these predictions can provide useful guidance for future experimental searches.

This paper is organized as follows. The formalism of effective interactions and complex scaling method are

briefly introduced in Sec. II. We present the numerical results and discussions for the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems in Sec. III. A summary is given in the last section.

II. FORMALISM

A. The effective interactions

In the present work, we employ the OBE model to obtain the effective potential of the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems. The OBE model describes the meson-meson interactions through the exchange of light scalar, pseudoscalar, and vector mesons, and has been successfully applied to explain the dynamical mechanisms of molecular states. Under both heavy quark symmetry and chiral symmetry, the relevant effective Lagrangians can be expressed as [50–53]

$$\begin{aligned} \mathcal{L} = & g_\sigma \langle H_a^{(Q)} \sigma \bar{H}_a^{(Q)} \rangle + g_\sigma \langle \bar{H}_a^{(Q)} \sigma H_a^{(Q)} \rangle \\ & + ig \langle H_b^{(Q)} \mathcal{A}_{ba} \gamma_5 \bar{H}_a^{(Q)} \rangle + ig \langle \bar{H}_a^{(Q)} \mathcal{A}_{ab} \gamma_5 H_b^{(Q)} \rangle \\ & + \langle iH_b^{(Q)} (\beta v^\mu (\mathcal{V}_\mu - \rho_\mu) + \lambda \sigma^{\mu\nu} F_{\mu\nu}(\rho))_{ba} \bar{H}_a^{(Q)} \rangle \\ & - \langle i\bar{H}_a^{(Q)} (\beta v^\mu (\mathcal{V}_\mu - \rho_\mu) - \lambda \sigma^{\mu\nu} F_{\mu\nu}(\rho))_{ab} H_b^{(Q)} \rangle, \end{aligned} \quad (1)$$

where a, b are flavor indices, $v^\mu = (1, \mathbf{0})$ is the four-velocity, and σ and ρ are the light scalar and vector fields, respectively. The $H_a^{(Q)}$, $H_a^{(\bar{Q})}$, $\bar{H}_a^{(Q)}$, and $\bar{H}_a^{(\bar{Q})}$ represent the fields of heavy-light mesons and can be written as

$$H_a^{(Q)} = (1 + \not{v})(\mathcal{P}_a^{*(Q)\mu} \gamma_\mu - \mathcal{P}_a^{(Q)} \gamma_5)/2, \quad (2)$$

$$H_a^{(\bar{Q})} = (\bar{\mathcal{P}}_a^{*(\bar{Q})\mu} \gamma_\mu - \bar{\mathcal{P}}_a^{(\bar{Q})} \gamma_5)(1 - \not{v})/2, \quad (3)$$

$$\bar{H} = \gamma_0 H^\dagger \gamma_0, \quad (4)$$

where heavy mesons with $J^P = 0^-$ and $J^P = 1^-$ are denoted by $\mathcal{P}$ and $\mathcal{P}^*$, respectively. The axial current is

$$A_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \quad (5)$$

the vector current is

$$\mathcal{V}_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \quad (6)$$

and the strength tensor of the vector field is

$$F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu], \quad (7)$$

where $\xi = \exp(i\mathbb{P}/f_\pi)$ and the vector meson fields $\rho_\mu = ig_V \mathbb{V}_\mu / \sqrt{2}$. The $f_\pi = 132$ MeV is the pion decay constant and then $g_V = m_\rho / f_\pi = 5.8$ [54]. The $\mathbb{P}$ and $\mathbb{V}$ stand for the matrices of light pseudoscalar and vector mesons, respectively,

$$\begin{aligned} \mathbb{P} = & \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \\ \mathbb{V} = & \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \end{aligned} \quad (8)$$

More explicitly, the effective Lagrangian depicting the couplings of light mesons and heavy-light mesons can be written as [55–57]

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\mathbb{V}} = -\sqrt{2}\beta g_V \mathcal{P}_b \mathcal{P}_a^\dagger v \cdot \mathbb{V}_{ba} + \sqrt{2}\beta g_V \tilde{\mathcal{P}}_a^\dagger \tilde{\mathcal{P}}_b v \cdot \mathbb{V}_{ab}, \quad (9)$$

$$\begin{aligned} \mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{V}} = & -2\sqrt{2}\lambda g_V v^\lambda \varepsilon_{\lambda\mu\alpha\beta} (\mathcal{P}_b \mathcal{P}_a^{*\mu\dagger} + \mathcal{P}_b^{*\mu} \mathcal{P}_a^\dagger) (\partial^\alpha \mathbb{V}^\beta)_{ba} \\ & - 2\sqrt{2}\lambda g_V v^\lambda \varepsilon_{\lambda\mu\alpha\beta} (\tilde{\mathcal{P}}_a^{*\mu\dagger} \tilde{\mathcal{P}}_b + \tilde{\mathcal{P}}_a^\dagger \tilde{\mathcal{P}}_b^{*\mu}) (\partial^\alpha \mathbb{V}^\beta)_{ab}, \end{aligned} \quad (10)$$

$$\begin{aligned} \mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{V}} = & \sqrt{2}\beta g_V \mathcal{P}_b^* \cdot \mathcal{P}_a^{*\dagger} v \cdot \mathbb{V}_{ba} \\ & - i2\sqrt{2}\lambda g_V \mathcal{P}_b^{*\mu} \mathcal{P}_a^{*\nu\dagger} (\partial_\mu \mathbb{V}_\nu - \partial_\nu \mathbb{V}_\mu)_{ba} \\ & - \sqrt{2}\beta g_V \tilde{\mathcal{P}}_a^{*\dagger} \tilde{\mathcal{P}}_b^* v \cdot \mathbb{V}_{ab} \\ & - i2\sqrt{2}\lambda g_V \tilde{\mathcal{P}}_a^{*\mu\dagger} \tilde{\mathcal{P}}_b^{*\nu} (\partial_\mu \mathbb{V}_\nu - \partial_\nu \mathbb{V}_\mu)_{ab}, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{P}} = & -i\frac{2g}{f_\pi} v^\beta \varepsilon_{\beta\mu\alpha\nu} \mathcal{P}_b^{*\mu} \mathcal{P}_a^{*\nu\dagger} \partial^\alpha \mathbb{P}_{ba} \\ & + i\frac{2g}{f_\pi} v^\beta \varepsilon_{\beta\mu\alpha\nu} \tilde{\mathcal{P}}_a^{*\mu\dagger} \tilde{\mathcal{P}}_b^{*\nu} \partial^\alpha \mathbb{P}_{ab}, \end{aligned} \quad (12)$$

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\sigma} = -2g_s \mathcal{P}_b \mathcal{P}_b^\dagger \sigma - 2g_s \tilde{\mathcal{P}}_b \tilde{\mathcal{P}}_b^\dagger \sigma, \quad (13)$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\sigma} = 2g_s \mathcal{P}_b^* \cdot \mathcal{P}_b^{*\dagger} \sigma + 2g_s \tilde{\mathcal{P}}_b^* \cdot \tilde{\mathcal{P}}_b^{*\dagger} \sigma. \quad (14)$$

In the equations above, for example, $\mathcal{L}_{\mathcal{P}\mathcal{P}\mathbb{V}}$ denotes the interaction Lagrangian describing the coupling between two heavy pseudoscalar mesons and one light vector meson. The relevant parameters and further details can be found in Refs. [56,58].

Using the given Lagrangians, we can deduce the relevant interaction potentials of these investigated systems in the momentum space straightforwardly. Based on the Breit approximation, the effective potential reads,

$$\mathcal{V}^{M_1 M_2 \rightarrow M_3 M_4}(\mathbf{q}) = -\frac{\mathcal{M}(M_1 M_2 \rightarrow M_3 M_4)}{4\sqrt{m_1 m_2 m_3 m_4}}, \quad (15)$$

where $\mathcal{M}(M_1 M_2 \rightarrow M_3 M_4)$ denotes the scattering amplitude for the $M_1 M_2 \rightarrow M_3 M_4$ process, and m_i is the mass of

the meson M_i . One can obtain the final effective potential in position space after performing the Fourier transformation

$$\mathcal{V}(\mathbf{r}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \mathcal{V}(\mathbf{q}) \mathcal{F}^2(q^2, m_i^2), \quad (16)$$

$$\mathcal{F}(q^2, m_i^2) = (\Lambda^2 - m_i^2)/(\Lambda^2 - q^2), \quad (17)$$

Since hadrons are not pointlike particles, a form factor with a cutoff parameter Λ is introduced to account for their internal structure. Then, we can obtain the potentials

$$V_v^a = -\frac{1}{2}\beta^2 g_V^2 Y(\Lambda, m_v, r), \quad (18a)$$

$$V_\sigma^a = -g_s^2 Y(\Lambda, m_\sigma, r), \quad (18b)$$

for the $D_{(s)}B_{(s)} \rightarrow D_{(s)}B_{(s)}$ process,

$$V_p^b = -\frac{1}{3}\frac{g^2}{f_\pi^2} \left(\mathcal{E}_{ij} \nabla^2 + \mathcal{F}_{ij} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) Y(\Lambda, m_p, r), \quad (19a)$$

$$V_v^b = \frac{2}{3}\lambda^2 g_V^2 \left(2\mathcal{E}_{ij} \nabla^2 - \mathcal{F}_{ij} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) Y(\Lambda, m_v, r), \quad (19b)$$

for the $D_{(s)}B_{(s)} \rightarrow D_{(s)}^*B_{(s)}^*$ process,

$$V_\sigma^c = -g_s^2 \mathcal{D}_{ij} Y(\Lambda, m_\sigma, r), \quad (20a)$$

$$V_v^c = -\frac{1}{2}\beta^2 g_V^2 \mathcal{D}_{ij} Y(\Lambda, m_v, r), \quad (20b)$$

for the $D_{(s)}B_{(s)}^* \rightarrow D_{(s)}B_{(s)}^*$ process,

$$V_p^d = -\frac{1}{3}\frac{g^2}{f_\pi^2} \left(\mathcal{E}_{ij} \nabla^2 + \mathcal{F}_{ij} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) Y(\Lambda, m_p, r), \quad (21a)$$

$$V_v^d = \frac{2}{3}\lambda^2 g_V^2 \left(2\mathcal{E}_{ij} \nabla^2 - \mathcal{F}_{ij} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) Y(\Lambda, m_v, r), \quad (21b)$$

for the $D_{(s)}B_{(s)}^* \rightarrow D_{(s)}^*B_{(s)}^*$ process,

$$V_p^e = -\frac{1}{3}\frac{g^2}{f_\pi^2} \left(\mathcal{E}_{ij} \nabla^2 + \mathcal{F}_{ij} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) Y(\Lambda_i, m_p, r), \quad (22a)$$

$$V_v^e = \frac{2}{3}\lambda^2 g_V^2 \left(-2\mathcal{E}_{ij} \nabla^2 - (\mathcal{F}'_{ij} - \mathcal{F}''_{ij}) r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) \times Y(\Lambda, m, r), \quad (22b)$$

for the $D_{(s)}B_{(s)}^* \rightarrow D_{(s)}^*B_{(s)}^*$ process,

$$V_p^f = -\frac{1}{3}\frac{g^2}{f_\pi^2} \left(\mathcal{E}_{ji} \nabla^2 + \mathcal{F}_{ji} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) Y(\Lambda_i, m_p, r), \quad (23a)$$

$$V_v^f = \frac{2}{3}\lambda^2 g_V^2 \left(-2\mathcal{E}_{ji} \nabla^2 - (\mathcal{F}'_{ji} - \mathcal{F}''_{ji}) r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) \times Y(\Lambda, m, r), \quad (23b)$$

for the $D_{(s)}^*B_{(s)} \rightarrow D_{(s)}^*B_{(s)}^*$ process,

$$V_p^g = -\frac{1}{3}\frac{g^2}{f_\pi^2} \left(\mathcal{E}_{ij} \nabla^2 + \mathcal{F}_{ij} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) Y(\Lambda, m_p, r), \quad (24a)$$

$$V_\sigma^g = -g_s^2 \mathcal{D}_{ij} Y(\Lambda, m_v, r), \quad (24b)$$

$$V_v^g = -\frac{1}{2}\beta^2 g_V^2 \mathcal{D}_{ij} Y(\Lambda, m_v, r) + \frac{2}{3}\lambda^2 g_V^2 \left(2\mathcal{E}_{ij} \nabla^2 - \mathcal{F}_{ij} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right) Y(\Lambda, m_v, r), \quad (24c)$$

for the $D_{(s)}^*B_{(s)}^* \rightarrow D_{(s)}^*B_{(s)}^*$ process. The explicit formula for the $Y(\Lambda, m, r)$ be expressed as

$$Y(\Lambda, m, r) = \frac{1}{4\pi r} (e^{-mr} - e^{-\Lambda r}) - \frac{\Lambda^2 - m^2}{8\pi\Lambda} e^{-\Lambda r}. \quad (25)$$

Here, $\mathcal{D}_{ij}$, $\mathcal{E}_{ij}$, and $\mathcal{F}_{ij}(\mathcal{F}'_{ij}, \mathcal{F}''_{ij})$ represent the operators for spin-spin coupling and tensor forces, respectively, and they are related to the polarization vectors ϵ_i and the Pauli matrix σ . These operators are taken from Ref. [56]. Finally, the total effective potentials for the $D_{(s)}^*B_{(s)}^*$ systems can be obtained by combining the above sub-potentials, following Refs. [55–57], and are summarized in Table I. Also, the relevant parameters are collected in Table II [54,56,59–61].

In above equations, the $\mathcal{I}$ is the isospin factor; $3/2$ for the isoscalar system, and $-1/2$ for the isovector system.

B. Numerical solution methods

In this work, we use GEM to solve the coupled-channel Schrödinger equation, which is widely applied to solve the problem of a few-body system [46,47]. In the GEM, the orbital wave functions are expanded in terms of a set of Gaussian basis functions,

$$\phi_{nl}(\mathbf{r}) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm_l}(\hat{\mathbf{r}}), \quad N_{nl} = \left(\frac{2^{l+2} (2\nu_n)^{l+3/2}}{\sqrt{\pi} (2l+1)!!} \right)^{1/2}, \quad (26)$$

where N_{nl} is the normalization constant, ν_n denotes the Gaussian range parameter, and $Y_{lm_l}(\hat{\mathbf{r}})$ represents the spherical harmonics. Following Refs. [46,47], the Gaussian size parameters are most effectively chosen in geometric progression, which provides both high flexibility and numerical stability in the calculations,

$$\nu_n = 1/r_n^2, \quad r_n = r_1 a^{n-1} (n = 1, \dots, n_{\max}), \quad (27)$$

TABLE I. Effective potentials for all $D_{(s)}^{(*)}B_{(s)}^{(*)}$ coupled-channel processes.

$V_{DB \rightarrow DB} = V_\sigma^a + \mathcal{I}V_\rho^a + \frac{1}{2}V_\omega^a$	$V_{DB \rightarrow D_s B_s} = \sqrt{2}V_{K^*}^a$	$V_{DB \rightarrow D^* B^*} = \frac{1}{6}V_\eta^b + \mathcal{I}V_\pi^b + \mathcal{I}V_\rho^b + \frac{1}{2}V_\omega^b$
$V_{DB \rightarrow D_s^* B_s^*} = \sqrt{2}V_{K^*}^b + \sqrt{2}V_K^b$	$V_{D_s B_s \rightarrow D_s B_s} = V_\sigma^a + V_\phi^a$	$V_{D_s B_s \rightarrow D^* B^*} = \sqrt{2}V_{K^*}^b + \sqrt{2}V_{K^*}^b$
$V_{D_s B_s \rightarrow D_s^* B_s^*} = \frac{2}{3}V_\eta^b + V_\phi^b$	$V_{D^* B^* \rightarrow D^* B^*} = V_\sigma^g + \mathcal{I}V_\pi^g + \frac{1}{6}V_\eta^g + \mathcal{I}V_\rho^g + \frac{1}{2}V_\omega^g$	$V_{D^* B^* \rightarrow D_s^* B_s^*} = \sqrt{2}V_{K^*}^g + \sqrt{2}V_K^g$
$V_{D_s^* B_s^* \rightarrow D_s^* B_s^*} = V_\sigma^g + \frac{2}{3}V_\eta^g + V_\phi^g$	$V_{DB^* \rightarrow DB^*} = V_\sigma^c + \mathcal{I}V_\rho^c + \frac{1}{2}V_\omega^c$	$V_{DB^* \rightarrow D^* B} = \mathcal{I}V_\pi^d + \frac{1}{6}V_\eta^d + \mathcal{I}V_\rho^d + \frac{1}{2}V_\omega^d$
$V_{DB^* \rightarrow D^* B^*} = \mathcal{I}V_\pi^e + \frac{1}{6}V_\eta^e + \mathcal{I}V_\rho^e + \frac{1}{2}V_\omega^e$	$V_{DB^* \rightarrow D_s B_s^*} = \sqrt{2}V_{K^*}^c$	$V_{DB^* \rightarrow D_s B_s^*} = \sqrt{2}V_{K^*}^c$
$V_{DB^* \rightarrow D_s^* B_s^*} = \sqrt{2}V_{K^*}^e + \sqrt{2}V_{K^*}^e$	$V_{D^* B \rightarrow D^* B} = V_\sigma^c + \mathcal{I}V_\rho^c + \frac{1}{2}V_\omega^c$	$V_{D^* B \rightarrow D^* B^*} = \mathcal{I}V_\pi^f + \frac{1}{6}V_\eta^f + \mathcal{I}V_\rho^f + \frac{1}{2}V_\omega^f$
$V_{D^* B \rightarrow D_s B_s^*} = \sqrt{2}V_{K^*}^d + \sqrt{2}V_K^d$	$V_{D^* B \rightarrow D_s^* B_s^*} = \sqrt{2}V_{K^*}^c$	$V_{D^* B \rightarrow D_s^* B_s^*} = \sqrt{2}V_{K^*}^f + \sqrt{2}V_K^f$
$V_{D^* B^* \rightarrow D^* B^*} = V_\sigma^g + \mathcal{I}V_\pi^g + \frac{1}{6}V_\eta^g + \mathcal{I}V_\rho^g + \frac{1}{2}V_\omega^g$	$V_{D^* B^* \rightarrow D_s B_s^*} = \sqrt{2}V_{K^*}^e + \sqrt{2}V_{K^*}^e$	$V_{D^* B^* \rightarrow D_s^* B_s^*} = \sqrt{2}V_{K^*}^f + \sqrt{2}V_K^f$
$V_{D^* B^* \rightarrow D_s^* B_s^*} = \sqrt{2}V_{K^*}^g + \sqrt{2}V_K^g$	$V_{D_s B_s^* \rightarrow D_s B_s^*} = V_\sigma^c + V_\phi^c$	$V_{D_s B_s^* \rightarrow D_s^* B_s^*} = \frac{2}{3}V_\eta^d + V_\phi^d$
$V_{D_s B_s^* \rightarrow D_s^* B_s^*} = \frac{2}{3}V_\eta^e + V_\phi^e$	$V_{D_s^* B_s \rightarrow D_s^* B_s} = V_\sigma^c + V_\phi^c$	$V_{D_s^* B_s \rightarrow D_s^* B_s^*} = \frac{2}{3}V_\eta^f + V_\phi^f$
$V_{D_s^* B_s^* \rightarrow D_s^* B_s^*} = V_\sigma^g + V_\phi^g + \frac{2}{3}V_\eta^g$		

TABLE II. The relevant parameters adopted in this work.

Parameters	Value
g_V	5.8
g	0.59
g_s	2.82
β	0.9
f_π	0.132 (GeV)
λ	0.56 (GeV ⁻¹)

where the Gaussian parameters $\{r_1, r_{\max}, n_{\max}\} = \{0.5 \text{ fm}, 25 \text{ fm}, 25\}$ are applied in the present work. In order to obtain resonances, the complex scaling method (CSM) introduced [62–64], which offers a practical framework for extracting many-body resonances by solving the rotated Schrödinger equation $H_\theta \Psi_\theta = E \Psi_\theta$. Under a real scaling angle θ , bound and resonant states appear as S -matrix poles whose energies and widths follow from the complex

eigenvalues $E = E_r - i\Gamma/2$. Since the rotated resonant states become square-integrable, they can be handled on the same footing as bound states. The CSM may be viewed as a natural extension of Berggren's representation [65–68], providing a unified treatment of bound and resonant configurations. In the CSM, the Hamiltonian $\hat{H}$ and wave function $|\Phi\rangle$ are transformed with complex rotation operator $\hat{U}(\theta)$,

$$\hat{H}_\theta = \hat{U}(\theta) \hat{H} \hat{U}^{-1}(\theta), |\Phi_\theta\rangle = \hat{U}(\theta) |\Phi\rangle. \quad (28)$$

The coordinate $\mathbf{r}$ and conjugate momentum $\mathbf{p}$ are transformed to be

$$\mathbf{r} \rightarrow \mathbf{r} e^{i\theta}, \mathbf{p} \rightarrow \mathbf{p} e^{-i\theta}, \quad (29)$$

where the θ is variable. Correspondingly, the complex scaling Schrödinger equation can be written as

$$\begin{pmatrix} e^{-2i\theta} T_{11} + V_{11}(r e^{i\theta}) + W_1 & V_{12}(r e^{i\theta}) & \cdots & V_{1j}(r e^{i\theta}) \\ V_{21}(r e^{i\theta}) & e^{-2i\theta} T_{22} + V_{22}(r e^{i\theta}) + W_2 & \cdots & V_{2j}(r e^{i\theta}) \\ \vdots & \vdots & \vdots & \vdots \\ V_{j1}(r e^{i\theta}) & V_{j2}(r e^{i\theta}) & \cdots & e^{-2i\theta} T_{jj} + V_{jj}(r e^{i\theta}) + W_j \end{pmatrix} \cdot \begin{pmatrix} \psi_1(r) \\ \psi_2(r) \\ \vdots \\ \psi_j(r) \end{pmatrix} = E \begin{pmatrix} \psi_1(r) \\ \psi_2(r) \\ \vdots \\ \psi_j(r) \end{pmatrix}, \quad (30)$$

where j denotes the number of coupled channels. The operator T_{ij} is defined as

$$T_{ij} = \frac{1}{2\mu_i} \left(-\frac{d^2}{dr^2} + \frac{l_i(l_i + 1)}{r^2} \right), \quad (31)$$

and W_j represents the threshold of the j th two meson channel, i.e., $W_1 = m_D + m_B$ for $DB \rightarrow DB$ scattering process. The wave function $\psi_j(r)$ represents the coupled-channel state in coordinate space and is related to the basis expansion coefficients $\phi_{nl}(\mathbf{r})$ by $\psi_j(r) = \sum_n^{n_{\max}} c_{jnl} \phi_{nl}(\mathbf{r})$, where j denotes the channel index.

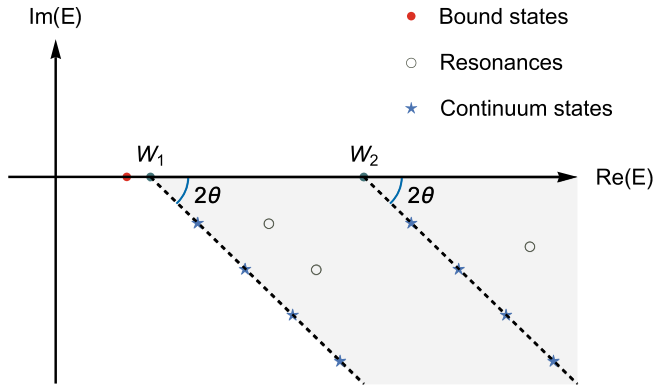


FIG. 2. Schematic eigenvalue distribution of H^θ in the coupled-channel two-body system. Continuum states are discretized along the 2θ -line as solid circles.

As shown in Fig. 2, on the complex-energy plane, the eigenvalues of the scattering continuum states align along the so-called 2θ line. According to the ABC theorem [62,63], when the scaling angle θ exceeds the critical value $\theta_r = \tan^{-1}(\Gamma/2E)$ corresponding to a resonance located at $E_r - i\Gamma/2$, where E_r and Γ denote the resonance energy and decay width, respectively, the resonance wave function becomes normalizable. Importantly, the resonance eigenvalue itself remains fixed on the complex-energy plane and is independent of the scaling angle. For example, whether $\theta = 15^\circ$, 20° , or 25° , the pole position of a given resonance does not change. This property allows one to clearly distinguish bound states and resonances from the continuum states. For a resonance, the RMS radius r_{RMS} and the composed proportion P are given by the c-product [69,70],

$$r_{\text{RMS}}^2 = (\psi^\theta | r^2 | \psi^\theta) = \sum_j \int r^2 \psi_j^\theta(\mathbf{r})^2 d^3\mathbf{r},$$

$$P = (\psi_j^\theta | \psi_j^\theta) = \int \psi_j^\theta(\mathbf{r})^2 d^3\mathbf{r}, \quad (32)$$

where the ψ_j is the wave function of the j th channel and obeys the normalization condition,

$$\sum_i (\psi_i^\theta | \psi_i^\theta) = 1. \quad (33)$$

The r_{RMS} radius represents the average distance between the constituent mesons, and P denotes the probability of a specific channel in the coupled-channel wave function. The r_{RMS} of resonant states can become complex. In such cases, one may apply the framework introduced by Berggren, which extends the concept of expectation values from bound states to resonances [68]. According to this framework, the real part of a complex r_{RMS} corresponds to the usual expectation value, while its imaginary part reflects the uncertainty associated with the measurement.

This generalized interpretation has been supported by numerical calculations of r_{RMS}^2 [71–73]. Moreover, numerical solutions with $r_{\text{RMS}} < 1$ fm are discarded, as they lie outside the region where the OBE model is applicable. In this framework, hadrons are treated as pointlike particles characterized by a cutoff scale, so the model reliably describes only loosely bound or resonant molecular states rather than compact exotic configurations. Imposing this criterion eliminates unphysical solutions and helps ensure the robustness of our predictions.

Additionally, the proportion P is defined through the square of the complex-scaled wave function $\psi_j^\theta(\mathbf{r})$ rather than its modulus squared. The real part of P may be positive or negative, and in some cases even exceed unity. This occurs because weak channels with small negative contributions can reduce the total norm, allowing the dominant channel to have $\text{Re}(P) > 1$. Importantly, the channel wave functions do not have a strict probabilistic interpretation; therefore, $P > 0$ or $P < 0$ should not be interpreted as physical probabilities, but rather as a mathematical measure of the channel's relative contribution to the resonance wave function. More details can be found in the book “Gamow Shell Model: The Unified Theory of Nuclear Structure and Reactions” [74]. For convenience, we denote the probabilities of finding the system in different $D_{(s)}^{(*)}B_{(s)}^{(*)}$ channels as P_j , where the subscript j labels specific channels. As an illustration, in the $I(J^P) = 1(1^+)$ case, $P_{D^*B(^3S_1)}$, $P_{D^*B(^3D_1)}$, and $P_{DB^*(^3S_1)}$ represent the contributions from the $D^*B(^3S_1)$, $D^*B(^3D_1)$, and $DB^*(^3S_1)$ components, respectively. This notation will be used consistently throughout Sec. III.

III. RESULTS AND DISCUSSIONS

A. Isospin vector systems

By following the above procedures, one can explore the possible bound states and resonances of the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems with various combinations by solving the coupled-channel Schrödinger equation using the complex scaling method. In this work, the only free parameter is the UV cutoff Λ in Eq. (17), which reflects the internal structure of the interacting hadrons. This parameter may vary for different coupled systems and lies within the range of 800–5000 MeV. A reasonable cutoff for deuteron studies is estimated to be around 1000 MeV, but it may vary across different scenarios [75,76]. It is worth noting that in Ref. [56], the authors studied the $D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}$ interactions, revealing charmoniumlike resonances with quantum numbers $I(J^{PC}) = 0(0^{++})$ and $0(1^{+-})$ for cutoff values between 1000–2000 MeV, with poles observed around 1500 MeV. In our previous work [44], we predicted several molecular hadron states for the $B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}$ systems by varying the cutoff in the range 800–1200 MeV. Since the interaction

TABLE III. The molecular states for $I(J^P) = 1(1^+)$ system. The numbers in the bracket are the components proportion of $DB^* + \text{H.c.}$, $D^*B + \text{H.c.}$, and D^*B^* channels.

Λ (MeV)	E (MeV)	r_{RMS} (fm)	$(P_{DB^*({}^3S_1)}(\%))$	$(P_{D^*B({}^3D_1)}(\%))$	$(P_{D^*B({}^3S_1)}(\%))$	$(P_{D^*B({}^3D_1)}(\%))$	$(P_{D^*B^*({}^3S_1)}(\%))$	$(P_{D^*B^*({}^3D_1)}(\%))$	$(P_{D^*B^*({}^3D_1)}(\%))$
1400	7287.71−0.36i	3.92−1.23i	(0.00−0.05i)	−0.19+0.25i	98.38−0.49i	0.00+0.01i	1.68+0.24i	0.03+0.00i	0.10+0.04i
1450	7287.07−0.53i 7333.22−7.46i	2.74−1.11i 1.22−0.21i	(0.00−0.07i) (−0.80−1.35i)	−0.23+0.47i −0.33−0.13i	96.71−0.70i −1.86−2.66i	0.01+0.00i −0.13+0.49i	3.30+0.24i 102.16+3.96i	0.04+0.01i 0.93−0.26i	0.17+0.05i 0.03−0.05i
1500	7286.03−0.67i 7332.50−9.68i 7191.81	2.11−0.75i 1.23−0.22i 3.86	(0.00−0.07i) (−1.11−1.64i) (99.54)	−0.19+0.71i −0.39−0.12i 0.00	94.53−0.74i −4.53−2.32i 0.10	0.02+0.00i −0.08+0.57i 0.07	5.36+0.03i 105.02+3.99i 0.22	0.04+0.01i 1.06−0.42i 0.01	0.24+0.04i 0.03−0.06i 0.06

potential between the mesons is similar, the heavier mesons effectively reduce the contribution from the kinetic energy. Consequently, the cutoff value for the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems is expected to lie between that of the $B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}$ systems and the $D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}$ systems. According to heavy quark symmetry, molecular hadron states should exist that correspond to the partners of the charmoniumlike and bottomoniumlike states in the bottom-charmed sector. Therefore, we first investigate the $I(J^P) = 1(1^+)$ $DB^*/D^*B/D^*B^*$ channel, which helps determine the cutoff value within a phenomenologically reasonable range. The numerical results as functions of the cutoff parameter are listed in Table III. The complex-energy eigenvalues as functions of the scaling angle are shown in Fig. 3, while the stability of the eigenvalues with respect to the scaling angle is illustrated in Fig. 4. The variation of the energy with the cutoff is presented in Fig. 5.

For the $I(J^P) = 1(1^+)$ $DB^*/D^*B/D^*B^*$ channel, no poles are found when the cutoff Λ varies from 800 to

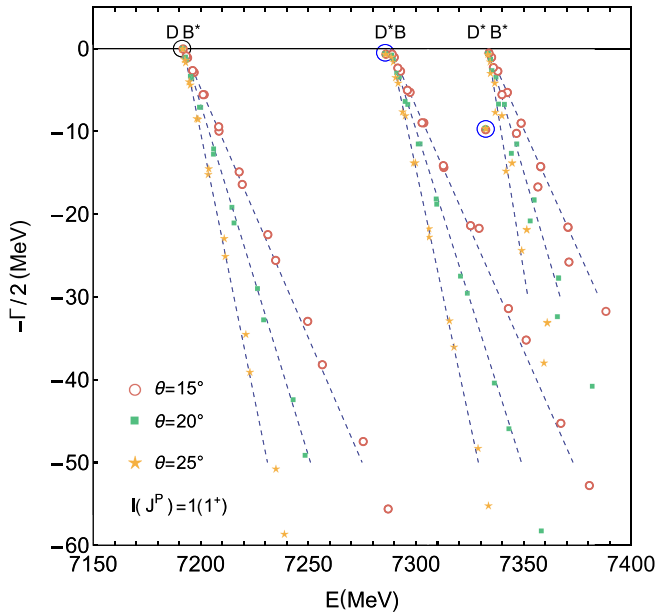


FIG. 3. The complex energy eigenvalues of $I(J^P) = 1(1^+)$ system with varying the angle θ from 15° to 25° .

1300 MeV. However, at $\Lambda = 1400$ MeV, a narrow resonance appears near the D^*B threshold, predominantly composed of the $D^*B({}^3S_1)$ component. When the cutoff increases to 1450 MeV, another resonance emerges below the D^*B^* threshold with a complex energy of $E - i\Gamma/2 = 7333 - 7i$ MeV. This resonance is mainly dominated by the $D^*B^*({}^3S_1)$ channel, and may be considered a promising partner to both the $Z_c(4020)$ and $Z_c(10650)$ states. As the cutoff further increases to around 1500 MeV, a loosely bound state with a predicted mass of 7192 MeV is found, which could correspond to the $Z_c(3900)$ and $Z_c(10610)$ states. It is noteworthy that, for similar cutoff values, previous studies considering only single-channel interactions identified solely bound states [37], highlighting the crucial influence of coupled-channel effects on molecular state formation.

In Fig. 3, we present the complex energy eigenvalues for the $I(J^P) = 1(1^+)$ system at $\Lambda = 1500$ MeV, with the scaling angle θ varied from 15° to 25° . As shown, the bound states (enclosed by black circles) and resonances (enclosed by blue circles) remain invariant with respect to the scaling angle θ , whereas the continuum states rotate along the so-called 2θ line. In Fig. 4, we illustrate the dependence of the complex energies of the two predicted resonances on θ , showing that numerical errors induced by the choice of θ are minimal and quickly converge as θ increases. The detailed numerical results are summarized in Table III and Fig. 5(e).

For the three predicted molecular states, the root-mean-square radius (r_{RMS}) is consistent with that of typical hadronic molecules such as the deuteron. Therefore, a cutoff range of 1200 ~ 1500 MeV is adopted for investigations of other $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems. The corresponding numerical results, depending on the cutoff parameter Λ , are presented in Fig. 5.

For the $I(J^P) = 1(0^+)$ DB/D^*B^* systems, a loosely bound state is predicted when the cutoff Λ is within the range of 1400 ~ 1500 MeV. In this region, the root-mean-square radius r_{RMS} decreases from 4 to 2 fm, suggesting that it can be considered a promising molecular state candidate. However, for the D^*B^* system with spin-parity $I(J^P) = 1(2^+)$, no structure is observed in the complex

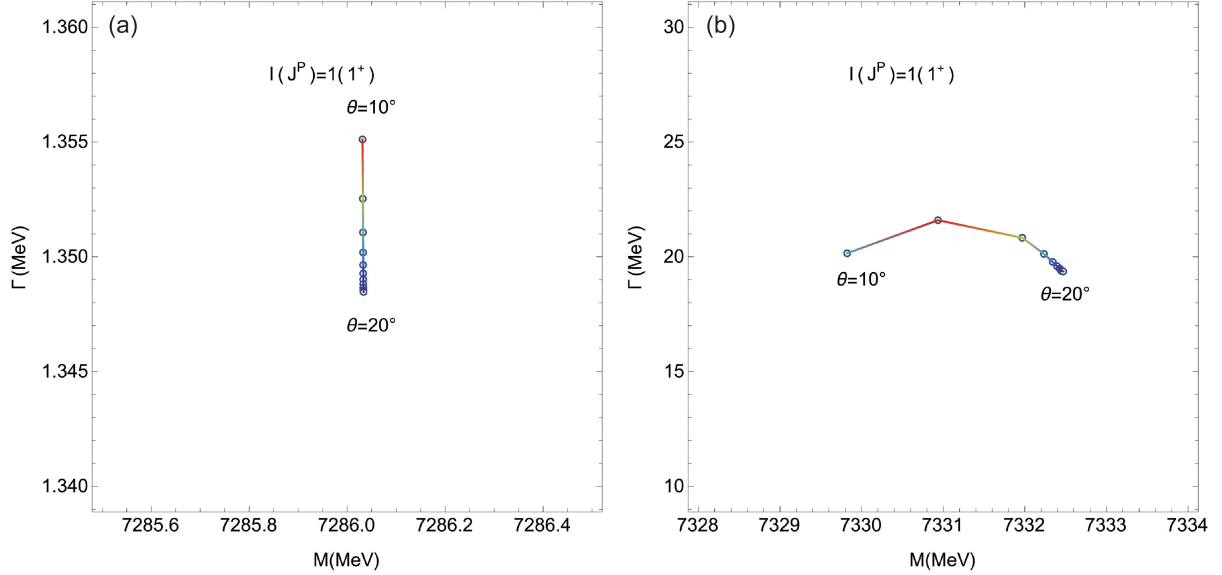


FIG. 4. The variations of complex energies for the two predicted $1(1)^+$ resonances with θ .

energy plane when the cutoff is set to 1200 ~ 1500 MeV. It is important to note that the $I(J^P) = 1(2^+)$ system consists solely of the D^*B^* channel, where the attractive interaction is not strong enough to form a bound state compared to other isospin-vector systems. Nevertheless, by increasing the cutoff to approximately 2500 MeV, one may explore the possibility of a bound state.

B. Isospin scalar systems

Except for isospin vector systems, we also have isospin scalar systems for $D_{(s)}^{(*)}B_{(s)}^{(*)}$. Unlike the $I = 1$ systems, the $I = 0$ case allows for more coupled channels. As shown in Fig. 5(a), we investigate the coupled-channel system $DB/D^*B^*/D_sB_s/D_s^*B_s^*$ with spin-parity $I(J^P) = 0(0^+)$. A narrow resonance, predominantly composed of $D^*B^*(P_{D^*B^*}(^1S_0) = 58.78 - 7.24i)$ and $D_sB_s(P_{D_sB_s}(^1S_0) = 29.74 + 6.81i)$, appears near the D_sB_s threshold with a complex energy of $E - i\Gamma/2 = 7332 - 0.5i$ MeV when the cutoff value is set to $\Lambda = 1200$ MeV. By increasing the cutoff, a bound state solution emerges below the DB threshold, meanwhile, the resonance evolves to $7387 - 0.3i$ MeV. Both states can be considered candidates for molecular hadronic states. Additionally, we identify a pole near the $D_s^*B_s^*$ threshold with a narrow width.

For the $I(J^P) = 0(1^+)$ channel, the dependence of the system on the cutoff parameter is illustrated in Fig. 5(b). As the cutoff increases from 1200 to 1500 MeV, several notable features are observed. When the cutoff lies within the range 1200–1300 MeV, two poles are excluded because they are highly sensitive to the cutoff. Their corresponding root-mean-square radii r_{RMS} are small, indicating that these states are too compact to be interpreted as molecular states. For visual clarity, these discarded poles are represented by gray points in the figures, distinguishing them from the

physically relevant states. In particular, a cutoff-sensitive resonance located below the D^*B threshold evolves into a bound state as the cutoff increases to 1280 MeV; however, due to its relatively small r_{RMS} in the range 1300–1500 MeV, it is not regarded as a suitable molecular candidate in the present analysis. Furthermore, when the cutoff increases to 1320 MeV, a new resonance with energy $7323 - 4i$ MeV, predominantly composed of the $D^*B^*(^3S_1)$ component, emerges and can be interpreted as a possible partner of the $X(4014)$ particle. Meanwhile, for the pole located below the $D_sB_s^*$ threshold, its complex energy $E_r - i\Gamma/2$ shifts from $7374 - 5i$ MeV to $7322 - 4i$ MeV as the cutoff increases from 1200 to 1500 MeV, indicating a gradual stabilization of this state.

Finally, for the $I(J^P) = 0(2^+)D^*B^*/D_s^*B_s^*$ system, a deeply bound state is found at $\Lambda = 1200$ MeV. However, its corresponding r_{RMS} value is too small to support a molecular interpretation. As the cutoff increases to approximately $\Lambda = 1400$ MeV, a resonance with a predicted energy of $E_r - i\Gamma/2 = 7530 - 5i$ MeV emerges near the $D_s^*B_s^*$ threshold. This state is predominantly composed of the $D_s^*B_s^*(^3S_2)$ component and, according to our analysis, can be considered a promising molecular candidate.

C. Further discussion

In the earlier hidden-bottom study, only two bound states were obtained in the $I(J^P) = 1(1^{+-})$ coupled channel. In contrast, the current bottom-charm study reveals both bound and resonant states in the $I(J^P) = 0, 1(1^+)$ sectors. This difference arises mainly because the bottom-charmed systems allow more coupled channels, and coupled-channel effects play a crucial role in generating resonances. For instance, in the single-channel DB^* or D^*B system with $I(J^P) = 1(1^+)$, no pole is found for cutoffs ranging

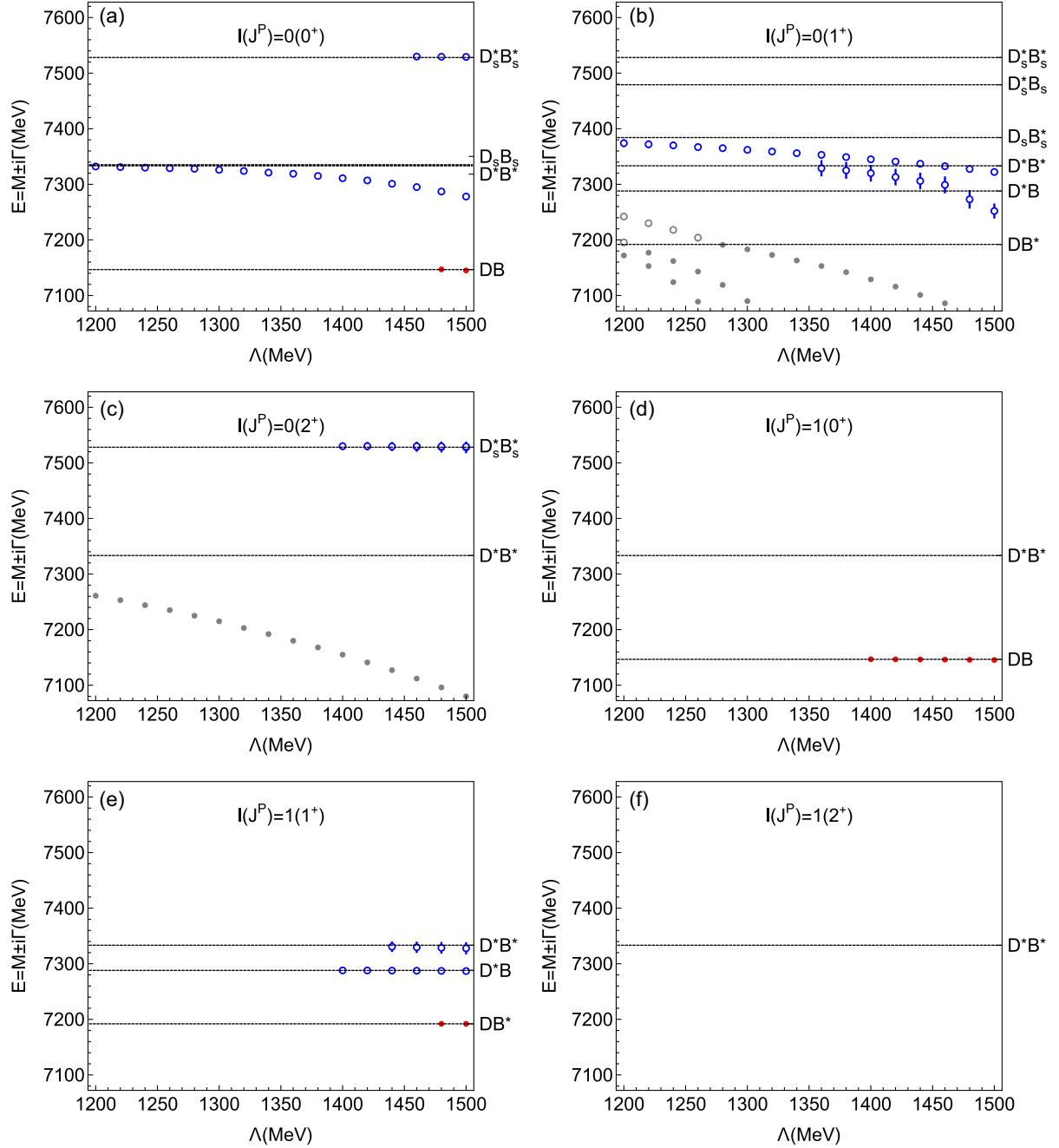


FIG. 5. The dependence of the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems on the cutoff parameter Λ . Red points indicate the predicted bound states, while blue open circles represent the resonances, with vertical bars showing their total decay widths. Gray points denote poles that are excluded from molecular interpretation. Panels (a)–(f) correspond to (a) the $DB/D^*B^*/D_sB_s/D_s^*B_s^*$ system with $I(J^P) = 0(0^+)$, (b) the $DB^*/D^*B/D_sB_s^*/D_s^*B_s^*$ system with $I(J^P) = 0(1^+)$, (c) the $D^*B^*/D_sB_s^*$ system with $I(J^P) = 0(2^+)$, (d) the $DB/D_s^*B_s^*$ system with $I(J^P) = 1(0^+)$, (e) the $DB^*/D^*B/D^*B^*$ system with $I(J^P) = 1(1^+)$, and (f) the D^*B^* system with $I(J^P) = 1(2^+)$, respectively.

from 800 to 1500 MeV, as the interaction is too weak to form a molecular state. However, when the full set of coupled channels is considered, bound and resonant states emerge, demonstrating that S -wave dominated coupled-channel effects are essential for the formation of these molecular states. Even when D -wave contributions are

neglected, the main features persist, confirming the dominant role of coupled-channel dynamics. Moreover, while no molecular states were identified near the $B_s^{(*)}\bar{B}_s^{(*)}$ threshold in the previous study, we now obtain molecular states close to the $D_s^{(*)}B_s^*$ thresholds, highlighting new structural features specific to the bottom–charm sector.

TABLE IV. The summary of our predictions for $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems with cutoff Λ in a range of 1200–1500 MeV. Here, the “...” means nonexistence, “✓”(“×”) represents that the corresponding state may (may not) form a molecular state.

$I(J^{PC})$	Mass (MeV)	Width (MeV)	r_{RMS} (fm)	Status	Selected decay mode
$0(0^+)$	7326.00 ~ 7277.68	0.76–0.82	$3.94 - 1.26i \sim 1.23 - 0.30i$	✓	$B_c\eta^{(l)}/B_c^*\omega/DB$
	7529.63 ~ 7528.91	6.16–6.48	$4.64 + 0.73i \sim 3.52 + 1.04i$	✓	$B_c\eta^{(l)}/B_c^*\phi/D_s^{(*)}B_s^{(*)}$
	7146.74 ~ 7145.49	...	$3.53 \sim 2.60$	✓	$B_c\eta^{(l)}/B_c^*\omega$
$0(1^+)$	7330.73 ~ 7251.56	24.06 ~ 23.62	$4.74 - 1.07i \sim 2.89 - 0.37i$	✓	$B_c\omega/B_c^*\eta^{(l)}/DB^*/D^*B$
	7374.10 ~ 7322.14	10.48 ~ 8.48	$2.32 + 0.08i \sim 2.40 + 0.62i$	✓	$B_c\omega/B_c^*\eta^{(l)}/DB^*/D^*B$
	7182.86 ~ 7049.72	...	$1.00 \sim 0.48$	✗	...
$0(2^+)$	7214.52 ~ 7079.97	...	$0.52 \sim 0.42$	✗	...
	7530.45 ~ 7527.80	9.46–17.24	$4.20 + 1.51i \sim 1.15 - 0.86i$	✓	$B_c\phi/B_c^*\phi/D_s^{(*)}B_s^{(*)}$
$1(0^+)$	7146.69 ~ 7145.31	...	$4.33 \sim 2.46$	✓	$B_c\pi/B_c^*\rho$
$1(1^+)$	7288.05 ~ 7286.03	0.58–1.34	$3.92 - 1.23i \sim 2.11 - 0.75i$	✓	$B_c\rho/B_c^*\pi/DB^*$
	7333.25 ~ 7332.39	14.92–19.6	$1.22 - 0.21i \sim 1.23 + 0.26i$	✓	$B_c\rho/B_c^*\pi/DB^*/D^*B$
	7191.94 ~ 7191.83	...	$4.47 \sim 4.19$	✓	$B_c\rho/B_c^*\pi$
$1(2^+)$	...	...	...	✗	...

Moreover, we prefer to collect the final results of $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems in Table IV, where we predict ten molecular particles. As plotted in Fig. 5(b) and Table IV, for $I(J^P) = 0(1^+)$ system, we predict two molecular states near D^*B , and D^*B^* thresholds, respectively, which may be regarded as good molecular state and be counterparts to the molecular states already observed in both the charm and bottom sectors. Also, for $I(J^P) = 1(1^+)$ system, three molecular states are found below their thresholds, respectively, which are shown in Fig. 5(e), Table III, and Table IV. However, whether the $Z_c(10610)$ and $Z_c(10650)$ are located above or below the corresponding thresholds is still an open question. For the bottom-charmed molecular tetraquark states, they may also encounter the same question, thus, the further theoretical studies and experimental explorations are needed. Finally, based on the quantum numbers, and available phase space of the predicted states, we collect some possible decay modes in Table IV. We can select the primary decay channels, which satisfy the conservation of quantum numbers such as angular momentum and parity, involve no change in quark flavor but only quark rearrangement, and have sufficient phase space. For the bound states, which are relatively more stable but still prone to decay, the favored final states are B_c or B_c^* plus light mesons; these decays occur via quark rearrangement into kinematically allowed two-meson states that conserve $I(J^P)$. For the resonances, which are less stable, the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ channels are preferred, as these states typically first decay into intermediate two-meson configurations satisfying the same quantum number and phase space constraints, before forming stable final states. These considerations provide a systematic and physically motivated basis for the decay channels listed in Table IV.

IV. SUMMARY

A. Summary

In this work, we perform a systematic investigation of the $D_{(s)}^{(*)}B_{(s)}^{(*)}$ systems by solving the coupled-channel Schrödinger equation within the one-boson-exchange framework, employing the Gaussian expansion method together with the complex scaling technique. We first explore the coupled channel $I(J^P) = 1(1^+) DB^*/D^*B/D^*B^*$, which helps to constrain the cutoff value within a phenomenologically reasonable range. Our analysis predicts ten molecular candidates with different quantum numbers. For the $I(J^P) = 1(1^+) DB^*/D^*B/D^*B^*$ system, we identify a loosely bound state and two resonant states located below the DB^* , D^*B , and D^*B^* thresholds, respectively. In $0(0^+)$ system, we predict a bound state and two resonance located to DB , D^*B^* , and $D_s^*B_s^*$ thresholds, respectively. In the $I(J^P) = 0(1^+)$ system, two resonant states are observed. For $I(J^P) = 0(2^+)D^*B^*/D_s^*B_s^*$ system, we obtain a resonance with strangeness. For DB/D^*B^* system with $I(J^P) = 0(0^+)$, a bound state located below DB threshold is revealed. The details of all the predicted molecular states are collected in Table IV.

Compared with earlier studies in the hidden-bottom sector, the bottom charm systems exhibit a richer coupled-channel structure, giving rise not only to bound states but also to dynamically generated resonances. For the predicted molecular states, the possible decay channels are also summarized in Table IV. Bound states, being relatively more stable, are expected to dominantly decay into B_c or B_c^* accompanied by light mesons through quark rearrangement

processes, whereas resonant states typically couple first to intermediate $D_{(s)}^{(*)}B_{(s)}^{(*)}$ channels before transitioning into their final, more stable configurations. These features provide a physically well motivated guidance for future experimental searches for bottom charmed molecular tetraquarks.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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