

Gravitational wave production and baryogenesis in a simple left-right model

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The left-right model with the simplest Higgs structure has a $SU(2)_L$ doublet H_L , a $SU(2)_R$ doublet H_R , and a singlet σ that couples to the doublets as $\sigma(|H_L|^2 - |H_R|^2)$. The left-right symmetry has $H_L \leftrightarrow H_R$ and $\sigma \leftrightarrow -\sigma$. We study gravitational wave production and baryogenesis in the electroweak phase transitions in the model. For two benchmark points, σ first gets a vev, followed by H_R and then H_L . An interesting feature is that the vev of H_L is initially much higher than its zero temperature value, leading to a more strongly first-order transition and higher frequency gravitational waves. An unusual benchmark point has the σ vev at zero when the electroweak symmetry breaks. This results in both H_L and H_R vevs being equal and in the multi-TeV range. At a lower temperature, the σ vev turns on, breaking the left-right symmetry and H_L drops to its standard model value. Since the electroweak symmetry is broken at the multi-TeV scale, the frequency of gravitational waves will be much higher than usual. Baryogenesis is also discussed.

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I. INTRODUCTION

One of the mysteries of cosmology is the matter-antimatter asymmetry. The current value of the asymmetry is $n_B/n_\gamma \simeq 6 \times 10^{-10}$ [1]. Since baryon number violation is required to generate such an asymmetry, initial discussions of baryogenesis focused on grand unified theories. However, it was recognized that electroweak sphalerons could wipe out an initial baryon asymmetry, which led to interest in baryogenesis on the electroweak scale, where sphalerons can generate the asymmetry [2–7]. The out-of-equilibrium Sakharov condition [8] is most easily satisfied via a first-order phase transition.

The Standard Model does not have a first-order phase transition [9] and does not have sufficient CP violation. The simplest and most well studied extension is the two-Higgs-doublet model (2HDM) [10]. This can have a first-order phase transition and there is substantial literature on baryogenesis in 2HDMs, in the CP conserving case [11–26]; see Ref. [27] for an extensive review. In addition, studies of electroweak baryogenesis have been carried out in the

2HDM with explicit CP violation [28–35]; see Ref. [36] for a recent review.

Are there any signatures of electroweak baryogenesis? Of course, many models do have collider signatures that can be tested at the LHC. With the recent discovery of gravitational waves, however, the possibility of gravitational wave signatures from bubble wall collisions becomes attractive [37–53]. The frequency range of these signatures [44] is in the range of LISA, BBO, or DECIGO and might be detectable.

Generally a baryon asymmetry can best be produced from a slow moving bubble wall, whereas gravitational wave production best requires a rapidly moving bubble wall. While some successful models do exist [54–57], they generally require complicated Higgs sectors and in 2HDM models may be incompatible with electron electric dipole moment (EDM) measurements (although some regions of parameter space may in some cases be allowed) [31,58–60]. This led to an interest in multistep electroweak phase transitions in which baryogenesis and gravitational wave production can be generated from different transitions.

However, Cline and Kainulainen [61] recently have shown that a fast moving wall can still generate a sufficient baryon asymmetry.¹ See Ref. [63] for a detailed discussion and numerous references. As a result, it is possible that both

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¹A precise determination of the bubble wall velocity is crucial. Very recently, Carena *et al.* [62] have studied this in great detail. We will use the method of Cline and Kainulainen, but the reader is referred to Carena *et al.* for a more precise calculation.

the baryon asymmetry and gravitational wave production could occur at the same transition although this will not occur in the Standard Model or typical 2HDMs. We will therefore not require that baryogenesis and gravitational wave production occur at different transitions.

Some of the earliest extensions of the Standard Model are left-right symmetric models [64–67]. Here, one step can break the left-right symmetry and the other can break the electroweak gauge symmetry. These models are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ and have received a great deal of attention. They differ in having a variety of fermion and scalar sectors. The most common scenario has two $SU(2)$ triplet Higgs with one breaking the $SU(2)_R$ symmetry and also has a bidoublet field which includes the Standard Model Higgs and provides masses for the fermions. This is a fairly complicated scalar sector that has doubly charged scalars and many other states. The phase transition in this model was studied long ago [68] and the production of gravitational waves has also been examined and baryogenesis briefly discussed [69–71]. The simplest possible Higgs sector involves just two doublets, with fermion masses generated by a see-saw mechanism. Gravitational wave production in this model was studied in Ref. [72], but the model in that case is not left-right symmetric [since the $SU(2)_L$ and $SU(2)_R$ gauge couplings are different]; one can impose left-right symmetry in the model at the cost of nonrenormalizable terms. Another scalar sector has two doublets plus a bidoublet [73,74]. Gravitational wave production in this model was recently studied in Ref. [75]. Another model has four doublets [76,77].

In this paper, we will focus on a left-right symmetric model without scalar triplets or bidoublets. The model was discussed by Gu [78] and contains two Higgs doublets, a left-handed doublet H_L , and right-handed doublet H_R and a singlet σ . The left-right symmetry requires $H_L \leftrightarrow H_R$ and $\sigma \leftrightarrow -\sigma$. It is a multistep transition in which σ gets a vev breaking the left-right symmetry (since it is a singlet, this vev does not break the electroweak symmetry), and the electroweak symmetry is broken when H_L or H_R (or both) get a vev. Fermion masses are generated by integrating out heavy fermions. Although the definition of “minimal left-right symmetric model” is subjective, we consider this model to have the minimal scalar sector—it is the only such sector with no charged or pseudoscalar Higgs bosons.

Gu studied the baryon asymmetry in the model. Here we will focus on both the baryon asymmetry and gravitational wave production. As we will see, the model has a very unusual history in that the phase transition breaking the electroweak symmetry can occur while the model is still left-right symmetric, leading to H_L and H_R both getting vev at a fairly high temperature. In Sec. II, the model is presented with the various stages of symmetry breaking. The symmetry-breaking pattern is discussed in Sec. III. Gravitational wave production is discussed in Sec. IV and baryogenesis in Sec. V. Section VI contains our conclusions.

TABLE I. Here we tabulate the particle content for the left-right model with a minimal scalar sector.

	n_f	$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}$	3	(3, 2, 1, 1/3)
$q_R = \begin{pmatrix} u \\ d \end{pmatrix}$	3	($\bar{3}$, 1, 2, 1/3)
$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}$	3	(1, 2, 1, -1)
$\ell_R = \begin{pmatrix} \nu \\ e \end{pmatrix}$	3	(1, 1, 2, 1)
U_L	3	(3, 1, 1, 1/3)
U_R	3	($\bar{3}$, 1, 1, 1/3)
D_L	3	(3, 1, 1, 1/3)
D_R	3	($\bar{3}$, 1, 1, 1/3)
E_L	3	(1, 1, 1, -1)
E_R	3	(1, 1, 1, -1)
N_L	3	(1, 1, 1, -1)
N_R	3	(1, 1, 1, -1)
H_L	1	(1, 2, 1, 0)
H_R	1	(1, 1, 2, 0)
σ	1	(1, 1, 1, 0)

II. MODEL

The particle content for this left-right $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model [78] is given in Table I.

The scalar potential is given by

$$\begin{aligned}
 V_0 = & -\mu_1^2 \sigma^2 - \mu_2^2 (H_L^\dagger H_L + H_R^\dagger H_R) \\
 & + \mu_3 \sigma (H_L^\dagger H_L - H_R^\dagger H_R) + \lambda_\sigma \sigma^4 \\
 & + \lambda_H [(H_L^\dagger H_L)^2 + (H_R^\dagger H_R)^2] + \lambda_{LR} H_L^\dagger H_L H_R^\dagger H_R \\
 & + \lambda_{\sigma H} \sigma^2 (H_L^\dagger H_L + H_R^\dagger H_R),
 \end{aligned} \tag{1}$$

and the Yukawa Lagrangian is

$$\mathcal{L}_Y = -y_D (\bar{q}_L H_L D_R + \bar{q}_R H_R D_L) - M_D \bar{D}_L D_R \tag{2}$$

$$-y_U (\bar{q}_L \tilde{H}_L U_R + \bar{q}_R \tilde{H}_R U_L) - M_U \bar{U}_L U_R \tag{3}$$

$$-y_E (\bar{\ell}_L H_L E_R + \bar{\ell}_R H_R E_L) - M_E \bar{E}_L E_R \tag{4}$$

$$-y_N (\bar{\ell}_L \tilde{H}_L E_R + \bar{\ell}_R \tilde{H}_R E_L) - M_N \bar{N}_L N_R. \tag{5}$$

Integrating out the heavy fermions, the masses of the SM fermions are as follows:

$$m_u = y_U \frac{v_L v_R}{M_U} y_U^\dagger, \tag{6}$$

$$m_d = y_D \frac{v_L v_R}{M_D} y_D^\dagger, \tag{7}$$

$$m_l = y_E \frac{v_L v_R}{M_E} y_E^\dagger, \tag{8}$$

$$m_\nu = y_N \frac{v_L v_R}{M_N} y_N^\dagger. \quad (9)$$

A. Bounded from below

One can constrain the parameters by requiring that the vacuum is stable at large field values. These constraints then apply to the quartic parameters. We have two doublets and a singlet. The bounds on the neutral components in a three-Higgs model are given explicitly in [79,80]. It has been noted [81] that there is no simple expression for bounded-from-below constraints including both the charged and neutral directions; however, our model contains no charged or pseudoscalar Higgs bosons since they are eaten by the left- and right-handed gauge bosons. Thus, we focus on the neutral components and the resulting bounds are necessary and sufficient. The bounds are

$$\lambda_\sigma \geq 0, \quad (10)$$

$$\lambda_H \geq 0, \quad (11)$$

$$\lambda_H + \frac{1}{2}\lambda_{LR} \geq 0, \quad (12)$$

$$2\sqrt{\lambda_\sigma \lambda_H} + \lambda_{\sigma H} \geq 0, \quad (13)$$

$$\left(\lambda_H + \frac{1}{2}\lambda_{LR}\right)\sqrt{2\lambda_\sigma} + \lambda_{\sigma H}\sqrt{2\lambda_H} + (2\sqrt{\lambda_\sigma \lambda_H} + \lambda_{\sigma H})\sqrt{\left(\lambda_H + \frac{1}{2}\lambda_{LR}\right)} \geq 0, \quad (14)$$

and we will restrict our study to parameters that satisfy these conditions.

B. Scalar masses

One can eliminate the μ^2 parameters in terms of the vevs of H_L , H_R , and σ :

$$\begin{aligned} \mu^2 &= \frac{1}{4}(\lambda_{LR} + 2\lambda_H)(v_L^2 + v_R^2) + \lambda_{\sigma H}v_\sigma^2, \\ \mu_1^2 &= \frac{1}{2}\lambda_{\sigma H}(v_L^2 + v_R^2) + \frac{(\lambda_{LR} - 2\lambda_H)(v_L^2 - v_R^2)^2}{16v_\sigma^2} + 2\lambda_\sigma v_\sigma^2, \\ \mu_3 &= \frac{(\lambda_{LR} - 2\lambda_H)(v_L^2 - v_R^2)}{4v_\sigma}. \end{aligned} \quad (15)$$

Then we compute the scalar mass matrix, defining $\lambda_v \equiv \frac{(\lambda_{LR} - 2\lambda_H)(v_L^2 - v_R^2)}{4v_\sigma}$,

$$m_h^2 = 2 \begin{pmatrix} 2\lambda_H v_L^2 & \lambda_{LR} v_L v_R & 2\lambda_{\sigma H} v_L v_\sigma + \lambda_v v_L \\ \lambda_{LR} v_L v_R & 2\lambda_H v_R^2 & 2\lambda_{\sigma H} v_R v_\sigma - \lambda_v v_R \\ 2\lambda_{\sigma H} v_L v_\sigma + \lambda_v v_L & 2\lambda_{\sigma H} v_R v_\sigma - \lambda_v v_R & 8\lambda_\sigma v_\sigma^2 - \lambda_v \frac{(v_L^2 - v_R^2)}{2v_\sigma} \end{pmatrix} = \mathcal{O}.m_D.\mathcal{O}^T, \quad (16)$$

$$m_D = \begin{pmatrix} m_{\text{Higgs}}^2 & 0 & 0 \\ 0 & m_{H_R}^2 & 0 \\ 0 & 0 & m_\sigma^2 \end{pmatrix}; \quad \mathcal{O} = \begin{pmatrix} c_{12}c_{13} & -c_{13}s_{12} & s_{13} \\ c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} + s_{12}s_{13}s_{23} & c_{13}s_{23} \\ -c_{12}c_{23}s_{13} - s_{12}s_{23} & c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix}. \quad (17)$$

Of course, one must ensure that mixing between the 125 GeV Higgs and the other neutral scalars is small enough to satisfy LHC bounds. This will generally be the case since $v_L \ll v_R$.

C. Phenomenological bounds

A significant constraint on many multi-Higgs models arises from the S and T parameters. These do not provide a major constraint here because we have no charged scalars or pseudoscalars.

However, there are bounds on right-handed W masses. Langacker and Sankar provided a detailed discussion of the

bounds under a wide variety of assumptions [82]. We will simply use the strongest bound² in the Particle Data Group [83] listing of 715 GeV [84]. This implies (since the gauge couplings are very similar) $v_R > 8.8v_L$. In the next two sections, we will use a benchmark point with a smaller W_R mass for illustrative purposes, but one must keep in mind that only masses in excess of 715 GeV are phenomenologically acceptable.

²In many models, detailed LHC analyses will provide stronger bounds. We can accommodate these bounds, although the allowed parameter space will shrink.

III. PHASE TRANSITIONS AND BREAKING PATTERN

In order to explore the transitions in detail, let us first focus on the effective potential. The complete effective potential consists of the tree-level contribution, the Coleman-Weinberg (CW) one-loop part, and then the corrections coming from the background temperature, which are as follows:

$$V_{\text{effective}} = V_0(v_L, v_R, v_\sigma) + V_{CW}(v_L, v_R, v_\sigma) + V_{FT}(v_L, v_R, v_\sigma, T) + V_{\text{daisy}}(v_L, v_R, v_\sigma, T). \quad (18)$$

The CW part is stemming from the complete one-loop effective potential which in general is written as

$$V_{CW}(v_L, v_R, v_\sigma) = \frac{1}{64\pi^2} \left[\text{Tr} \left[m_D^4 \left(\log \frac{m_D^2}{\mu^2} - \frac{3}{2} \right) \right] + 6 \sum_{i=L,R} m_{W_i}^4 \left(\log \frac{m_{W_i}^2}{\mu^2} - \frac{5}{6} \right) + 3 \sum_{i=L,R} m_{Z_i}^4 \left(\log \frac{m_{Z_i}^2}{\mu^2} - \frac{5}{6} \right) - 6 \sum_{f=u,d,l,\nu} m_f^4 \left(\log \frac{m_f^2}{\mu^2} - \frac{3}{2} \right) \right]. \quad (19)$$

The temperature dependent part is given as

$$V_{FT} = \frac{T^4}{2\pi^2} \left[J_B \left(\frac{\text{Tr}[m_D^2]}{T^2} \right) + 6 \sum_{i=L,R} J_B \left(\frac{m_{W_i}^2}{T^2} \right) + 3 \sum_{i=L,R} J_B \left(\frac{m_{Z_i}^2}{T^2} \right) - 6 \sum_{f=u,d,l,\nu} J_F \left(\frac{m_f^2}{T^2} \right) \right], \quad (20)$$

where the thermal functions $J_{B/F}$ are the integrals for bosons/fermions given as

$$J_{B/F}(y^2) = \int_0^\infty dx x^2 \log(1 \mp e^{-\sqrt{x^2+y^2}}). \quad (21)$$

The μ in the above equation is given as $\mu = \sqrt{v_L^2 + v_R^2 + v_\sigma^2}$ and the last expression V_{daisy} comes from the daisy diagrams

$$V_{\text{daisy}} = -\frac{T}{12\pi} \sum_i [(m_i^2 + \Pi_i^2)^{3/2} - m_i^2], \quad (22)$$

where Π_i is the thermal self-energy correction to the field-dependent masses.

The scalar potential contains seven free parameters. These become the zero-temperature vevs v_L , v_R and the

TABLE II. Three representative benchmark points which satisfy the constraints of Sec. II except for the W_R mass for the first benchmark point. The vevs listed are the zero-temperature vevs.

	BP1	BP2	BP3
λ_σ	0.065	0.014	0.0047
λ_H	8	2.56	2.32
λ_{LR}	-6.73	-2.11	-2.12
$\lambda_{\sigma H}$	0.19	0.045	0.012
m_H (TeV)	3.1	4	5
m_S (TeV)	6.2	2	2.5
m_T (TeV)	1.5	2	3.2
m_{W_R} (GeV)	364	813	1070
v_S (TeV)	12	8	18
v_R (TeV)	1.11	2.5	3.3
T_{LR} (TeV)	1.2		
T_L (GeV)		485	668
T_R (TeV)		1.9	2.4

observed Higgs mass, giving four free parameters (plus v_R , which will give the W_R mass). We will choose parameters such that the singlet vev is around 10 TeV. A set of benchmark points that satisfies all the bounds of the last section (except in one case, as noted earlier, the W_R mass) as well as the temperatures and vevs for each of these points is given in Table II.

Primarily there are two paths for the vevs from the fully symmetric phase (0, 0, 0) to fully broken phase (v_L , v_R , v_σ). The first path is when the σ gets a vev v_σ and then gradually restores back while $v_L = v_R$ gets a vev. And then later the σ again gets a vev leading to the $L - R$ breaking. The second path, for which the mass of the W_R is not constrained, is when the σ gets a vev (break the $L - R$ symmetry) then the H_R gets a vev and then finally H_L gets a vev.

The temperature dependences of the vevs are shown in Fig. 1 for two particular benchmark points. For benchmark point 2 (the dashed line), the σ vev is nonzero at a very high temperature—this breaks the left-right symmetry and the H_R vev turns on next and the H_L vev last. It is interesting and unusual that the latter vev goes to a fairly high value before relaxing to its zero temperature value. We have not shown the curve for benchmark point 3 since it is very similar to that of benchmark point 2. Although not shown explicitly on the plot, the vev of sigma does drop to zero at higher temperatures.

The more interesting benchmark point is benchmark point 1 (the solid line). Here, the σ turns on first. The left-right symmetry is broken but both vevs of H_L and H_R are zero due to the high temperature. At a lower temperature, v_σ drops to zero, the left-right symmetry is restored, and both H_L and H_R get identical vevs, in the multi-TeV range. Then the σ vev returns, breaking the left-right symmetry and the vev of H_L drops to its zero temperature value. Although interesting, benchmark point 1 is phenomenologically unacceptable due to the light W_R mass.

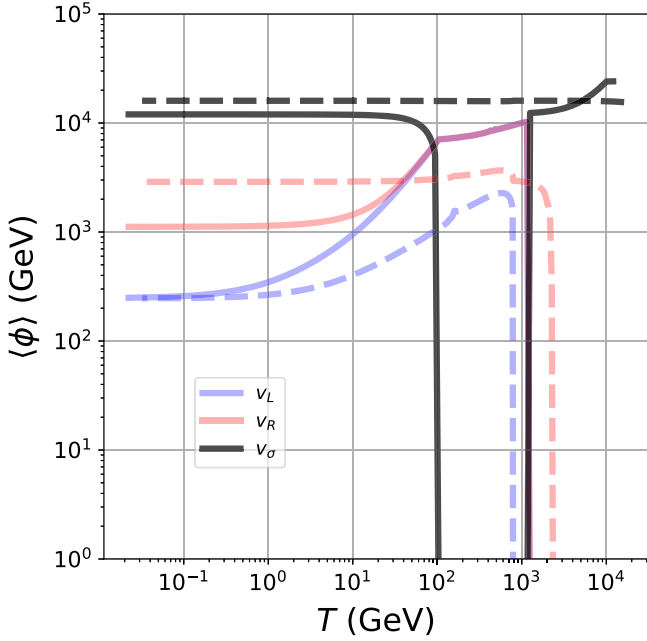


FIG. 1. The vevs as a function of temperature for two benchmark points. The solid lines correspond to benchmark point 1 and the dashed line to benchmark point 2. Benchmark point 3 is very similar to benchmark point 2. At extremely high temperatures, the vev of the σ is also zero in all cases considered.

Note that v_L/T_c for the transition where the vev of H_L turns on for benchmark points 2 and 3 is much larger than 1, and for benchmark point 1 is also substantially larger than 1 thus the transition is strongly first order and can give both baryogenesis and gravitational wave production.

Our point in this paper is not to rely on a specific model but to show that models exist in which the electroweak transition temperature is much higher than usual—in the multi-TeV range.³ That is the case for all three benchmark points. The most interesting pattern of benchmark point 1 is not phenomenologically acceptable, but in more complicated models this pattern could occur.

IV. GRAVITATIONAL WAVE PRODUCTION

The nucleation rate per unit volume is given by [85,86]

$$\Gamma(T) = [A(T)]^4 \exp[-S(T)], \quad (23)$$

where A is a prefactor of mass dimension one (see below), and S is the Euclidean bounce action. At zero temperature, the configuration minimizing the action is $O(4)$ symmetric, and $S \equiv S_4$, where

³After this calculation was completed, a more consistent resummation technique was developed by Bittar *et al.* [26]—this might change the specific results in this model but will not change the basic result.

$$S_4 = 2\pi^2 \int_0^\infty dr r^3 \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V_{\text{eff}}(\phi, 0) \right], \quad (24)$$

and can be estimated using the saddle-point approximation from the equation of motion:

$$\frac{d^2\phi}{dr^2} + \frac{3}{r} \frac{d\phi}{dr} - \frac{\partial V_{\text{eff}}}{\partial \phi} = 0, \quad (25)$$

with the boundary conditions

$$\frac{d\phi}{dr}(r=0) = 0, \quad \phi(r=\infty) = 0. \quad (26)$$

In the above equation, the first condition is for the solution to be regular at the center of the bubble, and the second one is to describe the initial false vacuum background far from the bubble. The bubble nucleation rate is eventually well approximated at low T by $\Gamma \equiv \Gamma_4$, where [cf. Eq. (23)]

$$\Gamma_4 \simeq \frac{1}{R_c^4} \left(\frac{S_4}{2\pi} \right)^2 \exp(-S_4), \quad (27)$$

with $R_c \sim 1/T$ being the bubble radius in the low T limit.

At finite temperature, the field becomes periodic in the time coordinate (or in $1/T$). The configuration minimizing the action in this case is $O(3)$ symmetric. Moreover, at sufficiently high temperatures, the minimum action configuration becomes constant in the time direction and $S \equiv S_3(T)/T$, where

$$S_3(T) = 4\pi \int_0^\infty dr r^2 \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + \Delta V_{\text{eff}}(\phi, T) \right], \quad (28)$$

where $\Delta V_{\text{eff}}(\phi, T) \equiv V_{\text{eff}}(\phi, T) - V_{\text{eff}}(\phi_{\text{false}}, T)$. Equation (28) represents bubble formation through classical field excitation over the barrier, with the corresponding equation of motion given by

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} - \frac{\partial V_{\text{eff}}}{\partial \phi} = 0, \quad (29)$$

with the same boundary conditions as in Eq. (26). The solution to Eq. (29) extremizes the action (28) that gives the exponential suppression of the false vacuum decay rate [87]. From Eq. (23), the nucleation rate $\Gamma \equiv \Gamma_3$ can be calculated as

$$\Gamma_3 \simeq T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} \exp \left[-\frac{S_3(T)}{T} \right]. \quad (30)$$

In practice, the exact solution with a nontrivial periodic bounce in the time coordinate, which corresponds to quantum tunneling at finite temperature, is difficult to evaluate. Following Ref. [88], we have taken the minimum

of the two actions S_3 and S_4 in our numerical calculation of the bubble nucleation rate, i.e., $\Gamma \approx \max(\Gamma_3, \Gamma_4)$. For a discussion of related theoretical uncertainties, see Ref. [89].

The nucleation temperature is defined as the inverse time of creation of one bubble per Hubble radius, i.e.,

$$\frac{\Gamma(T)}{H(T)^4} \Big|_{T=T_n} = 1, \quad (31)$$

where the Hubble expansion rate at temperature T is $H(T) \simeq 1.66\sqrt{g_*}T^2/M_{\text{Pl}}$, with $g_* \simeq 110$ being the relativistic degrees of freedom at high temperatures,⁴ and $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV being the reduced Planck mass.

The statistical analysis of the subsequent evolution of bubbles in the early Universe is crucial for strongly first order phase transition (SFOPT) [91]. The probability for a given point to remain in the false vacuum is given by $P(T) = \exp[-I(T)]$ [92–95], where $I(T)$ is the expected volume of true vacuum bubbles per comoving volume:

$$I(T) = \frac{4\pi}{3} \int_T^{T_c} \frac{dT'}{T'^4} \frac{\Gamma(T')}{H(T')} \left(\int_T^{T'} \frac{d\tilde{T}}{H(\tilde{T})} \right)^3. \quad (32)$$

The change in the physical volume of the false vacuum, $\mathcal{V}_{\text{false}} = a^3(t)P(T)$ [with $a(t)$ being the scale factor and t being the time], normalized to the Hubble rate, is given by [96]

$$\frac{1}{\mathcal{V}_{\text{false}}} \frac{d\mathcal{V}_{\text{false}}}{dt} = H(T) \left(3 + T \frac{dI(T)}{dT} \right). \quad (33)$$

We define the percolation temperature T_p as satisfying the condition $I(T_p) = 0.34$ [91], while ensuring that the volume of the false vacuum is decreasing, i.e., $d \log \mathcal{V}_{\text{false}}/dt < 0$, so that percolation is possible despite the exponential expansion of the false vacuum. Numerically, we find that the percolation temperature is only slightly smaller than the nucleation temperature, which in turn is smaller than the critical temperature. Moreover, as expected, both T_c and $T_n \simeq T_p$ grow monotonically, as either of the model parameters, $M_{Z'}$ or g' , increases. They are depicted in Fig. 2.

The strength of the phase transition is characterized by two quantities α and β defined as follows: $\alpha = \epsilon_*/\rho_{\text{rad}}$ is the ratio of the vacuum energy density ϵ_* released in the transition to the radiation energy density $\rho_{\text{rad}} = \pi^2 g_* T_*^4/30$, both evaluated at $T = T_*$ (where T_* is either T_n or T_p).⁵ The vacuum energy density is nothing but the free energy difference between the true and false vacua [97], thus yielding

⁴In our numerical analysis, we take into account the temperature dependence of g_* [90].

⁵Calculating α at T_p is better, as it accounts for the entropy dilution between T_n and T_p , although this distinction becomes irrelevant for the GW signal when $\alpha \gg 1$.

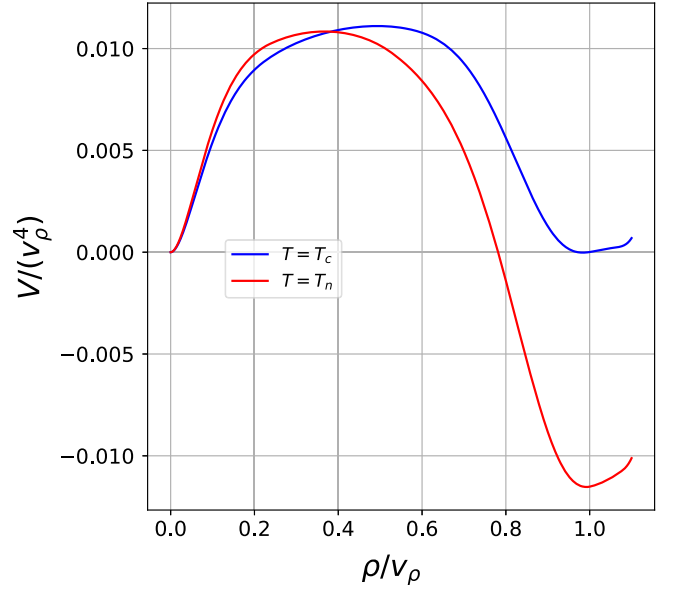


FIG. 2. The critical temperature and the nucleation temperature for a typical set of parameters.

$$\alpha = \frac{1}{\rho_{\text{rad}}} \left(-1 + T \frac{d}{dT} \right) \Delta V_{\text{min}}|_{T=T_*}, \quad (34)$$

where ΔV_{min} is the temperature-dependent minimum of the effective potential ΔV_{eff} defined below Eq. (28).

The second important parameter is β/H_* , where β is the (approximate) inverse timescale of the phase transition and H_* is the Hubble rate at T_* :

$$\frac{\beta}{H_*} = - \frac{T}{\Gamma} \frac{d\Gamma}{dT} \Big|_{T=T_*}. \quad (35)$$

For strong transitions, β is related to the average bubble radius R_* : $\beta = (8\pi)^{1/3}/R_*$ [98],⁶ where R_* defines the characteristic length scale of transition and is given by [95,96]

$$R_* = \left[T_* \int_{T_*}^{T_c} \frac{dT}{T^2} \frac{\Gamma(T)}{H(T)} \exp\{-I(T)\} \right]^{-1/3}. \quad (36)$$

The amplitude of the GW signal as a function of the frequency f is usually defined as

$$h^2 \Omega_{\text{GW}}(f) \equiv \frac{h^2}{\rho_c} \frac{d\rho_{\text{GW}}}{d \log f}, \quad (37)$$

where $h \sim 0.7$ is the dimensionless Hubble parameter (defined in terms of today's value of H , $H_0 = 100h$ km/s/Mpc), ρ_{GW} is the energy density released

⁶When identifying β/H_* with the average bubble radius, it is numerically found that taking $T_* = T_n$ gives a more accurate result [99].

in the form of GWs, and $\rho_c = 3H_0^2 M_{\text{Pl}}^2 \simeq 1.05 \times 10^{-5} h^2 \text{ GeV/cm}^3$ is the critical density of the Universe. The reason for multiplying Ω_{GW} by h^2 is to make sure that the GW amplitude is not affected by the experimental uncertainty [100] in the Hubble parameter H_0 .

There are three different mechanisms for producing GWs in an SFOPT from the expanding and colliding scalar-field bubbles, as well as from their interaction with the thermal plasma. These are (i) collisions of expanding bubble walls [101–111], (ii) compressional modes (or sound waves) in the bulk plasma [40,112–117], and (iii) vortical motion (or magnetohydrodynamic turbulence) in the bulk plasma [118–124]. The total GW signal can be approximated as a linear superposition of the signals generated from these three individual sources, denoted respectively by Ω_b (bubble wall), Ω_s (sound wave), and Ω_t (turbulence):

$$h^2 \Omega_{\text{GW}}(f) \simeq h^2 \Omega_b(f) + h^2 \Omega_s(f) + h^2 \Omega_t(f). \quad (38)$$

The three contributions can be parametrized in a model-independent way in terms of a set of characteristic SFOPT parameters, namely, α [cf. Eq. (34)], β/H_* [cf. Eq. (35)], T_* ,⁷ bubble-wall velocity v_w , and the three efficiency factors κ_b , κ_s , κ_t that characterize the fractions of the released vacuum energy that are converted into the energy of scalar field gradients, sound waves, and turbulence, respectively. The bubble-wall velocity in the plasma rest frame is given by [60]

$$v_w = \begin{cases} \sqrt{\frac{\Delta V_{\text{min}}}{\alpha \rho_{\text{rad}}}} & \text{for } \frac{\Delta V_{\text{min}}}{\alpha \rho_{\text{rad}}} < v_J \\ 1 & \text{for } \frac{\Delta V_{\text{min}}}{\alpha \rho_{\text{rad}}} \geq v_J \end{cases}, \quad (39)$$

where $v_J = (1 + \sqrt{3\alpha^2 + 2\alpha})/\sqrt{3}(1 + \alpha)$ is the Jouguet velocity [104,126,127]. As for the efficiency factors, it is customary to express κ_s and κ_t in terms of another efficiency factor κ_{kin} that characterizes the energy fraction converted into bulk kinetic energy and an additional parameter ε , i.e. [128]

$$\kappa_s = \kappa_{\text{kin}}, \quad \kappa_t = \varepsilon \kappa_{\text{kin}}. \quad (40)$$

While the precise numerical value of ε is still under debate, following Refs. [114,129], we will use $\varepsilon = 1$. The efficiency factor κ_b is taken from Ref. [104] and κ_{kin} is taken from Ref. [127], both of which were calculated in the so-called Jouguet detonation limit:

⁷We will use $T_* = T_n$, the nucleation temperature defined in Eq. (31). For subtleties, see Ref. [125].

$$\kappa_b = \frac{1}{(1 + 0.715\alpha)} \left(0.715\alpha + \frac{4}{27} \sqrt{\frac{3\alpha}{2}} \right),$$

$$\kappa_{\text{kin}} = \frac{\sqrt{\alpha}}{(0.135 + \sqrt{0.98 + \alpha})}. \quad (41)$$

Each of the three contributions in Eq. (38) is related to the SFOPT parameters discussed above, as follows [130]:

$$h^2 \Omega_i(f) = h^2 \Omega_i^{\text{peak}} \left(\alpha, \frac{\beta}{H_*}, T_*, v_w, \kappa_i \right) \mathcal{S}_i(f, f_i), \quad (42)$$

where $i \in \{b, s, t\}$, the peak amplitudes are given as [129]

$$h^2 \Omega_b^{\text{peak}} \simeq 1.67 \times 10^{-5} \left(\frac{v_w}{\beta/H_*} \right)^2 \left(\frac{100}{g_*(T_*)} \right)^{1/3} \times \left(\frac{\kappa_b \alpha}{1 + \alpha} \right)^2 \left(\frac{0.11 v_w}{0.42 + v_w^2} \right),$$

$$h^2 \Omega_s^{\text{peak}} \simeq 2.65 \times 10^{-6} \left(\frac{v_w}{\beta/H_*} \right) \left(\frac{100}{g_*(T_*)} \right)^{1/3} \left(\frac{\kappa_s \alpha}{1 + \alpha} \right)^2,$$

$$h^2 \Omega_t^{\text{peak}} \simeq 3.35 \times 10^{-4} \left(\frac{v_w}{\beta/H_*} \right) \left(\frac{100}{g_*(T_*)} \right)^{1/3} \left(\frac{\kappa_t \alpha}{1 + \alpha} \right)^{3/2}, \quad (43)$$

and the spectral shape functions are given as [129]

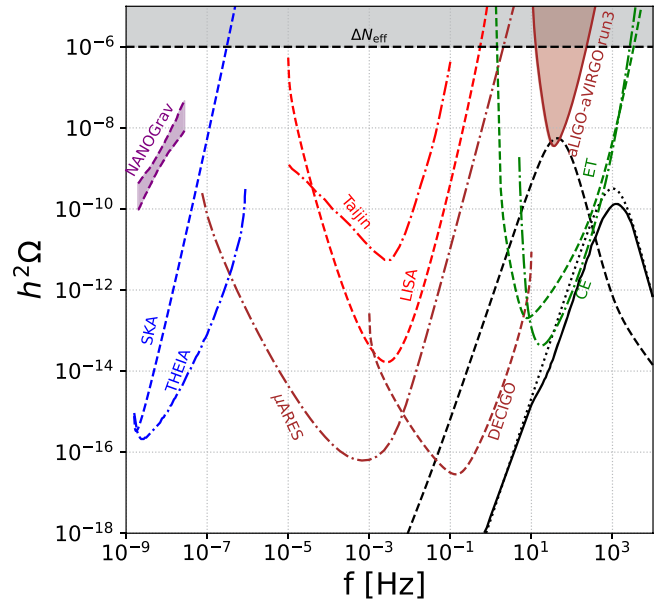


FIG. 3. The gravitational wave spectrum for our benchmark points. Benchmark point 1 is the solid line, benchmark point 2 is the dashed line, and benchmark point 3 is the dotted line. Note that since the electroweak transition temperature is much higher than in most models, the peak frequency is also substantially higher. CE and ET are the expected sensitivity of the Cosmic Explorer and Einstein Telescope, respectively.

$$\begin{aligned}
\mathcal{S}_b(f, f_b) &= \left(\frac{f}{f_b}\right)^{2.8} \left[\frac{3.8}{1 + 2.8(f/f_b)^{3.8}} \right], \\
\mathcal{S}_s(f, f_s) &= \left(\frac{f}{f_s}\right)^3 \left[\frac{7}{4 + 3(f/f_s)^2} \right]^{7/2}, \\
\mathcal{S}_t(f, f_t, h_*) &= \left(\frac{f}{f_t}\right)^3 \left[\frac{1}{1 + (f/f_t)} \right]^{11/3} \left(\frac{1}{1 + 8\pi f/h_*} \right).
\end{aligned} \tag{44}$$

Note that $\mathcal{S}_b$ and $\mathcal{S}_s$ are normalized to unity at their respective peak frequencies f_b and f_s , whereas $\mathcal{S}_t$ at f_t depends on the Hubble frequency

$$h_* = \frac{a_*}{a_0} H_* = 1.6 \times 10^{-2} \text{ mHz} \left(\frac{g_*(T_*)}{100} \right)^{1/6} \left(\frac{T_*}{100 \text{ GeV}} \right). \tag{45}$$

Finally, the peak frequencies are given as

$$\begin{aligned}
f_b &= 1.6 \times 10^{-2} \text{ mHz} \left(\frac{g_*(T_*)}{100} \right)^{1/6} \left(\frac{T_*}{100 \text{ GeV}} \right) \\
&\quad \times \left(\frac{\beta/H_*}{v_w} \right) \left(\frac{0.62 v_w}{1.8 - 0.1 v_w + v_w^2} \right), \\
f_s &= 1.9 \times 10^{-2} \text{ mHz} \left(\frac{g_*(T_*)}{100} \right)^{1/6} \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{\beta/H_*}{v_w} \right), \\
f_t &= 2.7 \times 10^{-2} \text{ mHz} \left(\frac{g_*(T_*)}{100} \right)^{1/6} \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{\beta/H_*}{v_w} \right).
\end{aligned} \tag{46}$$

The results are shown for our benchmark points in Fig. 3. Many gravitational wave signals from electroweak transitions are in the LISA/DECIGO range, but here we see that the frequency of the gravitational wave signal is higher, primarily due to the larger transition temperature. The signal would not have been seen by LIGO but might be in range of future gravitational wave detectors in that frequency range.

V. TRANSPORT EQUATION AND ASYMMETRY

Since sphalerons conserve $B - L$, a popular mechanism of baryogenesis is to generate a lepton asymmetry which sphalerons convert to a baryon asymmetry. One scenario for lepton asymmetry is a right-handed Majorana neutrino mass [131]. This has advantages and disadvantages—it does not require CP violation in the quark sector which may alleviate constraints from EDM experiments, but it also may not have any collider signatures since such a lepton asymmetry from a right-handed neutrino mass will typically arise at a very high scale. Although that could also work here, the fact that we have a strongly first-order phase transition makes it tempting to study whether the baryon

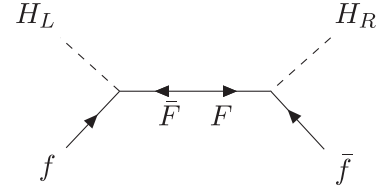


FIG. 4. Feynman diagrams for generating the baryon asymmetry.

asymmetry can be directly generated in this model. To this end, a complex phase must occur in the Higgs sector.

In a follow-up article to the Gu model described above,⁸ Gu and Lindner [132] added a complex singlet field ξ . The purpose of that work was to include a Peccei-Quinn symmetry. For our purposes, it is sufficient to include the phase of ξ . None of our previous results will be substantially changed by addition of this field. This is because the fact that ξ does not get a vev implies that its only effect is on undetermined parameters and masses.

The baryon asymmetry for our model is produced at the bubble walls specifically when $v_R = v_L \neq 0$ or when v_L gets the vev. We calculate this asymmetry using the method presented in [61].

For this method, transport equations are used which describe how particle asymmetries are transferred across bubble walls. The Boltzmann equations derived use the zeroth and first-order moment expansions of the Liouville operator to produce two first-order differential equations per particle species relating each species' CP -odd chemical potential, μ_i , and their velocity perturbations, u_i . Any coupling between particle species arises from the collision term δC_i . The collision terms include contributions from trilinear and quartic terms. The relevant Feynman diagrams are shown in Fig. 4.

The main source term for the CP violation comes from the mass term for each fermion which is given as

$$m_f = y_f^2 \frac{v_L v_R}{M_f}. \tag{47}$$

Furthermore, the complex part comes from M_f from the ξ field mentioned earlier. The relevant part comes from

$$\mathcal{L}_F \supset (M_F + i y_{\xi f} \xi) \bar{F} F, \tag{48}$$

$$M_f = M_F + i y_{\xi f} \xi = \tilde{M}_f e^{i\theta_f} \quad \theta_f = \tan^{-1} \left(\frac{y_{\xi f} \xi}{M_f} \right), \tag{49}$$

$$m_f = y^2 \frac{v_R v_L}{\tilde{M}_f} e^{-i\theta_f}, \tag{50}$$

⁸In Gu's work, a lepton asymmetry is generated through a heavy Majorana neutrino, which is then converted to a baryon asymmetry.

$$m_f^2 = y^4 \frac{v_R^2 v_L^2}{M_f^2} e^{-2i\theta_f}. \quad (51)$$

As noted, the ξ is an additional field whose only purpose is to provide us with the relevant complex phase. We have the freedom to choose the coupling of this scalar with the H_L such that the vev of $\langle \xi \rangle \neq 0$ around the transition temperature T_L . Now to analyze the phase transition we closely follow [133] and choose the kink profile for the scalars H_L, ξ given as

$$\xi(z) = \frac{\langle \xi \rangle}{2} \left[1 + \tanh\left(\frac{z}{L_w}\right) \right], \quad (52)$$

$$H_L(z) = \frac{\langle H_L \rangle}{2} \left[1 + \tanh\left(\frac{z}{L_w}\right) \right], \quad (53)$$

$$L_w^2 \simeq 2.7 \times \frac{1}{\kappa} \frac{v_L^2 + v_\xi^2}{v_L v_\xi} \left(1 + \frac{\kappa v_\xi^2}{4\lambda_H v_L^2} \right), \quad (54)$$

where L_w is the length of the bubble profile and κ is the effective coupling between the scalar fields ($\mathcal{L} \supset \frac{\kappa}{4} \xi^2 H_L^2$).

The source term S_f is given as [18]⁹

$$S_{fl} = -v_w \gamma_w h [(m_f^2 \theta')' Q_l^{8o} - (m_f^2)' m_f^2 \theta' Q_l^{9o}]. \quad (55)$$

The transport equations use the WKB approximation and treat the particle interactions as a semiclassical force. For this to be a valid approximation, the thickness of the bubble wall, L , should be greater than T^{-1} ,

$$A_i w_i' + (m_i^2)' B_i w_i = S_i + \delta C_i \quad (56)$$

where

$$A_i = \begin{pmatrix} -D_1 & 1 \\ -D_2 & R \end{pmatrix}, \quad B_i = \begin{pmatrix} v_w \gamma_w Q_1 & 0 \\ v_w \gamma_w Q_2 & \bar{R} \end{pmatrix}. \quad (57)$$

and all the integrals D_i, R , and $\bar{R}$ are given in [18].

The collision terms include Yukawa interactions, Yukawa interactions with an external gluon line, strong sphaleron rate, and W_L/W_R interactions. The temperature of the phase transition is much lower than the mass of the heavy fermions in our model, so the Yukawa interactions involving these and the SM fermions are kinematically forbidden. The relevant terms are shown below:

$$\delta C_1^t = \Gamma_{m_t}(\mu_t - \mu_{\bar{t}}) + \Gamma_{W_L}(\mu_t - \mu_b) + \tilde{\Gamma}_{SS}[\mu_i] + \tilde{\Gamma}_{W_L}[\mu_i], \quad (58)$$

TABLE III. The baryon asymmetry for BP2 and BP3 (BP1 is not phenomenologically acceptable so we do not consider it here). Each value of $\eta_B/\eta_{B_{\text{obs}}}$ should be multiplied by $\sin \arg(\xi)$.

	BP2	BP3
$\langle \xi \rangle$	8 TeV	18 TeV
κ	0.005	0.0001
$\eta_B/\eta_{B_{\text{obs}}}$	1.5	1.8

$$\delta C_1^{\bar{t}} = \Gamma_{m_t}(\mu_{\bar{t}} - \mu_t) + \Gamma_{W_R}(\mu_{\bar{t}} - \mu_{\bar{b}}) - \tilde{\Gamma}_{SS}[\mu_i] + \tilde{\Gamma}_{W_R}[\mu_i], \quad (59)$$

$$\delta C_1^b = \Gamma_{m_b}(\mu_b - \mu_{\bar{b}}) + \Gamma_{W_L}(\mu_b - \mu_t) + \tilde{\Gamma}_{SS}[\mu_i] + \tilde{\Gamma}_{W_L}[\mu_i], \quad (60)$$

$$\delta C_1^{\bar{b}} = \Gamma_{m_b}(\mu_{\bar{b}} - \mu_b) + \Gamma_{W_R}(\mu_{\bar{b}} - \mu_{\bar{t}}) - \tilde{\Gamma}_{SS}[\mu_i] + \tilde{\Gamma}_{W_R}[\mu_i], \quad (61)$$

$$\delta C_1^{h_L} = \Gamma_{h_L} \mu_{h_L}, \quad (62)$$

$$\delta C_1^{h_R} = \Gamma_{h_R} \mu_{h_R}. \quad (63)$$

To solve these coupled differential equations, we used relaxation methods that approximate the differential equations as stiff equations. After the chemical potential across the bubble wall is calculated using this method, the baryon asymmetry is calculated using the method of Ref. [134],

$$\eta_B = \frac{405 \Gamma_{\text{sph}}}{4\pi^2 v_w \gamma_w g_* T} \int dz \mu_{B_L} f_{\text{sph}} e^{-45 \Gamma_{\text{sph}} |z| / 4 v_w \gamma_w}, \quad (64)$$

$$\mu_{B_L} = \frac{1}{2} (1 + 4D_0^t) \mu_t + \frac{1}{2} (1 + 4D_0^b) \mu_b + 2D_0^{\bar{t}} \mu_{\bar{t}}, \quad (65)$$

$$h\sigma(\bar{D}_L D_R) + \text{H.c.} = h\sigma(\bar{D}_L D_R) - h\sigma(\bar{D}_R D_L) \\ = h\sigma(\bar{D}_L D_R - \bar{D}_R D_L),$$

$$\tilde{\Gamma}_{W_{R,L}}[\mu_i] = \Gamma_{W_{R,L}} ((1 + 9D_0^{\bar{t},t}) \mu_{\bar{t},t} \\ + (1 + 9D_0^{\bar{b},b}) \mu_{\bar{b},b}), \quad (66)$$

$$\tilde{\Gamma}_{SS}[\mu_i] = \Gamma_{SS} ((1 + 9D_0^t) \mu_t + (1 + 9D_0^b) \mu_b - (1 - 9D_0^{\bar{t}}) \mu_{\bar{t}}). \quad (67)$$

Our results for BP2 and BP3 are in Table III. The vev of ξ is large, but since it is a gauge singlet it will not affect the results of the previous section. For each point, the baryon asymmetry is in the acceptable range, depending on the phase of ξ .

For BP1, $v_L = v_R \neq 0$ earlier in time. Now, in this case the asymmetry will be created at that epoch T_{LR} (the ξ gets a vev at the same time creating the CP violation). At the

⁹The expression of Q_l^{8o} , Q_l^{9o} , and D_0 are mentioned in [18].

same epoch the sphaleron of both left and right will go out-of-equilibrium and hence the asymmetry created at that epoch will be frozen in. For this scenario one might need to consider right-handed sphalerons, but that is not necessary since BP1 is already excluded by the W_R mass.

VI. CONCLUSIONS

One of the most exciting discoveries of the past decade was the discovery of gravitational waves. Future experiments are planned that will provide an extraordinary increase in sensitivity. It had long been believed that the only experimental information about the electroweak phase transition was the single number giving the baryon asymmetry. Now, however, the gravitational wave spectrum might give much more information. A crucial input to understanding the spectrum is the temperature of the electroweak transition. We have considered a left-right model with two Higgs doublets and a singlet (although more complicated Higgs structures are possible) and shown that the transition temperature can be much higher than the current electroweak scale. This leads to a spectrum that peaks at higher frequencies than in previous studies, possibly in the range of the Cosmic Explorer or the

Einstein Telescope. We are not tied to a specific model and feel that this result might be generic. Certainly if a right-handed W is discovered at the LHC, a much more detailed analysis will be warranted. We also discuss electroweak baryogenesis in the model and show that the phenomenologically acceptable benchmark points can easily give the necessary baryon asymmetry if a complex singlet is added to provide a CP violating phase.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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