

From confinement to chaos in AdS/CFT via nonequilibrium local states

Dmitry S. Ageev^{*} and Vladimir A. Bykov[†]

*Department of Mathematical Methods for Quantum Technologies, Steklov Mathematical Institute of Russian Academy of Sciences, Gubkina str. 8, Moscow 119991, Russia
and Institute for Theoretical and Mathematical Physics, Lomonosov Moscow State University, 119991 Moscow, Russia*



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In this paper, we study excited states in anti-de Sitter (AdS) space prepared by local operator insertions of a massive scalar field, corresponding to local operator quenches in a free bulk scalar theory. Using the AdS/CFT correspondence, we compute the time evolution of boundary observables in the dual conformal field theory states. We then introduce a hard wall in AdS Poincare coordinates to impose an infrared cutoff (“hard wall”), thereby creating a confining deformation of the dual conformal field theory, and analyze the dynamics of excited states in this confining background. By comparing the evolution of boundary two-point correlation functions in the deformed theory with the statistics of Gaussian random matrix ensembles, we show that for sufficiently heavy operators, the spacing-ratio statistics of peaks in the temporal dynamics are closest to those of the Gaussian symplectic ensemble. Finally, we extend the analysis to the compact Bañados-Teitelboim-Zanelli black hole and to its hard-wall deformation, where we find qualitatively similar trends.

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I. INTRODUCTION

The study of complex quantum systems through multidisciplinary approaches has profoundly advanced our understanding of fundamental physical phenomena. Holography stands out as a powerful paradigm in this endeavor [1,2]. It bridges quantum field theory, gravity, and condensed matter physics [3,4] by mapping strongly coupled conformal field theories (CFTs) to weakly coupled gravitational theories in anti-de Sitter (AdS) spacetime.

This duality has shed light on intricate dynamics in out-of-equilibrium states. Quantum chaos manifests through rapid scrambling of information and thermalization, reminiscent of black hole physics [5–10]. Recent advancements underscore universal signatures, including exponential growth in out-of-time-order correlators [11] and spectral statistics akin to random-matrix theory [12–16].

These traits appear in holographic models and many-body quantum systems, such as Sachdev-Ye-Kitaev models, disordered spin chains [17–19], and spin superfluid currents [19]. See also holographic studies of quantum scars [20]. Essential to these explorations are local operator quenches [21–34].

They disrupt the system via the insertion of a localized operator, initiating nonequilibrium evolution. This framework probes relaxation timescales, entanglement propagation, and distinctions between chaotic and integrable regimes in finite-volume or gapped theories. In [35] we found that local quenches in massive free-field theories exhibit chaotic signatures in finite volume. In this paper we bring these results to a holographic setting, namely, to confining AdS backgrounds.

In high-energy physics, confining theories—like quantum chromodynamics (QCD) at low energies—display color confinement. Here, quarks and gluons form hadrons under a linear potential at large separations. This generates a mass gap and discrete spectrum, which curbs long-range correlations and modifies thermalization relative to gapless counterparts. Within holography, confinement corresponds to CFTs altered by relevant operators or infrared cutoffs [36–38]. These modifications produce dual gravitational setups with modified AdS boundaries or brane configurations that impose a characteristic scale. Such models replicate the confinement-deconfinement phase transition and facilitate investigations of chaotic dynamics in theories lacking scale invariance. The holographic model adopted here entails capping off AdS spacetime via a hard-wall cutoff [39,40]. This enforces Dirichlet boundary conditions on bulk fields to emulate confinement effects. Consequently, this discretizes Kaluza-Klein modes and induces a gapped spectrum on the boundary CFT.

In the present study, we undertake a comprehensive calculation of out-of-equilibrium dynamics induced by local scalar field quenches within both pure AdS and black hole

^{*}Contact author: ageev@mi-ras.ru

[†]Contact author: bykov.va20@physics.msu.ru

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backgrounds augmented with these confining cutoffs, employing exact analytical expressions for correlation functions in the bulk and on the boundary to track the system's evolution from initial perturbation to long-time behavior. Specifically, we derive two-point correlators after the quench using Green's function methods in deformed geometries, then focus on one-point functions of quadratic composite operators—such as ϕ^2 in the bulk or $\mathcal{O}^2$ on the boundary—to quantify the relaxation and oscillatory patterns, revealing how the quench propagates as wave fronts that reflect off the hard wall and interfere constructively or destructively over time. To probe chaotic signatures, we analyze peak spacing statistics in these temporal profiles extending the recent proposal of [15,16] from string amplitudes to the out-of-equilibrium setting (also see [41–45] for studies in this direction), comparing the distribution of spacing ratios to predictions from random matrix theory ensembles like the Gaussian Orthogonal Ensemble (GOE), Gaussian Unitary Ensemble (GUE), or Gaussian Symplectic Ensemble (GSE), thereby establishing quantitative measures of level repulsion and ergodicity. Our findings demonstrate a marked amplification of chaos with increasing scalar masses (corresponding to larger conformal dimensions Δ) or when the hard wall is positioned closer to the boundary, which intensifies the infrared truncation and enhances nonlinear interactions among modes, leading to stronger deviations from Poissonian statistics toward Wigner-Dyson distributions indicative of quantum chaos.

These results not only elucidate the interplay between confinement, mass gaps, and chaos in holographic settings but also provide benchmarks for comparing with lattice simulations of confining gauge theories or experimental realizations in quantum simulators, potentially guiding future explorations of thermalization in isolated quantum systems with tunable gaps.

The paper is structured as follows: Sec. II elaborates on local quenches in pure AdS, deriving correlation functions and boundary observables; Sec. III generalizes to capped-off AdS, scrutinizing chaotic traits through spectral statistics; Sec. IV investigates quenches in Bañados-Teitelboim-Zanelli (BTZ) black holes, encompassing both undeformed and deformed cases, alongside numerical assessments of dynamics; the appendixes furnish detailed derivations of Green's functions for the modified geometries.

II. LOCAL QUENCHES IN PURE ADS

We consider a massive scalar field theory with the action

$$S = \frac{1}{2} \int d^{d+1}x \sqrt{|g|} [g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2] \quad (1)$$

in AdS_{d+1} geometry, and for simplicity let us start with it in the Poincare coordinates. The metric of AdS_{d+1} in Poincare coordinates (Euclidean version) dual to d -dimensional CFT is given by

$$ds^2 = \frac{L^2}{z^2} (d\tau^2 + dx^2 + dz^2), \quad (2)$$

where $x = (x^1, \dots, x^{d-1})$, while the Lorentzian metric is obtained by the Wick rotation $\tau \rightarrow it$. The two-point correlation function of a massive scalar field in this geometry has the form [46]

$$\begin{aligned} \langle \phi(\tau_1, x_1, z_1) \phi(\tau_2, x_2, z_2) \rangle &= \frac{\Gamma(\Delta)}{2L^{d-1} \pi^{\frac{d}{2}} \Gamma(\Delta - \frac{d}{2} + 1)} \\ &\times \left(\frac{\sigma_{12}}{2} \right)^\Delta {}_2F_1(a, b; c; \sigma_{12}^2), \end{aligned} \quad (3)$$

where Δ ; parameters a, b, c ; and the geodesic distance σ_{12} are given by

$$\Delta = \frac{d}{2} + \nu, \quad \nu = \sqrt{m^2 L^2 + \frac{d^2}{4}}, \quad (4)$$

$$a = \frac{\Delta}{2}, \quad b = \frac{\Delta + 1}{2}, \quad c = \Delta - \frac{d}{2} + 1, \quad (5)$$

$$\sigma_{12} = \frac{2z_1 z_2}{z_1^2 + z_2^2 + (x_1 - x_2)^2 + (\tau_1 - \tau_2)^2}. \quad (6)$$

Following [28], we define local quench states $|\Psi(\tau)\rangle$ given by the insertion of the operator O at spacetime point (τ_q, x_q, z_q) :

$$|\Psi(\tau)\rangle = \mathcal{N}_0 \cdot e^{-H(\tau-\tau_q)} \cdot e^{-\epsilon H} O(\tau_q, x_q, z_q) |0\rangle, \quad (7)$$

so that the expectation value of a local operator A at Euclidean time τ is given by

$$\begin{aligned} \langle A(\tau, x, z) \rangle_O &= \frac{\langle 0 | O(-\epsilon + \tau_q, x_q, z_q) A(\tau, x, z) O(\epsilon + \tau_q, x_q, z_q) | 0 \rangle}{\langle 0 | O(-\epsilon + \tau_q, x_q, z_q) O(\epsilon + \tau_q, x_q, z_q) | 0 \rangle}, \end{aligned} \quad (8)$$

where ϵ is a regulator controlling the damping of divergent UV modes. Here and below, we continue to real time via $\tau \rightarrow it$ (for details, see Appendix A of [34]). The boundary dynamics corresponding to such a state can be defined via the Banks-Douglas-Horowitz-Martinez (BDHM) prescription [47] (see [48] for AdS/CFT dictionary out of equilibrium) by pushing the correlation function in the bulk to the boundary via

$$\langle \mathcal{O}(\tau_1, x_1) \mathcal{O}(\tau_2, x_2) \rangle = \lim_{z \rightarrow 0} z^{-2\Delta} \langle \phi(\tau_1, x_1, z) \phi(\tau_2, x_2, z) \rangle_O. \quad (9)$$

A natural candidate for the quench operator is simply field operator ϕ , which is dual to the primary field $\mathcal{O}$ in d -dimensional CFT. Then bulk two-point correlator of ϕ after local quench is given by

$$\langle \phi(\tau_1, x_1, z_1) \phi(\tau_2, x_2, z_2) \rangle_\phi = \frac{\langle 0 | \phi(-\epsilon + \tau_q, x_q, z_q) \phi(\tau_1, x_1, z_1) \phi(\tau_2, x_2, z_2) \phi(\epsilon + \tau_q, x_q, z_q) | 0 \rangle}{\langle 0 | \phi(-\epsilon + \tau_q, x_q, z_q) \phi(\epsilon + \tau_q, x_q, z_q) | 0 \rangle}. \quad (10)$$

A comment is in order. We are going to perform calculations in free scalar field theory in AdS in the context of an excited state which effectively reduces to four-point correlation function in the bulk. At leading order in large N , correlators of the single-trace operators dual to our free bulk scalar are generalized free; the four-point function that appears in the normalized quenched two-point function is therefore dominated by its disconnected part, with the subleading connected pieces of order $\mathcal{O}(1/N^2)$ neglected.

Thus, we consider the free theory and express the correlation function after the quench in the bulk explicitly as the sum of three Wick contractions:

$$\begin{aligned} \langle \phi(\tau_1, x_1, z_1) \phi(\tau_2, x_2, z_2) \rangle_\phi &= \frac{\Gamma(\Delta)}{2\pi^{\frac{d}{2}} L^{d-1} \Gamma(\Delta - \frac{d}{2} + 1)} \left(\left(\frac{\sigma_{12}}{2} \right)^\Delta {}_2F_1(a, b; c; \sigma_{12}^2) \right. \\ &\quad + \left(\frac{\sigma_1^+ \sigma_2^-}{2\sigma_q} \right)^\Delta \frac{{}_2F_1(a, b; c; (\sigma_1^+)^2) \cdot {}_2F_1(a, b; c; (\sigma_2^-)^2)}{{}_2F_1(a, b; c; \sigma_q^2)} \\ &\quad \left. + \left(\frac{\sigma_1^- \sigma_2^+}{2\sigma_q} \right)^\Delta \frac{{}_2F_1(a, b; c; (\sigma_1^-)^2) \cdot {}_2F_1(a, b; c; (\sigma_2^+)^2)}{{}_2F_1(a, b; c; \sigma_q^2)} \right), \end{aligned} \quad (11)$$

the first of which is the two-point correlation function without any quench, and the others control the dynamics in the bulk after the quench. Here σ_{12} is given by (6), while σ_q , $\sigma_1^\pm$, and $\sigma_2^\pm$ are defined as

$$\sigma_q = \frac{z_q^2}{z_q^2 + 2\epsilon^2}, \quad (12)$$

$$\sigma_{1,2}^\pm = \frac{2z_{1,2}z_q}{z_{1,2}^2 + z_q^2 + (\tau_{1,2} \pm \epsilon - \tau_q)^2 + (x_{1,2} - x_q)^2}. \quad (13)$$

After applying the BDHM rule we are left with the boundary correlation function corresponding to the quantum out-of-equilibrium bulk quench state

$$\langle \mathcal{O}(\tau_1, x_1) \mathcal{O}(\tau_2, x_2) \rangle_\phi = \frac{\Gamma(\Delta)}{2\pi^{\frac{d}{2}} L^{d-1} \Gamma(\Delta - \frac{d}{2} + 1)} \left(\frac{1}{[(\tau_1 - \tau_2)^2 + (x_1 - x_2)^2]^\Delta} + \frac{(\eta_1^+ \eta_2^-)^\Delta + (\eta_1^- \eta_2^+)^\Delta}{(2\sigma_q)^\Delta \cdot {}_2F_1(a, b; c; \sigma_q^2)} \right), \quad (14)$$

where the first term is the equilibrium two-point correlation function of primary operators, and $\eta_{1,2}^\pm$ is given by

$$\eta_{1,2}^\pm = \frac{2z_q}{z_q^2 + (\tau_{1,2} \pm \epsilon - \tau_q)^2 + (x_{1,2} - x_q)^2}. \quad (15)$$

$$\begin{aligned} \langle \phi^2(t, x, z) \rangle_\phi &= \frac{\Gamma(\Delta)}{\pi^{\frac{d}{2}} L^{d-1} \Gamma(\Delta - \frac{d}{2} + 1)} \left(\frac{\sigma^+ \sigma^-}{2\sigma_q} \right)^\Delta \\ &\quad \times \frac{{}_2F_1(a, b; c; (\sigma^+)^2) \cdot {}_2F_1(a, b; c; (\sigma^-)^2)}{{}_2F_1(a, b; c; \sigma_q^2)}, \end{aligned} \quad (16)$$

We focus on the dynamics of the one-point correlator of the composite operators $\phi^2(\tau, x, z)$ and $\langle \mathcal{O}^2(\tau, x) \rangle_\phi$, to avoid issues related to analytic continuation. Often in such problems, the dynamics of an equal-time two-point correlation function is studied, and we checked that it resembles ϕ^2 dynamics. After taking the coincident points limit of (11) and subtracting the divergences in a minimal way, we obtain

where we analytically continue $\tau \rightarrow it$ and

$$\sigma^\pm = \frac{2zz_q}{z^2 + z_q^2 + (it \pm \epsilon - it_q)^2 + (x - x_q)^2}. \quad (17)$$

Doing the same for (14) we get

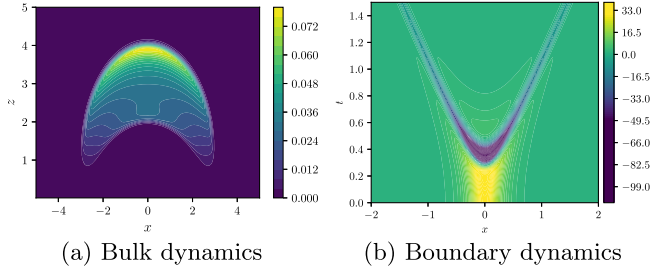


FIG. 1. (a) Bulk dynamics for the condensate $\langle \phi^2 \rangle$ for the mass parameter corresponding to $\Delta = 5$, $m = 3.87$ at the moment of time $t = 3$ with the quench point $z_q = 1$. (b) Boundary dynamics of $\mathcal{O}^2$ for the mass parameter $\Delta = 2$, $m = 0$ with the quench point $z_q = 0.35$. Parameters $d = 2$, $x_q = t_q = 0$, $\epsilon = 0.1$, $L = 1$ are fixed for all figures.

$$\langle \mathcal{O}^2(t, x) \rangle_\phi = \frac{(\eta^+ \eta^-)^\Delta}{\pi^{\frac{d}{2}} L (2\sigma_q)^\Delta \cdot {}_2F_1(a, b; c; \sigma_q^2)}. \quad (18)$$

In the bulk we observe a localized perturbation along the geodesics (i.e., circles) that propagates into the bulk (Fig. 1).

On the boundary it corresponds to forming a compact pulse that rapidly splits into two localized configurations of energy counterpropagating from the quench point, i.e., in some sense it resembles the canonical local quench in the flat space. However, there are different features making a difference with flat CFT quench, namely

- (1) The quench state in AdS depends less sensitively on the mass than in flat space. For all masses, in AdS we first observe a fast localized increase of correlations near the quench point, followed by a rapid decay into two compact configurations moving away from x_q , while in flat space for zero mass the initial configuration splits and propagates freely (increasing the mass introduces diffusion and damping). Increasing the mass in the AdS quench primarily adds oscillations near the peaks of configurations moving away from the quench point.
- (2) The parameters z_q and ϵ control how deep in the infrared the quench is made and how smeared it is. Mostly, this is controlled by z_q . Because the perturbation takes time to propagate from the quench point into the bulk, z_q sets how quickly the initial compact distribution stops growing and decays into two pulses.

III. CAPPING OFF ADS AND CHAOS IN CONFINING THEORY

We now deform the initial geometry of Poincare AdS by imposing a Dirichlet boundary condition on the scalar field at the plane $z = z_0$ in the bulk, introducing a hard-wall IR cutoff. This deforms the IR degrees of freedom in such a way that their dynamics is strongly restricted—in other words, this theory acquires a confining mechanism [39].

Let us calculate a two-point correlation function for this theory. For convenience, define boundary coordinates $y = (\tau, x)$. For the free massive scalar field, the two-point correlator $\langle \phi(y, z) \phi(y', z') \rangle = G(y, z; y', z')$ satisfies the same equation of motion, but the boundary conditions are

$$G(y, z_0; y', z') = G(y, z; y', z_0) = 0, \quad (19)$$

$$G(y, 0; y', z') = G(y, z; y', 0) = 0. \quad (20)$$

The details of the calculation can be found in Appendix A. The solution for $G(y, z; y', z')$ has the form

$$G(y, z; y', z') = \sum_n \int \frac{d^d k}{(2\pi)^d} e^{ik(y-y')} \times \frac{z^{\frac{d}{2}} z'^{\frac{d}{2}} J_\nu(\alpha_n z) J_\nu(\alpha_n z')}{N_n} G_n(k), \quad (21)$$

where α_n , N_n , $G_n(k)$ are defined as follows:

$$J_\nu(\alpha_n z_0) = 0, \quad (22)$$

$$N_n = \int_0^{z_0} dz z J_\nu^2(\alpha_n z) = \frac{z_0^2}{2} J_{\nu+1}^2(\alpha_n z_0), \quad (23)$$

$$G_n(k) = \frac{1}{L^{d-1} (k^2 + \alpha_n^2)}. \quad (24)$$

It is straightforward to see that (21) satisfies the boundary conditions (19). Performing the integration over momentum k , we obtain the two-point correlation function expressed as a sum over integers

$$G(\tau, x, z; \tau', x', z') = \frac{z^{\frac{d}{2}} z'^{\frac{d}{2}}}{(2\pi)^{\frac{d}{2}} L^{d-1}} \sum_n \frac{J_\nu(\alpha_n z) J_\nu(\alpha_n z')}{N_n} \times \left(\frac{\alpha_n}{r} \right)^{\frac{d}{2}-1} K_{\frac{d}{2}-1}(\alpha_n r), \quad (25)$$

where $r = \sqrt{(\tau - \tau')^2 + (x - x')^2}$, (see Appendix A for details). Having obtained Green's function for the deformed geometry, we proceed to define quench dynamics as we did earlier.

To obtain the deformed boundary correlation function, we first apply the BDHM rule for the deformed bulk correlator (25):

$$G_B(\tau, x; \tau', x') = \frac{1}{(2\pi)^{\frac{d}{2}} L^{d-1}} \sum_n \frac{\alpha_n^{2\nu}}{\Gamma^2(\nu + 1) 2^{2\nu} N_n} \times \left(\frac{\alpha_n}{r} \right)^{\frac{d}{2}-1} K_{\frac{d}{2}-1}(\alpha_n r). \quad (26)$$

To simplify notation for the deformed boundary quench dynamics, let us define $\tilde{G}_B$ as

$$\tilde{G}_B(\tau, x; \tau', x', z) = \frac{1}{(2\pi)^{\frac{d}{2}} L^{d-1}} \sum_n \frac{z^{\frac{d}{2}} J_\nu(\alpha_n z) \alpha'_n}{\Gamma(\nu+1) 2^\nu N_n} \times \left(\frac{\alpha_n}{r}\right)^{\frac{d}{2}-1} K_{\frac{d}{2}-1}(\alpha_n r), \quad (27)$$

where the z dependence corresponds to inserting the quench operator at a bulk point. Then the deformed boundary two-point correlation function after the local quench is given by

$$\begin{aligned} \langle \mathcal{O}(\tau_1, x_1) \mathcal{O}(\tau_2, x_2) \rangle_{D\phi} &= G_B(\tau_1, x_1; \tau_2, x_2) \\ &+ \frac{\tilde{G}_B(\tau_1, x_1; -\epsilon + \tau_q, x_q, z_q) \tilde{G}_B(\tau_2, x_2; \epsilon + \tau_q, x_q, z_q)}{G(-\epsilon + \tau_q, x_q, z_q; \epsilon + \tau_q, x_q, z_q)} \\ &+ \frac{\tilde{G}_B(\tau_2, x_2; -\epsilon + \tau_q, x_q, z_q) \tilde{G}_B(\tau_1, x_1; \epsilon + \tau_q, x_q, z_q)}{G(-\epsilon + \tau_q, x_q, z_q; \epsilon + \tau_q, x_q, z_q)}. \end{aligned} \quad (28)$$

The finite part of the boundary one-point correlator (after the Wick rotation $\tau \rightarrow it$) after the quench is written as

$$\begin{aligned} \langle \mathcal{O}^2(t, x) \rangle_{D\phi} &= \frac{2\tilde{G}_B(it, x; -\epsilon + it_q, x_q, z_q) \tilde{G}_B(it, x; \epsilon + it_q, x_q, z_q)}{G(-\epsilon + it_q, x_q, z_q; \epsilon + it_q, x_q, z_q)}. \end{aligned} \quad (29)$$

Dynamics of one-point correlation functions in the bulk and on the boundary are illustrated in Figs. 2 and 3, respectively. To analyze the chaotic behavior of (29) we examine the statistical properties of peak spacing ratios in correlation functions following a local quench. This approach uses random matrix theory as a framework to analyze chaotic quantum dynamics. The probability density function (PDF) of spacing ratio r_n , defined as

$$r_n = \frac{\delta_{n+1}}{\delta_n}, \quad (30)$$

is compared with those of Gaussian ensembles of random matrices (GUE, GOE and GSE), where $\delta_n = \lambda_{n+1} - \lambda_n$ is the spacing of peaks λ_n . The statistic of r_n is less sensitive to folding/unfolding procedures which are intrinsic to level spacing analyses [49]. We compare with the standard Gaussian ensemble distributions of ratios, which are given by [50]

$$f_\beta(r) = \frac{3^{\frac{3+\beta}{2}} \Gamma(1 + \frac{\beta}{2})^2}{2\pi \Gamma(1 + \beta)} \frac{(r + r^2)^\beta}{(1 + r + r^2)^{1+\frac{3+\beta}{2}}}, \quad (31)$$

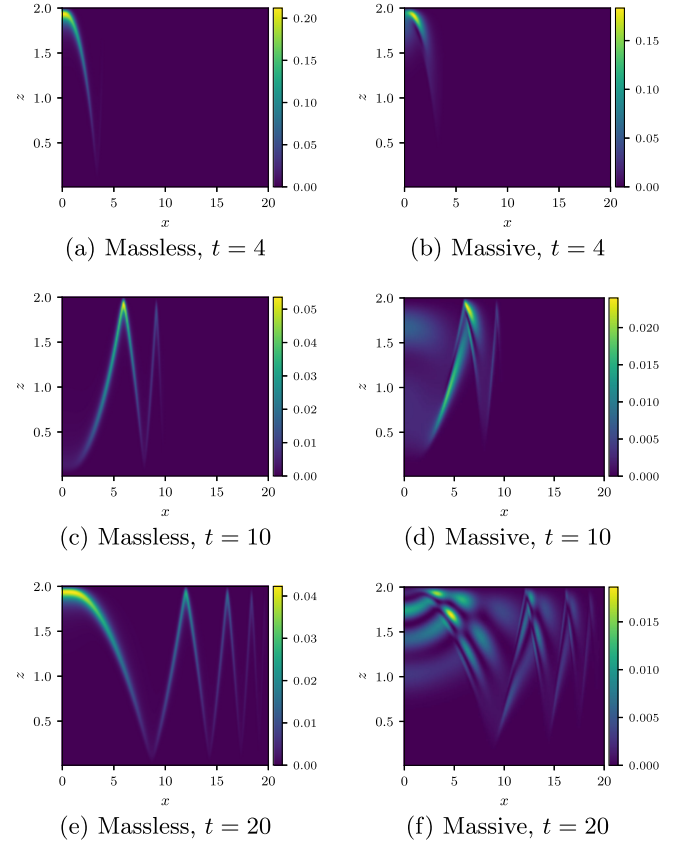


FIG. 2. Bulk dynamics for the condensate $\langle \phi^2 \rangle$ for the mass parameters corresponding $\nu = 1$, $m = 0$ (a), (c), (e) and $\nu = 7$, $m = 6.93$ (b), (d), (f). Parameters $d = 2$, $z_0 = 2$, $x_q = t_q = 0$, $z_q = 1.99$, $\epsilon = 0.1$, $L = 1$, $N_{\max} = 150$ are fixed for all figures.

with GOE corresponding to $\beta = 1$, GUE to $\beta = 2$, and GSE to $\beta = 4$.

In Fig. 4 we show the distribution of peak spacing ratios for $\mathcal{O}^2(\tau, x)_{D\phi}$ for different masses in a $d = 2$ dimensional theory. For large masses, we observe good agreement with the GSE distribution (green line), whereas for small masses we observe the absence of such dynamics. A chiral cousin of the symplectic ensemble is known to be closely related to

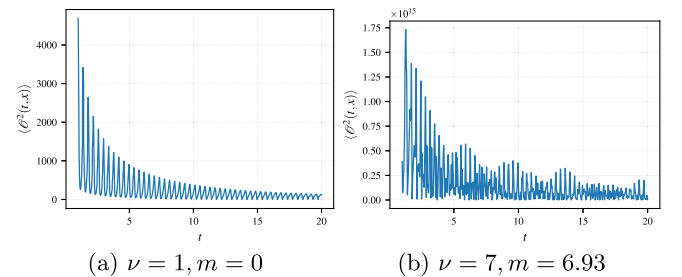


FIG. 3. Boundary dynamics of primary $\mathcal{O}^2$ at point $x = 0$. (a) $\nu = 1$, $m = 0$; (b) $\nu = 7$, $m = 6.93$. Parameters $d = 2$, $z_0 = 0.2$, $x_q = t_q = 0$, $z_q = 1.999$, $\epsilon = 0.1$, $L = 1$, $N = 100$ are fixed for all figures, where N is the number of terms taken in the sum over n .

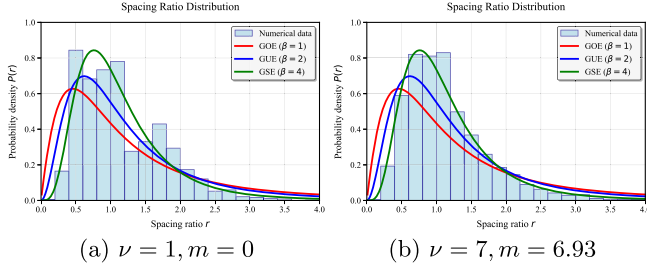


FIG. 4. Distributions of peak spacing ratios for boundary dynamics of primary $\mathcal{O}^2$ for different masses: (a) massless $\nu = 1$, (b) large mass $\nu = 7$. Parameters $d = 2$, $z_0 = 0.2$, $z_q = 0.199$, $x_q = t_q = 0$, $\varepsilon = 0.1$, $L = 1$, $N = 300$ are fixed for all figures.

the spectrum of the Dirac operator in adjoint QCD. Let us highlight important features and points concerning this:

- (1) For small masses (i.e., close to $\nu = 1$), we see that no desired PDF close enough to any ensemble is present. For larger masses, it starts to converge to desirable distribution.
- (2) A characteristic feature of the chaotic dynamics is the increase in chaotic behavior as the hard wall approaches the conformal boundary $z_0 \rightarrow 0$. This corresponds to the truncation of infrared degrees of freedom and strong deformation of the boundary conformal field theory. The more confining the theory, the more pronounced the chaotic behavior. By chaotic behavior we mean that it tends to get closer to GSE.
- (3) If the quench point is closer to the wall, chaos is present in a more pronounced form, the fit improves, and smaller masses and fewer series terms are needed to approach the GSE PDF.
- (4) An interesting fact is that for large masses and a quench point sufficiently close to the wall, a good fit to the chaotic PDFs can be obtained with as few as $N = 5$ terms of the series.
- (5) A larger ν (and thus larger Δ) corresponds to a more irrelevant operator in the CFT, which suppresses low-energy fluctuations and enhances nonlinear interactions and strong coupling effects.

Now it is in order to discuss quantum models which show similar behavior after turning on a truncation of degrees of freedom, Hilbert space, or spatial confinement, in order to understand the general mechanism leading to Random Matrix Theory (RMT) statistics in spacing-ratio behavior. We consider an initial quantum system (a d -dimensional CFT, for example, as in our case), and the introduction of a bulk cutoff in AdS space corresponds to some restriction imposed on the Hilbert space or on the infrared degrees of freedom of the starting system. One might think, in general, that the existence of constraints in a quantum system could move the system closer to the integrable sector due to the association between charges and constraints. However, this is not the case: there exist

many examples of systems where such restrictions (sometimes due to the existence of a sector in the Hilbert space associated with some symmetry) push the system toward the chaotic region. Let us briefly list cases where stronger restrictions in the system (in a broad sense) lead us to chaos, including RMT behavior:

- (1) A one-dimensional chain of Rydberg atoms with a nearest-neighbor blockade described by the PXP Hamiltonian $H_{\text{PXP}} = \sum_i P_{i-1} \sigma_i^x P_{i+1}$, with P_i being a projection operator onto the ground state and where the local constraint forbids two adjacent excitations [51–53]. This projects the full 2^L spin Hilbert space onto a much smaller “Fibonacci” subspace of allowed configurations, yet in appropriate symmetry sectors the level statistics are Wigner-Dyson (GOE), and typical eigenstates obey Eigenstate Thermalization Hypothesis (ETH). Thus a strong kinematic restriction of the Hilbert space is precisely what turns a trivial integrable model into a largely quantum-chaotic one.
- (2) A similar phenomenon is seen in Hamiltonian $\text{SU}(2)$ lattice gauge theory [54,55], where Gauss’s law projects the naive tensor-product link space onto a much smaller gauge-invariant physical Hilbert space, and one often further truncates the local electric flux to $j \leq j_{\text{max}}$ (e.g., $j_{\text{max}} = 1/2$). Despite these strong constraints and truncations, numerical studies show that, upon additionally fixing global quantum numbers (such as lattice momentum and reflection parity), the spectrum within a single sector exhibits clear GOE Wigner-Dyson level statistics, indicating nonintegrability and quantum chaos. Thus, here again a heavily restricted Hilbert space—defined by local constraints and cutoffs—enhances the visibility of chaotic spectral correlations.
- (3) This is exactly what we observed in a model of local quench in a large mass Klein-Gordon field on the cylinder—only the addition of confining volume in this nonequilibrium process turns it to be chaotic in the sense of level spacing ratio counting [35].
- (4) In [56] the authors showed that for a generic local Hamiltonian built from projectors onto fusion channels (the pinned Fibonacci anyon chain which is relevant for quantum matter and gauge theories), this highly constrained space still exhibits standard quantum chaos and ETH: the influence of local perturbations rapidly decays as in our case (a notion of locality exists), and within fixed translation/topological sectors the level statistics and eigenstate matrix elements are indistinguishable from those of a generic chaotic spin chain.
- (5) In studies of bosonization [57] within the Fermi liquids context, one often restricts to density modes with momenta $|q| \leq \Lambda$, defining a Λ -restricted Hilbert space of collective excitations. They show that, within this truncated space, low-energy levels

exhibit near-Poisson statistics, while at higher energies the spacing statistics cross over to GOE-like Wigner-Dyson behavior as interactions mix more modes. Thus even in a cutoff, restricted space, random-matrix universality emerges in appropriate parts of the spectrum, much like our finding, for a sufficiently strong IR wall and heavy bulk fields.

- (6) So far from previous items one can see that constraints in general and in Hilbert space do not contradict the existence of chaotic and RMT behavior; moreover, they often support it. More examples include subsystem/dipole symmetries [58], different versions of anyon spin chains [59], and nuclear shell models in a truncated valence space [60].
- (7) Last but definitely not least, turning on stronger confinement in coordinate space may lead us to chaotic behavior. This is exemplified by Sinai billiards: adding a circular wall inside a rectangular billiard reduces the available volume for the particle and yields stronger chaos, whereas enlarging the free volume yields more “integrable” behavior (see for example [61,62]). In some sense, a billiard is the best example of a “confining” theory leading to chaotic patterns—indeed a free particle is not chaotic, while a system consisting of a particle plus a specially tuned wall is. In general, one can state that at least “trajectories” (for example for quarks, quasiparticles, in a broad sense depending on the model we are investigating) should not run too far away from each other to create a complicated pattern giving us chaotic behavior in any reasonable way. We note that the thermal states in billiard systems and their chaotic properties (as well as an interpretation of results in our BTZ thermal setup) are more subtle; we would like to leave their interpretation for future research. It seems that in this case we may obtain complicated interplay between chaotic and thermal properties.

In summary, for many quantum systems including spin chains, anyon models, gauge theories models, and many others, RMT behavior often becomes clear only after restricting to a smaller symmetry sector; the larger “unresolved” Hilbert space can look less chaotic. The same logic applies for restriction in coordinate space—billiard systems are confined from the outset.

While the relation between a confining mechanism (in a broad sense) and chaos discussed here is far from understanding and could be model dependent let us outline some statements about it to finalize this discussion. The following conclusion albeit being hand waving is based on the models we just enumerated and calculations in this paper.

A restriction (hard IR cutoff, sector projection, kinematic/gauge constraint) acts in three model-independent ways. First, it increases mean spacing $\bar{\Delta}$ of the relevant spectrum, so the Heisenberg time $t_H = 2\pi/\bar{\Delta}$ decreases, bringing the system into the universal (RMT) time window sooner.

Second, it desymmetrizes the spectrum by effectively projecting onto fewer dynamical sectors, so level repulsion is not washed out by any kind of superposition. Finally, it can alter the periodic-orbit content if such is present in our theory (or configuration-graph connectivity), favoring the universal sine-kernel correlations underlying Wigner-Dyson (WD) statistics. These effects explain why tightening confinement can make WD/GSE features appear earlier and more cleanly in our one-point time series, even though the total microscopic degrees of freedom are smaller.

IV. TOWARD FINITE TEMPERATURE REGIMES: QUENCHES IN CAPPED BTZ

After we demonstrated the matching between level spacing ratios in temporal dynamics and GSE ensemble distribution in the previous section for Poincare AdS let us move to a more complicated case. We now consider the massive scalar field theory with the action (1) in a BTZ black hole background with metric

$$ds^2 = \frac{r^2 - r_h^2}{L^2} d\tau^2 + \frac{L^2}{r^2 - r_h^2} dr^2 + r^2 d\varphi^2, \quad r > r_h, \quad (32)$$

which is dual to 2D CFT at finite temperature $T = r_h/(2\pi L^2)$. Here we study the version of the BTZ black hole which has a periodic spatial coordinate, and in general it is more intricate and complicated from the physical viewpoint than its planar cousin because it is dual to a theory at finite volume and temperature simultaneously. The BTZ black hole is locally equivalent to AdS_3 space, so the two-point correlation function of the massive scalar field has the same local form as in AdS_3 space. Globally, the BTZ black hole is a discrete quotient of AdS_3 with the equivalence relation imposed on the angular variable ($\varphi \sim \varphi + 2\pi k$), where k is an integer.

A fully general study of boundary correlation functions and their statistics, in the spirit of the previous section, is a much more technically complicated task due to the number of free parameters and the large mode sums required for numerical stability, even in the compact case of BTZ with confining cutoff. Curiously, the planar black hole turns out to be an even more challenging object to study numerically when introducing an IR deformation. Here we demonstrate behavior qualitatively similar to that in the previous section and describe the general trends in the resulting distributions. We emphasize that this is not a full-scale study of chaos in thermal “confining” theories and leave a more detailed study to future research.

Hence a two-point correlation function in BTZ is obtained from the one of AdS_3 using the method of images:

$$G(\tau_1, \varphi_1, r_1; \tau_2, \varphi_2, r_2) = \sum_n \frac{1}{2L\pi} \left(\frac{\sigma_n^{12}}{2} \right)^\Delta \times {}_2F_1 \left(\frac{\Delta}{2}, \frac{\Delta+1}{2}; \Delta; (\sigma_n^{12})^2 \right), \quad (33)$$

where σ_n^{12} is the geodesic distance corresponding to the metric (32) and given by

$$\sigma_n^{12} = \left(\frac{r_1 r_2}{r_h^2} \cosh \left[\frac{r_h(\varphi_1 - \varphi_2 + 2\pi n)}{L} \right] - \sqrt{\left(\frac{r_1^2}{r_h^2} - 1 \right) \left(\frac{r_2^2}{r_h^2} - 1 \right)} \cos \left[\frac{r_h(\tau_1 - \tau_2)}{L^2} \right] \right)^{-1}. \quad (34)$$

Following the same logic as in the previous section, we obtain the bulk correlator after a local quench in a compact BTZ black hole as

$$\langle \phi^2(t, \varphi, r) \rangle_\phi = \frac{2G(-\epsilon + it_q, \varphi_q, r_q; it, \varphi, r) G(\epsilon + it_q, \varphi_q, r_q; it, \varphi, r)}{G(-\epsilon + it_q, \varphi_q, r_q; \epsilon + it_q, \varphi_q, r_q)}. \quad (35)$$

To define the boundary one-point observable, we set

$$G_B(\tau_1, \varphi_1; \tau_2, \varphi_2, r) = \sum_n \frac{1}{2L\pi} \left(\frac{\xi_n^{12}}{2} \right)^\Delta, \quad (36)$$

where ξ_n^{12}

$$\xi_n^{12} = \left(\frac{r}{r_h^2} \cosh \left[\frac{r_h(\varphi_1 - \varphi_2 + 2\pi n)}{L} \right] - \frac{1}{r_h} \sqrt{\frac{r^2}{r_h^2} - 1} \cos \left[\frac{r_h(\tau_1 - \tau_2)}{L^2} \right] \right)^{-1} \quad (37)$$

In terms of G_B , we define (after subtracting/extracting the divergences) boundary one-point function as

$$\langle \mathcal{O}^2(t, \varphi) \rangle_\phi = \frac{2G_B(it, \varphi; -\epsilon + it_q, \varphi_q, r_q) G_B(it, \varphi; \epsilon + it_q, \varphi_q, r_q)}{G(-\epsilon + it_q, \varphi_q, z_q; \epsilon + it_q, \varphi_q, z_q)}. \quad (38)$$

We now deform the BTZ black hole geometry by imposing a Dirichlet boundary condition on a brick-wall surface and compute the required two-point correlation function G_D . The technicalities are the same as in the previous section (for more details of the derivation, see Appendix B). The solution for $G_D(\tau, \varphi, r; \tau', \varphi', r')$ has the

form

$$G_D(\tau, \varphi, r; \tau', \varphi', r') = \sum_n \sum_J \int \frac{d\lambda}{(2\pi)^2} e^{i\lambda(\tau - \tau')} \times e^{iJ(\varphi - \varphi')} \frac{2L^2}{r_h \sqrt{rr'}} \frac{F_{n,J}(r) F_{n,J}(r')}{(\lambda^2 + \omega_{n,J}^2) N_{n,J}}, \quad (39)$$

where J is an integer and $F_{n,J}(r)$, $N_{n,J}$ are defined as follows:

$$F_{n,J}(r) = \left(1 - \frac{r_h^2}{r^2} \right)^{\alpha_n} \left(\frac{r_h^2}{r^2} \right)^\beta {}_2F_1 \left(a_{n,J}, b_{n,J}; c; \frac{r_h^2}{r^2} \right), \quad (40)$$

$$N_{n,J} = \frac{L^4}{r_h^2} \int_0^{z_0} (1-z)^{2\alpha_{n,J}-1} z^\nu {}_2F_1^2(a_{n,J}, b_{n,J}; c; z) dz, \quad (41)$$

where $z = r_h^2/r^2$. Parameters a_n , b_n , c , α_n , and β are given by

$$a_n, \quad b_n = \frac{1}{2} \left(1 + \nu + \frac{iL^2 \omega_n}{r_h} \mp \frac{iLJ}{r_h} \right), \quad c = \Delta, \quad (42)$$

$$\alpha_n = \frac{iL^2 \omega_n}{2r_h}, \quad \beta = \frac{1}{4} + \frac{\nu}{2}, \quad (43)$$

where ω_n are given as solutions of the equation

$${}_2F_1 \left(a_n, b_n; c; \frac{r_h^2}{r_0^2} \right) = 0. \quad (44)$$

Note that $\omega_{n,J}$ depends on J . Performing the integration over λ we get

$$G_D(\tau, \varphi, r; \tau', \varphi', r') = \sum_n \sum_J \frac{e^{-\omega_{n,J}|\tau - \tau'|}}{4\pi \omega_{n,J}} e^{iJ(\varphi - \varphi')} \times \frac{2L^2}{r_h \sqrt{rr'}} \frac{F_{n,J}(r) F_{n,J}(r')}{N_{n,J}}. \quad (45)$$

Having obtained Green's function for the deformed geometry, we proceed as before and define

$$G_{DB}(\tau, \varphi; \tau', \varphi') = \frac{L^2 r_h^{4\beta-1}}{2\pi} \sum_n \sum_J \frac{e^{-\omega_{n,J}|\tau - \tau'|}}{\omega_{n,J} N_{n,J}} e^{iJ(\varphi - \varphi')}, \quad (46)$$

as well as

$$\tilde{G}_{DB}(\tau, \varphi; \tau', \varphi', r) = \frac{L^2 r_h^{2\beta-1}}{2\pi} \sum_n \sum_J \frac{e^{-\omega_{n,J}|\tau - \tau'|}}{4\pi \omega_{n,J}} \times e^{iJ(\varphi - \varphi')} \frac{1}{\sqrt{r}} \frac{F_{n,J}(r)}{N_{n,J}}, \quad (47)$$

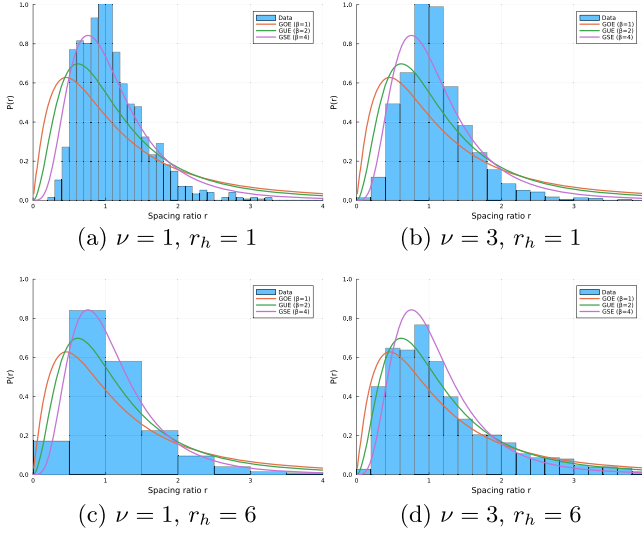


FIG. 5. Distributions of peak spacing ratios for boundary dynamics of primary $\mathcal{O}^2$ for different mass (massless $\nu = 1$ and large mass $\nu = 3$) parameters in deformed BTZ geometry. Panels (a)–(b) correspond to $r_h = 1$ and (c)–(d)— $r_h = 6$ (high temperature regime). Parameters $\varphi_q = t_q = 0$, $\epsilon = 0.1$, $L = 1$, $N = 25$, $J_{\max} = 40$, $z_0 = 0.99$, $z_q = 0.98$ are fixed for all figures.

then the finite part of boundary one-point quench dynamics has the form

$$\begin{aligned} \langle \mathcal{O}^2(t, \varphi) \rangle_{\phi, D} \\ = \frac{2\tilde{G}_{DB}(it, \varphi; -\epsilon + it_q, \varphi_q, r_q) \tilde{G}_{DB}(it, \varphi; \epsilon + it_q, \varphi_q, r_q)}{G_D(-\epsilon + it_q, \varphi_q, r_q; \epsilon + it_q, \varphi_q, r_q)}. \end{aligned} \quad (48)$$

In Fig. 5 we show the distribution for the peak ratio in the deformed BTZ geometry, regulated using a brick wall model with the wall placed very close to the horizon at $z_0 = r_h^2/r_0^2 \approx 1$, and with the quench point positioned near the wall at $z_q \approx z_0$. A key finding is that, in contrast to the Poincare AdS case, even massless fields exhibit a distribution in strong agreement with the GSE (pink line). Let us highlight the important features:

- (1) The observation of GSE statistics for massless fields is a significant and main departure from the confining AdS case, where chaos was exclusive to heavy fields. This aligns with known results for the spectral form factor of BTZ black holes, where normal modes exhibit chaotic behavior for both massive and massless fields [63,64].
- (2) The high-temperature regime of the BTZ black hole strongly enhances chaotic dynamics. We find that at elevated temperatures, the peak ratio distribution lies significantly closer to the GSE prediction than at lower temperatures, indicating a more pronounced level of quantum chaos.

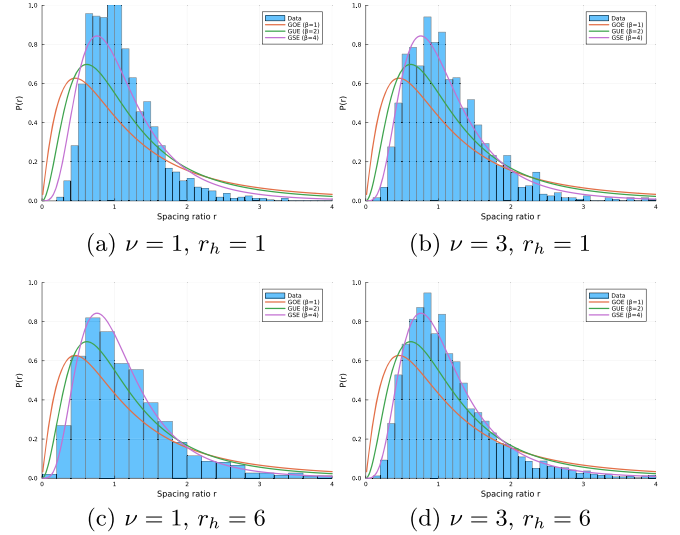


FIG. 6. Peak spacing ratio distributions for the boundary operator $\mathcal{O}^2$ in deformed BTZ geometry, demonstrating the persistence of GSE statistics when the local quench is farther from the wall. The quench point is positioned farther from the wall ($z_q = 0.79$) with the same wall placement ($z_0 = 0.99$). The results are shown for both massless ($\nu = 1$) and massive ($\nu = 3$) fields. Panels (a)–(b) correspond to $r_h = 1$, while (c)–(d) show the high-temperature regime ($r_h = 6$). Other parameters are fixed as $\varphi_q = t_q = 0$, $\epsilon = 0.1$, $L = 1$, $N = 25$, $J_{\max} = 40$.

- (3) In this thermal background, a quite significant number of modes must be taken into account to approach the RMT statistics. This contrasts with the IR-deformed Poincare AdS case, where for certain parameters, as few as $N = 5$ series terms were sufficient.

Following the analysis in deformed Poincare AdS, we initially placed the quench point very close to the brick wall. We now move the quench point farther from the wall, decreasing z_q , with the results shown in Fig. 6. Crucially, we find that chaotic behavior persists. This marks another major difference from the Poincare AdS case, where moving the quench point away from the wall led to a significant decrease in chaos. This resilience underscores that the black hole horizon itself, rather than just the proximity to the wall, is the crucial element driving the chaotic behavior of the system.

V. CONCLUSION

In this paper, we studied the evolution of boundary correlation functions for states excited via a path integral on AdS spaces (Poincare AdS and compact BTZ) with operator insertions, corresponding to local operator quenches in the bulk.

We deformed the Poincare AdS geometry with an infrared hard wall cutoff, a well-known setup in holographic QCD that induces confinement. In this scenario, we

confirmed and extended the findings of [35], observing a clear trend where the temporal dynamics of boundary one-point observables approach the GSE distribution. As in that work, this chaotic behavior occurs exclusively for heavy fields, as indicated by their peak-level spacing statistics, and is sensitive to the quench location relative to the wall.

Our study of the compact BTZ black hole, deformed by a brick wall placed near the horizon, reveals significant departures and establishes a more robust form of quantum chaos. In this thermal background, we demonstrate that even massless fields exhibit statistics consistent with the GSE distribution, a result not observed in the confining case. Crucially, this chaotic behavior persists even when the quench point is moved away from the brick wall, indicating that the horizon itself, rather than just the wall proximity, is the fundamental driver of chaos. Furthermore, the chaotic behavior is markedly temperature dependent. The higher temperature regime drives the level statistics substantially closer to the GSE distribution than at lower temperatures, underscoring the role of thermal excitations in strengthening quantum chaos.

We leave for future work detailed studies of planar and extremal BTZ black holes in this context. Furthermore, it would be interesting to apply recent proposals, such as the generalization of peak statistics to higher-dimensional patterns considered in [65], to our setup.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: DERIVATION OF GREEN’S FUNCTION FOR DEFORMED ADS GEOMETRY

In this section we will derive Green’s function for deformed AdS geometry. The Green’s function $G(y, z; y', z')$ for this case satisfies the following equation:

$$\left(\frac{z^2}{L^2} \partial_z^2 + z \frac{1-d}{L^2} \partial_z + \frac{z^2}{L^2} \eta^{\mu\nu} \partial_\mu \partial_\nu - m^2 \right) G(y, z; y', z') = -\frac{z^{d+1}}{L^{d+1}} \delta(y-y') \delta(z-z'), \quad (\text{A1})$$

with boundary conditions

$$G(y, z_0; y', z') = G(y, z; y', z_0) = 0, \quad (\text{A2})$$

$$G(y, 0; y', z') = G(y, z; y', 0) = 0, \quad (\text{A3})$$

where $y = (\tau, x)$ is a set of boundary coordinates. The ansatz for $G(y, z; y', z')$ has the form

$$G(y, z; y', z') = \sum_n \int \frac{d^d k}{(2\pi)^d} e^{ik(y-y')} \times \frac{z^{\frac{d}{2}} z'^{\frac{d}{2}} J_\nu(\alpha_n z) J_\nu(\alpha_n z')}{N_n} G_n(k), \quad (\text{A4})$$

where α_n are defined as follows:

$$J_\nu(\alpha_n z_0) = 0. \quad (\text{A5})$$

To satisfy boundary conditions (A2), the sum goes over all solutions of Eq. (A5). Our goal is to find such $G_n(k)$ and N_n that (A4) satisfies (A1). Substituting the ansatz into the equation, we get

$$\sum_n \int \frac{d^d k}{(2\pi)^d} \frac{z^{\frac{d}{2}} z'^{\frac{d}{2}} J_\nu(\alpha_n z')}{N_n} G_n(k) e^{ik(y-y')} \frac{z^{\frac{d}{2}}}{L^2} \times \left(z^2 J_\nu''(\alpha_n z) + z J_\nu'(\alpha_n z) + \left(-z^2 k^2 - m^2 L^2 - \frac{d^2}{4} \right) J_\nu(\alpha_n z) \right) = -\frac{z^{d+1}}{L^{d+1}} \delta(y-y') \delta(z-z'). \quad (\text{A6})$$

Using the equation for $J_\nu(\alpha_n z)$, we rewrite (A6) as

$$\sum_n \int \frac{d^d k}{(2\pi)^d} \frac{z^{\frac{d}{2}} z'^{\frac{d}{2}} J_\nu(\alpha_n z')}{N_n} G_n(k) e^{ik(y-y')} \frac{z^{\frac{d}{2}}}{L^2} \times (-z^2 k^2 - z^2 \alpha_n^2) J_\nu(\alpha_n z) = -\frac{z^{d+1}}{L^{d+1}} \delta(y-y') \delta(z-z'). \quad (\text{A7})$$

We choose $G_n(k)$ as

$$G_n(k) = \frac{1}{L^{d-1}(k^2 + \alpha_n^2)}, \quad (\text{A8})$$

so

$$z'^{\frac{d}{2}} \int \frac{d^d k}{(2\pi)^d} e^{ik(y-y')} \sum_n \frac{z J_\nu(\alpha_n z) J_\nu(\alpha_n z')}{N_n} = z^{\frac{d}{2}} \delta(y-y') \delta(z-z'), \quad (\text{A9})$$

which is consistent with the definition of $\delta(y-y')$ and the completeness relation for Bessel functions

$$\sum_n \frac{z J_\nu(\alpha_n z) J_\nu(\alpha_n z')}{N_n} = \delta(z-z'), \quad (\text{A10})$$

where

$$N_n = \int_0^{z_0} dz z J_\nu^2(\alpha_n z) = \frac{z_0^2}{2} J_{\nu+1}^2(\alpha_n z_0). \quad (\text{A11})$$

Then Green's function has the form

$$G(y, z; y', z') = \sum_n \int \frac{d^d k}{(2\pi)^d L^{d-1}} \frac{e^{ik(y-y')}}{k^2 + \alpha_n^2} \times \frac{z^{\frac{d}{2}} z'^{\frac{d}{2}} J_\nu(\alpha_n z) J_\nu(\alpha_n z')}{N_n}. \quad (\text{A12})$$

We perform the integration over k in spherical coordinates

$$\int \frac{d^d k}{(2\pi)^d} \frac{e^{ik(y-y')}}{k^2 + \alpha_n^2} = \int_0^\infty \frac{k^{d-1} dk}{(2\pi)^d (k^2 + \alpha_n^2)} \times \int_0^\pi d\theta e^{ikr \cos \theta} \sin^{d-2} \theta \Omega_{d-2}, \quad (\text{A13})$$

where $r = \sqrt{(\tau - \tau')^2 + (x - x')^2}$ and Ω_{d-2} is the area of the $(d-2)$ -dimensional sphere

$$\Omega_{d-2} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}. \quad (\text{A14})$$

Integration over angle θ gives us the Bessel function

$$\int_0^\pi d\theta e^{ikr \cos \theta} \sin^{d-2} \theta = \frac{2^{\frac{d-1}{2}} \sqrt{\pi} \Gamma(\frac{d-1}{2})}{(kr)^{\frac{d-1}{2}}} J_{\frac{d-1}{2}}(kr), \quad (\text{A15})$$

so we are left with

$$\int \frac{d^d k}{(2\pi)^d} \frac{e^{ik(y-y')}}{k^2 + \alpha_n^2} = \frac{1}{(2\pi)^{\frac{d}{2}} r^{\frac{d-1}{2}}} \int_0^\infty dk \frac{k^{\frac{d}{2}}}{k^2 + \alpha_n^2} J_{\frac{d-1}{2}}(kr). \quad (\text{A16})$$

The remaining integral evaluates to

$$\int_0^\infty dk \frac{k^{\frac{d}{2}}}{k^2 + \alpha_n^2} J_{\frac{d-1}{2}}(kr) = \alpha_n^{\frac{d-1}{2}} K_{\frac{d-1}{2}}(\alpha_n r). \quad (\text{A17})$$

The final result for Green's function is given by

$$G(\tau, x, z; \tau', x', z') = \sum_n \left(\frac{\alpha_n}{r} \right)^{\frac{d-1}{2}} \frac{z^{\frac{d}{2}} z'^{\frac{d}{2}} J_\nu(\alpha_n z) J_\nu(\alpha_n z')}{(2\pi)^{\frac{d}{2}} L^{d-1} N_n} K_{\frac{d-1}{2}}(\alpha_n r). \quad (\text{A18})$$

APPENDIX B: DERIVATION OF GREEN'S FUNCTION FOR DEFORMED BTZ GEOMETRY

In this section we will derive Green's function for deformed BTZ geometry. Green's function $G_D(\tau, \varphi, r; \tau', \varphi', r')$ for this case satisfies the following equation:

$$\left(\frac{r^2 - r_h^2}{L^2} \partial_r^2 + \frac{3r^2 - r_h^2}{rL^2} \partial_r + \frac{L^2}{r^2 - r_h^2} \partial_\tau^2 + \frac{1}{r^2} \partial_\varphi^2 - m^2 \right) G_D = -\frac{1}{r} \delta(\tau - \tau') \delta(\varphi - \varphi') \delta(r - r'), \quad (\text{B1})$$

with boundary conditions

$$G_D(\tau, \varphi, r_0; \tau', \varphi', r') = G_D(\tau, \varphi, r; \tau', \varphi', r_0) = 0, \quad (\text{B2})$$

$$G_D(\tau, \varphi, \infty; \tau', \varphi', r') = G_D(\tau, \varphi, r; \tau', \varphi', \infty) = 0, \quad (\text{B3})$$

$$G_D(\tau, \varphi + 2\pi n, r; \tau', \varphi', r') = G_D(\tau, \varphi, r; \tau', \varphi', r'). \quad (\text{B4})$$

The ansatz for $G_D(\tau, \varphi, r; \tau', \varphi', r')$ has the form

$$G_D(\tau, \varphi, r; \tau', \varphi', r') = \sum_n \sum_J \int d\lambda e^{i\lambda(\tau - \tau')} \times e^{iJ(\varphi - \varphi')} \frac{F_{n,J}(r) F_{n,J}(r')}{(2\pi)^2 N_{n,J} \sqrt{rr'}} G_{n,J}(\lambda), \quad (\text{B5})$$

where J is an integer and the index n numerates a set of solutions to the equation

$$F_{n,J}(r_0) = 0, \quad (\text{B6})$$

which enforces the boundary condition (B2). We comment on this later. Substituting the ansatz into the equation, we obtain

$$\sum_n \sum_J \int d\lambda e^{i\lambda(\tau - \tau')} e^{iJ(\varphi - \varphi')} \frac{F_{n,J}(r')}{(2\pi)^2 N_{n,J} \sqrt{rr'} L^2} \times G_{n,J}(\lambda) \left[(r^2 - r_h^2) \partial_r^2 + 2r \partial_r - \left(\frac{r_h^2}{4r^2} + \frac{3}{4} + \frac{L^2 J^2}{r^2} + m^2 L^2 \right) - \frac{L^4 \lambda^2}{r^2 - r_h^2} \right] F_{n,J}(r) = -\frac{1}{r} \delta(\tau - \tau') \delta(\varphi - \varphi') \delta(r - r'). \quad (\text{B7})$$

We choose function $F_{n,J}(r)$ to solve

$$\left[(r^2 - r_h^2) \partial_r^2 + 2r \partial_r - \left(\frac{r_h^2}{4r^2} + \frac{3}{4} + \frac{L^2 J^2}{r^2} + m^2 L^2 \right) + \frac{L^4 \omega_{n,J}^2}{r^2 - r_h^2} \right] F_{n,J}(r) = 0. \quad (\text{B8})$$

Then Eq. (B7) takes form

$$\begin{aligned} & \sum_n \sum_J \int \frac{d\lambda}{(2\pi)^2} e^{i\lambda(\tau-\tau')} e^{iJ(\varphi-\varphi')} \frac{1}{\sqrt{rr'}} \frac{F_{n,J}(r')}{N_{n,J}} \\ & \times G_{n,J}(\lambda) \frac{L^2(\lambda^2 + \omega_{n,J}^2)}{r^2 - r_h^2} F_{n,J}(r) \\ & = \frac{1}{r} \delta(\tau - \tau') \delta(\varphi - \varphi') \delta(r - r'). \end{aligned} \quad (\text{B9})$$

We take $G_{n,J}(\lambda)$ to be

$$G_{n,J}(\lambda) = \frac{2L^2}{r_h(\lambda^2 + \omega_{n,J}^2)}, \quad (\text{B10})$$

so (B9) takes form

$$\begin{aligned} & \frac{1}{\sqrt{rr'}} \sum_n \sum_J \int \frac{d\lambda}{(2\pi)^2} e^{i\lambda(\tau-\tau')} e^{iJ(\varphi-\varphi')} \\ & \times \frac{F_{n,J}(r) F_{n,J}(r')}{N_{n,J}} \frac{2L^4}{(r^2 - r_h^2) r_h} \\ & = \frac{1}{r} \delta(\tau - \tau') \delta(\varphi - \varphi') \delta(r - r'). \end{aligned} \quad (\text{B11})$$

Next, we analyze Eq. (B8). Taking into account the boundary condition (B3), the solution is given by

$$F_{n,J}(r) = \left(1 - \frac{r_h^2}{r^2} \right)^{\alpha_n} \left(\frac{r_h^2}{r^2} \right)^{\beta} {}_2F_1 \left(a_{n,J}, b_{n,J}; c; \frac{r_h^2}{r^2} \right), \quad (\text{B12})$$

where parameters a_n , b_n , c , α_n , and β are equal to

$$a_n, b_n = \frac{1}{2} \left(1 + \nu + \frac{iL^2 \omega_n}{r_h} \mp \frac{iLJ}{r_h} \right), \quad c = \Delta, \quad (\text{B13})$$

$$\alpha_n = \frac{iL^2 \omega_n}{2r_h}, \quad \beta = \frac{1}{4} + \frac{\nu}{2}. \quad (\text{B14})$$

To obtain a completeness relation for the functions $F_{n,J}(r)$ introduce coordinates $z = r_h^2/r^2$ and the ansatz $F_{n,J}(z) = z^{3/4} f_{n,J}(z)$. Then Eq. (B8) takes form

$$\begin{aligned} & 4z^2(1-z)f_{n,J}''(z) + 4z(2-3z)f_{n,J}'(z) \\ & - \left(4z + m^2 L^2 + \frac{L^2 J^2 z}{r_h^2} \right) f_{n,J}(z) \\ & + \frac{L^4 z}{r_h^2(1-z)} \omega_{n,J}^2 f_{n,J}(z) = 0, \end{aligned} \quad (\text{B15})$$

with boundary conditions

$$f_{n,J}(z_0) = f_{n,J}(0) = 0, \quad (\text{B16})$$

which are the re-expressed forms of conditions (B3) and (B6). Equations (B15) and (B16) define a regular Sturm-Liouville problem with eigenvalue $\omega_{n,J}^2$ and weight factor

$$\rho(z) = \frac{L^4 z}{r_h^2(1-z)}, \quad (\text{B17})$$

where eigenvalues $\omega_{n,J}$ are defined as the solutions to boundary equation (B6), which may be expressed as

$${}_2F_1(a_{n,J}, b_{n,J}; c; z_0) = 0. \quad (\text{B18})$$

Functions $f_{n,J}(z)$, as solutions to a Sturm-Liouville problem, form a complete set of functions with the completeness relation

$$\sum_n \rho(z) \frac{f_{n,J}(z) f_{n,J}(z')}{N_{n,J}} = \delta(z - z'), \quad (\text{B19})$$

where normalization factors $N_{n,J}$ are defined as

$$\begin{aligned} N_{n,J} &= \int_0^{z_0} \rho(z) f_{n,J}^2(z) dz \\ &= \frac{L^4}{r_h^2} \int_0^{z_0} (1-z)^{2\alpha_{n,J}-1} z^\nu {}_2F_1^2(a_{n,J}, b_{n,J}; c; z) dz. \end{aligned} \quad (\text{B20})$$

Returning to the r coordinates, we use

$$\delta(z - z') = \frac{\delta(r - r')}{\left| \frac{dz}{dr} \right|} = \frac{\delta(r - r')}{\frac{2r_h^2}{r^3}}. \quad (\text{B21})$$

Then (B19) takes the form

$$\delta(r - r') = \sum_n \frac{F_{n,J}(r) F_{n,J}(r')}{N_{n,J}} \frac{2L^4}{(r^2 - r_h^2) r_h}, \quad (\text{B22})$$

so expression (B11) is consistent with (B22) and basic definitions of $\delta(\tau - \tau')$ and $\delta(\varphi - \varphi')$, which concludes the derivation.

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