

Singly heavy omega baryon spectroscopy in the relativistic framework of an independent quark model

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The independent quark model, formulated for a three-body system within a relativistic framework, is applied to singly heavy baryons Ω_c^0 and Ω_b^- to investigate their spectroscopic properties. A Martin-like potential with an equal mixture of scalar and vector components is employed, with potential parameters fitted using ground-state experimental data. The resulting mass spectra include both radial and orbital excitations. The spin-parity for the recently observed states $\Omega_c^0(3000)$, $\Omega_c^0(3050)$, $\Omega_c^0(3067)$, $\Omega_c^0(3120)$, and $\Omega_c^0(3185)$, as well as possible spin assignments for the four newly observed excited states of Ω_b^- ($\Omega_b^-(6316)$, $\Omega_b^-(6330)$, $\Omega_b^-(6340)$, and $\Omega_b^-(6350)$), are proposed. The magnetic moments of the ground and first excited states are also calculated, along with radiative decay widths and transition magnetic moments. The nonleptonic weak decays of Ω_c^0 are analyzed, with decay widths and branching ratios computed and compared with experimental data to validate the predictive power of the model. The branching ratios for the nonleptonic decays of Ω_b^- are also predicted for future observations.

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I. INTRODUCTION

Heavy hadron spectroscopy has recently become more important and interesting due to the discovery of many new states. The classification of baryons with a single heavy quark (Qqq') where Q is a heavy quark such as bottom (b) or charm (c) and q, q' are lighter quarks such as up (u), down (d), or strange (s), is effectively done using the heavy quark-diquark concept. Heavy quark effective theory (HQET) helps simplify the study of these baryons by isolating the heavy-quark dynamics with corrections that are proportional to the inverse of the heavy-quark mass ($1/m_Q$). Observing new baryons and measuring their properties provide insights into the role of diquarks within baryons, and this information can also refine models of

more complex states such as tetraquarks and pentaquarks. Besides the challenges of exotic particles, many states are related to the excited states of known hadrons or mixtures of nearby states. For a long time, only two ground states had been observed for the Ω_c baryons: $\Omega_c^0(J^P = 1/2^+)$ and $\Omega_c(2770)^0(J^P = 3/2^+)$. The latter was detected by BABAR through electromagnetic decay $\Omega_c(2770) \rightarrow \Omega_c \gamma$ [1]. The Ω_c^0 baryon has become especially interesting after new resonances were found in the decay $\Omega_c^0 \rightarrow \Xi_c^+ K^-$ [2]. These new states are $\Omega_c^0(3000)$, $\Omega_c^0(3050)$, $\Omega_c^0(3067)$, $\Omega_c^0(3090)$, and $\Omega_c^0(3120)$. The LHCb Collaboration also suggested possible spins for these states and published their findings in 2021 [3]. However, the very recent observation of $\Omega_c^0(3185)$ and $\Omega_c^0(3327)$ made it necessary to study and apply the models to these baryons to understand the internal dynamics of the same [4]. The simplest explanation for the recent observation of four narrow peaks in the $\Xi_b^0 K^-$ mass spectrum at LHCb are excited states of Ω_b^- [5]. These are likely $L = 1$ orbital excitations of the ground state or possibly $n = 2$ radial excitations [5].

Over the past 27 years, singly heavy baryons, such as Ω_c^0 and Ω_b^- , have been the subject of extensive theoretical

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investigations [6–62]. Various frameworks, including relativistic quark models, the QCD sum rule method, heavy quark effective theory, diverse quark models, and lattice QCD simulations, have been employed to study their fundamental properties. These efforts have primarily focused on predicting their masses, strong couplings, transition amplitudes, magnetic moments, radiative decay rates, and a variety of other decay characteristics [6–62]. Despite these extensive studies, no approach has comprehensively and accurately predicted the spin-parity assignments of all recently observed excited states of these baryons. Recognizing this significant gap in understanding their properties, we investigate their mass spectra, magnetic moments, and decay characteristics, aiming to provide a deeper insight into these systems and assess the predictive success of our theoretical model.

The independent quark model was developed by Kobushkin [63] and Ferreira [64] to describe the linear confinement of quarks. Their framework proposed that individual quarks within a baryon obey a Dirac-type equation, with an average potential centered on the hadron’s center of mass. In later developments, it was demonstrated that expressing the average potential as an equal mix of scalar and vector components simplifies calculations. This transformation turns the single-quark Dirac equation into an effective Schrödinger equation [65–67]. The IQM treats all quarks—light and heavy—relativistically without using HQET formalism or $1/m_Q$ expansions, offering a unified framework for modeling both light and heavy baryons and capturing dynamics beyond HQET’s constraints. The study of quark confinement within a baryon can be carried out using the Martin-like potential, which includes an equal combination of scalar and vector components. This potential has been applied in the relativistic context of the IQM for various mesons [68–74]. Due to its favorable results and effectiveness in predicting and validating experimental observations for mesons, we recently adapted and refined this model for the use of different types of baryons [75,76].

This paper presents a comprehensive discussion of the general methodology in Sec. II, designed to be applicable to any baryonic system. The outlined approach involves solving the Dirac equations for individual quarks within a baryon, enabling the determination of spin-averaged masses and contributions arising from spin-spin, spin-orbit, and tensor interactions. Furthermore, we calculated key static properties, including the magnetic moment, radiative decay width of a baryon, along with the calculation for the nonleptonic weak decays of Ω_c^0 and nonleptonic decays of Ω_b^- , which are provided in Sec. III. The detailed conclusion is presented in Sec. IV.

II. METHODOLOGY

The independent quark model was developed in detail for the system of two quarks, i.e., mesons [68–74]. We have

developed it further to include three quark systems [75–78]. In this model, the properties of individual quarks are governed by a Dirac equation formulated in the hadron’s rest frame. The potential in this equation showcases a Lorentz structure featuring an even blend of scalar and vector components. The initial step in this formalism is to calculate the spin-averaged masses of the multiple quark systems; for the baryons, which can be calculated as

$$M_{SA}^{q_1 q_2 q_3} = E_{q_1}^D + E_{q_2}^D + E_{q_3}^D - E_{CM}. \quad (1)$$

Here, E_{CM} incorporates the center-of-mass corrections removing all the irrelevant effects from the dynamics of the center of mass. $E_{q_i}^D$ ($i = 1, 2, 3$) represents the Dirac energies of the individual quark in the baryon system. These energies calculated by solving the Dirac equation for a quasi-independent quark in the center-of-mass frame that has the form of

$$[E_q^D - \hat{\alpha} \cdot \hat{p} - \hat{\beta} m_q - V(r)]\psi_q(\vec{r}) = 0, \quad (2)$$

where E_q^D represents the Dirac energy of a quark, m_q is the current quark mass and $\psi_q(\vec{r})$ is the four-component quark wave function which is a spinor. Quarks within a hadron are hypothesized to move independently under the influence of a flavor-independent central potential. The central potential is described by a Martin-like functional form given as

$$V(r) = \frac{(1 + \gamma_0)}{2} (\Lambda r^{0.1} + V_0), \quad (3)$$

where Λ represents the strength of the potential, and V_0 denotes its depth. As discussed in Ref. [79], the solution of Eq. (2) can be expressed as

$$\psi_q(\vec{r}) = \begin{pmatrix} ig(r)\Omega_{jlm}(\frac{r}{r_s}) \\ -f(r)\Omega_{j'l'm}(\frac{r}{r_s}) \end{pmatrix}, \quad (4)$$

for the potential of Eq. (3). Here, the spinor spherical harmonics Ω_{jlm} defined as given in Ref. [79],

$$\Omega_{jlm} = \sum_{m', m_s} \left(l \frac{1}{2} j | m' m_s m \right) Y_{lm'} \chi_{\frac{1}{2} m_s}, \quad (5)$$

with parity $\hat{P}_0 \Omega_{jlm} = (-1)^l \Omega_{jlm} \chi_{\frac{1}{2} m_s}$ being eigenfunctions of $\hat{S}^2$ & $\hat{S}_3$ and $Y_{lm'}$ being the spherical harmonics. In order to estimate the Dirac energies, we need to mathematically rearrange the equations for $f(r)$ and $g(r)$ to make them equivalent to Ordinary differential equations (ODEs) obeyed by the reduced radial part of the Schrödinger wave function [65], as follows:

$$\frac{d^2 R^{\text{Sch}}(r)}{dr^2} + \left[m_q (E_q^{\text{Sch}} - V(r)) - \frac{l(l+1)}{r^2} \right] R^{\text{Sch}}(r) = 0. \quad (6)$$

The radial parts of Dirac spinors follow second-order ordinary differential equations:

$$\frac{d^2 g(r)}{dr^2} + \left[(E_q^D + m_q)[E_q^D - m_q - V(r)] - \frac{k(k+1)}{r^2} \right] g(r) = 0, \quad (7)$$

$$\frac{d^2 f(r)}{dr^2} + \left[(E_q^D + m_q)[E_q^D - m_q - V(r)] - \frac{k(k-1)}{r^2} \right] f(r) = 0, \quad (8)$$

where k is the eigenvalue of the operator $\hat{k} = (1 + \hat{\mathbf{L}} \cdot \hat{\boldsymbol{\sigma}})$ having the value

$$k = \begin{cases} -(l+1) = -(j + \frac{1}{2}) & \text{for } j = l + \frac{1}{2} \\ l = +(j + \frac{1}{2}) & \text{for } j = l - \frac{1}{2} \end{cases}. \quad (9)$$

For the potential of Martin-like for Eq. (3), these ODEs can be made equivalent to ODEs obeyed by the reduced radial part of the Schrödinger wave function by defining a dimensionless variable $\rho = \frac{r}{r_0}$ that will reduce Eqs. (6)–(8) to the equivalent form

$$\frac{d^2 g(\rho)}{d\rho^2} + \left[\epsilon^D - \rho^{0.1} - \frac{k(k+1)}{\rho^2} \right] g(\rho) = 0, \quad (10)$$

$$\frac{d^2 f(\rho)}{d\rho^2} + \left[\epsilon^D - \rho^{0.1} - \frac{k(k-1)}{\rho^2} \right] f(\rho) = 0, \quad (11)$$

$$\frac{d^2 R^{\text{Sch}}(\rho)}{d\rho^2} + \left[\epsilon^{\text{Sch}} - \rho^{0.1} - \frac{l(l+1)}{\rho^2} \right] R^{\text{Sch}}(\rho) = 0. \quad (12)$$

Here,

$$\epsilon^D = (E_q^D - m_q - V_0)(m_q + E_q^D)^{\frac{0.1}{2.1}} \left(\frac{2}{\Lambda} \right)^{\frac{2}{2.1}} \quad (13)$$

and

$$\epsilon^{\text{Sch}} = m_q (E_q^{\text{Sch}} - V_0) (m_q)^{\frac{2}{2.1}} \left(\frac{1}{\Lambda} \right)^{\frac{2}{2.1}}. \quad (14)$$

In the Schrodinger case, $r_0 = (m_q \Lambda)^{\frac{1}{2.1}}$ and $r_0 = [(m_q + E_q^D) \frac{\Lambda}{2}]^{\frac{1}{2.1}}$ in the Dirac case [65]. The Schrödinger equation can be solved numerically using the code given in Ref. [80], and the Dirac energies for the individual quarks can be found by equating ϵ^D to ϵ^{Sch} .

The parameters of the potential are determined by matching the theoretical spin-averaged mass, as given by Eq. (1), to the experimental spin-averaged mass of the S

wave. The experimental spin-averaged mass is calculated using the expression

$$M_{SA} = \frac{\sum_J (2J+1) M_{nJ}}{\sum_J (2J+1)} \quad (15)$$

for the S waves of the baryon, which simplifies to $(M_{1/2} + 2M_{3/2})/3$. Once the parameters are fitted, the spin-averaged masses for the excited S waves can also be evaluated.

In this model, spin-spin (hyperfine), spin-orbit, and tensor interactions are treated as perturbative corrections to the predominantly central potential, a standard approach in quark models; their contributions cause fine and hyperfine splittings in the baryon spectrum (of tens of MeV compared to GeV-level eigenenergies), and are naturally suppressed for heavy quarks due to their inverse proportionality to relativistic energy terms, justifying their perturbative treatment and subsequent addition to the spin-averaged mass. To account for spin degeneracy, the spin-spin interaction is incorporated into M_{SA} by considering the total spin of the quark system, defined as $\vec{J}_{3q} = \vec{J}_1 + \vec{J}_2 + \vec{J}_3$. The spin-spin interaction is described by the expression

$$\langle V_{q_1 q_2 q_3}^{jj}(r) \rangle = \sum_{i=1, i < k}^{i,k=3} \frac{\sigma \langle j_i \cdot j_k JM | \hat{j}_i \cdot \hat{j}_k | j_i \cdot j_k JM \rangle}{(E_{q_i}^D + m_{q_i})(E_{q_k}^D + m_{q_k})}, \quad (16)$$

where the interactions are expressed as the sum of the contributions from individual quark pairs. Here, σ represents the $j-j$ coupling constant, which is determined by fitting to experimental data. The fitted values for the potential parameters, center-of-mass corrections, and the $j-j$ coupling constant for the Ω_c^0 and Ω_b^- baryons are summarized in Tables I and II. In IQM, all quarks are treated equally with independent confinement, allowing central potential parameters to vary for different heavy quarks; this differs from

TABLE I. Fitted parameters for the Ω_c^0 .

Parameter	Value (with 5% variation)
Depth of the potential (V_0)	-1.011 ± 0.050 GeV
Potential strength (Λ)	1.250 ± 0.062 GeV ^{1.1}
Center-of-mass correction (E_{CM})	0.236 ± 0.012 GeV
$j-j$ coupling constant (σ)	0.075 ± 0.004 GeV ³

TABLE II. Fitted parameters for the Ω_b^- .

Parameter	Value (with 5% variation)
Depth of the potential (V_0)	-1.110 ± 0.055 GeV
Potential strength (Λ)	1.450 ± 0.072 GeV ^{1.1}
Center-of-mass correction (E_{CM})	0.102 ± 0.005 GeV
$j-j$ coupling constant (σ)	0.069 ± 0.003 GeV ³

HQET expectations of parameters being the same, but IQM does not invoke HQET.

To derive the masses of the P , D , and F states from the spin-averaged mass, we incorporate three interactions: spin-spin, spin-orbit, and tensor interactions. The spin-spin interaction term is defined in Eq. (16). However, based on a phenomenological current confinement models for gluons, a closed analytical expression for the confined gluon propagator (CGP) in coordinate space was derived for

small frequencies using a translationally invariant ansatz [81]. Utilizing the CGP, the complete one-gluon exchange potential (COGEP) between quarks has been formulated through the Fermi-Breit formalism. This formulation provides a valuable framework for exploring hadron spectroscopy and hadron-hadron interactions. The spin-orbit and tensor interaction terms emerge as integral components of the COGEP [81], which are also considered to be the sum of interactions between the pairs of quarks, as follows:

$$V_{q_1 q_2 q_3}^{LS}(r) = \frac{\alpha_s}{4} \sum_{i=1, i < k}^{i,k=3} \frac{N_{q_i}^2 N_{q_k}^2}{(E_{q_i}^D + m_{q_i})(E_{q_k}^D + m_{q_k})} \frac{\lambda_i \cdot \lambda_j}{2r} \otimes [[r \times (\hat{p}_{q_i} - \hat{p}_{q_k}) \cdot (\hat{\sigma}_{q_i} + \hat{\sigma}_{q_k})] \cdot [(D_0'(r) + 2D_1'(r))] + [r \times (\hat{p}_{q_i} + \hat{p}_{q_k}) \cdot (\hat{\sigma}_{q_i} - \hat{\sigma}_{q_k})] \cdot [(D_0'(r) - D_1'(r))]], \quad (17)$$

$$V_{q_1 q_2 q_3}^T(r) = -\frac{\alpha_s}{4} \sum_{i=1, i < k}^{i,k=3} \frac{N_{q_i}^2 N_{q_k}^2}{(E_{q_i}^D + m_{q_i})(E_{q_k}^D + m_{q_k})} \otimes \lambda_i \cdot \lambda_j \left(\left(\frac{D_1''(r)}{3} - \frac{D_1'(r)}{3r} \right) S_{q_i q_k} \right), \quad (18)$$

where $\lambda_i \cdot \lambda_j$ represents the color factor of the baryon and $S_{q_i q_k} = [3(\sigma_{q_i} \hat{r})(\sigma_{q_k} \hat{r}) - \sigma_{q_i} \sigma_{q_k}]$, and the running coupling constant can be calculated as

$$\alpha_s = \frac{\alpha_s(\mu_0)}{1 + \frac{33-2n_f}{12\pi} \alpha_s(\mu_0) \ln \left(\frac{E_{q_1}^D + E_{q_2}^D + E_{q_3}^D}{\mu_0} \right)}, \quad (19)$$

where $\alpha_s(\mu_0 = 1 \text{ GeV}) = 0.6$ is considered in the present study. We keep the parametric form of the confined gluon propagators (D_0 and D_1) as it is mentioned in Ref. [81]:

$$D_0(r) = \left(\frac{\alpha_1}{r} + \alpha_2 \right) \exp \left(\frac{-r^2 c_0^2}{2} \right) \quad (20)$$

$$D_1(r) = \frac{\gamma}{r} \exp \left(\frac{-r^2 c_1^2}{2} \right) \quad (21)$$

with $\alpha_1 = 0.036$, $\alpha_2 = 0.056$, $c_0 = 0.1017 \text{ GeV}$, $c_1 = 0.1522 \text{ GeV}$, and $\gamma = 0.0139$. While the gluon propagator in QCD is theoretically flavor independent due to uniform coupling with quark color charge, the phenomenological model expresses confined gluon effects through fixed functional forms but adjustable parameters $\alpha_1, \alpha_2, c_0, c_1, \gamma$, which incorporate nonperturbative QCD effects and system-specific dynamics. These effective parameters can vary between hadron sectors to better reproduce experimental data, reflecting the model's phenomenological nature rather than contradicting QCD principles, as supported by hadron spectroscopy studies employing COGEP [68,72,74,82]. After deriving the wave function, the normalization constant N_{q_i} for the individual quark wave function can be determined. This enables the evaluation of $\langle \psi | V^{LS} | \psi \rangle$ and $\langle \psi | V^T | \psi \rangle$ for all permutations of q_1, q_2 , and q_3 . Summing these contributions across all permutations provides the total

TABLE III. S state masses of Ω_c^0 (in GeV).

nL	J^P	State	$\langle V_{q_1 q_2 q_3}^{jj} \rangle$	Our	Exp. [83]	[22]	[58]	[60]	[15]	[84]
1S	$\frac{1}{2}^+$	$1^2 S_{\frac{1}{2}}$	-0.044	2.698 ± 0.048	2.695 ± 0.001	2.695	2.695	2.699	2.698	2.520
1S	$\frac{3}{2}^+$	$1^4 S_{\frac{3}{2}}$	0.026	2.769 ± 0.048	2.765 ± 0.002	2.767	2.766	2.762	2.768	2.611
2S	$\frac{1}{2}^+$	$2^2 S_{\frac{1}{2}}$	-0.034	3.047 ± 0.064	...	3.100	3.171	3.150	3.088	3.170
2S	$\frac{3}{2}^+$	$2^4 S_{\frac{3}{2}}$	0.020	3.102 ± 0.064	...	3.126	3.180	3.197	3.123	3.194
3S	$\frac{1}{2}^+$	$3^2 S_{\frac{1}{2}}$	-0.030	3.248 ± 0.073	...	3.436	3.522	3.308	3.489	3.560
3S	$\frac{3}{2}^+$	$3^4 S_{\frac{3}{2}}$	0.018	3.296 ± 0.072	...	3.450	3.524	3.346	3.510	3.576
4S	$\frac{1}{2}^+$	$4^2 S_{\frac{1}{2}}$	-0.027	3.392 ± 0.079	...	3.737	...	3.526	3.814	3.878
4S	$\frac{3}{2}^+$	$4^4 S_{\frac{3}{2}}$	0.016	3.436 ± 0.079	...	3.745	...	3.557	3.830	3.891
5S	$\frac{1}{2}^+$	$5^2 S_{\frac{1}{2}}$	-0.025	3.504 ± 0.084	...	...	...	...	4.102	...
5S	$\frac{3}{2}^+$	$5^4 S_{\frac{3}{2}}$	0.015	3.545 ± 0.084	...	...	...	...	4.114	...

TABLE IV. P State masses Ω_c^0 (in GeV).

$n^{2S+1}L_J$	$\langle V_{q_1 q_2 q_3}^{jj} \rangle$	$\langle V_{q_1 q_2 q_3}^{L.S} \rangle$	$\langle V_{q_1 q_2 q_3}^T \rangle$	Our	Exp. [83]	[60]	[15]	[22]	[58]	[84]
$1^2P_{\frac{1}{2}}$	-0.039	-0.003	-0.001	2.944 ± 0.059	...	3.057	3.055	3.011	3.003	...
$1^2P_{\frac{3}{2}}$	0.025	-0.0004	0.0001	3.012 ± 0.059	...	3.062	3.054	2.976	3.063	3.051
$1^4P_{\frac{1}{2}}$	-0.031	-0.004	-0.003	2.949 ± 0.058	3.000 ± 0.0002	3.045	2.966	3.028	3.011	...
$1^4P_{\frac{3}{2}}$	-0.043	-0.001	0.001	2.944 ± 0.059	...	3.039	3.029	2.993	3.107	...
$1^4P_{\frac{5}{2}}$	0.057	0.001	-0.0002	3.045 ± 0.059	3.050 ± 0.0001	3.067	3.051	2.947	3.145	3.073
$2^2P_{\frac{1}{2}}$	-0.032	-0.002	-0.001	3.177 ± 0.070	...	3.426	3.435	3.345	3.414	...
$2^2P_{\frac{3}{2}}$	0.020	-0.0003	0.00004	3.233 ± 0.069	...	3.431	3.433	3.315	3.437	3.453
$2^4P_{\frac{1}{2}}$	-0.026	-0.002	-0.002	3.182 ± 0.070	3.185 ± 0.001	3.416	3.384	3.359	3.421	...
$2^4P_{\frac{3}{2}}$	-0.034	-0.001	0.001	3.179 ± 0.070	...	3.411	3.415	3.330	3.459	...
$2^4P_{\frac{5}{2}}$	0.046	0.001	-0.0002	3.260 ± 0.069	...	3.435	3.427	3.290	3.471	3.469
$3^2P_{\frac{1}{2}}$	-0.029	-0.001	-0.0005	3.338 ± 0.077	...	3.562	3.754	3.644	3.62	...
$3^2P_{\frac{3}{2}}$	0.018	-0.0002	0.000	3.386 ± 0.076	...	3.566	3.752	3.620	3.644	3.779
$3^4P_{\frac{1}{2}}$	-0.023	-0.001	-0.001	3.343 ± 0.077	...	3.554	3.717	3.656	3.632	...
$3^4P_{\frac{3}{2}}$	-0.030	-0.001	0.0004	3.339 ± 0.077	...	3.550	3.737	3.632	3.656	...
$3^4P_{\frac{5}{2}}$	0.041	0.0005	-0.0001	3.410 ± 0.076	...	3.569	3.744	3.601	3.601	3.791
$4^2P_{\frac{1}{2}}$	-0.027	-0.001	-0.0003	3.461 ± 0.082	...	3.735	4.037	3.926	...	...
$4^2P_{\frac{3}{2}}$	0.016	-0.0001	0.000	3.505 ± 0.082	...	3.739	4.036	3.903	...	4.065
$4^4P_{\frac{1}{2}}$	-0.021	-0.001	-0.0005	3.465 ± 0.082	...	3.728	4.009	3.938	...	...
$4^4P_{\frac{3}{2}}$	-0.027	-0.0004	0.0002	3.461 ± 0.082	...	3.725	4.023	3.915	...	...
$4^4P_{\frac{5}{2}}$	0.038	0.0004	-0.000	3.527 ± 0.082	...	3.742	4.028	3.884	...	4.075

contribution for a given state. Including the spin-spin interaction terms in this total allows for the calculation of the masses of the corresponding P , D , and F states. We have mentioned our predictions, with corresponding experimental observation, and the other theoretical predictions for the S , P , and D state masses of Ω_c^0 in Tables III, IV, and V, respectively, and the S , P , and D state masses of Ω_b^- in Tables VI, VII, and VIII, respectively.

III. MAGNETIC MOMENTS AND DECAY PROPERTIES

A. Magnetic moment

The magnetic moment of baryons is expressed in relation to its constituent quarks [85] as follows:

$$\mu_B = \sum_q \langle \phi_{sf} | \vec{\mu}_{qz} | \phi_{sf} \rangle, \quad (22)$$

where

$$\mu_q = \frac{e_q}{2m_q} \sigma_q. \quad (23)$$

Here, e_q and σ_q represent the charge and the spin of the quark, and $|\phi_{sf}\rangle$ is the spin-flavor wave function.

The spin-flavor wave functions of Ω_c^0 & Ω_b^- baryons with $J^P = 1/2^+$ are

$$|\phi_{sf}\rangle_{\Omega_c^0} = \frac{\sqrt{2}}{6} (2c_-s_+s_+ - s_-c_+s_+ - c_+s_-s_+ + 2s_+c_-s_+ - s_+s_-c_+ - s_-s_+c_+ - s_+c_+s_- - c_+s_+s_- + 2s_+s_+c_-), \quad (24)$$

$$|\phi_{sf}\rangle_{\Omega_b^-} = \frac{\sqrt{2}}{6} (2b_-s_+s_+ - s_-b_+s_+ - b_+s_-s_+ + 2s_+b_-s_+ - s_+s_-b_+ - s_-s_+b_+ - s_+b_+s_- - b_+s_+s_- + 2s_+s_+b_-), \quad (25)$$

and the spin-flavor wave functions of Ω_c^0 & Ω_b^- baryons with $J^P = 3/2^+$ are

$$|\phi_{sf}\rangle_{\Omega_c^0} = \frac{1}{\sqrt{3}} (s_+s_+c_+ + s_+c_+s_+ + c_+s_+s_+), \quad (26)$$

$$|\phi_{sf}\rangle_{\Omega_b^-} = \frac{1}{\sqrt{3}} (s_+s_+b_+ + s_+b_+s_+ + b_+s_+s_+). \quad (27)$$

Here we have used the notations for spin operator $\hat{S}|q\uparrow\rangle = \hat{S}|q+\rangle$ & $\hat{S}|q\downarrow\rangle = \hat{S}|q-\rangle$. Due to the interactions

TABLE V. D state masses Ω_c^0 (in GeV).

$n^{2S+1}L_J$	$\langle V_{q_1 q_2 q_3}^{jj} \rangle$	$\langle V_{q_1 q_2 q_3}^{L.S} \rangle$	$\langle V_{q_1 q_2 q_3}^T \rangle$	Our	Exp. [83]	[60]	[15]	[22]	[58]
$1^2 D_{\frac{3}{2}}$	-0.072	-0.002	-0.0002	3.073 ± 0.067	3.065 ± 0.21	3.313	3.298	3.231	3.323
$1^2 D_{\frac{5}{2}}$	-0.011	-0.000	0.000	3.136 ± 0.067	...	3.314	3.297	3.188	3.364
$1^4 D_{\frac{1}{2}}$	-0.029	-0.004	-0.001	3.113 ± 0.067	...	3.304	3.287	3.215	3.290
$1^4 D_{\frac{3}{2}}$	-0.027	-0.003	-0.0004	3.118 ± 0.067	3.118 ± 0.0001	3.304	3.282	3.262	3.334
$1^4 D_{\frac{5}{2}}$	0.054	-0.001	0.000	3.201 ± 0.066	...	3.304	3.286	3.173	3.396
$1^4 D_{\frac{7}{2}}$	0.085	0.002	-0.000	3.234 ± 0.066	...	3.315	3.283	3.136	3.423
$2^2 D_{\frac{3}{2}}$	-0.059	-0.001	-0.000	3.258 ± 0.075	...	3.650	3.627	3.538	...
$2^2 D_{\frac{5}{2}}$	-0.008	-0.000	0.000	3.310 ± 0.074	...	3.651	3.626	3.502	...
$2^4 D_{\frac{1}{2}}$	-0.025	-0.003	-0.001	3.290 ± 0.075	...	3.641	3.623	3.524	...
$2^4 D_{\frac{3}{2}}$	-0.023	-0.002	-0.000	3.294 ± 0.077	...	3.641	3.613	3.565	...
$2^4 D_{\frac{5}{2}}$	0.045	-0.000	0.000	3.362 ± 0.074	...	3.640	3.614	3.488	...
$2^4 D_{\frac{7}{2}}$	0.070	0.001	-0.000	3.390 ± 0.074	...	3.652	3.611	3.456	...
$3^2 D_{\frac{3}{2}}$	-0.052	-0.001	-0.000	3.394 ± 0.080	...	3.771	...	...	...
$3^2 D_{\frac{5}{2}}$	-0.007	-0.000	0.000	3.441 ± 0.080	...	3.772	...	...	...
$3^4 D_{\frac{1}{2}}$	-0.022	-0.002	-0.000	3.422 ± 0.080	...	3.764	...	...	...
$3^4 D_{\frac{3}{2}}$	-0.021	-0.001	-0.000	3.425 ± 0.080	...	3.764	...	...	...
$3^4 D_{\frac{5}{2}}$	0.040	-0.000	0.000	3.487 ± 0.080	...	3.764	...	...	...
$3^4 D_{\frac{7}{2}}$	0.063	0.001	-0.000	3.511 ± 0.079	...	3.773	...	...	...
$4^2 D_{\frac{3}{2}}$	-0.048	-0.001	-0.000	3.502 ± 0.085	...	3.917	...	...	...
$4^2 D_{\frac{5}{2}}$	-0.006	-0.000	0.000	3.545 ± 0.085	...	3.918	...	...	...
$4^4 D_{\frac{1}{2}}$	-0.021	-0.001	-0.0002	3.528 ± 0.085	...	3.909	...	...	...
$4^4 D_{\frac{3}{2}}$	-0.019	-0.001	-0.000	3.531 ± 0.085	...	3.909	...	...	...
$4^4 D_{\frac{5}{2}}$	0.037	-0.000	0.000	3.588 ± 0.085	...	3.908	...	...	...
$4^4 D_{\frac{7}{2}}$	0.058	0.000	-0.000	3.610 ± 0.084	...	3.919	...	...	...

TABLE VI. S state masses of Ω_b^- (in GeV).

nL	J^P	State	$\langle V_{q_1 q_2 q_3}^{jj} \rangle$	Our	Exp. [83]	[62]	[57]	[15]	[60]
1S	$\frac{1}{2}^+$	$1^2 S_{\frac{1}{2}}$	-0.023	6.047 ± 0.066	6.045 ± 0.0008	6.065	6.046	6.064	6.043
1S	$\frac{3}{2}^+$	$1^4 S_{\frac{3}{2}}$	0.014	6.083 ± 0.068	...	6.081	6.082	6.088	6.069
2S	$\frac{1}{2}^+$	$2^2 S_{\frac{1}{2}}$	-0.018	6.436 ± 0.085	...	6.492	6.438	6.450	6.446
2S	$\frac{3}{2}^+$	$2^4 S_{\frac{3}{2}}$	0.011	6.465 ± 0.084	...	6.497	6.462	6.461	6.466
3S	$\frac{1}{2}^+$	$3^2 S_{\frac{1}{2}}$	-0.016	6.661 ± 0.095	...	6.840	6.740	6.804	6.633
3S	$\frac{3}{2}^+$	$3^4 S_{\frac{3}{2}}$	0.010	6.687 ± 0.095	...	6.841	6.753	6.811	6.650
4S	$\frac{1}{2}^+$	$4^2 S_{\frac{1}{2}}$	-0.015	6.823 ± 0.102	...	7.141	7.022	7.091	6.790
4S	$\frac{3}{2}^+$	$4^4 S_{\frac{3}{2}}$	0.009	6.846 ± 0.102	...	7.142	7.030	7.096	6.804
5S	$\frac{1}{2}^+$	$5^2 S_{\frac{1}{2}}$	-0.014	6.949 ± 0.108	...	7.411	7.290	7.338	...
5S	$\frac{3}{2}^+$	$5^4 S_{\frac{3}{2}}$	0.008	6.971 ± 0.108	...	7.412	7.296	7.343	...

of three quarks inside the bound system like a baryon, the masses of the quarks may vary according to the interaction. We refer to them as the effective quark mass. The effective quark masses m_q^{eff} in our model is defined as

$$m_q^{\text{eff}} = E_q^D \left(1 + \frac{\langle H \rangle - E_{CM}}{\sum_q E_q^D} \right). \quad (28)$$

Here, the $\langle H \rangle$ includes the strength of spin-spin interactions only as we are calculating the magnetic field of S waves.

TABLE VII. P state masses of Ω_b^- (in GeV).

$n^{2S+1}L_J$	$\langle V_{q_1 q_2 q_3}^{jj} \rangle$	$\langle V_{q_1 q_2 q_3}^{L,S} \rangle$	$\langle V_{q_1 q_2 q_3}^T \rangle$	Our	[62]	[57]	[15]	[60]
$1^2P_{\frac{1}{2}}$	-0.021	-0.002	-0.001	6.323 ± 0.080	6.322	6.344	6.339	6.334
$1^2P_{\frac{3}{2}}$	0.013	-0.000	0.000	6.360 ± 0.080	6.372	6.341	6.340	6.336
$1^4P_{\frac{1}{2}}$	-0.017	-0.003	-0.002	6.325 ± 0.080	6.350	6.345	6.330	6.329
$1^4P_{\frac{3}{2}}$	-0.023	-0.001	0.001	6.324 ± 0.080	6.443	6.343	6.331	6.326
$1^4P_{\frac{5}{2}}$	0.031	0.001	-0.000	6.379 ± 0.079	6.456	6.339	6.334	6.339
$2^2P_{\frac{1}{2}}$	-0.017	-0.001	-0.001	6.584 ± 0.092	6.730	6.596	6.710	6.662
$2^2P_{\frac{3}{2}}$	0.011	-0.0002	0.000	6.614 ± 0.091	6.751	6.594	6.705	6.664
$2^4P_{\frac{1}{2}}$	-0.014	-0.001	-0.001	6.586 ± 0.092	6.744	6.597	6.706	6.658
$2^4P_{\frac{3}{2}}$	-0.018	-0.001	0.000	6.585 ± 0.091	6.783	6.595	6.699	6.655
$2^4P_{\frac{5}{2}}$	0.025	0.001	-0.000	6.628 ± 0.091	6.787	6.592	6.700	6.666
$3^2P_{\frac{1}{2}}$	-0.015	-0.001	-0.000	6.763 ± 0.010	7.051	6.829	7.009	6.844
$3^2P_{\frac{3}{2}}$	0.009	-0.000	0.000	6.789 ± 0.099	7.065	6.827	7.002	6.846
$3^4P_{\frac{1}{2}}$	-0.012	-0.001	-0.001	6.766 ± 0.010	7.061	6.830	7.003	6.841
$3^4P_{\frac{3}{2}}$	-0.016	-0.000	0.000	6.764 ± 0.010	7.087	6.828	6.998	6.839
$3^4P_{\frac{5}{2}}$	0.022	0.000	-0.000	6.802 ± 0.099	7.089	6.826	6.996	6.848
$4^2P_{\frac{1}{2}}$	-0.014	-0.0005	-0.0002	6.901 ± 0.105	7.333	7.044	7.265	6.969
$4^2P_{\frac{3}{2}}$	0.009	-0.000	0.000	6.924 ± 0.105	7.343	7.043	7.258	6.970
$4^4P_{\frac{1}{2}}$	-0.011	-0.001	-0.000	6.903 ± 0.106	7.341	7.043	7.257	6.966
$4^4P_{\frac{3}{2}}$	-0.014	-0.0003	0.000	6.901 ± 0.106	7.361	7.043	7.250	6.964
$4^4P_{\frac{5}{2}}$	0.020	0.0002	-0.000	6.936 ± 0.105	7.363	7.042	7.251	6.972

The effective quark mass follows the property of $M_J = \sum_{q=1}^3 m_q^{\text{eff}}$. Our prediction and comparison with other different approaches are given in Table IX.

B. Radiative decay

Radiative decays of baryons provide valuable insights into their intrinsic structure and the relationship between the decay process and the masses of their constituent quarks. The radiative decay width of singly heavy baryons, such as Ω_c^0 & Ω_b^- , is comparatively larger than that of light baryons, hence it is significant enough to warrant detailed investigation.

The electromagnetic radiative decay width can be expressed in terms of the radiative transition magnetic moment (TMM) (in μ_N) and the photon energy ($q = M_{3/2} - M_{1/2}$), as follows [90,91]:

$$\Gamma_R = \frac{q^3}{4\pi} \frac{2}{2J+1} \frac{e^2}{m_p^2} |\mu_{\frac{3}{2}^+ \rightarrow \frac{1}{2}^+}|^2, \quad (29)$$

where the transition magnetic moment is given by

$$\mu_{\frac{3}{2}^+ \rightarrow \frac{1}{2}^+} = \sum_i \langle \phi_{sf}^{\frac{3}{2}^+} | \mu_i \cdot \vec{\sigma}_i | \phi_{sf}^{\frac{1}{2}^+} \rangle = \frac{2\sqrt{2}}{3} (\mu_u - \mu_s). \quad (30)$$

A key aspect of this calculation is the determination of the quark magnetic moments, which are obtained by taking

the geometric mean of their effective masses. This is expressed as [91,92]

$$m_i^{\text{eff}} = \sqrt{m_{i(\frac{3}{2}^+)}^{\text{eff}} m_{i(\frac{1}{2}^+)}^{\text{eff}}}. \quad (31)$$

Our calculated transition magnetic moments and decay widths are mentioned in Table X, and calculated decay widths from other approaches are mentioned in the table for comparison.

C. Nonleptonic weak decays of Ω_c^0

The final products of nonleptonic weak decays of Ω_c^0 involve octet (B) or decuplet (B^*) baryons along with pseudoscalar (M) or vector mesons (V). To establish the strict $SU(3)_f$ relations for possible decays and to parametrize the W - exchange processes and W - emission, the topological diagram approach is used in Ref. [93]. These two body decays of Ω_c^0 involve, $c \rightarrow s\bar{u}d$ & $c \rightarrow qu\bar{q}$ weak quark-level transitions, where $q = (d, s)$. The effective Hamiltonian is

$$\mathcal{H}_{\text{eff}} = \sum_{i=+,-} \frac{G_F}{\sqrt{2}} c_i (V_{cs}^* V_{ud} O_i + V_{cq}^* V_{uq} O_i^q), \quad (32)$$

where O s represent the four quark operators,

TABLE VIII. D state masses of Ω_b^- (in GeV).

$n^{2S+1}L_J$	$\langle V_{q_1 q_2 q_3}^{jj} \rangle$	$\langle V_{q_1 q_2 q_3}^{L,S} \rangle$	$\langle V_{q_1 q_2 q_3}^T \rangle$	Our	[62]	[15]	[57]	[60]
$1^2 D_{\frac{3}{2}}$	-0.039	-0.002	-0.000	6.488 ± 0.089	6.631	6.549	6.480	6.561
$1^2 D_{\frac{5}{2}}$	-0.006	-0.000	0.000	6.523 ± 0.088	6.727	6.529	6.476	6.561
$1^4 D_{\frac{1}{2}}$	-0.016	-0.003	-0.001	6.509 ± 0.089	6.620	6.540	6.485	6.556
$1^4 D_{\frac{3}{2}}$	-0.014	-0.002	-0.000	6.512 ± 0.089	6.663	6.530	6.482	6.556
$1^4 D_{\frac{5}{2}}$	0.029	-0.001	0.000	6.558 ± 0.089	6.673	6.520	6.478	6.555
$1^4 D_{\frac{7}{2}}$	0.046	0.001	-0.000	6.576 ± 0.087	6.736	6.517	6.472	6.562
$2^2 D_{\frac{3}{2}}$	-0.031	-0.001	-0.000	6.691 ± 0.097	6.971	6.863	6.726	6.852
$2^2 D_{\frac{5}{2}}$	-0.004	-0.000	0.000	6.718 ± 0.097	7.033	6.846	6.723	6.850
$2^4 D_{\frac{1}{2}}$	-0.013	-0.002	-0.000	6.707 ± 0.097	6.965	6.857	6.730	6.846
$2^4 D_{\frac{3}{2}}$	-0.012	-0.001	-0.000	6.709 ± 0.097	6.992	6.846	6.727	6.846
$2^4 D_{\frac{5}{2}}$	0.024	-0.000	0.000	6.746 ± 0.096	6.998	6.837	6.724	6.846
$2^4 D_{\frac{7}{2}}$	0.037	0.001	-0.000	6.761 ± 0.096	7.038	6.834	6.720	6.853
$3^2 D_{\frac{3}{2}}$	-0.028	-0.001	-0.000	6.841 ± 0.104	7.262	...	6.953	7.026
$3^2 D_{\frac{5}{2}}$	-0.003	-0.000	0.000	6.865 ± 0.103	7.311	...	6.951	7.026
$3^4 D_{\frac{1}{2}}$	-0.012	-0.001	-0.000	6.856 ± 0.104	7.258	...	6.956	7.021
$3^4 D_{\frac{3}{2}}$	-0.011	-0.001	-0.000	6.857 ± 0.104	7.280	...	6.954	7.022
$3^4 D_{\frac{5}{2}}$	0.021	-0.000	0.000	6.890 ± 0.103	7.283	...	6.951	7.021
$3^4 D_{\frac{7}{2}}$	0.033	0.001	-0.000	6.903 ± 0.103	7.313	...	6.948	7.027
$4^2 D_{\frac{3}{2}}$	-0.026	-0.001	-0.000	6.961 ± 0.109	...	...	7.164	7.124
$4^2 D_{\frac{5}{2}}$	-0.003	-0.000	0.000	6.984 ± 0.109	...	...	7.162	7.124
$4^4 D_{\frac{1}{2}}$	-0.011	-0.001	-0.000	6.975 ± 0.109	...	...	7.166	7.121
$4^4 D_{\frac{3}{2}}$	-0.010	-0.001	-0.000	6.976 ± 0.109	...	...	7.164	7.121
$4^4 D_{\frac{5}{2}}$	0.020	-0.000	0.000	7.006 ± 0.109	...	...	7.162	7.121
$4^4 D_{\frac{7}{2}}$	0.031	0.000	-0.000	7.018 ± 0.108	...	...	7.160	7.125

TABLE IX. Magnetic moments (in μ_N).

State	Our	[86]	[87]	[88]	[89]	[85]	[57]	[22]
Ω_c^{01+}	-0.860	-0.688	-0.69	-0.73	0.774	-0.916	...	-0.842
Ω_c^{03+}	-0.667	-0.730	-0.70	-0.70	-0.547	-0.827	...	-0.625
Ω_b^{-1+}	-0.539	...	-0.87	-0.545	...	...	-0.761	...
Ω_b^{-3+}	-0.906	...	-0.61	0.919	...	...	-1.236	...

TABLE X. Transition magnetic moment (in μ_N) and decay width (DW) (in keV).

Baryon	TMM	DW	[22]	[57]
$\Omega_c^0(1S_{\frac{3}{2}}^{43+} \rightarrow 1S_{\frac{1}{2}}^{21+})$	-0.893674	1.172	1.441	...
$\Omega_b^-(1S_{\frac{3}{2}}^{43+} \rightarrow 1S_{\frac{1}{2}}^{21+})$	-0.166325	0.0057	...	0.01

$$O_{\pm} = \frac{1}{2}[(\bar{u}d)(\bar{s}c) \pm (\bar{s}d)(\bar{u}c)],$$

$$O_{\pm}^q = \frac{1}{2}[(\bar{u}q)(\bar{q}c) \pm (\bar{q}q)(\bar{u}c)], \quad (33)$$

with $(\bar{q}_a q_b) \equiv \bar{q}_a \gamma_{\mu} (1 - \gamma_5) q_b$. $c_i = \{c_+, c_-\}$ represents the scale-dependent Wilson coefficients accounting for perturbative QCD corrections. The values of them at the scale that we are using (i.e., $\mu = 1$ GeV) are $c_+ = 1.422 + (-0.742) = 0.68$, and $c_- = 1.422 - (-0.742) = 2.164$ [94,95]. The effective Hamiltonian in two different $SU(3)_f$ representations, namely, the topological diagram approach (TDA) and the irreducible $SU(3)_f$ approach (IRA), can be expressed as $\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{TDA}} G_F / \sqrt{2}$ & $\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{IRA}} G_F / \sqrt{2}$.

In the context of the topological diagram approach, the focus is kept on flavor-changing transitions of the type $c \rightarrow q_k q_i \bar{q}_j$, where $q_i = (u, d, s)$ is treated as a triplet under $SU(3)_f$ symmetry. This transition is described by $H_{\text{TDA}} = H_j^{ki}$. The nonzero components of this effective operator are identified as $H_2^{31} = 1$ for the process $c \rightarrow su\bar{d}$; $H_2^{21} = -\sin\theta_c$ for $c \rightarrow ud\bar{d}$; and $H_3^{31} = \sin\theta_c$ for $c \rightarrow us\bar{s}$, with the Cabibbo–Kobayashi–Maskawa matrix (CKM) factors approximated as $V_{cs}^* V_{ud} \simeq 1$ and $V_{cd}^* V_{us} = -V_{cs}^* V_{us} \simeq -\sin\theta_c$, where θ_c is the Cabibbo angle.

On the other hand, within the irreducible representation approach, the operators in Eq. (33) are treated as a flavor tensor $(\bar{3} \times 3 \times \bar{3})_c = (\bar{3} + \bar{3}' + 6 + \bar{15})_c$. Specifically, the operators $O_- \sim (\bar{u}d\bar{s} - \bar{s}d\bar{u})c$ and $O_+ \sim (\bar{u}d\bar{s} + \bar{s}d\bar{u})c$ are

identified as belonging to the sextet (6) and the 15-plet ($\bar{15}$) representations, respectively.

Consequently, the effective Hamiltonian takes the form $H_{\text{IRA}} = c_- \frac{\epsilon^{ijl}}{2} H(6)_{lk} + c_+ H(\bar{15})_k^{ij}$. The nonzero entries for $H(6)_{lk}$ are found to be $H(6)_{22} = 2$, and $H(6)_{23} = H(6)_{32} = 2\sin\theta_c$, while for $H(\bar{15})_k^{ij}$, the components are $H(\bar{15})_2^{13} = H(\bar{15})_2^{31} = 1$, and $H(\bar{15})_2^{12,21} = -H(\bar{15})_3^{13,31} = -\sin\theta_c$. To parametrize the topological diagrams and derive the TDA amplitudes of $\Omega_c^0 \rightarrow \mathbf{B}M$, the sextet charmed baryon $\mathbf{B}_{6c}$ is connected to $\mathcal{H}_{\text{TDA}}$, the octet baryon $\mathbf{B}_8$, and M ; and the amplitudes of $\Omega_c^0 \rightarrow \mathbf{B}^*M$, the sextet charmed baryon $\mathbf{B}_{6c}$ is connected to $\mathcal{H}_{\text{TDA}}$, the decuplet baryon $\mathbf{B}_{10}$, and M , as follows:

$$\begin{aligned} \mathcal{M}_{\text{TDA}}(\Omega_c^0 \rightarrow \mathbf{B}M) = & t(\mathbf{B}_{6c})^{ab} H_j^{ki}(\mathbf{B}_8)_{kab}(M)_j^i + c(\mathbf{B}_{6c})^{ab} H_j^{ki}(\mathbf{B}_8)_{iab}(M)_j^k + c'(\mathbf{B}_{6c})^{ab} H_j^{ki}(\mathbf{B}_8)_{ikb}(M)_j^a \\ & + c''(\mathbf{B}_{6c})^{ab} H_j^{ki}(\mathbf{B}_8)_{ibk}(M)_j^a + e'(\mathbf{B}_{6c})^{jb} H_j^{ki}(\mathbf{B}_8)_{ika}(M)_a^b + e''(\mathbf{B}_{6c})^{jb} H_j^{ki}(\mathbf{B}_8)_{iak}(M)_a^b \\ & + e_{\mathbf{B}}(\mathbf{B}_{6c})^{jb} H_j^{ki}(\mathbf{B}_8)_{kab}(M)_a^i + e'_{\mathbf{B}}(\mathbf{B}_{6c})^{jb} H_j^{ki}(\mathbf{B}_8)_{kba}(M)_a^i + e_M(\mathbf{B}_{6c})^{jb} H_j^{ki}(\mathbf{B}_8)_{iba}(M)_a^k \\ & + e'_M(\mathbf{B}_{6c})^{jb} H_j^{ki}(\mathbf{B}_8)_{iab}(M)_a^k. \end{aligned} \quad (34)$$

$$\begin{aligned} \mathcal{M}_{\text{TDA}}(\Omega_c^0 \rightarrow \mathbf{B}^*M) = & t_d(\mathbf{B}_{6c})^{ab} H_j^{ki}(\mathbf{B}_{10})_{kab}(M)_j^i + c_d(\mathbf{B}_{6c})^{ab} H_j^{ki}(\mathbf{B}_{10})_{iab}(M)_j^k + c'_d(\mathbf{B}_{6c})^{ab} H_j^{ki}(\mathbf{B}_{10})_{ikb}(M)_j^a \\ & + e'_d(\mathbf{B}_{6c})^{jb} H_j^{ki}(\mathbf{B}_{10})_{ika}(M)_a^b + e_{d\mathbf{B}}(\mathbf{B}_{6c})^{jb} H_j^{ki}(\mathbf{B}_{10})_{kab}(M)_a^i \\ & + e_{dM}(\mathbf{B}_{6c})^{jb} H_j^{ki}(\mathbf{B}_{10})_{iba}(M)_a^k. \end{aligned} \quad (35)$$

Similar to TDA, $\mathbf{B}_{6c}$ is connected to $\mathcal{H}_{\text{IRA}}$, $\mathbf{B}_{8(10)}$, and M to establish the IRA amplitudes of $\Omega_c^0 \rightarrow \mathbf{B}^{(*)}M$. For $\Omega_c^0 \rightarrow \mathbf{B}M$, the amplitudes are given by

$$\mathcal{M}_{\text{IRA}}(\Omega_c^0 \rightarrow \mathbf{B}M) = \mathcal{M}_6 + \mathcal{M}_{\bar{15}}, \quad (36)$$

where

$$\begin{aligned} \mathcal{M}_6 = & a_{12} H_{ij}(6)(\mathbf{B}_{6c})^{ij}(\mathbf{B}_8)_k^l(M)_l^k + a_{13} H_{ij}(6)(\mathbf{B}_{6c})^{kl}(\mathbf{B}_8)_k^i(M)_l^j + a_{14} H_{ij}(6)(\mathbf{B}_{6c})^{jk}(\mathbf{B}_8)_k^l(M)_l^i \\ & + a_{15} H_{ij}(6)(\mathbf{B}_{6c})^{jk}(\mathbf{B}_8)_l^i(M)_k^l, \\ \mathcal{M}_{\bar{15}} = & a_{16}(\mathbf{B}_8)_j^i(M)_l^k H(\bar{15})_i^{jm}(\mathbf{B}_{6c})^{ln} \epsilon_{kmn} + a_{17}(\mathbf{B}_8)_j^i(M)_l^k H(\bar{15})_i^{lm}(\mathbf{B}_{6c})^{jn} \epsilon_{kmn} \\ & + a_{18}(\mathbf{B}_8)_n^m(M)_j^n H(\bar{15})_k^{ij}(\mathbf{B}_{6c})^{kl} \epsilon_{ilm} + a_{19}(\mathbf{B}_8)_l^j(M)_n^k H(\bar{15})_m^{il}(\mathbf{B}_{6c})^{mn} \epsilon_{ijk} \\ & + a_{20}(\mathbf{B})_n^j(M)_l^k H(\bar{15})_m^{il}(\mathbf{B}_{6c})^{mn} \epsilon_{ijk}. \end{aligned} \quad (37)$$

The parameters a_i (with i ranging from 12 to 20) represent the $SU(3)_f$ invariant amplitudes. For $\Omega_c^0 \rightarrow \mathbf{B}^*M$, the amplitudes are expressed as

$$\mathcal{M}_{\text{IRA}}(\Omega_c^0 \rightarrow \mathbf{B}^*M) = \mathcal{M}'_6 + \mathcal{M}'_{\bar{15}}, \quad (38)$$

where $\mathcal{M}'_6$ and $\mathcal{M}'_{\bar{15}}$ are defined similarly to $\mathcal{M}_6$ and $\mathcal{M}_{\bar{15}}$, respectively, given by

$$\begin{aligned} \mathcal{M}'_6 = & a_{21}(\mathbf{B}_{10})_{lkm}(M)_n^i H_{ij}(6)(\mathbf{B}_{6c})^{lk} \epsilon^{jmn} + a_{22}(\mathbf{B}_{10})_{klm}(M)_n^l H_{ij}(6)(\mathbf{B}_{6c})^{jk} \epsilon^{imn}, \\ \mathcal{M}'_{\bar{15}} = & a_{23}(\mathbf{B}_{10})_{ijk}(M)_l^m H(\bar{15})_m^{lk}(\mathbf{B}_{6c})^{ij} + a_{24}(\mathbf{B}_{10})_{ijk}(M)_m^k H(\bar{15})_l^{ij}(\mathbf{B}_{6c})^{lm} \\ & + a_{25}(\mathbf{B}_{10})_{ijk}(M)_m^l H(\bar{15})_l^{ij}(\mathbf{B}_{6c})^{km} + a_{26}(\mathbf{B}_{10})_{ijk}(M)_l^j H(\bar{15})_m^{lk}(\mathbf{B}_{6c})^{im}. \end{aligned} \quad (39)$$

The parameters a_i (with i ranging from 21 to 26) are another set of the $SU(3)_f$ parameters.

Using $\mathcal{M}_{\text{TDA}}(\Omega_c^0 \rightarrow \mathbf{B}^{(*)}M) = \mathcal{M}_{\text{IRA}}(\Omega_c^0 \rightarrow \mathbf{B}^{(*)}M)$, the unification of the two $SU(3)_f$ approaches and the equivalence relations are established. The relations are

$$\begin{aligned} (t, c, c', c'') &= (-2a_{14} + a_{17}, 2a_{14} + a_{17}, -2a_{13} - 2a_{14}, 0), \\ (e', e'') &= (-2a_{12} - 2a_{15} + a_{19}, -2a_{19}), \\ (e_{\mathbf{B}}, e'_{\mathbf{M}}) &= (-a_{17} - a_{20}, -a_{17} - a_{20}), \\ (e_{\mathbf{M}}, e'_{\mathbf{B}}) &= (2a_{12} + a_{18}, -2a_{12} + a_{18}), \end{aligned} \quad (40)$$

for $\Omega_c^0 \rightarrow \mathbf{B}M$ and

$$\begin{aligned} (t_d, c_d, c'_d, e'_d) &= (2a_{21} + a_{23}, -2a_{21} + a_{23}, 2a_{25}, 2a_{24}), \\ (e_{d\mathbf{B}}, e_{d\mathbf{M}}) &= (2a_{22} + a_{26}, -2a_{22} + a_{26}), \end{aligned} \quad (41)$$

for $\Omega_c^0 \rightarrow \mathbf{B}^*M$.

The topological diagram approach provides a clearer, more intuitive picture of the decay mechanisms for $\Omega_c^0 \rightarrow \mathbf{B}M$ and $\Omega_c^0 \rightarrow \mathbf{B}^*M$. When combined with the irreducible representation approach through Eqs. (40) and (41), it enables the merging or omission of certain topological parameters, thereby simplifying the parameter space for practical data analysis.

In the original study [93], the simplification of the effective Hamiltonian $\mathcal{H}_{\text{IRA}} \propto c_- H(6) + c_+ H(\overline{15})$, relies on the ratio $c_+/c_- \approx 35\%$, which highlights the dominance of the sextet component. This dominance allows the

parameters associated with the sextet to be treated as leading contributions, while those linked to the $c_+ H(\overline{15})$ term provide only a subleading correction of about 35%, on the order of typical $SU(3)_f$ symmetry-breaking effects (20%–30%), and are thus often neglected. In our analysis, performed at the scale of 1 GeV, the ratio c_+/c_- is found to be approximately 31%. Given the proximity of this value to the originally cited 35%, we consider the same hierarchical scheme applicable. Therefore, we adopt the parameter reduction and approximations introduced in the work [93] for our study.

By adopting this approximation, the following near-equality relations hold: for $\Omega_c^0 \rightarrow \mathbf{B}M$: $t \simeq -c \simeq -2a_{14}$, $c' \simeq -2a_{13} - 2a_{14}$, $e' \simeq -e_{\mathbf{M}} \simeq e'_{\mathbf{B}} \simeq -2a_{12}$, with $(e'', e_{\mathbf{B}}, e'_{\mathbf{M}}) \simeq 0$ and $c'' = 0$; and for $\Omega_c^0 \rightarrow \mathbf{B}^*M$: $t_d \simeq -c_d \simeq 2a_{21}$, $e_{d\mathbf{B}} \simeq -e_{d\mathbf{M}} \simeq 2a_{22}$, with $(c'_d, e'_d) \simeq 0$. Hence, the simplified TDA amplitudes, $\mathcal{M}_{\text{STDA}}$, are given in Table XI, and the methodology to obtain the numerical values of the parameters is given in Ref. [93]. In summary, the parameters are obtained by using the Wolfenstein parametrization, with parameter $\lambda = s_c = 0.225$, for CKM matrix elements and extracting theoretical branching fractions, particularly the semileptonic decay $\Omega_c^0 \rightarrow \Omega^- e^+ \nu_e$, which is more reliable due to minimal QCD corrections. This branching fraction is used to derive other absolute branching ratios (BRs), which, combined with experimental data on various decay modes, allow the extraction of topological parameters in the model. Factorizable amplitudes (expressed as, $\mathcal{M}_{\text{fac}}(\Omega_c^0 \rightarrow \mathbf{B}M, \mathbf{B}V) = \frac{G_F}{\sqrt{2}} V_{cq}^* V_{q_1 q_2} a_1 \langle M, V | (\bar{q}_1 q_2) | 0 \rangle \langle \mathbf{B} | (\bar{q} c) | \Omega_c^0 \rangle$) and transition

TABLE XI. Nonleptonic weak decay branching ratio and relative branching ratios for Ω_c^0 decays along with corresponding $\mathcal{M}_{\text{STDA}}$.

Decay	$\mathcal{M}_{\text{STDA}}$	Our BR (10^{-4})	[93]	Our RBR	Exp. RBR [83,96]
$\Omega_c^0 \rightarrow \Xi^0 \overline{K}^0$	$-(c + c')$	52.74	47 ± 7.2	1.62	1.64 ± 0.29
$\Omega_c^0 \rightarrow \Lambda^0 \overline{K}^0$	$\frac{2}{\sqrt{6}} c' s_c$	10.60	9 ± 0.4	0.33	...
$\Omega_c^0 \rightarrow \Xi^- \pi^+$	$-t s_c$	49.74	4.7 ± 0.4	0.15	0.25 ± 0.06
$\Omega_c^0 \rightarrow \Xi^0 \pi^0$	$-\frac{1}{\sqrt{2}} c s_c$	2.48	2.3 ± 0.2	0.08	...
$\Omega_c^0 \rightarrow \Xi^0 \eta^0$	$\frac{1}{\sqrt{6}} (3c + 2c') s_c$	0.20	0.2 ± 0.1	0.006	...
$\Omega_c^0 \rightarrow \Xi^- \rho^+$	$-\bar{t} s_c$	17.55	16.4 ± 1.3	0.54	...
$\Omega_c^0 \rightarrow \Xi^{*0} \overline{K}^0$	$\frac{1}{\sqrt{3}} c_d$	10.72	9.8 ± 1.3	0.33	...
$\Omega_c^0 \rightarrow \Omega^- \pi^+$	t_d	32.47	28.1 ± 3.6	1	1.8
$\Omega_c^0 \rightarrow \Omega^- K^+$	$(t_d - e_{d\mathbf{M}} + \delta e_{d\mathbf{B}}^s) s_c$	2.03	1.8 ± 0.3	0.06	< 0.29
$\Omega_c^0 \rightarrow \Sigma^{*+} K^-$	$\frac{1}{3} e_{d\mathbf{M}} s_c$	2.92	2.7 ± 0.2	0.09	...
$\Omega_c^0 \rightarrow \Xi^{*-} \pi^+$	$-\frac{1}{\sqrt{3}} (t_d + e_{d\mathbf{M}}) s_c$	6.12	5.9 ± 0.3	0.19	...
$\Omega_c^0 \rightarrow \Xi^{*0} \pi^0$	$\frac{1}{\sqrt{6}} (c_d - e_{d\mathbf{M}}) s_c$	3.05	2.9 ± 0.2	0.09	...
$\Omega_c^0 \rightarrow \Xi^{*0} \eta^0$	$-\frac{1}{\sqrt{2}} (c_d + e_{d\mathbf{M}} + \frac{2}{3} \delta e_{d\mathbf{M}}^s) s_c$	1.12	1.1 ± 0.2	0.03	...
$\Omega_c^0 \rightarrow \Omega^- \rho^+$	$\bar{t}_d$	55.80	46.3 ± 1.72	1.72	> 1.3
$\Omega_c^0 \rightarrow \Omega^- K^{*+}$	$(\bar{t}_d - \bar{e}_{d\mathbf{M}} + \delta \bar{e}_{d\mathbf{B}}^s) s_c$	3.01	2.5 ± 1.1	0.09	...
$\Omega_c^0 \rightarrow \Sigma^{*+} K^{*-}$	$\frac{1}{\sqrt{3}} \bar{e}_{d\mathbf{M}} s_c$	6.03	5.7 ± 1.3	0.18	...
$\Omega_c^0 \rightarrow \Sigma^{*0} K^{*0}$	$\frac{1}{\sqrt{6}} \bar{e}_{d\mathbf{M}} s_c$	3.15	2.8 ± 0.6	0.1	...
$\Omega_c^0 \rightarrow \Xi^{*-} \rho^{*+}$	$-\frac{1}{\sqrt{3}} (\bar{t}_d + \bar{e}_{d\mathbf{M}}) s_c$	11.99	11.7 ± 1.8	0.37	...

form factors aid in determining parameters such as a_1 , enabling simultaneous fits of multiple parameters. The numerical values are

$$\begin{aligned} &\text{for } \Omega_c^0 \rightarrow \mathbf{B}M, \\ &(t, c') = (0.25 \pm 0.01, 0.44 \pm 0.01) \text{ GeV}^3, \\ &(\bar{t}, \bar{c}') = (0.52 \pm 0.02, 0.69 \pm 0.01) \text{ GeV}^3, \\ &\text{for } \Omega_c^0 \rightarrow \mathbf{B}^*M, \\ &(t_d, e_{dM}) = (0.16 \pm 0.01, 0.35 \pm 0.01) \text{ GeV}^3, \\ &(\bar{t}_d, \bar{e}_{dM}) = (0.26 \pm 0.02, 0.57 \pm 0.06) \text{ GeV}^3, \end{aligned} \quad (42)$$

with the assumption $\delta e_{dB,dM}^s = \delta \bar{e}_{dB,dM}^s = 0$. The branching fraction in terms of these parametrized amplitudes is given by

$$\begin{aligned} &B(\Omega_c^0 \rightarrow B^{(*)}M, B^{(*)}V) \\ &= \frac{G_F^2 |\vec{p}_{cm}| \tau_{\Omega_c}}{16\pi m_{\Omega_c}^2} |\mathcal{M}_{STDA}(\Omega_c^0 \rightarrow B^{(*)}M, B^{(*)}V)|^2, \end{aligned} \quad (43)$$

where the magnitude of the three-momentum of the final state is

$$|\vec{p}_{cm}| = \frac{[(m_{\Omega_c}^2 - m_+^2)(m_{\Omega_c}^2 - m_-^2)]^{\frac{1}{2}}}{2m_{\Omega_c}}, \quad (44)$$

with $m_{\pm} = m_B^* \pm m_M(V)$, τ_{Ω_c} is the lifetime of Ω_c^0 baryon. Our predictions for the nonleptonic weak decay branching ratios and relative branching ratio (RBR) of Ω_c^0 , along with the corresponding experimental values and theoretical predictions from alternative approaches, are presented in Table XI.

D. Nonleptonic decays of Ω_b baryons

The color-allowed two-body nonleptonic decays of bottom baryons Ω_b with the emission of a pseudoscalar (π^- , K^- , D^- , and D_s^-) or a vector meson (ρ^- , K^{*-} , D^{*-} , and D_s^{*-}) are calculated within naive factorization approach. Under this approximation, the hadronic transition matrix element is factorized into a product of two independent matrix elements [97]. Accordingly, we can express

$$\begin{aligned} &\langle \mathcal{B}_c^{(*)}(P', J'_z) M^- | \mathcal{H}_{\text{eff}} | \mathcal{B}_b(P, J_z) \rangle \\ &= \frac{G_F}{\sqrt{2}} V_{cb} V_{qq'}^* \langle M^- | \bar{q}' \gamma_\mu (1 - \gamma_5) q | 0 \rangle \\ &\quad \times \langle \mathcal{B}_c^{(*)}(P', J'_z) | \bar{c} \gamma^\mu (1 - \gamma_5) b | \mathcal{B}_b(P, J_z) \rangle, \end{aligned} \quad (45)$$

where the meson transition term is given by

$$\langle M | \bar{q}' \gamma_\mu (1 - \gamma_5) q | 0 \rangle = \begin{cases} if_P q_\mu, & M = P \\ if_V \epsilon_{\mu\nu}^* m_V, & M = V \end{cases} \quad (46)$$

In the case where the final state includes a pseudoscalar meson, the decay width [97] takes the following form:

$$\Gamma = \frac{|p_c|}{8\pi} \left(\frac{(M + M')^2 - m^2}{M^2} |A|^2 + \frac{(M - M')^2 - m^2}{M^2} |B|^2 \right), \quad (47)$$

$$\alpha = \frac{2\kappa \text{Re}(A^* B)}{|A|^2 + \kappa^2 |B|^2}, \quad (48)$$

whereas for the transitions emitting vector meson in final states [97], we can write

$$\begin{aligned} \Gamma &= \frac{|p_c| (E' + M')}{4\pi M} \left(2(|S|^2 + |P_2|^2) \right. \\ &\quad \left. + \frac{E_m^2}{m^2} (|S + D|^2 + |P_1|^2) \right), \end{aligned} \quad (49)$$

$$\alpha = \frac{4m^2 \text{Re}(S^* P_2) + 2E_m^2 \text{Re}(S + D)^* P_1}{2m^2 (|S|^2 + |P_2|^2) + E_m^2 (|S + D|^2 + |P_1|^2)}. \quad (50)$$

Here, p_c represents the momentum of the daughter baryon measured in the rest frame of the parent baryon, and $\kappa = |p_c|/(E' + M')$. Additionally, M (E) and M' (E') denote the masses (energies) of the parent and daughter baryons, respectively, while m (E_m) corresponds to the mass (energy) of the final-state meson. The amplitudes A and B in Eq. (47) are given by

$$A = \frac{G_F}{\sqrt{2}} V_{cb} V_{qq'}^* a_i f_P (M - M') f_1^V(m^2), \quad (51)$$

$$B = -\frac{G_F}{\sqrt{2}} V_{cb} V_{qq'}^* a_i f_P (M + M') g_1^A(m^2), \quad (52)$$

and S , $P_{1,2}$, and D in Eq. (49) are expressed as

$$S = A_1, \quad (53)$$

$$P_1 = -\frac{|p_c|}{E_m} \left(\frac{M + M'}{E' + M'} B_1 + M B_2 \right), \quad (54)$$

$$P_2 = \frac{|p_c|}{E' + M'} B_1, \quad (55)$$

$$D = \frac{|p_c|^2}{E_m (E' + M')} (A_1 - M A_2) \quad (56)$$

with

TABLE XII. Branching ratio for $\Omega_b^- \rightarrow \Omega_c^0 M^-$. All the values are multiplied by a factor of 10^{-3} .

	This work	[98]	[100]	[101,102]	[103]	[104]	[99]
$\Omega_b^- \rightarrow \Omega_c^0 \pi^-$	2.54	2.82	4.92	5.81	4.00	1.88	$1.10^{+0.85}_{-0.55}$
$\Omega_b^- \rightarrow \Omega_c^0 K^-$	0.20	0.22	...	...	0.326	...	$0.08^{+0.07}_{-0.04}$
$\Omega_b^- \rightarrow \Omega_c^0 D^-$	0.46	0.52	...	...	0.636	...	$0.15^{+0.14}_{-0.08}$
$\Omega_b^- \rightarrow \Omega_c^0 D_s^-$	12.2	13.5	17.9	...	17.1	...	$4.03^{+3.72}_{-2.21}$
$\Omega_b^- \rightarrow \Omega_c^0(2S)\pi^-$	0.27	0.30	...	...	...	...	...
$\Omega_b^- \rightarrow \Omega_c^0(2S)K^-$	0.02	0.02	...	...	...	...	...
$\Omega_b^- \rightarrow \Omega_c^0(2S)D^-$	0.03	0.03	...	...	...	...	...
$\Omega_b^- \rightarrow \Omega_c^0(2S)D_s^-$	0.53	0.62	...	...	...	...	...
$\Omega_b^- \rightarrow \Omega_c^0 \rho^-$	7.43	7.92	12.8	...	10.8	5.43	$3.07^{+2.41}_{-1.53}$
$\Omega_b^- \rightarrow \Omega_c^0 K^{*-}$	0.32	0.41	...	...	0.544	...	$0.16^{+0.12}_{-0.08}$
$\Omega_b^- \rightarrow \Omega_c^0 D^{*-}$	0.39	0.48	...	...	0.511	...	$0.16^{+0.13}_{-0.08}$
$\Omega_b^- \rightarrow \Omega_c^0 D_s^{*-}$	9.44	9.73	11.5	...	11.7	...	$3.18^{+2.69}_{-1.61}$
$\Omega_b^- \rightarrow \Omega_c^0(2S)\rho^-$	0.63	0.70	...	...	...	...	...
$\Omega_b^- \rightarrow \Omega_c^0(2S)K^{*-}$	0.03	0.03	...	...	...	...	...
$\Omega_b^- \rightarrow \Omega_c^0(2S)D^{*-}$	0.02	0.02	...	...	...	...	...
$\Omega_b^- \rightarrow \Omega_c^0(2S)D_s^{*-}$	0.22	0.36	...	...	...	...	...

$$A_1 = \frac{G_F}{\sqrt{2}} V_{cb} V_{qq'}^* a_i f_V m_V \left(g_1^A(m^2) + g_2^A(m^2) \frac{M - M'}{M} \right), \quad (57)$$

$$A_2 = \frac{G_F}{\sqrt{2}} V_{cb} V_{qq'}^* a_i f_V m_V (2g_2^A(m^2)), \quad (58)$$

$$B_1 = \frac{G_F}{\sqrt{2}} V_{cb} V_{qq'}^* a_i f_V m_V \left(f_1^V(m^2) - f_2^V(m^2) \frac{M + M'}{M} \right) \quad (59)$$

$$B_2 = \frac{G_F}{\sqrt{2}} V_{cb} V_{qq'}^* a_i f_V m_V (2f_2^V(m^2)). \quad (60)$$

For numerical evaluation, we adopted the values of the form factors from the three-body light-front quark model from Ref. [98]. For the color-allowed transition, we consider the coefficient $a_1 = 1.018$ [99]. The lifetime is taken as $\tau_{\Omega_b^-} = 1.65$ ps [83]. The CKM matrix elements used are [83]

$$\begin{aligned} V_{cb} &= 0.0405, & V_{ud} &= 0.9740, & V_{us} &= 0.2265, \\ V_{cd} &= 0.2264, & V_{cs} &= 0.9732. \end{aligned}$$

The decay constants of pseudoscalar and vector mesons are taken from Ref. [83], as follows:

$$\begin{aligned} f_\pi &= 130.2, & f_K &= 155.6, & f_D &= 211.9, & f_{D_s} &= 249.0, \\ f_\rho &= 216, & f_{K^*} &= 210, & f_{D^*} &= 220, & f_{D_s^*} &= 230. \end{aligned}$$

In the present work, the branching ratios are predicted using the baryon masses calculated within our formalism.

Table XII compares our results for the branching fractions of $\Omega_b^- \rightarrow \Omega_c^0 M^-$ decays with those from various theoretical models, including the nonrelativistic quark model [100], the relativistic three-quark model [101,102], the light-front quark model [99,103], and the covariant confined quark model [104]. This comparison highlights the consistency and differences between our predictions and previous studies.

IV. DISCUSSION AND CONCLUSION

We have investigated the applicability of the modified independent quark model within a relativistic framework to predict the masses, magnetic moments, and decay properties of singly heavy baryons, specifically Ω_c^0 and Ω_b^- . These baryons have been recently observed, yet their spin-parity assignments remain uncertain. Our predictions for the ground and first excited states of Ω_c^0 show good agreement with experimental observations. Additionally, we have attempted to determine the spin-parity of five out of the seven observed states. We propose the spin-parity (J^P) assignments of $\Omega_c^0(3050)$, $\Omega_c^0(3065)$, $\Omega_c^0(3120)$, and $\Omega_c^0(3185)$ as $\frac{5}{2}^-$, $\frac{3}{2}^+$, $\frac{3}{2}^+$, and $\frac{1}{2}^-$, respectively, based on our mass predictions presented in Tables III–V. These results align with observational constraints on spin-parity assignments [3]; for instance, the $J = \frac{1}{2}$ hypothesis is rejected at 2.2σ for $\Omega_c^0(3050)$ and at 3.6σ for $\Omega_c^0(3065)$. Due to the close mass spacing of the P -wave states, the spin-parity of $\Omega_c^0(3000)$ remains uncertain and could be either $\frac{1}{2}^-$ or $\frac{3}{2}^-$. Our predicted masses for the $L = 1$ orbital excitation states of Ω_b^- (Table VII) fall within the range of the four recently observed excited states of Ω_b^- [i.e., $\Omega_b^-(6316)$, $\Omega_b^-(6330)$, $\Omega_b^-(6340)$, and $\Omega_b^-(6350)$], in

agreement with experimental results [5]. While we could not assign specific spin-parity values to these states, they are most likely P -wave excitations of Ω_b^- . Additionally, our predictions for higher excited states may serve as a reference for future observations. The mass uncertainties presented in Tables III–VIII arise solely from a 5% variation in the fitted potential parameters listed in Tables I and II. These variations were introduced to assess the sensitivity of our predictions to changes in the model parameters.

We have calculated the magnetic moments of the ground and first radially excited states of Ω_c^0 and Ω_b^- using the effective masses of quarks. The different ordering of magnetic moments for Ω_c and Ω_b baryons arises naturally within our model due to effective quark masses, internal quark dynamics, and wave function structure. While no experimental data currently exist, we have included comparisons with other theoretical models and lattice QCD results to provide a broader context. Our predictions fall within the range of those obtained from other quark models and lattice QCD as given in Table IX.

We have calculated the radiative decay width and electromagnetic transition magnetic moment of these baryons and mentioned them in Table X. Despite the small decay widths, we observe that the decay width of Ω_b^- is significantly smaller than that of Ω_c^0 . While these decays are scarcely accessible to the experimental measurement, they offer valuable insights into the electromagnetic interactions within singly heavy baryons. Finally, it is essential to study and calculate other decay properties of a baryon that can be experimentally verified to assess the validity of our model. To this end, we have computed the decay widths and branching ratios for various nonleptonic decays of Ω_c^0 . For simplification, we have normalized the branching ratio of $\Omega_c^0 \rightarrow \Xi^- \pi^+$ to 1 and determined the relative branching ratios of other decay modes accordingly. As shown in Table XI, our predicted branching ratio for the decay $\Omega_c^0 \rightarrow \Xi^0 \bar{K}^0$ is 1.62, which is in good agreement with the experimental value of 1.64 ± 0.29 . Similarly, the calculated branching ratio for $\Omega_c^0 \rightarrow \Xi^- \pi^+$ is 0.15, which aligns well with the experimental result of 0.25 ± 0.060 . For the decay $\Omega_c^0 \rightarrow \Omega^- \pi^+$, our predicted branching ratio is 1, whereas the experimental value is 1.8. In the case of $\Omega_c^0 \rightarrow \Omega^- K^+$, our calculation yields 0.06, which is consistent with the experimental upper limit of <0.29 . For $\Omega_c^0 \rightarrow \Omega^- \rho^+$ we obtain a branching ratio of 1.72, which is compatible with the experimental lower bound of >1.3 . Overall, the theoretical prediction exhibits good agreement with the experimental measurement, reinforcing the validity of our approach.

We analyze the color-allowed two-body nonleptonic decays of the bottom baryon Ω_b^- involving pseudoscalar and vector meson emission within the naive factorization framework in which the hadronic matrix element is

represented as a product of the meson decay constant and the heavy-baryon transition form factors associated with the relevant current. In the Cabibbo-allowed two-body decays of bottom baryons, the dominant contributions arise from factorizable diagrams, namely external (color-allowed) and internal (color-suppressed) W -emission processes [105]. In such transitions, the weak current directly produces the final meson, while the two light quarks in the baryon act as a tightly bound spectator diquark and interact only minimally with the outgoing meson [106,107]. Nonfactorizable contributions such as W -exchange and loop diagrams are strongly suppressed because of the large bottom-quark mass reduces wave function overlap with the light quarks, and these effects scale as $1/N_c$ in the large N_c limit of QCD [106,107]. For this reason, the naive factorization approximation becomes applicable in the present study of bottom-baryon decays. In contrast, for light hadrons, nonfactorizable effects due to soft-gluon exchanges are typically of the same order as the factorizable contributions and factorization alone does not provide an accurate description of the decay dynamics [102,106,107].

The decay widths for both final states are expressed in terms of the parent and daughter baryon masses. For numerical evaluation, form factors are adopted from the light-front quark model, while CKM matrix elements, meson decay constants, and the Ω_b^- lifetime are taken from current experimental data. The decay mode $\Omega_b^- \rightarrow \Omega_c^0 \pi^-$ has been observed experimentally; however, precise measurements of its decay width and corresponding branching ratio remain unavailable. Consequently, it becomes essential to compare our predicted branching ratios with those obtained from other theoretical frameworks to assess the degree of model dependence in these decay processes. In Table XII, we present a comparative analysis of our predicted branching ratios with various quark model predictions. The predicted branching ratios span a wide range from 0.02 to 12.2, indicating notable variation across decay channels. This is consistent with the range of 0.02 to 13.5 reported in Ref. [98], supporting the overall agreement with existing theoretical models. For the decay $\Omega_b^- \rightarrow \Omega_c^0 \pi^-$, our predicted value is approximately half of that reported in Refs. [100–103], but nearly twice that predicted in Refs. [99,104], highlighting the model dependence for this channel. In contrast, our results for $\Omega_b^- \rightarrow \Omega_c^0 K^-$, $\Omega_b^- \rightarrow \Omega_c^0 D^-$, $\Omega_b^- \rightarrow \Omega_c^0 K^{*-}$, and $\Omega_b^- \rightarrow \Omega_c^0 D^{*-}$ show close agreement with Refs. [98,103], suggesting these modes are relatively insensitive to the choice of theoretical framework. The decay $\Omega_b^- \rightarrow \Omega_c^0 D_s^-$ consistently emerges as the most favorable channel, with the highest branching ratio in our analysis, in agreement with other theoretical predictions. Our calculated branching ratios are in good agreement with those reported in Ref. [98]. However, substantial deviations from other theoretical approaches are evident, underscoring the

importance of future experimental validation. In particular, confirmation of any one of the predicted decay modes would provide critical insight into the reliability of the various models used in describing nonleptonic bottom baryon decays. Our approach successfully predicts experimental data by fixing model parameters (λ , V_0 , and σ), demonstrating its potential for broader application. In our previous study [77], we applied this method to light baryons with extensive data. Here, we extend it to singly heavy baryons using the independent quark model in a relativistic framework with a Martin-like potential. Our results not only align with experimental observations but also provide reliable predictions for future discoveries. Given this success, the approach can be applied to other well-studied baryons, allowing parameter correlations with mass and enabling predictions for baryons with limited or no experimental data.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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