

Updated analysis of minimal supersymmetric $SO(10)$ with a universal soft spectrum

Cheng-Wei Chiang^{1,2,*} and Takeshi Fukuyama^{3,†}

¹*Department of Physics and Center for Theoretical Physics, National Taiwan University, Taipei, Taiwan 10617, Republic of China*

²*Physics Division, National Center for Theoretical Sciences, Taipei, Taiwan 10617, Republic of China*

³*Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan*



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We update the analysis of minimal supersymmetric $SO(10)$ grand unification with a universal, constrained-minimal supersymmetric Standard Model (MSSM)-like soft-breaking spectrum. A pair of Yukawa matrices in the $\mathbf{10} \oplus \mathbf{\overline{126}}$ Higgs sector fixes the charged fermion masses and quark mixing, the baryon number-violating dimension-five operators that mediate proton decay, and the right-handed neutrino Majorana masses of the type-I seesaw. The simultaneously imposed constraints include: (i) gauge coupling unification and vacuum stability; (ii) dimension-five proton decay (the dimension-six gauge mode also computed and negligible), the 125-GeV Higgs mass, and the Large Electron-Positron chargino limit; and (iii) $\mu \rightarrow e\gamma$, the muon $g - 2$, and electric dipole moments. We perform a global scan over the soft-breaking parameters, complemented by a constrained- μ cross-check that fixes the electroweakino sector from the grand unified theory (GUT)-scale boundary conditions through radiative electroweak symmetry breaking. Subject to a single effective colored Higgs scale M_{H_C} for the proton decay normalization (common to unification and decay, though in a general color triplet mass matrix the two can differ) and to a constrained- μ treatment, what survives is a narrow minisplit region: heavy, multitensof-TeV scalars with $m_0 \gtrsim 11.0$ TeV for a dressing electroweakino mass $m_\chi = 1$ TeV and $\tan\beta \lesssim 8.0^{+5.5}_{-2.0}$, the uncertainty being the Higgs mass theory error. We also examine the fate of the lightest supersymmetric particle as a dark matter candidate, in both the universal model and its free- μ nonuniversal-Higgs-mass extension. The surviving region is sharply bounded and testable in the coming decade, most directly by Hyper-Kamiokande proton decay and electroweakino searches, with dark matter direct detection and electric dipole moment experiments.

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I. INTRODUCTION

Grand unified theories (GUTs) embed the Standard Model (SM) gauge group into a single simple group, so that the three gauge couplings meet at a common high energy scale [1]. Of the candidate groups, $SO(10)$ is the most economical for matter: a single irreducible $\mathbf{16}$ accommodates all fifteen SM Weyl fermions of one generation together with a right-handed neutrino, which could give rise to Majorana masses for the light neutrinos through the type-I seesaw mechanism. When the model is made supersymmetric, the superpartners stabilize the electroweak scale against the GUT scale and, equally

important, sharpen the unification: the minimal supersymmetric Standard Model (MSSM) particle content drives the three running couplings to a strikingly accurate crossing near 2×10^{16} GeV, a result that historically turned supersymmetric grand unification from an aesthetic preference into a quantitative success [2–4]. The most predictive realization of the matter sector couples each $\mathbf{16}$ to just two Higgs representations, the $\mathbf{10}$ and the $\mathbf{\overline{126}}$, through a single pair of symmetric Yukawa matrices [5–7].

These Yukawa matrices ($Y_{\mathbf{10}}, Y_{\mathbf{\overline{126}}}$) control several observables simultaneously. Once they are fitted to the measured charged-fermion masses and the Cabibbo-Kobayashi-Maskawa (CKM) matrix, the baryon number-violating dimension-five operators that mediate proton decay in any supersymmetric GUT [8–10] are no longer adjustable: the same couplings that reproduce the fermion spectrum also fix the flavor structure of these operators. Through the $(B - L)$ -breaking vacuum expectation value (VEV) of the $\mathbf{\overline{126}}$ Higgs, the same pair also fixes the right-handed-neutrino Majorana masses and hence the type-I

* Contact author: chengwei@phys.ntu.edu.tw

† Contact author: fukuyama@rcnp.osaka-u.ac.jp

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seesaw. The proton decay rate is thus a prediction of the fitted model, up to the overall factor $1/M_{H_C}^2$ set by the effective colored Higgs scale, rather than an adjustable parameter. The dimension-five operator generated by the exchange of the heavy color triplet Higgs is dressed to a physical $\Delta B = 1$ amplitude by a loop of electroweak gauginos and Higgsinos, whose strength depends on the superpartner spectrum [11,12], so the nucleon decay rate ties the GUT-scale Yukawa couplings directly to the soft supersymmetry-breaking masses to be constrained.

Many independent measurements bear on this tightly constrained model. They fall into four groups by physical origin: theoretical consistency, including gauge coupling unification and vacuum stability against charge- and color-breaking minima; the direct experimental limits, including dimension-five and dimension-six operators for proton decay, the measured Higgs boson mass, and the Large Electron-Positron (LEP) chargino bound; dark matter (DM), including the thermal relic density and direct detection limits on the lightest supersymmetric particle (LSP); and the flavor and precision observables, such as the $\mu \rightarrow e\gamma$ bound, the muon anomalous magnetic moment $(g-2)_\mu$, and electric dipole moments (EDMs). In this work, we take the simplest and most predictive soft spectrum: a universal, constrained-MSSM-like (CMSSM) pattern in which a single soft scalar mass m_0 governs all squarks and sleptons, with no intergeneration hierarchy.

Taken together, these constraints leave only a narrow, low-to-moderate- $\tan\beta$ region of the minisplit type. It is bounded from below by the proton lifetime bound, which forces the scalars heavy, and from above by the observed Higgs mass. The surviving region demands a heavy universal scalar mass and a bounded $\tan\beta$. Within it, the lightest neutralino is a bino whose thermal relic overcloses the Universe, so the universal model does not naturally provide a viable thermal neutralino DM candidate over most of the surviving parameter space, and DM enters as a model-dependent input. Only in the minimal nonuniversal-Higgs-mass (NUHM) extension, where μ is freed, does a light neutralino relic reappear, and present direct detection data then exclude the well-tempered strip and leave only a ~ 1.1 TeV Higgsino.

In summary, we work with the effective $\mathbf{10} \oplus \overline{\mathbf{126}}$ Yukawa sector: the charged-fermion fit, the seesaw, and the dimension-five proton decay operators all follow from the fitted Yukawa couplings, while the colored Higgs sector is reduced to a single effective triplet scale M_{H_C} that sets the proton decay normalization and is pinned by gauge unification. A unique proton lifetime in a fully renormalizable completion would instead require the complete color triplet mass matrix together with the accompanying doublet-mixing factors, which lie beyond the effective treatment adopted here.

The paper is organized as follows. Section II sets out the model, including its Lagrangian, the $\mathbf{10} \oplus \overline{\mathbf{126}}$ Yukawa

sector, the universal soft terms, and our notation. Section III covers the theoretical consistency constraints and Sec. IV the direct experimental limits, notably dimension-five and dimension-six proton decay operators. Section V treats the flavor and precision observables. Section VI presents the electroweakino sector in the free- μ and constrained- μ schemes and the resulting allowed parameter space, and Sec. VII the fate of the lightest neutralino as DM in each scheme. Section VIII interprets the surviving region, compares it with prior work, and lists its near-future signatures. Section IX concludes with our findings.

II. THE MODEL

This section fixes the model and the notation: the field content and Yukawa sector of the minimal supersymmetric $SO(10)$, the color triplet and doublet-triplet sector that mediates proton decay and is tied to gauge coupling unification, the universal (CMSSM-like) soft supersymmetry-breaking sector that is our working hypothesis, and the inputs to the analysis.

A. Field content and the Yukawa sector

The matter sector of $SO(10)$ is maximally economical: a single irreducible spinorial $\mathbf{16}$ chiral superfield per generation accommodates all fifteen SM Weyl fermions of that generation, along with a right-handed neutrino N^c . We write the three generations as $\mathbf{16}_a$ with a family index $a = 1, 2, 3$ acting on the generation space. Because a single multiplet carries all the matter, the entire flavor structure of the theory is encoded in how the three $\mathbf{16}_a$ couple to the Higgs sector.

In the *minimal* model, the Yukawa couplings of the matter spinors arise from just two Higgs representations, a $\mathbf{10}$ and a $\overline{\mathbf{126}}$, through the renormalizable superpotential [5]:

$$W_Y = \mathbf{16}_a \left[(Y_{10})_{ab} \mathbf{10} + (Y_{126})_{ab} \overline{\mathbf{126}} \right] \mathbf{16}_b, \quad (1)$$

where the two Yukawa matrices Y_{10} and Y_{126} act on the family indices $a, b = 1, 2, 3$. The group theory of $SO(10)$ forces both matrices to be *symmetric*,¹ which is the source of the model's predictive power.

The single pair of symmetric matrices (Y_{10}, Y_{126}) therefore controls three distinct sectors at once. When the $\mathbf{10}$ and $\overline{\mathbf{126}}$ acquire electroweak-scale VEVs along their SM doublet directions, Eq. (1) produces the up-type, down-type, charged-lepton, and Dirac-neutrino mass matrices as fixed linear combinations of the same Y_{10} and Y_{126} . Explicitly, writing $v_{u,d}^{10}$ and $v_{u,d}^{126}$ for the up- and down-type Higgs doublet VEVs residing in the $\mathbf{10}$ and the $\overline{\mathbf{126}}$, we have

¹The product $\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \overline{\mathbf{126}}$ contains the $\mathbf{10}$ and the $\overline{\mathbf{126}}$ in its symmetric part, while the antisymmetric $\mathbf{120}$ is absent in the minimal model.

$$M_u = Y_{10}v_u^{10} + Y_{126}v_u^{126}, \quad (2)$$

$$M_{\nu_D} = Y_{10}v_u^{10} - 3Y_{126}v_u^{126}, \quad (3)$$

$$M_d = Y_{10}v_d^{10} + Y_{126}v_d^{126}, \quad (4)$$

$$M_e = Y_{10}v_d^{10} - 3Y_{126}v_d^{126}, \quad (5)$$

where the factor -3 multiplying the Y_{126} contribution to the leptonic masses is the $SU(4)_c$ Clebsch-Gordan coefficient by which the $\overline{126}$ distinguishes leptons from quarks (leptons carry $B - L = -1$, three times the quark value in magnitude), and is the origin of the $SO(10)$ analog of the Georgi-Jarlskog relations [5,13]. The two symmetric matrices Y_{10} , Y_{126} thus appear in all four charged- and neutral-fermion mass matrices at once, which is what makes the fit overdetermined. Fitting these two matrices to the measured charged-fermion masses and the CKM matrix therefore leaves essentially no freedom (up to residual fit ambiguities²). The same matrices thus control, through the unified $SO(10)$ structure, the flavor structure of the B -violating dimension-five Wilson coefficients C_L (for the operator $QQQL$) and C_R (for $u^c u^c d^c e^c$) that drive proton decay [8–10]; and, through the large $SO(10)$ -breaking VEV of the $\overline{126}$, the heavy Majorana mass matrix of the right-handed neutrinos. The flavor structure that reproduces the quark and lepton masses also fixes the proton decay amplitude, and the decay rate is a prediction of the fitted theory rather than an adjustable parameter.

The right-handed-neutrino Majorana mass deserves separate mention. The $\overline{126}$ contains a SM singlet whose VEV $v_R \equiv \langle \overline{126} \rangle$ breaks $B - L$ at a high scale and, with M_{ν_D} the Dirac neutrino mass of Eq. (3), realizes the type-I seesaw,

$$M_R = Y_{126}v_R, \quad m_\nu \simeq -M_{\nu_D}^T M_R^{-1} M_{\nu_D}, \quad (6)$$

a Majorana mass matrix for the right-handed neutrinos N^c together with the light neutrino mass matrix it induces. The same $\overline{126}$ VEV that feeds proton decay through the GUT-scale dynamics is therefore also responsible for the small neutrino masses.

We obtain the dimension-five Wilson coefficients C_L , C_R and the realized colored Higgs phase ϕ_{13} (the relevant CP phase of the color triplet coupling that enters the dimension-five amplitude) by fitting Y_{10} and Y_{126} to the charged-fermion masses and the CKM matrix at M_{GUT} and contracting the fitted matrices through the doublet-triplet vacuum. Crucially, and in contrast to analyses that leave the colored Higgs phase free and scan over it, we *derive* the phase in the effective fit, under the phase convention fixed

²These include discrete sign and phase branches, and the split of each fermion mass between the $\mathbf{10}$ and $\overline{\mathbf{126}}$, which leave the charged-fermion spectrum unchanged but are fixed, for the proton decay phase, by the color triplet sector below.

below: the measured third-generation b and τ Yukawas are real, which forces $(Y_{10})_{33}$ and $(Y_{126})_{33}$ to be real, so their relative sign is a discrete π flip rather than a tunable CP phase and $\phi_{13} \approx 0$. Varying the residual fit ambiguities still leaves $\phi_{13} \approx 0$, a robust prediction of the model.

More concretely, the third-generation down-quark and charged-lepton masses fix the relevant couplings through the $SU(4)_c$ relations $m_b \propto (Y_{10})_{33} + (Y_{126})_{33}$ and $m_\tau \propto (Y_{10})_{33} - 3(Y_{126})_{33}$. So two real measured masses fix two real couplings, $(Y_{10})_{33}, (Y_{126})_{33} \in \mathbb{R}$. The color triplet ratio inherits this reality, $C_R/C_L = 1 + \mathcal{O}((Y_{126}/Y_{10})_{33}) \in \mathbb{R}$ with $(Y_{126}/Y_{10})_{33} \simeq 0.06$, so $\phi_{13} = \arg(C_R/C_L)$ is fixed to the discrete values 0 or π , not a continuous phase, the realized signs giving $\phi_{13} \approx 0$. The color triplet couplings also carry mixing-matrix phases that the real diagonal third-generation masses do not by themselves fix. An explicit matrix-level fit of Y_{10} and Y_{126} including the triplet mixing nonetheless confirms $\phi_{13} \approx 0$: at the GUT-scale benchmark, the third-generation entries come out $(Y_{10})_{33} \approx 0.060$ and $(Y_{126})_{33} \approx -0.0038$ (both real, with $\arg[(Y_{126}/Y_{10})_{33}] = \pi$ a discrete sign flip and $|(Y_{126}/Y_{10})_{33}| \approx 0.06$), giving $|C_R/C_L| \approx 1.07$ and $\phi_{13} = \arg(C_R/C_L) \approx 0$. To go beyond the third-generation argument, we fit Y_{10} and Y_{126} to the charged fermion masses together with the complete CKM matrix, its magnitudes and its CP phase, and evaluate $\arg(C_R/C_L)$ over the residual fit ambiguities. The realized phase has a central branch sharply clustered near zero, with 96% of the fits within 10° , together with a small $\sim 4\%$ population on the discrete π branch; the overall circular spread (standard deviation) is 0.40 rad, dominated by that π -branch tail, with central value $\phi_{13} = -0.003$ rad and $|C_R/C_L| \approx 1.1$. The CKM phase enters C_R/C_L only through off-diagonal terms weighted by $|(Y_{126}/Y_{10})_{33}| \approx 0.06$ and the small quark mixing, so it broadens the distribution but does not move its center off the real axis. The prediction $\phi_{13} \approx 0$ is therefore a property of the full flavor structure, not of the third generation alone.

B. The color triplet sector and unification

The light Higgs doublets of the MSSM sit inside the $\mathbf{10}$ (and, through mixing, the $\overline{\mathbf{126}}$). Their unavoidable companions are the color triplet, weak-singlet Higgs states and, due to supersymmetry, the corresponding colored Higgsino of mass M_{H_c} . It is the *exchange* of this colored Higgsino between two quark-squark legs that generates the B -violating dimension-five operators, encoded in the effective superpotential [8–10],

$$W_5 = \frac{1}{M_{H_c}} \left[\frac{1}{2} C_L^{ijk\ell} (Q_i Q_j) (Q_k L_\ell) + C_R^{ijk\ell} u_i^c e_j^c u_k^c d_\ell^c \right], \quad (7)$$

where i, j, k, ℓ are the generation indices and the Wilson coefficients $C_L \propto Y_{10,126}^2$ and $C_R \propto Y_{10,126}^2$ are fixed in both

their overall size and their flavor structure by the same fitted Yukawa matrices, divided by the colored Higgsino mass M_{H_C} . Being dimension-five rather than dimension-six, these are suppressed by only a single power of the heavy mass, $\sim 1/M_{H_C}$, rather than two, which is why supersymmetric proton decay tends to be too fast. The dressed decay rate scales as $\Gamma \propto M_{H_C}^{-2}$, making M_{H_C} the single most important GUT-scale input to the proton lifetime to be examined in Sec. IV.

In a fully specified GUT Higgs sector, M_{H_C} is a particular combination of the **210/126** VEVs and superpotential couplings that break $SO(10)$, and the doublet-triplet splitting problem is the requirement that the doublets remain light while the triplets are heavy. Rather than commit to one such sector, we fix M_{H_C} from gauge-coupling unification: in the supersymmetric $SO(10) \rightarrow$ MSSM desert, the three SM gauge couplings run from their measured low-scale values, meet at the unification scale $M_{\text{GUT}} \approx 1.4 \times 10^{16}$ GeV with a unified coupling $\alpha_{\text{GUT}} \approx 1/24$ [1–4], and the color triplet threshold modifies the running through the specific combination of $3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1}$ [12] that isolates the triplet’s contribution alone. Demanding that the couplings still unify, given the precisely measured $\alpha_s(M_Z)$, confines M_{H_C} to a two-sided band rather than leaving it free. Because the same M_{H_C} sets the proton decay rate ($\Gamma \propto M_{H_C}^{-2}$), unification and proton decay are correlated. We construct the band, and work out how it interlocks with the proton decay bound, in Sec. III. Though in a complete Higgs sector M_{H_C} can exceed M_{GUT} , we impose $M_{H_C} \leq M_{\text{GUT}}$ as a calculational convention, treating the scale at which the couplings meet as the natural upper limit for any state in the unified multiplets.

C. The universal soft supersymmetry-breaking sector

Supersymmetry must be broken, and the breaking is parametrized by the soft Lagrangian

$$-\mathcal{L}_{\text{soft}} = \sum_i m_i^2 |\tilde{\phi}_i|^2 + \left(\frac{1}{2} \sum_{a=1}^3 M_a \lambda_a \lambda_a + \sum_f A_f \tilde{f} \tilde{f} H + \text{H.c.} \right), \quad (8)$$

collecting the scalar mass-squareds m_i^2 of the squarks, sleptons and Higgs scalars $\tilde{\phi}_i$; the Majorana masses M_a of the bino, wino and gluino λ_a ; and the trilinear scalar couplings A_f . We adopt the simplest and most predictive pattern, the *universal* or CMSSM ansatz, which fixes these soft terms at the GUT scale to

$$m_{\tilde{f}}^2 = m_0^2 \mathbb{1}, \quad M_1 = M_2 = M_3 = M_{1/2}, \quad A_f = A_0 Y_f, \quad (9)$$

with the universal scalar mass m_0 , the universal gaugino mass $M_{1/2}$, and the universal trilinear A_0 . The scalar masses, the gaugino mass, and A_0 in Eq. (9) are the boundary values imposed at M_{GUT} . The low-scale spectrum follows by renormalization-group running, and $|\mu|$ (and the soft parameter B) are the low-scale outputs fixed by electroweak symmetry breaking (EWSB), not inputs. Here, A_f is the trilinear soft coupling of the sfermion $\tilde{f}$, Y_f the corresponding SM fermion Yukawa coupling, and the boundary condition fixes $A_t(M_{\text{GUT}}) = A_0$ (and likewise for the other flavors). Here $m_0^2 \mathbb{1}$ is proportional to the identity in flavor space, so the squark and slepton soft masses are flavor-blind: there is no intergeneration hierarchy and no new flavor structure beyond the Yukawa (CKM-induced) one. This is what “universal” means, and, as noted in Sec. I, it is the most predictive soft sector because it leaves no room to weaken proton decay by decoupling certain generations, yielding a clean lower bound on m_0 . Two further parameters complete the specification: $\tan \beta \equiv v_u/v_d$, the ratio of the up- and down-type Higgs VEVs; and the sign of the supersymmetric Higgs mass parameter μ , since under the CMSSM boundary conditions $|\mu|$ is not free but fixed by the radiative EWSB minimization condition.

Three low-scale consequences of Eq. (9) matter for what follows. First, the common gaugino mass runs to the low-scale ratio

$$M_1 : M_2 : M_3 \simeq 1 : 2 : 6, \quad (10)$$

fixed by the SM gauge β -functions, where M_1 , M_2 , and M_3 are the bino, wino, and gluino soft masses. Both the wino and the gluino that dress the proton decay loop are therefore tied to the single scale $M_{1/2}$. Second, the universal trilinear sets the stop mixing

$$X_t = A_t - \mu \cot \beta, \quad (11)$$

where A_t is the low-scale stop trilinear (A_0 being its GUT-scale input), and X_t acts in the 2×2 stop mass-squared matrix. The parameter X_t drives the radiative correction to the Higgs mass and is essential to reaching $m_h = 125$ GeV to be detailed in Sec. IV. Third, because the scalars are free to be heavy while the gauginos and Higgsinos are kept near the TeV scale, the natural regime once the proton bound pushes m_0 up is a *minisplit* spectrum [14,15]: one in which the scalars are far heavier than the electroweakinos, which both dress the proton loop and preserve unification.

For the phenomenological scan we treat $\{m_0, M_2, \mu, \tan \beta, X_t\}$, together with M_{H_C} , as the working variables, with M_1 and M_3 following from Eq. (10). We sample M_2 and μ directly as effective low-scale inputs and impose only the sign requirement $\mu > 0$ for radiative EWSB. Solving the EWSB conditions point by point, under which M_2 and $|\mu|$ are determined by $\{m_0, M_{1/2}, A_0, \tan \beta\}$ rather than

TABLE I. Inputs to the analysis. Top block: the scanned soft-breaking parameters and their ranges (sampled uniformly in $\log_{10}$, except $\tan\beta$, which is linear), with M_1, M_3 fixed by the GUT ratio Eq. (10). Bottom block: the fixed SM and hadronic inputs, each with its source. For the observed Higgs mass m_h , a wider acceptance window is used instead in the scan, reflecting the *theory* uncertainty of the resummed m_h , not an experimental error.

Symbol	Value/range	Unit	Source/remark
<i>Scanned soft-breaking inputs</i>			
m_0	0.5–3000	TeV	Universal scalar mass
M_2	0.3–5	TeV	Wino mass; $M_1:M_2:M_3 \simeq 1:2:6$
$ \mu $	0.3–5	TeV	Effective input; sign $\mu > 0$ imposed
$\tan\beta$	2–50	...	Ratio of Higgs VEVs, v_u/v_d
X_t	$ X_t \leq \min(\text{pos.}, 6 m_0)$	...	Stop mixing = $A_t - \mu \cot\beta$; imposing stop-positivity bound and charge- and color-breaking stability imposed
M_{H_C}	Up to M_{GUT}	GeV	Colored Higgsino mass; pinned by unification
<i>Fixed SM and hadronic inputs</i>			
$\alpha_s(M_Z)$	0.1180 ± 0.0009	...	PDG 2026 [16]
m_t	172.6 ± 0.27	GeV	PDG 2026 [16]
m_h	125.13 ± 0.11	GeV	PDG 2026 [16] (listing average)
β_H	0.0144	GeV^3	Lattice [17] (proton decay matrix element)
$f_\pi^{(0)}$	86.8	MeV	Lattice [17]

chosen independently, is imposed separately in the constrained- μ cross-check in Sec. VI.

D. Inputs, benchmarks, and notation

Table I collects the inputs to the analysis: the scanned soft-breaking parameters with their ranges (top block) and the fixed SM and hadronic inputs that enter the proton decay rate and the Higgs mass (bottom block). The scanned parameters are exactly the working set of Sec. IIC.

We treat the electroweakino sector in two ways: (i) the constrained- μ scheme, the prediction of the universal hypothesis, in which $|\mu|$ and the spectrum are determined point by point from the GUT-scale boundary conditions Eq. (9) by renormalization-group (RG) running plus a tree-level EWSB treatment (Sec. VIA), and (ii) a free- μ scan, a minimal NUHM extension of the previous one, in which M_2 and $|\mu|$ are sampled directly as low-scale inputs (with $\mu > 0$ standing in for radiative EWSB) so that a light electroweakino can be probed (Sec. VIB). Two ingredients of the calculation are approximate. The Higgs boson mass is evaluated with an effective field theory-resummed (EFT-resummed) treatment, which uses a one-loop leading-log expression calibrated point-by-point to FeynHiggs 2.19 [18], which lowers it by ~ 6 –9 GeV for stop masses of tens of TeV (where the bare leading-log overshoots). The proton decay rate is normalized following the benchmark calculation of [19]. The dimension-five $p \rightarrow K^+ \bar{\nu}$ rate for the minimal $SO(10)$ color triplet sector is not available from a public code. The amplitude is evaluated from the fitted Yukawa couplings, contracted through the doublet-triplet vacuum, and normalized to the published Goto-Nihei rate, as detailed in Sec. IVA. The dimension-six gauge-boson decay mode is unobservably slow at these scales and is not

imposed. The individual constraints are developed in Secs. III–V, the electroweakino sector in Sec. VI, and DM in Sec. VII.

III. GAUGE UNIFICATION AND VACUUM STABILITY

The constraints in this section are theoretical consistency conditions, required for the universal minimal supersymmetric $SO(10)$ spectrum to be physical. A point in the parameter space that violates any of them describes a tachyonic stop, a charge- and color-breaking (CCB) vacuum, or a spectrum whose gauge couplings do not unify, and is thus removed. Gauge coupling unification plays a second role as well: it fixes the color triplet (colored Higgsino) mass band M_{H_C} that enters the proton decay rate, linking this section to the direct experimental limits of Sec. IV.

A. Gauge-coupling unification and the M_{H_C} band

In the supersymmetric $SO(10) \rightarrow \text{MSSM}$ desert, the three SM gauge couplings must, given their precisely measured low-scale values, converge to a single value at the unification scale $M_{\text{GUT}} \approx 1.4 \times 10^{16}$ GeV [1–4]. Here M_{GUT} is not an input but an output, the scale at which α_1 and α_2 are computed to cross under the supersymmetric running of the measured electroweak couplings. With only a weak dependence on the sparticle scale, it comes out at 1.427×10^{16} GeV for $m_0 = 12$ TeV, and varies by only 1.5% over the whole surviving region $m_0 = 10$ –30 TeV. This is the value used throughout, in the cap $M_{H_C} \leq M_{\text{GUT}}$ of Eq. (13) and in the dimension-six rate of Sec. IV B alike. Writing $\alpha_i^{-1}(\mu)$ for the inverse couplings of $U(1)_Y, SU(2)_L$

and $SU(3)_c$ ($i = 1, 2, 3$) in the standard grand unified normalization, the color triplet threshold modifies the running through the specific linear combination [12]

$$3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1}, \quad (12)$$

which is constructed to be blind to the heavy GUT-breaking gauge bosons and adjoints and to isolate exactly the contribution of the color triplet, weak-singlet Higgs partner. Reference [12] gives the systematic treatment of the dimension-five proton decay dressing together with the color triplet threshold matching used throughout this study. Because Eq. (12) retains the triplet but cancels the rest of the GUT spectrum, its measured value translates the requirement of unification directly into a condition on the triplet mass. The blindness to M_{GUT} is twofold. The coefficients in Eq. (12) sum to zero, so the unified coupling and every *complete* GUT multiplet, including the super-heavy gauge bosons whose mass defines M_{GUT} , contribute equally to the three α_i^{-1} and cancel; and those same coefficients annihilate the slope of the running between M_{H_C} and M_{GUT} (where the spectrum is the MSSM plus the color triplet), which removes the $\ln M_{\text{GUT}}$ term. What survives is set by the color triplet alone, an *incomplete* multiplet split from its light doublet partner. Consequently, M_{H_C} , and with it the proton decay bound, is independent of M_{GUT} and of the GUT-scale thresholds. Instead, M_{GUT} enters only the subdominant dimension-six mode (to be discussed in Sec. IV) and the condition $M_{H_C} \leq M_{\text{GUT}}$ below.

The practical consequence is sharp. Demanding that the three couplings still meet, given the accurately determined strong coupling $\alpha_s(M_Z)$ whose LEP-era determination first sharpened supersymmetric unification into a quantitative success [20–22], fixes the colored Higgsino mass M_{H_C} to a two-sided band rather than leaving it free. Our working window is

$$M_{H_C} \in [0.58, 1.43] \times 10^{16} \text{ GeV}, \quad (13)$$

a factor- ~ 2 band fixed by a two-loop running of Eq. (12) following the threshold analysis of [12]. The lower edge is the value below which the couplings overshoot before meeting, the upper edge the value above which they undershoot. The width of this band comes from the measured strong coupling. Propagating $\alpha_s(M_Z) = 0.1180 \pm 0.0009$ [16] through the unification condition gives $M_{H_C} \in [0.70, 1.35] \times 10^{16} \text{ GeV}$, a factor 1.9, at the reference scalar mass $m_0 \simeq 12 \text{ TeV}$ near the proton decay bound. Each edge is then widened by a factor 1.22, a conservative allowance for the two-loop and GUT threshold corrections of the size found in Ref. [12], and the upper edge is capped at M_{GUT} ; this gives the quoted Eq. (13), a factor 2.5. The band narrows as m_0 rises, since the upper edge saturates the cap while the lower edge continues to

rise. Because $\tau_p \propto M_{H_C}^2$ and $m_0 \propto \tau_p^{1/4} \propto M_{H_C}^{1/2}$, it maps to a factor $\sqrt{1.43/0.58} \simeq 1.6$ on the proton decay lower bound on m_0 , collected with the other uncertainties in Sec. IV A. That bound is evaluated at the most favorable in-band value $M_{H_C} = \min(M_{H_C}^{\text{hi}}, M_{\text{GUT}})$, where $M_{H_C}^{\text{hi}}$ is the upper edge of Eq. (13), which gives the weakest bound, so any smaller M_{H_C} within the band would only strengthen it. A complete, model-dependent GUT-threshold calculation would require the full colored Higgs sector and lies beyond the effective single- M_{H_C} treatment adopted here.

This band is the most consequential entry in this section. The same M_{H_C} that fixes unification also sets the proton decay rate, $\tau_p \propto M_{H_C}^2$. So the obvious way to lengthen the proton lifetime by raising M_{H_C} is strongly disfavored: a larger M_{H_C} spoils unification because the couplings then fail to meet. The normalization of the proton decay bound is therefore fixed by unification rather than left adjustable, a tie made quantitative in Sec. IV.

We additionally impose an upper bound on the colored Higgsino mass at the unification scale, $M_{H_C} \leq M_{\text{GUT}}$. In a fully specified GUT-Higgs sector, however, the triplet mass is a particular combination of VEVs and superpotential couplings and can in principle exceed M_{GUT} . But in the absence of such a detailed model, we treat M_{GUT} as a natural upper limit for any state living in the unified multiplets. This bound ties the upper edge of the band to the same running that defines M_{GUT} . Throughout this work, M_{H_C} denotes an effective single scale standing for the color triplet combination that controls both unification and the dimension-five rate. In a complete GUT-Higgs sector, these are entries of the inverse color triplet mass matrix together with doublet–triplet mixing. The single- M_{H_C} treatment is the leading parametrization, and resolving the full matrix structure is beyond the scope of the present analysis.

To estimate the effect of this assumption, take the simplest extension beyond a single triplet: two color triplet pairs with masses $M_1 \leq M_2$ and spread $r \equiv M_2/M_1$. Gauge unification constrains the sum of their logarithms, so it fixes their geometric mean, $M_{H_C}^{\text{unif}} = (M_1 M_2)^{1/2}$, whereas the dimension-five amplitude runs through the inverse mass matrix, $1/M_{H_C}^{\text{dec}} = |w_1^2/M_1 \pm w_2^2/M_2|$ with $w_1^2 + w_2^2 = 1$, the weights with which the operator couples to the two triplets. For comparable couplings and no cancellation, the decay scale is the harmonic mean, and the two effective scales differ by

$$\frac{M_{H_C}^{\text{unif}}}{M_{H_C}^{\text{dec}}} = \frac{1+r}{2\sqrt{r}} = \cosh\left(\frac{1}{2} \ln r\right), \quad (14)$$

a factor of 1.15, 1.7, and 2.8 for $r = 3, 10$, and 30 . Since $m_0 \propto M_{H_C}^{1/2}$, such a decoupling shifts the proton decay lower bound on m_0 by at most $(M_{H_C}^{\text{unif}}/M_{H_C}^{\text{dec}})^{1/2} \approx 1.1\text{--}1.7$

over the same range. In the generic case, the decay scale is the *smaller* of the two (the harmonic mean lies below the geometric mean), which would *strengthen* the bound. The one loophole is a relative sign between the two terms, $1/M_{H_C}^{\text{dec}} = |w_1^2/M_1 - w_2^2/M_2|$: a tuned near-cancellation can raise $M_{H_C}^{\text{dec}}$ and suppress the rate, the known freedom by which an extended color triplet sector can evade proton decay. We do not exploit this tuning in this work. The single- M_{H_C} choice is an effective parametrization of the minimal, most predictive scenario, not a general property of every supersymmetric $SO(10)$ model, and a more general sector would multiply the rate by a model-dependent factor that we do not scan.

B. Vacuum stability: Stop positivity and the CCB bound

The stop trilinear coupling $X_t = A_t - \mu \cot \beta$ controls the top-squark left–right mixing and enters the one-loop Higgs mass through the polynomial $f(x) = x^2(1 - x^2/12)$ with $x = X_t/M_S$, where M_S is the stop mass scale and here taken to be m_0 . The function $f(x)$ peaks at the maximal-mixing point $|X_t| = \sqrt{6}M_S \approx 2.449M_S$, with $f(\sqrt{6}) = 3$ and turns *negative* for $|X_t| > \sqrt{12}M_S \approx 3.46M_S$. The sign of f is what matters physically: for $|X_t|/M_S < \sqrt{12}$ the mixing *raises* m_h (most strongly at $\sqrt{6}$), while in the negative- f window $|X_t|/M_S > \sqrt{12}$ it *lowers* m_h . Reaching $m_h = 125$ GeV needs the positive- f region, so the negative- f window must be excluded; otherwise, an arbitrarily heavy m_0 could be tuned back to 125 GeV by over-large mixing, spuriously erasing the upper bound on m_0 . The restriction on the mixing comes instead from the following two physical criteria.

Stop positivity requires both eigenvalues of the full 2×2 top-squark mass-squared matrix to be positive (no tachyonic stop). In the universal scenario, with both soft stop masses equal to m_0 and the electroweak D -terms negligible against m_0^2 , the smaller eigenvalue is $\approx m_0^2 + m_t^2 - m_t|X_t|$ in the off-diagonal-dominant approximation. So the positivity reads $|X_t| \lesssim (m_0^2 + m_t^2)/m_t$. This is essentially nonbinding: because the stops are heavy here ($m_0 \gg m_t$), they stay nontachyonic even under enormous mixing, the bound being far above the $\sqrt{12} \approx 3.46$ where the Higgs physics matters (e.g., $|X_t|/m_0 \lesssim m_0/m_t \sim 17$ at $m_0 \sim 3$ TeV).

Charge- and color-breaking metastability is the operative condition. Physically, the ordinary electroweak vacuum must not be destabilized by a deeper minimum along a CCB field direction: a large soft trilinear can tilt the scalar potential so that a vacuum where the stop fields acquire VEVs, thereby breaking color and electric charge at once, lies below the standard one, which would be catastrophic. The constraint on the soft trilinears was first developed by Frère-Jones-Raby and Claudson-Hall-Hinchliffe [23,24].

We impose it in the empirical sufficient-stability form of Casas, Lleyda and Muñoz [25],

$$|A_t|^2 + 3\mu^2 < 3(m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2), \quad A_t = X_t + \mu \cot \beta, \quad (15)$$

evaluated in the universal scenario $m_{Q_3}^2 = m_{U_3}^2 = m_0^2$. Its right-hand side still depends on the up-type Higgs soft mass $m_{H_u}^2$, which runs and is not fixed at the stop scale. So we bracket it: the primary cut uses $m_{H_u}^2 = m_0^2$, and we also keep the conservative choice $m_{H_u}^2 = 0$, which drops that term and gives the tightest bound. With $m_{H_u}^2 = m_0^2$, the constraint maximizes the mixing at $|X_t|/m_0 < 3.06$; the conservative $m_{H_u}^2 = 0$ value would tighten this only to $\sqrt{6} \approx 2.45$. Either upper bound sits below the negative- f threshold $|X_t|/m_0 = \sqrt{12} \approx 3.46$. Therefore, CCB stability forbids the entire window where the mixing would lower m_h . In other words, it is the vacuum stability, not the stop positivity, that closes off that region.

C. The $\mu > 0$ sign requirement

We sample the Higgsino mass parameter μ as an effective input and impose only its *sign*, $\mu > 0$, as a conservative constraint. Such a sign choice, which does not solve the electroweak-minimization condition, reproduce M_Z , or test spectrum viability, is a weak condition, but it connects to a deeper issue. Under the universal (CMSSM) boundary conditions, $|\mu|$ is not a free input but is determined by $\{m_0, M_{1/2}, A_0, \tan \beta\}$ through the electroweak minimization condition. A more rigorous scan for radiative EWSB, and the question of whether the light- μ region is actually realized, is examined in the constrained- μ cross-check in Sec. VI.

D. The consistency cuts in the parameter space

Figures 1 and 2 display the two consistency conditions in isolation, before any experimental limit is overlaid. Neither is a bound on m_0 by itself: unification fixes the M_{H_C} band (the empty region in Fig. 1), which in turn will normalize the proton decay bound of Sec. IV, while stop positivity together with CCB limits the stop mixing (Fig. 2), which controls where the upper edge of the Higgs mass constraint falls rather than bounding m_0 directly.

The lower edge of the unification band in Fig. 1 rises with m_0 because heavier scalars push the color triplet mass required by gauge unification upward toward the unification scale (the minisplit spectrum sharpens the meeting of the couplings), until the band is forced to saturate the bound $M_{H_C} \leq M_{\text{GUT}}$ and falls with the slowly decreasing M_{GUT} . The upper (lower) edge reaches that limit around $m_0 \sim 7$ TeV (~ 166 TeV).

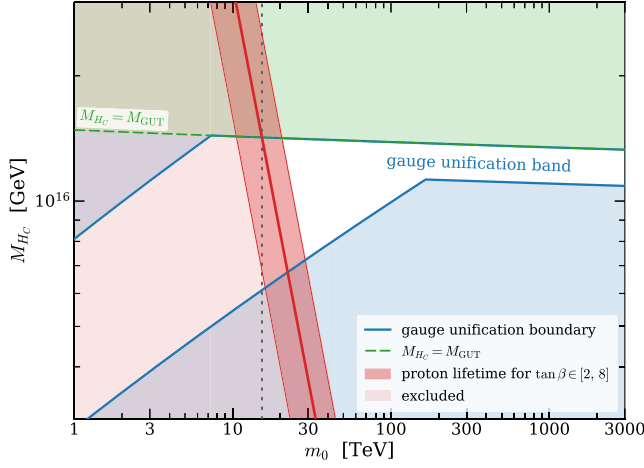


FIG. 1. Gauge unification and proton decay constraints together in the (m_0, M_{H_C}) plane. Demanding that the combination $3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1}$ unify (given the measured α_s) fixes the colored Higgsino mass to the band $M_{H_C} \in [0.58, 1.43] \times 10^{16}$ GeV of Eq. (13), subject to the assumed bound $M_{H_C} \leq M_{GUT}$. The proton lifetime lower bound rises with $\tan\beta$ and is drawn in red for $\tan\beta \in [2, 8]$ (thick red at 5). Here, we take $M_2 = \mu = 1$ TeV. The colored regions are excluded: when M_{H_C} lies outside the unification band (light blue), when $M_{H_C} > M_{GUT}$ (light green), and when the proton lifetime $\tau_p < 6.61 \times 10^{33}$ yr (light red).

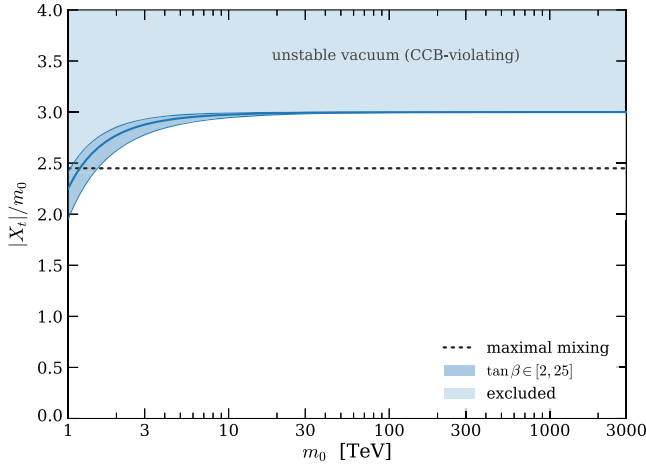


FIG. 2. The vacuum stability consistency condition [stop positivity together with the Casas-Lleyda-Muñoz CCB bound, Eq. (15)]. The dotted line marks the maximal-mixing reference value $|X_t|/m_0 = \sqrt{6}$, the stop mixing that maximizes the one-loop contribution to the light-Higgs mass, shown for reference. The CCB bound of Casas, Lleyda and Muñoz [25] is given by the solid curve, sitting just above the dotted line and reaching the mixing at $|X_t|/m_0 \approx 3$, still below the negative- f threshold of $\sqrt{12} \approx 3.46$. The region above the CCB curve has an unstable (CCB-violating) vacuum and is excluded. In this plot, we take $\tan\beta \in [2, 25]$ (thick blue at 5), $\mu = 1$ TeV, and top mass $m_t = 172.6$ GeV.

IV. PROTON DECAY AND THE HIGGS-MASS BOUND

The direct experimental limits considered in this section constrain the universal scalar mass m_0 from both sides. Supersymmetric proton decay sets a lower bound on m_0 that rises with $\tan\beta$; the measured 125 GeV Higgs mass sets an upper bound that falls with $\tan\beta$. The two cross at $\tan\beta \approx 8.0$, so the allowed region is confined to low-to-moderate $\tan\beta$ and heavy scalars. The LEP chargino limit constrains only the light electroweakino sector and does not bound m_0 .

A. Dimension-five proton decay

In a supersymmetric GUT, the dominant B -violating interactions are not gauge-boson exchange but a pair of dimension-five four-fermion operators,³

$$\mathcal{O}_L = (QQQL), \quad \mathcal{O}_R = (u^c u^c d^c e^c), \quad (16)$$

with Q, L the quark and lepton $SU(2)_L$ doublet superfields and u^c, d^c, e^c the conjugate singlets. They arise from exchange of the colored Higgsino—the color triplet, weak-singlet partner of the Higgs doublets, of mass M_{H_C} fixed by gauge-coupling unification in Sec. III—and, being dimension five, are suppressed by a single power $1/M_{H_C}$ rather than the $1/M_{GUT}^2$ of the gauge mode, which makes supersymmetric proton decay fast.

Wilson coefficients. Integrating the color triplet out of the superpotential generates the two operators with seesawlike Yukawa sandwiches [9] in the *same* matrices Y_{10}, Y_{126} that reproduce the charged-fermion masses and the CKM matrix. With the $\mathbf{10} \oplus \overline{\mathbf{126}}$ Higgs fields the left-handed coefficient is [13]

$$C_L = (Y_{10}, Y_{126}) \tilde{\mathcal{M}}_C^{-1} (Y_{10}, Y_{126})^T, \quad (17)$$

with $\tilde{\mathcal{M}}_C$ the color triplet mass matrix (lightest eigenvalue M_{H_C}). The right-handed coefficient shares this structure, with the left vertex dressed by the triplet mixing,

$$C_R = (Y_{10} - aY_{126}, bY_{126}) \tilde{\mathcal{M}}_C^{-1} \begin{pmatrix} Y_{10} \\ Y_{126} \end{pmatrix}, \quad (18)$$

$$a = \frac{(\mathcal{M}_C)_{13}}{(\mathcal{M}_C)_{33}}, \quad b = 1 - \frac{(\mathcal{M}_C)_{32}}{(\mathcal{M}_C)_{33}},$$

the dressing a, b being proportional to the $\overline{\mathbf{126}}$ admixture. The two coefficients are nearly equal for a symmetry

³These operators were identified in supersymmetric grand unification in 1981–82 by Weinberg, Sakai and Yanagida, and Dimopoulos, Raby and Wilczek [8–10]. Their appearance makes the supersymmetric proton lifetime far shorter than the dimension-six estimate, the recurrent “dimension-five problem” of supersymmetric GUTs.

reason: $SO(10)$ D-parity (the $L \leftrightarrow R$ exchange of the Pati-Salam chain) maps $\mathcal{O}_L \leftrightarrow \mathcal{O}_R$, and the dominant mediator—the color triplet of $\mathbf{10}_H$, a Pati-Salam $(6, 1, 1)$ and hence D-parity even—couples to the two bilinears identically, giving $C_R = C_L$ in the $\mathbf{10}_H$ limit. Only the $\overline{\mathbf{126}}_H$ breaks D-parity, so C_R/C_L departs from unity linearly in the (real) third-generation admixture $|Y_{126}/Y_{10}| \simeq 0.06$; we obtain $|C_R/C_L| \simeq 1.07$ with $\phi_{13} \simeq 0$.

Dressing into a physical amplitude. The bare operators must be dressed into a physical $\Delta B = 1$ amplitude by closing the sfermion line into a one-loop diagram with a gaugino or Higgsino [11,12]. Although the gluino carries the largest coupling, the gluino and neutralino loops are flavor-diagonal and subject to a super-Glashow-Iliopoulos-Maiani (super-GIM) cancellation [10,12]: they vanish for exactly degenerate squarks and switch on only with inter-generation squark splitting. In the universal spectrum the squarks are degenerate up to small RG running, so these contributions are strongly suppressed and the *chargino* dressing dominates—a wino loop for $\mathcal{O}_L$ and a Higgsino loop for $\mathcal{O}_R$. Both chargino dressings are built from the Goto-Nihei one-loop function [19,26]

$$F(m_a, m_b) = \frac{\ln(m_a^2/m_b^2)}{m_a^2 - m_b^2} \xrightarrow{m_a \rightarrow m_b} \frac{1}{m_a^2}, \quad (19)$$

evaluated on the two internal sfermion masses, and read

$$D_L = \frac{g_2^2}{16\pi^2} \left(\tan\beta + \frac{1}{\tan\beta} \right) M_2 F(m_{\tilde{q}}, m_{\tilde{\tau}}), \quad (20)$$

$$D_R = \frac{y_t y_\tau}{16\pi^2} \left(\tan\beta + \frac{1}{\tan\beta} \right)^2 \mu F(m_{\tilde{t}}, m_{\tilde{\tau}}). \quad (21)$$

Here g_2 is the $SU(2)_L$ coupling, $M_2(\mu)$ the wino (Higgsino) chargino mass, and $y_{t,\tau}$ the loop Yukawa vertices. The wino piece carries one power of the chirality factor $(\tan\beta + 1/\tan\beta)$ and the Higgsino piece two, so the right-handed dressing is more $\tan\beta$ -enhanced.

Decay width. The physical decay amplitudes are $\mathcal{A}_{LLLL} = \eta_L D_L \mathcal{A}_L^{(0)}$ and $\mathcal{A}_{RRRR} = \eta_R D_R \mathcal{A}_R^{(0)}$, with $\mathcal{A}_X^{(0)}$ the Cabibbo-weighted ($\lambda_C \simeq 0.22$) combination of the Wilson coefficients C_X [26]. The decay width is the sum of the two contributions [19,26],

$$\Gamma(p \rightarrow K^+ \bar{\nu}) = \Gamma_L + \Gamma_R, \quad \Gamma_X = \frac{m_p}{32\pi f_\pi^2} |\beta_H|^2 (A_{LD} A_{SD})^2 |\mathcal{A}_{XXX}|^2, \quad (22)$$

with m_p the proton mass, f_π the pion decay constant, β_H the lattice hadronic matrix element of the three-quark proton-to-vacuum operator (dimension GeV^3), and A_{LD}, A_{SD} the long- and short-distance quantum chromodynamics (QCD) renormalization factors. The color triplet

mass band that normalizes the amplitude is set by two-loop gauge coupling running [12]. The long- and short-distance QCD factors are $A_{LD} = 0.32$ and $A_{SD} = 0.93$ [13], common to both chiralities, and $\beta_H = 0.0144 \text{ GeV}^3$ is taken from Ref. [17]. The factors η_L and η_R introduced above carry the dimension-five Wilson coefficients from M_{GUT} down to the SUSY-breaking scale M_{SUSY} , and they are evaluated separately for the two operators [19],

$$\eta_X = \prod_{i=1}^3 \left[\frac{\alpha_i(M_{\text{GUT}})}{\alpha_i(M_{\text{SUSY}})} \right]^{-c_i^X/2b_i} \exp \left[-\frac{1}{16\pi^2} \int_{\ln M_{\text{SUSY}}}^{\ln M_{\text{GUT}}} \mathcal{I}_X dt \right], \quad (23)$$

where b_i are the one-loop MSSM beta-function coefficients, $c^L = (2/5, 6, 8)$ and $c^R = (12/5, 0, 8)$ for $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$, and the Yukawa insertions $\mathcal{I}_L = y_\tau^2$ and $\mathcal{I}_R = 2y_t^2 + 2y_\tau^2$. The two differ for a group-theoretic reason: the right-handed operator $u^c u^c d^c e^c$ carries no $SU(2)_L$ charge, so $c_2^R = 0$ and it does not have the weak running that acts on $QQQL$ through $c_2^L = 6$, and it picks up two additional top-Yukawa insertions. At $M_{\text{SUSY}} = 1 \text{ TeV}$, this gives $\eta_L = 5.11$ and $\eta_R = 2.88$ at $\tan\beta = 5$, so the left-handed coefficient is enhanced by a factor $\eta_L/\eta_R \simeq 1.8$ relative to the right-handed one.

Because κ is fixed by the calibration point, the size of this effect is best quoted with κ recalibrated for each choice. Doing so, replacing the two separate factors by a single common one lowers the proton decay bound at $\tan\beta = 2$ from $m_0 \gtrsim 11.0 \text{ TeV}$ to 9.3 TeV , an 18% shift and not a refinement that can be dropped.

The quark mixing that enters the Wilson coefficients is likewise taken at the unification scale rather than at the weak scale. Running the CKM matrix up through the MSSM [27] suppresses the two third-generation angles by about 7%, $\sin\theta_{13}:0.00369 \rightarrow 0.00342$ and $\sin\theta_{23}:0.0405 \rightarrow 0.0375$, while θ_{12} is stable under the running; the phase convention $\delta_{13} = 90^\circ$ is unchanged. Its net effect on the bound is small once κ is recalibrated, $11.1 \rightarrow 11.0 \text{ TeV}$ at $\tan\beta = 2$, because the same suppression acts on the calibration point and on the physical point. We use the unification-scale values because that is the scale at which the Wilson coefficients are generated. Both refinements are included in every number quoted below. Through D_X and F , each piece inherits the parametric scaling

$$\Gamma_X \propto \frac{(m_X^X)^2}{m_0^4 M_{H_C}^2} \left(\tan\beta + \frac{1}{\tan\beta} \right)^{2p_X} |\mathcal{A}_X^{(0)}|^2, \quad \text{with } p_L = 1, \quad p_R = 2, \quad (24)$$

where $m_X^X = M_2, \mu$ and the heavy-sfermion limit $F \sim 1/m_0^2$. The factor common to both, $1/(m_0^4 M_{H_C}^2)$, makes the lifetime rise as the fourth power of the scalar mass; this is what turns the proton lifetime limit into a *lower* bound on

m_0 that strengthens with $\tan\beta$, the strongest lower bound in the model and the reason a light, natural supersymmetric spectrum is excluded. The rate Eq. (24) is evaluated point by point over the $(\tan\beta, m_0)$ plane and over the electroweakino dressing mass, so the resulting bound on m_0 is a curve rather than a single point. The form $\Gamma_X \propto (m_\chi^X)^2/m_0^4$, and with it $m_0 \propto (m_\chi^X)^{1/2}$, is the *asymptotic* behavior reached only once the electroweak mixing in the chargino mass matrix is negligible. It is the mass insertion $M_2 = \mu$ that the dressing amplitude is proportional to, not the physical chargino mass m_χ^X . For $M_2 = \mu$, the two chargino eigenvalues remain split by $\sqrt{2}m_W(\sin\beta + \cos\beta) \simeq 134$ GeV. At $m_\chi^X = 0.1$ TeV, the Higgsino dressing mass is therefore 2.3 times m_χ^X , which raises the rate above a strict $(m_\chi^X)^2$ law and flattens the bound to $m_0 \propto (m_\chi^X)^{0.45}$. The quoted values come from the point-by-point evaluation, not from the asymptotic form. At $\tan\beta = 5$ the bound moves from about 5.8 to 26.8 TeV as m_χ^X ranges over 0.1–3 TeV (the light orange band in Fig. 5), and we quote the reference value $m_\chi^X \simeq 1$ TeV throughout.

We collect here, in one place, everything that moves this bound. Every number in this paragraph is quoted at $\tan\beta = 2$ and, where a dressing mass is not stated, at $m_\chi = 1$ TeV, m_χ being the mass of the lighter chargino. Two are theoretical uncertainties. The overall normalization of the dimension-five rate, the constant κ set by the calibration reference point, carries a factor $\lesssim 1.7$, which through $m_0 \propto \tau_p^{1/4}$ is $\pm 14\%$ on m_0 , a band of about [9.6, 12.5] TeV around the 11.0 TeV central value. The color triplet mass band of Sec. III A gives a further factor $\simeq 1.6$ through $m_0 \propto M_{H_C}^{1/2}$. The lattice matrix element β_H and the residual spread over the color triplet phase ϕ_{13} , which moves the $\tan\beta = 2$ bound within [9.9, 11.6] TeV, are both subdominant. The dressing mass m_χ is by contrast an input rather than an uncertainty, and it is the largest single lever on the number. At $\tan\beta = 2$ the bound runs

$$m_0 \gtrsim 4.1, \quad 11.0, \quad 18.4 \text{ TeV} \quad \text{for } m_\chi = 0.1, \quad 1, \quad 3 \text{ TeV}, \quad (25)$$

well described by $m_0 \propto m_\chi^{0.45}$ over that range. The LEP chargino limit already allows the lowest of these, $m_\chi \simeq 0.1$ TeV. The corresponding numbers at $\tan\beta = 5$, 5.8 and 26.8 TeV at the two ends, are larger throughout because the bound rises with $\tan\beta$. So every quoted value of m_0 carries the reference choice $m_\chi \simeq 1$ TeV with it.

Left/right balance. Because $C_R/C_L \simeq 1$, the amplitude ratio

$$\frac{|\mathcal{A}_{RRRR}|}{|\mathcal{A}_{LLLL}|} = \frac{D_R |\mathcal{A}_R^{(0)}|}{D_L |\mathcal{A}_L^{(0)}|} \quad (26)$$

TABLE II. The ratio $|\mathcal{A}_{RRRR}|/|\mathcal{A}_{LLLL}|$ of the right-handed (Higgsino-dressed) to left-handed (wino-dressed) $p \rightarrow K^+ \bar{\nu}$ amplitudes, which is independent of m_0 . It grows with $\tan\beta$ and crosses unity at $\tan\beta \simeq 7.6$, below (above) which the left-handed (right-handed) piece dominates.

$\tan\beta$	2.5	6	10
$ \mathcal{A}_{RRRR} / \mathcal{A}_{LLLL} $	0.146	0.672	1.795

has its dominant growth from the extra power of the chirality factor $(\tan\beta + 1/\tan\beta)$ in D_R/D_L , with a milder further rise from the Yukawa and Wilson-coefficient weighting (so the computed ratio grows somewhat faster than $(\tan\beta + 1/\tan\beta)$ alone). It is independent of m_0 , grows with $\tan\beta$, and crosses unity at $\tan\beta \simeq 7.6$ (Table II). Across the allowed window $\tan\beta \lesssim 8.0$ (to be seen in Sec. IV C), the left-handed piece dominates below $\tan\beta \simeq 7.6$ and the right-handed piece above. Because this ratio varies by roughly one order of magnitude over the allowed window, neither chirality amplitude of the $p \rightarrow K^+ \bar{\nu}$ decay can be neglected, and the customary wino-dressed-only estimate is inadequate here. Several observations follow. As the two pieces add coherently and, since the real third-generation Yukawas force $\phi_{13} \simeq 0$ (below), constructively, their comparable size *raises* the total rate above a left-handed-only calculation and strengthens the proton-decay lower bound on m_0 ; dropping the right-handed operator would instead give a too-loose bound. The $\mathcal{O}(1)$ ratio is also what makes the relative phase ϕ_{13} physically consequential: the interference term is comparable to the diagonal ones, so although the *total* rate shifts by only tens of percent under $\phi_{13} \rightarrow \pi$ (diluted because the $\bar{\nu}_\mu, \bar{\nu}_e$ channels carry the left-handed operator alone), the $\bar{\nu}_\tau$ partial rate is strongly phase-sensitive. Because the two operators carry different flavor structures, the decay *pattern* is $\tan\beta$ -dependent, and the region above the crossing is right-handed dominated. We do not, however, put this forward as a signature to be tested. The right-handed region inside the allowed window runs only from the crossing at $\tan\beta \simeq 7.6$ to the upper limit at 8.0, whereas that upper limit carries the asymmetric uncertainty $^{+5.5}_{-2.0}$ of Eq. (37). The interval is therefore contained well within the uncertainty of its own boundary, and no measurement of the branching composition could resolve it. Finally, the comparable right-handed piece is a fingerprint of the **126**, whose couplings generate both C_R and the Majorana/seesaw structure, tying the high- $\tan\beta$ proton-decay edge to the same $\tan\beta$ -enhanced down-quark and charged-lepton Yukawas that fix m_b and m_τ . The same trend appears in a strongly hierarchical sfermion spectrum, where the light-generation left-handed loops decouple and the right-handed piece dominates throughout.

Phase, modes, and experimental limits. The two pieces add coherently in the $\bar{\nu}_\tau$ channel (the $\bar{\nu}_\mu, \bar{\nu}_e$ channels carry only $\mathcal{O}_L$) with a relative phase ϕ_{13} . We derive

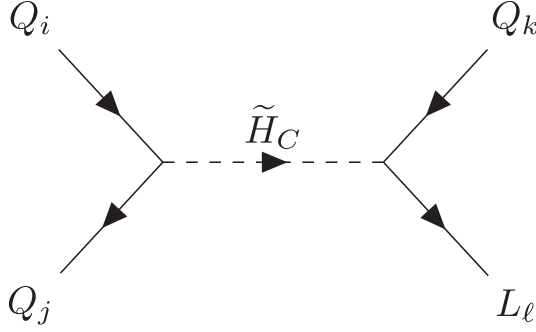


FIG. 3. The dimension-five amplitude that drives $p \rightarrow K^+ \bar{\nu}$ in minimal supersymmetric $SO(10)$. The colored Higgsino is exchanged between the external matter superfield legs, generating the B -violating four-fermion operator $\mathcal{O}_L = QQQ\bar{L}$ of Eq. (16). (The squark $\rightarrow$ gaugino/Higgsino loop dressing that produces the physical $\Delta B = 1$ amplitude is not drawn.)

this phase rather than scan over it: the measured third-generation b , τ Yukawas are real, forcing a real relative sign, so the realized $\phi_{13} \simeq 0$ and the interference is *constructive*; flipping $\phi_{13} \rightarrow \pi$ shifts the total rate by only tens of percent, so the bound is robust against it. The most important mode is $p \rightarrow K^+ \bar{\nu}$ (the strange final state is favored because the dominant operator carries a second-generation index and the kaon is kinematically accessible). As a cross-check, we also compute $p \rightarrow \mu^+ K^0$ from the same coefficients and overall normalization [19,26]. We impose the current limits

$$\begin{aligned} \tau_p(p \rightarrow K^+ \bar{\nu}) &> 6.61 \times 10^{33} \text{ yr}, \\ \tau_p(p \rightarrow \mu^+ K^0) &> 3.6 \times 10^{33} \text{ yr}, \end{aligned} \quad (27)$$

with the first from the Particle Data Group [16] and the second from Super-Kamiokande [28], the former superseding the earlier Super-Kamiokande value $> 5.9 \times 10^{33}$ yr [29]. We write $\tau_p = \kappa / \Gamma_{\text{shape}}$, computing the parameter dependence Γ_{shape} and fixing the single overall constant κ once to the published Goto-Nihei rate (a factor-of- ~ 2

calibration [19]). We fix the overall constant κ so that our computation gives $\tau_p = 3 \times 10^{31}$ yr at the Goto-Nihei reference point ($m_0 = 1$ TeV, $M_2 = 125$ GeV, $\mu = 500$ GeV, $\tan \beta = 2.5$). Figure 3 shows the mechanism. This absolute normalization is calibrated to the Goto-Nihei rate and uses the single effective M_{H_C} ; it does not invoke the color triplet doublet-mixing factors of specific complete-model constructions. Because κ is fixed in this way, the Goto-Nihei point is a calibration and not a test of our amplitude. What the calibration does fix externally is the overall scale of the rate. The dependence on m_0 , $\tan \beta$ and the electroweakino masses, and the relative weight of the left- and right-handed operators, are ours. The dressed amplitude has been evaluated using Python and Mathematica codes, which agree at the three points ($\tan \beta, m_0/\text{TeV}$) = (2, 12), (5, 20) and (8, 25) inside the allowed region to better than 10^{-8} (see Table III). The two codes are written based on the same formulas, as a check of the coding. No public code computes this observable for the minimal $SO(10)$ color triplet sector, and the amplitude that gives the main result therefore has no check from outside the work of the authors.

One further comparison suggests itself, and we explain why we do not make it. Reproducing the published Goto-Nihei lifetimes at parameter points *away* from the calibration point would be a real test rather than a calibration. Their calculation, however, is for minimal $SU(5)$ supergravity. The hadronic matrix element β_H , the QCD renormalization factors $A_{\text{LD}} A_{\text{SD}}$, and the dressing loop function $F(m_a, m_b)$ of Eq. (19) are common to the two models, but the Wilson coefficients are not. Ours follow from the $\mathbf{10} \oplus \overline{\mathbf{126}}$ fit of Eqs. (17) and (18) and differ from the $SU(5)$ ones both in flavor structure and in the chirality ratio, which is $|C_R/C_L| \simeq 1.07$ here and is generated with a different weight in minimal $SU(5)$. Evaluating our amplitude at their parameter points would therefore mix a difference of implementation with a difference of model, and neither outcome could be read cleanly. A meaningful comparison would be outside the scope of the present analysis.

TABLE III. Public-code benchmark comparisons underlying the analysis: for each observable, the benchmark point, this work's value, and the public-code value. Benchmark BM is $m_0 = 45$ TeV, $M_{1/2} = 1.3$ TeV, $\tan \beta = 5$, $A_0 = 0$. The last three rows are the dimension-five proton lifetime at the listed benchmark points, with $M_2 = \mu = 1$ TeV and $\phi_{13} = 0$. The two entries in the third column are the values computed by our Python and Mathematica codes, in that order.

Observable	Benchmark	This work	Public code
m_h (fixed order)	BM, CMSSM input	~ 142 GeV	SPheno: 142.3 GeV
m_h (resummed)	BM	≈ 125 GeV	FeynHiggs: 124.7 GeV
Ωh^2 (Higgsino)	$\mu = 1.1$ TeV	0.12	micrOMEGAs: 0.122
σ_{SI} (Higgsino)	$\mu \leq 1.1$ TeV	$(2-3) \times 10^{-47} \text{ cm}^2$	micrOMEGAs: $(2.2-2.6) \times 10^{-47} \text{ cm}^2$
$ \mu $ (constrained- μ)	$\tan \beta = 5$	Multi-TeV	SPheno: 7–33 TeV
$\tau_p(p \rightarrow K^+ \bar{\nu})$ [10^{34} yr]	(2, 12)	2.14879727/2.14879728	None available (see the text)
	(5, 20)	3.78359687/3.78359684	
	(8, 25)	2.88037069/2.88037071	

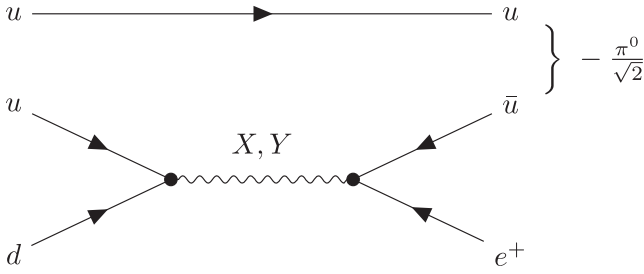


FIG. 4. The dimension-six $p \rightarrow e^+ \pi^0$ decay, mediated by the heavy X, Y grand unified gauge bosons and suppressed by $1/M_{\text{GUT}}^2$, as computed in Eq. (28). At $M_{\text{GUT}} = 1.43 \times 10^{16}$ GeV found in Sec. III A, the lifetime is 3.3×10^{35} yr, far longer than the dimension-five mode of Fig. 3.

The dimension-five rate scales as $\Gamma \propto M_{H_C}^{-2}$, so a longer proton lifetime demands a heavier colored Higgsino: proton decay therefore bounds M_{H_C} from *below*. With grand unification bounding M_{H_C} from above and proton decay raising it from below, M_{H_C} is pinched from both sides, leaving m_0 as the only free quantity, as shown by the red region for $\tan \beta \in [2, 8]$ in Fig. 1. In the “central” case of $\tan \beta = 5$ given by the thick red curve, the allowed strip first opens at $m_0 \simeq 15.3$ TeV (marked by the vertical dotted line), where the descending proton decay lower bound meets the upper edge of the grand unification band, and widens to the full band for $m_0 \gtrsim 22.3$ TeV. One cannot raise M_{H_C} to evade the proton bound, because the same M_{H_C} also sets the unification condition.

B. Dimension-six proton decay

Another grand unified proton decay mode is the *dimension-six* channel $p \rightarrow e^+ \pi^0$, mediated by tree-level exchange of the heavy X, Y gauge bosons that acquire mass $\sim M_{\text{GUT}}$ when $SO(10)$ breaks, as shown in Fig. 4. Integrating them out gives dimension-six four-fermion operators suppressed by *two* powers of the heavy scale, so the rate scales as

$$\Gamma(p \rightarrow e^+ \pi^0) \sim \frac{g_{\text{GUT}}^4}{M_{\text{GUT}}^4} m_p^5, \quad \tau_p(p \rightarrow e^+ \pi^0) \sim 10^{35} \text{ to } 10^{36} \text{ yr}, \quad (28)$$

where g_{GUT} is the unified gauge coupling and the numerical estimate uses the unification scale of Sec. III A, $M_{\text{GUT}} = 1.43 \times 10^{16}$ GeV, and $\alpha_{\text{GUT}} = g_{\text{GUT}}^2/4\pi \approx 1/24$, together with the lattice hadronic matrix element. The two chiral structures generated by gauge boson exchange do not carry the same weight: with the operators in the form given in Ref. [12], the CKM multiplicity factor $(1 + |V_{ud}|^2)$ multiplies the structure with a left-handed diquark alone. Hence, the squared amplitude carries $|W_0|^2 [A_1^2 + A_2^2 (1 + |V_{ud}|^2)^2]$ rather than a common factor, with $W_0 = 0.131 \text{ GeV}^2$ being the lattice matrix element of Ref. [17] (equal for the two

structures by parity) and $A_{1,2}$ being the renormalization factors from M_{GUT} down to 2 GeV. Taking $A_1 = 2.78$ and $A_2 = 2.91$ [30], we obtain

$$\tau_p(p \rightarrow e^+ \pi^0) = 5.15 \times 10^{35} \text{ yr } [SU(5)], \quad 3.34 \times 10^{35} \text{ yr } [SO(10)]. \quad (29)$$

The $SO(10)$ value is shorter because the additional $(3, 2, 1/6)$ leptoquark gauge boson of the left-right completion feeds the structure that the $SU(5)$ bosons leave with unit weight. In the limit of degenerate heavy gauge bosons, both structures then carry the same factor $(1 + |V_{ud}|^2)^2$, so the squared amplitude becomes $|W_0|^2 (A_1^2 + A_2^2) (1 + |V_{ud}|^2)^2$ and the two rates stand in the ratio

$$\frac{\Gamma_{SO(10)}}{\Gamma_{SU(5)}} = \frac{(A_1^2 + A_2^2)(1 + |V_{ud}|^2)^2}{A_1^2 + A_2^2(1 + |V_{ud}|^2)^2} = 1.54, \quad (30)$$

which is the ratio of the $SU(5)$ lifetime to the $SO(10)$ one in Eq. (29). Only when the two renormalization factors are taken equal does this reduce to $2(1 + |V_{ud}|^2)^2 / [1 + (1 + |V_{ud}|^2)^2] = 1.58$, so the realized enhancement at $A_1 = 2.78$ and $A_2 = 2.91$ is a factor of 1.54. The structure promoted by the extra boson is the one carrying the smaller factor A_1 , and promoting it therefore gains less than it would if the two factors coincided. If the second boson is lighter, the weight of the promoted structure grows from $(1 + |V_{ud}|^2)^2$ to $(1 + |V_{ud}|^2 (M_X/M_Q)^2)^2$, so that the ratio above becomes $[A_1^2 (1 + |V_{ud}|^2 (M_X/M_Q)^2)^2 + A_2^2 (1 + |V_{ud}|^2)^2] / [A_1^2 + A_2^2 (1 + |V_{ud}|^2)^2]$, which rises steeply: it is 5.3 at $M_X/M_Q = 2$ and 18 at $M_X/M_Q = 3$, where the $SO(10)$ lifetime would fall to 2.8×10^{34} yr, only 1.2 times the present limit. Therefore, degeneracy is an assumption rather than a prediction here. The normalization is checked externally against the detailed minimal $SU(5)$ calculation of Ref. [12]: evaluated with their inputs, our expression reproduces their published $\tau_p = 1.1 \times 10^{36} (M_V/10^{16} \text{ GeV})^4 (0.003 \text{ GeV}^3/\alpha_H)^2 \text{ yr}$ to better than 0.1%. The residual uncertainty is dominated by the renormalization factors, which move the lifetime by about 25% across the range quoted in the literature. This is the channel that originally motivated proton decay searches, and it would dominate in a nonsupersymmetric GUT. Here the $1/M_{\text{GUT}}^2$ suppression makes it orders of magnitude slower than the dimension-five mode of Eq. (24), and $p \rightarrow e^+ \pi^0$ lies well below the present Super-Kamiokande sensitivity to set a bound: the $SO(10)$ value of Eq. (29) is 14 times the current limit $\tau_p > 2.4 \times 10^{34} \text{ yr}$ [31] and 3 times the projected Hyper-Kamiokande reach of $\sim 10^{35} \text{ yr}$, and it stays above the current limit by a factor of 7 even when every choice that shortens the lifetime is made at once. We compute it for completeness and compare the two channels in Sec. VI, but do not impose it as a constraint.

C. The Higgs mass imposing an upper bound on m_0

In the MSSM, the mass of the lightest CP -even Higgs boson is not an input but a *prediction*. Its quartic self-coupling is not free but fixed by the electroweak gauge couplings (the D -term potential), which at tree level bounds it from above by

$$m_h^{\text{tree}} \leq m_Z |\cos 2\beta| \leq m_Z. \quad (31)$$

Hence, a tree-level MSSM Higgs mass cannot exceed $m_Z \simeq 91$ GeV. The gap up to the observed 125 GeV must be supplied by radiative corrections, dominated by top/stop loops that grow logarithmically with the stop mass plus a contribution from the stop mixing X_t [32–34]:

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3m_t^4}{2\pi^2 v^2} \left[\ln \frac{m_t^2}{m_i^2} + \frac{X_t^2}{m_i^2} \left(1 - \frac{X_t^2}{12m_i^2} \right) \right]. \quad (32)$$

To reach 125 GeV, one needs either heavy stops or large mixing. Equation (32) is the one-loop leading-logarithm result, shown here for intuition; for the multiteens-of-TeV stops in this model, its logarithm grows too large, so the analysis instead uses the resummed Higgs mass calibrated to FeynHiggs , which levels off as the stop mass rises. That leveling-off keeps the low- $\tan\beta$ part of the allowed region open and places the upper limit on $\tan\beta$ at 8.0.

Because the correction in Eq. (32) is bounded, this places an upper bound on m_0 : too heavy a stop, and hence too large an m_0 , drives the loop above 125 GeV, and the stop mixing X_t cannot compensate, since it only raises m_h . The tree piece $\propto |\cos 2\beta|$ saturates by $\tan\beta \sim 5$ –10. So beyond that, larger $\tan\beta$ raises the tree-level mass no further and the loop contribution must fill the gap; the upper bound on m_0 therefore tightens as $\tan\beta$ grows. It meets the rising proton decay lower bound of Sec. IV A at $\tan\beta \simeq 8.0$, which is what makes the low-to-moderate- $\tan\beta$ a consequence of the data rather than an assumption. The physically allowed window closes by $\tan\beta \simeq 8.0$; the illustrative figures below span $\tan\beta \in [2, 25]$, while the global scan samples $\tan\beta \in [2, 50]$ (Table I).

The precise value of the Higgs mass used in our analysis is the world average $m_h = 125.13 \pm 0.11$ GeV [16]. We evaluate m_h with an EFT-resummed treatment: a one-loop leading-log expression calibrated point-by-point to FeynHiggs [18], which at the multiteens-of-TeV stop masses of the surviving region lies ~ 6 –9 GeV below the bare leading-log (the latter omits the negative two-loop and resummation terms). This calibration sets the upper reach of $\tan\beta$; the bare leading-log, being several GeV too high at these stop masses, would misplace it. This resummed value matches a native FeynHiggs computation and lies about 17 GeV below the fixed-order SPheno 4.0.7 [35] result at the same heavy-stop spectrum. The fixed-order calculation omits the resummation of the large stop logarithm $\ln(M_S^2/m_t^2) \approx 11$ (at $M_S \sim 45$ TeV), which the EFT treatment sums, so in Table III the SPheno entry is this fixed-order

value while the FeynHiggs entry is the EFT-resummed value used in our analysis. The ~ 17 GeV separation is therefore the expected fixed-order overshoot at these masses, and we do not treat it as an uncertainty of our Higgs-mass prediction; the uncertainty we assign is the residual ~ 1.5 GeV theory error of the resummed Higgs mass. The calibration is done point by point against FeynHiggs 2.19 over the domain $\tan\beta \in [2, 20]$ and $M_S \in [3, 36]$ TeV, which contains the entire surviving region. The ± 1.5 GeV uncertainty is built and rounded from:

$$\delta m_h = [(0.65)_{\text{FH}}^2 + (0.72)_{\text{calib}}^2 + (0.27)_{m_t}^2 + (0.20)_{\alpha_s}^2 + (0.11)_{m_h}^2 + (1.0)_{\text{match}}^2]^{1/2} = 1.44 \text{ GeV}, \quad (33)$$

The first term is the uncertainty FeynHiggs itself reports at the $m_0 = 45$ TeV spectrum of Table III (0.57–0.96 GeV over the four heavy-stop benchmarks). The second is the largest departure of our calibrated expression from FeynHiggs anywhere in the calibration domain. The next three are parametric, from m_t , from α_s , and from the measured m_h . The last allows for the higher orders missing from the matching of the effective theory at the stop scale, following Ref. [36], which finds ~ 0.5 GeV at small stop mixing. We take 1.0 GeV to be conservative. Propagated through the meeting of the proton and Higgs bounds, this ~ 1.5 GeV error gives the best-estimate upper limit $\tan\beta \lesssim 8.0$ a large, asymmetric systematic uncertainty, quantified in Sec. VI A. It does not enter the bounding curves of Fig. 5, which are drawn at $m_h = 125$ GeV exactly, and only the ± 1.5 GeV theory error propagates into the quoted limits.

D. The LEP chargino bound

In the minisplit spectrum, the electroweakinos are the only light superpartners, and they are the states that dress the proton decay loop of Sec. IV A. The nonobservation of chargino pair production at LEP2 excludes a light chargino by requiring

$$m_{\chi_1^\pm} > 103.5 \text{ GeV} \quad (34)$$

in the general MSSM [16]. The limit is conditional: it assumes a noncompressed spectrum with standard decays, and softens toward the LEP kinematic reach $\simeq 92$ GeV when the lightest chargino and neutralino are nearly degenerate (compressed Higgsino- or winolike states) or when the sneutrino-mediated t -channel interference depletes the production cross section. In the constrained- μ spectrum, the lighter chargino is winolike (mass $\sim M_2$) and sits well above the bino LSP, since gaugino-mass unification gives $M_1 < M_2$. It therefore decays promptly rather than forming a compressed, long-lived state. In any case, the electroweakinos in the allowed region lie at the TeV scale, far above the LEP reach, so the chargino bound is not binding. The bound

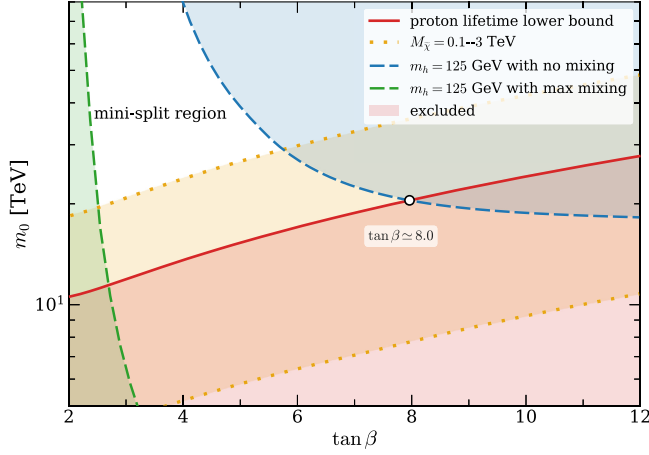


FIG. 5. The allowed region in the $(\tan\beta, m_0)$ plane. The proton decay lower bound (red solid), shown for the reference dressing mass $m_{\tilde{\chi}} \simeq 1$ TeV, and the $m_h = 125$ GeV upper bound (blue dashed) bound the minisplit region, which closes at $\tan\beta \simeq 8.0$ ($m_0 \simeq 20.5$ TeV). The two dashed m_h curves are the edges of the $m_h = 125$ GeV band traced as the stop mixing X_t is varied: the upper edge (blue dashed) is the largest m_0 reaching $m_h = 125$ GeV at zero or minimal stop mixing, and the lower edge (green dashed) is the smallest m_0 reaching it at maximal mixing $|X_t| = \sqrt{6}m_0$. The region to the left of this green dashed curve (light green) is excluded because there the tree-level contribution $m_h \propto |\cos 2\beta|$ is too small for even maximal mixing to reach 125 GeV, so the Higgs is too light. The shaded light orange region bounded by the two dotted curves is obtained by varying $m_{\tilde{\chi}} = 0.1\text{--}3$ TeV. The shaded pink, light blue, and light green regions are excluded. Here we fix $M_2 = \mu = 1$ TeV.

removes the corner where the dressing electroweakino would be too light to be consistent with collider data. It does not constrain m_0 , but sets the lower end of the electroweakino axis of the projection in Sec. IV E.

E. The individual constraints in the parameter space

We now show how these limits bound the $(\tan\beta, m_0)$ plane, deferring the full global scan to Sec. VI. Figure 5 shows the two that determine the allowed region: the rising proton decay lower bound and the falling $m_h = 125$ GeV upper bound on m_0 , which close at $\tan\beta \simeq 8.0$ and $m_0 \simeq 20.5$ TeV, with the lower bound reaching down to $m_0 \gtrsim 11.0$ TeV at low $\tan\beta$. Gauge unification enters indirectly, normalizing the proton decay bound through the M_{H_C} band of Sec. III, while flavor and DM (Secs. V and VII) do not constrain this plane. The two edges depend on $\{m_0, M_{H_C}, \tan\beta\}$ and, through the dressing loop, on the electroweakino mass $m_{\tilde{\chi}}$. The fixed $M_2 = \mu = 1$ TeV in Figs. 5 and 6 fixes only the parameter values plotted in each figure, not the edges themselves. The sole residual dependence of the edges on $m_{\tilde{\chi}}$ is shown as the $m_{\tilde{\chi}} = 0.1\text{--}3$ TeV band.

Figure 6 projects onto the electroweakino plane $(m_{\tilde{\chi}}, m_0)$, showing the LEP bound of Eq. (34) and the

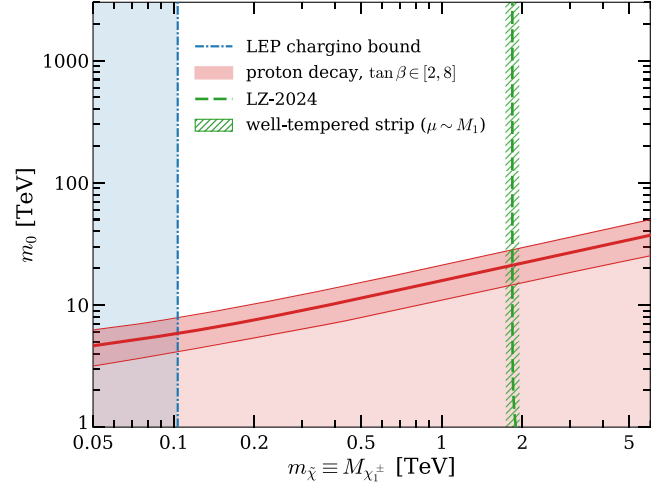


FIG. 6. The electroweakino projection in the $(m_{\tilde{\chi}}, m_0)$ plane: the LEP chargino bound ($m_{\tilde{\chi}_1^\pm} > 103.5$ GeV) and the proton decay bound, plus the DM direct detection bound. The direct detection exclusion (green hatched region) applies only when the lightest neutralino is the thermal DM (the well-tempered strip; see Sec. VII). In this plot, we take $\tan\beta = 5$ and $M_2 = \mu = 1$ TeV. The proton bound uses $\tau_p(p \rightarrow K^+ \bar{\nu}) > 6.61 \times 10^{33}$ yr.

proton decay bound together with the direct detection bound to be discussed in Sec. VII. The proton decay bound is unconditional. The LEP chargino bound is conditional on the electroweakino spectrum (Sec. IV D), though nonbinding here. The direct detection bound is conditional, applying only to the well-tempered strip and only under the thermal-neutralino DM hypothesis.

V. FLAVOR PHYSICS AND PRECISION OBSERVABLES

The flavor- and CP -violating observables, together with the muon anomalous magnetic moment, probe structure beyond the universality assumed for the soft scalar masses, and exact universality would switch them all off. Two small but irreducible pieces survive: the RG-induced slepton mixing that drives $\mu \rightarrow e\gamma$, and the residual SM CP violation that feeds the EDMs. We compute all of them over the allowed region, and none are restricted in the heavy, CP -conserving (real-soft) regime that the proton bound forces, under our universality assumption of the soft scalar masses. The multi-TeV scalars required by proton decay (Sec. IV) suppress every supersymmetric loop, and the real soft terms leave the CKM phase as the sole source of CP violation, keeping the model CKM-safe.

$\mu \rightarrow e\gamma$. The lepton flavor-violating (LFV) process, $\mu \rightarrow e\gamma$, is not absent even with a flavor-blind universal scalar mass m_0 . The neutrino Dirac Yukawa coupling Y_D , tied by the unified multiplets to the up-type Yukawa and hence large, drives an off-diagonal entry in the left-handed slepton soft-mass matrix through RG running between M_{GUT} and the seesaw scale. The process therefore tests the

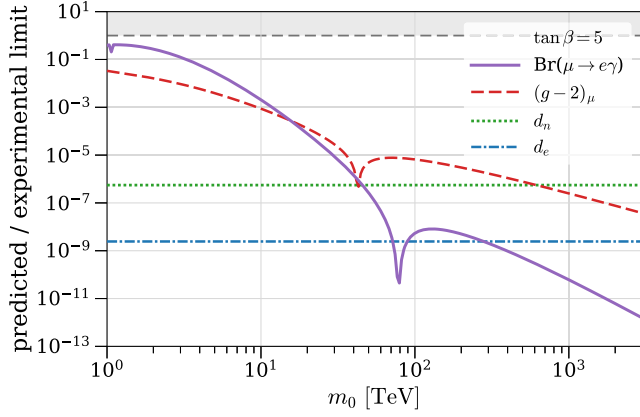


FIG. 7. Flavor and precision observables ($\mu \rightarrow e\gamma$, muon $g-2$, electron and neutron EDM): each sits far below its experimental limit (ratio $\ll 1$) across the allowed region. The very heavy scalars suppress the supersymmetric loops, and the real soft sector keeps the model CKM-safe. We fix $\tan\beta = 5$ in this plot.

same Yukawa structure that fixes the fermion masses and the proton decay operators, and the off-diagonal slepton entry is radiatively irreducible and cannot be removed by a choice of boundary conditions. A neutralino-slepton (or chargino-sneutrino) loop carrying one δ_{12} insertion emits the photon, giving a magnetic-dipole (photon-penguin) transition. The branching ratio is [37]

$$\begin{aligned} \text{Br}(\mu \rightarrow e\gamma) &\simeq \frac{\alpha^3}{G_F^2 m_0^4} |\delta_{12}|^2 \tan^2 \beta, \\ \delta_{12} &\sim \frac{1}{8\pi^2} (Y_D^\dagger Y_D)_{12} \ln \frac{M_{\text{GUT}}}{M_R}, \end{aligned} \quad (35)$$

where δ_{12} is the radiatively generated left-handed slepton mass insertion, set by the neutrino Dirac Yukawa Y_D running between M_{GUT} and the seesaw scale M_R . Because $\text{Br} \propto 1/m_0^4$, the heavy scalars demanded by the proton bound suppress it far below the current MEG II limit [38] across the entire allowed region. The predicted-to-limit ratio is $\ll 1$ everywhere in Fig. 7.

Muon $g-2$. The supersymmetric contribution to the muon anomalous magnetic moment a_μ^{SUSY} (with $a_\mu = (g-2)_\mu/2$) arises at one loop from the chargino-sneutrino and neutralino-smuon diagrams [39]. Because the smuon and sneutrino in the loop sit at the heavy scale m_0 , the contribution is

$$a_\mu^{\text{SUSY}} \simeq \frac{\alpha m_\mu^2 \tan \beta}{8\pi \sin^2 \theta_W m_0^2} \text{sgn}(\mu), \quad (36)$$

Equation (36) is the light-sfermion limit; in the heavy scalar regime here, the contribution decouples one power faster, $\propto M_2 \mu m_\mu^2 \tan \beta / m_0^4$, and the bound quoted next is the full one-loop result evaluated numerically. It is small: over the allowed region $|a_\mu^{\text{SUSY}}| \leq 7.9 \times 10^{-13}$, about 1.1×10^{-3} of

the present experimental sensitivity (Brookhaven E821 [40], now superseded in precision by Fermilab [41]). So the model is SM-like in $(g-2)_\mu$. Under the 2025 lattice-based muon $g-2$ theory initiative evaluation [42], which uses lattice QCD for the hadronic vacuum polarization, the SM prediction sits within $\sim 0.4\sigma$ of experiment (the data-driven e^+e^- evaluation remains in tension), so there is little or no anomaly to accommodate, and the absence of a sizable supersymmetric shift in the heavy scalar regime is consistent with this.

Electric dipole moments (electron, neutron). Supersymmetry with CP -violating soft terms generically induces one-loop EDMs far above the experiment, the “supersymmetric CP problem” first identified in Refs. [43–45]. Under the universal, CP -conserving soft sector assumed throughout (real $m_0, M_{1/2}, A_0$), the only source of CP violation in the model is the CKM phase. The CKM-induced SM values are $\sim 10^{-38}$ and $\sim 10^{-32}$ e-cm for the electron and neutron EDMs [46,47], respectively, some $\sim 2 \times 10^{-9}$ and $\sim 6 \times 10^{-7}$ of the experimental limits (electron $|d_e| < 4.1 \times 10^{-30}$ e-cm [48]; neutron $|d_n| < 1.8 \times 10^{-26}$ e-cm [49]). This follows from the assumed real soft terms and is not automatic: if an $\mathcal{O}(1)$ CP phase were to reside instead in the light electroweakino sector, the one-loop sfermion EDM would still decouple as $1/m_0^2$, but the two-loop Barr-Zee diagram, in which a heavy inner loop attaches to the fermion through a photon and a Higgs or gauge boson, would not, so the dipole survives the sfermion decoupling. A maximal-phase Barr-Zee estimate, computed following the split-supersymmetry analysis of Ref. [50] (a maximal CP phase $\sin \phi = 1$ on the electroweakino sector, the heavy Higgs states decoupled at $M_A \sim m_0$), can reach the electron EDM limit, $d_e \sim 10^{-27}$ e-cm for TeV-scale electroweakinos ($M_2, |\mu| \sim 0.6\text{--}0.8$ TeV), above the current bound of 4.1×10^{-30} e-cm, and so constrains only a light-gaugino corner. We caution that this Barr-Zee number is an $\mathcal{O}(1)$ -factor estimate, not a full two-loop computation; the conclusion that EDMs are CKM-safe and nonbinding under the real soft sector is unaffected.

Figure 7 collects these flavor and precision observables, each shown as a ratio to its experimental limit across the allowed region. Every ratio sits far below unity. The heavy scalars required by the proton bound suppress the loops, and the real (CP -conserving) soft terms keep the model CKM-safe. The sharp downward spikes are accidental cancellations: as m_0 increases, the competing chargino- and neutralino-loop contributions to each amplitude shift in relative size and momentarily cross zero. The muon $g-2$ ratio dips where Δa_μ changes sign (around $m_0 \sim 45$ TeV; the curve plots $|\Delta a_\mu|$), and the $\mu \rightarrow e\gamma$ ratio plunges much more sharply where the flavor-violating dipole amplitude vanishes (around $m_0 \sim 80$ TeV), since the branching ratio scales as that amplitude squared.

Being nonbinding in this regime, the flavor and precision observables leave the main result unchanged. The allowed

region is set by the theoretical and internal consistency requirements (Sec. III) together with the direct experimental limits: proton decay, the Higgs mass, and the LEP chargino bound (Sec. IV). The DM data (Sec. VII) act only in the electroweakino sector.

VI. THE ELECTROWEAKINO SECTOR

The survival wedge from proton decay, the Higgs mass, and unification (Secs. III and IV) fix $\{m_0, \tan\beta, M_{HC}\}$ but leave the electroweakino sector unspecified. We fix it in two ways that share the same physics modules. Under the universal hypothesis, this sector is *not* free: as anticipated in Sec. III C, EWSB fixes $|\mu|$ from the high-scale set $\{m_0, M_{1/2}, A_0, \tan\beta\}$. We therefore give the constrained- μ result first (Sec. VI A), as the prediction of the universal model, and then a free- μ low-scale scan (Sec. VI B) that samples μ directly, as the minimal NUHM extension of the universal model. The two agree on the survival wedge and differ only in what they imply for the Higgsino.

A. The constrained- μ sector

Under the universal boundary conditions, $|\mu|$ is not a free input: it is fixed, up to its sign, by the two electroweak-minimization conditions once $\{m_0, M_{1/2}, A_0, \tan\beta\}$ are given, so the electroweakino sector is a prediction of the model. We fix the universal set of variables at the GUT scale, integrate the RG equations down, and read $|\mu|$ off the minimization condition.

Figure 8 shows the result, computed with SPheno using $\tan\beta = 5$ as an example. The EWSB-determined $|\mu|$ is heavy everywhere on the plane (never below 1.5 TeV) and rises to 7–33 TeV across the light-red surviving region, in which $|\mu|$ always exceeds M_2 by a factor of $\mathcal{O}(10)$. So the Higgsino is the heaviest of the three. Since gaugino mass unification gives $M_1 < M_2$, the lightest electroweakino is then the bino, not the wino. The spectrum shown in Fig. 9 along a representative surviving slice makes the ordering explicit: $|\mu|$ sits far above M_1 and M_2 throughout. We note that this is a SPheno-based constrained- μ spectrum result: the other spectrum generators (SOFTSUSY 4.1.22, FlexibleSUSY 2.9.0 [51,52]) reproduce the low- $\tan\beta$ edge but frequently fail to converge in the heavy-scalar region under identical constrained-MSSM inputs as a known problem associated with focus-point supersymmetry.⁴ So we quote the constrained- μ spectrum as obtained with SPheno rather than as generator-independent. Even where the generators do

⁴This is the expected behavior of a focus-point boundary, where $|\mu|^2$ comes out as a small residual of a large cancellation in the running of the up-type Higgs soft mass. So percent-level differences between codes (in the top-quark Yukawa coupling, the superpartner thresholds, or the matching scale) can readily flip its sign. Where the other generators differ, they return *no* valid EWSB at those points rather than a light Higgsino. Hence, the conclusion that the Higgsino is heavy is firm in direction.

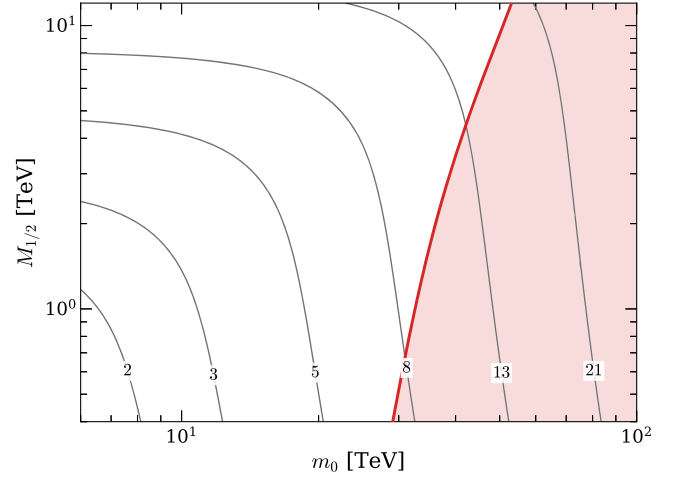


FIG. 8. Contours of $|\mu|$ (in TeV) as the EWSB output over the $(m_0, M_{1/2})$ input plane at $\tan\beta = 5$ and $A_0 = 0$ from a scan using SPheno: $|\mu| \geq 1.5$ TeV everywhere, excluding the light-Higgsino scenario. The light-red region passes the proton, Higgs mass, unification, and LEP cuts, giving $|\mu| = 7\text{--}33$ TeV. The Higgs mass uses the EFT resummation calibrated to FeynHiggs [18], and the proton rate uses the $\tau_p > 6.61 \times 10^{33}$ yr limit [16].

converge, on the low- $\tan\beta$ edge, the EWSB-determined $|\mu|$ is strongly calculator-dependent: the values from SPheno and SOFTSUSY differ by a median factor of about 4 (range 2–12), the expected sensitivity of a focus-point $|\mu|^2$ that is a small residual of a large cancellation. We therefore consider the *ordering* $M_1 < M_2 \ll |\mu|$, a heavy Higgsino with a bino lightest neutralino, as the robust output, not the precise value of $|\mu|$. In every converged case, $|\mu|$ comes out multi-TeV and well above M_2 , and none of our conclusions depends on its exact size. The $(\tan\beta, m_0)$ bounds are

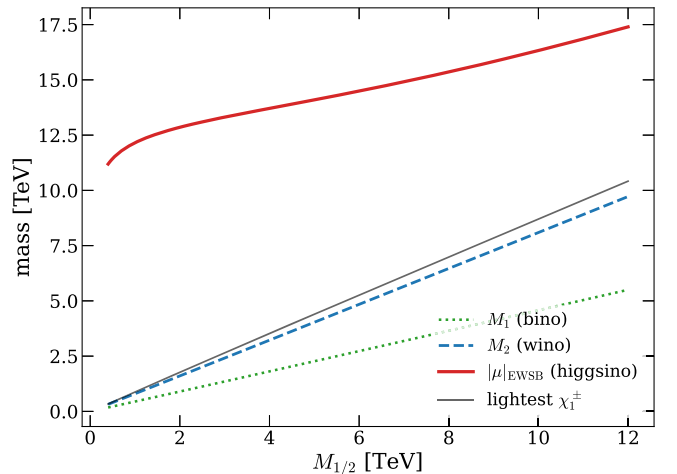


FIG. 9. The constrained- μ electroweakino spectrum along a surviving $m_0 = 45$ TeV slice ($\tan\beta = 5$ and $A_0 = 0$): the EWSB-determined $|\mu|$ lies far above M_1 and M_2 , implying that the Higgsino is heavy and the lightest neutralino is the bino, and that the lighter chargino is winolike.

μ -independent at leading order, and the bino remains a pure, overabundant thermal relic for any $|\mu| \gg M_1$.

The surviving region is the same narrow minisplit strip whether or not μ is constrained:

$$\begin{aligned} \tan\beta &\lesssim 8.0^{+5.5}_{-2.0}, & m_0 &\gtrsim 11.0 \text{ TeV} \quad (m_\chi = 1 \text{ TeV}), \\ M_{H_C} &\in [0.58, 1.43] \times 10^{16} \text{ GeV} \quad (\text{input}), \end{aligned} \quad (37)$$

with heavy, tens-of-TeV scalars sitting above light gauginos, signaling the high-scale (minisplit) supersymmetry pattern. The first two entries are results of the analysis, whereas the third is not, since the M_{H_C} band is fixed beforehand by gauge coupling unification [Eq. (13)] and enters as an input that normalizes the proton decay rate. The boundaries are exactly those derived in Secs. III and IV and drawn in the $(\tan\beta, m_0)$ plane of Fig. 5: the proton (lower) and Higgs-mass (upper) edges meet at $\tan\beta \approx 8.0$, with unification fixing M_{H_C} (Fig. 1). At the lowest $\tan\beta$, by contrast, the Higgs mass does not bound m_0 from above because m_h grows only logarithmically with the scalar scale. At $\tan\beta \approx 2$, the zero-mixing value lies below the 125 GeV window even for very heavy scalars. Thus, $m_h = 125$ GeV is reached only with stop mixing and stays attainable up to arbitrarily large m_0 . In other words, the scalars are bounded only by the requirement that they remain below the unification scale at low $\tan\beta$. Of the four families of constraints, only these two draw an edge in this plane; DM and the flavor observables act in the electroweakino and scalar sectors treated in Secs. V and VII and leave the $(\tan\beta, m_0)$ window untouched.

This constrained- μ treatment sharpens the bounds rather than weakening them. The three edges of Eq. (37), from the proton decay, the M_{H_C} band, and the Higgs mass, depend at leading order only on $\{m_0, M_{H_C}, \tan\beta\}$ and not on μ . So the lower bound $m_0 \gtrsim 11.0$ TeV and the upper bound of $\tan\beta \approx 8.0$ stand in either scheme. What the constrained- μ treatment removes is the light electroweakino, and with it the light-Higgsino DM strip (to be seen in Sec. VII): both are features of treating μ as a free parameter or of the NUHM extension of Sec. VIB. Strictly speaking, μ reenters the three edges at subleading order: through the Higgsino dressing of the dimension-five operators, through $X_t = A_t - \mu \cot\beta$, and through the supersymmetric thresholds in unification.

With the Higgsino driven heavy, the lightest superpartner is the bino ($M_1 < M_2$). Its DM fate is analyzed in the constrained- μ scheme of Sec. VII B. The winolike states sit above the bino as a near-degenerate pair (split by ~ 160 MeV [53]) but decay promptly to it, so standard electroweakino searches apply rather than disappearing-track searches. Under the constrained- μ treatment, the neutralino is thus not a unique thermal candidate, and DM enters as a model-dependent input rather than as part of the core exclusion.

The uncertainties on the m_0 lower bound, and the separate dependence on the dressing mass m_χ , are already collected in Sec. IV A. The upper bound of $\tan\beta \approx 8.0$ inherits the residual uncertainty of the Higgs mass bound, the least certain of the three: m_h is an EFT-resummed leading logarithm calibrated to FeynHiggs (the bare leading logarithm, ~ 6 – 9 GeV too high, would have closed the window prematurely at $\tan\beta \approx 5$), and the residual FeynHiggs theory error is ~ 1.5 GeV. This last error dominates the systematic on the upper limit of $\tan\beta$. Because the zero-mixing Higgs mass grows only logarithmically with the stop scale, $m_h^2 \simeq M_Z^2 \cos^2 2\beta + (3m_t^4/2\pi^2 v^2) \ln(M_S^2/m_t^2)$, a few GeV of Higgs mass uncertainty translates into a wide range of M_S , and hence of the point where the rising proton lower bound meets the falling Higgs upper bound. This leading-log expression is used only to show the logarithmic sensitivity; the numerical limits quoted are obtained from the EFT-resummed calculation. Shifting the accepted m_h by ± 1.5 GeV moves that meeting point from the best-estimate $\tan\beta \approx 8.0$ ($m_0 \approx 20.5$ TeV) to $\tan\beta \approx 6.0$ ($m_0 \approx 17.0$ TeV, at -1.5 GeV) and $\tan\beta \approx 13.5$ ($m_0 \approx 30.6$ TeV, at $+1.5$ GeV). The shift is strongly one-sided, climbing about 3.7 units of $\tan\beta$ per GeV on the upper side. We therefore report $\tan\beta \approx 8.0$ as a best estimate carrying this large one-sided theory systematic uncertainty. The proton decay lower bound $m_0 \gtrsim 11.0$ TeV and the minisplit character are set by proton decay alone and do not depend on the Higgs window.

B. The free- μ extension

A light Higgsino is realized only if we relax the universal hypothesis in its Higgs sector: letting the two Higgs soft masses $m_{H_u}^2, m_{H_d}^2$ differ from the common sfermion mass m_0^2 at the GUT scale, leading to the NUHM generalization [54–56], trades one or two of the electroweak minimization conditions so that $|\mu|$ (and m_A) become free low-scale inputs while the sfermions stay universal. This is natural in $SO(10)$: matter sits in the **16**, but the light Higgs doublets descend from the **10** (mixing with the **126** and higher representations), and no symmetry forces their soft masses to equal m_0 . Keeping the sfermions universal preserves the flavor safety of Sec. V; and the proton, Higgs, and unification edges are unchanged. We therefore read the free- μ scan below as this minimal NUHM extension of the universal model.

We locate the region with a unified Monte Carlo scan of 10^7 points over the soft-breaking box of Sec. II ($m_0, M_2, |\mu|, \tan\beta, X_t, M_{H_C}$), recording a deterministic pass/fail flag for each of the nine cuts of Secs. III to V and VII. The scan reproduces the same survival wedge Eq. (37), an independent confirmation of its μ -independence, and in addition opens the light-Higgsino window that the constrained- μ model closes. Our amplitude and spectrum are benchmarked against public codes observable by observable, at

the points and to the agreement listed in Table III, rather than as a single global comparison: the fixed-order and EFT-resummed Higgs mass against SPheno and FeynHiggs, the Higgsino relic density and spin-independent cross section against micrOMEGAs 6.0, and the constrained- μ EWSB spectrum against SPheno, with SOFTSUSY, and FlexibleSUSY reproducing the low- $\tan\beta$ edge [18,35,51,52,57].

The free- μ scan thus leaves the survival wedge intact and adds one feature the universal model lacks, i.e., a light electroweakino, whose DM fate (a well-tempered strip and a Higgsino branch) is taken up in Sec. VII C.

VII. LIGHTEST SUPERSYMMETRIC PARTICLE

Assuming exact R -parity symmetry, the LSP is stable, and its *fate* is a test of the model: it must neither over-close the Universe nor be seen by a direct detection experiment. In our spectrum, the LSP is the lightest neutralino $\tilde{\chi}_1^0$, a weakly interacting massive particle (WIMP), whose gauge content (and hence its relic abundance and scattering rate) is fixed by the electroweakino sector (M_1, M_2, μ). That sector is realized, as discussed in Sec. VI, in two ways that give qualitatively different fates for the LSP: the constrained- μ scheme (Sec. VII B) and the free- μ scheme (Sec. VII C), where a light electroweakino (and hence a thermal WIMP candidate) reappears. We first set up the relic abundance and the direct detection cross section for comparison with the data, then take the two schemes in turn. Throughout, the LSP analysis lives in the electroweakino sector and leaves the $(\tan\beta, m_0)$ bounds of Sec. IV untouched.

A. Relic density and direct detection

We impose two DM conditions with the following data. First, if $\tilde{\chi}_1^0$ is a thermal relic, its abundance must not exceed the measured cold DM density $\Omega h^2 = 0.120 \pm 0.001$ [58]; we impose $\Omega h^2 \leq 0.122$, the one-sided 95% confidence-level upper limit on the measured value, saturated when the neutralino is all of the DM. Because the squarks and sleptons here sit at tens of TeV and decouple, the abundance is fixed entirely by the electroweakino sector (M_1, M_2, μ) and their mixing. Thermal freeze-out sets it through the thermally averaged annihilation cross section $\langle\sigma v\rangle$,

$$\Omega h^2 \simeq 0.12 \times \frac{2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}}{\langle\sigma v\rangle}. \quad (38)$$

A weakly coupled (small- $\langle\sigma v\rangle$) state over-closes the Universe. A pure bino, generic when M_1 is the smallest mass, has no $SU(2)_L$ or Higgsino gauge coupling to annihilate through. So with only the very heavy sfermions for t -channel exchange, it over-closes the Universe, $\Omega h^2 \gg 0.12$. In this case, efficient annihilation requires a Higgsino (or wino) admixture, which depends on the scheme.

Second, a neutralino that makes up the DM also scatters off nuclei [59,60]; the strongest current bound is the 2024 LUX-ZEPLIN (LZ) spin-independent limit [61], $\sigma_{\text{SI}} \lesssim 2\text{--}3 \times 10^{-47} \text{ cm}^2$ near 1 TeV. The cross section is controlled by Higgs exchange and scales with the *Higgsino fraction* of $\tilde{\chi}_1^0$: a pure bino barely scatters, a pure Higgsino more, and a *mixed* bino-Higgsino scatters *strongly*,

$$\sigma_{\text{SI}} = \frac{4}{\pi} \mu_{\text{red}}^2 f_p^2, \quad f_p \propto \frac{g_{\tilde{\chi}\tilde{\chi}h}}{m_h^2}, \quad g_{\tilde{\chi}\tilde{\chi}h} \propto N_{11} N_{1H}, \quad (39)$$

where μ_{red} is the neutralino-nucleon reduced mass and $g_{\tilde{\chi}\tilde{\chi}h} \propto N_{11} N_{1H}$ (bino $\times$ Higgsino component) is largest for a comparably mixed state. Only the light-Higgs exchange survives: with the heavy-Higgs sector at the squark scale, we fix $M_A = 20 \text{ TeV}$, so the H piece is suppressed by $\sim m_h^2/M_A^2 \approx 4 \times 10^{-5}$ and is negligible; the relic density is likewise far from any s -channel A -pole. The LSP makes up a fraction $f \equiv \min(\Omega h^2/0.12, 1)$ of the local DM density, up to $f = 1$, so the limit reads $f\sigma_{\text{SI}} \leq \sigma_{\text{SI}}^{\text{LZ}}$. An under-abundant relic ($f < 1$) has its rate lowered below the raw σ_{SI} , whereas an overabundant relic has $f = 1$ and its raw σ_{SI} is tested directly. The largest fraction it can constitute at a given mass is then $f_{\text{max}} = \min(1, \sigma_{\text{SI}}^{\text{LZ}}/\sigma_{\text{SI}})$. The relic abundance and σ_{SI} are computed with a Sommerfeld-corrected analytic freeze-out and a one-loop σ_{SI} , cross-checked against micrOMEGAs; the constrained- μ bino relic below is taken directly from micrOMEGAs.

B. The constrained- μ scheme

When μ is fixed by radiative EWSB (Sec. VI), it is driven to the multi-TeV scalar scale, $|\mu| = 7\text{--}33 \text{ TeV}$. Therefore, $M_1 < M_2 \ll |\mu|$, and the LSP is a nearly pure bino ($\gtrsim 99.9\%$ bino). Evaluated with micrOMEGAs along the constrained- μ surviving region, swept over the $\tan\beta \in [2, 8.0]$ in Fig. 10(a), bino mass $M_{\tilde{\chi}_1^0} \simeq 0.1\text{--}7 \text{ TeV}$, sfermions at tens of TeV, the thermal relic is hugely *overabundant*:

$$\Omega h^2 \sim (10^4 \text{ to } 10^5) \times 0.12, \quad (40)$$

because the tiny $\sim 0.1\%$ Higgsino admixture leaves only a feeble $\tilde{\chi}\tilde{\chi} \rightarrow W^+W^-, ZZ, hh$ annihilation through the 7–33 TeV Higgsino. As a standard thermal relic, the constrained- μ bino thus over-closes the Universe by some five orders of magnitude. The heavy μ , the split $M_1 < M_2$, and the tens-of-TeV sfermions leave no efficient annihilation channel. In this case, $\Omega h^2 \leq 0.122$ could be reached only through, for example, nonthermal dilution, after which it becomes a direct detection-safe subdominant relic.

Direct detection is mostly blind to it. Over most of the region, the LSP-nucleon cross section $\sigma_{\text{SI}} \simeq 2\text{--}6 \times 10^{-49} \text{ cm}^2$, two to three orders *below* LZ [Fig. 10(a)]. The only exception is the high- $\tan\beta$ ($\simeq 8.0$), low-mass ($M_{\tilde{\chi}_1^0} \lesssim 0.2 \text{ TeV}$) corner, where the $\tan\beta$ -enhanced coupling lifts σ_{SI} up to $\sim 10^{-46} \text{ cm}^2$ (reaching 8×10^{-46} at the

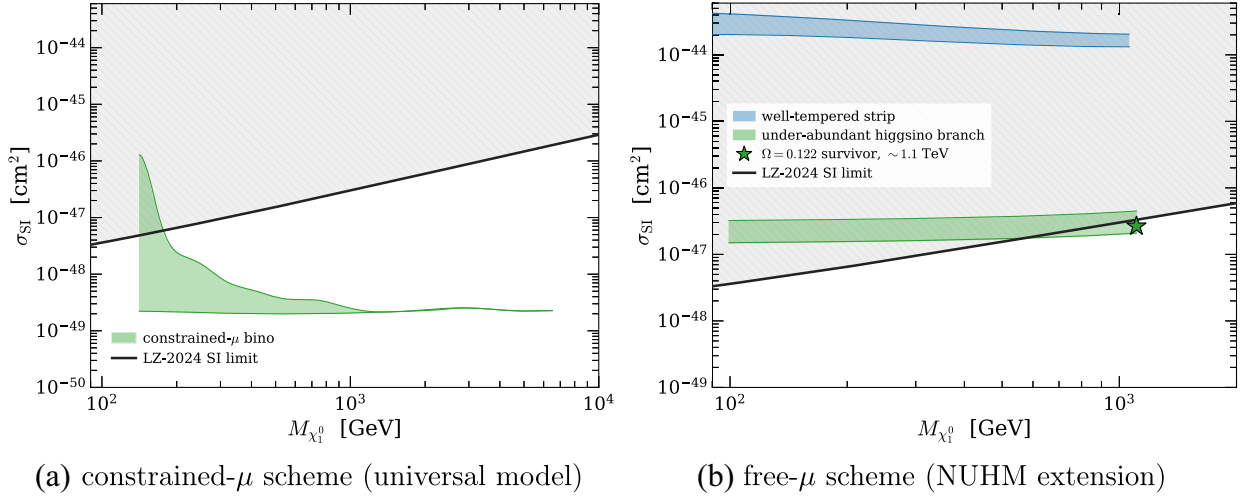


FIG. 10. Fate of the LSP: spin-independent cross section with the nucleon as a function of $M_{\chi_1^0}$ against the LZ-2024 limit [61], swept over the $\tan\beta \in [2, 8.0]$ and taking $M_A = 20$ TeV. (a) Constrained- μ (the universal model): the bino (green), with $\mu = 7\text{--}33$ TeV fixed by EWSB, is thermally *overabundant*, so its local density fraction is capped at $f = 1$ and its *raw* σ_{SI} is what LZ tests. (b) Free- μ (the NUHM extension): the two strips are the *same* neutralino with different compositions (dialed by μ/M_1), running from the LEP chargino mass lower bound ($M_{\chi_1^\pm} \gtrsim 100$ GeV) to the ~ 1.1 TeV thermal-Higgsino point where they merge (marked by the green star). The blue region is the well-tempered $\Omega h^2 = 0.12$ strip ($f = 1$, $\sigma_{\text{SI}} \simeq 10^2\text{--}10^3$ above LZ, excluded, not rescalable). The green region is the $\Omega h^2 \leq 0.122$ Higgsino branch (raw σ_{SI} touching/above LZ, but *allowed* if underabundant). Relic and σ_{SI} are computed by micrOMEGAs.

extreme). There, the bino cannot be 100% of the DM and would have to be diluted to a subdominant fraction. Under exact R -parity, the relic excess is removed by a standard late entropy injection, a modulus or saxion decaying after bino freeze-out but before the big bang nucleosynthesis dilutes Ωh^2 by the required $\sim 10^5$. After the dilution, the bino is a harmless, direct detection-safe subdominant relic and the observed $\Omega h^2 = 0.12$ should be supplied by another sector (such as an axion). The same excess is equally avoided if the bino is not the true LSP: a lighter axino or gravitino, natural in the Peccei-Quinn or supergravity sectors that would also supply the axion, makes the bino a next-to-lightest state that decays into it. Consequently, the surviving abundance is set by the lighter LSP rather than the overabundant bino. The constrained- μ scheme therefore offers no thermal WIMP DM candidate.

C. The free- μ scheme

If instead we relax the universal hypothesis in the Higgs sector, we have the NUHM extension of Sec. VI B, where μ becomes a free low-scale input while the sfermions stay universal. In such a scheme, a light electroweakino and a thermal WIMP candidate reappear. The relic condition $\Omega h^2 = 0.12$ is then met on two thin regions, drawn in Fig. 10(b) as *strips* because each is swept over the $\tan\beta \in [2, 8.0]$. Since σ_{SI} carries a $\tan\beta$ -enhanced down-type Higgs coupling, a fixed neutralino mass broadens into a band. The two strips occupy the *same* mass range but differ in the neutralino *composition*, which is a third axis that the $(M_{\chi_1^0}, \sigma_{\text{SI}})$ projection hides and that is dialed by

the ratio μ/M_1 . For $\mu \simeq M_1$, the LSP is a well-tempered bino-Higgsino mixture [62] (blue), while for $\mu \ll M_1$ (a heavy bino) it is a nearly pure Higgsino (green). Both strips run from the LEP chargino bound on the left ($m_{\chi_1^\pm} > 103.5$ GeV forces $M_{\chi_1^0} \gtrsim 100$ GeV, since either configuration has a light chargino near the LSP mass) to the ~ 1.1 TeV thermal-Higgsino point on the right, where they merge. Above $M_1 \simeq 1.1$ TeV no amount of mixing brings the bino down to $\Omega h^2 = 0.12$, so it becomes the pure Higgsino, which itself saturates $\Omega h^2 = 0.12$ at $\mu \simeq 1.1$ TeV. Heavier neutralinos overclose the Universe.

The two strips meet direct detection very differently. The well-tempered strip sits at $\Omega h^2 = 0.12$ ($f = 1$), but its $\mathcal{O}(1)$ Higgsino admixture gives a raw $\sigma_{\text{SI}} \sim 1.4 \times 10^{-44}$ cm², $10^2\text{--}10^3$ above LZ. It is thus excluded by direct detection and, being already 100% of the DM, cannot be rescued by density rescaling. The Higgsino branch is instead underabundant. For $|\mu| < M_1$, the pure Higgsino has mass $\simeq |\mu|$ and $\Omega h^2 \simeq 0.12(\mu/1.1 \text{ TeV})^2 < 0.12$, the leading-order thermal scaling ($\Omega h^2 \propto 1/\langle\sigma v\rangle$, $\langle\sigma v\rangle \propto 1/\mu^2$) normalized to the 1.1 TeV thermal Higgsino (up to a mild logarithmic and Sommerfeld/coannihilation correction folded into the calibration). Its raw $\sigma_{\text{SI}} \simeq 2\text{--}3 \times 10^{-47}$ cm² is close to the LZ bound and lies above it at low mass. Since the Higgsino is only a fraction $f = \Omega h^2/0.12 = (\mu/1.1 \text{ TeV})^2 < 1$ of the local density, the direct detection rate is $f\sigma_{\text{SI}}$, which drops below LZ across the whole branch. Hence, the Higgsino can be a viable (generally subdominant) DM component: up to 100% at the ~ 1.1 TeV saturation (marked by the green star sitting right below the LZ bound) and a fraction f below it. Every $\Omega h^2 \leq 0.122$ point

is Higgsino-dominated, and no bino-dominated LSP is ever underabundant. This thermal-WIMP candidate is a feature of the NUHM extension.

VIII. DISCUSSIONS AND OUTLOOK

The surviving region of Eq. (37) is small and sharply bounded, and each of its edges is the target of a near-future experiment, as detailed below.

Proton decay. The lower bound on m_0 is not a comfortable distance below the current limit: the lower edge sits at $\tau_p(p \rightarrow K^+ \bar{\nu}) \approx 6.61 \times 10^{33}$ yr, just above the present Super-Kamiokande bound it was forced to clear [16]. Hyper-Kamiokande [63], with roughly an order-of-magnitude greater sensitivity in this channel, will probe lifetimes into the $\sim 10^{34}$ – 10^{35} yr range and so test the low- m_0 edge directly. The prediction is sharp: either Hyper-Kamiokande (and DUNE in the complementary K^+ channel) sees the $K^+ \bar{\nu}$ mode at a rate consistent with m_0 near the lower edge, or a null result pushes that edge upward, tightening the region toward smaller $\tan\beta$, subject to the factor-few hadronic uncertainty of Sec. IV.

Electroweakinos. The minisplit pattern places the gauginos and Higgsinos at $\sim \text{TeV}$, far below the scalars, so they are the only directly accessible superpartners. The heavy Higgs states $H, A, H^\pm$ also sit at $\gtrsim m_0$, leaving the SM-like Higgs boson as the sole accessible scalar at low energies. In the constrained- μ scheme, the ordering of gauginos is fixed: a bino $\tilde{\chi}_1^0 \simeq M_1$, a winolike $\tilde{\chi}_1^\pm, \tilde{\chi}_2^0 \simeq M_2 \simeq 2M_1$ (the lighter chargino mass ranging up to a few TeV in the allowed region), and a decoupled Higgsino ($|\mu| = 7$ – 33 TeV). Since $M_1 < M_2$, the winos decay promptly to the bino, giving the golden channel $p\bar{p} \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow W^{(*)} \tilde{\chi}_1^0 (Z/h) \tilde{\chi}_1^0$, seen as a trilepton or $\ell\ell b\bar{b}$ final state with missing transverse energy. The high-luminosity LHC (HL-LHC) reaches wino-bino masses near 1 TeV [64], and a 100-TeV collider (FCC-hh) [65,66] extends the search to several TeV, covering the full electroweakino window predicted here. In the free- μ (NUHM) scheme, the surviving ~ 1.1 TeV Higgsino is nearly degenerate and appears as soft leptons with a mono-jet or a disappearing track, out of HL-LHC pure-electroweak reach but a clean FCC-hh target.

Gluino. A sharp signature is the gluino placed at $m_{\tilde{g}} \simeq (4$ – $7)M_1$ by gaugino-mass unification, a few TeV when the electroweakinos are light. Because the squarks are very heavy, $\tilde{g} \rightarrow q\bar{q}\tilde{\chi}$ proceeds only through an off-shell squark of mass $\sim m_0$, with the width scaling as $m_{\tilde{g}}^5/m_0^4$ and leaving the gluino long-lived, a hallmark of split spectra. Taking $m_0 \sim 45$ TeV and $m_{\tilde{g}} \sim 3$ TeV as an example, the decay length is macroscopic, and the leading probes are long-lived particle searches (R-hadrons tagged by anomalous ionization and time-of-flight, and stopped-gluino searches), with prompt multijet-plus-missing-energy when the decay is fast. The HL-LHC reaches $m_{\tilde{g}} \sim 2.5$ – 3 TeV [64] and FCC-hh ~ 10 – 15 TeV [65,66], spanning the entire allowed

range predicted here. The model thus predicts a compact electroweakino system with a possibly long-lived multi-TeV gluino. Therefore, a $\sim \text{TeV}$ electroweakino or R-hadron signal with no accompanying light squarks or sleptons would point directly to the minisplit spectrum and select the light- $M_{1/2}$ corner.

Dark matter. The free- μ DM target of Sec. VII, a ~ 1.1 TeV nearly pure Higgsino (a feature of the NUHM extension, not of the universal model, whose own LSP is an over-abundant bino requiring dilution), is a clean, fixed-mass prediction (relaxed to a Higgsino branch $\mu \lesssim 1.1$ TeV if one requires only $\Omega h^2 \leq 0.122$; Sec. VII). Its spin-independent cross section lies below the current LZ bound but within the projected sensitivity of the next-generation multiton xenon experiments (XLZD/DARWIN), which will cover essentially the entire Higgsino-mass window, making the Higgsino DM a falsifiable hypothesis.

Electric dipole moments. With strictly real, universal soft terms, the electric dipole moments vanish (with only the SM CKM, $d_e \sim 10^{-38}$ e cm), so they do not constrain the baseline scan [48,49]. The heavy-scalar/light-gaugino corner is nonetheless sensitive: a two-loop Barr-Zee contribution survives the $1/m_0^2$ decoupling of the one-loop sfermion piece. Hence, even a small residual CP -violating phase in the gaugino sector would generate an electron EDM within reach of next-generation searches. It is tight enough that the present electron-EDM limit [48] already requires roughly $\sin\varphi \lesssim 10^{-3}$. The next round of electron- and neutron-EDM experiments therefore probes the CP structure of the light-gaugino corner: a nonzero EDM at their sensitivity would point to this regime, while continued nulls tighten the bound on gaugino-sector phases.

Taken together, these four probes reach the relevant sensitivity within the coming decade: proton decay tests the lower edge, colliders the electroweakinos, direct detection the Higgsino, and EDMs the CP structure.

IX. CONCLUSIONS

We have delineated the allowed parameter space of the minimal supersymmetric $SO(10)$ with a universal soft spectrum by requiring, on every point of a dense scan over $\{m_0, M_2, \mu, \tan\beta, X_t, M_{H_C}\}$, that all constraints hold at once. The constraints span four physically distinct categories: theory (gauge coupling unification and vacuum stability), the robust experimental limits (proton lifetime limit, $m_h = 125$ GeV, and the LEP chargino bound), dark matter data, and the flavor and precision observables. The model is not excluded, but, under a single effective colored Higgs scale for the proton decay normalization and the SPheno-based constrained- μ treatment, it survives only in a sharply bounded, low-to-moderate- $\tan\beta$ minisplit region: multiten-of-TeV scalars, $m_0 \gtrsim 11.0$ TeV for a dressing electroweakino mass $m_\chi = 1$ TeV and $\tan\beta \lesssim 8.0_{-2.0}^{+5.5}$, subject to the systematic uncertainty in the EFT-resummed calculation of m_h , above $\sim \text{TeV}$ gauginos, with the

colored Higgs mass pinned by unification to $M_{H_C} \in [0.58, 1.43] \times 10^{16}$ GeV.

Proton decay inverts the usual decoupling intuition into a lower bound on m_0 that rises with $\tan\beta$. The Higgs mass sets an upper bound that descends with $\tan\beta$. These two constraints meet near $\tan\beta \approx 8.0$ (best estimate) with no extra assumption. Unification ties the proton rate to the colored Higgs mass through $\tau_p \propto M_{H_C}^2$, closing, within the single effective- M_{H_C} parametrization adopted here, the obvious escape of raising M_{H_C} : a lighter scalar cannot be traded for a heavier triplet without spoiling unification. In a complete color triplet sector, where the dimension-five decay depends on the inverse triplet mass matrix and unification on threshold combinations, this is a model assumption. We estimate the resulting decoupling of the unification-effective and decay-effective triplet scales to be a factor $\lesssim 2$ on M_{H_C} , hence $\lesssim 1.5$ on the m_0 bound, for a color triplet mass spread up to an order of magnitude [Eq. (14)], barring a tuned cancellation in the inverse mass matrix.

Solving the electroweak symmetry-breaking conditions point by point, rather than treating M_2 and $|\mu|$ as free parameters, sharpens this picture. The three edges above depend at leading order only on $\{m_0, M_{H_C}, \tan\beta\}$, so the lower bound and the $\tan\beta$ upper bound are unchanged. What the constrained- μ treatment removes is the light electroweakino, since radiative symmetry breaking drives $|\mu|$ to the multi-TeV scalar scale, well above M_2 at every surviving point. Therefore, the lightest electroweakino is the bino (since $M_1 < M_2$) and no light Higgsino is realized.

Dark matter leaves the $(\tan\beta, m_0)$ bounds untouched but sharply constrains the fate of the lightest supersymmetric particle. In the universal model, the lightest neutralino is a bino and radiative electroweak symmetry breaking drives $|\mu|$ heavy, so its thermal relic is hugely overabundant yet invisible to direct detection: under exact R -parity, a late entropy dilution is required to render $\Omega h^2 \leq 0.122$, with the observed density supplied by another sector (Sec. VII B). A light thermal-relic neutralino reappears only in the minimal nonuniversal-Higgs-mass extension in which μ is a free parameter; there LUX-ZEPLIN direct detection [61]

excludes the entire well-tempered bino-Higgsino strip and collapses the viable candidate to a single ~ 1.1 TeV nearly pure Higgsino. Because this acts orthogonally to the $(\tan\beta, m_0)$ plane, the allowed region and its bounds are unaffected. Equivalently, relaxing the requirement that the lightest neutralino be the dark matter or that it be a thermal relic at all does not qualitatively alter the minisplit conclusion, since the bounds $m_0 \gtrsim 11.0$ TeV (again at $m_\chi = 1$ TeV) and $\tan\beta \lesssim 8.0$ rest entirely on the proton decay limit, the Higgs mass, and gauge unification. The flavor and precision observables ($\mu \rightarrow e\gamma$, $(g-2)_\mu$, and electric dipole moments) are also checked to be nonbinding: the Y_D -induced slepton mixing behind $\mu \rightarrow e\gamma$ falls below MEG II once $m_0 \gtrsim 5$ TeV, comparable to the heaviness the proton lifetime lower bound already demands.

In short, the minimal supersymmetric $SO(10)$, if realized at all, is pushed to the cornered minisplit and, as detailed in Sec. VIII, testable in the coming decades, most directly through Hyper-Kamiokande proton decay and electroweakino searches at the HL-LHC and a future 100-TeV collider, with dark matter direct detection and electric dipole moment experiments covering parts of the region.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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