

Chiral limit of fully nonlinear minimal massive gravity

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We study 3D anti-de Sitter minimal massive gravity in two regimes: a) at the chiral limit where one of the boundary Brown-Henneaux central charges vanishes and two modes become null and b) in the regime where one of the two charges is much larger than the other. At the chiral point, we go beyond the known free-theory analysis to prove that these modes decouple completely also in the full MMG Lagrangian and field equations. We also use the full action to ascertain if the interacting theory becomes infinitely strongly coupled in the neighborhood of the chiral limit, where the theory is not unitary. We show that this is not the case at tree level but that a strong coupling pathology appears in loops, starting at a loop level determined by the number of external legs in the bulk Feynman diagrams. Finally, we show that there is no strong coupling problem in the second regime where, instead, the two boundary fields and the field propagating in the bulk decouple from one another.

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I. INTRODUCTION

Gravity in three spacetime dimensions is interesting both as a simplified model of gravity possessing some of the most interesting features of the more complex four-dimensional theory—e.g. black holes for negative cosmological constant $\Lambda = -1/l^2$ —and also for $\Lambda < 0$, as a holographic dual of two dimensional conformal field theories (CFTs). 3D pure gravity with negative Λ does not propagate any local degree of freedom in the bulk but it possesses boundary excitations that give rise to a boundary Virasoro $\times$ Virasoro algebra with a classical nonzero central charge $c = 3l/2G_3$ ($G_3 =$ Newton's constant) [1]. The simplest formulation of this theory is in terms of an $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ Chern-Simons theory [2]. The Chern-Simons theory depends on the two couplings of the two gauge factors, but the equations of motion for any nonzero coupling are always equivalent to Einstein's equation for pure gravity.

The $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ theory does change if the zero-torsion constraint is added to the Chern-Simons theory. The resulting theory is topologically massive gravity (TMG) [3], which possesses two independent central charges, c_+ , c_- . Generically, TMG contains ghosts whenever the Newton

constant's sign is chosen to give positive boundary Brown-Henneaux [1] central charges, but ghosts appear to be absent at the chiral point, where either c_+ or c_- vanish, provided that the theory is restricted to the singlet subsector of the Virasoro algebra with vanishing central charge [4,5].

The holographic duality [6–9] between gravity in AdS_3 and 2D CFTs can be checked in great detail thanks to the rich symmetry structure of 2D CFTs. One interesting question one can ask is: what is the bulk dual of a renormalization group (RG) flow from some high energy (UV) CFT with central charges ($c_+ \gg 1$, $c_- \gg 1$) to a chiral CFT with either $c_+ = 0$ or $c_- = 0$? A candidate for such bulk dual is TMG, which in fact has been studied as a holographic model of a chiral ($c_- - c_+ \neq 0$) RG flow in [10]. The problem with such bulk dual is that TMG propagates ghosts away from the strict chiral limit. A proper bulk dual must be unitary even away from the chiral point. The simplest such model is minimal massive gravity [11] (MMG), which can be ghost and tachyon free for generic positive central charges.

For generic values of $c_+ > 0$, $c_- > 0$, MMG propagates two boundary and one (chiral) bulk degree of freedom. At the chiral point where either $c_- = 0$ or $c_+ = 0$, the bulk degree of freedom decouples together with one of the two boundary ones. At the level of the free, quadratic action, such decoupling was verified in [12]. That paper worked in the metric formalism and showed that the Ostrogradsky energy of the unwanted modes vanishes at the chiral points. This is of course a harbinger of potential problems away from the strict chiral limit, because a theory with vanishing energy (or, equivalently, vanishing kinetic term) can become infinitely coupled near the limit. If the kinetic

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term of a field is infinitesimal, e.g. $O(\eta)$ for $\eta \rightarrow 0$, interaction terms can become $O(\eta^{-1})$. This problem is distinct from another known problem of MMG, namely, that it not unitary near the chiral point [13].

To understand if MMG possesses a strong coupling pathology we must go beyond the free, quadratic action and study the fully nonlinear MMG. This is done in this letter, which begins in Sec. II with a review of the MMG action written in first-order formalism, its equations of motion, and its linearization around an AdS_3 background. Section III describes the chiral limit of the quadratic part of the MMG action, where one of the two central charges, which we choose to be c_- for definiteness, vanishes. Section IV is the first truly new part of this work. It contains a detailed study of the full MMG action near the chiral limit. An appropriate rescaling of the fluctuations around the classical AdS_3 background allows one to recover in a very simple fashion the results of [12] and show how interactions behave as a function of a constant, η , which parametrizes the distance from the chiral limit. The results of Sec. IV are used in Sec. V to show that the boundary mode that survives in the chiral limit, called f_-^a , is not strongly coupled at bulk tree level. This is sufficient to establish the consistency of MMG as a holographic (nonunitary) dual of a large-central-charge CFT, because

in the large central charge regime (also known as “large- N ”) the holographic dual is a classical gravity bulk action. Section V also shows that certain loop diagrams containing only f_-^a external legs *can* be strongly interacting at loop level. The minimum number of loops for this pathology to appear is related to the number of external f_-^a legs by an equation given in the section. Section V concludes with a brief discussion of the interpretation of, and possible solutions to, the strong coupling pathology. Section VI studies another region of MMG, in which the two boundary central charges c_+ and $c_- < c_+$ are very different, but still in the region allowed by bulk unitarity. Precisely, we study the region where $(c_+ - c_-/c_+ + c_-) = 1 - \epsilon$, with $0 < \epsilon \ll 1$. We find that no strong coupling pathology exists there. The two boundary degrees of freedom and the bulk degree of freedom are healthy and propagate in the limit $(c_+ - c_-/c_+ + c_-) \rightarrow 1$, but decouple from one another, and one of them becomes free.

II. THE MMG ACTION IN FIRST ORDER FORMALISM

The Lagrangian density of MMG in first order formalism is the three form

$$16\pi G_3 \mathcal{L}_{\text{MMG}} = -\sigma e^a \wedge R(\omega)_a + \frac{\Lambda_0}{6} \epsilon_{abc} e^a \wedge e^b \wedge e^c + h^a \wedge T(\omega)_a + \frac{1}{2\mu} \left(\omega^a \wedge d\omega_a + \frac{1}{3} \epsilon_{abc} \omega^a \wedge \omega^b \wedge \omega^c \right) + \frac{\alpha}{2} \epsilon_{abc} e^a \wedge h^b \wedge h^c. \quad (1)$$

Here e^a is the dreibein, ω^a is the spin connection and h^a is a nonpropagating degree of freedom [11]. Λ_0 is a parameter. Furthermore, μ is a mass parameter while $\sigma = \pm 1$. The sign can be chosen freely without loss of generality and we set $\sigma = +1$ in this paper. The torsion tensor $T(\omega)_a$ and Riemann tensor $R(\omega)_a$ are [11]

$$T(\omega)_a = de_a + \epsilon_{abc} \omega^a \wedge e^b, \quad R(\omega)_a = d\omega_a + \frac{1}{2} \epsilon_{abc} \omega^a \wedge \omega^b. \quad (2)$$

The MMG Lagrangian density differs from the TMG one because of a single term, namely $16\pi G_3 \mathcal{L}_{\text{MMG}} = 16\pi G_3 \mathcal{L}_{\text{TMG}} + \frac{\alpha}{2} \epsilon_{abc} e^a \wedge h^b \wedge h^c$ [11]. The extra term transforms the Lagrange multiplier h^a into a nonpropagating degree of freedom appearing quadratically in the action. The Lagrange multiplier appears linearly in the TMG action, so its equations of motion enforce the zero torsion constraint $T(\omega)_a = 0$. The MMG action is instead quadratic in h^a , so the equations of motion of this nonpropagating degree of freedom remove it from the action without

imposing any constraint. Two interesting limits of MMG are $\alpha = 0$ and $\alpha = \infty$. In the first limit MMG reduces to TMG; in the second it reduces to the $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ Chern-Simons theory formulation of pure gravity [2]. The first property follows from the definition of MMG while the second is seen by solving the h^a equations of motion, which give a term proportional to $\alpha^{-1} T^2$, which vanishes in the limit $\alpha \rightarrow \infty$.

The classical field equations for MMG are [11],

$$\begin{aligned} 0 &= T(\omega)_a + \alpha \epsilon_{abc} e^b \wedge h^c, \\ 0 &= R(\omega)_a + \mu(1 + \alpha\sigma) \epsilon_{abc} e^b \wedge h^c, \\ 0 &= Dh_a + \frac{\alpha}{2} \epsilon_{abc} h^b \wedge h^c + \frac{\Lambda_0}{2} \epsilon_{abc} e^b \wedge e^c \\ &\quad + \mu\sigma(1 + \alpha\sigma) \epsilon_{abc} e^b \wedge h^c, \end{aligned} \quad (3)$$

where $D = d + \epsilon_{abc} \omega^b$ is the covariant derivative with respect to the spin-connection field ω^a .

The MMG action can be linearized around an AdS_3 background that solves the classical equations of motion by expanding the one form fields e^a , ω^a and h^a into a

background, $\bar{e}^a, \bar{\omega}^a, \bar{h}^a \propto \bar{e}^a$, and three propagating modes, f_+^a, f_-^a and p^a [11], according to

$$\begin{aligned} e^a &= \bar{e}^a + l(f_+^a + f_-^a) + \frac{1}{\mu(1-2C)} p^a, \\ \omega^a &= \bar{\omega}^a - \alpha C \mu \bar{e}^a + (1 - \alpha C \mu l) f_+^a - (1 + \alpha C \mu l) f_-^a \\ &\quad + \frac{M + \alpha \mu(C-1)}{\mu(1-2C)} p^a, \\ h^a &= C \mu \bar{e}^a + C \mu l f_+^a + C \mu l f_-^a + \frac{\mu(1-C)}{\mu(1-2C)} p^a, \end{aligned} \quad (4)$$

where

$$\begin{aligned} \sigma \frac{\Lambda_0}{\mu^2} &\equiv \frac{\Lambda}{\mu^2} - \alpha(1 + \sigma\alpha)C^2, & C &\equiv -\frac{\Lambda + \alpha\Lambda_0}{2\mu^2(1 + \sigma\alpha)^2}, \\ M &\equiv \mu[\sigma(1 + \sigma\alpha) - \alpha C], & 1 - 2C &\equiv \frac{(Ml)^2 - 1}{(1 + \sigma\alpha)^2(l\mu)^2}. \end{aligned} \quad (5)$$

The new parameter Λ is the cosmological constant of the

AdS₃ background, so $\Lambda = -\frac{1}{l^2}$ and l is the AdS₃ radius [11]. The identity relating C to M in Eq. (5) immediately implies (when $\sigma = +1$)

$$[C^2\alpha^2 + 2\alpha C + 2C]l^2\mu^2 - 1 = 0, \quad (6)$$

which is crucially important when deriving the linearized MMG Lagrangian and analyzing the region near the chiral limit.

The boundary central charge for MMG is [11]

$$c_{\pm} \propto \Sigma_{\pm} \equiv \sigma \pm \frac{1}{\mu l} + \alpha C. \quad (7)$$

When $\sigma = +1$ Eqs. (6) and (7) further yield

$$\Sigma_- \Sigma_+ = 1 - 2C. \quad (8)$$

The MMG Lagrangian expanded to quadratic order around the AdS₃ background (with $\sigma = +1$) is [11],

$$\begin{aligned} 16\pi G_3 \mathcal{L}_{\text{MMG}}^{(2)} &= -l\Sigma_-[f_+^a \wedge \bar{D}f_{+a} + l^{-1}\epsilon_{abc}\bar{e}^a \wedge f_+^b \wedge f_+^c] + l\Sigma_+[f_-^a \wedge \bar{D}f_{-a} - l^{-1}\epsilon_{abc}\bar{e}^a \wedge f_-^b \wedge f_-^c] \\ &\quad + \frac{1}{2\mu\Sigma_- \Sigma_+}[p^a \wedge \bar{D}p_a + M\epsilon_{abc}\bar{e}^a \wedge p^b \wedge p^c], \end{aligned} \quad (9)$$

where $\bar{D} = d + \epsilon_{abc}\bar{\omega}^a$.

The equations of motion obeyed by $\mathcal{L}_{\text{MMG}}^{(2)}$ are [11],

$$\bar{D}f_{+a} + l^{-1}\epsilon_{abc}\bar{e}^b \wedge f_+^c = 0, \quad \bar{D}f_{-a} - l^{-1}\epsilon_{abc}\bar{e}^b \wedge f_-^c = 0, \quad \bar{D}p_a + M\epsilon_{abc}\bar{e}^b \wedge p^c = 0. \quad (10)$$

A. Alternative formulation

The MMG action can be written in a second-order metric formulation. Expanding around the AdS₃ background with metric

$$ds^2 \equiv \bar{g}_{\mu\nu} dx^\mu dx^\nu = -\frac{1}{\Lambda}(-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\phi^2), \quad (11)$$

one obtains an action, whose quadratic form was given in [12] in the gauge $\bar{\nabla}_\mu h^{\mu\nu} = 0$. Writing $h = \bar{g}^{\mu\nu} h_{\mu\nu}$ the result is

$$S = -\frac{1}{4\pi G_3} \int d^3x \sqrt{-\bar{g}} \left[\left(\sigma - \frac{\gamma\Lambda}{2\mu^2} \right) [\bar{\nabla}_\alpha h_{\mu\nu} \bar{\nabla}^\alpha h^{\mu\nu} + 2\Lambda h_{\mu\nu} h^{\mu\nu}] + \frac{1}{\mu} \epsilon_\mu^{\alpha\beta} \bar{\nabla}_\alpha h_{\mu\nu} (\bar{\square} - 2\Lambda) h_{\beta\nu} \right]. \quad (12)$$

The definition of Λ and σ is as before, while γ is a dimensionless parameter. The boundary central charges are [12],

$$c_L = \frac{3l}{2G_3} \left(\sigma + \frac{\gamma}{2l^2\mu^2} - \frac{1}{l\mu} \right), \quad c_R = \frac{3l}{2G_3} \left(\sigma + \frac{\gamma}{2l^2\mu^2} + \frac{1}{l\mu} \right). \quad (13)$$

Furthermore, $h_{\mu\nu}$ is expanded into three propagating modes $h_{\mu\nu} = h_{\mu\nu}^M + h_{\mu\nu}^R + h_{\mu\nu}^L$ and their field equations are [12],

$$\epsilon_\mu^{\alpha\beta} \bar{\nabla}_\alpha h_{\beta\nu}^M + \mu \left(\sigma - \frac{\gamma\Lambda}{2\mu^2} \right) h_{\beta\nu}^M = 0, \quad \epsilon_\mu^{\alpha\beta} \bar{\nabla}_\alpha h_{\beta\nu}^L + l^{-1} h_{\beta\nu}^L = 0, \quad \epsilon_\mu^{\alpha\beta} \bar{\nabla}_\alpha h_{\beta\nu}^R - l^{-1} h_{\beta\nu}^R = 0. \quad (14)$$

III. THE CHIRAL LIMIT

We are only concerned here with the case where $\sigma = +1$ and we choose $c_- = 0$ for the chiral limit. Analogous statements can be made for the other limit, $c_+ = 0$ by replacing $l \rightarrow -l$ and $f_+^a \leftrightarrow -f_-^a$ etc. in the formulas presented in this paper. After substituting $c_- = 0$ into Eq. (5), we find the values for the parameters of the theory at the chiral limit,

$$C = \frac{1}{2}, \quad M = l^{-1}, \quad \alpha = \frac{2}{l\mu} - 2, \quad \Sigma_- = 0, \quad \Sigma_+ = 2. \quad (15)$$

It is clear that at the chiral limit any term involving the p mode will have infinite coupling strength. This strong coupling can be real or just an effect of a bad normalization. To see which case it is, we rescale the mode p by defining $p^a = (1 - 2C)q^a = \Sigma_+ \Sigma_- q^a$. Then, Eq. (9) becomes

$$16\pi G_3 \mathcal{L}_{\text{MMG}}^{(2)} = \frac{2}{\mu} [f_-^a \wedge \bar{D}f_{-a} - l^{-1} \epsilon_{abc} \bar{e}^a \wedge f_-^b \wedge f_-^c]. \quad (16)$$

Notice that the two modes f_+^a and q^a have disappeared completely from the action, so the only propagating degree of freedom is the chiral boundary graviton f_-^a and the equations of motion for f_+^a and q^a are not determined by the action. If we derive the equations of motion before taking the chiral limit, we do get that three field equations in (10) become,

$$\begin{aligned} \bar{D}f_{+a} + l^{-1} \epsilon_{abc} \bar{e}^b \wedge f_+^c &= 0, \quad \bar{D}f_{-a} - l^{-1} \epsilon_{abc} \bar{e}^b \wedge f_-^c = 0, \\ \bar{D}q_a + l^{-1} \epsilon_{abc} \bar{e}^b \wedge q^c &= 0. \end{aligned} \quad (17)$$

The important point here is that the equations of motion for f_+^a and q^a decouple from f_-^a , which is necessary for the consistency of the chiral limit.

Our results are in agreement with the analysis of the chiral limit of MMG done in [12] which we briefly summarize in III A. The first-order formalism used in this letter makes it easy to extend the analysis, done in [12] only at quadratic order and in a Hamiltonian formalism, to the

full interacting action. The linearization plus rescaling of the action done here is, in our opinion, the simplest way to check that the unwanted excitations f_+^a, q^a do in fact decouple in the full theory.

A. Alternative formulation at the chiral limit

We choose $c_L = 0$ for the alternative formulation at the chiral limit. Then, the MMG action becomes,

$$S = -\frac{1}{2\pi l\mu G_3} \int d^3x \epsilon_\mu^{\alpha\beta} \bar{\nabla}_\alpha h^{R\mu\nu} \square h_{\beta\nu}^R + \frac{2}{l^2} \epsilon_\mu^{\alpha\beta} (\bar{\nabla}_\alpha h^{R\mu\nu}) h_{\beta\nu}^R. \quad (18)$$

The field equations obeyed the three propagating modes at the chiral limit are,

$$\begin{aligned} \epsilon_\mu^{\alpha\beta} \bar{\nabla}_\alpha h_{\beta\nu}^M + l^{-1} h_{\beta\nu}^M &= 0, \quad \epsilon_\mu^{\alpha\beta} \bar{\nabla}_\alpha h_{\beta\nu}^L + l^{-1} h_{\beta\nu}^L = 0, \\ \epsilon_\mu^{\alpha\beta} \bar{\nabla}_\alpha h_{\beta\nu}^R - l^{-1} h_{\beta\nu}^R &= 0. \end{aligned} \quad (19)$$

It is clear that $h_{\mu\nu}^M, h_{\mu\nu}^L$ and $h_{\mu\nu}^R$ correspond to q^a, f_+ and f_- , respectively.

IV. THE NEAR-CHIRAL LIMIT

Even chiral TMG [4,5] appears to be consistent and ghost-free in the singlet sector of the “−” boundary Virasoro algebra—which has $c_- = 0$ so it does admit a singlet unitary representation. On the other hand, TMG does contain ghosts for any $c_- > 0$, so it is not consistent near the chiral point $c_- = 0$. MMG, on the other hand, makes sense also for $c_- > 0$, at least at the level of the noninteracting quadratic action. To investigate whether MMG is consistent and computable near the chiral limit is the aim of this section and it is the main new result of this letter.

We begin by writing in full the MMG action expanded around the AdS_3 background. In first order formalism, it contains cubic terms besides the quadratic ones. The cubic-order MMG Lagrangian for $\sigma = +1$ is

$$\begin{aligned} 16\pi G_3 \mathcal{L}_{\text{MMG}}^{(3)} &= -\frac{2l}{3} \Sigma_- \epsilon_{abc} f_+^a \wedge f_+^b \wedge f_+^c + \frac{lM+1}{2\mu \Sigma_- \Sigma_+} \epsilon_{abc} f_+^a \wedge p^b \wedge p^c - \frac{2l}{3} \Sigma_+ \epsilon_{abc} f_-^a \wedge f_-^b \wedge f_-^c \\ &\quad + \frac{lM-1}{2\mu \Sigma_- \Sigma_+} \epsilon_{abc} f_-^a \wedge p^b \wedge p^c + \frac{(2+\alpha)\mu + 2M}{6\mu^2 (\Sigma_- \Sigma_+)^2} \epsilon_{abc} p^a \wedge p^b \wedge p^c. \end{aligned} \quad (20)$$

To analyze the region near the chiral limit we take the central charge $c_- = \eta \ll 1$ and expand in powers of η ,

$$c_- \propto \Sigma_- = 1 - \frac{1}{\mu l} + \alpha C = \eta. \quad (21)$$

Next we use Eqs. (6) and (21), to find that, near the chiral limit, C, α and M have the following expansion in powers of η

$$C = \frac{1}{2} - \frac{\eta}{l\mu}, \quad \alpha = \frac{2l\mu\eta + 2 - 2l\mu}{l\mu - 2\eta}, \quad M = \frac{\mu[-l\mu + 4 - \frac{2}{l\mu}\eta - 1]}{2\eta - l\mu}. \quad (22)$$

We see that after rescaling the p^a mode as $p^a = (1 - 2C)q^a = \Sigma_+ \Sigma_- q^a$ and keeping only the lowest order of η in the numerator and the zeroth order in the denominator, the cubic-order MMG Lagrangian near the chiral limit is

$$\begin{aligned} 16\pi G_3 \mathcal{L}_{\text{MMG}}^{(3)} = & -\frac{2l\eta}{3} \epsilon_{abc} f_+^a \wedge f_+^b \wedge f_+^c + \frac{2\eta}{l\mu^2} \epsilon_{abc} f_+^a \wedge q^b \wedge q^c \\ & - \left(\frac{4}{3\mu} + \frac{2l\eta}{3} \right) \epsilon_{abc} f_-^a \wedge f_-^b \wedge f_-^c + \left(\frac{1}{\mu} - \frac{4}{l\mu^2} + \frac{4}{l^2\mu^3} \right) \eta^2 \epsilon_{abc} f_-^a \wedge q^b \wedge q^c \\ & + \frac{4\eta}{3l^2\mu^3} \epsilon_{abc} q^a \wedge q^b \wedge q^c. \end{aligned} \quad (23)$$

Two points to keep in mind here are: a) all cubic terms except the third one, which contains only f_-^a , vanish at $\eta = 0$; b) There exist no terms in (23) which contain two f_-^a 's and either one q^a or one f_+^a . These two points are crucial to show the consistency of the tree level expansion of MMG near the chiral limit, which we shall prove in the next section.

At the chiral point $\eta = 0$ the full Lagrangian is

$$16\pi G_3 \mathcal{L}_{\text{MMG}} = \frac{2}{\mu} [f_-^a \wedge \bar{D}f_{-a} - l^{-1} \epsilon_{abc} \bar{e}^a \wedge f_-^b \wedge f_-^c] - \frac{4}{3\mu} \epsilon_{abc} f_-^a \wedge f_-^b \wedge f_-^c. \quad (24)$$

As for the quadratic action (16), the full action also depends only on f_-^a . The additional modes f_+^a , q^a have completely decoupled.

The chiral-point limit of the three field equations obeyed by the full Lagrangian is

$$\begin{aligned} 0 = & \bar{D}f_{+a} + l^{-1} \epsilon_{abc} \bar{e}^b \wedge f_+^c + \epsilon_{abc} f_+^b \wedge f_+^c - \frac{1}{l\mu} \epsilon_{abc} q^b \wedge q^c, \\ 0 = & \bar{D}f_{-a} - l^{-1} \epsilon_{abc} \bar{e}^b \wedge f_-^c - \epsilon_{abc} f_-^b \wedge f_-^c, \\ 0 = & \bar{D}q_a + l^{-1} \epsilon_{abc} \bar{e}^b \wedge q^c + \frac{2}{l\mu} \epsilon_{abc} q^b \wedge q^c + 2\epsilon_{abc} f_+^b \wedge q^c. \end{aligned} \quad (25)$$

As in (17), the modes f_+^a , q^a decouple from f_-^a not only in the action but also in the equations of motion.

V. UNITARITY AND STRONG COUPLING

MMG is not unitary near the chiral limit $c_- = 0$ for $\eta \geq 0$ [13]. It is still interesting to study this region in view of possible applications where the holographic dual is non unitary, as in statistical mechanics models and in view of other considerations explained at the end of this section. Besides that, it is instructive to check the region since $c_- = 0$ (equivalently, $c_+ = 0$) is a potentially dangerous point, because it is there that the kinetic term of the two modes f_+ and p^a vanishes. We saw that the rescaling of $p^a = \Sigma_+ \Sigma_- q^a$ makes it manifest that at $\eta = 0$ the whole action for f_+ and q^a vanish. So these modes disappear from the theory without producing infinite strong coupling

pathologies. The behavior of the theory near $\eta = 0$ is different. There, all modes propagate and the presence of one power of η in the kinetic term of q^a and f_+^a makes it possible that strong coupling pathologies are present. In fact a simple power counting argument shows that they do exist, but they are not present at tree level.

The first feature that we need to prove that MMG is (classically) not strongly coupled is that the only field that can appear as an external state in a Feynman diagram expansion of the action is f_-^a . This is the field that remains physical at the chiral point. After all, even in TMG at the chiral point, there exist nonunitary modes that are non-singlets of the chiral left Virasoro algebra [5,14]. Certainly, there could be a strongly coupled sector in the near-chiral MMG, but it would be harmless if it was sequestered from the physical sector, whose propagating field is f_-^a only. The previous remark shows that at tree level MMG is safe, because a tree-level diagram in which some of the dangerous modes q^a , f_+^a propagate and with only f_-^a s as external legs must contain at least two vertices with two f_-^a s and either one q^a or one f_+^a . But Eq. (23) shows that such vertices do not exist.

Beyond tree level, the theory still makes sense up to a loop order that depends on the number of external f_-^a legs. A standard Feynman diagram counting shows that the number of vertices, V , external f_-^a legs, E , propagators P , loops L , and leading powers of η , H , are related by

$$2P + E = 3V, \quad L = 1 + P - V, \quad H = V + E - P. \quad (26)$$

These equations give $H = 1 + E - L$, so a diagram with E external f^a legs exhibits a string coupling pathology—i.e. negative powers of η —only at $L > 1 + E$ loops.

It is our opinion that neither the loop divergences of MMG, nor indeed its lack of unitarity near the chiral limit necessarily signal a lethal pathology of the theory, but only of its classical action. First of all, if we use MMG as a bulk dual of a large- N boundary 2D CFT, it is the classical limit that matters in the description of the large- N limit. Finite N corrections involve genuine bulk quantum gravity effects. So the chiral point, where one of the central charges is very small, is not under perturbative control. In the same vein, at finite N (e.g. finite Newton constant) the action is not power-counting renormalizable so it requires counterterms not present in the classical action. Those counterterms could cancel the negative- H divergences order by order in the loop expansion. Whether this is indeed possible is an interesting question, but quite out of reach with perturbative bulk gravity techniques. So, we move instead to another region, where both c_+ and c_- are positive *and large* but where the ratio

$$\frac{c_+ - c_-}{c_+ + c_-} \lesssim 1. \quad (27)$$

This is the limit where the ratio approaches 1 from below.

VI. LEFT-RIGHT ASYMMETRIC REGIME

This is the regime where the two boundary central charges are “as chiral as possible,” that is when they differ by a large amount while lying in the unitary region of MMG parameters. In this regime it is convenient to use the parametrization of [13], which defines

$$x = 2C, \quad y = \frac{2}{l\mu}. \quad (28)$$

The central charges become [13]

$$c_{\pm} = \frac{3l}{2G_3} \Sigma_{\pm} = \frac{3l}{2G_3} \left(1 + \frac{\alpha x}{2} \pm \frac{y}{2} \right), \quad (29)$$

and Eq. (8) is transformed to

$$\Sigma_+ \Sigma_- = 1 - x. \quad (30)$$

Combining Eqs. (29) and (8) we get the equation of a hyperbola in the x, y plane [13]

$$y^2 = \alpha^2 x^2 + 4(1 + \alpha)x. \quad (31)$$

While minimal massive gravity is not unitary near a chiral point, it is unitary in a region where the two central charges take very different values, namely where

$$\frac{c_+ - c_-}{c_+ + c_-} = \frac{y}{2 + \alpha x} = 1 - \epsilon, \quad 0 < \epsilon \ll 1. \quad (32)$$

Since $\alpha < 0$, $y > 0$ [13], this limit is reached for negative values of x and $\alpha x \rightarrow \infty$, where

$$y = \alpha x + \frac{2(1 + \alpha)}{\alpha} + O(1/x). \quad (33)$$

Here we are interested in studying minimal massive gravity, in which α is finite and nonzero. So, we study the limit $x \rightarrow -\infty$, $\alpha = \text{constant}$. Substituting Eq. (33) into $c_{\pm}$ we find

$$c_+ = \frac{3l}{2G_3} \left(2 + \alpha x + \frac{1}{\alpha} \right), \quad c_- = \frac{3l}{2G_3} \left(-\frac{1}{\alpha} \right). \quad (34)$$

Both charges are positive and c_+ diverges in the limit.

The quadratic Lagrangian written in the new parametrization in terms of α, x, y is

$$\begin{aligned} 16\pi G_3 \mathcal{L}_{\text{MMG}}^{(2)} = & -l \left(1 + \frac{\alpha x}{2} - \frac{y}{2} \right) [f_+^a \wedge \bar{D}f_{+a} + l^{-1} \epsilon_{abc} \bar{e}^a \wedge f_+^b \wedge f_+^c] \\ & + l \left(1 + \frac{\alpha x}{2} + \frac{y}{2} \right) [f_-^a \wedge \bar{D}f_{-a} - l^{-1} \epsilon_{abc} \bar{e}^a \wedge f_-^b \wedge f_-^c] \\ & + \frac{ly}{4(1-x)} p^a \wedge \bar{D}p_a + \frac{(2+2\alpha-\alpha x)}{4(1-x)} \epsilon_{abc} \bar{e}^a \wedge p^b \wedge p^c. \end{aligned} \quad (35)$$

We see that in the limit $x \rightarrow -\infty$, $\alpha = \text{constant} < 0$ the kinetic terms of f_+^a and p^a remain constant while the kinetic term of f_-^a diverges proportionally to αx . The mass term of p^a approaches a finite value. The cubic part of the Lagrangian is

$$\begin{aligned}
16\pi G_3 \mathcal{L}_{\text{MMG}}^{(3)} = l \left[-\frac{2+\alpha x-y}{3} \epsilon_{abc} f_+^a \wedge f_+^b \wedge f_+^c + \frac{2+2\alpha-\alpha x+y}{4(1-x)} \epsilon_{abc} f_+^a \wedge p^b \wedge p^c \right. \\
- \frac{2+\alpha x+y}{3} \epsilon_{abc} f_-^a \wedge f_-^b \wedge f_-^c + \frac{2+2\alpha-\alpha x-y}{4(1-x)} \epsilon_{abc} f_-^a \wedge p^b \wedge p^c \\
\left. + \frac{(4+3\alpha-\alpha x)y}{12(1-x)^2} \epsilon_{abc} p^a \wedge p^b \wedge p^c \right]. \quad (36)
\end{aligned}$$

All interactions are finite in the limit, except the $(f_-^a)^3$ term, which diverges proportionally to αx . The interaction term $f_+^a (p^a)^2$ vanishes. By redefining $f_-^a = (\alpha x)^{-1/2} \phi_-^a$ and taking the limit $x \rightarrow -\infty$ we finally arrive at the following simple result:

- (1) All interaction terms with at least one ϕ_-^a vanish, so ϕ_-^a becomes a decoupled free field.
- (2) f_+^a and p^a become two decoupled self-interacting fields, without any interaction with each other of ϕ_-^a .

We conclude that in extreme asymmetric limit, which is well inside the unitary region of parameters for MMG [13], the theory exhibits no pathologies and in fact becomes quite simple. The limit can be taken while keeping both c_+ and c_- large, so it is well within the validity regime of a

semiclassical holographic interpretation. It is certainly interesting to see if the simplification in the 3D bulk theory corresponds to a simplification of the dual boundary conformal field theory, but we leave this question for future study.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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