

Thermal modifications of mesons and energy-energy correlators from real-time simulations of a $U(1)$ lattice gauge theory

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We investigate thermal properties of a $U(1)$ lattice gauge theory in $1 + 1$ dimensions through real-time simulations. We extract the spectral functions directly coupling to the pseudoscalar and scalar mesons, demonstrating the thermal modifications of these states with increasing temperatures. Introducing the notion of energy-flow operators, we quantify the temporal buildup of correlations in the energy flows across the lattice. We demonstrate that energy-energy correlators fail to factorize to products of energy flows, both in the vacuum and at nonzero temperature, indicating the presence of nontrivial correlations in the quantum states. Our results constitute a first real-time *ab initio* study of bound-state thermal broadening and finite temperature energy-flow correlations in a gauge theory, providing a benchmark for future studies of hadronic matter under extreme conditions.

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I. INTRODUCTION

Characterizing the properties and dynamics of quantum systems under extreme conditions remains one of the central open theoretical problems across physics, from condensed matter [1–3] to high-energy theory [4,5]. A deeper understanding of these regimes is not only crucial for unraveling the inner workings of quantum field theories, but also essential to describe a myriad of phenomena observed in nature across all scales, from the formation of the early Universe [6] to the dynamics of atomic systems [7].

In high-energy nuclear physics, elucidating the behavior of quantum chromodynamics (QCD) at finite temperature and baryon density is essential for explaining how matter forms and evolves within the Standard Model. Over the past five decades a succession of high-energy nuclear experiments—culminating in RHIC’s and LHC’s ultra-relativistic heavy-ion physics programs [8,9]—has provided an unprecedented opportunity to test the properties of hot and dense QCD, including its far-from-equilibrium dynamics. These efforts led to the first experimental observation of a quark-gluon plasma state [10], which filled the primordial Universe in the first few microseconds after the big bang.

Despite the numerous theoretical and experimental advances motivated by these physics programs, fundamental questions about the properties of QCD matter under extreme conditions remain open. In this respect one could argue that two theoretical limitations stand out. First, although finite temperature (T) equilibrium (Euclidian) lattice QCD calculations have allowed precise mapping of several properties of the theory, see e.g. [11], notorious sign problems prevent the access to the real-time dynamics and systems at finite density, needed to describe the spacetime evolution of matter and the phase structure of the theory [12]. Second, and as a result of the previous point, such dynamical observables are thus studied through effective descriptions, e.g. QCD effective kinetic theory [13] or relativistic viscous hydrodynamics [14]. In practice, the respective weak-coupling and near-equilibrium assumptions underlying these pictures are typically pushed beyond their theoretical applicability, preventing a true first-principle description of the observed phenomena. Moreover, how these different effective descriptions can be properly combined is still an open question.

Therefore, bridging this gap requires novel theory tools capable of following real-time dynamics of QCD at finite T from a first-principle formulation. In this regard, emerging technologies and methods from quantum information science (QIS)—such as digital quantum computers or analog quantum simulators—promise exactly the capability of tracking the real-time dynamics of quantum systems from first principles, sidestepping the sign problem, see e.g. [15,16]. Extending these QIS techniques to gauge theories at finite temperature could, one day, allow for an entirely new way to describe and understand the complex dynamics

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of heavy-ion collisions and ultimately the properties of QCD matter under extreme conditions.

In this work, we take a first step in this direction by performing real-time simulations in the Schwinger model, a $U(1)$ gauge theory in $1+1$ dimensions, at finite T . This extends a large range of recent works on this model at $T=0$, see e.g. [17,18] for recent reviews. We present results for the temperature-dependent spectral functions of the theory's lightest mesonic excitations, revealing how the interfermion potential is thermally screened by the hot medium. We directly observe that the mass of the bound states remain largely unchanged at finite T , indicating that the real part of the respective interfermionic potential is unmodified. Conversely, the width of these states grows with temperature, indicating the rise of the imaginary part of the potential similarly to what is seen in lattice QCD studies. The pattern of sequential melting, paradigmatic as thermometer of the bulk matter produced in heavy-ion collisions [19,20], is not cleanly observed although there is distinction between the studied bound states. We complement the discussion by studying the behavior of detector observables at finite T , computing energy-correlation functions that play the role of calorimeter cells and track how short-range correlations propagate to asymptotic distances, see e.g. [21] for a recent review. We find that at finite T more nontrivial correlations develop compared to the $T=0$ scenario. Technical aspects of our calculations and details of the lattice model considered are introduced beforehand.

II. LATTICE SCHWINGER MODEL AT FINITE T

The Schwinger model corresponds to a $U(1)$ gauge quantum field theory in $1+1$ spacetime dimensions [22], analog to the QED in $3+1$ D. This theory describes the interaction of a single-flavor fermion ψ , with mass m_c , with the gauge field A^μ . The matter-gauge coupling is g_c ; the subscript “c” in the model parameters stands for “continuum.” While this theory is in many ways simpler than QCD in $3+1$ D, they share several key qualitative features. In particular, among other features, the Schwinger model possesses an interfermionic confining potential, spontaneous chiral symmetry breaking, and chiral anomaly; see e.g. [22–26]. Unlike higher-dimensional theories, the Schwinger model is dual to a bosonic interacting theory [23], where the fundamental particle is the so-called Schwinger boson. In the $m_c=0$ limit, this massive boson, with mass $g_c/\sqrt{\pi}$, becomes noninteracting and the theory can be solved analytically through Abelian bosonization methods.

Although for general m_c/g_c the theory cannot be solved analytically, its real-time properties can be numerically studied using lattice simulations in the Hamiltonian picture, see e.g. [16,17]. Recently, such numerical approaches have become practically feasible, with the progress of new quantum technologies and related tools. Currently they

are particularly suited for the simulation of lower-dimensional theories.¹

In the continuum, the Schwinger model in the temporal gauge $A_0=0$ is described by the Hamiltonian,

$$H_{\text{cont}} = \int dx \left[\frac{1}{2} F_{01}^2 + \bar{\psi}(-i\gamma^1 \partial_1 + g_c \gamma^1 A_1 + m_c) \psi \right], \quad (1)$$

where $F_{01} = \partial_0 A_1$. It can be mapped to a Kogut-Susskind lattice Hamiltonian [30,31],

$$H_{\text{lat}} = -\frac{i}{2a} \sum_{n=1}^{N-1} (U_n^\dagger \chi_n^\dagger \chi_{n+1} - U_n \chi_{n+1}^\dagger \chi_n) + m \sum_{n=1}^N (-1)^i \chi_n^\dagger \chi_n + \frac{ag^2}{2} \sum_{n=1}^{N-1} L_n^2, \quad (2)$$

where we introduced staggered fermionic variables χ_n , gauge link operators $U_n = e^{-iagA_1(an)}$, and electric field strength operators $L_n = E(an)/g$. The system has volume Na given in terms of lattice spacing a and the total number of sites N . We use m and g to denote the mass and coupling in the lattice model, respectively. We further employ the finite lattice spacing correction to the mass, $m \rightarrow m - g^2 a/8$, introduced in [32] to preserve chiral symmetry in the massless limit.

The Hamiltonian in Eq. (2) possesses a residual gauge freedom, namely $\chi_n \rightarrow \Omega_n \chi_n$, $U_n \rightarrow \Omega_{n+1}^\dagger U_n \Omega_n$. Performing a gauge transformation with $\Omega_n = \prod_{i=1}^{n-1} U_i$, we eliminate the link operators from the Hamiltonian. Furthermore, the electric field operators must obey the discretized version of Gauss's law,

$$L_n - L_{n-1} = Q_n, \quad (3)$$

where $Q_n = \chi_n^\dagger \chi_n + \frac{(-1)^{n-1}}{2}$ is the electric charge operator. With open boundary conditions, no background electric field, and in the total charge zero sector, Gauss's law can be solved as $L_n = \sum_{i=1}^n Q_i$. Thus the system is described exclusively in terms of fermionic variables with a finite-dimensional Hilbert space, at the expense of introducing an all-to-all interaction in the electric field energy.

To make contact with the language of QIS, fermionic degrees of freedom can be mapped to spins by the means of the Jordan-Wigner transformation [33],

$$\chi_n = \frac{(X_n - iY_n)}{2} \prod_{k=1}^{n-1} (-iZ_k), \quad (4)$$

where X_i , Y_i , Z_i are the Pauli matrices acting in the spin-1/2 Hilbert space of site i . The respective lattice Hamiltonian reads

¹See e.g. [27–29] for recent studies of higher-dimensional theories.

$$H = \frac{1}{4a} \sum_{i=1}^{N-1} (X_i X_{i+1} + Y_i Y_{i+1}) + \frac{m}{2} \sum_{i=1}^N (-1)^i Z_i + \frac{ag^2}{2} \sum_{i=1}^{N-1} \left(\sum_{j=1}^i \frac{Z_j + (-1)^j}{2} \right)^2. \quad (5)$$

In this work we explore properties of the theory at finite T . In general, a finite temperature state is a (statistically) mixed quantum state described by the thermal density matrix

$$\rho_\beta = \frac{e^{-\beta H}}{\text{Tr} e^{-\beta H}}, \quad (6)$$

where $\beta = T^{-1}$ is the inverse temperature. Thermal expectation value of any operator $\mathcal{O}$ is given by

$$\langle \mathcal{O} \rangle_\beta = \text{Tr}(\rho_\beta \mathcal{O}). \quad (7)$$

Because of the presence of a conserved charge—the total electric charge of the system—the thermal density matrix is block diagonal in the basis where the charge operator is diagonal, for instance in the computational basis where the Pauli operators take their familiar form. Furthermore, since the electric charge is confined in the Schwinger model, nonzero charge sectors decouple in the infinite volume limit. Thus the charge zero sector is of main interest, and in the following we will always restrict ourselves to the chargeless sector. Any operators we consider also commute with charge, and Eq. (7) holds if ρ_β and $\mathcal{O}$ are understood as their respective charge zero blocks.

For sufficiently small systems it is possible to perform a full diagonalization of the Hamiltonian, i.e., find the eigenstates $|n\rangle$ and the spectrum E_n such that

$$H|n\rangle = E_n|n\rangle, \quad (8)$$

so that the thermal expectation value of an operator $\mathcal{O}$ is

$$\langle \mathcal{O} \rangle_\beta = \sum_n p_n \langle n | \mathcal{O} | n \rangle, \quad (9)$$

where $p_n = e^{-\beta E_n} / Z$, with $Z = \sum_n e^{-\beta E_n}$, are the Boltzmann weights. As noted above, sums over n include only the eigenvalues with zero charge. The dimension of the corresponding Hilbert space is $D = \binom{N/2}{N}$ for a system with N lattice sites. All the results reported below are obtained with this method of exact diagonalization.

III. SPECTRAL FUNCTIONS AND THERMAL MODIFICATIONS OF MESONS

Spectral functions are an indispensable tool for understanding the properties of the excitation spectrum in an interacting quantum theory; in QCD they play a critical role

in e.g. describing the bulk matter properties in heavy-ion experiments. In particular, they have been used to study quarkonium melting [19,34,35], dilepton and photon emission rates [36–40], and transport coefficients [41]. Since a spectral function is related to a real-time two-point correlator of the mesonlike operators, it is most natural to study it with Hamiltonian methods. This has previously been performed in a nongauge theory, see [42]. In what follows we discuss the spectral properties of such real-time two-point correlation functions in the lattice Schwinger model at finite T .

Consider a time-ordered two-point function of an arbitrary local operator $\mathcal{O}$,

$$D_{\mathcal{O}}(x, t, x_0, t_0) \equiv \langle T \mathcal{O}(x, t) \mathcal{O}^\dagger(x_0, t_0) \rangle_\beta, \quad (10)$$

where in the context of QCD, the operator $\mathcal{O}$ is often chosen as a meson operator, $M(x, t) = \bar{q}(x, t) \Gamma q(x, t)$, with Γ denoting an appropriate combination of Dirac matrices corresponding to the meson's quantum numbers. The correlation function can be decomposed into commutator and anticommutator parts, reflecting the statistical function F and the spectral function ρ [19],

$$D_{\mathcal{O}}(x, t, x_0, t_0) = F_{\mathcal{O}}(x, t, x_0, t_0) - \frac{i}{2} \text{sign}(t - t_0) \rho_{\mathcal{O}}(x, t, x_0, t_0). \quad (11)$$

In the Schwinger model, there are several species of mesons, with the lowest-lying one being the pseudoscalar state. In the massless fermion case, as noted above, the Schwinger model is exactly solvable via bosonization, with the free Schwinger boson being precisely this pseudoscalar meson. Naturally, the two-point function of axial charge operators, $q_5(x) \equiv \bar{\psi}(x) \gamma^0 \gamma^5 \psi(x)$, receives contributions from this pseudoscalar state.² We thus consider the two-point correlator

$$D_p(x, t) = \langle T q_5(x, t) q_5(0, 0) \rangle_\beta, \quad (12)$$

where the subscript p stands for pseudoscalar. In momentum space, focusing on the zero spatial momentum mode, we find

$$D_p(\omega) = \frac{1}{V} \lim_{\epsilon \rightarrow 0} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle T Q_5(t) Q_5(0) \rangle_\beta e^{-\epsilon|t|}, \quad (13)$$

where we suppressed $k = 0$ in the notation; V is the one-dimensional spatial volume of the system and Q_5 is the

²Note that, due to the Dirac matrix algebra in $1+1$ dimensions, $q_5(x) = j(x) = \bar{\psi}(x) \gamma^1 \psi(x)$, so one can identify the pseudoscalar and the vector meson state, as sometimes found in the literature.

global axial charge: $Q_5(t) = \int_0^L dx q_5(x, t)$. To target the second excited state, corresponding to the a scalar meson at strong coupling [23,43], we further consider the higher weight correlator,

$$D_s(\omega) = \frac{1}{V} \lim_{\epsilon \rightarrow 0} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle T Q_5^2(t) Q_5^2(0) \rangle_{\beta} e^{-\epsilon|t|}. \quad (14)$$

This structure is motivated by the strong-coupling limit where one can understand the scalar state as a bound state of two pseudoscalar mesons. Indeed, in this limit $m_S = 2m_P + \mathcal{O}(mg^{-1})$ [43], where m_S and m_P are the scalar and pseudoscalar meson masses, respectively.

The procedure to extract either of these correlators is similar; we focus the discussion now on the pseudoscalar sector. We define the finite T correlator

$$C_{Q_5}(t) = \langle T Q_5(t) Q_5(0) \rangle_{\beta} \equiv \text{tr}(\rho_{\beta} Q_5(t) Q_5(0)), \quad (15)$$

where $Q_5(t)$ evolves according to the Heisenberg picture

$$Q_5(t) = e^{iHt} Q_5(0) e^{-iHt}. \quad (16)$$

The discretized version of the global axial charge operator is

$$Q_5(0) = \sum_n \frac{Y_n X_{n+1} - X_n Y_{n+1}}{4}, \quad (17)$$

after applying the Jordan-Wigner representation of the fermionic fields. Defining the matrix elements of the axial charge operator

$$c_{mn} = \langle m | Q_5(0) | n \rangle, \quad (18)$$

where $\{|n\rangle\}$ are the Hamiltonian eigenstates in Eq. (8), one finds an exact expression for the time dependence of the axial charge correlator

$$\langle T Q_5(t) Q_5(0) \rangle_{\beta} = \frac{1}{Z} \sum_{n,m} |c_{nm}|^2 e^{-\beta E_n} e^{i(E_n - E_m)|t|}, \quad (19)$$

where at zero temperature the double sum is replaced by a single sum, setting $n = 0$. Furthermore, the frequency domain expression for the spectral function can also be determined analytically [19]

$$\begin{aligned} \rho(\omega) &= \sum_{n,m} |c_{nm}|^2 p_n [\delta(\Delta E_{nm} + \omega) - \delta(\Delta E_{mn} + \omega)] \\ &= \sum_{n,m} |c_{nm}|^2 p_n \frac{4\epsilon\omega\Delta E_{mn}}{(\Delta E_{nm}^2 - \omega^2)^2 + 2\epsilon^2(\Delta E_{nm}^2 + \omega^2) + \epsilon^4} \Big|_{\epsilon \rightarrow 0}, \end{aligned} \quad (20)$$

where we defined energy differences $\Delta E_{nm} \equiv E_n - E_m$ and in the second line introduced a regulator ϵ for convergence. From a practical real-time simulation viewpoint, the ϵ regulator suppresses the contributions from $|t| \gtrsim 1/\epsilon$ reflecting the finite time intervals accessible in a simulation. Note that, with exact diagonalization, one can directly compute c_{nm} and ΔE_{nm} , so performing a real-time evolution numerically is not necessary. However, for a simulation with quantum hardware or with tensor network methods an actual time evolution would be required. Our early attempts using the latter revealed a rapid increase of the bond dimension of the matrix product state during the real-time evolution of a thermal state in the purification approach [44–46], even for modest ($N \sim 40$) system sizes. Thus we leave a detailed tensor network study to the future and focus on exact diagonalization in this work.

From the structure of Eq. (20) it is clear that the peaks in the spectral function correspond to the differences in the energies of the Hamiltonian eigenstates, with contributions from different pairs of states weighed by the thermal coefficients $e^{-\beta E_n}$ and the matrix elements of the axial charge operator c_{mn} . In Fig. 1 we display the spectral function in the vacuum in the pseudoscalar (with $\mathcal{O} = Q_5$)

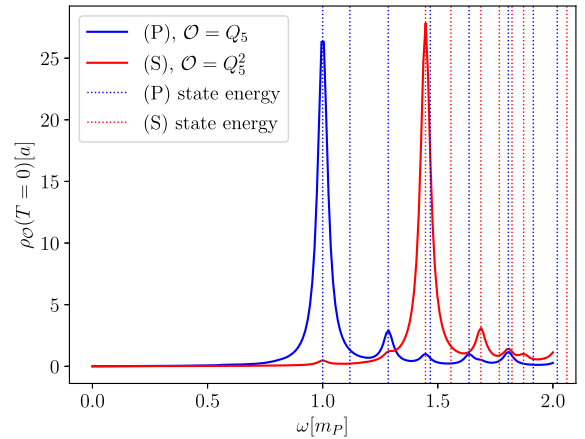


FIG. 1. Spectral function in the vacuum in the pseudoscalar (P) and scalar (S) sectors. Energies of the first few excited states are shown in dotted lines with blue color for negative (P) and red color for positive (S) $\mathcal{C}$ -parity. The corresponding energy levels are presented in Table I. The parameters of the system are $m/g = 0.5$, $N = 12$, $ga = 0.5$, $\epsilon a = 0.02$. Energy is measured in the units of the first excited-state energy that corresponds to the mass of the pseudoscalar meson m_P .

TABLE I. Low-lying spectrum of the Schwinger model with $m/g = 0.5$, $N = 12$, $ga = 0.5$. n denotes the state level, starting from 0 for the vacuum. Energies E have vacuum energy subtracted and are given in the units of inverse lattice spacing a . The sign of expectation values of the approximate C -parity operator $\langle S_R \rangle$ determines whether the state is shown as a pseudoscalar or scalar in Fig. 1.

n	$E[1/a]$	$\langle S_R \rangle$	n	$E[1/a]$	$\langle S_R \rangle$	n	$E[1/a]$	$\langle S_R \rangle$
0	0.0000	0.8063	5	1.1306	-0.3715	10	1.3918	-0.4487
1	0.7705	-0.6196	6	1.1996	0.3136	11	1.4041	0.0808
2	0.8615	-0.6428	7	1.2605	-0.1209	12	1.4430	0.0951
3	0.9899	-0.4005	8	1.3000	0.2920	13	1.4754	-0.4685
4	1.1145	0.7293	9	1.3618	0.0945	14	1.5555	-0.0997

and scalar ($\mathcal{O} = Q_5^2$) sectors and show the energies of several low-lying excited states for comparison. We differentiate between the excited states $|\Psi\rangle$ by the sign of the $\langle S_R \rangle_\Psi$, where $S_R \equiv \bigotimes_{i=1}^N X_i T^{(1)}$, with $T^{(1)}$ denoting the cyclic translation by one site. In the case of periodic boundary conditions this operator commutes with the Hamiltonian and measures C -parity of the state. With open boundary conditions, charge conjugation is no longer an exact symmetry of the Hamiltonian, but the expectation value of this operator can serve as an approximate way to distinguish scalar and pseudoscalar states [47,48]. Namely, we assign a positive C -parity to states in the Hamiltonian spectrum $|\psi\rangle$ such that $\langle \psi | S_R | \psi \rangle > 0$ and vice versa for negative parity. Energies and parity expectation values for a few low-lying states in the Hamiltonian at $N = 12$, $ga = 0.5$, $m/g = 0.5$ are listed in Table I. As is observed in Fig. 1, the peaks in the pseudoscalar spectral function originate mainly from the pseudoscalar states, with the similar matching in the scalar sector. We note that the low amplitude peaks for both operators correspond to excited states with the same quantum numbers of the respective meson, but with a finite momentum.

The spectral functions in the vacuum and at finite T are shown in Fig. 2 for a variety of m/g and temperature values. In the vacuum, the leading peaks are at the corresponding meson masses, m_P and m_S . The fate of these peaks at high temperatures depends on the fermion mass, which we follow to analyze. When $m/g = 0$ the theory is that of free bosons, and thus the thermal medium cannot screen the interfermionic potential of the Schwinger boson. Note that in this case, as discussed above, the peak in the spectral function for the scalar state is very close to 2 times the mass of the pseudoscalar meson. In the opposite limit, when $g/m \rightarrow 0$ ($m/g = 2$ in the numerical results), the theory approaches a free fermion model, where bound states are not formed, thus e.g. the pseudoscalar peak appears at $2m$. For both of these cases, we observe that the peaks at $T = 0$ survive at finite temperature, while no lower frequency mode appears. Note that while for heavy fermions the peaks' positions do not shift, for the massless case we observe an overall shift toward lower frequencies with increasing temperature.

In contrast, in the scenario where $m/g = 0.5$, the theory is genuinely interacting. Indeed, in this scenario we observe that both peaks in the spectral functions decay as T

increases, virtually disappearing once $T/m_P = 4$. At the same time, we observe that both the pseudoscalar and the scalar cases develop a low frequency peak.

We can further quantify the connection between the thermal modifications to mesons in this theory to what is observed in the QCD context. In the latter, it is expected that, at finite T , the interquark potential develops an imaginary component, associated with the suppression (melting) of the bound state. At the same time, the real part of the potential should remain mostly unchanged at these temperatures, as a thermal medium does not alter the state's intrinsic properties. This behavior has been observed in nonperturbative extractions of the complex heavy-quark potential from (Euclidian) lattice QCD, which find a temperature-dependent imaginary part but a nearly temperature-independent real part, see e.g. [49–57].

In the present context, the previous observations imply that the characteristic frequency of the spectral function

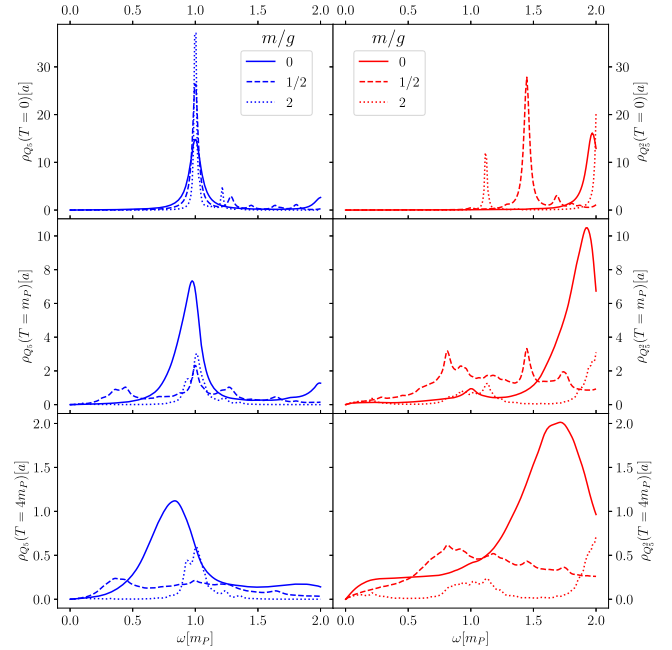


FIG. 2. Spectral function at zero and finite temperature for the pseudoscalar and scalar operators, with three values of the fermion mass compared. In all cases, $N = 12$, $ga = 0.5$, $\epsilon a = 0.02$.

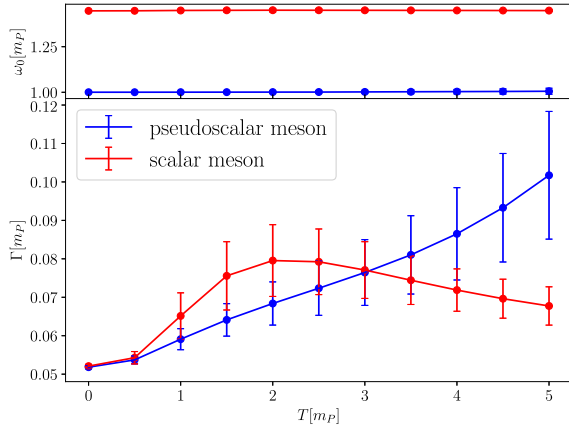


FIG. 3. Frequency (upper) and width (lower) of the pseudo-scalar and scalar meson peaks in the corresponding spectral functions, as functions of temperature. Thermal modifications broaden both peaks but leave the meson mass intact. Here we considered the same set conditions as in Fig. 2 with $m/g = 0.5$.

peak should not evolve with T , while its width should increase with temperature. Figure 3 corroborates this expectation: the upper panel shows no statistically significant drift of the peak frequency ω_0 for either the scalar or the pseudoscalar state, while the lower panel displays a clear broadening of both peaks with widths Γ obtained from Breit-Wigner fits. This behavior provides *ab initio* evidence that bound-state thermal modification is governed by the emergence of a finite imaginary part of the interquark potential, without an appreciable change in its real part. Interestingly, the pseudoscalar width grows faster than the scalar one. While this trend is opposite to a strong-coupling-based intuition, where one expects the size of the scalar state to be larger than the pseudoscalar, the large error bars at the highest temperatures preclude a definite conclusion; simulations on larger lattices will be required to better understand the detailed evolution with temperature. Finally, we note that the magnitude of the spectral function depends strongly on the choice of an operator in Eq. (10) and does not have a direct physical meaning. In particular, it cannot be compared to any experimental observables in QCD. For the same reason, no physical conclusions are to be drawn from a comparison between the heights of the peaks in the spectral function for the different m/g and T .

IV. ENERGY-ENERGY CORRELATORS

High-energy collider experiments allow one to probe the small scale properties of quantum field theories by measuring the final state of scattering experiments through the use of energy-flow detectors. On the theory side, the role played by calorimeter detectors is captured by the notion of asymptotic detector observables. The properties of the theory are encoded in the many-point correlation functions of such objects, see e.g. [58–61], which have seen a revamped development in recent years [21].

In the context of real-time simulations, one can study the properties of these asymptotic operators beyond the limitations of real detectors and traditional theoretical methods, which either rely on perturbation theory or that the theory has sufficient underlying symmetries. Here, following [62], we consider the one- and two-point energy-correlation functions

$$\begin{aligned} C_{1,\pm}(\beta) &= \text{Tr}[\rho_\beta \mathcal{O}^\dagger \mathcal{C}_\pm \mathcal{O}], \\ C_2(\beta) &= \text{Tr}[\rho_\beta \mathcal{O}^\dagger \mathcal{C}_+ \mathcal{C}_- \mathcal{O}], \end{aligned} \quad (21)$$

where $\pm$ indicates the position of the detector/flow operator $\mathcal{C}$ on the left or right edges of the system, and the $\mathcal{O}$ operator generates a local excitation on top of the equilibrium thermal state. The flow operators are obtained by light-transforming locally conserved currents [58]; in the case of the energy and electric charge they take the simplified form in $1+1\text{D}$,

$$\begin{aligned} \mathcal{Q}_\pm &\equiv \pm \int_0^\infty dt J^1(t, x \rightarrow \pm\infty), \\ \mathcal{E}_\pm &\equiv \pm \int_0^\infty dt T^{01}(t, x \rightarrow \pm\infty). \end{aligned} \quad (22)$$

Since in the continuum and infinite volume limits the states of the theory are charge neutral, here we restrict the discussion to energy correlators. For this, the off-diagonal component of the energy momentum tensor can be written as [63]

$$\begin{aligned} T^{01}(t, x_n) &= e^{iHt} T^{01}(t=0, x_n) e^{-iHt} \\ &= e^{iHt} \left(\frac{Y_n Z_{n+1} X_{n+2} - X_n Z_{n+1} Y_{n+2}}{2a^2} \right) e^{-iHt}, \end{aligned} \quad (23)$$

after performing the Jordan-Wigner transformation.

To excite the system, we consider a (scalar) quench proportional to the local kinetic energy term at the center of the lattice; the excitation operator reads

$$\mathcal{O} = X_{N/2} X_{N/2+1} + Y_{N/2} Y_{N/2+1}. \quad (24)$$

In the strong-coupling regime this corresponds to the creation operator of a spatially localized meson state, with X and Y operators flipping the spins at the corresponding sites. This particular combination of X and Y has the virtue of conserving the global charge, i.e., $[\mathcal{O}, \sum_n Z_n] = 0$, and thus corresponds to a gauge-invariant operator in the Schwinger model language. We note that, even though one could, for example, consider a vector excitation, in a $1+1\text{D}$ model such a quench would not introduce any new element, unlike what occurs in higher dimensions where correlation functions acquire nontrivial angular modulations. We also consider the linear rescaling of the scalar excitation

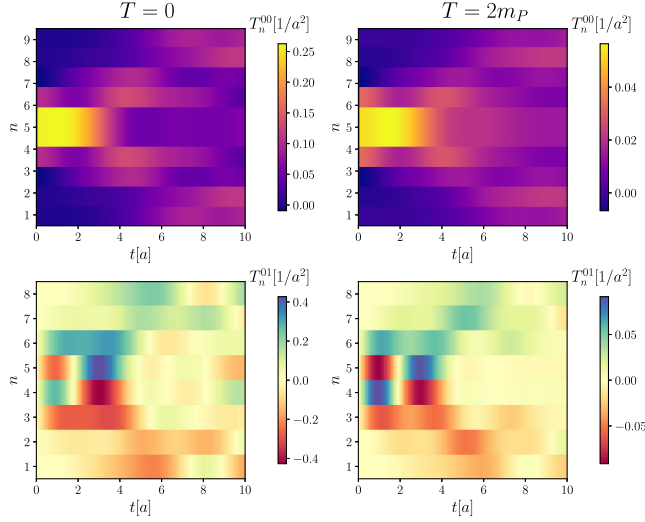


FIG. 4. Energy (top row) and momentum (bottom row) densities as functions of space and time for a strong quench in the vacuum ($T = 0$, left column) and at finite temperature ($T = 2m_P$, right column).

$$\mathcal{O}_\alpha \equiv (1 - \alpha)I + \alpha\mathcal{O}, \quad (25)$$

which allows one to adjust the amount of energy injected into the system by changing the parameter α . In what follows, we shall refer to the choice $\alpha = 1$ as “strong quench” and we reserve the term “weak quench” for the choice $\alpha = 0.17$, which is empirically found to deposit the same amount of energy in the vacuum as the strong quench does in the thermal state with $T = 2m_P$.

In Fig. 4 we show the spacetime evolution of the energy (T^{00}) and momentum (T^{01}) densities, after the initial quench, at zero (left) and finite T (right). For both quantities, after the initial quench they spread along a light-cone trajectory, until reaching the lattice’s edge, after which a boundary induced oscillatory pattern ensues. Despite the small size of the lattice, we observe that the finite and zero T results show a similar light-cone structure, although slightly smeared at finite temperature. Throughout this section we present the results for a system with $N = 10$, $ga = 0.5$, and $m/g = 0.5$. We do not show the data for $t > 10$, where edge effects become prevalent.

To capture the evolution of correlations in the energy flows, we compute the “time differential” energy-energy correlators

$$D(t, t') \equiv \mathcal{N} \text{Tr}[\rho_\beta \mathcal{O}^\dagger T^{01}(t, +)(-T^{01})(t', -)\mathcal{O}], \quad (26)$$

whose time integral corresponds to $\mathcal{C}_2$, $\mathcal{N} \equiv [\text{Tr}(\rho_\beta \mathcal{O}^\dagger \mathcal{O})]^{-1}$ is the normalization prefactor, and $+$ and $-$ in the arguments of T^{01} correspond to the physical locations at the right and left edges of the system, respectively.

In Fig. 5 we present the results for the time differential correlators. Correlations remain small at early times, when

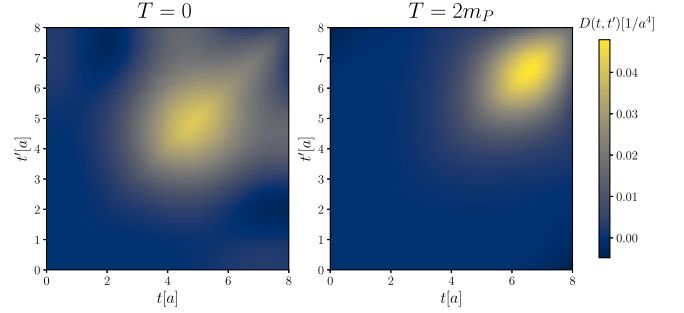


FIG. 5. Energy-energy correlator for a strong quench in the vacuum (left) and at finite T (right).

the excitations have not yet reached the detectors. In the subsequent times, we observe a peak along the line $t = t'$ centered around the time the light cone reaches the detector and decreasing once the signal propagates past it. We only show data up to $t, t' = 8a$, before the edge effect is visible. Comparing the zero and finite temperature results, we observe that the vacuum energy-energy correlator is rather sensitive to out of time correlations, while in the finite T scenario the buildup of correlations is more peaked, occurring at later times and more concentrated along $t = t'$.

We further quantify how nontrivial correlations build up after the quench by computing the connected two-point correlator

$$D_c(t, t') = D(t, t') - D_+(t)D_-(t'), \quad (27)$$

where $D_\pm(t)$ denotes the one-point time differential correlator along one of the lattice’s edges. If this quantity is vanishing at all points, or in a nonzero measure set, then one would conclude that the energy-energy correlator factorizes: $\langle \mathcal{E}_+ \mathcal{E}_- \rangle = \langle \mathcal{E}_+ \rangle \langle \mathcal{E}_- \rangle$. In Fig. 6 we show the evolution of the connected two-point correlator at zero and finite T . Remarkably the connected correlator at finite T exhibits larger values (around 2–4 times larger at the peak), compared to the vacuum case. This seems to suggest that the inclusion of the finite T bulk results in the buildup of nontrivial correlations in the state, which are reflected in the outgoing momentum flows. The small

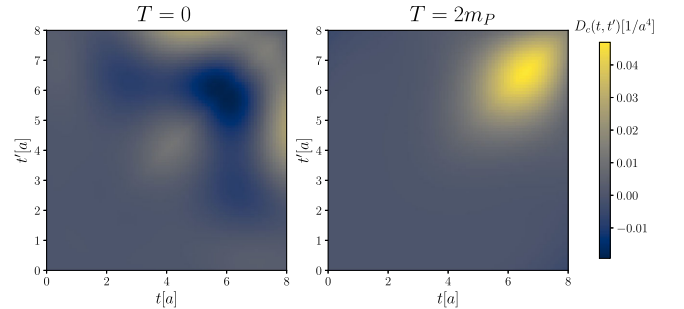


FIG. 6. Connected energy-energy correlator for a strong quench in the vacuum (left) and at finite T (right).

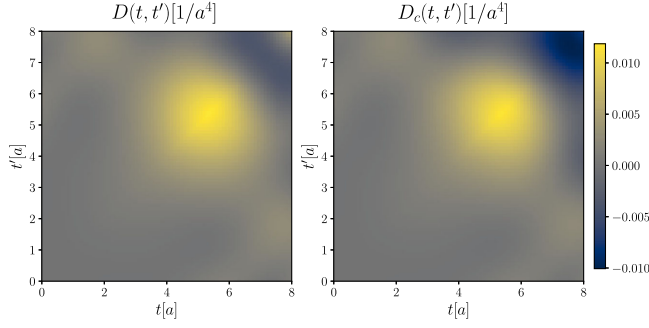


FIG. 7. Left: energy-energy correlator for a weak quench in the vacuum. Right: corresponding connected energy-energy correlator for a weak quench in the vacuum.

values observed in the vacuum are perhaps not surprising due to the $1 + 1\text{D}$ nature of the theory and the low level of entanglement generated by the quench used. A better quantification of these observations requires a larger system size lattice study, which we leave for future work.

Finally, in Fig. 7 we consider a weak quench which allows one to compare the vacuum and finite temperature cases when the same energy is deposited into the system. We note that, although $D \approx D_c$ in this case, the single-point correlators are numerically rather small. Still, the structure observed for the connected correlator matches more closely what is seen for the strong quench at finite temperature, although the magnitudes are different.

We stress that the quench that we used has no particular meaning from the viewpoint of QCD and we do not expect that it corresponds to any real collider process. For this reason, added to the fact that we work in a toy model, the numerical values of the energy correlators bear no meaning for the $(3 + 1)$ -dimensional QCD. Furthermore, the real particle detectors do not provide sufficient resolution to study time differential energy correlators, introduced in Eq. (26). Nevertheless, our results help frame the general expectations about temperature dependence and factorization properties of energy correlators that can be measured in real processes.

V. CONCLUSION

In this work, we have studied the properties of the Schwinger model at finite temperature. In particular, we have focused on extracting the spectral functions of the lowest-lying meson excitations and the correlations between asymptotic energy-flow operators, which have clear connections to QCD in the context of heavy-ion collisions. To this end, we have employed real-time lattice simulations, which allow one to explore the real-time properties of the model underlying these observables without encountering sign problems.

Analogous to what is observed in heavy-ion collisions, we have found that, when placing mesons in a finite temperature plasma, the corresponding spectral widths increase with T . This indicates the enhancement of

the imaginary part of the respective interfermionic potential, responsible for the suppression of bound states as observed in QCD. Further, in the weak- and strong-coupling limits, where the theory is effectively free, we observe that the mesons' spectral functions do not develop a low frequency peak, as what occurs in the interacting theory. Our result corresponds to a first demonstration of thermal broadening of mesons, from real-time simulations, in a gauge theory.

We have further studied how correlations in the momentum flows build up after a sudden quench, also in direct analogy to what occurs in high-energy scattering experiments. We have found that at finite T the connected correlator is enhanced compared to the vacuum case, indicating the buildup of nontrivial correlations in the hot bulk; this matches the expectations gained from experimental results in high-energy nuclear scattering experiments, see e.g. [64,65].

Looking forward, it would be interesting to go beyond the small system sizes here considered, either using quantum simulation or tensor network techniques, following the recent interest in exploring the finite T dynamics of gauge theories using these methods, see e.g. [66–74]. On the modeling side, it would be interesting to extend the current considerations to multiflavor models [75–77] or including a topological term [78,79] to the action. Such modifications would allow for the formation of closer analogs to the QCD mesons states, due to mass splitting in the fermions, or provide a new way to explore other aspects of bound-state modification at fixed T . It would also be valuable to understand how the meson observables considered here, which are frequently used in QCD, relate to QIS inspired measures, as explored in e.g. [42]. Another interesting opportunity lies in relating the observations made for the finite T connected energy-energy correlator to what has been observed in heavy-ion collisions, where both classical and quantum correlations play an important role, see discussion in e.g. [80,81].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- [1] I. Bloch, J. Dalibard, and W. Zwerger, Many-body physics with ultracold gases, *Rev. Mod. Phys.* **80**, 885 (2008).
- [2] A. Mitra, Quantum quench dynamics, *Annu. Rev. Condens. Matter Phys.* **9**, 245 (2018).
- [3] D. Bernard and B. Doyon, Conformal field theory out of equilibrium: A review, *J. Stat. Mech.* **06** (2016) 064005.
- [4] W. Busza, K. Rajagopal, and W. van der Schee, Heavy ion collisions: The big picture, and the big questions, *Annu. Rev. Nucl. Part. Sci.* **68**, 339 (2018).
- [5] J. Berges, M.P. Heller, A. Mazeliauskas, and R. Venugopalan, QCD thermalization: *Ab initio* approaches and interdisciplinary connections, *Rev. Mod. Phys.* **93**, 035003 (2021).
- [6] M.S. Turner, The road to precision cosmology, [arXiv:2201.04741](https://arxiv.org/abs/2201.04741).
- [7] A.D. Ludlow, M.M. Boyd, J. Ye, E. Peik, and P.O. Schmidt, Optical atomic clocks, [arXiv:1407.3493](https://arxiv.org/abs/1407.3493).
- [8] J. Adams *et al.* (STAR Collaboration), Experimental and theoretical challenges in the search for the quark gluon plasma: The STAR Collaboration's critical assessment of the evidence from RHIC collisions, *Nucl. Phys.* **A757**, 102 (2005).
- [9] K. Aamodt *et al.* (ALICE Collaboration), Elliptic flow of charged particles in Pb-Pb collisions at 2.76 TeV, *Phys. Rev. Lett.* **105**, 252302 (2010).
- [10] U.W. Heinz and M. Jacob, Evidence for a new state of matter: An Assessment of the results from the CERN lead beam program, [arXiv:nucl-th/0002042](https://arxiv.org/abs/nucl-th/0002042).
- [11] C. Ratti, Lattice QCD and heavy ion collisions: A review of recent progress, *Rep. Prog. Phys.* **81**, 084301 (2018).
- [12] K. Nagata, Finite-density lattice QCD and sign problem: Current status and open problems, *Prog. Part. Nucl. Phys.* **127**, 103991 (2022).
- [13] P.B. Arnold, G.D. Moore, and L.G. Yaffe, Effective kinetic theory for high temperature gauge theories, *J. High Energy Phys.* **01** (2003) 030.
- [14] P. Romatschke and U. Romatschke, *Relativistic Fluid Dynamics In and Out of Equilibrium*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2019).
- [15] E.A. Martinez *et al.*, Real-time dynamics of lattice gauge theories with a few-qubit quantum computer, *Nature (London)* **534**, 516 (2016).
- [16] U.-J. Wiese, Ultracold quantum gases and lattice systems: Quantum simulation of lattice gauge theories, *Ann. Phys. (Berlin)* **525**, 777 (2013).
- [17] M.C. Bañuls *et al.*, Simulating lattice gauge theories within quantum technologies, *Eur. Phys. J. D* **74**, 165 (2020).
- [18] M.C. Bañuls and K. Cichy, Review on novel methods for lattice gauge theories, *Rep. Prog. Phys.* **83**, 024401 (2020).
- [19] A. Rothkopf, Heavy quarkonium in extreme conditions, *Phys. Rep.* **858**, 1 (2020).
- [20] T. Matsui and H. Satz, J/ψ suppression by quark-gluon plasma formation, *Phys. Lett. B* **178**, 416 (1986).
- [21] I. Moulton and H. X. Zhu, Energy correlators: A journey from theory to experiment, [arXiv:2506.09119](https://arxiv.org/abs/2506.09119).
- [22] J.S. Schwinger, On gauge invariance and vacuum polarization, *Phys. Rev.* **82**, 664 (1951).
- [23] S.R. Coleman, More about the massive Schwinger model, *Ann. Phys. (N.Y.)* **101**, 239 (1976).
- [24] S. Mandelstam, Soliton operators for the quantized Sine-Gordon equation, *Phys. Rev. D* **11**, 3026 (1975).
- [25] S.R. Coleman, R. Jackiw, and L. Susskind, Charge shielding and quark confinement in the massive Schwinger model, *Ann. Phys. (N.Y.)* **93**, 267 (1975).
- [26] T. Banks, L. Susskind, and J.B. Kogut, Strong coupling calculations of lattice gauge theories: (1 + 1)-dimensional exercises, *Phys. Rev. D* **13**, 1043 (1976).
- [27] D. Gonzalez-Cuadra *et al.*, Observation of string breaking on a (2 + 1)D Rydberg quantum simulator, *Nature (London)* **642**, 321 (2025).
- [28] F. Di Marcantonio, S. Pradhan, S. Vallecorsa, M.C. Bañuls, and E.R. Ortega, Roughening and dynamics of an electric flux string in a (2 + 1)D lattice gauge theory, [arXiv:2505.23853](https://arxiv.org/abs/2505.23853).
- [29] T. Felser, P. Silvi, M. Collura, and S. Montangero, Two-dimensional quantum-link lattice quantum electrodynamics at finite density, *Phys. Rev. X* **10**, 041040 (2020).
- [30] J.B. Kogut and L. Susskind, Hamiltonian formulation of Wilson's lattice gauge theories, *Phys. Rev. D* **11**, 395 (1975).
- [31] C.J. Hamer, W.-h. Zheng, and J. Oitmaa, Series expansions for the massive Schwinger model in Hamiltonian lattice theory, *Phys. Rev. D* **56**, 55 (1997).
- [32] R. Dempsey, I.R. Klebanov, S.S. Pufu, and B. Zan, Discrete chiral symmetry and mass shift in the lattice Hamiltonian approach to the Schwinger model, *Phys. Rev. Res.* **4**, 043133 (2022).
- [33] L. Susskind, Lattice fermions, *Phys. Rev. D* **16**, 3031 (1977).
- [34] N. Brambilla, M.A. Escobedo, J. Ghiglieri, J. Soto, and A. Vairo, Heavy quarkonium in a weakly-coupled quark-gluon plasma below the melting temperature, *J. High Energy Phys.* **09** (2010) 038.
- [35] N. Brambilla, M.A. Escobedo, J. Soto, and A. Vairo, Quarkonium suppression in heavy-ion collisions: An open quantum system approach, *Phys. Rev. D* **96**, 034021 (2017).
- [36] X.-Y. Wu, L. Du, C. Gale, and S. Jeon, Probing the equilibration of the QCD matter created in heavy-ion collisions with dileptons, *Phys. Rev. C* **110**, 054904 (2024).
- [37] O. Garcia-Montero, P. Plaschke, and S. Schlichting, Scaling of pre-equilibrium dilepton production in QCD kinetic theory, *Phys. Rev. D* **111**, 034036 (2025).
- [38] O. Garcia-Montero, A. Mazeliauskas, P. Plaschke, and S. Schlichting, Pre-equilibrium photons from the early stages of heavy-ion collisions, *J. High Energy Phys.* **03** (2024) 053.
- [39] G. David, Direct real photons in relativistic heavy ion collisions, *Rep. Prog. Phys.* **83**, 046301 (2020).
- [40] S. Turbide, R. Rapp, and C. Gale, Hadronic production of thermal photons, *Phys. Rev. C* **69**, 014903 (2004).
- [41] O. Kaczmarek and H.-T. Shu, Spectral and transport properties from lattice QCD, *Lect. Notes Phys.* **999**, 307 (2022).
- [42] M.C. Bañuls, M.P. Heller, K. Jansen, J. Knaute, and V. Svensson, Quantum information perspective on meson melting, *Phys. Rev. D* **108**, 076016 (2023).
- [43] C. Adam, Improved vector and scalar masses in the massive Schwinger model, *Phys. Lett. B* **555**, 132 (2003).

- [44] F. Verstraete, J.J. García-Ripoll, and J.I. Cirac, Matrix product density operators: Simulation of finite-temperature and dissipative systems, *Phys. Rev. Lett.* **93**, 207204 (2004).
- [45] M. Zwolak and G. Vidal, Mixed-state dynamics in one-dimensional quantum lattice systems: A time-dependent superoperator renormalization algorithm, *Phys. Rev. Lett.* **93**, 207205 (2004).
- [46] A.E. Feiguin and S.R. White, Finite-temperature density matrix renormalization using an enlarged Hilbert space, *Phys. Rev. B* **72**, 220401 (2005).
- [47] M.C. Bañuls, K. Cichy, K. Jansen, and J.I. Cirac, The mass spectrum of the Schwinger model with matrix product states, *J. High Energy Phys.* **11** (2013) 158.
- [48] E. Itou, A. Matsumoto, and Y. Tanizaki, Calculating composite-particle spectra in Hamiltonian formalism and demonstration in 2-flavor QED_{1+1d}, *J. High Energy Phys.* **11** (2023) 231.
- [49] R. Larsen, S. Meinel, S. Mukherjee, and P. Petreczky, Excited bottomonia in quark-gluon plasma from lattice QCD, *Phys. Lett. B* **800**, 135119 (2020).
- [50] Y. Burnier, O. Kaczmarek, and A. Rothkopf, Static quark-antiquark potential in the quark-gluon plasma from lattice QCD, *Phys. Rev. Lett.* **114**, 082001 (2015).
- [51] A. Rothkopf, T. Hatsuda, and S. Sasaki, Complex heavy-quark potential at finite temperature from lattice QCD, *Phys. Rev. Lett.* **108**, 162001 (2012).
- [52] Z. Tang, B. Wu, A. Hanlon, S. Mukherjee, P. Petreczky, and R. Rapp, Quarkonium spectroscopy in the quark-gluon plasma, *Phys. Rev. Lett.* **135**, 142302 (2025).
- [53] H.-T. Ding, W.-P. Huang, R. Larsen, S. Meinel, S. Mukherjee, P. Petreczky, and Z. Tang, In-medium bottomonium properties from lattice NRQCD calculations with extended meson operators, *J. High Energy Phys.* **05** (2025) 149.
- [54] A. Bazavov, D. Hoyer, R. N. Larsen, S. Mukherjee, P. Petreczky, A. Rothkopf, and J.H. Weber (HotQCD Collaboration), Unscreened forces in the quark-gluon plasma?, *Phys. Rev. D* **109**, 074504 (2024).
- [55] D. Bala, O. Kaczmarek, R. Larsen, S. Mukherjee, G. Parkar, P. Petreczky, A. Rothkopf, and J.H. Weber (HotQCD Collaboration), Static quark-antiquark interactions at non-zero temperature from lattice QCD, *Phys. Rev. D* **105**, 054513 (2022).
- [56] S. Shi, K. Zhou, J. Zhao, S. Mukherjee, and P. Zhuang, Heavy quark potential in the quark-gluon plasma: Deep neural network meets lattice quantum chromodynamics, *Phys. Rev. D* **105**, 014017 (2022).
- [57] R. Larsen, S. Meinel, S. Mukherjee, and P. Petreczky, Thermal broadening of bottomonia: Lattice nonrelativistic QCD with extended operators, *Phys. Rev. D* **100**, 074506 (2019).
- [58] D.M. Hofman and J. Maldacena, Conformal collider physics: Energy and charge correlations, *J. High Energy Phys.* **05** (2008) 012.
- [59] N. A. Sveshnikov and F. V. Tkachov, Jets and quantum field theory, *Phys. Lett. B* **382**, 403 (1996).
- [60] C.L. Basham, L.S. Brown, S.D. Ellis, and S.T. Love, Energy correlations in electron-positron annihilation: Testing QCD, *Phys. Rev. Lett.* **41**, 1585 (1978).
- [61] F. V. Tkachov, Measuring multi-jet structure of hadronic energy flow or what is a jet?, *Int. J. Mod. Phys. A* **12**, 5411 (1997).
- [62] J. Barata and S. Mukherjee, Probing celestial energy and charge correlations through real-time quantum simulations: Insights from the Schwinger model, *Phys. Rev. D* **111**, L031901 (2025).
- [63] R. A. Janik, M. A. Nowak, M.M. Rams, and I. Zahed, Universality and emergent effective fluid from jets and string breaking in the massive Schwinger model using tensor networks, *arXiv:2502.12901*.
- [64] CMS Collaboration, Energy-energy correlators from PbPb and pp collisions at 5.02 TeV, Technical Report No. CMS-PAS-HIN-23-004, CERN, Geneva, 2023.
- [65] A. Nambrath, Energy-energy correlators in jets across collision systems and parton flavors, *EPJ Web Conf.* **339**, 02001 (2025).
- [66] T. Angelides, Y. Guo, K. Jansen, S. Kühn, and G. Magnifico, Meson thermalization with a hot medium in the open Schwinger model, *J. High Energy Phys.* **04** (2025) 195.
- [67] X. Du and W. Qian, Accelerated quantum circuit Monte Carlo simulation for heavy quark thermalization, *Phys. Rev. D* **109**, 076025 (2024).
- [68] K. Ikeda, Z.-B. Kang, D.E. Kharzeev, W. Qian, and F. Zhao, Real-time chiral dynamics at finite temperature from quantum simulation, *J. High Energy Phys.* **10** (2024) 031.
- [69] W. Qian and B. Wu, Quantum computation in fermionic thermal field theories, *J. High Energy Phys.* **07** (2024) 166.
- [70] H. Saito, M. C. Bañuls, K. Cichy, J. I. Cirac, and K. Jansen, The temperature dependence of the chiral condensate in the Schwinger model with matrix product states, *Proc. Sci., LATTICE2014* (2014) 302 [arXiv:1412.0596].
- [71] J. W. Pedersen, E. Itou, R.-Y. Sun, and S. Yunoki, Quantum simulation of finite temperature Schwinger model via quantum imaginary time evolution, *Proc. Sci., LATTICE2023* (2024) 220 [arXiv:2311.11616].
- [72] A. Florio, D. Frenklakh, S. Griener, D. E. Kharzeev, A. Palermo, and S. Shi, Thermalization from quantum entanglement: Jet simulations in the massive Schwinger model, *Phys. Rev. D* **112**, 094502 (2025).
- [73] J. Barata, X. Du, M. Li, W. Qian, and C. A. Salgado, Quantum simulation of in-medium QCD jets: Momentum broadening, gluon production, and entropy growth, *Phys. Rev. D* **108**, 056023 (2023).
- [74] Y. Kuramashi and Y. Yoshimura, Three-dimensional finite temperature Z₂ gauge theory with tensor network scheme, *J. High Energy Phys.* **08** (2019) 023.
- [75] M. Sadzikowski and P. Węgrzyn, Meson bound states in multi-flavor massive Schwinger model, *Mod. Phys. Lett. A* **11**, 1947 (1996).
- [76] M. C. Bañuls, K. Cichy, J. I. Cirac, K. Jansen, and S. Kühn, Density induced phase transitions in the Schwinger model: A study with matrix product states, *Phys. Rev. Lett.* **118**, 071601 (2017).
- [77] J.E. Hetrick, Y. Hosotani, and S. Iso, The massive multi-flavor Schwinger model, *Phys. Lett. B* **350**, 92 (1995).

- [78] E. Itou, A. Matsumoto, and Y. Tanizaki, DMRG study of the theta-dependent mass spectrum in the 2-flavor Schwinger model, *J. High Energy Phys.* **09** (2024) 155.
- [79] R. Dempsey, I. R. Klebanov, S. S. Pufu, B. T. Søgaaard, and B. Zan, Phase diagram of the two-flavor Schwinger model at zero temperature, *Phys. Rev. Lett.* **132**, 031603 (2024).
- [80] J. Barata, I. Moul, A. V. Sadofyev, and J. M. Silva, Dissecting jet modification in the QGP with multi-point energy correlators, [arXiv:2503.13603](#).
- [81] Z. Yang, Y. He, I. Moul, and X.-N. Wang, Probing the short-distance structure of the quark-gluon plasma with energy correlators, *Phys. Rev. Lett.* **132**, 011901 (2024).