

Dark energy and QCD instanton vacuum in a Friedmann-Lemaître-Robertson-Walker universe

M. M. Musakhanov^{ID*}*Institute of Theoretical Physics, National University of Uzbekistan, Tashkent 100174, Uzbekistan*

(Received 29 July 2025; accepted 9 November 2025; published 1 December 2025)

The standard model of the universe, λ CDM, is based on the Friedmann-Lemaître-Robertson-Walker metric with a flat three-dimensional coordinate space and the Friedmann equations Navas *et al.* [*Phys. Rev. D* **110**, 030001 (2024)]. The cosmological constant λ provides the cancellation of the matter field contributions in the flat (Minkowski) space, as was proposed long ago in 1967 by Zeldovich for the first time to our knowledge Zeldovich [*JETP Lett.* **6**, 316 (1967)]; see also Krasinski and Zeldovich [*Sov. Phys. Usp.* **11**, 381 (1968)]. The dynamical dark energy appears on the surface of the vacuum energy of matter fields at the flat (Minkowski) space. Within the Standard Model, the gluon Yang-Mills (YM) fields are playing a specific role since the properties of their vacuum, where there is the presence of the gluon condensate, provide the nonperturbative vacuum energy. It is natural to apply the successful instanton liquid model of the QCD vacuum and its lowest excitations. Our aim is to calculate the contribution of gluon YM fields to the dark energy density. We find that the universe metric is generating the QCD vacuum excitation, which gives the contribution to the dark energy density. But this one may hardly play a central role in the dynamics of the universe, since its timescale is too small. We also find the equation-of-state parameters $w_0 = -1, w_a = 0$ in accordance with λ CDM, while the newest data, analyzed at Shajib and Frieman [*Phys. Rev. D* **112**, 063508 (2025)], give them at least in the range $-0.91 < w_0 < -0.73, -1.05 < w_a < -0.65$. They are requesting a contribution from an ultralight scalar such as an axion, or from YM field topological configurations with the nontrivial holonomy due to the deviation from a pure de Sitter state Van Waerbeke and Zhitnitsky [DESI results and dark energy from QCD topological sectors, [arXiv:2506.14182](https://arxiv.org/abs/2506.14182)].

DOI: [10.1103/x2b7-5y1r](https://doi.org/10.1103/x2b7-5y1r)

I. INTRODUCTION

The impact of matter (particles and fields) on cosmological observables of the universe can be quantified in terms of its equation of state w and the ratio of its pressure p to its energy density ϵ , $w = p/\epsilon$. A phenomenological description of the redshift (time) evolution is represented by a two-parameter function $w(z) = w_0 + w_a z/(1+z)$ where $w_0 \sim$ the present value, $w_a \sim$ the time evolution, and z is the redshift. The standard cosmological λ cold dark matter (λ CDM) model, with 70% vacuum energy (equivalently the cosmological constant λ) and 30% matter (CDM plus baryons), is fixing $w_0 = -1$ and $w_a = 0$.

λ CDM is based on the Friedmann-Lemaître-Robertson-Walker (FLRW) space metric:

$$\begin{aligned} ds^2 &= dt^2 - a^2(t) d\vec{x}^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2), \quad dt = a(\tau)d\tau, \\ g_{00} &= g^{00} = 1, \quad g_{mn} = -a^2(t)\delta_{mn}, \quad g^{mn} = -a^{-2}(t)\delta_{mn}, \\ g &\equiv \det g_{\mu\nu} = -a^6(t). \end{aligned} \quad (1)$$

dt is proper while $d\tau = dt/a(t)$ is conformal time. We define the FLRW space with conformal time τ as FLRW conformal space. The basic Friedmann equations of the FLRW universe (see recent PDG review [1]) are

$$\begin{aligned} H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\epsilon_{\text{tot}} + \frac{\lambda}{3}, \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\epsilon_{\text{tot}} + 3p_{\text{tot}}) + \frac{\lambda}{3}, \end{aligned} \quad (2)$$

where ϵ_{tot} is the total energy density of the universe and p_{tot} is the total pressure. Here and in the following, we are using $\hbar = c = 1$, Planck mass $M_{\text{Pl}} = 1.2 \times 10^{22}$ MeV, Newton's gravity constant $G = M_{\text{Pl}}^{-2}$, and present time Hubble parameter $H_0 = \dot{a}(t_0)/a(t_0) \approx 1.4 \times 10^{-39}$ MeV [1]. Also, we will use the definition of metric field $h(x)$ as $a(x) = \exp(h(x)/2)$.

*Contact author: musakhanov@gmail.com

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

The total energy density and pressure of the universe has an essential part given by the Standard Model matter fields vacuum contributions as perturbative vacuum fields fluctuations and nonperturbative Higgs potential and QCD vacuum condensates. Current data indicate that all vacuum contributions are the largest contributors to the cosmological energy density budget (0.685 ± 0.007) [1], and we for a moment will neglect the contributions of the pressureless matter and relativistic particles.

In the flat (Minkowski) space, the left sides of Eq. (2) are zero, which correspond to the total compensation of the flat (Minkowski) space matter fields vacuum contributions $\epsilon_{\text{vac}}(0)$ by the cosmological constant λ and $\epsilon_{\text{vac}}(0) + p_{\text{vac}}(0) = 0$. It means at least that gravitating dynamical dark energy is appearing on top of the vacuum energy of matter fields at the flat (Minkowski) space. Such an interpretation of the cosmological constant λ was proposed long ago in 1967 by Zeldovich for the first time to our knowledge [2,3] and are widely accepted now [4–14].

Certainly, first of all, we have to take into account Standard Model fields. Among them, the gluon Yang-Mills (YM) fields are playing the specific role since the properties of their vacuum, where the presence of the gluon condensate and YM conformal (scale) anomaly, provide the nonperturbative vacuum energy.

In QCD vacuum perturbative quantum fluctuations are living over spontaneously created nonperturbative classical YM fields, which are the paths of the quantum tunneling processes—(anti)instantons, randomly populating the total space. These fields are giving nonperturbative contribution ϵ_{vac} to the vacuum energy density of YM fields. The nonperturbative part of the energy-momentum tensor vacuum expectation in the space with the metric $g_{\mu\nu}$ is given by $\langle T_{\mu\nu} \rangle_{np} = \epsilon_{\text{vac}} g_{\mu\nu}$. The vacuum energy density ϵ_{vac} can be taken from the trace of this tensor as $\langle T_{\mu}^{\mu} \rangle_{np} = 4\epsilon_{\text{vac}}$. This trace in QCD is nonzero only due to the quantum loops, which are generating scale anomaly. This one is related to the renormalization of quantum loops, which leads to the dimensional transmutation and provides dimension-full parameter Λ_{QCD} .

In the flat (Minkowski) space, the field vacuum energy density $\epsilon_{\text{vac}}(0)$ is canceled by the cosmological constant λ on the right side of the Friedmann equations (2). In the FLRW space it was assumed [6] the metric depended on an additional contribution $\sim O(H\Lambda_{\text{QCD}}^3)$ induced by the gluon condensate. But this assumption was ruled out at the work [15].

More detailed further studies of the QCD in a curved space [9–14] showed that such a type contribution can appear due to the YM topological configurations with the nontrivial holonomy, compactified to a Euclidean time circle $\mathbb{S}_{\kappa^{-1}}^1$ with the radius κ^{-1} . This nontrivial holonomy is a source of a $O(\kappa\Lambda_{\text{QCD}}^3)$ contribution to the YM vacuum energy. This phenomena is quite similar to the nonzero temperature QCD (KvBLL calorons) [16,17]. It is important that these deviations from a pure de Sitter state can explain the current

data from the DESI Collaboration [18] as well as other data. The analysis of them [19] gives the equation-of-state parameters in the range $-0.91 < w_0 < -0.73$ and $-1.05 < w_a < -0.65$, which is also a well corresponding phenomenological massive scalar field model with the mass just slightly below the current value of the Hubble parameter [for instance, $m \approx 0.82H(t_0) = 1.2 \times 10^{-39}$ MeV for a free, massive scalar], corresponding to a current value of the equation-of-state parameter $w_0 \approx -0.9$ [19]. Such an ultra-light scalar mass is a natural in a model like QCD axions.

The name dark energy has become usual for such types of contributions.

In the present work we are staying within the λ CDM FLRW metric (1). In this case, it is natural to apply the successful instanton liquid model (ILM) of the QCD vacuum and its lowest excitations. Originally, ILM was formulated in Euclidean flat space, taking into account essential topological properties of the YM fields [20,21] (see reviews [22,23]). We are considering only gluon YM fields as a matter field living in nonflat space with the FLRW metric (1). Our framework is the ILM extended to this space, which is described in Sec. II. It is important here that perturbative loops are generating the dimensional transmutation and, essential for us, the metric-gluons interaction term, leading to the conformal (scale) anomaly properties of QCD. These properties are presented by scalar glueball field effective Lagrangian, which describes the fluctuations of instantons' density. In Sec. III the effective Lagrangian is used for the calculations of the energy density and the pressure of the excitations of the scalar glueball field ξ induced by the metric h , which is YM dark energy density $\epsilon_{\text{dark}}(h)$ and pressure $p_{\text{dark}}(h)$ of the FLRW universe.

II. GLUON YM FIELDS IN THE FLRW UNIVERSE

We take the YM action in Weyl gauge ($A_0^a = 0$) at FLRW space (1), change the time t to the conformal time τ , and have

$$\begin{aligned} S &= -\frac{1}{4e^2} \int d^4x (-g)^{1/2} F_{\mu\nu}^a F^{a\mu\nu} \\ &= \frac{1}{2e^2} \int d\tau \int d^3x (\vec{E}^{a2} - \vec{B}^{a2}), \\ E_i^a &= \frac{\partial A_i^a}{\partial \tau}, \quad B_i^a = \frac{1}{2} \epsilon_{ijk} (\partial_j A_k^a - \partial_k A_j^a + \epsilon_{abc} A_j^b A_k^c). \end{aligned} \quad (3)$$

We see the YM action in the FLRW conformal space $(\tau, \vec{x})$ completely coincide with the action in the flat (Minkowski) space $(t, \vec{x})$. We expect that YM fields in the FLRW conformal space have the same properties as in the flat (Minkowski) space.

A. Conformal time

The conformal time τ in FLRW space, filled by constant positive density energy, at present time t_0 is defined by the

solution of the Friedmann equation (2) $a(t_0) \sim \exp(Ht_0)$ and for the whole period can be taken as $a(t) = \sinh(Ht)$. Then, the conformal time is given by

$$\tau(t) = \int_{t_0}^t \frac{dt'}{a(t')} = \frac{1}{H} \int_{Ht_0}^{Ht} \frac{dx}{\sinh(x)} = -\frac{1}{2H} \ln \left(\frac{e^{Ht'} + e^{-Ht'} + 2}{e^{Ht'} + e^{-Ht'} - 2} \right) \Big|_{t'=t_0}^{t'=t}. \quad (4)$$

We see $\tau(t \rightarrow 0) \rightarrow -\infty$, $\tau(t \rightarrow \pm\infty) \rightarrow 0$. So, $\tau = -\infty$ correspond to the initial time $t = 0$, while $\tau = 0$ —final one $t = \infty$.

We will also need the conformal time in Euclidean FLRW space. Define $t = i\tilde{t}$, $\tilde{x} = (\tilde{t}, \vec{x})$, and the metrics $\tilde{g}_{\alpha\beta}$ with Euclidean signature $(++++)$ as $g_{\mu\nu}(x) = -\tilde{g}_{\alpha\beta}(\tilde{x}) \frac{\partial \tilde{x}_\alpha}{\partial x_\mu} \frac{\partial \tilde{x}_\beta}{\partial x_\nu}$. Then $-\det g_{\mu\nu} \equiv -g = \tilde{g}$.

The FLRW Euclidean metric is given by

$$d\tilde{s}^2 = d\tilde{t}^2 + a^2(\tilde{t})d\vec{x}^2 = a^2(\tilde{t})(d\tilde{\tau}^2 + d\vec{x}^2), \quad (5)$$

where $d\tilde{t}$ is proper while $d\tilde{\tau} = d\tilde{t}/a(\tilde{t})$ is conformal times. At $a = 1$ we have the Euclidean flat metric. The conformal Euclidean time $\tilde{\tau}$ is given by

$$\tilde{\tau}(\tilde{t}) = \int_{\pi/2}^{H\tilde{t}} \frac{d\tilde{t}'}{\sin(H\tilde{t}')} = -\frac{1}{2H} \ln \left(\frac{1 + \cos H\tilde{t}}{1 - \cos H\tilde{t}} \right). \quad (6)$$

We have $\tilde{\tau}(H\tilde{t} \rightarrow 0) = -\infty$ and $\tilde{\tau}(H\tilde{t} \rightarrow \pi) = +\infty$.

B. Topology of YM fields in FLRW space

We expect that the topology of YM fields will be the same in both spaces—flat and FLRW conformal ones. We introduce in FLRW conformal space the topological collective coordinate of the YM fields—the Chern-Simons (CS) number as

$$N_{\text{CS}}(\tau) = \frac{1}{16\pi} \int d^3x \epsilon^{ijk} \left(A_i^a \partial_j A_k^a + \frac{1}{3} \epsilon_{abc} A_i^a A_j^b A_k^c \right) \quad (7)$$

and take a large gauge transformation of the YM fields $A_i \rightarrow U^+(A_i + i\partial_i)U$ with topological winding number

$$N_W = \frac{1}{24\pi^2} \int d^3x \epsilon^{ijk} [(U^+ \partial_i U)(U^+ \partial_j U)(U^+ \partial_k U)] \quad (8)$$

with $SU(2)$ matrix U , which is providing mapping of three-dimensional $SU(2)$ internal space S^3 to the external coordinate space S^3 . Under this transformation the CS number is changed as

$$N_{\text{CS}} \rightarrow N_{\text{CS}} + N_W. \quad (9)$$

Let us take the potential energy of YM fields as a function of CS number N_{CS} ,

$$V(N_{\text{CS}}) = \frac{1}{2e^2} \int d^3x (\vec{B}^a(\tau, \vec{x}))^2. \quad (10)$$

We arrive at the conclusion first pointed out in [24–26], to the best of knowledge, that the potential $V(N_{\text{CS}})$ is periodic on the collective coordinate N_{CS} like a one-dimensional crystal and oscillatorlike in all other directions in functional space. The quasiclassical quantization leads to the band structure of the energies. The width of the band is defined by the amplitude of tunneling $\sim e^{-S_E}$, where the action S_E is calculated on the Euclidean classical trajectory between the nearest minima. This trajectory is a YM (anti)instanton with topological charge $Q_T = \pm 1$, which is the solution of YM equations in Euclidean FLRW conformal space.

C. (Anti)instanton in conformal Euclidean FLRW space

Euclidean YM action in Euclidean conformal FLRW metric $\tilde{g}_{\mu\nu} = \exp(h(\tilde{\tau}))\delta_{\mu\nu}$ and at Weyl gauge $A_0 = 0$ can be easily found from (3) as

$$S_E = \frac{1}{2e^2} \int d\tilde{\tau} \int d^3x (\vec{E}^{a2} + \vec{B}^{a2}), \quad E_i^a = \frac{\partial A_i^a}{\partial \tilde{\tau}}, \quad B_i^a = \frac{1}{2} \epsilon_{ijk} (\partial_j A_k^a - \partial_k A_j^a + \epsilon_{abc} A_j^b A_k^c), \quad (11)$$

which completely coincides with the YM action in Euclidean flat space. It means we may take the known solution [27] for the (anti)instanton $A^\pm$ with $Q_T = \pm 1$ in singular gauge. In the general case, the solution is given by $U_i^\pm A_i^\pm(\tilde{\tau} - \tilde{\tau}_i, \vec{x} - \vec{x}_i)U_i$ the collective coordinates: four-dimensional position $(\tilde{\tau}_i, \vec{x}_i)$, color orientation U_i , and size ρ_i .

D. Instanton liquid model of a QCD vacuum in FLRW conformal space

ILM in the flat space for the QCD vacuum and its lowest excitations—hadrons [20,21] (see reviews [22,23])—can easily be translated to the FLRW conformal space. The quantum tunneling processes, randomly distributed over FLRW conformal space, are corresponding to the sum ansatz for the total YM field

$$A_i^{\text{ILM}}(\tilde{\tau}, \vec{x}) = \sum_{\pm} \sum_{i=1}^{N_{\pm}} U_i^\pm A_i^\pm(\tilde{\tau} - \tilde{\tau}_i, \vec{x} - \vec{x}_i)U_i. \quad (12)$$

The variational [21] and lattice [28] calculations and phenomenological estimations [20] demonstrated the concentration of the (anti)instanton sizes around an average of one ρ as an effect of instanton-(anti)instanton interactions. In the following, we take the same size ρ for all (anti)instantons. Also, we take $N_+ = N_- = N/2$, since CP invariance is requesting this one. Another important parameter of ILM is the interinstanton distance R , which is certainly related to the density of instanton media as

$N/V = 1/R^4$. The renormalization properties of the QCD are controlling the dispersion of the number of instantons N . At color number $N_c \rightarrow \infty$, the dispersion becomes zero [29]. The various estimations of ILM main parameters (variational [21], phenomenological [20], and lattice [28]) give $\rho \approx 0.33$ fm and $R \approx 1$ fm.

Finally, ILM's remaining problems are the integrations over the (anti)instantons collective coordinates [four-dimensional position $(\tau_i, \vec{x}_i)$ and color orientation U_i] and over quantum fluctuations living above the classical background (12). The calculation of these perturbative quantum corrections—gluon (and ghost) loops—is requesting the renormalization of them at the specific scale, which is in ILM given by the instanton size ρ . As a result, we have the dimensional transmutation, which provides dimension-full QCD parameters Λ_{QCD} as

$$\frac{2\pi}{\alpha_s(\rho)} = b_1 \ln \frac{1}{\Lambda_{\text{QCD}} \rho}, \quad \alpha_s(\rho) = \frac{e^2(\rho)}{4\pi}. \quad (13)$$

Here one-loops provide $b_1 = 11N_c/3$ (in QCD $N_c = 3$) and $\Lambda_{\text{QCD}} \approx 100$ MeV. This dimensional transmutation leads to the QCD conformal (scale) anomaly, and we have a nonzero trace of the YM energy-momentum tensor, which is in the ILM given by

$$\langle T^\mu_\mu \rangle_{\text{ILM}} \approx -b_1 \frac{\langle F^{a\mu\nu} F^\mu_{\nu} \rangle_{\text{ILM}}}{32\pi^2} = -b_1/R^4. \quad (14)$$

It is obvious that the nonperturbative QCD vacuum energy in the flat (Minkowski) space case

$$\epsilon_{\text{vac}}(0) = \langle T^\mu_\mu \rangle_{\text{ILM}}/4 = -\frac{11}{4} \frac{1}{R^4} \approx -550 \text{ MeV/fm}^3. \quad (15)$$

From previous discussion, we remember that λ must completely cancel $\epsilon_{\text{vac}}(0)$ in the right sides of the Friedmann equations. It looks strange that the FLRW metric did not modify the YM fields energy density.

E. YM effective action in $D = (4 - \epsilon)$ -dim Euclidean FLRW conformal space

To clarify the above-mentioned problem, let us consider YM fields in the Euclidean FLRW conformal D -dimensional space with the metric [where $\tilde{x} = (\tilde{\tau}, \vec{\tilde{x}})$] $\tilde{g}_{\mu\nu}(\tilde{x}) = \exp(h(\tilde{x}))\delta_{\mu\nu}$. The classical YM action has h dependence as

$$S = \frac{1}{4e_0^2} \int d^D \tilde{x} (e^{h/2})^{D-4} F_{\mu\lambda}^a F_{\mu\lambda}^a, \quad (16)$$

and dimensional regularization $\epsilon = 4 - D$ of the one-loop contributions [30] leads to the renormalization and dimensional transmutation (13) as

$$\frac{1}{e_0^2} = \frac{1}{e(\rho)^2} \left(1 + \frac{b_1}{8\pi^2} \frac{e(\rho)^2}{\epsilon} \right). \quad (17)$$

There, the second term represents gluon (and ghost) loops contribution. So, we have the effective action S_{eff} as

$$\begin{aligned} S_{\text{eff}} &= \frac{1}{4e^2(\rho)} \int d^D \tilde{x} (e^{h/2})^{D-4} F_{\mu\lambda}^a F_{\mu\lambda}^a + \Delta S, \\ \Delta S &= \lim_{\epsilon \rightarrow 0} \int d^4 \tilde{x} [(e^{h/2})^{-\epsilon} - 1] \frac{1}{\epsilon} \frac{b_1}{32\pi^2} F_{\mu\lambda}^a F_{\mu\lambda}^a \\ &= - \int d^4 \tilde{x} \frac{h}{2} \frac{b_1}{32\pi^2} F_{\mu\lambda}^a F_{\mu\lambda}^a, \end{aligned} \quad (18)$$

where the gluon fields are interacting with the external field h in the conformal FLRW space. Their coupling is given by ΔS . It is natural to assume that a metric field h can generate a scalar glueball field as the excitations of instanton liquid with vacuum quantum numbers.

F. Scalar glueball field as excitations of instanton liquid in the conformal FLRW space

Fluctuations of the numbers of instantons in the flat space are controlled by the renormalization (scale dependence) properties of QCD [29,31].

Again we explore one-to-one correspondence of the flat space effective action to the same one in the conformal FLRW space. The partition function Z in that space with the account of Eqs. (16) and (18) is given by

$$\begin{aligned} Z &= \int DA \exp(-S_{\text{eff}}) \\ &= \int DA \exp\left(-\frac{1}{4e^2(\rho)} \int d^4 \tilde{x} F_{\mu\lambda}^a F_{\mu\lambda}^a - \Delta S\right). \end{aligned} \quad (19)$$

Since in QCD we have only one dimensional-full parameter, Λ_{QCD} the vacuum average of $F^2 = F_{\mu\lambda}^a F_{\mu\lambda}^a$, which is given by

$$\langle F^2 \rangle = -\frac{1}{V} \frac{d \ln Z}{d(\frac{1}{4e^2(\rho)})} \sim \Lambda_{\text{QCD}}^4 = \rho^{-4} \exp\left(-\frac{1}{b_1} \frac{32\pi^2}{e^2(\rho)}\right), \quad (20)$$

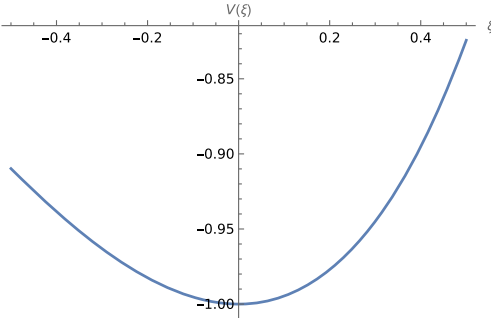
is defined only by its dimensionality. Repeating the differentiations of both sides of (20) we get the (infinite) series of low-energy theorems for the series of F^2 correlators. These simple arguments can even be extended to the similar series of the correlators of local colorless QCD operators with F^2 operators [31] (see review [32]).

As a result, the fluctuations of the instanton density N/V are controlled by these theorems [29]. These features of the QCD can be reproduced by means of corresponding scalar glueball field effective Lagrangian $L(\xi)$ [30,33,34], where the scalar glueball field is defined as

$$|\sigma_0| \exp \xi = -T_\mu^\mu = \frac{b_1}{32\pi^2} F^2. \quad (21)$$

This field ξ corresponds to the density fluctuations of the ILM instanton liquid. The kinetic part $L(\xi)$ must be scale invariant, while the potential part must be scale

noninvariant to reproduce the QCD scale anomaly and mentioned low-energy theorems [30,32]. In the ILM $|\sigma_0| = -4\epsilon_{\text{vac}}(0) = b_1/R^4$. So, we have a Lagrangian $L(\xi)$ for the real scalar glueball field ξ with glueball mass m , which realize low-energy theorems, as

$$L(\xi) = \frac{|\sigma_0|}{4m^2} \frac{1}{2} (\partial_\mu \xi)^2 \exp \left\{ \frac{1}{2} \xi \right\} + V(\xi), \quad V(\xi) = \frac{|\sigma_0|}{4} (\xi - 1) \exp \xi$$

(22)

III. GLUEBALL DARK ENERGY DENSITY AND PRESSURE OF THE FLRW UNIVERSE

Let us consider the effective action $S_{\text{eff}}[\xi, h]$ in the conformal FLRW space with Euclidean time $\tilde{\tau}$. The effective action has the term ΔS describing the interaction of the metric field h with the scalar glueball field ξ . With the account of the ΔS , Eq. (18), the effective action $S_{\text{eff}}[\xi, h]$ and the partition function Z become

$$\begin{aligned} S_{\text{eff}}[\xi, h] &= \int d^3x d\tilde{\tau} \left(L(\xi) - \frac{h}{2} |\sigma_0| \exp \xi \right) \\ &= \int d^3x d\tilde{\tau} \exp(-h/2) \left(L(\xi) - \frac{h}{2} |\sigma_0| \exp \xi \right), \\ Z &= \int D\xi \exp(-S_{\text{eff}}[\xi, h]). \end{aligned} \quad (23)$$

We see that the metric h is playing the role of the external scalar field in the flat space $(\tilde{\tau}, \vec{x})$. The calculations of the Z is reduced to finding a saddle-point $\xi(h)$ from the equation

$$\frac{\delta S_{\text{eff}}[\xi, h]}{\delta \xi} = 0. \quad (24)$$

The explicit form of the saddle-point equation (24) is given by

$$-\frac{1}{m^2} \left(\frac{d^2 \xi}{d\tilde{\tau}^2} + \frac{1}{4} \left(\frac{d\xi}{d\tilde{\tau}} \right)^2 \right) \exp \left(-\frac{\xi}{2} \right) + (\xi - 2h) = 0, \quad (25)$$

where $h = h(\tilde{\tau})$ and m is the scalar glueball mass. The ILM estimation of the mass m [35] gave

$$m = 2.16/\rho, \quad (26)$$

which at $\rho \approx 0.30$ fm gives $m \approx 1290$ MeV. This one is in good consistency with lattice results [36] $m \approx 1470$ MeV.

It is natural to assume that $H = \frac{1}{2} \frac{dh}{dt}$ is much less than scalar glueball mass m ($H/m \ll 1$). Then, we may find the solution of Eq. (25) as a series on $\frac{H^2}{m^2}$. With the account of the zero and next order we have $\xi(h) = \xi_0(h) + \xi_1(h)$ where

$$\begin{aligned} \xi_0(h) &= 2h, \quad \xi_1(h) = \frac{1}{m^2} \left[2 \frac{d^2 h}{d\tilde{\tau}^2} + \left(\frac{dh}{d\tilde{\tau}} \right)^2 \right] \exp(-h) \\ &= \frac{1}{m^2} \left[2 \frac{d^2 h}{d\tilde{\tau}^2} + \left(\frac{dh}{d\tilde{\tau}} \right)^2 \right] = -\frac{1}{m^2} \left(\frac{dH(t)}{dt} + \frac{H^2(t)}{4} \right). \end{aligned} \quad (27)$$

The standard calculations of energy-momentum tensor $T_{\mu\nu}$ from the action

$$S = \int d^3x dt L(\phi(t), \dot{\phi}(t)), \quad L = \frac{1}{2} \dot{\phi}(t)^2 - V(\phi) \quad (28)$$

give the nontrivial components of energy-momentum tensor as

$$T_{00} = \frac{1}{2} \dot{\phi}(t)^2 + V(\phi), \quad T_{ij} = \left[\frac{1}{2} \dot{\phi}(t)^2 - V(\phi) \right] \delta_{ij}, \quad (29)$$

where $\phi(t)$ must be taken from the solution of the equations of motion $\frac{\delta S}{\delta \phi} = 0$. Now we have to calculate the YM fields vacuum energy density $\epsilon_{\text{vac}}(h)$ and pressure $p_{\text{vac}}(h)$ of the FLRW universe from Eqs. (23) and (27), where we have to turn to the Minkowski signature and the time $dt = \exp(h/2)d\tau$ as

$$S_{M,\text{eff}}[\xi, h] = \int d^3x d\tau L_{M,\text{eff}}(\xi, h) = \int d^3x dt \exp(-h/2) \times L_{M,\text{eff}}(\xi, h), \quad (30)$$

$$L_{M,\text{eff}}(\xi, h) = \frac{|\sigma_0|}{4} \frac{1}{2m^2} \left(\frac{d\xi}{dt} \right)^2 \exp\left(h + \frac{\xi}{2}\right) - V_{\text{eff}}(\xi, h),$$

$$V_{\text{eff}}(\xi, h) = \frac{|\sigma_0|}{4} (\xi - 1 - 2h) \exp \xi. \quad (31)$$

Comparing with the standard calculations (29) and using the solution of the saddle-point Eq. (27), we have YM fields vacuum energy density, and pressure of FLRW universe as

$$T_{00}(h) = \epsilon_{\text{vac}}(h) = \exp(-h/2) \left[\frac{|\sigma_0|}{4m^2} \frac{1}{2} \left(\frac{d\xi(h)}{dt} \right)^2 \exp\left(h + \frac{\xi(h)}{2}\right) + V_{\text{eff}}(\xi(h), h) \right]$$

$$= \frac{|\sigma_0|}{4} \left(\frac{H^2(t)}{2m^2} - 1 \right) \exp(3h(t)/2) + O\left(\frac{H^4}{m^4}\right), \quad (32)$$

$$T_{ij}(h) = \delta_{ij} p_{\text{vac}}(h), \quad p_{\text{vac}}(h) = \exp(-h/2) \left[\frac{|\sigma_0|}{4m^2} \frac{1}{2} \left(\frac{d\xi(h)}{dt} \right)^2 \exp\left(h + \frac{\xi(h)}{2}\right) - V_{\text{eff}}(\xi(h), h) \right]$$

$$= \frac{|\sigma_0|}{4} \left(\frac{H^2(t)}{2m^2} + 1 \right) \exp(3h(t)/2) + O\left(\frac{H^4}{m^4}\right), \quad (33)$$

since $T_{\mu\nu}^{\text{dark}}(h) = T_{\mu\nu}(h) - T_{\mu\nu}(0)$ we have the gluon contributions to the dark energy density-pressure as

$$\epsilon_{\text{dark}}(h(t)) = \frac{|\sigma_0|}{4} \left(\frac{H^2(t)}{2m^2} - 1 \right) (\exp(3h(t)/2) - 1) + O\left(\frac{H^4}{m^4}\right),$$

$$p_{\text{dark}}(h(t)) = \frac{|\sigma_0|}{4} \left(\frac{H^2(t)}{2m^2} + 1 \right) (\exp(3h(t)/2) - 1) + O\left(\frac{H^4}{m^4}\right), \quad (34)$$

and the equation of state

$$w(t) = p_{\text{dark}}(h(t))/\epsilon_{\text{dark}}(h(t)) = -\frac{1 + \frac{H^2(t)}{2m^2}}{1 - \frac{H^2(t)}{2m^2}} \leq -1. \quad (35)$$

We see from Eq. (34) that at the present time t_0 , where $h(t_0)$ is small and negative and $\frac{H^2(t_0)}{m^2} \ll 1$, the main contribution to the $\epsilon_{\text{dark}}(h(t_0))$ and $p_{\text{dark}}(h(t_0))$ is given by the potential term ($V_{\text{eff}}(\xi(h), h) - V_{\text{eff}}(0, 0)$) as

$$\epsilon_{\text{dark}}(h(t_0)) = \frac{|\sigma_0|}{4} (1 - \exp(3h(t_0)/2)) + O\left(\frac{H^2(t_0)}{m^2}\right),$$

$$p_{\text{dark}}(h(t_0)) = -\frac{|\sigma_0|}{4} (1 - \exp(3h(t_0)/2)) + O\left(\frac{H^2(t_0)}{m^2}\right). \quad (36)$$

Also, accordingly Eqs. (35) and (36) at the present time equation of state are given by

$$w(t_0) = p_{\text{dark}}(h(t_0))/\epsilon_{\text{dark}}(h(t_0)) = -1. \quad (37)$$

The data (in MeV) are

$$\frac{|\sigma_0|}{4} = |\epsilon_{\text{vac}}(0)| = 550 \text{ MeV/fm}^3 = 4.4 \times 10^9 \text{ MeV}^4, \quad m = 1200 \text{ MeV},$$

$$H(t_0) = 1.4 \times 10^{-39} \text{ MeV}, \quad G = M_{\text{Pl}}^{-2}, \quad M_{\text{Pl}} = 1.2 \times 10^{22} \text{ MeV}. \quad (38)$$

Here $|\epsilon_{\text{vac}}|, m$ are given by ILM, while $H(t_0)$ and M_{Pl} are present astrophysical data [1].

The estimations (in MeV) give the present status of the FLRW metric and of the equation-of-state parameter.

The Friedmann equation (2) for the metric $h(t)$ with the account of Eq. (34) looks like

$$\dot{h}^2(t) = \frac{8\pi G|\sigma_0|}{3}(1 - \exp(3h(t)/2)) + O\left(\frac{\dot{h}^4}{4m^4}\right), \quad (39)$$

where we neglected by the factor $\frac{\pi G|\sigma_0|}{3m^2} \ll 1$. This equation requests $h(t) \leq 0$ and in useful notation looks like

$$\begin{aligned} \dot{f}(t) &= \alpha(1 - \exp(f(t)))^{1/2}, \quad f(t) = 3h(t)/2, \\ \alpha &= 3/2 \left(\frac{8\pi G|\sigma_0|}{3} \right)^{1/2}. \end{aligned} \quad (40)$$

The solution of this equation is given by

$$(1 - \exp f(t))^{1/2} = \tanh(-\alpha t/2 - C). \quad (41)$$

At present time t_0 the metric is close to Minkowski one, which means $f(t_0) \sim h(t_0) \approx 0$ and $H(t_0) \sim \alpha \tanh(-\alpha t_0/2 - C) \approx 0$, $(-\alpha t_0/2 - C) \approx 0$.

At the initial time $f(t_{\text{in}}) \sim h(t_{\text{in}}) \rightarrow -\infty$ means $\tanh(-\alpha(t_{\text{in}} - t_0)/2) \approx 1$. This one corresponds to $\alpha(t_{\text{in}} - t_0) \rightarrow -\infty$. Finally, the solution is given by

$$\frac{3}{2}\dot{h}(t) = \alpha(1 - \exp(3h(t)/2))^{1/2} = \alpha \tanh(-\alpha(t - t_0)/2). \quad (42)$$

The solution (42) of Eqs. (39) and (40) at the present time t_0 with the value of $H(t_0)$ from (38) gives the present time metric

$$h(t_0) = -\frac{H^2(t_0)}{\pi G|\sigma_0|} = -5 \times 10^{-45}. \quad (43)$$

At the initial time t_{in} , when the spatial size of the FLRW universe $a(t_{\text{in}}) \rightarrow 0 \sim h(t_{\text{in}}) \rightarrow -\infty$, we have from Eqs. (34), (39), (40), and (42)

$$H(t_{\text{in}}) = \left(\frac{2\pi G|\sigma_0|}{3} \right)^{1/2} = 1.6 \times 10^{-17} \text{ MeV} \sim 10^{22} H(t_0). \quad (44)$$

The lifetime scale of the universe is given by

$$2/\alpha = 0.42 \times 10^{17} \text{ MeV}^{-1} \approx 3 \times 10^{10} \text{ sec} \approx 10^3 \text{ years}, \quad (45)$$

which is too small. We also see that $w(t \leq t_0) = -1$ with high accuracy.

IV. CONCLUSION

The above given QCD ILM calculations (35) lead to the equation-of-state parameters $w_0 = -1$ and $w_a = 0$ in accordance with λ CDM. The detailed study of the solution of Friedmann equations (39) and (40) demonstrated that the contribution to the dark energy from the QCD vacuum excitations, generated by the universe metric, hardly plays a central role in the dynamics of the universe, since their timescale (45) is too small. As mentioned in Sec. I the newest data, analyzed at [19], prefer a dark energy from an ultralight scalar field, corresponding to a current value of the equation-of-state parameter $w_0 \approx -0.9$ [19]. Such a small scalar mass is a natural in models of QCD axions. Another very interesting alternative [14] of the understanding of the current data is based on the topological properties of the QCD in a curved space [9–14], where it was shown that YM topological configurations with the nontrivial holonomy, compactified to a Euclidean time circle $\mathbb{S}_{\kappa^{-1}}^1$ with the radius κ^{-1} is providing $O(\kappa\Lambda_{\text{QCD}}^3)$ contribution to the dark energy density.

ACKNOWLEDGMENTS

I am very grateful to the referee of my work for the valuable suggestions and criticisms, which improved the quality of my work very much. Also, I thank B. Faizullaev for many fruitful discussions. The work was partially supported by the Ministry of Higher Education, Science, and Innovation of the Republic of Uzbekistan (Grant No. ALM-2023-1006-2528).

DATA AVAILABILITY

The data that support the findings of this article are openly available [1], and embargo periods may apply.

- [1] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [2] Y. B. Zeldovich, Cosmological constant and elementary particles, *JETP Lett.* **6**, 316 (1967), http://jetpletters.ru/ps/1673/article_25521.shtml.
- [3] A. Krasinski and Y. B. Zeldovich, The cosmological constant and the theory of elementary particles, *Sov. Phys. Usp.* **11**, 381 (1968).
- [4] J. Bjorken, Emergent gauge bosons, [arXiv:hep-th/0111196](https://arxiv.org/abs/hep-th/0111196).
- [5] J. D. Bjorken, The classification of universes, [arXiv:astro-ph/0404233](https://arxiv.org/abs/astro-ph/0404233).
- [6] R. Schutzhold, Small cosmological constant from the QCD trace anomaly?, *Phys. Rev. Lett.* **89**, 081302 (2002).
- [7] F. R. Klinkhamer and G. E. Volovik, Self-tuning vacuum variable and cosmological constant, *Phys. Rev. D* **77**, 085015 (2008).
- [8] F. R. Klinkhamer and G. E. Volovik, Dynamic vacuum variable and equilibrium approach in cosmology, *Phys. Rev. D* **78**, 063528 (2008).
- [9] F. R. Urban and A. R. Zhitnitsky, The cosmological constant from the QCD Veneziano ghost, *Phys. Lett. B* **688**, 9 (2010).
- [10] F. R. Urban and A. R. Zhitnitsky, The QCD nature of dark energy, *Nucl. Phys.* **B835**, 135 (2010).
- [11] A. R. Zhitnitsky, Inflaton as an auxiliary topological field in a QCD-like system, *Phys. Rev. D* **89**, 063529 (2014).
- [12] A. R. Zhitnitsky, Dynamical de Sitter phase and nontrivial holonomy in strongly coupled gauge theories in an expanding universe, *Phys. Rev. D* **92**, 043512 (2015).
- [13] A. O. Barvinsky and A. R. Zhitnitsky, Inflation and gauge field holonomy, *Phys. Rev. D* **98**, 045008 (2018).
- [14] L. Van Waerbeke and A. Zhitnitsky, DESI results and dark energy from QCD topological sectors, [arXiv:2506.14182](https://arxiv.org/abs/2506.14182).
- [15] I. L. Shapiro, Effective action of vacuum: Semiclassical approach, *Classical Quantum Gravity* **25**, 103001 (2008).
- [16] T. C. Kraan and P. van Baal, Periodic instantons with nontrivial holonomy, *Nucl. Phys.* **B533**, 627 (1998).
- [17] K. M. Lee and C. h. Lu, SU(2) calorons and magnetic monopoles, *Phys. Rev. D* **58**, 025011 (1998).
- [18] M. Abdul Karim *et al.* (DESI Collaboration), DESI DR2 results. II. Measurements of baryon acoustic oscillations and cosmological constraints, *Phys. Rev. D* **112**, 083515 (2025).
- [19] A. J. Shajib and J. A. Frieman, Scalar-field dark energy models: Current and forecast constraints, *Phys. Rev. D* **112**, 063508 (2025).
- [20] E. V. Shuryak, The role of instantons in quantum chromodynamics. 1. Physical vacuum, *Nucl. Phys.* **B203**, 93 (1982).
- [21] D. Diakonov and V. Y. Petrov, Instanton based vacuum from Feynman variational principle, *Nucl. Phys.* **B245**, 259 (1984).
- [22] D. Diakonov, Instantons at work, *Prog. Part. Nucl. Phys.* **51**, 173 (2003).
- [23] E. Shuryak, Lectures on nonperturbative QCD (nonperturbative topological phenomena in QCD and related theories), [arXiv:1812.01509](https://arxiv.org/abs/1812.01509).
- [24] L. D. Faddeev, In search of multidimensional solitons, (JINR, Dubna, 1976), in *40 Years in Mathematical Physics* (World Scientific Publishing, Singapore, 1995), pp. 369–381.
- [25] L. D. Faddeev, Some comments on the many dimensional solitons, *Lett. Math. Phys.* **1**, 289 (1976).
- [26] R. Jackiw and C. Rebbi, Vacuum periodicity in a Yang-Mills quantum theory, *Phys. Rev. Lett.* **37**, 172 (1976).
- [27] G. V. Dunne and B. Tekin, Calorons in Weyl gauge, *Phys. Rev. D* **63**, 085004 (2001).
- [28] M. C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, Evidence for the role of instantons in hadron structure from lattice QCD, *Phys. Rev. D* **49**, 6039 (1994).
- [29] D. Diakonov, M. V. Polyakov, and C. Weiss, Hadronic matrix elements of gluon operators in the instanton vacuum, *Nucl. Phys.* **B461**, 539 (1996).
- [30] A. A. Migdal and M. A. Shifman, Dilaton effective Lagrangian in gluodynamics, *Phys. Lett.* **114B**, 445 (1982).
- [31] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Are all hadrons alike?, *Nucl. Phys.* **B191**, 301 (1981).
- [32] M. A. Shifman, Anomalies and low-energy theorems of quantum chromodynamics, *Sov. Phys. Usp.* **32**, 289 (1989).
- [33] J. R. Ellis and J. Lanik, Is scalar gluonium observable?, *Phys. Lett. B* **150**, 289 (1985).
- [34] D. Kharzeev, E. Levin, and K. Tuchin, QCD in curved space-time: A conformal bag model, *Phys. Rev. D* **70**, 054005 (2004).
- [35] M. C. Tichy and P. Faccioli, The scalar glueball in the instanton vacuum, *Eur. Phys. J. C* **63**, 423 (2009).
- [36] H. B. Meyer and M. J. Teper, Glueball Regge trajectories and the Pomeron: A lattice study, *Phys. Lett. B* **605**, 344 (2005).