

Wilson loop in $\text{AdS}_3 \times S^3 \times T^4$ from quantum M2 brane

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Type-IIB string theory on $\text{AdS}_3 \times S^3 \times T^4$ with Ramond–Ramond flux as the near-horizon limit of the D1-D5 solution is expected to be dual to a (4, 4) supersymmetric 2D conformal field theory (CFT) parametrized by the integers Q_1 , Q_5 , and other moduli. It is related by T-duality to type-IIA string theory in the near-horizon limit of the D2-D4 solution, which admits an uplift to the 11D $\text{AdS}_3 \times S^3 \times T^5$ background which is the near-horizon limit of the M2-M5 solution. We point out that this relation allows one to use the quantum M2-brane description to probe “nonplanar” corrections in the dual 2D CFT, in close analogy with the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory case (described by M-theory on $\text{AdS}_4 \times S^7/\mathbb{Z}_k$). We consider an analog of a supersymmetric Wilson loop (line defect) expectation value represented by a type-IIA string partition function expanded around an $\text{AdS}_2 \subset \text{AdS}_3$ minimal surface. Its M-theory analog is the M2-brane partition function expanded near $\text{AdS}_2 \times S^1$. We compute the one-loop contribution Z_1 to the M2-brane partition function and find that in contrast to the ABJM case in arXiv:2303.15207 [where $Z_1 = (2 \sin \frac{2\pi}{k})^{-1} = \frac{k}{4\pi} + \frac{\pi}{6k} + \dots$ contains an infinite series of higher-genera string corrections, $k^{-1} \sim \frac{q_{\text{m}}}{\sqrt{t}}$, here it is given solely by the leading string theory contribution $Z_1 = \frac{\kappa}{\sqrt{2\pi}}$, where $\kappa \sim \sqrt{Q_5}$ plays a role analogous to k . We also discuss a generalization to the mixed-flux case which is straightforward from the 11D perspective.

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I. INTRODUCTION

Most symmetric examples of AdS/CFT [1–3] related to $\text{AdS}_5 \times S^5$, $\text{AdS}_4 \times \text{CP}^3$, and $\text{AdS}_3 \times S^3 \times T^4$ share the common feature that their spectrum is expected to be controlled by integrability in the large- N (tree-level string) limit. Going beyond this planar limit at strong coupling is challenging, as this requires computing string loops.

Remarkably, in the $\text{AdS}_4 \times \text{CP}^3$ or Aharony-Bergman-Jafferis-Maldacena (ABJM) theory case, one is able to compute certain nonplanar strong-coupling corrections using its M-theory uplift in terms of quantum M2 branes in $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ [4–10]. Here, we point out that an analogous approach is possible also in the $\text{AdS}_3 \times S^3 \times T^4$ type-IIA case that admits an 11D M-theory embedding.

The standard discussion of the $\text{AdS}_3/\text{CFT}_2$ duality¹ starts with the D1-D5 combination in type-IIB string theory, which

has $\text{AdS}_3 \times S^3 \times T^4$ as its near-horizon limit. Guided by a weak-coupling (flat-space) open-string description of the D1-D5 combination, one expects that the dual 2D conformal field theory (CFT) should be related to an IR limit of a coupled system of three (4,4) supersymmetric multiplets (see, e.g., Refs. [2,11,12]): a $U(Q_1)$ adjoint vector multiplet, a $U(Q_1)$ adjoint hypermultiplet, and a $U(Q_1) \times U(Q_5)$ bifundamental hypermultiplet. It should correspond to a certain deformation of symmetric orbifold $(T^4)^N/S_N$ 2D CFT, where $N = Q_1 Q_5$ (see, e.g., Refs. [13,14]).

In general, details of this duality (in particular, interpolation from weak to strong coupling) beyond the Bogomol’nyi-Prasad-Sommerfield (BPS)-protected sector are still poorly understood (for some progress, see, e.g., Ref. [15]).

The simplest “protected” observable is the free energy on S^2 related to the central charge. The central charge count on the gauge theory side² gives $c = 6Q_1 Q_5 + 6$. On the string-theory side, the free energy is proportional to the logarithmically IR-divergent volume of AdS_3 and is thus related to

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¹We will be mostly interested in the case of $\text{AdS}_3 \times S^3 \times T^4$ supported by Ramond–Ramond (RR) flux.

²The contribution to 2D conformal anomaly of a single free (4, 4) hypermultiplet (four real scalars and four real fermions) is $c = 6$. The 2D vector multiplet has an irrelevant kinetic term and thus contributes to the anomaly only through a ghost factor, with a single vector giving $c = -1$. The $U(1)$ part of the vector multiplet is decoupled (representing the c.m. of the bound D1-D5 combination), so that finally $c = 6Q_1 Q_5 + Q_1^2 - 6(Q_1^2 - 1) = 6Q_1 Q_5 + 6$. The same result is found by counting the $SU(2)$ chiral anomaly of the (4,4) superconformal algebra [2,11].

the UV conformal anomaly in the boundary theory. The leading $6Q_1Q_5$ term is reproduced by the classical action of the 10D supergravity on $\text{AdS}_3 \times S^3 \times T^4$ [16], while the subleading +6 term comes from the one-loop 10D supergravity contribution [17] (with all subleading in large $N = Q_1Q_5$ contributions expected to cancel out).³

Another special observable that will be our main focus here is the expectation value of a supersymmetric conformal one-dimensional defect operator, which is a formal analog of the $\frac{1}{2}$ -BPS Wilson loop (WL) in gauge theories, and on the string-theory side should be represented by the partition function expanded near the $\text{AdS}_2 \subset \text{AdS}_3$ minimal surface [19–21].

Performing T-duality along one D5 direction (a circle in T^4), one gets the D2-D4 combination in type-IIA string theory that also has $\text{AdS}_3 \times S^3 \times T^4$ space (supported by the RR 4-form flux) as its near-horizon limit. The corresponding dual 2D CFT should be equivalent to the one in the type-IIB case, up to a redefinition of parameters (in particular, $Q_5 \rightarrow Q_4$, $Q_1 \rightarrow Q_2$).

The key point here will be that the D2-D4 solution admits the uplift to the M2-M5 solution of 11D supergravity [22] that has the $\text{AdS}_3 \times S^3 \times T^5$ as its near-horizon limit. This opens up a possibility to use the quantum M2-brane probe method to shed light on nonplanar corrections in the dual 2D CFT.

In particular, the AdS_2 minimal surface representing the WL in the type-IIA string on $\text{AdS}_3 \times S^3 \times T^4$ supported by RR flux gets uplifted to the $\text{AdS}_2 \times S^1$ M2-brane solution in $\text{AdS}_3 \times S^3 \times T^4 \times S^1$, where M2 is wrapped on $S^1 \subset T^4$. This is analogous to how the $\frac{1}{2}$ -BPS WL in ABJM theory is represented by the $\text{AdS}_2 \times S^1$ M2 brane in $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ [4,23].

Our aim below will be to compute the one-loop M2-brane correction to the expectation value of the circular WL in $\text{AdS}_3 \times S^3 \times T^4 \times S^1$ following the same steps as in the $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ case in Ref. [4].

A. Review

Let us first review the known expression for the one-loop WL expectation value at strong coupling represented by the string partition function on the disc that can be computed using semiclassical expansion near the AdS_2 minimal surface [20]. The three cases of a string with AdS_2 -induced geometry in AdS_3 subspace of $\text{AdS}_p \times M^{10-p}$ supported by RR fluxes, i.e.,

- (i) $p = 5$: $\text{AdS}_5 \times S^5$; (ii) $p = 4$: $\text{AdS}_4 \times \text{CP}^3$;
- (iii) $p = 3$: $\text{AdS}_3 \times S^3 \times T^4$;

³This is analogous to the $N^2 \rightarrow N^2 - 1$ shift in the $\text{AdS}_5 \times S^5$ case [18] and the $4N^3 - \frac{9}{4}N \rightarrow 4N^3 - \frac{9}{4}N - \frac{7}{4}$ shift in the $\text{AdS}_7 \times S^4$ case [17].

can be treated on an equal footing. The main parameters are the AdS_p scale L , the string coupling g_s , and the effective dimensionless string tension T :

$$T = L^2 T = \frac{L^2}{2\pi\alpha'}. \quad (1.1)$$

The WL expectation value is given by the string partition function with a disklike world sheet ending on a circle at the boundary of AdS_p

$$\langle \mathcal{W} \rangle = \frac{1}{g_s} Z_1 + \mathcal{O}(g_s) = W_1[1 + \mathcal{O}(T^{-1})] + \mathcal{O}(g_s), \quad (1.2)$$

$$Z_1 = \int [dx \dots] e^{-T \int d^2\sigma \mathcal{L}} = e^{-\Gamma}, \quad \Gamma = \Gamma_0 + \Gamma_1 + \Gamma_2 + \dots \quad (1.3)$$

Here, $\Gamma_0 = -2\pi T$ is the classical string action proportional to the renormalized AdS_2 volume $\text{vol}(\text{AdS}_2) = -2\pi$. The one-loop correction $\Gamma_1 = \mathcal{O}(T^0)$ found from the expansion of the Green-Schwarz string action in a static and κ -symmetry gauge is given by the determinants of the fluctuation operators in AdS_2 world-sheet geometry ($R^{(2)} = -2$) [19,20]:

$$\begin{aligned} e^{-\Gamma_1} &= \frac{[\det(-\nabla^2 + \frac{1}{4}R^{(2)} + 1)]^{p-1} [\det(-\nabla^2 + \frac{1}{4}R^{(2)})]^{5-p}}{[\det(-\nabla^2 + 2)]^{\frac{1}{2}p-\frac{3}{2}} [\det(-\nabla^2 + R^{(2)} + 4)]^{\frac{1}{2}} [\det(-\nabla^2)]^{5-\frac{1}{2}p}} \\ &= \frac{[\det(-\nabla^2 + \frac{1}{2})]^{p-1} [\det(-\nabla^2 - \frac{1}{2})]^{5-p}}{[\det(-\nabla^2 + 2)]^{\frac{1}{2}p-1} [\det(-\nabla^2)]^{5-\frac{1}{2}p}}. \end{aligned} \quad (1.4)$$

In the case of $\text{AdS}_3 \times S^3 \times T^4$ supported by RR flux, there are four massless fermion modes (which are partners of T^4 bosonic modes) and four massive ones [19] (the result is the same in T-dual type-IIB and type-IIA embeddings). For general p , one finds that

$$\Gamma_1 = \frac{1}{2} \sum \log \det \Delta = \Gamma_{1\infty} + \bar{\Gamma}_1, \quad \Gamma_{1\infty} = -\zeta_{\text{tot}}(0) \log \Lambda, \quad \zeta_{\text{tot}}(0) = 1, \quad (1.5)$$

$$\bar{\Gamma}_1 = -\frac{1}{2} \zeta'_{\text{tot}}(0) = \frac{1}{2} (p-4) \log(2\pi), \quad (1.6)$$

where Λ is a 2D UV cutoff (the radius of AdS_2 is set to 1). As was argued in Ref. [20], the universal divergent term $\Gamma_{1\infty} = -\log \Lambda$ [Eq. (1.5)] should be canceled against the contribution of the Green-Schwarz (GS) string path-integral measure, so that at the end it gets effectively replaced by a finite term

$$\Gamma_{1\infty} \rightarrow -\log \frac{\sqrt{T}}{\sqrt{2\pi}}. \quad (1.7)$$

As a result, the one-loop contribution W_1 in Eq. (1.2) is given by

$$W_1 = \frac{1}{g_s} \frac{\sqrt{T}}{\sqrt{2\pi}} e^{-\tilde{\Gamma}_1} e^{2\pi T} = c_p \frac{\sqrt{T}}{g_s} e^{2\pi T}, \quad c_p = \frac{1}{(\sqrt{2\pi})^{p-3}}. \quad (1.8)$$

In the $p = 3$ case corresponding to $\text{AdS}_3 \times S^3 \times T^4$, we get $c_p = 1$.

The expression (1.8) matches the known gauge theory localization results in the $p = 5$ (SYM) [24,25] and $p = 4$ (ABJM) [26] cases, where one has

$$\text{SYM: } g_s = \frac{g_{\text{YM}}^2}{4\pi} = \frac{\lambda}{4\pi N}, \quad \lambda = g_{\text{YM}}^2 N, \quad T = \frac{\sqrt{\lambda}}{2\pi}, \quad (1.9)$$

$$\text{ABJM: } g_s = \frac{\sqrt{\pi}(2\lambda)^{5/4}}{N}, \quad \lambda = \frac{N}{k}, \quad T = \frac{\sqrt{\lambda}}{\sqrt{2}}, \quad k^{-1} = \frac{g_s}{\sqrt{8\pi T}}. \quad (1.10)$$

Higher-genus string corrections to Eq. (1.2) are expected to have the structure [20] (χ is the 2D Euler number)

$$\begin{aligned} \langle \mathcal{W} \rangle &= \sum_{\chi=1,-1,\dots} b_\chi \left(\frac{\sqrt{T}}{g_s} \right)^\chi e^{2\pi T} [1 + \mathcal{O}(T^{-1})] \\ &= \frac{\sqrt{T}}{g_s} e^{2\pi T} \left(c_p + c_{1p} \frac{g_s^2}{T} [1 + \mathcal{O}(T^{-1})] \right. \\ &\quad \left. + c_{2p} \frac{g_s^4}{T^2} [1 + \mathcal{O}(T^{-1})] + \dots \right), \end{aligned} \quad (1.11)$$

which is indeed consistent with that of the $1/N$, strong-coupling expansion found from localization in the corresponding 4D and 3D gauge theories in Refs. [25] and [26].

In the AdS_4 or ABJM case, one is able also to reproduce the leading large- T corrections to all orders in g_s by semiclassically quantizing the M2 brane [27] wrapped on AdS_2 and the 11D circle of radius $\sqrt{\frac{2}{k}}$ in $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ [4]. Localization predicts [28] that the $\frac{1}{2}$ -BPS WL expanded in large N at fixed k is given by [cf. Eqs. (1.8) and (1.10)]

$$\langle \mathcal{W} \rangle = \frac{1}{2 \sin \frac{2\pi}{k}} e^{\pi \sqrt{\frac{2N}{k}}} \left[1 + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right) \right] = \frac{1}{2 \sin(\sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}})} e^{2\pi T} [1 + \mathcal{O}(T^{-1})] \quad (1.12)$$

$$= \frac{1}{\sqrt{2\pi}} \frac{\sqrt{T}}{g_s} e^{2\pi T} \left(1 + \frac{\pi}{12} \frac{g_s^2}{T} [1 + \mathcal{O}(T^{-1})] + \frac{7\pi^2}{1440} \frac{g_s^4}{T^2} [1 + \mathcal{O}(T^{-1})] + \mathcal{O}(g_s^6) \right). \quad (1.13)$$

Written in terms of the AdS_4 radius L and the radius $R = 2L$ of S^7 , the effective M2-brane tension and its classical action on $\text{AdS}_2 \times S^1$ is the same as the string action appearing in the exponent in Eq. (1.12):

$$T_2 = L^2 R T_2 = \frac{1}{\sqrt{8\pi}} \sqrt{Nk}, \quad L^6 = \frac{1}{2} \pi^2 N k \ell_p^6, \quad T_2 = \frac{1}{(2\pi)^2 \ell_p^3}, \quad (1.14)$$

$$S_{\text{M2}} = (-2\pi) \left(\frac{2\pi}{k} \right) T_2 = -\pi \sqrt{\frac{2N}{k}}. \quad (1.15)$$

The M2-brane semiclassical expansion is performed for large T_2 for fixed k (fixed 11D background)—i.e., the $\frac{1}{2 \sin \frac{2\pi}{k}}$ prefactor in Eq. (1.12) should be the one-loop M2-brane contribution [which, according to Eqs. (1.11) and (1.7), sums up all the leading large string tension contributions at each order in g_s]. The $\mathcal{O}(\frac{1}{\sqrt{N}})$ term in Eq. (1.12) should correspond to the two-loop contribution.

Indeed, summing up the contributions of the M2-brane fluctuations near $\text{AdS}_2 \times S^1$, one finds a generalization of the string one-loop contribution in Eq. (1.4) [4]:

$$\langle \mathcal{W} \rangle = e^{-\hat{\Gamma}_1} e^{-S_{\text{M2}}} [1 + \mathcal{O}(T_2^{-1})], \quad \hat{\Gamma}_1 = -\frac{1}{2} \sum_{n=-\infty}^{\infty} \log X_n, \quad (1.16)$$

$$X_n = \frac{[\det(-\nabla^2 + \frac{R^{(2)}}{4} + (\frac{kn}{2} + 1)^2)]^{\frac{3}{2}} [\det(-\nabla^2 + \frac{R^{(2)}}{4} + (\frac{kn}{2} - 1)^2)]^{\frac{3}{2}} \det(-\nabla^2 + \frac{R^{(2)}}{4} + (\frac{kn}{2})^2)}{\det(-\nabla^2 + \frac{1}{4}(kn-2)(kn-4)) [\det(-\nabla^2 + \frac{1}{4}kn(kn+2))]^3}. \quad (1.17)$$

Here, n labels the modes of the M2-brane fluctuations expanded in σ_3 (identified with an S^1 coordinate with radius $\sqrt{\frac{2}{k}}$) that propagate on AdS_2 . The $n = 0$ level corresponds to the string-mode contribution in Eq. (1.4) in the AdS_4 case ($p = 4$).

In contrast to the string case in Eqs. (1.5) and (1.6), $\hat{\Gamma}_1$ in Eq. (1.16), which corresponds to a 3D theory should not contain log UV divergences—i.e., it should be given by a finite expression (once computed using ζ -function regularization that removes power divergences). Indeed, the coefficient of the log Λ term in Eq. (1.5) here is replaced by

$$\begin{aligned}\hat{\Gamma}_{1\infty} &= -\zeta_{\text{tot}}(0) \log \Lambda, \\ \zeta_{\text{tot}}(0) &= \sum_{n=-\infty}^{\infty} 1 = 1 + 2\zeta_R(0) = 0.\end{aligned}\quad (1.18)$$

The remaining terms take the form (for $k > 2$) [4]

$$\begin{aligned}\hat{\Gamma}_1 &= \sum_{n=1}^{\infty} \log\left(\frac{k^2 n^2}{4} - 1\right) \\ &= 2 \sum_{n=1}^{\infty} \log \frac{kn}{2} + \sum_{n=1}^{\infty} \log\left(1 - \frac{4}{k^2 n^2}\right) \\ &= -\log \frac{k}{4\pi} + \log\left(\frac{k}{2\pi} \sin \frac{2\pi}{k}\right),\end{aligned}\quad (1.19)$$

$$e^{-\hat{\Gamma}_1} = \frac{1}{2 \sin \frac{2\pi}{k}}, \quad (1.20)$$

where we have used $\sin(\pi x) = \pi x \prod_{n=1}^{\infty} (1 - \frac{x^2}{n^2})$ and

$$\begin{aligned}2 \sum_{n=1}^{\infty} \log \frac{kn}{2} &= 2\zeta_R(0) \log \frac{k}{2} - 2\zeta'_R(0) = -\log \frac{k}{4\pi}, \\ \zeta_R(0) &= -\frac{1}{2}, \quad \zeta'_R(0) = -\frac{1}{2} \log(2\pi).\end{aligned}\quad (1.21)$$

Equation (1.20) reproduces the prefactor in Eq. (1.12)—and, in particular, the leading string one-loop factor in Eqs. (1.8) (with $p = 4$) and (1.7).

Thus, the M2-brane “uplift” of the GS string [29] effectively provides its consistent definition—i.e., the required measure contribution that cancels the log Λ term in (1.5) is at the same time replacing it with the required $\sqrt{\frac{T}{2\pi}}$ term in Eq. (1.8), in agreement with Eq. (1.7) [which here automatically includes the $\frac{1}{g_s}$ factor; cf. Eq. (1.13)].

As we shall see below, a similar result is found also in the AdS_3 case, where the M2-brane probe will be wrapping $\text{AdS}_2 \times S^1$ in $\text{AdS}_3 \times S^3 \times T^4 \times S^1$.

B. Results

Our starting point will be the $\frac{1}{4}$ -BPS M2-M5 solution of 11D supergravity [22,30] with the harmonic functions

$$\begin{aligned}H_2 &= 1 + \frac{Q_2}{r^2}, & H_5 &= 1 + \frac{Q_5}{r^2}, \\ Q_2 &= \ell_P^2 \hat{v}^{-1} Q_2, & Q_5 &= \ell_P^2 u^{-1} Q_5,\end{aligned}\quad (1.22)$$

$$\hat{v} \equiv v_1 v_2 v_3 v_4, \quad y_p \equiv y_p + 2\pi v_p, \quad w \equiv w + 2\pi u, \quad (1.23)$$

where y_p ($p = 1, \dots, 4$) are the angular coordinates (with dimensionless radii v_p) of T^4 along M5 not shared with M2, u is a dimensionless radius of the S^1 coordinate w , and Q_2 and Q_5 are integers (see Sec. II).⁴

The M2-M5 solution reduces to the D2-D4 solution of type-IIA theory, provided the 11th coordinate is identified with one of the T^4 coordinates (denoted below as y_4), so that v_4 is related to the asymptotic value of the type-IIA string coupling g_s (see Appendix A):

$$\begin{aligned}R_{11} &= v_4 \ell_P = g_s \ell_s, & v_4 &= g_s^{2/3}, \\ \ell_P &= g_s^{1/3} \ell_s, & \ell_s &= \sqrt{\alpha'}.\end{aligned}\quad (1.24)$$

In the near-horizon limit, the 11D metric becomes that of $\text{AdS}_3 \times S^3 \times T^5$:

$$ds_{11}^2 = L^2 (ds_{\text{AdS}_3}^2 + ds_{S^3}^2) + R^2 dy_p dy_p + R_w^2 dw^2, \quad (1.25)$$

$$\begin{aligned}L^2 &= (Q_2 Q_5^2)^{1/3}, & R^2 &= \ell_P^2 (Q_2 Q_5^{-1})^{1/3}, \\ R_w^2 &= \ell_P^2 (Q_2 Q_5^{-1})^{-2/3}.\end{aligned}\quad (1.26)$$

The effective dimensionless tension and the action of the M2 brane wrapped on $\text{AdS}_2 \subset \text{AdS}_3$ and $y_4 \equiv y_4 + 2\pi v_4$ are given by [cf. Eq. (1.14)]

$$\begin{aligned}T_2 &= L^2 R T_2 = \frac{1}{(2\pi)^2} \frac{1}{\sqrt{\hat{v}u}} \sqrt{Q_2 Q_5}, \\ S_{\text{M2}} &= (-2\pi)(2\pi v_4) T_2 = -\frac{v_4}{\sqrt{\hat{v}u}} \sqrt{Q_2 Q_5}.\end{aligned}\quad (1.27)$$

Let us introduce the parameter κ proportional to v_4^{-1} that is the analog of k , which determines the square of the effective radius $\frac{2}{k}$ of the 11th angular direction in the $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ case—i.e.,

⁴The dimensionless moduli parameters v , v_4 , and u that will be related to similar parameters of D2-D4 and D1-D5 10D solutions (cf. Ref. [31]) may be set to 1, but it is useful to keep their values generic.

$$\frac{1}{\kappa_{\text{AdS}_4}} = \frac{R_{S^7}}{L_{\text{AdS}_4}} \frac{1}{k} = \frac{2}{k} \rightarrow \frac{1}{\kappa} \equiv \frac{R}{L} v_4 = \frac{v_4 \ell_P}{\sqrt{Q_5}},$$

$$\kappa = \frac{1}{v_4 \sqrt{u}} \sqrt{Q_5}. \quad (1.28)$$

The expansion in large T_2 for fixed κ (and moduli $\hat{v}$, v_4 , u) is thus the expansion in large Q_2 for fixed Q_5 . Taking κ large corresponds to shrinking the size v_4 of the 11th dimension or taking the type-IIA string coupling g_s in Eq. (1.24) small.

$$X_n = \frac{[\det(-\nabla^2 + \frac{1}{4}R^{(2)} + \frac{1}{4}(m_n + 1)^2)]^2 [\det(-\nabla^2 + \frac{1}{4}R^{(2)} + \frac{1}{4}(m_n - 1)^2)]^2}{[\det(-\nabla^2 + 1 + m_n + \kappa^2 n^2)]^{\frac{1}{2}} [\det(-\nabla^2 + 1 - m_n + \kappa^2 n^2)]^{\frac{1}{2}} [\det(-\nabla^2 + \kappa^2 n^2)]^3},$$

$$m_n \equiv \sqrt{1 + 4\kappa^2 n^2}. \quad (1.30)$$

The $n = 0$ part here is the same as the $\text{AdS}_3 \times S^3 \times T^4$ string contribution ($p = 3$) in Eq. (1.4).

As a result, using the ζ -function regularization as in Eq. (1.21), we conclude that the log-divergent part of $\hat{\Gamma}_1$ again vanishes as in Eq. (1.18) so that Γ_1 is finite.

Assuming that κ is large enough⁵ ($\kappa^2 > \frac{3}{4}$, so that $m_n > 2$)—which, according to Eq. (1.28), is the case if v_4 is small or Q_5 is large, which corresponds to the perturbative type-IIA string-theory regime—we will find that after some nontrivial cancellations [that are due to underlying extended (4,4) 2D supersymmetry of the spectrum in Eq. (1.30); see Secs. III and IV],⁶ $\hat{\Gamma}_1$ in Eq. (1.29) is given simply by [cf. Eqs. (1.20) and (1.21)]

$$\hat{\Gamma}_1 = -\frac{1}{2} \log(2\pi) + \sum_{n=1}^{\infty} \log(\kappa^2 n^2) = -\log \frac{\kappa}{\sqrt{2\pi}}. \quad (1.31)$$

The first ($n = 0$) term is the same as the finite part of the string-theory ($p = 3$) contribution in Eq. (1.6).

For smaller values of $\kappa^2 = v_4^{-2} u^{-1} Q_5 \leq \frac{3}{4}$ (corresponding to nonperturbative string-theory or M-theory regimes), $\hat{\Gamma}_1$ in Eq. (1.31) gets modified according to Eqs. (4.18) and (4.17), as discussed in Sec. IV below.

Thus, for $\kappa > \sqrt{\frac{3}{4}}$, the one-loop M2-brane partition function in the $\text{AdS}_3 \times S^3 \times T^5$ case is given by

$$\langle \mathcal{W} \rangle = e^{-S_{\text{M2}} - \hat{\Gamma}_1 + \dots} = \frac{\kappa}{\sqrt{2\pi}} e^{-S_{\text{M2}}} [1 + \mathcal{O}(T_2^{-1})]$$

$$= \frac{\sqrt{Q_5}}{\sqrt{2\pi u v_4}} e^{\frac{v_4}{\sqrt{v u}} \sqrt{Q_2 Q_5}} [1 + \mathcal{O}(T_2^{-1})], \quad (1.32)$$

⁵This condition is somewhat analogous to $k > 2$ in the AdS_4 case in Eq. (1.19).

⁶As discussed in Appendix D, an extended 2D supersymmetry is also responsible for the simplicity of the result in Eq. (1.19) in the AdS_4 case.

The computation of the one-loop correction to the M2-brane partition function described in Secs. III and IV below leads to the following analog of the $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ expressions in the case of Eqs. (1.16) and (1.17):

$$\langle \mathcal{W} \rangle = e^{-\hat{\Gamma}_1} e^{-S_{\text{M2}}} [1 + \mathcal{O}(T_2^{-1})],$$

$$\hat{\Gamma}_1 = -\frac{1}{2} \sum_{n=-\infty}^{\infty} \log X_n, \quad (1.29)$$

where $\mathcal{O}(T_2^{-1})$ stands for a possible two-loop M2-brane contribution.

As in the $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ case, we find again that the M2-brane generalization “regularizes away” the one-loop divergence of the GS string in Eq. (1.5), replacing it with the expected [20] finite one-loop prefactor proportional to $\sqrt{\frac{T}{2\pi}}$ [cf. Eqs. (1.7) and (1.8)]. Compared to the result in the $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ case [Eq. (1.20)], where $\frac{1}{2 \sin \frac{2\pi}{k}} = \frac{k}{4\pi} [1 + \mathcal{O}(\frac{1}{k^2})]$, we do not get any subleading $1/\kappa$ corrections in Eq. (1.32).

The effective string tension and the “near-horizon” value g_s of the string coupling (dilaton) in the corresponding $\text{AdS}_3 \times S^3 \times T^4$ type-IIA background, which is the limit of the D2-D4 solution, are given by [here, $Q_5 = Q_4$; see Eq. (1.24) and Appendixes A and B]

$$\text{IIA: } T = 2\pi v_4 T_2 = \frac{1}{2\pi} \sqrt{\frac{v_4}{v_1 v_2 v_3 u}} \sqrt{Q_2 Q_5}$$

$$= \frac{g_s}{2\pi \sqrt{v u}} \sqrt{Q_2 Q_5}, \quad (1.33)$$

$$S = S_{\text{M2}} = -2\pi T = -\frac{g_s}{\sqrt{v u}} \sqrt{Q_2 Q_5}, \quad (1.34)$$

$$v = v_1 v_2 v_3, \quad u = \ell_s^{-1} \ell_P u,$$

$$v_i = \ell_s^{-1} \ell_P v_i, \quad \ell_s^{-1} \ell_P = g_s^{1/3}, \quad i = 1, 2, 3, \quad (1.35)$$

$$g_s^2 = g_s^2 \sqrt{\frac{u}{v}} \sqrt{\frac{Q_2}{Q_5}}, \quad \frac{\sqrt{T}}{g_s} = \frac{\kappa}{\sqrt{2\pi}} = \frac{1}{\sqrt{2\pi g_s u}} \sqrt{Q_5}. \quad (1.36)$$

Here, $T = \frac{L_s^2}{2\pi \ell_s^2}$, with $L_s^2 = \sqrt{Q_2 Q_5}$ being the AdS_3 radius in the type-IIA metric in Eq. (B5), and $g_s = e^\phi$, with g_s being its “asymptotic” value, and we have redefined the moduli parameters so that the corresponding $T^3 \times S^1$ radii

in the 10D metric are “measured” in units of ℓ_s . Then, Eq. (1.32) takes the form [cf. Eq. (1.24)]

$$\langle \mathcal{W} \rangle = \frac{\sqrt{T}}{g_s} e^{2\pi T} [1 + \mathcal{O}(T_2^{-1})]. \quad (1.37)$$

Compared to the $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ case [Eqs. (1.12) and (1.13)]; here, surprisingly, we do not get any leading large tension corrections $(g_s^2 T^{-1})^p$ at higher genus orders.⁷

One may speculate that further T_2^{-1} contributions in Eq. (1.32) may also be absent so that the result for the $\frac{1}{2}$ -BPS WL in the $\text{AdS}_3 \times S^3 \times T^4$ string case may be one-loop exact (like the central charge).

Expressed in terms of the parameters of the $\text{AdS}_3 \times S^3 \times T^4$ solution in type-IIB string theory, which is the near-horizon limit of the D1-D5 background T-dual [along the w circle in the 10D limit of Eq. (1.25)] to the D2-D4 type-IIA solution, we get, instead of Eq. (1.36),

$$\text{IIB: } T' = \frac{g'_s}{2\pi\sqrt{\hat{v}'}} \sqrt{Q_2 Q_5}, \quad \hat{v}' = v u^{-1}, \quad g'_s = u^{-1} g_s, \quad (1.38)$$

$$g'_s = \frac{g'_s}{\sqrt{\hat{v}'}} \sqrt{\frac{Q_2}{Q_5}}, \quad \frac{\sqrt{T'}}{g'_s} = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{Q_5}{Q_2}} \sqrt{Q_5}, \quad (1.39)$$

where $Q_2 = Q_1$ (see Appendix B), and primes are used to indicate the type-IIB parameters. Under the T-duality, $u' = u^{-1}$ and $g_s'^{-2} u' = g_s^{-2} u$, where g'_s is the type-IIB asymptotic string coupling. We also defined $\hat{v}' = v u'$ as the dimensionless volume of $T^3 \times S^1$.

One may also consider an analog of the AdS_2 string minimal surface in a $\text{AdS}_3 \times S^3 \times T^4$ background supported by a “mixed” combination of RR and NSNS fluxes. We will discuss the corresponding 11D background and find the $\text{AdS}_2 \times S^1$ M2-brane solution that generalizes the string-theory one in Ref. [32] in Appendix E. The corresponding one-loop correction turns out to have the simple form (1.31) with a rescaled parameter κ .

There are several generalizations of the present work. One may compute the two-loop string and also two-loop M2-brane corrections to the WL expectation value following the approach of Ref. [33]. It would be interesting also to use the semiclassical M2-brane approach to find “non-planar” corrections to anomalous dimensions of non-BPS operators, as was done in the ABJM context in Refs. [6,8].

⁷It is interesting to note that a similar result was found in Ref. [10]. This work studied a $\frac{1}{2}$ -BPS Wilson loop for a class of $\mathcal{N} \geq 2$ 3D Chern-Simons matter theories dual to $\text{AdS}_4 \times \text{SE}^7/\mathbb{Z}_k$ backgrounds. In certain cases of $\text{SE}^7/\mathbb{Z}_k$ spaces, it was found that the one-loop $\text{AdS}_2 \times S^1$ M2-brane partition function is given just by the leading string-theory one-loop contribution.

II. M2-M5 SOLUTION AND ITS $\text{AdS}_3 \times S^3 \times T^5$ LIMIT

Our starting point will be the $\frac{1}{4}$ -supersymmetric 11D supergravity solution representing intersecting M2 and M5 branes [22]:

$$ds_{11}^2 = H_2^{-2/3} H_5^{-1/3} (dx_0^2 + dx_1^2) + H_2^{1/3} H_5^{2/3} (dr^2 + r^2 ds_{S^3}^2) + H_2^{1/3} H_5^{-1/3} dy_p^2 + H_2^{-2/3} H_5^{2/3} dw^2, \quad (2.1)$$

$$F_4 = dC_3 = (i dH_2^{-1} \wedge dx_0 \wedge dx_1 - *_4 dH_5) \wedge dw, \quad (2.2)$$

$$H_2 = 1 + \frac{Q_2}{r^2}, \quad H_5 = 1 + \frac{Q_5}{r^2}, \quad Q_2 = \ell_P^2 \hat{v}^{-1} Q_2, \quad Q_5 = \ell_P^2 u^{-1} Q_5. \quad (2.3)$$

We consider the Euclidean signature of the metric (see Appendix A) and assume that the four “noncommon” y_p ($p = 1, 2, 3, 4$) directions of M5 and the w direction of M2 are compactified on T^4 and S^1 , respectively—i.e.,

$$y_p = \ell_P y_p, \quad w = \ell_P w, \quad y_p \equiv y_p + 2\pi v_p, \quad w \equiv w + 2\pi u, \quad \hat{v} \equiv v_1 v_2 v_3 v_4. \quad (2.4)$$

Here, ℓ_P is the 11D scale, so that y_p , w , v_p , u , and v are dimensionless. The charge quantization condition implies that Q_2 and Q_5 are integers.

In the “near-horizon” limit—i.e., dropping 1 from H_2 and H_5 in Eq. (2.3)—we get the $\text{AdS}_3 \times S^3 \times T^5$ solution⁸

$$ds_{11}^2 = L^2 (ds_{\text{AdS}_3}^2 + ds_{S^3}^2) + R^2 dy_p^2 + R_w^2 dw^2, \quad F_4 = 2L^2 R_w (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}) \wedge dw, \quad (2.5)$$

$$L^2 = (Q_2 Q_5^2)^{\frac{1}{3}} = \ell_P^2 \left(\frac{Q_2 Q_5^2}{u^2 \hat{v}} \right)^{\frac{1}{3}}, \quad R^2 = \ell_P^2 (Q_2 Q_5^{-1})^{\frac{1}{3}} = \ell_P^2 \left(\frac{Q_2 u}{Q_5 \hat{v}} \right)^{\frac{1}{3}}, \quad R_w^2 = \ell_P^6 R^{-4}. \quad (2.6)$$

The (Euclidean) AdS_3 metric originates from Eq. (2.1) in Poincare coordinates and can then be transformed into global coordinates. We may rewrite Eq. (2.5) as ($i = 1, 2, 3$)

$$ds^2 = L^2 (ds_{\text{AdS}_3}^2 + ds_{S^3}^2 + dy_i'^2 + dw'^2 + \kappa^{-2} d\varphi^2), \quad F_4 = 2L^3 (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}) \wedge dw', \quad (2.7)$$

$$y_i' = L^{-1} R y_i, \quad w' = L^{-1} R w, \quad \varphi = v_4^{-1} y_4 \equiv \varphi + 2\pi, \quad \kappa^{-1} \equiv L^{-1} R v_4, \quad (2.8)$$

⁸We shall use the notations $\text{vol}(M)$ and vol_M , respectively, for the volume and the volume form of space M .

$$ds_{\text{AdS}_3}^2 = \frac{(1 + \frac{1}{4}z^2)^2}{(1 - \frac{1}{4}z^2)^2} ds_{\text{AdS}_2}^2 + \frac{dz^2}{(1 - \frac{1}{4}z^2)^2},$$

$$ds_{S^3}^2 = \frac{dq^a dq^a}{(1 + \frac{1}{4}q^a q^a)^2}, \quad a = 1, 2, 3, \quad (2.9)$$

where AdS_3 and S^3 in Eq. (2.9) have unit radii.

The M2-M5 solution is related by dimensional reduction (along a T^4 direction $y_4 = v_4 \varphi$) to the 10D type-IIA D2-D4 solution.⁹ It is, in turn, T-dual (along w) to the type-IIB D1-D5 solution having the same numbers of respective branes or integer charges and related values of moduli parameters (see Appendix B).

III. $\text{AdS}_2 \times S^1$ M2-BRANE SOLUTION

Our aim will be to compute the one-loop M2-brane partition function expanded near the $\text{AdS}_2 \times S^1$ minimal surface. We shall assume that S^1 corresponds to the $y_4 = v_4 \varphi$ direction in Eq. (2.4). If y_4 is identified with the 11th direction, then the “double-dimensional” reduction will lead to a type-IIA string wrapped on AdS_2 in $\text{AdS}_3 \times S^3 \times T^3 \times S^1$, which is the near-horizon limit of the D2-D4 solution.

We shall consider the M2-brane solution for which two of the world volume coordinates σ^1, σ^2 are identified with the AdS_2 coordinates inside AdS_3 in Eq. (2.9) (with AdS_2

located at $z = 0$), and the third one $\sigma^3 \equiv \sigma^3 + 2\pi$ with the angle φ in Eq. (2.7). One can check that such an M2-brane configuration solves the equations of motion and is a $\frac{1}{2}$ -BPS case (see Appendix C).

The value of the classical M2-brane action in Eq. (C1) is then

$$S_{\text{M2}} = T_2 L^3 (2\pi \kappa^{-1}) \text{vol}(\text{AdS}_2)$$

$$= T_2 (2\pi v_4) (-2\pi) = -\frac{v_4}{\sqrt{\hat{v}u}} \sqrt{Q_2 Q_5}, \quad (3.1)$$

where $T_2 = L^2 R T_2$ is the effective dimensionless tension, as in Eq. (1.27). Note that the potential C_3 corresponding to F_4 in Eq. (2.5) does not contribute to the classical value of the action.

We shall consider fluctuations near this $\text{AdS}_2 \times S^1$ solution fixing the static gauge, as in, e.g., Ref. [4]. Representing the induced 3D world-volume metric as

$$h_{mn} d\sigma^m d\sigma^n = \hat{h}_{\hat{m}\hat{n}} d\sigma^{\hat{m}} d\sigma^{\hat{n}} + h_{33} d\sigma^3 d\sigma^3,$$

$$h_{33} = \kappa^{-2}, \quad m, n = 1, 2, 3, \quad \hat{m}, \hat{n} = 1, 2, \quad (3.2)$$

where $\hat{h}_{\hat{m}\hat{n}}$ is the unit-radius AdS_2 metric and $\sigma^3 = \varphi$ is 2π -periodic, we get from Eqs. (2.7) and (2.9), for the “volume” part [cf. Eq. (C1)] of the bosonic quadratic fluctuation Lagrangian,¹⁰

$$\mathcal{L}_V^{(2)} = \frac{1}{2} (\partial^m z \partial_m z + 2z^2 + \partial^m q_a \partial_m q_a + \partial^m y'_i \partial_m y'_i + \partial^m w' \partial_m w'). \quad (3.3)$$

Here, z and q_a are fluctuations of the coordinates in Eq. (2.9). We thus get one massive and $3 + 3 + 1$ massless 3D fields propagating on $\text{AdS}_2 \times S^1$.

The contribution of the WZ term to the fluctuation part of the M2 action [Eq. (C2)] is found by choosing the potential C_3 , corresponding to F_4 in Eq. (2.7) as $C_3 = 2L^3 (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}) w'$, so that $\mathcal{L}_{\text{WZ}}^{(2)} = -2\kappa w' \partial_3 z + \dots$. Then, the total quadratic bosonic fluctuation action may be written as $T_2 v_4 \int d^3 \sigma \sqrt{\hat{h}} (\mathcal{L}_V^{(2)} + \mathcal{L}_{\text{WZ}}^{(2)})$, where

$$\mathcal{L}_B = \mathcal{L}_V^{(2)} + \mathcal{L}_{\text{WZ}}^{(2)} = \frac{1}{2} [\partial^{\hat{m}} z \partial_{\hat{m}} z + \kappa^2 (\partial_3 z)^2 + 2z^2 + \partial^{\hat{m}} w' \partial_{\hat{m}} w' + \kappa^2 (\partial_3 w')^2 - 4\kappa w' \partial_3 z$$

$$+ \partial^{\hat{m}} q_a \partial_{\hat{m}} q_a + \kappa^2 (\partial_3 q_a)^2 + \partial^{\hat{m}} y'_i \partial_{\hat{m}} y'_i + \kappa^2 (\partial_3 y'_i)^2]. \quad (3.4)$$

To diagonalize the (z, w') part of the action, we may expand the 3D fluctuation fields in Fourier modes in σ^3 , thus replacing $\partial_3 \rightarrow in$, $n = 0, \pm 1, \dots$. Then, the (z_n, w'_n) kinetic operator becomes

$$\begin{pmatrix} -\nabla^2 + 2 + \kappa^2 n^2 & i\kappa n \\ -i\kappa n & -\nabla^2 + \kappa^2 n^2 \end{pmatrix} \rightarrow \begin{pmatrix} -\nabla^2 + M_{n,+}^2 & 0 \\ 0 & -\nabla^2 + M_{n,-}^2 \end{pmatrix}, \quad (3.5)$$

$$M_{n,\pm}^2 = 1 + \kappa^2 n^2 \pm \sqrt{1 + 4\kappa^2 n^2}, \quad n \in \mathbb{Z}, \quad (3.6)$$

⁹Reduction along the w direction leads to the NS1-NS5 solution.

¹⁰We scale out the overall factor of T_2 in the quadratic action.

where ∇^2 is the AdS_2 Laplacian. As a result, the bosonic part of the one-loop M2-brane partition function is represented by the contributions of eight towers of bosonic modes in AdS_2 :

$$\hat{Z}_{1B} = \prod_{n=0,\pm 1,\dots} [\det(-\nabla^2 + M_{n,+}^2)]^{\frac{1}{2}} [\det(-\nabla^2 + M_{n,-}^2)]^{-\frac{1}{2}} [\det(-\nabla^2 + \kappa^2 n^2)]^{-3}. \quad (3.7)$$

The $n = 0$ contribution here is the same as the 10D string contribution in Eq. (1.4) with $p = 3$.

The quadratic fermionic part of the M2-brane action in Eq. (C3) may be written as¹¹

$$S_F^{(2)} = iT_2 \int d^3\sigma \sqrt{\bar{h}} \bar{\theta} \Gamma^m (1 - \Gamma_\kappa) \hat{D}_m \theta, \quad (3.8)$$

$$\Gamma_\kappa = \frac{1}{3!\sqrt{h}} \epsilon^{mnp} \Gamma_{mnp}, \quad \Gamma^m = h^{mn} \partial_n X^N \Gamma_N, \quad \hat{D}_m = \partial_m X^M \hat{D}_M. \quad (3.9)$$

The explicit form of the spinor derivatives is given in Appendix C. Using underlined indices for tangent-space components of Dirac matrices Γ^A and labeling φ in Eq. (2.7) as the ninth direction, we observe that $\Gamma_\kappa = \Gamma^{019}$, $[\Gamma_\kappa, \hat{D}_m] = 0$, so that the Lagrangian in Eq. (3.8) can be written as $\bar{\theta} \Gamma^m (1 - \Gamma_\kappa) \hat{D}_m \theta = \bar{\theta} \Gamma^m \hat{D}_m [(1 - \Gamma_\kappa) \theta]$. We fix the κ -symmetry gauge by the condition $\frac{1}{2}(1 - \Gamma_\kappa) \theta = \theta$. Then,

$$S_F^{(2)} = 2iT_2 \int d^3\sigma \sqrt{\bar{h}} \bar{\theta} \Gamma^m \hat{D}_m \theta \sim \int d^3\sigma \sqrt{\bar{h}} [\bar{\theta} \Gamma^m \nabla_m \theta - \frac{1}{2} \bar{\theta} \Gamma^{34510} (1 - i\Gamma^{012345}) \theta], \quad (3.10)$$

where $\Gamma^m \nabla_m = \Gamma^{\hat{m}} \nabla_{\hat{m}} + \Gamma^3 \partial_3$ corresponds to the Dirac operator for the $\text{AdS}_2 \times S^1$ metric in Eq. (3.2). As in the $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ case [23], expanding in Fourier modes in σ^3 the resulting fermionic Lagrangian may be written in terms of towers of complex 2D fermions ψ_n^r, χ_n^r ($r = 1, 2$) propagating on AdS_2 :

$$\mathcal{L}_F^{(2)} = \sum_{n=0,\pm 1,\dots} \left[\bar{\psi}_{-n}^r \rho^{\hat{m}} \nabla_{\hat{m}} \psi_n^r + \bar{\chi}_{-n}^r \rho^{\hat{m}} \nabla_{\hat{m}} \chi_n^r + \frac{1}{2} \mu_- \bar{\psi}_{-n}^r \rho_2 \psi_n^r + \frac{1}{2} \mu_+ \bar{\chi}_{-n}^r \rho_2 \chi_n^r - \frac{1}{2} \bar{\psi}_{-n}^r \chi_n^r - \frac{1}{2} \bar{\chi}_{-n}^r \psi_n^r \right], \quad (3.11)$$

$$\mu_\pm = \pm 1 - 2\kappa n.$$

Here, $\rho^0, \rho^1 = (\hat{\sigma}_2, \hat{\sigma}_1)$ and $\rho^2 = \hat{\sigma}_3$ are 3D Dirac matrices ($\hat{\sigma}_i$ are Pauli matrices).

Computing the fermionic determinant corresponding to Eq. (3.11), we get the following contribution to the partition function [cf. Eqs. (3.6) and (3.7)]:

$$\hat{Z}_{1F} = \prod_{n=0,\pm 1,\dots} \left[\det \left(-\nabla^2 + \frac{1}{4} R^{(2)} + \mathcal{M}_{n,+}^2 \right) \right]^2 \left[\det \left(-\nabla^2 + \frac{1}{4} R^{(2)} + \mathcal{M}_{n,-}^2 \right) \right]^2, \quad (3.12)$$

$$\mathcal{M}_{n,+} = \frac{1}{2} (\sqrt{1 + 4\kappa^2 n^2} + 1), \quad \mathcal{M}_{n,-} = \frac{1}{2} (\sqrt{1 + 4\kappa^2 n^2} - 1), \quad (3.13)$$

where $-\nabla^2 + \frac{1}{4} R^{(2)}$ is the square of the Dirac operator on AdS_2 ($R^{(2)} = -2$). The $n = 0$ contribution (with $\mathcal{M}_{0,\pm}^2 = 1$) in Eq. (3.12) is the same as in the $p = 3$ case of the string partition function in Eq. (1.4).

Combining Eqs. (3.7) and (3.12), we get for the one-loop M2-brane partition function

$$\hat{Z}_1 = \prod_{n=0,\pm 1,\dots} \frac{[\det(-\nabla^2 + \frac{1}{4} R^{(2)} + \mathcal{M}_{n,+}^2)]^2 [\det(-\nabla^2 + \frac{1}{4} R^{(2)} + \mathcal{M}_{n,-}^2)]^2}{[\det(-\nabla^2 + M_{n,+}^2)]^{\frac{1}{2}} [\det(-\nabla^2 + M_{n,-}^2)]^{\frac{1}{2}} [\det(-\nabla^2 + \kappa^2 n^2)]^3}. \quad (3.14)$$

Note that due to the residual AdS_2 supersymmetry that should be present in the quadratic fluctuation spectrum after fixing the static and κ -symmetry gauges (cf. Refs. [4,19,23,34]), the bosonic and fermionic modes for each level $n \neq 0$ should

¹¹Here, we formally continue to the Euclidean signature so that the resulting fermionic differential operator can be defined on Euclidean AdS_2 .

form long, massive $\mathcal{N} = (4, 4)$ supermultiplets of $\mathfrak{psu}(1, 1|2)$, with the bosonic and fermionic masses of the $8 + 8$ modes related by (see Appendix D)¹²

$$m_B^2: \Delta(\Delta - 1) \quad (1 \text{ mode}); \quad (\Delta + 1)\Delta \quad (6 \text{ modes}); \quad (\Delta + 2)(\Delta + 1) \quad (1 \text{ mode}); \quad (3.15)$$

$$|m_F|: \Delta(4 \text{ modes}); \quad \Delta + 1 \quad (4 \text{ modes}). \quad (3.16)$$

These relations are indeed satisfied for the spectrum in Eq. (3.14) for Δ equal to

$$\Delta_n = \frac{1}{2}(m_n - 1), \quad m_n \equiv \sqrt{1 + 4\kappa^2 n^2}. \quad (3.17)$$

The bosonic modes have 1D scaling dimensions $1 \times \Delta_n$, $6 \times (\Delta_n + 1)$, $1 \times (\Delta_n + 2)$. The condition $\Delta_n \geq \frac{1}{2}$, which corresponds to the choice of the $\Delta = \Delta_+ = \frac{1}{2}(1 + \sqrt{1 + 4m_B^2})$ branch above the BF threshold (i.e., the Dirichlet boundary conditions as appropriate for the 2D fluctuation fields) is satisfied, provided $m_n \geq 2$. This applies for all $n^2 > 0$ if $\kappa^2 \geq \frac{3}{4}$.

A check of consistency of the above spectrum with the AdS_2 supersymmetry is the vanishing of the total AdS_2 vacuum energy [4, 19]. Using the fact that $E_B = -\frac{1}{4}(m_B^2 + \frac{1}{6})$, $E_F = \frac{1}{4}(m_F^2 - \frac{1}{12})$, we get for the spectrum in Eq. (3.14) (for each value of $n = 0, \pm 1, \dots$)

$$E_{\text{tot}}^{(n)} = -\frac{1}{4} \left[M_{n,+}^2 + M_{n,-}^2 + 6\kappa^2 n^2 - 4\mathcal{M}_{n,+}^2 - 4\mathcal{M}_{n,-}^2 + 8 \times \left(\frac{1}{6} + \frac{1}{12} \right) \right] = 0. \quad (3.18)$$

IV. ONE-LOOP CORRECTION TO M2-BRANE PARTITION FUNCTION

Starting with $\hat{Z}_1$ in Eq. (3.14), we find [cf. Eqs. (1.5) and (1.18)]

$$\hat{\Gamma}_1 = -\frac{1}{2} \log \hat{Z}_1 = \hat{\Gamma}_{1\infty} + \bar{\Gamma}_1, \quad (4.1)$$

$$\hat{\Gamma}_{1\infty} = -\sum_{n=-\infty}^{\infty} B_2^{(n)} \log \Lambda, \quad B_2^{(n)} = \frac{1}{4\pi} \int d^2\sigma \sqrt{\hat{h}} b_2^{(n)}, \quad (4.2)$$

where the Seeley coefficient b_2 for a 2D operator $-\nabla^2 + X$ is given by $b_2 = \text{tr}(\frac{1}{6}R^{(2)} - X)$. Using $\text{vol}(\text{AdS}_2) = -2\pi$ and $R^{(2)} = -2$, we then get from Eq. (3.14) [cf. Eq. (3.18)]

$$b_{2,\text{tot}}^{(n)} = -\left[M_{n,+}^2 + M_{n,-}^2 + 6\kappa^2 n^2 - 4\mathcal{M}_{n,+}^2 - 4\mathcal{M}_{n,-}^2 - 8 \times \frac{1}{4} R^{(2)} \right] = -2, \quad B_{2,\text{tot}}^{(n)} = 1. \quad (4.3)$$

Thus, for each n , the contribution of the bosonic and fermionic modes to the logarithmic divergence has the same universal value as in the string ($n = 0$) case [20], and as in the M2 brane in the $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ case [4]. Summing over n , we then get, as in Ref. [4], that $B_{2,\text{tot}} = \zeta_{\text{tot}}(0) = 0$ —i.e.,¹³

$$\hat{\Gamma}_{1\infty} = -\log \Lambda \sum_{n=-\infty}^{\infty} 1 = -\log \Lambda [1 + 2\zeta_R(0)] = 0. \quad (4.4)$$

Using, as in Refs. [19, 20, 35], the AdS_2 spectral zeta-function regularization to compute finite parts of log determinants of differential operators in Eq. (3.14), we get

¹²Note that for bosons and fermions belonging to $\mathcal{N} = (1, 1)\text{AdS}_2$ supermultiplets, one has $m_B^2 = \Delta(\Delta - 1)$, $m_F = \Delta$, so that $m_B^2 = m_F^2 - m_F$.

¹³The use of Riemann zeta-function regularization is consistent with the expected absence of one-loop logarithmic divergences in a 3D theory and is part of the definition of the M2-brane path integral (cf. standard quantum mechanics path integral).

$$\hat{\Gamma}_1 = -\frac{1}{2} \sum_{n=-\infty}^{\infty} \zeta'_{\text{tot},n}(0), \quad (4.5)$$

$$\zeta'_{\text{tot},n}(0) = \zeta'_B(0; M_{n,+}^2) + \zeta'_B(0; M_{n,-}^2) + 6\zeta'_B(0; \kappa^2 n^2) + 4\zeta'_F(0; \mathcal{M}_{n,+}) + 4\zeta'_F(0; \mathcal{M}_{n,-}), \quad (4.6)$$

$$\zeta'_B(0; m_B^2) = -\frac{1}{12}(1 + \log 2) + \log A - \int_0^{m_B^2 + \frac{1}{4}} dx \psi\left(\sqrt{x} + \frac{1}{2}\right), \quad (4.7)$$

$$\zeta'_F(0; m_F) = -\frac{1}{6} + 2 \log A + |m_F| + \int_0^{m_F^2} dx \psi(\sqrt{x}), \quad (4.8)$$

where A is Glaisher's constant. The integrals of $\psi(x) = \frac{d}{dx} \log \Gamma(x)$ can be written (after integration by parts) in terms of the Barnes function $G(x)$ (see, e.g., Ref. [36]):

$$\int_0^{m_B^2 + \frac{1}{4}} dx \psi\left(\sqrt{x} + \frac{1}{2}\right) = 2\sqrt{m_B^2 + \frac{1}{4}} \log \Gamma\left(\sqrt{m_B^2 + \frac{1}{4}} + \frac{1}{2}\right) + \frac{5}{12} \log 2 + \frac{1}{2} \log \pi + 3 \log A - 2\psi^{(-2)}\left(\sqrt{m_B^2 + \frac{1}{4}} + \frac{1}{2}\right), \quad (4.9)$$

$$\int_0^{m_F} dx \psi(\sqrt{x}) = 2|m_F| \log \Gamma(|m_F|) - 2\psi^{(-2)}(|m_F|), \quad (4.10)$$

$$\psi^{(-2)}(z) = \int_0^z dx \log \Gamma(x) = \frac{1}{2}z(1-z) + \frac{1}{2}z \log(2\pi) + (z-1) \log \Gamma(z) - \log G(z). \quad (4.11)$$

Using the notation $m_n = \sqrt{1 + 4\kappa^2 n^2}$ and considering the case of $m_n > 2$, which applies for all $n^2 > 0$ if $\kappa^2 > \frac{3}{4}$, we may express all $\zeta'(0)$ values in Eq. (4.6) in terms of m_n as¹⁴

$$\begin{aligned} \zeta'_B(0; \kappa^2 n^2) &= -\frac{1}{12} + \frac{1}{2} m_n \log(2\pi) - 2 \log A - \frac{1}{4}(m_n^2 - 1) - \log \frac{m_n - 1}{2} - 3 \log \Gamma\left(\frac{m_n - 1}{2}\right) - 2 \log G\left(\frac{m_n - 1}{2}\right), \\ \zeta'_B(0; M_{n,-}^2) &= -\frac{1}{12} + \frac{1}{2} (m_n - 2) \log(2\pi) - 2 \log A - \frac{1}{4}(m_n - 3)(m_n - 1) - \log \Gamma\left(\frac{m_n - 1}{2}\right) - 2 \log G\left(\frac{m_n - 1}{2}\right), \\ \zeta'_B(0; M_{n,+}^2) &= -\frac{1}{12} + \frac{1}{2} (m_n + 2) \log(2\pi) - 2 \log A - \frac{1}{4}(m_n + 3)(m_n + 1) - \log \frac{m_n + 1}{2} \\ &\quad - 3 \log \frac{m_n - 1}{2} - 5 \log \Gamma\left(\frac{m_n - 1}{2}\right) - 2 \log G\left(\frac{m_n - 1}{2}\right), \end{aligned} \quad (4.12)$$

$$\begin{aligned} \zeta'_F(0; \mathcal{M}_{n,-}) &= -\frac{1}{6} - \frac{m_n - 1}{2} \log(2\pi) + 2 \log A + \frac{1}{4}(m_n - 1)^2 + 2 \log \Gamma\left(\frac{m_n - 1}{2}\right) + 2 \log G\left(\frac{m_n - 1}{2}\right), \\ \zeta'_F(0; \mathcal{M}_{n,+}) &= -\frac{1}{6} - \frac{m_n + 1}{2} \log(2\pi) + 2 \log A + \frac{1}{4}(m_n + 1)^2 + 2 \log \frac{m_n - 1}{2} + 4 \log \Gamma\left(\frac{m_n - 1}{2}\right) + 2 \log G\left(\frac{m_n - 1}{2}\right), \end{aligned}$$

where we have used $\Gamma(y+1) = y\Gamma(y)$, $G(y+1) = \Gamma(y) G(y)$.

Substituting these expressions into Eq. (4.6), we observe some remarkable cancellations, so that the final result for $\kappa^2 > \frac{3}{4}$ is just

$$\zeta'_{\text{tot},n=0}(0) = \log(2\pi), \quad \zeta'_{\text{tot},n \neq 0}(0) = -\log \left[\frac{1}{4}(m_n^2 - 1) \right] = -\log(n^2 \kappa^2). \quad (4.13)$$

¹⁴In general, the $\zeta'(0)$ values contain terms like $\log \Gamma\left(\frac{1+|m_n-2|}{2}\right)$ that are sensitive to the sign of $m_n - 2$. The $n = 0$ case corresponding to $m_0 = 1$ in Eq. (3.17) can also be treated separately [cf. Eq. (1.6)].

Thus, Eq. (4.5) takes a very simple form [cf. Eq. (1.21)],

$$\hat{\Gamma}_1 = -\frac{1}{2} \log(2\pi) + \sum_{n=1}^{\infty} \log(n^2 \kappa^2) = -\log \frac{\kappa}{\sqrt{2\pi}}, \quad (4.14)$$

where we have again used the Riemann zeta-function regularization [cf. Eq. (1.21)]. The one-loop partition function is then given by the same expression as in the string-theory case [cf. Eqs. (1.8) and (1.37)],

$$\hat{Z}_1 = e^{-\hat{\Gamma}_1} = \frac{\kappa}{\sqrt{2\pi}}. \quad (4.15)$$

The reason for the cancellations leading to Eq. (4.13) can be traced back to the (4, 4) supersymmetry of the mass spectrum in Eqs. (3.14)–(3.17). Indeed, considering the contribution of a single (4, 4) supermultiplet in Eqs. (3.15) and (3.16), we find for its one-loop contribution [cf. Eq. (4.6)]¹⁵

$$\begin{aligned} \zeta'_{\text{tot}, \Delta}(0) &= \zeta'_B(0; \Delta(\Delta - 1)) + 6\zeta'_B(0; (\Delta + 1)\Delta) + \zeta'_B(0; (\Delta + 2)(\Delta + 1)) + 4\zeta'_F(0; \Delta) + 4\zeta'_F(0; \Delta + 1) \\ &= 2 + 8\Delta - 4 \log 2\pi + (6\Delta + 1) \log \Gamma(\Delta) + (2 - 4\Delta) \log \Gamma(\Delta + 1) - (2\Delta + 3) \log \Gamma(\Delta + 2) \\ &\quad - 6\psi^{(-2)}(\Delta) + 4\psi^{(-2)}(\Delta + 1) + 2\psi^{(-2)}(\Delta + 2) = -\log(\Delta(\Delta + 1)), \end{aligned} \quad (4.16)$$

where we have assumed that $\Delta > \frac{1}{2}$. This result is equivalent to Eq. (4.13) if we set Δ equal to Δ_n in Eq. (3.17).¹⁶

Thus, the simplicity of the result in Eqs. (4.13) and (3.1) is due to the extended $\mathfrak{psu}(1, 1|2)\text{AdS}_2$ supersymmetry of the world-volume action. Similar observation applies (though in a more nontrivial way) to the case of an $\text{AdS}_2 \times S^1$ M2 brane in $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ considered in Ref. [4]. As we discuss in Appendix D, the nontrivial cancellations in the one-loop effective action in Eqs. (1.16) and (1.17) leading to Eq. (1.19) can be understood as a consequence of underlying $\mathfrak{su}(1, 1|3)$ supersymmetry.

When $\kappa^2 < \frac{3}{4}$, we find that $\hat{\Gamma}_1$ in Eq. (4.5) contains a finite number of terms with $|n| = 1, \dots, n_*$ [where $n_* = n_*(\kappa)$] such that $m_n < 2$. Then, instead of Eq. (4.13), we get¹⁷

$$\zeta'_{\text{tot}, n \leq n_*}(0) \equiv \zeta_n = (2 - m_n) \log(2\pi) - 2 \log \frac{G(\frac{1+m_n}{2})}{G(\frac{3-m_n}{2})} - \log \left[\Gamma\left(\frac{3+m_n}{2}\right) \Gamma\left(\frac{3-m_n}{2}\right) \right]. \quad (4.17)$$

Then [cf. Eq. (4.14)],

$$\hat{\Gamma}_1 = -\frac{1}{2} \log(2\pi) - \sum_{n=1}^{n_*} \left[\frac{1}{2} \zeta_n + \log(n^2 \kappa^2) \right] + \sum_{n=1}^{\infty} \log(n^2 \kappa^2) = -\log \frac{\kappa}{\sqrt{2\pi}} - \sum_{n=1}^{n_*} \left[\frac{1}{2} \zeta_n + \log(n^2 \kappa^2) \right]. \quad (4.18)$$

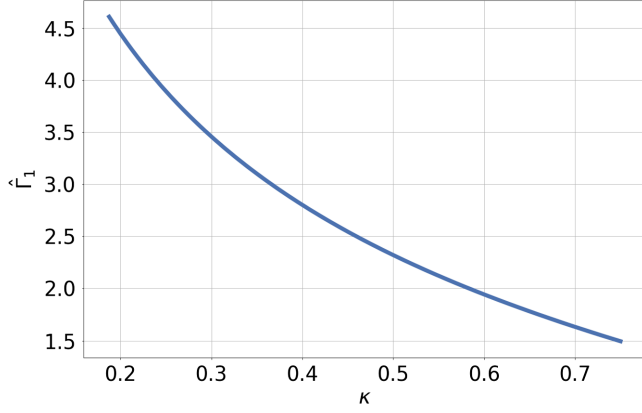
The plot of $\hat{\Gamma}_1$ in Eq. (4.18), as a function of κ in the special case of $\kappa \in (\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2})$ when $n_* = 1$, is shown in Fig. 1.

We conclude that, while for large enough values of the parameter κ in Eqs. (2.8) or (1.28) (corresponding, e.g., to a small value of $v_4 = g_s^{2/3}$ —i.e., to the perturbative type-IIA string regime) the one-loop M2-brane partition function is given by Eq. (4.15)—i.e., it does not contain subleading κ^{-1} corrections—for smaller values of $\kappa < \frac{\sqrt{3}}{2}$ (corresponding to a genuine M-theory regime), it is given by Eq. (4.18). Similar results are found in the mixed flux case, as discussed in Appendix E.

¹⁵Here, we use again Eqs. (4.7)–(4.11), with $\psi^{(-2)}(z+1) = \psi^{(-2)}(z) - z + \frac{1}{2} \log 2\pi + z \log z$ and $\Gamma(z+1) = z\Gamma(z)$.

¹⁶Note that in the case of an $\mathcal{N} = (1, 1)$ multiplet of one boson and one fermion in AdS_2 (cf. footnote 12), one finds $\zeta'_B(0; \Delta(\Delta - 1)) = -\frac{1}{12} - \frac{1}{2} \log 2\pi - 2 \log A - (2\Delta - 1) \log \Gamma(\Delta) + 2\psi^{(-2)}(\Delta) - 2\Delta - \log \Delta$, $\zeta'_F(0; \Delta) = -\frac{1}{6} + 2 \log A + \Delta + 2\Delta \log \Gamma(\Delta) - 2\psi^{(-2)}(\Delta)$, so that the analog of Eq. (4.16), $\zeta'_B(0; \Delta(\Delta - 1)) + \zeta'_F(0; \Delta) = -\frac{1}{4} + \Delta + \frac{1}{2} \log 2\pi - \log \Gamma(\Delta) - \log \Delta$, still contains a $\log \Gamma(\Delta)$ term.

¹⁷This case is somewhat analogous to the special cases of $k = 1, 2$ values in the AdS_4 case [4].

FIG. 1. $\hat{\Gamma}_1(\kappa)$ in the interval $\frac{\sqrt{3}}{4} < \kappa < \frac{\sqrt{3}}{2}$.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: 11D SUPERGRAVITY ACTION AND REDUCTION TO 10D

The bosonic part of the action of 11D supergravity is given by

$$S_{11} = \frac{1}{2\kappa_{11}^2} \left[\int d^{11}x \sqrt{-G} \left(R - \frac{1}{2} |F_4|^2 \right) - \frac{1}{6} \int C_3 \wedge F_4 \wedge F_4 \right], \quad (\text{A1})$$

$$F_4 = dC_3, \quad |F_4|^2 = \frac{1}{4!} F_{MNPQ} F^{MNPQ}, \quad 2\kappa_{11}^2 = (2\pi)^8 \ell_P^9. \quad (\text{A2})$$

The 11D metric of $M^{10} \times S^1$ may be written as

$$ds_{11}^2 = e^{-\frac{2}{3}\tilde{\phi}} ds_{10}^2 + e^{\frac{4}{3}\tilde{\phi}} (dx^{11} + C_\mu dx^\mu)^2, \quad x^{11} \equiv x^{11} + 2\pi R_{11}, \quad (\text{A3})$$

where ds_{10}^2 is a type-IIA string-frame metric, and $\tilde{\phi} = \phi - \phi_0$, where $\phi_0 = \log g_s$ is the asymptotic (large-distance) value of the dilaton, so that in the flat-space limit, both ds_{11}^2 and ds_{10}^2 have the standard Minkowski form.

Then, the relations between R_{11} and the asymptotic string coupling g_s , and also the string and membrane tensions are¹⁸

$$R_{11} = g_s \ell_s = g_s^{2/3} \ell_P, \quad \ell_P = g_s^{1/3} \ell_s, \quad \frac{2\pi R_{11}}{2\kappa_{11}^2} = \frac{1}{2\kappa_0^2 g_s^2}, \quad 2\kappa_0^2 \equiv (2\pi)^7 \ell_s^8, \quad (\text{A4})$$

$$T = \frac{1}{2\pi \ell_s^2}, \quad T_2 = \frac{1}{(2\pi)^2 \ell_P^3}, \quad T = 2\pi R_{11} T_2. \quad (\text{A5})$$

The resulting 10D type-IIA supergravity action may be written as

$$S_{10A} = \frac{1}{2\kappa_0^2 g_s^2} \int d^{10}x \sqrt{-G} \left[e^{-2\tilde{\phi}} \left(R + 4(\partial\tilde{\phi})^2 - \frac{1}{2} |H_3|^2 \right) - \frac{1}{2} (|F_2| + |F_4|^2) + \dots \right] \\ = \frac{1}{2\kappa_0^2} \int d^{10}x \sqrt{-G} \left[e^{-2\phi} \left(R + 4(\partial\tilde{\phi})^2 - \frac{1}{2} |H_3|^2 \right) - \frac{1}{2} (|F_2| + |F_4|^2) + \dots \right], \quad (\text{A6})$$

where $e^\phi = g_s e^{\tilde{\phi}}$ and $F_p = g_s^{-1} F_p$.

In Sec. II, we consider the M2-M5 11D solution [Eq. (2.1)] with five isometric toroidal directions. We use the following notation for the coordinates of T^5 with a flat asymptotic metric¹⁹:

$$ds_{T^5}^2 = dy_p dy_p + dw^2, \quad (\text{A7})$$

$$y_p = \ell_P y_p \equiv y_p + 2\pi \ell_P v_p, \quad w \equiv w + 2\pi \ell_P u, \quad p = (i, 4), \quad i = 1, 2, 3, \quad (\text{A8})$$

$$y_p = \ell_P y_p, \quad w = \ell_P w, \quad y_p \equiv y_p + 2\pi v_p, \quad w \equiv w + 2\pi u, \quad \hat{v} \equiv v_1 v_2 v_3 v_4. \quad (\text{A9})$$

In reducing to 10D, we shall identify y_4 direction with x^{11} in Eq. (A3), so that according to Eq. (A5),

$$R_{11} = \ell_P v_4, \quad v_4 = g_s^{2/3}. \quad (\text{A10})$$

¹⁸Alternatively, one may replace $\tilde{\phi}$ in Eq. (A3) with the full ϕ , and then do constant rescaling of the 11D coordinates to put the 11D metric into the canonical flat-space form. In general, the vacuum expectation value of the metric may be chosen as $c_0 \eta_{MN}$, where c_0 will be related to ϕ_0 if the 10D metric has canonical form. In that case, $R_{11} = \ell_P = \ell_s$ (see, e.g., Appendix A in Ref. [37]).

¹⁹For notational simplicity, we will not distinguish between y_p and y^p , etc.

We shall also define the following dimensionless parameters:

$$v_i \equiv \ell_s^{-1} \ell_P v_i = g_s^{1/3} v_i, \quad v_4 \equiv \ell_s^{-1} \ell_P v_4 = g_s, \quad u \equiv \ell_s^{-1} \ell_P u = g_s^{1/3} u, \quad (\text{A11})$$

$$\hat{v} = v_1 v_2 v_3 v_4 = g_s^{-1/3} v, \quad v \equiv v_1 v_2 v_3. \quad (\text{A12})$$

APPENDIX B: D2-D4 AND D1-D5 SOLUTIONS

The dimensional reduction of the M2-M5 solution [Eqs. (2.1) and (2.2)] on the y_4 direction gives the D2-D4 solution of type-IIA supergravity action [Eq. (A6)] that can be written as ($i = 1, 2, 3$; $\ell_s = \sqrt{\alpha'}$) [22]

$$ds_{10A}^2 = H_2^{-1/2} H_4^{-1/2} (dx_0^2 + dx_1^2) + H_2^{1/2} H_4^{1/2} (dr^2 + r^2 ds_{S^3}^2) + H_2^{1/2} H_4^{-1/2} dy_i^2 + H_2^{-1/2} H_4^{1/2} dw^2, \quad (\text{B1})$$

$$F_4 = g_s^{-1} (idH_2^{-1} \wedge dx_0 \wedge dx_1 - *_4 dH_4) \wedge dw, \quad e^{2\phi} = g_s^2 H_2^{1/2} H_4^{-1/2}, \quad (\text{B2})$$

$$H_2 = 1 + \frac{Q_2}{r^2}, \quad H_4 = 1 + \frac{Q_4}{r^2}, \quad Q_2 = \frac{g_s \ell_s^2 Q_2}{v}, \quad Q_4 = \frac{g_s \ell_s^2 Q_4}{u}, \quad v = v_1 v_2 v_3. \quad (\text{B3})$$

Here, Q_2 and $Q_4 \equiv Q_5$ are the same as in Eq. (2.3), and we follow the notation introduced in Eq. (1.35) and Eqs. (A11), (A12)—i.e.,

$$y_i \equiv y_i + 2\pi \ell_s v_i, \quad w \equiv w + 2\pi \ell_s u, \quad v_i = g_s^{1/3} v_i, \quad u = g_s^{1/3} u. \quad (\text{B4})$$

Taking the near-horizon limit and rescaling x_0 and x_1 , we get the $\text{AdS}_3 \times S^3 \times T^3 \times S^1$ type-IIA solution²⁰:

$$ds_{10A}^2 = \sqrt{Q_2 Q_4} (ds_{\text{AdS}_3}^2 + ds_{S^3}^2) + \sqrt{\frac{Q_2}{Q_4}} dy_i^2 + \sqrt{\frac{Q_4}{Q_2}} dw^2, \quad e^{2\phi} = g_s^2 = g_s^2 \sqrt{\frac{Q_2}{Q_4}}, \quad (\text{B5})$$

$$F_4 = 2g_s^{-1} Q_4 (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}) \wedge dw. \quad (\text{B6})$$

The corresponding string coupling g_s and dimensionless tension T are thus as in Eqs. (1.33) and (1.36)²¹:

$$g_s^2 = g_s^2 \sqrt{\frac{u}{v}} \sqrt{\frac{Q_2}{Q_4}}, \quad T = \frac{L_s^2}{2\pi \ell_s^2} = \frac{\sqrt{Q_2 Q_4}}{2\pi \ell_s^2} = \frac{g_s}{2\pi \sqrt{uv}} \sqrt{Q_2 Q_4}. \quad (\text{B7})$$

Similarly, the D1-D5 solution of type-IIB supergravity may be represented as

$$ds_{10B}^2 = H_1^{-1/2} H_5^{-1/2} (dx_0^2 + dx_1^2) + H_1^{1/2} H_5^{1/2} (dr^2 + r^2 ds_{S^3}^2) + H_1^{1/2} H_5^{-1/2} dy_i^2 + H_1^{-1/2} H_5^{1/2} dw'^2, \quad (\text{B8})$$

$$F_3 = g_s'^{-1} (idH_1^{-1} \wedge dx_0 \wedge dx_1 - *_4 dH_5), \quad e^{2\phi'} = g_s'^2 H_1 H_5^{-1}, \quad (\text{B9})$$

$$H_1 = 1 + \frac{Q_1}{r^2}, \quad H_5 = 1 + \frac{Q_5}{r^2}, \quad Q_1 = \frac{g_s' \ell_s^2 Q_1}{\hat{v}'}, \quad Q_5 = g_s' \ell_s^2 Q_5, \quad (\text{B10})$$

where g_s' is the type-IIB asymptotic string coupling, and

$$y_i \equiv y_i + 2\pi \ell_s v_i, \quad w' \equiv w' + 2\pi \ell_s u', \quad v = v_1 v_2 v_3, \quad \hat{v}' = v u'. \quad (\text{B11})$$

²⁰Note that the volume of the 4-torus (y_i, w) in the near-horizon limit, given by $\text{vol}(T^4) = (2\pi)^4 \ell_s^4 \frac{Q_2}{Q_4} v u = (2\pi)^4 \ell_s^4 \frac{Q_2}{Q_4} u^2$, does not depend on the original 3-torus volume parameter v .

²¹Note that if one formally chooses $Q_2 = Q_4$ —i.e., $v^{-1} Q_2 = u^{-1} Q_4$, so that $g_s = g_s$ —then $T = g_s u^{-1} Q_4$.

The D2-D4 [Eq. (B1)] and D1-D5 [Eq. (B8)] solutions are related by T-duality along the $w \rightarrow w'$ direction, with the radii measured in units of ℓ_s being related by $u' = u^{-1}$, and the dilatons (and thus also the asymptotic string couplings) transforming as $ue^{-2\phi} = u'e^{-2\phi'}$ (and the RR potentials related according to the standard rule; see, e.g., Ref. [38])—i.e.,

$$u' = u^{-1}, \quad \hat{v}' = vu^{-1}, \quad g'_s = u^{-1}g_s. \quad (\text{B12})$$

The harmonic functions are related by $H_2 = H_1$, $H_4 = H_5$.

The near-horizon limit of Eqs. (B8) and (B9) gives the $\text{AdS}_3 \times S^3 \times T^3 \times S^1$ IIB solution supported by the RR flux²²:

$$ds_{10B}^2 = \sqrt{Q_1 Q_5} \left(ds_{\text{AdS}_3}^2 + ds_{S^3}^2 \right) + \sqrt{\frac{Q_1}{Q_5}} (dy_i^2 + dw'^2), \quad e^{2\phi'} = g_s'^2 \frac{Q_1}{Q_5}, \quad (\text{B13})$$

$$F_3 = 2g_s'^{-1} Q_5 \left(i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3} \right). \quad (\text{B14})$$

The corresponding string coupling g'_s and dimensionless tension T' here are as in Eqs. (1.38) and (1.39)²³:

$$g'_s = \frac{g'_s}{\sqrt{\hat{v}'}} \sqrt{\frac{Q_1}{Q_5}}, \quad T' = \frac{\sqrt{Q_1 Q_5}}{2\pi \ell_s^2} = \frac{g'_s}{2\pi \sqrt{\hat{v}'}} \sqrt{Q_1 Q_5}. \quad (\text{B15})$$

APPENDIX C: SUPERSYMMETRY OF $\text{AdS}_2 \times S^1$ M2-BRANE SOLUTION

The bosonic and quadratic fermionic parts of the M2-brane action in the 11D supergravity background may be written as (using Euclidean signature) [27]

$$S_B = S_V + S_{WZ}, \quad S_V = T_2 \int d^3 \sigma \sqrt{h}, \quad h_{mn} = \partial_m X^M \partial_n X^N G_{MN}(X), \quad (\text{C1})$$

$$S_{WZ} = -\frac{i}{3!} T_2 \int d^3 \sigma \epsilon^{mnp} C_{MNP}(X) \partial_m X^M \partial_n X^N \partial_p X^P, \quad (\text{C2})$$

$$S_F = iT_2 \int d^3 \sigma \left[\sqrt{h} h^{mn} \partial_m X^M \bar{\theta} \Gamma_M \hat{D}_n \theta - \frac{i}{2} \epsilon^{mnk} \partial_m X^M \partial_n X^N \bar{\theta} \Gamma_{MN} \hat{D}_k \theta \right], \quad \Gamma_M = E_M^A \Gamma_A, \\ \hat{D}_m = \partial_m X^M \hat{D}_M, \quad \hat{D}_M = D_M - \frac{1}{288} (\Gamma^{PQRS}{}_M + 8\Gamma^{PQR} \delta_M^S) F_{PQRS}, \quad D_M = \partial_M + \frac{1}{4} \Gamma_{AB} \omega_M^{AB}. \quad (\text{C3})$$

To preserve supersymmetry, an M2-brane solution $X^M(\sigma)$ should satisfy

$$\Gamma_x \epsilon = \epsilon, \quad \Gamma_x = \frac{i}{3! \sqrt{h}} \epsilon^{mnp} \partial_m X^M \partial_n X^N \partial_p X^P \Gamma_{MNP}, \quad (\text{C4})$$

where ϵ is a Killing spinor of the background,

$$\delta \psi_M = \hat{D}_M \epsilon \equiv (\partial_M - \Omega_M) \epsilon = 0. \quad (\text{C5})$$

To show that the $\text{AdS}_2 \times S^1$ M2-brane solution preserves half of supersymmetry, let us first find explicitly the solution of Eq. (C5) for the Killing spinor in the case of an $\text{AdS}_3 \times S^3 \times T^5$ background in Eq. (2.5), with AdS_3 and S^3 parametrized as [cf. Eq. (2.9)]

$$ds_{\text{AdS}_3}^2 = \cosh^2 \zeta (\cosh^2 \xi d\eta^2 + d\xi^2), \quad ds_{S^3}^2 = d\rho^2 + \sin^2 \rho d\alpha^2 + \cos^2 \rho d\beta^2. \quad (\text{C6})$$

²²Like in the above D2-D4 case, the volume of the 4-torus (y_i, w) in the near-horizon limit takes the form $\text{vol}(T^4) = (2\pi)^4 \ell_s^4 \frac{Q_1}{Q_5} \hat{v}' = (2\pi)^4 \ell_s^4 \frac{Q_1}{Q_5}$ and thus is fixed—i.e., it does not depend on the original 4-torus volume parameter $\hat{v}'$ [1,2].

²³If one formally sets $Q_1 = Q_5$ —i.e., $\hat{v}'^{-1} Q_5 = Q_1$ and $g_6 \equiv \frac{1}{\sqrt{\hat{v}'}} g'_s$ (as, e.g., in Ref. [14])—then $g'_s = g_s$ and $T' = g'_s Q_5 = g_6 \sqrt{Q_1 Q_5}$.

We shall label the 11D coordinates as $X^M = (X^0, X^1, \dots, X^{10}) = (\eta, \xi, \zeta, \rho, \alpha, \beta, y^1, y^2, y^3, y^4, w)$. The underlined indices will stand for tangent-space components. Then, Eq. (C5) may be written as

$$\delta\psi_\eta = \partial_\eta \epsilon + \frac{1}{2} \sinh \xi \Gamma^{01} \epsilon + \frac{1}{2} \cosh \xi \sinh \zeta \Gamma^{02} \epsilon - \frac{i}{6} \cosh \zeta \cosh \xi \Gamma^{1210} (\Gamma_* - 2) \epsilon = 0, \quad (\text{C7})$$

$$\delta\psi_\xi = \partial_\xi \epsilon + \frac{1}{2} \sinh \zeta \Gamma^{12} \epsilon - \frac{i}{6} \cosh \zeta \Gamma^{0210} (\Gamma_* - 2) \epsilon = 0, \quad \Gamma_* \equiv i\Gamma_{012345}, \quad (\text{C8})$$

$$\delta\psi_\zeta = \partial_\zeta \epsilon - \frac{i}{6} \Gamma^{0110} (\Gamma_* - 2) \epsilon = 0, \quad \delta\psi_\rho = \partial_\rho \epsilon - \frac{1}{6} \Gamma^{4510} (\Gamma_* - 2) \epsilon = 0, \quad (\text{C9})$$

$$\delta\psi_\alpha = \partial_\alpha \epsilon - \frac{1}{2} \cos \rho \Gamma^{34} \epsilon + \frac{1}{6} \sin \rho \Gamma^{3510} (\Gamma_* - 2) \epsilon = 0, \quad (\text{C10})$$

$$\delta\psi_\beta = \partial_\beta \epsilon + \frac{1}{2} \sin \rho \Gamma^{35} \epsilon - \frac{1}{6} \cos \rho \Gamma^{3410} (\Gamma_* - 2) \epsilon = 0, \quad (\text{C11})$$

$$\delta\psi_{y_p} = \partial_{y_p} \epsilon - \frac{R}{6L} \Gamma_{y_p} \Gamma^{34510} (\Gamma_* + 1) \epsilon = 0, \quad \delta\psi_w = \partial_w \epsilon - \frac{1}{6LR^2} \Gamma^{345} (\Gamma_* + 1) \epsilon = 0. \quad (\text{C12})$$

We shall consider the Killing spinor to be constant in the T^5 directions (y_p, w) by choosing a projection

$$\Gamma_* \epsilon = -\epsilon, \quad (\text{C13})$$

so that the equations in (C12) are satisfied. It remains to solve the six equations in (C7)–(C11). As the two components of Ω_M in Eq. (C5), given by $\Omega_\zeta = -\frac{i}{2} \Gamma^{0110}$ and $\Omega_\rho = -\frac{1}{2} \Gamma^{4510}$, are constant, we get

$$\epsilon = \exp[\Omega_\zeta \zeta] \exp[\Omega_\rho \rho] \chi, \quad (\text{C14})$$

where χ depends on the other four coordinates. The remaining equations written in terms of χ are

$$\partial_\eta \chi = \hat{\Omega}_\eta \chi, \quad \partial_\xi \chi = \hat{\Omega}_\xi \chi, \quad \partial_\alpha \chi = \hat{\Omega}_\alpha \chi, \quad \partial_\beta \chi = \hat{\Omega}_\beta \chi, \quad (\text{C15})$$

$$\hat{\Omega}_\xi = e^{-\Omega_\zeta \zeta} \Omega_\xi(\zeta) e^{\Omega_\zeta \zeta} = \frac{i}{2} \Gamma^{0210}, \quad \hat{\Omega}_\eta = e^{-\hat{\Omega}_\xi \xi} e^{-\Omega_\zeta \zeta} \Omega_\eta(\zeta) e^{\Omega_\zeta \zeta} e^{\hat{\Omega}_\xi \xi} = -\frac{i}{2} \Gamma^{1210}, \quad (\text{C16})$$

$$\hat{\Omega}_\beta = e^{-\Omega_\rho \rho} \Omega_\beta(\rho) e^{\Omega_\rho \rho} = -\frac{1}{2} \Gamma^{3410}, \quad \hat{\Omega}_\alpha = e^{-\hat{\Omega}_\beta \beta} e^{-\Omega_\rho \rho} \Omega_\alpha(\rho) e^{\Omega_\rho \rho} e^{\hat{\Omega}_\beta \beta} = \frac{1}{2} \Gamma^{34}, \quad (\text{C17})$$

and thus are solved by $\chi = \exp[\hat{\Omega}_\xi \xi] \exp[\hat{\Omega}_\eta \eta] \exp[\hat{\Omega}_\beta \beta] \exp[\hat{\Omega}_\alpha \alpha] \epsilon_0$. The final expression for the Killing spinor is then

$$\epsilon = \exp[\Omega_\zeta \zeta] \exp[\hat{\Omega}_\xi \xi] \exp[\hat{\Omega}_\eta \eta] \exp[\Omega_\rho \rho] \exp[\hat{\Omega}_\beta \beta] \exp[\hat{\Omega}_\alpha \alpha] \epsilon_0, \quad \Gamma_* \epsilon_0 = -\epsilon_0, \quad (\text{C18})$$

where we have imposed the projection on ϵ_0 to satisfy the condition (C13). This demonstrates that the $\text{AdS}_3 \times S^3 \times T^5$ background has 16 superisometries parametrized by ϵ_0 —i.e., it preserves 16 supercharges.

The $\text{AdS}_2 \times S^1$ M2-brane solution corresponds to η and ξ in Eq. (C6), identified with σ^1 and σ^2 in world-volume coordinates; φ in Eq. (2.7), with σ^3 ; and

$$\zeta = 0, \quad \rho = 0, \quad \beta = 0, \quad \alpha = \alpha_0. \quad (\text{C19})$$

The condition (C4) that it must preserve supersymmetry may then be written as

$$\Gamma_x \epsilon = i\Gamma_{019} \epsilon = \epsilon. \quad (\text{C20})$$

Using Eq. (C18), one can check that Eq. (C20) is satisfied, provided ϵ_0 is subject to the same projection [Eq. (C20)], and also the one in Eq. (C18)—i.e.,

$$\Gamma_x \epsilon_0 = \epsilon_0, \quad \Gamma_* \epsilon_0 = -\epsilon_0, \quad [\Gamma_x, \Gamma_*] = 0. \quad (\text{C21})$$

As a result, ϵ_0 has eight independent components—i.e., the $\text{AdS}_2 \times S^1$ M2-brane solution preserves half of the supersymmetry of the $\text{AdS}_3 \times S^3 \times T^5$ background.

APPENDIX D: AdS_2 SUPERSYMMETRY AND MASS SPECTRUM

The $\text{AdS}_3 \times S^3 \times T^5$ solution of 11D supergravity preserves 16 real supercharges, corresponding to $\mathcal{N} = (4, 4)$ supersymmetry from the dual 2D CFT point of view (see, e.g., Ref. [39]). The bosonic isometry group has the algebra $\mathfrak{so}(2, 2) \oplus \mathfrak{so}(4) \oplus \mathfrak{u}(1)^5$. Ignoring the last torus symmetry factor, this background has global superisometry algebra²⁴ $\mathfrak{psu}(1, 1|2)_L \times \mathfrak{psu}(1, 1|2)_R$.

The presence of an $\text{AdS}_2 \times S^1$ M2-brane defect breaks it to $\mathfrak{psu}(1, 1|2) \times \mathfrak{su}(2)_A$ (see, e.g., Ref. [21]). The bosonic part of $\mathfrak{psu}(1, 1|2)$ is $\mathfrak{sl}(2, R) \times \mathfrak{su}(2)_R$, where $\mathfrak{sl}(2, R)$ is the global 1D conformal algebra (with generators D, P, K) corresponding to AdS_2 , and $\mathfrak{su}(2)_R$ is the R-symmetry (with generators R_b^a , where $a, b = 1, 2$). $\mathfrak{su}(2)_A$ acts as an automorphism with generators $A_b^{\dot{a}}$. Omitting the commutators involving R_b^a and $A_b^{\dot{a}}$, we have

$$[D, P] = P, \quad [D, K] = -K, \quad [P, K] = -2D, \\ [D, Q_{\dot{a}\dot{a}}] = \frac{1}{2} Q_{\dot{a}\dot{a}}, \quad [D, S_{\dot{a}\dot{a}}] = -\frac{1}{2} S_{\dot{a}\dot{a}}, \quad (\text{D1})$$

$$[P, S_{\dot{a}\dot{a}}] = -Q_{\dot{a}\dot{a}}, \quad [K, Q_{\dot{a}\dot{a}}] = S_{\dot{a}\dot{a}}, \\ \{Q_{\dot{a}\dot{a}}, Q_{\dot{b}\dot{b}}\} = -\epsilon_{\dot{a}\dot{b}} \epsilon_{\dot{a}\dot{b}} P, \quad (\text{D2})$$

$$\{Q_{\dot{a}\dot{a}}, S_{\dot{b}\dot{b}}\} = -\epsilon_{\dot{a}\dot{b}} \epsilon_{\dot{a}\dot{b}} D - \epsilon_{\dot{a}\dot{b}} R_{\dot{a}\dot{b}}, \\ \{S_{\dot{a}\dot{a}}, S_{\dot{b}\dot{b}}\} = -\epsilon_{\dot{a}\dot{b}} \epsilon_{\dot{a}\dot{b}} K. \quad (\text{D3})$$

The four 2D spinor generators $Q_{\dot{a}\dot{a}}$ transform as $(2, 2)$ under $SU(2)_R \times SU(2)_A$. The states in supermultiplets can be labeled by their 1D scaling dimension Δ —i.e., $D|\Delta\rangle = \Delta|\Delta\rangle$ —as well as quantum numbers corresponding to R_b^a and $A_b^{\dot{a}}$ that we shall omit (see, e.g., Ref. [39]).

The long $\mathcal{N} = (4, 4)$ supermultiplet of $\mathfrak{psu}(1, 1|2)$ is obtained by acting on a given state $|\Delta\rangle$ with $Q_{\dot{a}\dot{a}}$:

²⁴This is the global part of the “small” $\mathcal{N} = (4, 4)$ superconformal symmetry in two dimensions.

$$1 \times |\Delta\rangle, \quad 4 \times |\Delta + \frac{1}{2}\rangle, \quad 6 \times |\Delta + 1\rangle, \\ 4 \times |\Delta + \frac{3}{2}\rangle, \quad 1 \times |\Delta + 2\rangle. \quad (\text{D4})$$

The corresponding masses of scalars and fermions propagating in AdS_2 are related to the values of Δ as $m_B^2 = \Delta(\Delta - 1)$, $|m_F| = \Delta - \frac{1}{2}$ —i.e.,

$$\text{scalar: } \Delta = \frac{1}{2} \left(1 + \sqrt{1 + 4m_B^2} \right), \\ \text{spinor: } \Delta = \frac{1}{2} (1 + 2|m_F|). \quad (\text{D5})$$

The resulting spectrum of $8 + 8$ fields in AdS_2 is then

$$\text{scalars } m_B^2: \Delta(\Delta - 1) \text{ (1 state)}, \quad (\Delta + 1)\Delta \text{ (6 states)}, \\ (\Delta + 1)(\Delta + 2) \text{ (1 state)}; \quad (\text{D6})$$

$$\text{spinors } |m_F|: \Delta \text{ (4 states)}, \quad \Delta + 1 \text{ (4 states)}. \quad (\text{D7})$$

This agrees with the quadratic fluctuation spectrum of the M2 brane represented in terms of the S^1 Fourier modes propagating on AdS_2 if [see Eq. (3.14)]

$$\Delta_n = \frac{1}{2} (m_n - 1), \quad m_n = \sqrt{1 + 4\kappa^2 n^2}, \quad n \neq 0. \quad (\text{D8})$$

Note that the $n = 0$ fluctuations fit into short multiplets corresponding to the displacement and tilt multiplets from the boundary-theory point of view.

Let us now comment on the supermultiplet structure of quadratic fluctuations in the case of an $\text{AdS}_2 \times S^1$ M2 brane in $\text{AdS}_4 \times S^7/\mathbb{Z}_k$, considered in Refs. [4, 23]. Here, for $k > 2$, the relevant supersymmetry algebra is expected to be $\mathfrak{su}(1, 1|3)$ [with the bosonic part $\mathfrak{su}(1, 1) \oplus \mathfrak{su}(3) \oplus \mathfrak{u}(1)$], which is the subalgebra of the $\mathfrak{osp}(6|4)$ symmetry of $\text{AdS}_4 \times \mathbb{CP}^3$ that is preserved by a 1D defect (see, e.g., Refs. [40, 41]). There is also an extra $\mathfrak{u}(1)_b$ symmetry associated with S^1 of the $\text{AdS}_2 \times S^1$ M2 brane in $\text{AdS}_4 \times S^7/\mathbb{Z}_k$, corresponding to the KK charge²⁵ n .

The spectrum of towers of quadratic fluctuations in AdS_2 that leads to the one-loop partition function in Eqs. (1.16) and (1.17) here is given by the following ($|kn| > 2$, $n \in \mathbb{Z}$): (i) Complex scalars η_n (two real for each n), with $m^2 = \frac{1}{4}(kn - 2)(kn - 4)$; three sets of complex scalars ζ_n^a ($a = 1, 2, 3$) with $m^2 = \frac{1}{4}kn(kn + 2)$. (ii) 8 two-component spinor fields ϑ_n^A , with $m = \frac{1}{2}kn \pm 1$ ($3 + 3$ modes) and $m = \frac{1}{2}kn$

²⁵For $k = 1, 2$, the symmetry of the background enhances from $\mathfrak{osp}(6|4)$ to $\mathfrak{osp}(8|4)$, and the maximal 1D $\frac{1}{2}$ -BPS subalgebra preserved by the defect is $\mathfrak{su}(1, 1|4) \oplus \mathfrak{u}(1)_b$.

(2 modes). These can be organized into $\mathcal{N} = 1$ multiplets with $m_B^2 = m_F(m_F - 1)$ [4].

The $n = 0$ modes correspond to the short multiplet of $\mathfrak{su}(1, 1|3)$, while the $n \neq 0$ modes are *a priori* expected to belong to long multiplets. The long multiplet of $\mathfrak{su}(1, 1|3)$ contains $32 + 32$ states obtained by acting with all six supercharges (Q_a and $\bar{Q}^a$, $a = 1, 2, 3$).²⁶ To exhibit the extended supersymmetry structure of the massive fluctuation mode spectrum, we may organize them for each n into “chiral” sectors (ζ_n^a, η_n) and $(\bar{\zeta}_n^a, \bar{\eta}_n)$, combined with the corresponding fermions. These will form conjugate “shortened” supermultiplets corresponding to the actions of Q_a and $\bar{Q}^a$, respectively—i.e., obtained by imposing $\bar{Q}^a|\Delta\rangle = 0$ or $Q_a|\Delta\rangle = 0$. The shortening condition [see Eqs. (C.26) and (C.30) in Ref. [40]] is $\Delta = \mp \frac{1}{3}j_0, j_1 = j_2 = 0$ [where j_0, j_1, j_2 are Dynkin labels of Cartan generators of $\mathfrak{u}(1)$ and $\mathfrak{su}(3)$].

The shortened multiplet contains $4 + 4$ states: $1 \times |\Delta\rangle$, $3 \times |\Delta + \frac{1}{2}\rangle$, $3 \times |\Delta + 1\rangle$, $1 \times |\Delta + \frac{3}{2}\rangle$. If the lowest Δ state is bosonic (so we have $1_B \rightarrow 3_F \rightarrow \bar{3}_B \rightarrow 1_F$), then the corresponding masses are $m_B^2: 1 \times \Delta(\Delta - 1), 3 \times (\Delta + 1)\Delta$ and $m_F: 3 \times \Delta, 1 \times (\Delta + 1)$. The above quadratic fluctuation spectrum at given n can be organized into the sum of two such shortened multiplets (corresponding to Q_a and $\bar{Q}^a$ and related by the conjugation $n \rightarrow -n$), with the lowest $\Delta > \frac{1}{2}$ given by $\Delta_{1,n}$ or $\Delta_{2,n}$ [cf. Eq. (D8)]:

$$\Delta_{1,n} = \frac{1}{2}(kn + 1), \quad \Delta_{2,n} = \frac{1}{2}(kn - 2), \quad kn \geq 3. \quad (\text{D9})$$

Computing the value of the one-loop contribution corresponding to the above shortened multiplet using Eqs. (4.5)–(4.12), one finds, as in Eq. (4.16), that all transcendental parts cancel out, and one gets simply $\zeta'_{\text{tot},\Delta}(0) = -\log \Delta$ (if $|\Delta\rangle$ is a bosonic state), and $\zeta'_{\text{tot},\Delta}(0) = -\log(\Delta + \frac{1}{2})$ (if $|\Delta\rangle$ is a fermionic state). Considering the contribution to the effective action in Eq. (4.5) from all the states at levels n and $-n$ (that form two multiplets with $\Delta_{1,n}$ and two multiplets with $\Delta_{2,n}$) and using Eq. (D9), we get ($k > 2$)

$$\begin{aligned} \zeta'_{\text{tot},n} + \zeta'_{\text{tot},-n} &= -2 \log \left(\Delta_{1,n} + \frac{1}{2} \right) - 2 \log \Delta_{2,n} \\ &= -2 \log \left[\frac{1}{4}(n^2 k^2 - 4) \right], \end{aligned} \quad (\text{D10})$$

in agreement with Eq. (1.19). We conclude that, like in the $\text{AdS}_3 \times S^3$ case discussed in Sec. IV here, the nontrivial

cancellations in the one-loop effective action in Eqs. (1.16) and (1.17) leading to Eq. (1.19) can be understood as a consequence of the corresponding extended AdS_2 supersymmetry.

APPENDIX E: MIXED FLUX CASE

The D1-D5 solution is related by S-duality to the NS1-NS5 one, and more generally to the (p, q) family of $\frac{1}{4}$ -supersymmetric type-IIB solutions supported by a combination of RR and NSNS fluxes. This background is T-dual to a generalization of the type-IIA D2-D4 solution supported by a combination of fluxes.

From the 11D point of view, the S-duality of type-IIB theory on S^1 is a modular transformation on a 2-torus [38,42,43]. Indeed, as we shall discuss below, the (p, q) generalization of the D2-D4 solution can be obtained from the M2-M5 solution in Eqs. (2.1)–(2.3) upon dimensional reduction along the 11th direction, which is a linear combination of the isometric circular directions y_4 and w [44,45].

The near-horizon limit of the above type-IIB solution is $\text{AdS}_3 \times S^3 \times T^4$, supported by mixed 3-form flux; its type-IIA T-dual is supported by F_4 and H_3 flux; and their 11D counterpart is the same $\text{AdS}_3 \times S^3 \times T^5$ metric, supported by F_4 , as in Eqs. (2.5) and (2.7), but with the S^1 direction w replaced by its linear combination with y_4 . It is then straightforward to find the corresponding generalization of the $\text{AdS}_2 \times S^1$ M2-brane solution discussed in Sec. III. Its +IIA string counterpart will be the AdS_2 minimal surface, representing WL in the “mixed flux” background.

1. (D1-D5)_{p,q} background and its type-IIA counterpart

Let us start with the (p, q) family of solutions that includes the NS1-NS5 [46] and D1-D5 [47] solutions of type-IIB supergravity—i.e., which is a generalization of Eqs. (B8)–(B10). It is obtained by applying a special $SO(2) \subset SL(2, \mathbb{R})$ transformation [42], which is a symmetry of the type-IIB equations of motion²⁷:

$$F_3 \rightarrow \cos \gamma F_3 - \sin \gamma H_3, \quad H_3 \rightarrow \cos \gamma H_3 + \sin \gamma F_3, \quad F_3 = g'_s F_3, \quad (\text{E1})$$

$$\begin{aligned} \tau &\equiv C + i e^{-\tilde{\phi}} \rightarrow \frac{\cos \gamma \tau - \sin \gamma}{\sin \gamma \tau + \cos \gamma} = \frac{p\tau - g_s'^{-1}q}{g_s'^{-1}q\tau + p}, \\ \cos \gamma &\equiv \frac{p}{\sqrt{p^2 + g_s'^{-2}q^2}}, \quad p, q \in \mathbb{Z}. \end{aligned} \quad (\text{E2})$$

²⁶The full long multiplet is given by $[\Delta; 1]_B \oplus [\Delta + \frac{1}{2}; 3 \oplus \bar{3}]_F \oplus [\Delta + 1; 8 \oplus 1 \oplus 3 \oplus \bar{3}]_B \oplus [\Delta + \frac{3}{2}; 2 \times 1 \oplus 3 \oplus 3 \oplus 6 \oplus \bar{6}]_F \oplus [\Delta + 2; 8 \oplus 1 \oplus 3 \oplus \bar{3}]_B \oplus [\Delta + \frac{5}{2}; 3 \oplus \bar{3}]_F \oplus [\Delta + 3; 1]_B$. To get the full long multiplet, one would need to combine states from different n -levels, as at each level, we have only $8 + 8$ states.

²⁷The type-IIB action which is the analog of Eq. (A6) has the structure (the prime on g_s and $\tilde{\phi}$ is used to indicate type-IIB quantities): $S_{10B} = \frac{1}{2\kappa_0^2 g_s'^2} \int d^{10}x \sqrt{-G} [e^{-2\tilde{\phi}} (R + 4(\partial\tilde{\phi})^2 - \frac{1}{2}|H_3|^2) - \frac{1}{2}|F_3 - CH_3|^2 + \dots]$, where $F_3 = g'_s F_3$.

Here, we follow the same notation as in Eqs. (B8)–(B10): g'_s is the asymptotic coupling of the type-IIB string (which is not transformed, as we are looking for a more general solution in the same theory), and $\tilde{\phi}'$ is the “nonconstant” part of the type-IIB dilaton—i.e., $e^{\phi'} = g'_s e^{\tilde{\phi}'}$. The resulting generalization of the D1-D5 solution is then²⁸

$$ds_{10B}^2 = H_5^{-1/2} [H_1^{-1} H_5^{-1/2} (dx_0^2 + dx_1^2) + H_5^{1/2} (dr^2 + r^2 ds_{S^3}^2) + H_5^{-1/2} (dy_i^2 + dw^2)], \quad (E3)$$

$$F_3 = \cos \gamma g'_s {}^{-1} (idH_1^{-1} \wedge dx_0 \wedge dx_1 - *_4 dH_5), \quad H_3 = \sin \gamma (idH_1^{-1} \wedge dx_0 \wedge dx_1 - *_4 dH_5), \quad (E4)$$

$$C = \cos \gamma \sin \gamma (H_5 - H_1) H_5^{-1}, \quad e^{2\phi'} = g_s^2 H_5^2 H_1^{-1} H_5^{-1}, \quad (E5)$$

$$H_{1,5} = 1 + \frac{\bar{Q}_{1,5}}{r^2}, \quad \bar{Q}_{1,5} \equiv \sqrt{p^2 + g_s'^{-2} q^2} Q_{1,5}, \quad H_{15} \equiv \cos^2 \gamma H_1 + \sin^2 \gamma H_5, \quad (E6)$$

where $Q_1 = g'_s \ell_s^2 \hat{v}'^{-1} Q_1$ and $Q_5 = g'_s \ell_s^2 Q_5$ are the same as in Eq. (B10).

In general, this background describes a system with four sets of integer charges:

$$Q_{1RR} = p Q_1, \quad Q_{1NS} = q Q_1, \quad Q_{5RR} = p Q_5, \quad Q_{5NS} = q Q_5. \quad (E7)$$

The original D1-D5 solution corresponds to the special case of $p = 1, q = 0$ ($\gamma = 0$), while the NS1-NS5 one corresponds to $p = 0, q = 1$ ($\gamma = \frac{\pi}{2}$) (in the latter case, $\bar{Q}_1 = \ell_s^2 \hat{v}'^{-1} Q_1$, $\bar{Q}_5 = \ell_s^2 Q_5$).

The near-horizon limit of this solution is represented by the $\text{AdS}_3 \times S^3 \times T^4$ case supported by a mixed 3-form flux (with the RR scalar C and dilaton being constant)²⁹:

$$ds_{10B}^2 = \tilde{L}_s^2 [ds_{\text{AdS}_3}^2 + ds_{S^3}^2 + \bar{Q}_5^{-1} (dy_i^2 + dw^2)], \quad (E8)$$

$$\tilde{L}_s^2 = (\cos^2 \gamma \bar{Q}_1 + \sin^2 \gamma \bar{Q}_5)^{1/2} \bar{Q}_5^{1/2} = (p^2 Q_1 Q_5 + g_s'^{-2} q^2 Q_5^2)^{1/2} = (p^2 \hat{v}'^{-1} g_s^2 Q_1 Q_5 + q^2 Q_5^2)^{1/2} \ell_s^2, \quad (E9)$$

$$F_3 = 2 \cos \gamma g'_s {}^{-1} \bar{Q}_5 (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}) = 2p Q_5 (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}) \ell_s^2, \quad (E10)$$

$$H_3 = 2 \sin \gamma \bar{Q}_5 (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}) = 2q Q_5 (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}) \ell_s^2, \quad (E11)$$

$$C = \frac{\cos \gamma \sin \gamma (Q_5 - Q_1)}{\cos^2 \gamma Q_1 + \sin^2 \gamma Q_5}, \quad e^{2\phi'} = g_s^2 \frac{(\cos^2 \gamma Q_1 + \sin^2 \gamma Q_5)^2}{Q_1 Q_5}. \quad (E12)$$

This reduces to Eq. (B13) when $p = 1, q = 0$ ($\cos \gamma = 1$), and thus $\bar{Q}_{1,5} = Q_{1,5}$.

Applying the same T-duality transformation along the w direction that related type-IIA [Eqs. (B1)–(B3)] and type-IIB [Eqs. (B8)–(B10)] solutions, we find for the type-IIA counterpart of Eq. (E3)–(E6)

$$ds_{10A}^2 = H_{24}^{-1/2} [H_2^{-1} H_4^{-1/2} (dx_0^2 + dx_1^2) + H_4^{1/2} (dr^2 + r^2 ds_{S^3}^2) + H_4^{-1/2} dy_i^2] + H_{24}^{-1/2} H_4^{1/2} dw^2, \quad (E13)$$

$$F_4 = \cos \gamma g_s {}^{-1} (idH_2^{-1} \wedge dx_0 \wedge dx_1 - *_4 dH_4) \wedge dw, \quad H_3 = \sin \gamma (idH_2^{-1} \wedge dx_0 \wedge dx_1 - *_4 dH_4), \quad (E14)$$

$$C_w = \cos \gamma \sin \gamma (H_4 - H_2) H_{24}^{-1}, \quad e^{2\phi} = g_s^2 H_2^{-1} H_4^{-1/2} H_{24}^{3/2}, \quad (E15)$$

$$H_{2,4} \equiv H_{1,5} = 1 + \frac{\bar{Q}_{2,4}}{r^2}, \quad \bar{Q}_{2,4} \equiv \bar{Q}_{1,5}, \quad H_{24} \equiv H_{15}, \quad (E16)$$

²⁸It is equivalent to the solution in Eq. (2.15) of Ref. [44], where g'_s was set to 1. As in Eq. (B8) here, the metric in Eq. (E3) is the string-frame one.

²⁹We again rescale x_0 and x_1 , as in AdS_3 in Eq. (B13).

where C_w is the w component of the RR vector potential. This background interpolates between the D2-D4 solution ($p = 1, q = 0$) and the type-IIA NS1-NS5 solution ($p = 0, q = 1$). Its near-horizon limit is T-dual to Eqs. (E8)–(E12):

$$ds_{10A}^2 = \tilde{L}_s^2 [ds_{\text{AdS}_3}^2 + ds_{S^3}^2 + \bar{Q}_4^{-1} dy_i^2] + \tilde{L}_s^{-2} \bar{Q}_4 dw^2, \quad \tilde{L}_s^2 = (\cos^2 \gamma \bar{Q}_2 + \sin^2 \gamma \bar{Q}_4)^{1/2} \bar{Q}_4^{1/2}, \quad (\text{E17})$$

$$F_4 = 2 \cos \gamma g_s^{-1} \bar{Q}_4 (\text{ivol}_{\text{AdS}_3} + \text{vol}_{S^3}) \wedge dw, \quad H_3 = 2 \sin \gamma \bar{Q}_4 (\text{ivol}_{\text{AdS}_3} + \text{vol}_{S^3}), \quad (\text{E18})$$

$$C_w = \frac{\cos \gamma \sin \gamma (\mathcal{Q}_4 - \mathcal{Q}_2)}{\cos^2 \gamma \mathcal{Q}_2 + \sin^2 \gamma \mathcal{Q}_4}, \quad e^{2\phi} = g_s^2 \frac{(\cos^2 \gamma \mathcal{Q}_2 + \sin^2 \gamma \mathcal{Q}_4)^{3/2}}{\mathcal{Q}_2 \mathcal{Q}_4^{1/2}}. \quad (\text{E19})$$

2. Rotated M2-M5 background

Let us now discuss how the 11D uplift of the type-IIA solution [Eqs. (E17)–(E19)] is related to the original M2-M5 solution [Eqs. (2.1)–(2.4)]. There, we had [for notation, see also Refs. (A7)–(A12)]

$$y_p = (y_i, y_4), \quad y_4 \equiv y_4 + 2\pi \ell_s v_4, \quad w \equiv w + 2\pi \ell_s u, \quad v_4 = \ell_s^{-1} \ell_P v_4 = g_s, \quad g'_s = u^{-1} g_s = u^{-1} v_4, \quad (\text{E20})$$

where y_4 plays the role of x_{11} in the reduction to Eqs. (B1)–(B3), and g'_s is the “T-dual” type-IIB coupling. Let us perform the following rotation on these 2-torus coordinates with radii v_4 and u :

$$y_4 = \cos \gamma \tilde{y}_4 - \sin \gamma \tilde{w}, \quad w = \sin \gamma \tilde{y}_4 + \cos \gamma \tilde{w}, \quad \cos \gamma = \frac{p}{\sqrt{p^2 + \frac{u^2}{v_4^2} q^2}}, \quad p, q \in \mathbb{Z}. \quad (\text{E21})$$

This effectively aligns the new 11th direction $\tilde{y}_4$ with the (p, q) cycle of the 2-torus. It will then have the period $2\pi \tilde{v}_4$, where $\tilde{v}_4 = \sqrt{p^2 v_4^2 + q^2 u^2}$ [cf. Eq. (A5)], so that

$$R_{11} = \tilde{v}_4 \ell_s, \quad \tilde{v}_4 = \tilde{g}_s = g_s \sqrt{p^2 + \frac{u^2}{v_4^2} q^2} = \sqrt{p^2 v_4^2 + q^2 u^2}, \quad \ell_P = \tilde{g}_s^{1/3} \ell_s. \quad (\text{E22})$$

Note that ℓ_P is now related to $\tilde{g}_s$, while ℓ_s is unchanged [cf. Eq. (A5)]. The M2-M5 solution [Eqs. (2.1)–(2.3)] written in terms of the transformed coordinates in Eq. (E21) takes the following form:

$$ds_{11}^2 = H_2^{-2/3} H_5^{-1/3} (dx_0^2 + dx_1^2) + H_2^{1/3} H_5^{2/3} (dr^2 + r^2 ds_{S^3}^2) + H_2^{1/3} H_5^{-1/3} dy_i^2 + H_2^{1/3} H_5^{2/3} H_{25}^{-1} d\tilde{w}^2 \\ + H_2^{-2/3} H_5^{-1/3} H_{25} [d\tilde{y}_4 + \cos \gamma \sin \gamma (H_5 - H_2) H_{25}^{-1} d\tilde{w}]^2, \quad (\text{E23})$$

$$F_4 = dC_3 = (idH_2^{-1} \wedge dx_0 \wedge dx_1 - *_4 dH_5) \wedge (\cos \gamma d\tilde{w} + \sin \gamma d\tilde{y}_4), \quad (\text{E24})$$

$$H_2 = 1 + \frac{\mathcal{Q}_2}{r^2}, \quad H_5 = 1 + \frac{\mathcal{Q}_5}{r^2}, \quad H_{25} \equiv \cos^2 \gamma H_2 + \sin^2 \gamma H_5. \quad (\text{E25})$$

Here, $\mathcal{Q}_2$ and $\mathcal{Q}_5$ are defined as in the original solution in Eqs. (1.22), (1.23), (2.3), and (2.4), but with ℓ_P now related to rescaled type-IIA string coupling as in Eq. (E22), so that [cf. Eq. (A12)]

$$\mathcal{Q}_2 = \ell_P^2 \hat{v}^{-1} Q_2 = \ell_P^3 \ell_s^{-1} v^{-1} Q_2 \rightarrow \tilde{g}_s \ell_s^2 v^{-1} Q_2 = \sqrt{p^2 v_4^2 + q^2 u^2} \ell_s^2 v^{-1} Q_2, \quad (\text{E26})$$

$$\mathcal{Q}_5 = \ell_P^2 u^{-1} Q_5 = \ell_P^3 \ell_s^{-1} u^{-1} Q_5 \rightarrow \tilde{g}_s \ell_s^2 u^{-1} Q_5 = \sqrt{p^2 v_4^2 + q^2 u^2} \ell_s^2 u^{-1} Q_5. \quad (\text{E27})$$

Since $g'_s = u^{-1} v_4 = u^{-1} g_s$, these coefficients are thus the same as $\bar{Q}_1$ and $\bar{Q}_5$ in Eq. (E6) or $\bar{Q}_2$ and $\bar{Q}_4$ in Eq. (E16). One can then check that Eqs. (E23)–(E25), reduced to 10D along $x^{11} = \tilde{y}_4$ according to Eq. (A3), gives precisely the type-IIA background in Eqs. (E13)–(E16).

The near-horizon limit of Eqs. (E23)–(E25) can be equivalently obtained directly from Eqs. (2.5) and (2.6) by performing the rotation (E21) there. The result can be written as

$$ds_{11}^2 = (Q_2 Q_5^2)^{1/3} (ds_{\text{AdS}_3}^2 + ds_{S^3}^2) + (Q_2 Q_5^{-1})^{1/3} dy_i^2 + (Q_2 Q_5^{-1})^{1/3} (\cos^2 \gamma + \sin^2 \gamma Q_2^{-1} Q_5) d\tilde{y}_4^2 \\ + (Q_2 Q_5^{-1})^{-2/3} (\cos^2 \gamma + \sin^2 \gamma Q_2 Q_5^{-1}) d\tilde{w}^2 + 2(Q_2^2 Q_5)^{-1/3} \cos \gamma \sin \gamma (Q_5 - Q_2) d\tilde{w} d\tilde{y}_4, \quad (\text{E28})$$

$$F_4 = dC_3 = 2Q_5 (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}) \wedge (\cos \gamma d\tilde{w} + \sin \gamma d\tilde{y}_4). \quad (\text{E29})$$

Dimensional reduction along the $x_{11} = \tilde{y}_4$ direction then gives the mixed IIA background in Eqs. (E17)–(E19).

3. $\text{AdS}_2 \times S^1$ M2 brane solution and one-loop partition function

Let us now consider a generalization of the M2-brane solution discussed in Sec. III to the case of $\gamma \neq 0$. We shall assume that the M2 brane is stretched along a 2-surface in AdS_3 (ending on a circle at the boundary) and is wrapping the 11th dimension $\tilde{y}_4$, so that double dimensional reduction will give a string solution in the mixed-flux type-IIA background [Eqs. (E17)–(E19)].

We shall parametrize AdS_3 as in Eqs. (2.9) and (C6)—i.e.,

$$ds_{\text{AdS}_3}^2 = \cosh^2 \zeta ds_{\text{AdS}_2}^2 + d\zeta^2, \quad z = 2 \tanh \frac{\zeta}{2}. \quad (\text{E30})$$

We shall choose the first two of the M2-brane world-volume coordinates σ_1, σ_2 to be identified again with the AdS_2 coordinates and assume that $\tilde{y}_4 \equiv \tilde{y}_4 + 2\pi \tilde{v}_4 \ell_s$ is proportional to $\sigma_3 \equiv \sigma_3 + 2\pi$. We then set

$$\zeta = \zeta(\sigma_1, \sigma_2), \quad \tilde{y}_4 = \tilde{v}_4 \ell_s \sigma_3, \quad \tilde{w} = 0, \quad y_i = 0, \quad (\text{E31})$$

and also assume that S^3 coordinates are trivial (i.e., the brane is localized at a point in S^3). Equivalently, one may start with the original “unrotated” background in Eq. (2.5) and consider the more general M2-brane solution with $y_4 = \ell_s \tilde{v}_4 \cos \gamma \sigma_3$, $w = \ell_s \tilde{v}_4 \sin \gamma \sigma_3$, corresponding to $\tilde{y}_4 = \tilde{v}_4 \ell_s \sigma_3$, $\tilde{w} = 0$.

It is straightforward to check that all the M2-brane equations of motion following from the action in Eqs. (C1) and (C2) in the background [Eqs. (E28) and (E29)]—i.e., $\frac{1}{\sqrt{h}} \partial_m (h^{mn} \partial_n X) \sim F_4 \partial X \partial X \partial X$ —are indeed satisfied by the ansatz (E31) with the remaining differential equation for ζ , also solved by a constant ζ :

$$\zeta = \zeta_0, \quad \tanh \zeta_0 = -\frac{\sin \gamma (Q_2^{-1} Q_5)^{1/2}}{(\cos^2 \gamma + \sin^2 \gamma Q_2^{-1} Q_5)^{1/2}}, \quad (\text{E32})$$

$$\cosh^2 \zeta_0 = \Lambda, \quad \Lambda \equiv 1 + \tan^2 \gamma Q_2^{-1} Q_5. \quad (\text{E33})$$

For $q = 0$ or $\gamma = 0$ (pure RR case), this reduces to the solution with $\zeta = 0$ discussed in Sec. III. In the opposite

pure NSNS case $p = 0$, $\gamma = \frac{\pi}{2}$, the solution becomes singular ($\zeta_0 \rightarrow \infty$).

The corresponding induced metric is again $\text{AdS}_2 \times S^1$, but with rescaled radii [cf. Eqs. (1.36), (2.5), (2.6), (2.8), and (3.2)]:

$$(ds^2)_{\text{M2}} = \Lambda (L^2 ds_{\text{AdS}_2}^2 + \tilde{R}^2 d\sigma_3^2) \\ = \Lambda L^2 (ds_{\text{AdS}_2}^2 + \tilde{\kappa}^{-2} d\sigma_3^2), \quad (\text{E34})$$

$$L^2 = (Q_2 Q_5^2)^{1/3}, \quad \tilde{R}^2 = \cos^2 \gamma R^2, \\ R^2 = \ell_p^2 (Q_2 Q_5^{-1})^{1/3}, \quad (\text{E35})$$

$$\tilde{\kappa}^{-1} = L^{-1} \tilde{R} \tilde{v}_4 = \cos \gamma \sqrt{p^2 + q^2 v_4^{-2} u^2} \kappa^{-1} = p \kappa^{-1}, \\ \kappa^{-1} = L^{-1} R v_4 = g_s^{2/3} u^{1/2} Q_5^{-1/2}, \quad (\text{E36})$$

$$\tan^2 \gamma = v_4^{-2} u^2 p^{-2} q^2, \quad \tilde{v}_4 = \sqrt{p^2 v_4^2 + q^2 u^2}. \quad (\text{E37})$$

Here, $ds_{\text{AdS}_2}^2(\sigma_1, \sigma_2)$ is the unit-radius AdS_2 metric. To compute the value of the classical M2-brane action in Eqs. (C1) and (C2) on the solution in Eqs. (E31) and (E32), we shall assume that the gauge for the field C_3 is chosen so that it vanishes³⁰ for constant ζ . Then, the action is given just by the volume part in Eq. (C1), generalizing the $\gamma = 0$ expression in Eq. (3.1):

$$\tilde{S}_{\text{M2}} = T_2 \Lambda^{3/2} L^3 (2\pi \tilde{\kappa}^{-1}) \text{vol}(\text{AdS}_2) = p \Lambda^{3/2} S_{\text{M2}}, \\ S_{\text{M2}} = -\frac{v_4}{\sqrt{\tilde{v} u}} \sqrt{Q_2 Q_5}. \quad (\text{E38})$$

Let us now consider the type-IIA string-theory expressions for the corresponding AdS_2 string solution in the IIA background in Eqs. (E17)–(E19). From Eqs. (E17)–(E19),

³⁰One can find such a gauge by starting with Eq. (E29) and choosing a specific parametrization of the AdS_2 part of AdS_3 in Eq. (E30).

we get for the induced string metric and the string action value on the solution (here $Q_4 = Q_5$)³¹

$$(ds^2)_s = \hat{L}_s^2 ds_{\text{AdS}_2}^2, \quad \hat{L}_s^2 = \Lambda \tilde{L}_s^2 = p \Lambda^{3/2} (Q_2 Q_4)^{1/2}, \quad (\text{E39})$$

$$Q_2 = \ell_s^2 g_s v^{-1} Q_2, \quad Q_4 = \ell_s^2 g_s u^{-1} Q_4, \\ \tilde{g}_s^2 = e^{2\phi} = g_s^2 \cos^3 \gamma \Lambda^{3/2} (Q_2^{-1} Q_4)^{-1/2}, \quad (\text{E40})$$

$$\tilde{S}_s = -2\pi T \hat{L}_s^2 = -\ell_s^{-2} \hat{L}_s^2 = p \Lambda^{3/2} S_s, \\ S_s = -\ell_s^{-2} L_s^2 = -g_s (v^{-1} u^{-1} Q_2 Q_4)^{1/2}. \quad (\text{E41})$$

The value of the string action is the same as the M2-brane one in Eq. (E37) [cf. Eq. (1.33)].

In the special case of $Q_2 = Q_4$, the corresponding string solution can be shown to be equivalent to the one discussed in Ref. [32], which considered the string representing an analog of WL in the type-II $\text{AdS}_3 \times S^3$ background supported by the $H_3 = 2\hat{q}\tilde{L}_s^2 \text{vol}_{\text{AdS}_3}$ flux, with $\tilde{L}_s$ assumed to be independent of $\hat{q}$. The world surface ended on a circle at the boundary of AdS_3 and had an AdS_2 induced metric. When $Q_2 = Q_4 \equiv Q$, the type-IIA dilaton in Eq. (E19) is γ -independent, $e^{2\phi} = g_s^2$, and using Eqs. (E21), (E33), and (E39), we get³²

$$\Lambda = (\cos \gamma)^{-2}, \quad \hat{L}_s^2 = \Lambda \tilde{L}_s^2 = p (\cos \gamma)^{-3} L_s^2, \\ \tilde{L}_s^2 = p (\cos \gamma)^{-1} L_s^2, \quad L_s^2 = Q. \quad (\text{E42})$$

The NSNS flux H_3 in Eq. (E18) is given by

$$H_3 = 2h_3 (i \text{vol}_{\text{AdS}_3} + \text{vol}_{S^3}), \\ h_3 = \sin \gamma \bar{Q}_4 = \hat{q} \tilde{L}_s^2, \quad \hat{q} \equiv \sin \gamma. \quad (\text{E43})$$

The string action in Eq. (E41) written in terms of $\hat{q}$ and the radius $\tilde{L}_s$ in Eq. (E17) takes the same form as in Ref. [32]³³:

³¹The same value for ζ_0 in Eq. (E33) is, of course, found also by solving the string equations directly in the background [Eqs. (E17) and (E18)].

³²Equivalent expressions are found also by starting with the type-IIB background in Eqs. (E8)–(E11).

³³Note that in the discussions of the mixed flux case in Ref. [32] and, e.g., Ref. [48], the radius $\tilde{L}_s$ of the $\text{AdS}_3 \times S^3$ part of the type-IIA metric in Eq. (E17) was treated as an independent parameter, with the NSNS and RR fluxes proportional to $\hat{q}\tilde{L}_s^2$ and $\sqrt{1 - \hat{q}^2}\tilde{L}_s^2$, respectively. This is different from the parametrization used in Eqs. (E10) and (E11), where both Q_2 and Q_4 were assumed to be quantized, with $\tilde{L}$ expressed in terms of them.

$$\tilde{S}_s = p (\cos \gamma)^{-3} S_s(L_s) = \frac{1}{1 - \hat{q}^2} S_c(\tilde{L}_s),$$

$$S_c(L_s) \equiv -\ell_s^{-2} L_s^2. \quad (\text{E44})$$

The corresponding one-loop correction to the string partition function turns out to have the same form as in the pure RR case [Eq. (1.37)], but with the effective string tension rescaled like the action in Eq. (E44) by $(1 - \hat{q}^2)^{-1}$ [32].

Starting with M2 action in the rotated background [Eqs. (E28) and (E29)], it is straightforward to expand it near the M2-brane solution [Eqs. (E31) and (E32)] to quadratic order in fluctuations using the same static gauge as in Sec. III (i.e., setting the fluctuations of AdS_2 coordinates and of $\tilde{y}_4$ to zero). After a rescaling of the fluctuation fields, we find that the bosonic Lagrangian depends on just one parameter—it is the same as in Eq. (3.4), with κ replaced by $\tilde{\kappa} = p^{-1}\kappa$, defined in Eq. (E36).³⁴ As a result, the bosonic contribution to the one-loop M2-brane partition function is given again by Eqs. (3.7) and (3.6), where all the masses depend only on $\tilde{\kappa}$. Similarly, the fermionic contribution is found to be given by Eq. (3.12) with $\kappa \rightarrow \tilde{\kappa}$, so that the total is again given in Eq. (3.14) with

$$\tilde{M}_{n,\pm}^2 = 1 + \tilde{\kappa}^2 n^2 \pm \sqrt{1 + 4\tilde{\kappa}^2 n^2}, \\ \tilde{\mathcal{M}}_{n,\pm} = \frac{1}{2} \left(\sqrt{1 + 4\tilde{\kappa}^2 n^2} \pm 1 \right). \quad (\text{E45})$$

As a result, we find the same expressions as in Sec. IV. In particular, for $\tilde{\kappa}^2 > \frac{3}{4}$, we get, as in Eq. (4.15),

$$\hat{Z}_1 = \frac{\tilde{\kappa}}{\sqrt{2\pi}} = \frac{\kappa}{p\sqrt{2\pi}}. \quad (\text{E46})$$

Thus, the one-loop result is the same as in the 10D string-theory limit (cf. Ref. [32]) and does not depend on values of q and Q_1 , with the pure NSNS limit of $p = 0$ being singular. Note that instead of p , q , Q_2 , and Q_4 , both the classical action [Eq. (E41)] and the one-loop correction [Eq. (E46)] can be expressed in terms of the four integer charges in Eq. (E7) (with $Q_2 = Q_1$ and $Q_4 = Q_5$).

³⁴This may not be too surprising, given that the background [Eqs. (E28) and (E29)] is just a coordinate transformation of the original one, and the M2-brane solution still has an $\text{AdS}_2 \times S^1$ induced metric.

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