

Eikonal spin-dependent odderon and gluon Sivvers function of a proton and its small- x evolution

Sanjin Benić,¹ Adrian Dumitru^{2,3} , Florian Hechenberger⁴ , and Tomasz Stebel⁵ 

¹*Department of Physics, Faculty of Science, University of Zagreb, Bijenička c. 32, 10000 Zagreb, Croatia*

²*Department of Natural Sciences, Baruch College, CUNY, 17 Lexington Avenue, New York, New York 10010, USA*

³*The Graduate School and University Center, The City University of New York, 365 Fifth Avenue, New York, New York 10016, USA*

⁴*Center for Nuclear Theory, Stony Brook University, Stony Brook, New York 11794, USA*

⁵*Jagiellonian University, Institute of Theoretical Physics, Łojasiewicza 11, 30-348 Kraków, Poland*



(Received 30 March 2026; accepted 8 July 2026; published 4 August 2026)

The matrix element in the proton of the eikonal Odderon operator, with a helicity flip, has been shown to correspond to the dipole gluon Sivvers function. We employ a three quark light-front model of the proton to determine the Sivvers function at moderately small $x_0 \sim 0.1$ and transverse momentum $k_\perp \lesssim 1$ GeV. The model light-cone (LC) wave function predicts the properties of $xf_{1T}^{\perp g}(x, k_\perp)$ such as its overall magnitude, the position of its peak in $k_\perp$, and its behavior at small $k_\perp$. We then compute numerically the Balitsky-Fadin-Kuraev-Lipatov anomalous dimension characterizing the power-law tail at $k_\perp \gtrsim 1.5$ GeV of the gluon Sivvers function at small (but preasymptotic) LC momentum fractions $x \sim x_0 e^{-1/\alpha_s}$: $xf_{1T}^{\perp g}(x, k_\perp) \sim k_\perp^{-3.3}$.

DOI: 10.1103/wwmj-yxtg

I. INTRODUCTION

The Sivvers function describes a left-right asymmetry, relative to the spin vector, of the transverse momentum dependent parton distributions (TMDs) in a transversely polarized proton [1,2]. Finite parton transverse momentum allows for $\mathbf{S}_\perp \times \mathbf{k}_\perp$ “spin-orbit” correlations in the proton. These correlations are probed even in eikonal, high-energy scattering processes which involve transfer of transverse momentum to the partons.

We are interested here in the (dipole) gluon Sivvers function of the proton which we denote $xf_{1T}^{\perp g}(x, k_\perp)$. To date, this element of the structure of the proton is rather poorly known [3], its measurement represents one of the main goals of the physics program at the future Electron-Ion Collider (EIC) [4,5]. In the eikonal approximation the gluon Sivvers function is given by the C -odd component of the dipole scattering amplitude evaluated for a transversely polarized proton, i.e., the so-called spin dependent Odderon [6–9]. In fact, the eikonal contribution to the quark Sivvers function of the proton is also obtained from the spin-dependent Odderon [10–12]. The general correspondence

of TMDs and matrix elements of eikonal Wilson line operators at small x has been established in Ref. [13].

In this paper we determine the eikonal spin dependent Odderon and the associated gluon Sivvers function from an effective three-quark light-cone wave function (LCwf) of the proton, whose analytical form is given in Appendix A. In the helicity basis we can interpret the transversely polarized proton matrix element as involving helicity flips between the incoming and the outgoing proton states. Even though the eikonal color current operator $J^+ J^+ J^+$, sourcing the small- x gluon fields in the Odderon, conserves the helicities of the individual quarks, a proton helicity flip is made possible by the quark orbital angular momentum [14] which generates an overlap of proton wave functions differing by one unit of orbital angular momentum. The possibility of a proton helicity flip in scattering with Odderon exchange was realized a long time ago [15,16] albeit the corresponding matrix element vanishes in the nonrelativistic quark model. Also, the connection to the Sivvers function is more recent [6–9].

Our empirical knowledge of the proton suggests that the three quark LCwf should provide an adequate model for the partonic structure of the proton for moderately small $x \sim 0.1$, and transverse momenta up to $k_\perp \sim 1$ GeV. To assess the accuracy of this approach, we confront the Dirac and Pauli electromagnetic form factors obtained in this model with up to date data in Appendix B. We then employ the eikonal Odderon derived from the nonperturbative LC quark model wave function as initial condition for QCD

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

evolution to smaller x . That is, we solve the small- x dipole evolution equation [17] from this initial condition to determine the gluon Sivers function and the anomalous dimension associated with its transverse momentum dependence.

So far, the only model for the eikonal spin-dependent Odderon available in the literature was constructed by Zhou [7] within the framework of an extended McLerran-Venugopalan model with cubic Casimir [18]. In that model the random static sources for the soft gluon fields are assumed to reside in a high-dimensional representation of color-SU(3), and therefore also in a high-dimensional representation of spin-SU(2). Here, we construct a model where these sources are in the fundamental representations of both color SU(3) and spin SU(2).

The spin dependent Odderon features in a variety of processes in Deep Inelastic Scattering (DIS) at high energies such as the spin dependent part of the cross section for inclusive open charm production on transversely polarized protons [8,19], and furthermore, in exclusive forward production of C -even mesons [9,20] (see also [21]). Other observables for the spin dependent Odderon include a T -odd nucleon energy correlator which generates a single transverse spin asymmetry for the energy pattern in the target fragmentation region [22]; as well as transverse energy-energy correlations at small x in DIS [23]. Most recently, Hatta and Teryaev discussed a left-right asymmetry with respect to the spin of the *recoil* baryon in DIS [24].

The physics of the spin dependent Odderon and of the gluon Sivers function of a transversely polarized proton should not be confused with the physics of longitudinal spin. The latter pertains, for example, to the determination of helicity and orbital angular momentum Parton Distribution Functions (PDFs) such as $\Delta G(x, Q^2)$ and $L_G(x, Q^2)$. Indeed, these quantities, at small x , involve completely different (subeikonal) operators than the spin dependent Odderon considered here; see Sec. 3.3 of Ref. [25] for a recent overview.

II. THE SPIN DEPENDENT ODDERON(S)

The $\mathcal{S}$ -matrix for eikonal scattering of a $q - \bar{q}$ dipole from a target proton can be written, in dipole factorization, in the form [27,28]

$$1 - \mathcal{P}_{\Lambda'\Lambda}(\mathbf{r}_\perp, \mathbf{b}_\perp) - i\mathcal{O}_{\Lambda'\Lambda}(\mathbf{r}_\perp, \mathbf{b}_\perp) = \frac{1}{N_c} \frac{\langle P'\Lambda' | \text{tr} V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) | P\Lambda \rangle}{\langle P\Lambda | P\Lambda \rangle}. \quad (1)$$

Here, $\mathbf{x}_\perp = \mathbf{b}_\perp + \mathbf{r}_\perp/2$ and $\mathbf{y}_\perp = \mathbf{b}_\perp - \mathbf{r}_\perp/2$ are the transverse positions of the quark and the antiquark, respectively, and $V(\mathbf{x}_\perp)$ are eikonal Wilson lines at transverse position $\mathbf{x}_\perp$,

$$V(\mathbf{x}_\perp) = \hat{P} \exp \left(-ig \int dx^- A^{+a}(x^-, \mathbf{x}_\perp) t^a \right). \quad (2)$$

This fundamental Wilson line describes the eikonal propagation of a quark and corresponds to the covariant derivative $(D^\mu)_{ij} = \partial^\mu \delta_{ij} + igA^{\mu a}(t^a)_{ij}$.

In Eq. (1), $\Lambda, \Lambda' = \pm 1$ denote (twice) the helicities of the incoming and scattered proton, respectively. C -even Pomeron exchange relates to $\mathcal{P}_{\Lambda'\Lambda}(\mathbf{x}_\perp, \mathbf{y}_\perp)$, and the amplitude for $\Lambda' \neq \Lambda$ has been derived in Ref. [29]; this amplitude vanishes when the transverse momentum transfer $\mathbf{P}'_\perp - \mathbf{P}_\perp \rightarrow 0$. C -odd Odderon exchange is related to $i\mathcal{O}_{\Lambda'\Lambda}(\mathbf{x}_\perp, \mathbf{y}_\perp)$. A key point is that despite taking the eikonal limit, the helicity flip Odderon exchange amplitude is nonzero even in the forward limit, where it is integrated over the impact parameter $\mathbf{b}_\perp$ [7,9,20]. We shall confirm this by an explicit computation of the above matrix element at third order in gA^+ for a light-cone quark model of the proton $|P\Lambda\rangle$. Nonzero contributions to $\int d^2\mathbf{b}_\perp i\mathcal{O}_{\Lambda'\neq\Lambda}(\mathbf{r}_\perp, \mathbf{b}_\perp)$ arise when the three eikonal, t -channel gluons attach to three *different* quarks in the proton.

Expanding the Wilson lines to cubic order in the field gA^{+a} we can write the Odderon amplitude in the form [30]

$$\mathcal{O}_{\Lambda'\Lambda}(\mathbf{r}_\perp, \mathbf{b}_\perp) = \frac{-ig^6(N_c^2 - 4)(N_c^2 - 1)}{16N_c^2} \int_{\mathbf{q}_{1\perp}\mathbf{q}_{2\perp}\mathbf{q}_{3\perp}} \frac{G_{3\Lambda'\Lambda}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}, \mathbf{q}_{3\perp})}{q_{1\perp}^2 q_{2\perp}^2 q_{3\perp}^2} e^{i(\mathbf{q}_{1\perp} + \mathbf{q}_{2\perp} + \mathbf{q}_{3\perp}) \cdot \mathbf{b}_\perp} \times \left[\sin \left(\frac{1}{2} \mathbf{r}_\perp \cdot (\mathbf{q}_{1\perp} - \mathbf{q}_{2\perp} - \mathbf{q}_{3\perp}) \right) + \frac{1}{3} \sin \left(\frac{1}{2} \mathbf{r}_\perp \cdot (\mathbf{q}_{1\perp} + \mathbf{q}_{2\perp} + \mathbf{q}_{3\perp}) \right) \right]. \quad (3)$$

$G_{3\Lambda'\Lambda}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}, \mathbf{q}_{3\perp})$ represents the C -odd part of the eikonal coupling of three static t -channel gluons to the proton; we shall provide explicit expressions below. We abbreviate transverse momentum integrals as $\int_{\mathbf{q}_\perp} \equiv \int d^2\mathbf{q}_\perp / (2\pi)^2$, and position space integrals as $\int_{\mathbf{x}_\perp} \equiv \int d^2\mathbf{x}_\perp$.

Performing a Fourier transform from impact parameter to momentum transfer we obtain the Odderon in mixed representation,

$$\begin{aligned}
\mathcal{O}_{\Lambda'\Lambda}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) &= \int_{\mathbf{b}_\perp} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \mathcal{O}_{\Lambda'\Lambda}(\mathbf{r}_\perp, \mathbf{b}_\perp) \\
&= \frac{-ig^6(N_c^2 - 4)(N_c^2 - 1)}{48N_c^2} \int_{\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}} \frac{G_{3\Lambda'\Lambda}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}, \mathbf{\Delta}_\perp - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp})}{\mathbf{q}_{1\perp}^2 \mathbf{q}_{2\perp}^2 (\mathbf{\Delta}_\perp - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp})^2} \left[\sin\left(\frac{1}{2}\mathbf{r}_\perp \cdot (2\mathbf{q}_{1\perp} - \mathbf{\Delta}_\perp)\right) \right. \\
&\quad \left. + \sin\left(\frac{1}{2}\mathbf{r}_\perp \cdot (2\mathbf{q}_{2\perp} - \mathbf{\Delta}_\perp)\right) + \sin\left(\frac{1}{2}\mathbf{r}_\perp \cdot (\mathbf{\Delta}_\perp - 2\mathbf{q}_{1\perp} - 2\mathbf{q}_{2\perp})\right) + \sin\left(\frac{1}{2}\mathbf{r}_\perp \cdot \mathbf{\Delta}_\perp\right) \right]. \quad (4)
\end{aligned}$$

Forward scattering of a very small dipole scales like $\sim r_\perp^3$ as the coupling of each one of the exchanged gluons to the almost pointlike dipole operator brings about one factor of $r_\perp$.

A further Fourier transform from dipole vector $\mathbf{r}_\perp$ to transverse momentum $\mathbf{k}_\perp$ gives

$$\begin{aligned}
\mathcal{O}_{\Lambda'\Lambda}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp) &= -\frac{g^6(N_c^2 - 4)(N_c^2 - 1)}{32N_c^2} (2\pi)^2 \int_{\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}} \frac{G_{3\Lambda'\Lambda}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}, \mathbf{\Delta}_\perp - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp})}{\mathbf{q}_{1\perp}^2 \mathbf{q}_{2\perp}^2 (\mathbf{\Delta}_\perp - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp})^2} \left[\delta^{(2)}\left(\mathbf{q}_{1\perp} - \frac{1}{2}\mathbf{\Delta}_\perp - \mathbf{k}_\perp\right) \right. \\
&\quad \left. - \delta^{(2)}\left(\mathbf{q}_{1\perp} - \frac{1}{2}\mathbf{\Delta}_\perp + \mathbf{k}_\perp\right) + \frac{1}{3}\delta^{(2)}\left(\mathbf{k}_\perp - \frac{1}{2}\mathbf{\Delta}_\perp\right) - \frac{1}{3}\delta^{(2)}\left(\mathbf{k}_\perp + \frac{1}{2}\mathbf{\Delta}_\perp\right) \right]. \quad (5)
\end{aligned}$$

Note that in this expression the antisymmetry of the Odderon under $\mathbf{k}_\perp \rightarrow -\mathbf{k}_\perp$ is manifest. Hence, the $\mathbf{k}_\perp^2 d^2\mathbf{k}_\perp$ integral of this function is zero; it would represent the imaginary part of a twist-2 PDF which, however, must be real.

As is customary in the literature, when referring to the forward matrix element at zero momentum transfer we will also use the notation $\mathcal{O}_{\Lambda'\Lambda}(\mathbf{r}_\perp) \equiv \mathcal{O}_{\Lambda'\Lambda}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp = 0)$, or $\mathcal{O}_{\Lambda'\Lambda}(\mathbf{k}_\perp)$ for its Fourier transform to transverse momentum space. We have

$$\begin{aligned}
\mathcal{O}_{\Lambda'\Lambda}(\mathbf{k}_\perp) &= -\frac{g^6(N_c^2 - 4)(N_c^2 - 1)}{32N_c^2} (2\pi)^2 \int_{\mathbf{q}_\perp} \sum_{s=\pm 1} \frac{s G_{3\Lambda'\Lambda}(s\mathbf{k}_\perp, \mathbf{q}_\perp, -s\mathbf{k}_\perp - \mathbf{q}_\perp)}{\mathbf{k}_\perp^2 \mathbf{q}_\perp^2 (s\mathbf{k}_\perp + \mathbf{q}_\perp)^2} \\
&= -\frac{g^6(N_c^2 - 4)(N_c^2 - 1)}{16N_c^2} (2\pi)^2 \int_{\mathbf{q}_\perp} \frac{G_{3\Lambda'\Lambda}(\mathbf{k}_\perp, \mathbf{q}_\perp, -\mathbf{k}_\perp - \mathbf{q}_\perp)}{\mathbf{k}_\perp^2 \mathbf{q}_\perp^2 (\mathbf{k}_\perp + \mathbf{q}_\perp)^2}. \quad (6)
\end{aligned}$$

In the second step we used the antisymmetry of $G_{3\Lambda'\Lambda}$ under a sign flip of all its arguments, which applies when $\Lambda' = -\Lambda$, see the following Sec. II A.

The most general angular decomposition of the Odderon in the helicity basis is [9]

$$\begin{aligned}
\mathcal{O}_{\Lambda'\Lambda}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) &= \delta_{\Lambda\Lambda'} i \cos(\phi_{r\Delta}) \mathcal{O}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) + \delta_{\Lambda, -\Lambda'} i \Lambda e^{i\Lambda\phi_r} \mathcal{O}_S(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) \\
&\quad + \delta_{\Lambda, -\Lambda'} i \Lambda e^{i\Lambda\phi_\Delta} \cos(\phi_{r\Delta}) \mathcal{O}_S^\perp(\mathbf{r}_\perp, \mathbf{\Delta}_\perp). \quad (7)
\end{aligned}$$

There are three independent real functions in general. The first one is the usual spin-independent (also known as “b-dependent”) Odderon while the second and third ones are spin-dependent Odderons. The second one becomes the Sivers function in the forward limit, and that is our focus here.

We may also consider the matrix element of the Odderon operator in a (transverse) spin basis which is usually employed to relate it to the gluon Sivers function. To obtain a state of definite spin vector one constructs a superposition of states of definite LC helicity [31]

$$|\vec{S}\rangle = \cos\frac{\theta_S}{2} |\Lambda = +\rangle + \sin\frac{\theta_S}{2} e^{i\phi_S} |\Lambda = -\rangle. \quad (8)$$

Here, $\vec{S} = (\sin\theta_S \cos\phi_S, \sin\theta_S \sin\phi_S, \cos\theta_S)$. In case of a transverse spin vector $\mathbf{S}_\perp$ we have $\theta_S = \frac{\pi}{2}$ and

$$|\mathbf{S}_\perp\rangle = \frac{1}{\sqrt{2}} (|\Lambda = +\rangle + e^{i\phi_S} |\Lambda = -\rangle). \quad (9)$$

Then,

$$\begin{aligned}
\mathcal{O}_{S_\perp S_\perp}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) &= \sum_{\Lambda', \Lambda} \langle \mathbf{S}_\perp | \Lambda' \rangle \mathcal{O}_{\Lambda'\Lambda} \langle \Lambda | \mathbf{S}_\perp \rangle \\
&= \cos(\phi_{r\Delta}) i \mathcal{O}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) - \frac{\mathbf{S}_\perp \times \mathbf{r}_\perp}{r_\perp} \mathcal{O}_S(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) \\
&\quad - \frac{\mathbf{S}_\perp \times \mathbf{\Delta}_\perp}{\Delta_\perp} \cos(\phi_{r\Delta}) \mathcal{O}_S^\perp(\mathbf{r}_\perp, \mathbf{\Delta}_\perp), \quad (10)
\end{aligned}$$

with $\mathbf{S}_\perp \times \mathbf{r}_\perp = r_\perp \sin(\phi_r - \phi_S)$. In the $\Delta_\perp \rightarrow 0$ forward limit,

$$\mathcal{O}_{S_\perp S_\perp}(\mathbf{r}_\perp) \rightarrow -\frac{\mathbf{S}_\perp \times \mathbf{r}_\perp}{r_\perp} \mathcal{O}_S(\mathbf{r}_\perp, \mathbf{\Delta}_\perp = 0). \quad (11)$$

The gluon Sivers function of the eikonal dipole model can be obtained from its forward, helicity flip Odderon [6–8],

$$xf_{1T}^\perp(x, k_\perp) = \frac{N_c k_\perp M_p}{2\pi^3 g^2} e^{-i\phi_k} \mathcal{O}_{-+}(\mathbf{k}_\perp). \quad (12)$$

On the lhs of this expression, the Siverson function does not depend on the direction of $\mathbf{k}_\perp$. $M_p = 0.938$ GeV denotes the mass of the proton.

In Zhou's model [7] the integral of $xf_{1T}^\perp(x, k_\perp)$ over $dk_\perp^2$ vanishes,

$$\int_0^\infty dk_\perp k_\perp xf_{1T}^\perp(x, k_\perp) = 0. \quad (13)$$

Given the expressions for $xf_{1T}^\perp(x, k_\perp)$ obtained here, Eq. (13) does not appear to represent an exact “sum rule.” However, a numerical evaluation confirms it at a level better than 0.5%.

The above relation implies that the Siverson function is not sign definite, and one may expect substantial cancellations, in particular at relatively low scales μ^2 , also in the $k_\perp$ -integral for the twist-3 collinear function (also called the d -type trigluon PDF) given by [6,7,20,32]

$$xf_{1T}^{\perp(1)}(x, \mu^2) = 8\pi \mathcal{O}(x, \mu^2) = \pi \int_0^{\mu^2} dk_\perp^2 \frac{k_\perp^2}{2M_p^2} xf_{1T}^\perp(x, k_\perp). \quad (14)$$

We shall investigate the magnitude of this function via a specific physical process in Sec. III below.

A. Eikonal triple gluon coupling to the proton

The C -odd coupling of three small- x , t -channel gluons (in covariant gauge) to the proton is given by the expectation value of three light-front color charge density operators $\rho^a(\mathbf{q}_\perp) \equiv J^{+a}(\mathbf{q}_\perp)$, i.e., of the fundamental cubic Casimir of SU(3). These color charges source the soft gluon fields, A^{+a} , appearing in the Wilson lines in Eq. (1).

$$\begin{aligned} d^{abc} \langle P' \Lambda' | \rho^a(\mathbf{q}_{1\perp}) \rho^b(\mathbf{q}_{2\perp}) \rho^c(\mathbf{q}_{3\perp}) | P \Lambda \rangle \\ \equiv \frac{(N_c^2 - 1)(N_c^2 - 4)}{4N_c} G_{3\Lambda'\Lambda}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}, \mathbf{q}_{3\perp}) \\ = N_c C_{3F} G_{3\Lambda'\Lambda}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}, \mathbf{q}_{3\perp}). \end{aligned} \quad (15)$$

(We have stripped the rhs of this expression of the delta functions [33] for conservation of LC and transverse momentum.)

To proceed, we adopt an effective light-cone three-quark model of the proton intended to describe its partonic structure for LC momentum fractions of roughly 10^{-1} (and higher), and for transverse momenta up to approximately 1 GeV. We write the proton state in the light-front (LF) quark model in the form

$$|P, \Lambda\rangle = \int [dx_i] \int [d^2\mathbf{k}_{i\perp}] \sum_{j_1, j_2, j_3} \frac{\epsilon^{j_1 j_2 j_3}}{\sqrt{N_c!}} \sum_{\{\lambda_i\}} \Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i) | \{x_i P^+, \mathbf{k}_{i\perp} + x_i \mathbf{P}_\perp, \lambda_i, j_i\} \rangle. \quad (16)$$

Here, x_i denotes the LC momentum fraction of the i^{th} quark, where $i = 1 \dots N_c = 3$; $\mathbf{k}_{i\perp}$ is its transverse momentum relative to the center of mass transverse momentum $\mathbf{P}_\perp$ of the proton; j_i refers to its color; λ_i to its helicity; a flavor quantum number is not required for the present purposes. The explicit form of the momentum space integrals and of the three-quark LC wave function are provided in Appendix A. In Appendix B we confront the model with up to date data for the Dirac and Pauli electromagnetic form factors of the proton.

Given a light-front Fock space expansion of the proton, this matrix element can be computed explicitly, in principle. For a proton effectively made of three valence quarks, as also assumed here, such computations have previously been carried out in Ref. [30,33,34], however, without considering a helicity flip of the proton. An analogous matrix element for eikonal *two*-gluon exchange *with* helicity flip has been presented in Ref. [29]. We refer to those papers for technical details. Here we just quote our result,

$$\begin{aligned} G_{3\Lambda'\Lambda}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}, \mathbf{q}_{3\perp}) \\ = \sum_{\lambda_i} \sum_{j=1}^3 \int_{x_i} 4\pi \delta(1 - x_1 - x_2 - x_3) \int_{\mathbf{k}_{i\perp}} (2\pi)^2 \delta^{(2)}(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} + \mathbf{k}_{3\perp}) \\ \times \left[\Psi_{\Lambda'}^*(x_i, \mathbf{k}_{i(j)\perp}, \lambda_i) - \frac{1}{2} \sum_{l=1}^3 \sum_{j' \neq j} \Psi_{\Lambda'}^*(x_i, \mathbf{k}_{i(jj')\perp}^{(l)}, \lambda_i) + \sum_{j' \neq j} \sum_{j'' \neq j'} \Psi_{\Lambda'}^*(x_i, \mathbf{k}_{i(jj'j'')\perp}, \lambda_i) \right] \Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i), \end{aligned} \quad (17)$$

where

$$\begin{aligned} \mathbf{k}_{i(j)\perp} &= \mathbf{k}_{i\perp} + (x_i - \delta_{ij})(\mathbf{q}_{1\perp} + \mathbf{q}_{2\perp} + \mathbf{q}_{3\perp}), \\ \mathbf{k}_{i(jj')\perp}^{(l)} &= \mathbf{k}_{i\perp} + (x_i - \delta_{ij})(\mathbf{q}_{1\perp} + \mathbf{q}_{2\perp} + \mathbf{q}_{3\perp}) - (\delta_{ij'} - \delta_{ij})\mathbf{q}_{l\perp}, \\ \mathbf{k}_{i(jj'j'')\perp} &= \mathbf{k}_{i\perp} + x_i(\mathbf{q}_{1\perp} + \mathbf{q}_{2\perp} + \mathbf{q}_{3\perp}) - \delta_{ij}\mathbf{q}_{1\perp} - \delta_{ij'}\mathbf{q}_{2\perp} - \delta_{ij''}\mathbf{q}_{3\perp}. \end{aligned} \quad (18)$$

We note that $G_{3\Lambda'\Lambda}$ is symmetric under the exchange of any two of its transverse momentum arguments. Also, when $\Lambda = \Lambda'$ ($\Lambda \neq \Lambda'$), it is even (odd) under a simultaneous sign flip of all three $\mathbf{q}_{i\perp}$.

In this LCwf model the helicity transfer between the two proton states is made possible by the quark orbital angular momentum originating from the Melosh rotation $\sim \boldsymbol{\sigma}_\perp \times \mathbf{k}_\perp$, where $\boldsymbol{\sigma}_\perp = (\sigma^1, \sigma^2)$ are the transverse Pauli matrices, cf. Ref. [26]. This is a purely kinematical effect, connecting the quark spin to the orbital motion in the relativistic formulation and unrelated to the conventional spin-orbit coupling $L^z S^z$ which is of dynamical origin. The proton wave functions $\Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i)$ contain quark helicity contributions that go beyond the static quark-model spin structure. For example, a helicity $+1/2$ proton may correspond to quark helicities $(+1/2, -1/2, -1/2)$, since the mismatch is compensated by quark orbital angular momentum—the resulting proton wave function is a P wave [see the fourth line in (30)]. As the eikonal operator in (15) does not flip the quark helicities, the proton helicity flip matrix element $G_{3-+}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}, \mathbf{q}_{3\perp})$ relies on wave function overlap that is diagonal in quark helicities, and the outgoing proton with helicity $-1/2$ is in an S wave. In the full expression, the overlaps contain not only S -wave and P -wave interferences, but also P -wave and D -wave interferences. We note the distinction between the off-diagonal matrix element G_{3-+} sourcing the gluon Sivers function and the matrix element for the orbital angular momentum distribution (see, e.g., [35]) which is, by contrast, diagonal in the proton helicity and, in the LCwf model, diagonal in the total parton orbital angular momentum [35].

For forward scattering the first and second terms in Eq. (17), where the three t -channel gluons attach to one or two quarks in the proton, respectively, vanish when $\Lambda' \neq \Lambda$. It is impossible to transfer orbital angular momentum to one or two quarks when the total momentum transfer vanishes. Hence, it is required to consider a system of at least three target partons. Mathematically this comes about in Eq. (17) due to the odd parity of the helicity wave function overlap $\Psi_{\Lambda'}^*(x_i, \mathbf{k}_{i\perp}, \lambda_i) \Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i)$ under a simultaneous sign flip of all quark transverse momenta $\mathbf{k}_{i\perp}$; see Appendix A for explicit expressions for the LC helicity wave functions. For this same reason, the two-gluon exchange matrix element with helicity flip requires nonzero momentum transfer $\Delta_\perp$ [29].

As described thus far, Eq. (1) now pertains to scattering of a dipole from the soft gluon field sourced by the valence quarks Eq. (16). To improve this result, one may consider the emission of a gluon with moderate x from one of the quarks which adds to the fluctuating color charge sources in the proton. Such computations have been performed elsewhere [36,37], albeit for simplified helicity wave functions which did not allow nonzero helicity flip matrix elements of

the eikonal dipole. For moderate $x \simeq 0.1$ and dipole sizes $r_\perp \simeq 0.1\text{--}0.5$ fm, and $\alpha_s \lesssim 0.3$, the corrections have been found to amount to less than a factor of 2. Fixed order perturbative corrections to the helicity flip matrix elements are presently not known but are expected to be of similar magnitude. Finally, we mention that the emission of soft gluons with momentum fractions much less than those of the valence partons shall be resummed into the RG-evolved source via the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation in the following Subsection.

B. BFKL evolution of the helicity flip Odderon

With increasing energy or decreasing x the phase space for gluon emissions opens up. It becomes necessary then to resum pairwise interactions, i.e., gluon exchanges among the three gluons exchanged by the dipole and the proton. Such resummation is achieved by the small- x QCD evolution equation. The nonlinear evolution equation for the Odderon(s) has been derived in Refs. [38,39]. Here, however, we only require the linear BFKL evolution [40–42], since at the initial x_0 both the two- and three-gluon exchange amplitudes (for a perturbative dipole, $r_\perp \ll \Lambda_{\text{QCD}}^{-1}$) are small: $\mathcal{P}_{\Lambda'\Lambda}(\mathbf{r}_\perp), \mathcal{O}_{\Lambda'\Lambda}(\mathbf{r}_\perp) \ll 1$. Nonlinear effects in the evolution equation act to decrease the Odderon amplitude [8,43,44], however this only plays a role when scattering off the proton approaches the unitarity limit, $\mathcal{P}(\mathbf{r}_\perp) \rightarrow 1$. (The evolution with energy of the C -even scattering amplitude of the LC quark model has been studied in Ref. [45]. For a comparison of linear vs nonlinear evolution of the spin independent Odderon see Ref. [44].) Therefore, we expect that the quantitative impact on Odderon related observables will be small for perturbative dipoles, at the energies available at the future EIC.

In the leading $\log 1/x$ approximation, in the framework of Mueller's dipole approach [17] the evolution equation formulated in position space reads

$$\begin{aligned} & \frac{\partial}{\partial Y} \mathcal{O}_{\Lambda'\Lambda}(\mathbf{x}_\perp, \mathbf{y}_\perp) \\ &= \frac{\bar{\alpha}_s}{2\pi} \int_{\mathbf{z}_\perp} \frac{(\mathbf{x}_\perp - \mathbf{y}_\perp)^2}{(\mathbf{x}_\perp - \mathbf{z}_\perp)^2 (\mathbf{z}_\perp - \mathbf{y}_\perp)^2} \\ & \times [\mathcal{O}_{\Lambda'\Lambda}(\mathbf{x}_\perp, \mathbf{z}_\perp) + \mathcal{O}_{\Lambda'\Lambda}(\mathbf{z}_\perp, \mathbf{y}_\perp) - \mathcal{O}_{\Lambda'\Lambda}(\mathbf{x}_\perp, \mathbf{y}_\perp)]. \end{aligned} \quad (19)$$

Here, $\bar{\alpha}_s = \alpha_s N_c / \pi$ and $Y = \log x_0 / x$ where $x_0 \sim 0.1$ corresponds to the initial condition. For dipole scattering with Odderon exchange, the above is equivalent to the Bartels-Kwiecinski-Praszalowicz linear evolution equation [46,47], see the discussion in Ref. [38].

Assuming translational invariance, $\mathcal{O}_{\Lambda'\Lambda}(\mathbf{x}_\perp, \mathbf{y}_\perp) \rightarrow \mathcal{O}_{\Lambda'\Lambda}(\mathbf{x}_\perp - \mathbf{y}_\perp)$, and integrating over the impact parameter $\mathbf{b}_\perp = (\mathbf{x}_\perp + \mathbf{y}_\perp)/2$ (i. e. taking $\Delta_\perp \rightarrow 0$), we arrive at the evolution equation for the forward matrix element,

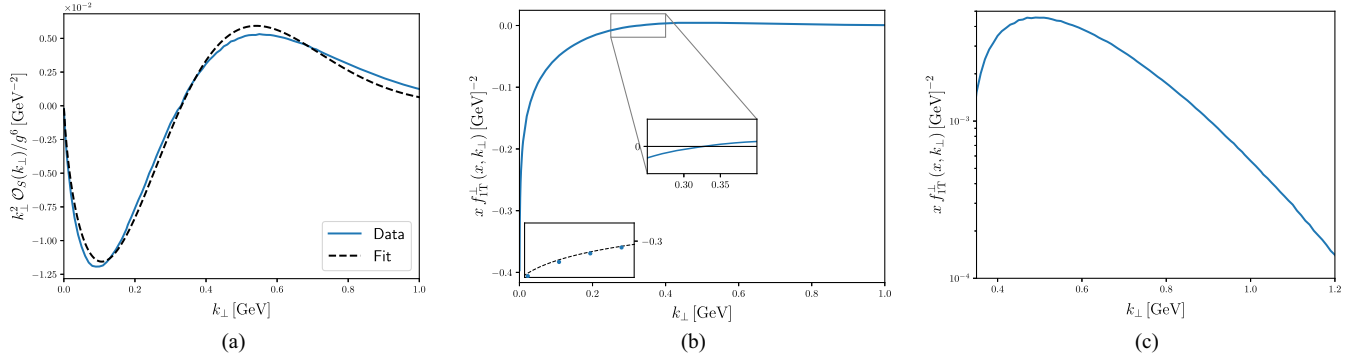


FIG. 1. (a) Forward Odderon $k_{\perp}^2 \mathcal{O}_S(k_{\perp})/g^6$ in transverse momentum space. The dashed line shows the best fit via Eq. (23). (b) and (c) Gluon Siverson function $x f_{1T}^+(x, k_{\perp})$ obtained from the helicity flip, forward Odderon for $\alpha_s = 0.25$. The dashed black line in the lower inset emphasizes the log-type divergence at low- $k_{\perp}$, see also (23).

$$\frac{\partial}{\partial Y} \mathcal{O}_{\Lambda\Lambda}(\mathbf{r}_{\perp}) = \frac{\bar{\alpha}_s}{2\pi} \int_{s_{\perp}} \frac{\mathbf{r}_{\perp}^2}{(\frac{\mathbf{r}_{\perp}}{2} - \mathbf{s}_{\perp})^2 (\frac{\mathbf{r}_{\perp}}{2} + \mathbf{s}_{\perp})^2} \left[\mathcal{O}_{\Lambda\Lambda}\left(\frac{\mathbf{r}_{\perp}}{2} - \mathbf{s}_{\perp}\right) + \mathcal{O}_{\Lambda\Lambda}\left(\frac{\mathbf{r}_{\perp}}{2} + \mathbf{s}_{\perp}\right) - \mathcal{O}_{\Lambda\Lambda}(\mathbf{r}_{\perp}) \right]. \quad (20)$$

We note that aside from the trivial fixed point $\mathcal{O}_{\Lambda\Lambda}(\mathbf{r}_{\perp}) = 0$, the equation also possesses a nontrivial fixed point for an Odderon that is linear in the dipole size: $\mathcal{O}_{\Lambda\Lambda}(\mathbf{r}_{\perp}) \sim \mathbf{S}_{\perp} \times \mathbf{r}_{\perp}$. To our knowledge this fixed point of the helicity flip Odderon has not been pointed out before. Deep in the perturbative regime of very small dipoles, however, the initial condition will be of cubic order in $\mathbf{r}_{\perp}$, as already alluded to above. The BFKL resummation is expected to generate an anomalous dimension $\gamma(Y) < 1$, which we determine numerically in Sec. III below.

Equation (20) can be simplified using translational and rotational invariance: we fix $\mathbf{r}_{\perp} = (r_{\perp} \cos \phi_r, r_{\perp} \sin \phi_r) = (r_{\perp}, 0)$ in the polar coordinates. The angular dependence of $\mathcal{O}_{\Lambda\Lambda}(\mathbf{r}_{\perp})$ is given by (7)¹:

$$\mathcal{O}_{-+}(\mathbf{r}_{\perp}) = i e^{i\phi_r} \mathcal{O}_S(r_{\perp}), \quad (21)$$

where the radial part $\mathcal{O}_S(r_{\perp})$ is real and satisfies the equation,

$$\begin{aligned} & \frac{\partial}{\partial Y} \mathcal{O}_S(Y, r_{\perp}) \\ &= \frac{\bar{\alpha}_s}{2\pi} \int_{z_{\perp}} \frac{\mathbf{r}_{\perp}^2}{(\mathbf{r}_{\perp} - \mathbf{z}_{\perp})^2 \mathbf{z}_{\perp}^2} \left[\frac{r_{\perp} - z_{\perp} \cos \phi_z}{|\mathbf{r}_{\perp} - \mathbf{z}_{\perp}|} \mathcal{O}_S(Y, |\mathbf{r}_{\perp} - \mathbf{z}_{\perp}|) \right. \\ & \quad \left. + \cos \phi_z \mathcal{O}_S(Y, z_{\perp}) - \mathcal{O}_S(Y, r_{\perp}) \right]. \end{aligned} \quad (22)$$

Here we explicitly indicate that $\mathcal{O}_S$ depends on the rapidity Y ; the initial condition at $Y = 0$ is given by the perturbative three gluon exchange of Eq. (4) with $\Lambda' = -\Lambda$ and $\Delta_{\perp} = 0$, using the three gluon to proton coupling $G_{3\Lambda\Lambda}$ obtained via

¹Similar formulas hold for $\mathcal{O}_{\Lambda\Lambda}(\mathbf{k}_{\perp})$.

Eq. (17) from the nonperturbative model LCwf. In the numerical implementation, we shall use $\alpha_s = 0.25$. At the current order the coupling enters the Siverson function only as an overall factor, $x f_{1T}^+(x, k_{\perp}) \sim \alpha_s^2$, and as a scale factor in the evolution “time” $t \sim \alpha_s Y$.

III. NUMERICAL RESULTS

In this Section we present and discuss our numerical results. We shall first address the helicity flip Odderon and the associated gluon Siverson function obtained from the perturbative computation of three gluon exchange. Then, we illustrate the properties of that Siverson function (such as its magnitude, sign definiteness, etc.) by the example of a specific physical process. Lastly, we show the effect of BFKL resummation of contributions $\sim (\alpha_s \log x_0/x)^n$. The figures shown in this Section were obtained with the exponential quark-level LCwf, see Appendix A. We have verified that the alternative power-law wave function of Appendix A produced very similar results.

Figure 1(a) shows the transverse momentum dependence of the forward, helicity flip Odderon at x_0 . We may guess the following functional behavior of the helicity flip Odderon:

$$k_{\perp}^2 \frac{|\mathcal{O}_{-+}(\mathbf{k}_{\perp})|}{g^6} = \mathcal{N} \frac{k_{\perp}}{Q_b} \log\left(\frac{k_{\perp}}{Q_b}\right) e^{-k_{\perp}^2/(4Q_c^2)}. \quad (23)$$

The best fit parameters for the exponential wave function in Appendix A are $\mathcal{N} = 0.036 \text{ GeV}^{-2}$, $Q_b = 0.331 \text{ GeV}$, $Q_c = 0.220 \text{ GeV}$. Note that the fit is not meant to describe the analytic form beyond $k_{\perp} \simeq 1 \text{ GeV}$.

Figure 1(b) shows the Siverson function in the regime of low $k_{\perp}$. The inset depicts its logarithmic divergence for

very low transverse momenta, which is evident from (23). Such logarithmic behavior at low $k_\perp$ is not manifest in the Sivers function from the eikonal, “cubic Casimir extended” MV model [7,8]. It arises from Eq. (5), in the $\Delta_\perp \rightarrow 0$ limit, if the three-gluon exchange is dominated by the exchange of two fairly hard gluons with nearly anticollinear transverse momenta, and a softer third gluon; and if in that regime the helicity flip three gluon coupling to the proton is $G_{3-+}(k_\perp, q_{2\perp}, -q_{2\perp}) \sim k_L q_{2R} q_{2L} = k_L q_{2\perp}^2$.² The integration over $q_{2\perp}$ will then result in a logarithmic behavior of the Sivers function. Intuitively, such behavior of G_{3-+} , which corresponds to the exchange of vector gauge bosons, appears natural. It is similar to the way in which the electromagnetic form factor F_{+-} is proportional to the transverse momentum of the exchanged photon, see Eq. (B8). However, this point may warrant more detailed investigation in the future.

Since $xf_{1T}^\perp(x, k_\perp)$, in the nonperturbative regime of low $k_\perp$, may not be suppressed $\sim k_\perp^2$ but may actually diverge logarithmically, therefore caution is advised with observables such as single spin asymmetries which integrate $xf_{1T}^\perp(x, k_\perp)$ over $k_\perp$. On the other hand, the collinear function $xf_{1T}^{\perp(1)}(x, \mu^2)$ is safe from logarithmic low- $k_\perp$ contributions, see Eq. (14).

The behavior for higher $k_\perp$ is shown in Fig. 1(c), on a logarithmic scale. The Sivers function peaks at about $k_\perp \simeq 0.5$ GeV where its magnitude is $(4-5) \times 10^{-3}$ GeV⁻². In the neighborhood of $k_\perp = 1$ GeV it falls off from a magnitude of about 10^{-3} GeV⁻² at $k_\perp = 0.9$ GeV to about 10^{-4} GeV⁻² at $k_\perp = 1.2$ GeV. Beyond that, the function is expected to exhibit a perturbative power-law tail $\sim 1/k_\perp^4$ which we mention below in conjunction with BFKL-resummed solutions.

In Fig. 2 we compare our model prediction for the eikonal gluon Sivers function $xf_{1T}^\perp(x, k_\perp)$ to other models from the literature. In particular, Ref. [48] performed a fit to transverse single spin asymmetry measurements in $pp \rightarrow \pi^0 X$ at RHIC [49]. Their “SIDIS2” fit involves an ansatz based on the generalized parton model approach whereby $f_{1T}^\perp(x, k_\perp)$ is parametrized in terms of the unpolarized collinear gluon distribution $f(x)$ times a Gaussian in $k_\perp$, and constrained by the Burkardt sum rule [50]. This trivially satisfies the positivity bound [$f_{1T}^\perp(x, k_\perp) \leq 2f(x)$], albeit at the cost of lacking a node in $k_\perp$. On the other hand, the low- x dipole approach of Ref. [8] leaves the sign and overall magnitude arbitrary but explicitly enforces a node via the initial condition $\mathcal{O}_S(r_\perp) \sim r_\perp^3 e^{-r_\perp^2 Q_{s0}^2}$ for the dipole Odderon, where $Q_{s0} = 0.7$ GeV was used for the saturation scale. Since the node structure in $k_\perp$ is unchanged by BFKL evolution, we restrict our comparison in Fig. 2 to the initial conditions at $Y = 0$. Lastly, we note that around $k_\perp \simeq 0.4$ GeV the Sivers function obtained here is suppressed

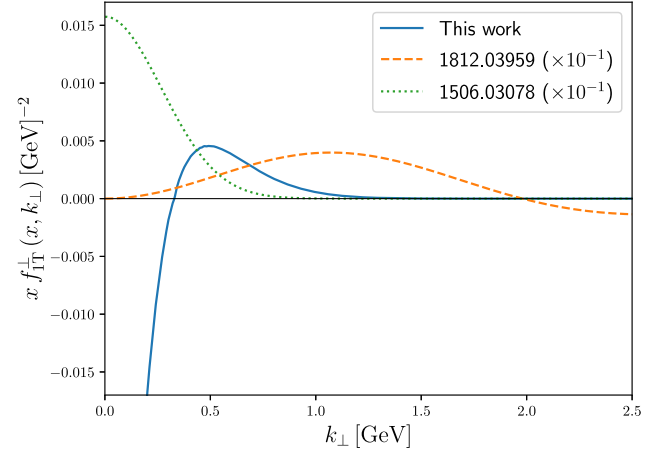


FIG. 2. Comparison of the eikonal gluon Sivers function $xf_{1T}^\perp(x, k_\perp)$ to the fit and model of Refs. [48] (green dotted) and [8] (orange dashed), respectively, with adjusted scales for easier comparison.

by about one order of magnitude as compared to the model by D’Alesio *et al.* [48], albeit this discrepancy diminishes with increasing $k_\perp$.

We scrutinize the phenomenological implications of the Sivers function we obtained by considering a specific physical process, i.e., the exclusive forward production of χ_{c1} axial vector mesons on unpolarized protons [20]. (Other observables for the spin dependent Odderon have been mentioned in the Introduction, and shall be considered in the future.) In the nonrelativistic limit this process probes the trigluon PDF of the proton (14). However, it occurs both due to the spin-dependent Odderon as well as due to photon exchange, a Primakoff-like process. For an axial-vector state such as the χ_{c1} the latter cross section does not exhibit the usual Coulomb singularity at $|t| \rightarrow 0$ thanks to the Landau-Yang theorem [44]. The finite ratio of three-gluon versus photon exchange cross sections in the nonrelativistic heavy-quark and collinear limits is [20]

$$r(x, \mu^2) = \frac{4\pi^2 \alpha_s^2 M_p^2}{q_c^2 N_c^2 \alpha^2 M_\chi^2} |xf_{1T}^{\perp(1)g}(x, \mu^2)|^2. \quad (24)$$

The scale μ should be in the range of $M_\chi/2 - M_\chi$, and $Q^2 = 0$ has been assumed. Our numerical evaluation of the function $xf_{1T}^{\perp(1)g}(x, \mu^2)$ results in a ratio to Primakoff of only $r \approx 10^{-6}$. This very small value of r at $x \simeq 0.1$ is due to substantial cancellations in the $k_\perp^2$ moment (14) of the Sivers function which satisfies the sum rule (13). Based on the “SIDIS2” and “5% positivity” models of Refs. [48,19], respectively, the r -ratio estimate of Ref. [20] at $x \simeq 0.1$ was three to four orders of magnitude higher. The small magnitude of the trigluon PDF obtained here implies that forward exclusive photoproduction produces transversely polarized χ_{c1} [20].

² $q_{R/L} \equiv q_x \pm iq_y$.

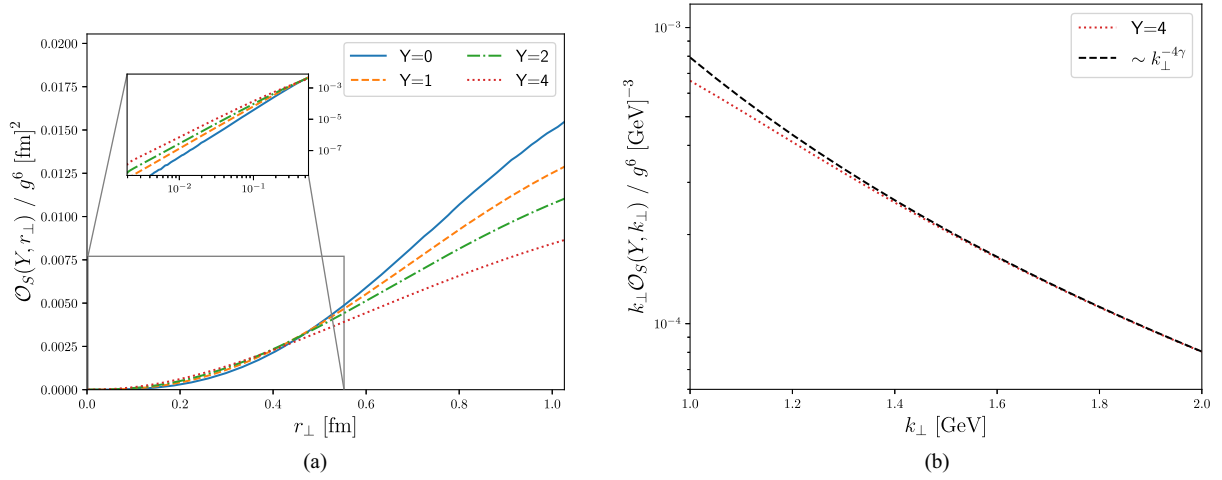


FIG. 3. (a) Radial part of the helicity flip Odderon amplitude at different rapidities, $Y = 0, 1, 2, 4$. The inset is a double logarithmic version of the plot to better show the small- $r_\perp$ regime. (b) High- $k_\perp$ tail of the momentum-space Odderon amplitude $k_\perp \mathcal{O}_S(k_\perp)$, which is proportional to $xf_{1T}^{\perp g}(x, k_\perp^2)$, at $Y = 4$. The dashed line indicates the $\sim 1/k_\perp^{3.3}$ power-law asymptotics.

We now proceed to discuss the solution of the BFKL equation. The above initial condition exhibits long tails toward large dipoles which complicate the numerical solution of Eq. (22). Therefore, we multiply the initial condition (4) by a Gaussian suppression factor,

$$\mathcal{O}_S(Y=0, r_\perp) \rightarrow e^{-r_\perp^2 \mu^2} \mathcal{O}_S(Y=0, r_\perp). \quad (25)$$

This “cutoff” suppresses contributions from large $r_\perp$ that correspond to nonperturbative, large dipoles. We choose $\mu = 0.1$ GeV, so that our $\mathcal{O}_S(Y=0, r_\perp)$ is not altered much for $r_\perp < 5$ GeV $^{-1}$, which is the region we shall focus on.

In Fig. 3(a), we show (the radial part of) the helicity flip Odderon amplitude for several values of rapidity Y : 0 (solid blue), 1 (dashed orange), 2 (dash-dotted green) and 4 (dotted red). We observe that small- x evolution increases $\mathcal{O}_S(Y, r_\perp)$ at small $r_\perp \lesssim 0.45$ fm, while it suppresses it for greater $r_\perp \gtrsim 0.45$ fm; this point corresponds to the fixed point of linear $\mathcal{O}_S(r_\perp)$ mentioned above. The suppression at moderately large $r_\perp$ was already noted in Ref. [8]. We find that the asymptotic behavior at small $r_\perp$, $\mathcal{O}_S(Y, r_\perp) \sim r_\perp^{3\gamma(Y)}$, changes with evolution: the anomalous dimension $\gamma(Y)$ decreases with Y , starting from $\gamma(Y=0) = 1$ for perturbative three gluon exchange and reaching $\gamma(Y=4) \approx 0.8$.

Figure 3(b) shows the high- $k_\perp$ power-law tail of the spin dependent Odderon times $k_\perp$; this is proportional to the gluon Siverson function $xf_{1T}^{\perp g}(x, k_\perp^2)$. At the initial rapidity $Y=0$, to exhibit the power-law tail of the perturbative three gluon exchange, we require a small anomalous dimension $1-\epsilon$ for $\mathcal{O}_S(r_\perp)$; this then leads to $xf_{1T}^{\perp g}(x, k_\perp^2) \sim 1/k_\perp^4$. Comparing to unpolarized gluon TMD with a perturbative $\sim 1/k_\perp^2$ tail, the stronger falloff of the gluon Siverson function is due to its connection to the Odderon, which is a three gluon exchange. This is similar to diffractive TMDs which are

described by four gluon exchanges, also leading to a stronger $\sim 1/k_\perp^4$ tail [51,52].

At $Y=4$ (for $\alpha_s = 0.25$) the power-law tail of the solution of BFKL becomes $\sim 1/k_\perp^{4\gamma(Y)}$ with $\gamma(Y) \approx 0.8$. This value of the anomalous dimension agrees with the position space behavior of $\mathcal{O}_S(Y, r_\perp)$ at small $r_\perp$. For asymptotic rapidity, $\gamma(Y)$ approaches the standard LO BFKL anomalous dimension, $\gamma = 0.5$. Numerically, we find that this requires $Y \gtrsim 16$ where nonlinear unitarity effects are expected to become important. Hence, we find it potentially more relevant to quote the value of γ at $\alpha_s Y = 1$ where BFKL evolution has made a substantial yet preasymptotic effect on the high- $k_\perp$ tail.

IV. SUMMARY AND DISCUSSION

To date the most commonly used parametrization for the gluon Siverson function of the proton is the model by D’Alesio *et al.* [48]. However, this model does not originate from the correspondence at small x to the helicity flip Odderon, and is not well suited as an initial condition for BFKL evolution. Hence, it is not straightforward to determine from it the BFKL anomalous dimension for the small- x gluon Siverson function.

Our goal here was to develop an alternative based on the eikonal approach, and on an effective quark model LCwf for the proton. Such a model incorporates empirical knowledge of the proton at moderate energy and transverse resolution scales, and can be tuned to provide a satisfactory description of Dirac and Pauli form factors. In particular, the model specifically involves sources (for the soft t -channel gluons exchanged in high-energy scattering) in the fundamental representations of both color-SU(3) and spin-SU(2).

The above light-cone quark model for the proton enables the explicit computation of the matrix element of the

eikonal dipole operator between proton states of opposite helicity. While the C -even part of this matrix element requires nonzero momentum transfer [29], the C -odd part is finite even in the limit of forward scattering, where it defines the “spin dependent Odderon” and the gluon Sivers function [6–9], a transverse momentum dependent parton distribution.

Our main results for the small- x dipole gluon Sivers function are as follows. We confirm the statements in Refs. [7,8] that $xf_{1T}^\perp(x, k_\perp)$, as a function of transverse momentum $k_\perp$, is not sign definite. This leads to substantial cancellations in the first $k_\perp^2$ moment of the Sivers function, $xf_{1T}^{\perp(1)}(x, \mu^2)$, which is related to the twist three collinear function frequently denoted the “tri-gluon PDF” $O(x, \mu^2)$; this is true especially for relatively low scales μ of order 1–3 GeV. A related phenomenological consequence is that forward exclusive photoproduction of χ_{c1} axial vector mesons is dominated by photon rather than Odderon exchange [20].

We provide the following parametrization of the eikonal gluon Sivers function at $x = x_0 \sim 0.1$:

$$k_\perp xf_{1T}^\perp(x, k_\perp) = \frac{24M_p}{\pi} \alpha_s^2 \mathcal{N} \frac{k_\perp}{Q_b} \log\left(\frac{k_\perp}{Q_b}\right) e^{-k_\perp^2/(4Q_b^2)}. \quad (26)$$

The fit parameters are $\mathcal{N} = 0.036 \text{ GeV}^{-2}$, $Q_b = 0.331 \text{ GeV}$, $Q_c = 0.220 \text{ GeV}$.³ This fit does not apply beyond $k_\perp \sim 1 \text{ GeV}$.

The gluon Sivers function in this model exhibits a peak (at $k_\perp > Q_b$) at a transverse momentum scale typical of the quarks in the proton. This is unlike the parametrization of $xf_{1T}^\perp(x, k_\perp)$ by D’Alesio *et al.* [48] which is a Gaussian, and has its maximum at $k_\perp = 0$. The peak obtained in our model resembles the one found in saturation models of the Odderon [8] albeit its physical origin in our purely perturbative computation of three gluon exchange is rather different. The forward matrix element, in the proton, of the *eikonal* dipole operator for reversed helicity, involves orbital angular momentum (OAM) of the sources of the soft gluons: despite the fact that the eikonal dipole operator conserves the helicities of the individual quarks, a proton helicity flip is possible due to nonvanishing overlap of quark configurations which differ by their orbital angular momentum. The peak of $xf_{1T}^\perp(x, k_\perp)$ corresponds to the scale where the transfer of OAM between initial and final proton state is most efficient. Numerically, for the model LCwf used here the peak occurs at about $k_\perp \simeq 0.5 \text{ GeV}$ where the magnitude of $xf_{1T}^\perp(x, k_\perp)$ is $(4\text{--}5) \times 10^{-3} \text{ GeV}^{-2}$, assuming $\alpha_s = 0.25$. While a comprehensive quantitative analysis of uncertainties is beyond the scope of this paper, we do recall that $\mathcal{O}(\alpha_s)$

perturbative corrections to the nonperturbative three quark LCwf could increase the magnitude of the gluon Sivers function at $x = x_0 = 0.1$ by up to a factor of two. Rescaling the coupling changes $xf_{1T}^\perp(x, k_\perp)$ at that initial x_0 by a factor of α_s^2 . Lastly, variation of the LC quark model wave function from exponential to power law, both constrained by proton form factors, was found to modify the Sivers function by factors of up to 1.5.

For small $k_\perp$ we obtain a logarithmic singularity of the Sivers function rather than power-law decay [8] or approach to a finite constant [48]. Hence, there could be potentially significant contributions from small, nonperturbative transverse momentum imbalances to charm single spin asymmetries. These would also be sensitive to parton shower effects and to transverse momentum acquired in parton fragmentation [53–56]. The resulting joint resummation framework is known to kinematically constrain small- x radiations [57,58], which would be important and interesting not only for quantitative phenomenology on single spin asymmetries for open charm at the EIC but also in view of the qualitative features of the gluon Sivers function found here.

The BFKL evolution of the spin dependent Odderon in this model generates an anomalous dimension of its power-law tail. At high $k_\perp \gtrsim 1.5 \text{ GeV}$ we find that $xf_{1T}^\perp(x, k_\perp) \sim k_\perp^{-4\gamma(Y)}$, with $\gamma(Y) \simeq 0.8$ at $Y \equiv \log x_0/x = 1/\alpha_s$. Also, given the above initial condition, we observe that the magnitude of the perturbative small- r tail of the BFKL spin-dependent Odderon $\mathcal{O}_s(Y, r_\perp)$ increases with Y as $Y \rightarrow 1/\alpha_s$. The nonforward, spin- independent dipole Odderon amplitude off the proton also increases with decreasing x if a gluon is added into the three quark wave function [59]. Asymptotically, the BFKL resummed Odderon amplitudes are expected to be (at most) independent of energy [38].

ACKNOWLEDGMENTS

We thank Leszek Motyka for collaboration in the initial stages of this work and Edmond Iancu for comments on the manuscript. T.S. thanks Piotr Korcyl for sharing his evolution code and Florian Cougoulic for discussions. S.B. was supported by the European Union – NextGenerationEU through the National Recovery and Resilience Plan 2021–2026 Institutional grants of University of Zagreb Faculty of Science (PMF-CROFUND). A.D. acknowledges support by the DOE Office of Nuclear Physics through Grant No. DE-SC0002307. F.H. is funded by the Austrian Science Fund (FWF) [10.55776/J4854]. T.S. kindly acknowledges the support of the Polish National Science Center (NCN) Grant No. 2021/43/D/ST2/03375.

DATA AVAILABILITY

To promote open and reproducible research, the data and computer code used to generate the numerical results and

³Allowing the exponent of $(k_\perp/Q_b) \rightarrow (k_\perp/Q_b)^a$ to vary as well, we obtain a statistically more significant fit with $\mathcal{N} = 0.029 \text{ GeV}^{-2}$, $a = 0.901$, $Q_b = 0.333 \text{ GeV}$, $Q_c = 0.232 \text{ GeV}$, albeit at the cost of an additional parameter.

figures in this paper are publicly available via Zenodo [60] and GitHub [61].

APPENDIX A: LC CONSTITUENT QUARK MODEL WAVE FUNCTION

In this appendix we present the detailed form of the three-quark LC wave function $\Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i)$ used in our numerical evaluation of the Odderon. We take the momentum space integrals in Eq. (16) as follows:

$$[dx_i] = 4\pi\delta(1 - x_1 - x_2 - x_3) \prod_{i=1\dots 3} \frac{dx_i}{4\pi\sqrt{x_i}},$$

$$[d^2\mathbf{k}_{i\perp}] = (2\pi)^2\delta(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} + \mathbf{k}_{3\perp}) \prod_{i=1\dots 3} \frac{d^2\mathbf{k}_{i\perp}}{(2\pi)^2}. \quad (\text{A1})$$

Next, we write $\Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i)$ as a product of a LC helicity wave function and a spatial (momentum space) wave function,

$$\Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i) = \Phi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i)\psi(x_i, \mathbf{k}_{i\perp}). \quad (\text{A2})$$

For the spatial wave function we use a simple model due to Schlumpf [62,63] whereby the suppression of the Fock space amplitude away from the minimal configuration is governed by the invariant mass (squared) of the three-quark system, $\mathcal{M}^2 = \sum_i(\mathbf{k}_{i\perp}^2 + m^2)/x_i$. Schlumpf considered

both a power-law as well as an exponential wave function

$$\psi_{\text{exp}}(x_i, \mathbf{k}_{i\perp}) = \mathcal{N}_{\text{exp}} e^{-\mathcal{M}^2/2\beta^2},$$

$$\psi_{\text{pwr}}(x_i, \mathbf{k}_{i\perp}) = \mathcal{N}_{\text{pwr}} \left(1 + \frac{\mathcal{M}^2}{\beta^2}\right)^{-p}, \quad (p = 3.5). \quad (\text{A3})$$

The effective quark mass m and the parameter β have been tuned in Refs. [62,63] to reproduce the electromagnetic “radius” and the anomalous magnetic moment of the proton. In the subsequent Appendix B we provide an update of these parameters in view of more recent data on the electromagnetic form factors. The normalization $\mathcal{N}_{\text{exp/pwr}}$ of the spatial wave function follows from the requirement that $\langle K, \Lambda' | P, \Lambda \rangle = 16\pi^3 P^+ \delta(P^+ - K^+) \delta^{(2)}(\mathbf{P}_\perp - \mathbf{K}_\perp) \delta_{\Lambda\Lambda'}$. The results of Sec. III of the main text were obtained with the exponential wave function. The power-law wave function did not produce large differences in observables where the quark transverse momenta involved were limited to about 1–1.5 GeV.

The helicity wave functions $\Phi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i)$ in (16) for a proton with helicity $\Lambda/2$ are obtained through a Melosh transformation of rest frame Pauli spinors to the LF. Their explicit expressions for $\Lambda = +1$ are given in Ref. [14], which we have confirmed. For completeness, we also list them here:

$$\begin{aligned} \Phi_\uparrow(\uparrow, \uparrow, \downarrow) &= \frac{1}{\sqrt{N_1 N_2 N_3}} \frac{1}{\sqrt{6}} (2a_1 a_2 a_3 + a_1 k_{2L} k_{3R} + k_{1L} a_2 k_{3R}), \\ \Phi_\uparrow(\uparrow, \downarrow, \uparrow) &= \frac{1}{\sqrt{N_1 N_2 N_3}} \frac{1}{\sqrt{6}} (-a_1 a_2 a_3 + k_{1L} k_{2R} a_3 - 2a_1 k_{2R} k_{3L}), \\ \Phi_\uparrow(\downarrow, \uparrow, \uparrow) &= \frac{1}{\sqrt{N_1 N_2 N_3}} \frac{1}{\sqrt{6}} (-a_1 a_2 a_3 + k_{1R} k_{2L} a_3 - 2k_{1R} a_2 k_{3L}), \\ \Phi_\uparrow(\uparrow, \downarrow, \downarrow) &= \frac{1}{\sqrt{N_1 N_2 N_3}} \frac{1}{\sqrt{6}} (a_1 a_2 k_{3R} - 2a_1 k_{2R} a_3 - k_{1L} k_{2R} k_{3R}), \\ \Phi_\uparrow(\downarrow, \uparrow, \downarrow) &= \frac{1}{\sqrt{N_1 N_2 N_3}} \frac{1}{\sqrt{6}} (a_1 a_2 k_{3R} - 2k_{1R} a_2 a_3 - k_{1R} k_{2L} k_{3R}), \\ \Phi_\uparrow(\downarrow, \downarrow, \uparrow) &= \frac{1}{\sqrt{N_1 N_2 N_3}} \frac{1}{\sqrt{6}} (a_1 k_{2R} a_3 + k_{1R} a_2 a_3 + 2k_{1R} k_{2R} k_{3L}), \\ \Phi_\uparrow(\uparrow, \uparrow, \uparrow) &= \frac{1}{\sqrt{N_1 N_2 N_3}} \frac{1}{\sqrt{6}} (-a_1 k_{2L} a_3 - k_{1L} a_2 a_3 + 2a_1 a_2 k_{3L}), \\ \Phi_\uparrow(\downarrow, \downarrow, \downarrow) &= \frac{1}{\sqrt{N_1 N_2 N_3}} \frac{1}{\sqrt{6}} (-a_1 k_{2R} k_{3R} - k_{1R} a_2 k_{3R} + 2k_{1R} k_{2R} a_3), \end{aligned} \quad (\text{A4})$$

with $a_i = m + x_i \mathcal{M}$ and $N_i \equiv \mathbf{k}_{i\perp}^2 + a_i^2$ and $(\uparrow, \downarrow) \equiv (+, -)$ in our notation. These helicity wave functions pertain to the $|uud\rangle$ state; there is no need here to consider quark flavor permutations since all the matrix elements we are interested in are “flavor blind.”

The helicity wave functions for $\Lambda = -1$ are obtained by a sign flip of Φ , and the exchange $k_{iL} \leftrightarrow -k_{iR}$, where $k_{R/L} = k_\perp^1 \pm ik_\perp^2 = k_\perp e^{\pm i\phi_k}$, and $\lambda_i \rightarrow -\lambda_i$.

APPENDIX B: DIRAC AND PAULI ELECTROMAGNETIC FORM FACTORS

To exhibit the connection of the form factors to generalized parton distributions we start from their usual definitions

$$P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P' \Lambda' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | P \Lambda \rangle = \bar{u}(P', \Lambda') \left[\gamma^+ H_q(x, \xi, \Delta_\perp) + \frac{i\sigma^{+\nu} \Delta_\nu}{2M_p} E_q(x, \xi, \Delta_\perp) \right] u(P, \Lambda), \quad (\text{B1})$$

where $\Delta = P' - P$. We integrate this over x and take the eikonal limit of vanishing skewness, $\xi \rightarrow 0$ or $P'^+ \rightarrow P^+$,

$$\langle P' \Lambda' | \bar{\psi}(0) \gamma^+ \psi(0) | P \Lambda \rangle = \bar{u}(P', \Lambda') \left[\gamma^+ F_1^q(\Delta_\perp) + \frac{i\sigma^{+\nu} \Delta_\nu}{2M_p} F_2^q(\Delta_\perp) \right] u(P, \Lambda), \quad (\text{B2})$$

where

$$F_1^q = \int dx H_q, \quad F_2^q = \int dx E_q, \quad (\text{B3})$$

are form factors per flavor.

Equation (B2) involves a local quark current. Its Fourier transform at $q^+ \rightarrow 0$ defines the eikonal LC charge operator

$$\sum_f e_f \int dz^- \int d^2z_\perp e^{iq^+z^- - iq_\perp \cdot z_\perp} \bar{\psi}_f(z) \gamma^+ \psi_f(z) = \int dz^- e^{iq^+z^-} J^+(z^-, \mathbf{q}_\perp) \approx \int dz^- J^+(z^-, \mathbf{q}_\perp) \equiv \rho(\mathbf{q}_\perp). \quad (\text{B4})$$

This charge operator can be written in terms of quark creation and annihilation operators [30] in order to evaluate explicitly the matrix element on the lhs of (B2). This leads to

$$\langle P' \Lambda' | \rho(\mathbf{q}_\perp) | P \Lambda \rangle = 2P^+ (2\pi) \delta(P'^+ - P^+) (2\pi)^2 \delta^{(2)}(\mathbf{q}_\perp + \Delta_\perp) F_{\Lambda' \Lambda}(\Delta_\perp), \quad (\text{B5})$$

where

$$F_{\Lambda' \Lambda}(\Delta_\perp) = \sum_{\lambda_i} \int_{x_i} 4\pi \delta(1 - x_1 - x_2 - x_3) \int_{\mathbf{k}_{i\perp}} (2\pi)^2 \delta(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} + \mathbf{k}_{3\perp}) [e_d \Psi_{\Lambda'}^*(x_i, \mathbf{k}_{i\perp} - x_i \Delta_\perp + \delta_{i3} \Delta_\perp, \lambda_i) \Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i) + e_u \Psi_{\Lambda'}^*(x_i, \mathbf{k}_{i\perp} - x_i \Delta_\perp + \delta_{i2} \Delta_\perp, \lambda_i) \Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i) + e_u \Psi_{\Lambda'}^*(x_i, \mathbf{k}_{i\perp} - x_i \Delta_\perp + \delta_{i1} \Delta_\perp, \lambda_i) \Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i)], \quad (\text{B6})$$

corresponds to a kind of helicity form factor. $F_{\Lambda' \Lambda}$ is related to F_1 and F_2 via

$$F_{\Lambda' \Lambda}(\Delta_\perp) = F_1(\Delta_\perp) \delta_{\Lambda' \Lambda} + \frac{\Delta_\perp}{2M_p} F_2(\Delta_\perp) \Lambda e^{i\Lambda \phi_\Delta} \delta_{\Lambda, -\Lambda'}, \quad (\text{B7})$$

or

$$F_1(\Delta_\perp) = F_{++}(\Delta_\perp) = F_{--}(\Delta_\perp), \quad \frac{1}{2M_p} F_2(\Delta_\perp) \Delta_R = F_{-+}(\Delta_\perp), \quad -\frac{1}{2M_p} F_2(\Delta_\perp) \Delta_L = F_{+-}(\Delta_\perp). \quad (\text{B8})$$

Importantly, the Pauli form factor of the *eikonal* current does not vanish thanks to the fact that nonzero overlaps of the type $\Psi_{-\Lambda}^*(x_i, \mathbf{k}'_{i\perp}, \lambda_i) \Psi_\Lambda(x_i, \mathbf{k}_{i\perp}, \lambda_i)$ exist. The helicity of the proton may flip due to the transfer of orbital angular momentum to its constituents.

Figure 4 shows our best fit to the experimental parametrization from Ref. [64], achieved by variation of the two parameters β and m in the three-quark LCwf. In particular, we obtain $m = 0.28$ GeV, $\beta = 0.88$ GeV, and $p = 3.5$ for the powerlike wavefunction, and $m = 0.24$ GeV, $\beta = 0.7$ GeV for the exponential wavefunction, respectively.

For reference, their normalizations are given by $\mathcal{N}_{pwr} = 87088.44$ GeV⁻² and $\mathcal{N}_{exp} = 7389.46$ GeV⁻², respectively. We note a satisfactory description of the electromagnetic form factors. For Q^2 greater than about 4 GeV² the model under-shoots the measured charge density $F_1(Q^2)$ somewhat. This could perhaps be improved in the future by incorporating perturbative QCD corrections, which tend to increase quark transverse momenta. Indeed, the model with the one gluon emission corrections and BFKL resummation has been shown to reproduce fairly well the inclusive charm cross section measured at HERA [45].

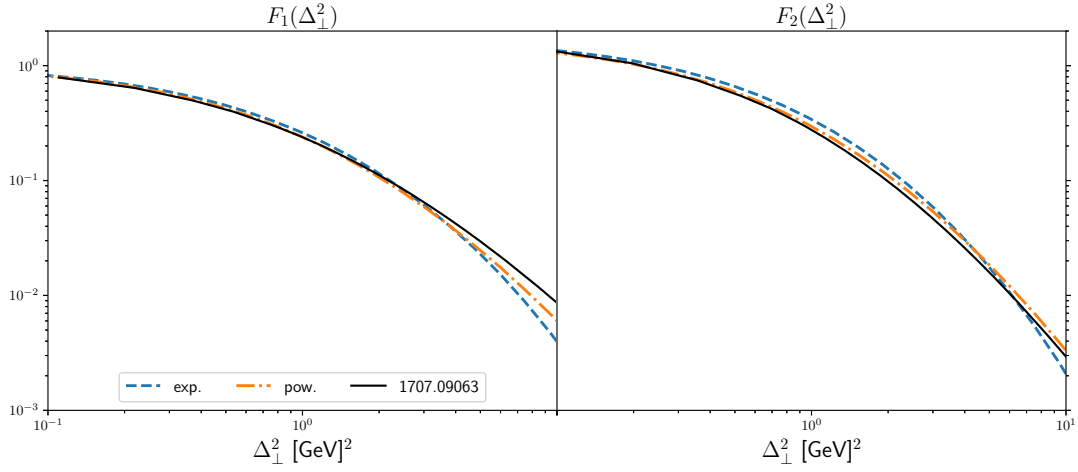


FIG. 4. Dirac (left) and Pauli (right) electromagnetic form factors obtained from the three-quark LCwf compared to a modern fit (solid black line) to experimental data [64].

-
- [1] D. W. Sivers, Single spin production asymmetries from the hard scattering of point-like constituents, *Phys. Rev. D* **41**, 83 (1990).
 - [2] R. Boussarie *et al.*, TMD handbook, [arXiv:2304.03302](#).
 - [3] D. Boer, C. Lorcé, C. Pisano, and J. Zhou, The gluon sivers distribution: Status and future prospects, *Adv. High Energy Phys.* **2015**, 371396 (2015).
 - [4] A. Accardi *et al.*, Electron Ion Collider: The next QCD frontier: Understanding the glue that binds us all, *Eur. Phys. J. A* **52**, 268 (2016).
 - [5] R. Abdul Khalek *et al.*, Science requirements and detector concepts for the electron-ion collider: EIC Yellow Report, *Nucl. Phys.* **A1026**, 122447 (2022).
 - [6] D. Boer, M. G. Echevarria, P. Mulders, and J. Zhou, Single spin asymmetries from a single Wilson loop, *Phys. Rev. Lett.* **116**, 122001 (2016).
 - [7] J. Zhou, Transverse single spin asymmetries at small x and the anomalous magnetic moment, *Phys. Rev. D* **89**, 074050 (2014).
 - [8] X. Yao, Y. Hagiwara, and Y. Hatta, Computing the gluon Sivers function at small- x , *Phys. Lett. B* **790**, 361 (2019).
 - [9] R. Boussarie, Y. Hatta, L. Szymanowski, and S. Wallon, Probing the gluon Sivers function with an unpolarized target: GTMD distributions and the odderons, *Phys. Rev. Lett.* **124**, 172501 (2020).
 - [10] H. Dong, D.-X. Zheng, and J. Zhou, Sea quark sivers distribution, *Phys. Lett. B* **788**, 401 (2019).
 - [11] Y. V. Kovchegov and M. G. Santiago, Quark sivers function at small x : Spin-dependent odderon and the sub-eikonal evolution, *J. High Energy Phys.* **11** (2021) 200; **09** (2022) 186(E).
 - [12] Y. V. Kovchegov and M. G. Santiago, T-odd leading-twist quark TMDs at small x , *J. High Energy Phys.* **11** (2022) 098.
 - [13] F. Dominguez, C. Marquet, B.-W. Xiao, and F. Yuan, Universality of unintegrated gluon distributions at small x , *Phys. Rev. D* **83**, 105005 (2011).
 - [14] B. Pasquini, S. Cazzaniga, and S. Boffi, Transverse momentum dependent parton distributions in a light-cone quark model, *Phys. Rev. D* **78**, 034025 (2008).
 - [15] M. G. Ryskin, Odderon and polarization phenomena in QCD, *Sov. J. Nucl. Phys.* **46**, 337 (1987).
 - [16] B. G. Zakharov, Quark—diquark model of nucleon and properties of Pomeron and Odderon NN scattering amplitudes in QCD Born approximation. (In Russian), *Sov. J. Nucl. Phys.* **49**, 860 (1989).
 - [17] A. H. Mueller, Soft gluons in the infinite momentum wave function and the BFKL pomeron, *Nucl. Phys.* **B415**, 373 (1994).
 - [18] S. Jeon and R. Venugopalan, A classical odderon in QCD at high energies, *Phys. Rev. D* **71**, 125003 (2005).
 - [19] L. Zheng, E. C. Aschenauer, J. H. Lee, B.-W. Xiao, and Z.-B. Yin, Accessing the gluon sivers function at a future electron-ion collider, *Phys. Rev. D* **98**, 034011 (2018).
 - [20] S. Benic, A. Dumitru, L. Motyka, and T. Stebel, Gluon Sivers function from forward exclusive $\chi c1$ photoproduction on unpolarized protons, *Phys. Rev. D* **111**, 054008 (2025).
 - [21] J. P. Ma, Diffractive photoproduction of eta(c), *Nucl. Phys.* **A727**, 333 (2003).
 - [22] H. Mäntysaari, Y. Tawabutr, and X.-B. Tong, Nucleon energy correlators for the odderon, *Phys. Rev. D* **112**, 114027 (2025).
 - [23] S. Bhattacharya, Z.-B. Kang, D. Padilla, and J. Penttala, Probing the Sivers asymmetry with transverse energy-energy correlators in the small- x regime, [arXiv:2504.10475](#).
 - [24] Y. Hatta and O. V. Teryaev, Single spin asymmetry in $e + p \rightarrow e' + B^\dagger + X$, [arXiv:2602.07682](#).

- [25] C. Alexandrou *et al.*, Precision QCD with the electron-ion collider, [arXiv:2604.04765](#).
- [26] B. Pasquini and F. Yuan, Sivers and Boer-Mulders functions in light-cone quark models, *Phys. Rev. D* **81**, 114013 (2010).
- [27] E. Iancu and R. Venugopalan, The color glass condensate and high-energy scattering in QCD, in *Quark-Gluon Plasma 4*, edited by R. C. Hwa and X.-N. Wang (World Scientific, Singapore, 2004), pp. 249–363.
- [28] Y. V. Kovchegov and E. Levin, *Quantum Chromodynamics at High Energy* (Oxford University Press, New York, 2013), Vol. 33.
- [29] S. Benić and A. Dumitru, Off forward non-s-channel helicity conserving contributions to exclusive vector quarkonium production from the spin dependent BFKL pomeron, *Phys. Rev. D* **112**, 034025 (2025).
- [30] A. Dumitru, G. A. Miller, and R. Venugopalan, Extracting many-body color charge correlators in the proton from exclusive DIS at large bjorken x, *Phys. Rev. D* **98**, 094004 (2018).
- [31] M. Diehl and P. Hagler, Spin densities in the transverse plane and generalized transversity distributions, *Eur. Phys. J. C* **44**, 87 (2005).
- [32] Z.-B. Kang and J.-W. Qiu, Single transverse-spin asymmetry for D-meson production in semi-inclusive deep inelastic scattering, *Phys. Rev. D* **78**, 034005 (2008).
- [33] A. Dumitru and T. Stebel, Multiquark matrix elements in the proton and three gluon exchange for exclusive η_c production in photon-proton diffractive scattering, *Phys. Rev. D* **99**, 094038 (2019).
- [34] A. Dumitru, V. Skokov, and T. Stebel, Subfemtometer scale color charge correlations in the proton, *Phys. Rev. D* **101**, 054004 (2020).
- [35] C. Lorce, B. Pasquini, X. Xiong, and F. Yuan, The quark orbital angular momentum from Wigner distributions and light-cone wave functions, *Phys. Rev. D* **85**, 114006 (2012).
- [36] A. Dumitru and R. Paatelainen, Sub-femtometer scale color charge fluctuations in a proton made of three quarks and a gluon, *Phys. Rev. D* **103**, 034026 (2021); **108**, 059901(E) (2023).
- [37] A. Dumitru, H. Mäntysaari, and R. Paatelainen, Cubic color charge correlator in a proton made of three quarks and a gluon, *Phys. Rev. D* **105**, 036007 (2022); **109**, 119901(E) (2024).
- [38] Y. V. Kovchegov, L. Szymanowski, and S. Wallon, Perturbative odderon in the dipole model, *Phys. Lett. B* **586**, 267 (2004).
- [39] Y. Hatta, E. Iancu, K. Itakura, and L. McLerran, Odderon in the color glass condensate, *Nucl. Phys. A* **760**, 172 (2005).
- [40] L. N. Lipatov, Reggeization of the vector meson and the vacuum singularity in nonabelian gauge theories, *Sov. J. Nucl. Phys.* **23**, 338 (1976).
- [41] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, The Pomeron singularity in non-Abelian gauge theories, *Sov. Phys. JETP* **45**, 199 (1977).
- [42] I. I. Balitsky and L. N. Lipatov, The Pomeron singularity in quantum chromodynamics, *Sov. J. Nucl. Phys.* **28**, 822 (1978).
- [43] L. Motyka, Nonlinear evolution of Pomeron and odderon in momentum space, *Phys. Lett. B* **637**, 185 (2006).
- [44] S. Benić, A. Dumitru, A. Kaushik, L. Motyka, and T. Stebel, Photon-Odderon interference in exclusive χ_c charmonium production at the electron-ion collider, *Phys. Rev. D* **110**, 014025 (2024).
- [45] A. Dumitru, H. Mäntysaari, and R. Paatelainen, High-energy dipole scattering amplitude from evolution of low-energy proton light-cone wave functions, *Phys. Rev. D* **107**, 114024 (2023).
- [46] J. Bartels, High-energy behavior in a non-Abelian gauge theory (II): First corrections to $T_{n \rightarrow m}$ beyond the leading $\ln s$ approximation, *Nucl. Phys. B* **175**, 365 (1980).
- [47] J. Kwiecinski and M. Praszalowicz, Three gluon integral equation and odd c singlet regge singularities in QCD, *Phys. Lett.* **94B**, 413 (1980).
- [48] U. D'Alesio, F. Murgia, and C. Pisano, Towards a first estimate of the gluon Sivers function from A_N data in pp collisions at RHIC, *J. High Energy Phys.* **09** (2015) 119.
- [49] A. Adare *et al.* (PHENIX Collaboration), Measurement of transverse-single-spin asymmetries for midrapidity and forward-rapidity production of hadrons in polarized p + p collisions at $\sqrt{s} = 200$ and 62.4 GeV, *Phys. Rev. D* **90**, 012006 (2014).
- [50] M. Burkardt, Sivers mechanism for gluons, *Phys. Rev. D* **69**, 091501 (2004).
- [51] E. Iancu, A. H. Mueller, D. N. Triantafyllopoulos, and S. Y. Wei, Gluon dipole factorisation for diffractive dijets, *J. High Energy Phys.* **10** (2022) 103.
- [52] S. Hauksson, E. Iancu, A. H. Mueller, D. N. Triantafyllopoulos, and S. Y. Wei, TMD factorisation for diffractive jets in photon-nucleus interactions, *J. High Energy Phys.* **06** (2024) 180.
- [53] L. Zheng, E. C. Aschenauer, J. H. Lee, and B.-W. Xiao, Probing gluon saturation through dihadron correlations at an electron-ion collider, *Phys. Rev. D* **89**, 074037 (2014).
- [54] D. Boer, Y. Hagiwara, J. Zhou, and Y.-j. Zhou, Scale evolution of T-odd gluon TMDs at small x, *Phys. Rev. D* **105**, 096017 (2022).
- [55] K. Cassar, Z. Wang, X. Chu, and E.-C. Aschenauer, Investigating the broadening phenomenon in two-particle correlations induced by gluon saturation, *Phys. Rev. D* **112**, 034034 (2025).
- [56] P. Caucal, Z.-B. Kang, P. Korcyl, F. Salazar, B. Schenke, T. Stebel, R. Venugopalan, and W. Zhao, Probing gluon saturation with forward di-hadron correlations in proton-nucleus collisions, *Phys. Lett. B* **879**, 140599 (2026).
- [57] G. Beuf, Improving the kinematics for low-x QCD evolution equations in coordinate space, *Phys. Rev. D* **89**, 074039 (2014).
- [58] P. Caucal, F. Salazar, B. Schenke, and R. Venugopalan, Back-to-back inclusive dijets in DIS at small x: Sudakov suppression and gluon saturation at NLO, *J. High Energy Phys.* **11** (2022) 169.
- [59] A. Dumitru, H. Mäntysaari, and R. Paatelainen, Stronger C-odd color charge correlations in the proton at higher energy, *Phys. Rev. D* **107**, L011501 (2023); **111**, 059902(E) (2025).

- [60] F. Hechenberger, The eikonal spin-dependent odderon and gluon sivers function of a proton, and its small- x evolution, Zenodo, (2026) [10.5281/zenodo.18703045](https://zenodo.org/record/18703045).
- [61] F. Hechenberger, Nucleon-lccqm, GitHub, <https://github.com/HechenMountain/nucleon-lccqm> (2026).
- [62] F. Schlumpf, Relativistic constituent quark model of electro-weak properties of baryons, *Phys. Rev. D* **47**, 4114 (1993); **49**, 6246(E) (1994).
- [63] S. J. Brodsky and F. Schlumpf, Wave function independent relations between the nucleon axial coupling $g(A)$ and the nucleon magnetic moments, *Phys. Lett. B* **329**, 111 (1994).
- [64] Z. Ye, J. Arrington, R. J. Hill, and G. Lee, Proton and neutron electromagnetic form factors and uncertainties, *Phys. Lett. B* **777**, 8 (2018).