

Complexity and the Hilbert space dimension of 3D gravity

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A central problem in formulating a theory of quantum gravity is to determine the size and structure of the Hilbert space of black holes. Here we use a quantum dynamical Krylov complexity approach to calculate the Hilbert space dimension of a black hole in $(2 + 1)$ -dimensional anti-de Sitter space. We achieve this by obtaining the spread of an initial thermofield double state over the Krylov basis. The associated Lanczos coefficients match those for chaotic motion on the $SL(2, \mathbb{R})$ group. By including nonperturbative effects in the path integral, which computes coarse-grained ensemble averages, we find that the complexity saturates at late times. The saturation value is given by the exponential of the Bekenstein-Hawking entropy plus logarithmic corrections. Our results introduce a new way to compute the Hilbert space dimension of complex interacting systems from the saturating value of spread complexity.

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Introduction. An outstanding problem in quantum gravity is to understand the size and structure of Hilbert spaces describing different gravitating objects. One way to determine the Hilbert space dimension is to work out the number of vectors in a complete, orthonormal basis. This is the procedure used in [1] to explain the entropy of certain extremal, supersymmetric black holes in string theory. A second approach is to consider any sufficiently large collection of distinct vectors. The rank of the Gram matrix of overlaps of these vectors equals the Hilbert space dimension. This is the procedure used in [2–8] to account for the entropy of general black holes. The latter approach employed the Euclidean gravity path integral and its saddle point approximation, assumed to be well-defined by some high-energy completion, perhaps by a string theory. There is a third approach which applies to chaotic, or at least sufficiently ergodic, systems; to compute the dimension of the span of all states explored by a generic initial vector as it evolves over time.

We apply this third approach to asymptotically three-dimensional anti-de Sitter space (AdS_3) gravity by computing spread complexity [9] of evolving thermofield double (TFD) states describing near-extremal eternal black holes. Spread complexity is computed from the support of quantum states on the Krylov basis, constructed to span the time evolution. Our Krylov basis will be related to the generalized TFD states of [4,5]. In chaotic theories, generic initial states are eventually completely delocalized in the Krylov basis [9–11], so that measuring the spread reveals the dimension of the accessible part of the Hilbert space. We work with thermal states of 3D gravity with support on the entire infinite-dimensional energy eigenspace, but the exponential suppression of high energies effectively restricts dynamics to a microcanonical band as in statistical mechanics.

We start with the return amplitude, i.e., the probability for a state to remain unchanged, processed into hopping amplitudes, known as Lanczos coefficients, between a chain of Krylov basis states [9]. For initial TFD states the return amplitude is an analytically continued partition sum determined by the density of states. Spread complexity can then be written in terms of the spectral density and certain orthogonal polynomials in the energy built from the Lanczos coefficients [12–15]. Identifying these polynomials as those of a random matrix ensemble, our method introduces a new physical notion of the eigenvalue statistics of the Hamiltonian by means of a random matrix theory

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(RMT) with matrix potential $e^{-V(E)} = |\phi_0(E)|^2 \rho(E)$ specified by the reference state $\phi_0(E)$ and microscopic spectral density $\rho(E)$ (see also [16]).

We apply this procedure to 3D gravity in a near-extremal limit, where the spectral density $\rho(E)$ is known [17–19]. This goes beyond the well-understood example of Jackiw-Teitelboim (JT) gravity in $1+1$ dimensions [20–22]. We show that the Lanczos coefficients match those of quantum particles moving on the $SL(2, \mathbb{R})$ manifold, with the associated Laguerre polynomials leading to monotonically increasing spread complexity. The spectral density $\rho(E)$ in [17–19] was derived from the Euclidean gravity path integral (GPI); many studies, e.g., [2,23,24] and [25] for 3D gravity, suggest that this $\rho(E)$ should be interpreted as a coarse-grained average over an underlying ensemble. As such, products like $\rho(E)\rho(E')$ in the formula for spread complexity should be understood as spectral correlators in this ensemble rather than as simple products. We assume that these correlators follow random matrix universality, as suggested by effects of wormholes contributing to the product of partition sums in 2D and 3D gravity [19,25]. The same wormholes imply a finite-dimensional black hole Hilbert space spanned by time-shifted TFD states [4,5]. Including this effect, spread complexity saturates at a value exponential in the Bekenstein-Hawking entropy plus logarithmic corrections.

The saturation of spread complexity is a signature of the finite dimensionality of the black hole Hilbert space. To our knowledge, this is the first work that derives the saturation of complexity for an explicit gravitational theory in dimensions higher than two. The calculation relies only on the semiclassical gravitational path integral and not on any UV completion. Spread complexity not only provides a measure of the dimensionality of the Hilbert space but also encodes the dynamical evolution of the state in the underlying system. Therefore, this formalism provides an explicit solution of the dynamical evolution of a TFD state in the Hilbert space. At early times, the evolution is governed by an $SL(2, \mathbb{R})$ symmetry, resulting in quadratic growth of complexity. At late times, when the quantum gravity effects become dominant, the complexity saturates, determining the black hole Hilbert space dimension.

Orthogonal polynomials and complexity. The idea behind spread complexity is to quantify the spread of a wave function in the Hilbert space as it evolves over time [9,26–28]. Consider an initial state $|\psi_0\rangle$, evolving as

$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle \equiv \sum_n \psi_n(t)|K_n\rangle, \quad (1)$$

where the Krylov basis $\{|K_n\rangle|n \in \mathbb{N}\}$ is obtained recursively from the set $\{H^n|\psi_0\rangle|n \in \mathbb{N}\}$ through an orthogonalization procedure. By construction, the basis vectors satisfy a recursion relation,

$$H|K_n\rangle = a_n|K_n\rangle + b_n|K_{n-1}\rangle + b_{n+1}|K_{n+1}\rangle, \quad (2)$$

where a_n and b_n are the Lanczos coefficients. Since the $|K_n\rangle$ are orthonormal, we obtain the coefficients $\psi_n(t)$ of the state (1) in the Krylov basis as $\psi_n(t) = \langle K_n|\psi(t)\rangle$. The spread complexity is then defined as

$$C_S(t) = \sum_n n |\psi_n(t)|^2. \quad (3)$$

We can also expand (1) in the energy basis as

$$|\psi(t)\rangle = \int dE \rho(E) \phi_0(E) e^{-iEt} |E\rangle, \quad (4)$$

where $\rho(E)$ is the spectral density and $\phi_0(E) = \langle E|\psi_0\rangle$. For a theory with a discrete spectrum $\rho(E) = \sum_i \delta(E_i - E)$, where the E_i are energy eigenvalues; we are working in a thermodynamic limit and approximating the spectral density as continuous. The energy eigenstates are normalized such that

$$\langle E|E'\rangle = \frac{\delta(E - E')}{\rho(E)}, \quad \mathbb{1} = \int dE \rho(E) |E\rangle \langle E|. \quad (5)$$

Projecting (4) onto $|K_n\rangle$ gives

$$\psi_n(t) = \int dE \rho(E) \phi_0(E) e^{-iEt} c_n^*(E), \quad (6)$$

where $c_n(E) \equiv \langle E|K_n\rangle$. It follows from (2) that the $c_n(E)$ also satisfy a recursion relation (S.1) with initial conditions $c_{-1}(E) = 0$ and $c_0(E) = \phi_0(E)$ (also see [29,30]). We show in the supplemental material [31] that the solution to this recursion relation takes the form,

$$c_n(E) = \phi_0(E) h_n(E), \quad (7)$$

where $h_n(E)$ is a polynomial of degree n with coefficients determined purely from the Lanczos coefficients (S.5). It follows from the orthonormality of the Krylov basis that

$$\int dE \rho(E) |\phi_0(E)|^2 h_n^*(E) h_m(E) = \delta_{nm}, \quad (8)$$

which we obtain by inserting the identity (5) into the overlap $\langle K_n|K_m\rangle$. Therefore, the polynomials $h_n(E)$ are orthogonal with respect to the measure $\rho(E) |\phi_0(E)|^2$. Moreover, from (6) and (7) we determine spread complexity (3) in terms of these polynomials as [12,14,15]

$$C_S(t) = \int dE dE' e^{-i(E-E')t} \rho(E') \rho(E) \times |\phi_0(E)|^2 |\phi_0(E')|^2 \sum_n n h_n(E') h_n^*(E). \quad (9)$$

Early time complexity growth. Now consider an eternal black hole in AdS_{d+1} . The Hartle-Hawking no-boundary Euclidean path integral constructs a vacuum wave function of the TFD form,

$$|\text{TFD}(t)\rangle = \frac{1}{\sqrt{Z(\beta)}} \int dE \rho(E) e^{-\frac{\beta}{2}E} e^{-iEt} |E\rangle, \quad (10)$$

where $Z(\beta)$ is the canonical partition function,

$$Z(\beta) = \int dE \rho(E) e^{-\beta E}, \quad (11)$$

and we use the shorthand $|E\rangle \equiv |E\rangle_L \otimes |E\rangle_R$, as in [22,32]. The Lanczos coefficients can be obtained from moments of the return amplitude [9],

$$S_{\text{TFD}}(t) := \langle \text{TFD}(0) | \text{TFD}(t) \rangle = \frac{Z(\beta + it)}{Z(\beta)}. \quad (12)$$

In the effective low-energy description, Z is obtained from the GPI over manifolds with boundary $S^1_\beta \times S^{d-1}$ [33], which computes a trace over the gravity Hilbert space [8].

For general spacetime dimension this path integral has to be approximated by the leading saddle point. However, for $d+1=3$ Maloney-Witten-Keller (MWK) evaluated the sum over all classical solutions to Einstein's equations with torus boundary conditions [17,18]. The torus parameter is $\tau = \frac{\theta + i\beta}{2}$, where θ is the periodicity of the spatial circle. τ is the dual variable to the angular momentum J of the black hole. Close to extremality [34], i.e., $|J| \gg \frac{L}{G_N}$ and $0 < E - |J| \ll \frac{L}{G_N}$, where L is the anti-de Sitter radius and G_N the Newton's constant, the leading contribution to the MWK density of states is [19]

$$\rho_J^{\text{MWK}}(E) \simeq \kappa_0 e^{S_0} \sqrt{E - |J|} + \frac{\kappa_1 (-1)^J e^{S_0/2}}{\sqrt{E - |J|}}, \quad (13)$$

where $S_0 \equiv S_0(J)$ is the Bekenstein-Hawking entropy of the Bañados-Teitelboim-Zanelli black hole. The return amplitude (12) for the MWK density is obtained from (11) and given in the supplemental material [31] as (S.15). As shown in the supplemental material, the resulting Lanczos coefficients (S.16)–(S.17) match those for a particle on the $SL(2, \mathbb{R})$ group manifold when the index n is large [9].

In fact the MWK density of states cannot be used to define an integration measure (8) for the polynomials h_n , since for exponentially small $E - |J|$ and odd spins, (13) becomes negative [35]. To make matters worse, this negativity implies that the underlying theory cannot be unitary. One proposal to resolve the negativity of the density of states includes additional off shell contributions to the gravitational path integral [19]. This shifts the ground-state energy and the full spectral density in the near-extremal limit of 3D gravity becomes

$$\rho_J^{\text{MT}}(E) \simeq \kappa_0 e^{S_0} \sqrt{E - E_0(J)}, \quad (14)$$

with $E_0(J) - |J| \propto -(-1)^J e^{-\frac{S_0}{2}}$. Again, the energy density determines the return amplitude (12) via (11),

$$S_{\text{MT}}(t) = \left(\frac{\beta}{\beta + it} \right)^{3/2} \exp[-iE_0(J)t]. \quad (15)$$

We can derive Hamiltonian moments in the initial state and Lanczos coefficients from the return amplitude [9]:

$$a_n = \frac{4n+3}{2\beta} + E_0(J), \quad b_n = \begin{cases} 1 & \text{if } n = 0, \\ \frac{1}{\beta} \sqrt{\frac{n(2n+1)}{2}} & \text{if } n > 0, \end{cases} \quad (16)$$

which have the same form as for particles on $SL(2, \mathbb{R})$ [9],

$$a_n = \gamma(h+n) + \xi, \quad b_n = \sigma \sqrt{n(2h+n-1)}. \quad (17)$$

The general $SL(2, \mathbb{R})$ case was worked out in [36] by directly solving (2) for a highest weight state of scaling dimension h , with an explicit Hamiltonian. One realization, the harmonic oscillator, gives (17) with $h = 1/2$ [9]. Here we find that the partition function of near-extremal 3D gravity leads to (17) with $h = 3/4$.

The coefficients in (16) match the $SL(2, \mathbb{R})$ form in (17) because the near-extremal limit of 3D gravity has a dimensional reduction to JT gravity with defects [19]. The reduced theory contains a Schwarzian sector with $SL(2, \mathbb{R})$ symmetry. In the near-extremal limit the energy density of the Schwarzian theory is approximated by $\sqrt{E - E_*}$, leading to the first term in (13). On the other hand, from [9] the spread complexity for the Schwarzian theory coincides with the $SL(2, \mathbb{R})$ model at late times or alternatively in the large C/β limit which is the regime where the Schwarzian theory describes JT gravity and C is interpreted as the boundary value of the dilaton.

We show in the supplemental material [31] that for the Lanczos coefficients (16) the polynomials in (7) take the form (S.13),

$$h_n(E) = \sqrt{\frac{\Gamma(\frac{3}{2})n!}{\Gamma(n+\frac{3}{2})}} (-1)^n L_n^{(1/2)}[\beta(E - E_0(J))], \quad (18)$$

where $L_n^{(\alpha)}$ are the generalized Laguerre polynomials. The polynomials $L_n^{(\alpha)}(y)$ are orthogonal with respect to the measure $\mu(y) \propto y^\alpha e^{-y}$. Therefore, (18) is indeed consistent with (8), since for the TFD state (10) we have $\phi_0(E) \propto e^{-\beta E/2}$, and (14) implies $\alpha = 1/2$. Thus, an alternative way to obtain the polynomials $L_n^{(\alpha)}$ is to directly determine them from the integration measure in (8). The generalized Laguerre polynomials were used to calculate operator complexity in [12], and for $\alpha = 1/2$ this leads to

$$C_S^{\text{MT}}(t) = \frac{3t^2}{2\beta^2}. \quad (19)$$

This agrees with the spread complexity growth for a particle on $SL(2, \mathbb{R})$ [9], after comparing (17) to (16) where we observe that $\gamma^2/4 - \sigma^2 = 0$.

Complexity saturation from quantum gravity. We expect spread complexity to saturate at late times at a value set by the dimension of the underlying Hilbert space. For black holes we are studying this implies that complexity should saturate at e^{S_0} . The reason why (19) does not saturate is that we obtained it from a continuous energy spectrum (14) with infinitely many states in any energy band. By contrast the full quantum gravity theory should have a discrete spectrum if the finite black hole entropy has a sensible microscopic interpretation. These considerations suggest that the computations above are only valid for timescales shorter than the inverse of the typical energy spacing $t \ll 1/\delta E \sim e^{S_0}$, and that to go to later times we have to find a way to resolve the discreteness of the spectrum.

The continuum spectral density in (14) was obtained by using the gravitational path integral (GPI), an effective field theory description of the underlying ultraviolet complete theory. By construction, the GPI only contains coarse grained information about the fundamental degrees of freedom, and is believed to compute quantities that act like averages in some underlying ensemble (see, e.g., [2,23–25,37–39]). This suggests that the density of states (14) should be interpreted as an ensemble average that coarse grains a fundamentally discrete spectrum.

From this perspective, we cannot simply substitute (14) into the integral expression for spread complexity (9). Rather, if we use the GPI to compute the integrand, the $\rho(E')\rho(E)$ factor should act like a density-density correlator in the effective ensemble. Thus, if we use the GPI to directly calculate the integrand in the complexity integral, we are effectively computing

$$\overline{C_S(t)} = \int dE dE' e^{-i(E-E')t} \overline{\rho(E')\rho(E)} \times |\phi_0(E)|^2 |\phi_0(E')|^2 A(E, E'), \quad (20)$$

where the overbar indicates that the quantity is computed by the GPI, and so acts effectively as an ensemble average. We also introduced the complexity kernel,

$$A(E, E') = \sum_n n h_n(E') h_n^*(E), \quad (21)$$

and assume that it is self-averaging, since $A(E, E')$ contains a sum over the orthogonal polynomials obtained from a fixed set of Lanczos coefficients. The fine grained structure of the spectrum is encoded in the density-density correlator $\overline{\rho(E')\rho(E)}$.

This spectral correlator can be computed as an inverse Laplace transform of the two-boundary partition function $Z(\beta', \beta)$, where β, β' are the periodicities of the two thermal

circles. The leading path-integral contribution to this partition function is disconnected [5], resulting in the factorized form $Z(\beta')Z(\beta)$, leading to spread complexity of the form (9), which does not saturate. The leading nonperturbative correction to the factorized partition function comes from a Euclidean wormhole that connects the two boundaries. For 3D gravity, this contribution has been calculated in [25]. In the near-extremal limit discussed above (13) the wormhole contribution has a universal random matrix form [40], consistent with the fact that the effective dimensionally reduced description has a known random matrix dual in the Gaussian unitary ensemble (GUE) universality class [19].

Thus, the spectral correlator in (20) shows random matrix universality for $E \approx E'$, so that [23]

$$\overline{\rho(E')\rho(E)} \approx \rho(E')\rho(E) - \frac{\sin^2(\pi\rho(\bar{E})(E' - E))}{\pi^2(E' - E)^2} + \dots \quad (22)$$

Here $\bar{E} = \frac{E+E'}{2}$ and “...” denotes contributions from contact terms proportional to $\delta(E - E')$, which lead to a constant offset in the spread complexity (20) that we can drop. The second term in (22), arising from the connected wormhole contribution to the path integral is called the sine kernel, and is universal for any matrix model in the GUE universality class [23]. Since (22) vanishes as $E \rightarrow E'$, the universal form of the spectral correlator indicates level repulsion between the energy eigenvalues, a signature of spectral discreteness in the fundamental theory [22,32].

We are now ready to evaluate the complexity integral (20); the two key ingredients are the density-density correlator that we just discussed, and the orthogonal polynomials (18) from which we can explicitly calculate the complexity kernel (21). Note that we are interested in the late-time behavior of (20), where the integral is dominated by frequencies $\omega := E - E'$ of order $1/t$. In this regime, the spectral correlator has the form (22), and we show in the supplemental material [31] that the kernel (21) is given by

$$A(E, E') = \frac{\mathcal{C}(\bar{E})}{(E - E')^2} + (\text{terms regular in } E - E'), \quad (23)$$

where $\mathcal{C}$ is a nonzero and smooth function (S.31).

We show in the supplemental material [31] that as $E - E' \rightarrow 0$ the spectral correlator (22) vanishes as $(E - E')^2$ [see (S.34)]; so the integrand is regular in $\omega = E - E'$ and will thus lead to a finite late-time limit of spread complexity. Without the nonperturbative corrections, which result in the sine kernel contribution to the spectral correlator, the double pole in (23) would have led a late-time divergence of complexity as in (19). Evaluating the ω integral in the supplemental material [31], following [20], where a similar integral appeared in the context of the wormhole length in

JT gravity, we find (S.35)

$$\overline{C_S(\hat{t})} = C_0 + \frac{2\pi^2 e^{S_0}}{3\hat{Z}(\beta)^2} \int_{\bar{E}_*(\hat{t})}^{\infty} d\bar{E} e^{-2\beta\bar{E}} \hat{\rho}(\bar{E})^3 \mathcal{C}(\bar{E}) \times (1 - X(\hat{t}; \bar{E}))^3, \quad (24)$$

where $e^{S_0} \hat{\rho}(\bar{E}) = \rho(\bar{E})$, $e^{S_0} \hat{Z}(\beta) = Z(\beta)$, and $e^{S_0} \hat{t} = t$. Moreover, $X(\hat{t}; \bar{E})$ and $\bar{E}_*(\hat{t})$ are defined in (S.36) and (S.37) and C_0 is a constant. We show in the supplemental material [31] that for $\hat{t} \rightarrow \infty$ the integral in (24) vanishes. Therefore, we obtain the late-time saturation value of complexity from the constant term in (24), which we determine in the supplemental material [31] and find (S.39)

$$C_0 = e^{S_0} \frac{\kappa_0 \pi^3}{8} (2E_0(J))^{\frac{3}{2}} e^{4\beta E_0(J)} \Gamma\left(-\frac{3}{2}, 4\beta E_0(J)\right). \quad (25)$$

This saturation relies on random matrix universality (22) and the double pole in the complexity kernel (23), similarly to observations in [29,32] motivated by JT gravity and its relation with the double-scaled Sachdev-Ye-Kitaev (SYK) model [20,22,41]. In the high-temperature limit, $E_0(J)\beta \ll 1$, the saturation value simplifies to

$$C_0 \simeq e^{S_0} \kappa_0 \frac{\pi^3}{12} (2\beta)^{-3/2}. \quad (26)$$

This result includes the familiar logarithmic temperature correction to the black hole entropy S_0 . In particular, the $3/2$ prefactor for the logarithmic correction agrees with the value derived from the Schwarzian mode contributing to the path integral of near-extremal black holes [42–44]. Our final result (25) shows that the Hilbert space of 3D gravity near extremality has a finite dimension that is exponential in the black hole entropy S_0 .

Summary and outlook. We used spread complexity to study the structure of the black hole Hilbert space in 3D gravity. For generic initial states of chaotic systems with discrete, bounded spectra, the saturating value of spread complexity is related to the Hilbert space dimension. In the context of gravity, the challenge is that without a complete microscopic theory, we can only compute complexity from the density of states in an effective theory described by the gravitational path integral. As this approach results in a continuous and unbounded spectrum, we find that naively complexity grows for all times, suggesting an infinite dimensional Hilbert space. We overcome this obstacle by extracting information about the discreteness of the spectrum in the microscopic theory from the density-density correlator appearing in the spectral representation of complexity. This correlator can be computed from the effective path integral, which averages over the microscopic details of the fundamental theory. The outcome is saturation of spread complexity at a value exponential in

the black hole entropy including the universal logarithmic corrections.

There is a conjecture that in holographic theories of gravity, the complexity of evolving TFD states is geometrized, possibly as the volume of distinguished spacelike slices passing through the Einstein-Rosen bridge in the gravitational description [45,46]. Alternative proposals suggest that complexity might be related to other diffeomorphism invariant quantities [47–50]. In most of these proposals, the complexity in question was conceived of in terms of a dual field theory. But since, in holography, the field theory state and the quantum gravity state must be dual to each other, we might as well compute complexity directly in the bulk gravity if we are able to do it. The question is what notion of complexity to use. Here we looked at a measure of complexity associated to the spreading quantum state [9] that has found numerous applications. In fact, recently the authors of [51,52] showed that this notion of spread complexity applied to TFD states of the double-scaled SYK model [53,54] precisely reproduces the growing classical length of wormholes in the dual 2D JT gravity. Then by invoking RMT correlations in the Lanczos coefficients, the authors of [21,55,56] showed that the spread complexity saturates at late times, implying large quantum corrections to the classical notion of length.

In a complementary approach, the authors of [20] defined a nonperturbative notion of wormhole length directly from the path integral of a class of 2D dilaton gravity theories, including JT gravity with defects, which is a dimensionally reduced description of the near-extremal sector of 3D gravity [19] (also see [22,41]). The resulting expression for 2D wormhole length is very similar to our result for spread complexity (24) and only differs in the functional form of the $\mathcal{C}(\bar{E})$ [compare (S.31) and (24) to Eqs. (3.10) and (4.6) in [20]]. An explanation for this discrepancy is that the nonperturbative wormhole length in [20] is defined by summing over geodesics in all Euclidean 2D geometries with a single circular boundary, the simplest example being the disk that analytically continues to the Lorentzian black hole. If we define the nonperturbative wormhole length in a similar way for 3D gravity, we have to sum over geometries with a single torus boundary $S^1_\beta \times S^1$. Some of these lead to the aforementioned 2D geometries after dimensional reduction. However, there are infinitely many 3D geometries which do not have this 2D interpretation. Therefore, the 2D result of [20] will receive corrections, which should lead to our 3D formula (24). We leave the nonperturbative study of wormhole length in 3D gravity to future work.

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