

Universality at next-to-leading power for jet associated processes

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The study of cross sections in the threshold limit at next-to-leading power has been a subject of sustained interest for many years. We demonstrate the universality of leading logarithms at next-to-leading power for the production of arbitrary massive colorless particles in association with a jet, contingent upon the identification of appropriate combination of helicity configurations.

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Introduction. The avalanche of data generated by the Large Hadron Collider (LHC), combined with the absence of compelling new physics signatures, necessitates a precise understanding of the well-established Standard Model. There are two primary approaches to enhance the accuracy of theoretical predictions: (i) calculating cross sections at higher orders in perturbation theory and (ii) performing resummation of certain large logarithmic terms to all orders in perturbation theory. For all physical processes occurring at the LHC, the differential cross section in the threshold limit takes the following form:

$$\frac{d\sigma}{d\xi} \approx \sum_{n=0}^{\infty} \alpha_s^n \left\{ \sum_{m=0}^{2n-1} D_{nm}^{\text{LP}} \left(\frac{\log^m \xi}{\xi} \right) + d_n \delta(\xi) + \sum_{m=0}^{2n-1} D_{nm}^{\text{NLP}} \log^m \xi \right\}, \quad (1)$$

where ξ is the threshold variable. In this equation, the first set of logarithms and the δ function arise when one considers leading power approximation of the soft radiation. These set of logarithms are universal due to the factorization properties of soft and collinear radiations. The groundbreaking works of Refs. [1–8], based on diagrammatic methods, were instrumental in developing methods for leading power (LP) resummation. Subsequently, several alternative methods of LP resummation were developed,

including those based on Wilson lines [9,10], renormalization group [11], and soft collinear effective theory [12–15]. A comparison of these various resummation techniques of LP logarithms can be found in Refs. [16–18]). The second set of logarithms in Eq. (1) originates due to next-to-leading power (NLP) approximation of the radiated particle's momentum, which is the focus of this article.

The structure of scattering amplitudes was first studied almost 60 years ago [19–21] and continues to be an area of active research. Given the significant numerical impacts of the NLP logarithms [22–29], substantial progress has been made toward understanding these logarithms through the infrared structure of gauge theory amplitudes. Over the past decade, several methods have been developed to study these logarithms [29–90]. However, none of the existing methods can conclusively confirm the factorization of next-to-soft radiation at all orders for all collider processes in perturbation theory. While the universality of NLP logarithms has been established in case of color-singlet particle production [45] and a generalized threshold factorization theorem—accounting for the full soft and collinear dynamics in the momentum fractions of the incoming partons and valid even when only one of them approaches unity—has been developed [90], no unique resummation formula exists for colored particles in the final state. One approach to recognize the patterns of NLP logarithms and develop a general resummation formula involves analyzing multiple processes with colored particles in the final state. In this context, the NLP logarithms for prompt photon plus jet production were investigated in [46]. Additionally, attempts have been made toward calculating these logarithms for vector boson plus jet production [51,91,92]. However, due to the complex nature of the phase space integration, none of these references provide complete expressions of the NLP logarithms.

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To remove the complexity and streamline the NLP calculations, we developed a straightforward formalism in Ref. [93] to obtain NLP corrections due to the radiation of gluons. Our formalism combines color-ordered helicity amplitudes and soft theorems of gauge theory [94–96]. Since soft theorems apply only to the emission of gluons, we could use them only to compute the NLP logarithms that arise due to the next-to-soft gluon radiation. Further, to account for soft quark contributions at NLP, we defined soft quark operators in terms of helicity spinors in Ref. [97]. Using these two approaches, we derived the NLP threshold corrections for all possible partonic channels in Higgs plus one jet production. Moreover, based on symmetry considerations and the similarity in structure between the color-ordered amplitudes of Higgs plus one jet and pseudoscalar Higgs plus one jet production, we established that these logarithms are universal for both processes.

The next natural step in investigating the universal structure of scattering cross sections at NLP accuracy is to calculate the NLP corrections for the production of a massive vector boson associated with a jet at the LHC. Prior to undertaking explicit computations of these processes, we propose in this article that the coefficients of NLP leading logarithms exhibit a universal analytic structure, provided the appropriate combination of helicity configurations is identified, for the production of any massive color-singlet particle—regardless of its spin—in association with a jet. These coefficients can be derived through the mass factorization formula, using the helicity-dependent Altarelli-Parisi splitting functions for both the quark and the gluon.

Next-to-soft gluon and soft quark. It is well established that soft gluon radiation is the sole source of threshold logarithms at leading power. In contrast, NLP logarithms receive contributions from two distinct sources: (i) next-to-soft gluon radiation and (ii) soft quark radiation. In this section, we describe the methods introduced in [93,97] for computing these contributions. To capture the next-to-soft gluon radiation effects, we use the soft theorem of gauge theory [94–96], which relates $(n+1)$ -particle helicity amplitude to the n -particle amplitude at LP and NLP as

$$\mathcal{A}_{n+1}^{\text{LP+NLP}}(\{\lambda|s\rangle, |s]\}, \{|1\rangle, |1']\}, \dots, \{|n\rangle, |n']\}) = \frac{1}{\lambda^2} \frac{\langle 1n \rangle}{\langle 1s \rangle \langle ns \rangle} \times \mathcal{A}_n(\{|1\rangle, |1']\}, \dots, \{|n\rangle, |n']\}), \quad (2)$$

where we consider the emission of a gluon with helicity “+” and use the holomorphic soft limit $|s\rangle \rightarrow \lambda|s\rangle$. The amplitude on the right-hand side of the above equation corresponds to the simple nonradiative n -particle amplitude, with a pair of square spinors being shifted as

$$|1'] = |1] + \Delta_s^{(1,n)}|s], \quad |n'] = |n] + \Delta_s^{(n,1)}|s], \quad \Delta_s^{(i,j)} = \lambda \frac{\langle js \rangle}{\langle ji \rangle}.$$

In the case where the radiated gluon carries negative helicity, the square spinors are naturally replaced by their corresponding angle spinors.

In order to compute the soft quark contribution using color-ordered helicity amplitudes, we define two soft quark operators in [97]. These operators are constructed by combining a soft quark with its neighboring hard color particle. This combination yields an effective hard particle which subsequently participates in the corresponding non-radiative amplitudes. As a result, the $(n+1)$ -particle color-ordered amplitude under soft quark emission is given by [97]

$$\mathcal{A}_{n+1}^{s^i h^j} = \mathcal{Q}^{s^i h^j \rightarrow c^k} \mathcal{A}_n^{c^k}, \quad (3)$$

where $\mathcal{Q}$ is the soft quark operator, and s^i denotes the “soft” (anti)quark with helicity i ; h^j is the “hard” particle with helicity j , and the “clubbed” color particle c^k with helicity k is being formed by combining the hard and soft particles of the $(n+1)$ -particle amplitude. Since the soft quark effect is fully captured within the operator, the helicity and momentum of the resulting clubbed particle are taken to be the same as those of the original hard particle; i.e., $j = k$ in the equation above. There are two distinct ways to merge a soft quark with a hard particle, resulting in the following two fundamental nonzero soft quark operators [97]:

$$\mathcal{Q}^{q_s^+ \bar{q}_h^- \rightarrow g_c^-} = -\frac{1}{\langle sh \rangle}, \quad \mathcal{Q}^{g_h^+ q_s^+ \rightarrow q_c^+} = -\frac{1}{\langle sh \rangle}, \quad (4)$$

and all other possible combinations can be derived from these two.

NLP corrections. In this article, we focus on the processes that involve production of one colorless particle associated with a jet at the LHC. The colorless particle can be either massive or massless, and the general form for these processes can be written as

$$P_C^a(p_1) + P_C^b(p_2) \rightarrow P_{\mathcal{C}}(p_3) + P_C^d(p_4), \quad (5)$$

where the suffix C and $\mathcal{C}$ denote colored and colorless particles (P), respectively. In a completely general setup $p_3^2 = m_{\mathcal{C}}^2$, where in the Standard Model $m_{\mathcal{C}} \in \{0_\gamma, m_W, m_Z, m_H\}$. Our interest lies in studying the threshold logarithms arising from the emission of an additional colored particle with momentum p_5 , leading to the following scattering process:

$$P_C^a(p_1) + P_C^b(p_2) \rightarrow P_{\mathcal{C}}(p_3) + P_C^d(p_4) + P_C^e(p_5). \quad (6)$$

Here p_5 can either be a quark, an antiquark, or a gluon. Since we are interested in obtaining scattering cross section up to NLP, we may express the scattering amplitude as

$$\mathcal{A} = \mathcal{A}_{\text{LP}} + \mathcal{A}_{\text{NLP}}, \quad (7)$$

and squaring this amplitude, we obtain

$$\mathcal{A}^2 = \mathcal{A}_{\text{LP}}^2 + 2\text{Re}(\mathcal{A}_{\text{NLP}}\mathcal{A}_{\text{LP}}^\dagger), \quad (8)$$

where the term $\mathcal{A}_{\text{NLP}}^2$ is being neglected, as it starts contributing at the next-to-next-to leading power. Throughout this article, we use the shorthand notation $[\mathcal{A}]_{\text{NLP}}^2$ to represent the second term. In terms of holomorphic scaling of spinors, the color-ordered helicity amplitudes and their squared parts scale at NLP as

$$\mathcal{A}_{\text{NLP}} \approx \frac{1}{[5]}, \quad [\mathcal{A}]_{\text{NLP}}^2 \approx \frac{1}{s_{i5}}, \quad (9)$$

where $s_{i5} = \{s_{15}, s_{25}, s_{45}\}$, since the momentum of the massive particle can always be expressed using four-momentum conservation as $p_3 = p_1 + p_2 - p_4 - p_5$. Once the squared amplitudes for each independent helicity configuration are obtained, the next step is to perform the phase space integration over the unobserved parton's momentum. To deal with the infrared singularities, we choose the space time dimension $d = (4 - 2\epsilon)$ and parametrize the particle momenta in the rest frame of p_4 and p_5 as

$$\begin{aligned} p_1 &= (E_1, 0, \dots, 0, E_1), \\ p_2 &= (E_2, 0, \dots, 0, p_3 \sin \psi, p_3 \cos \psi - E_1), \\ p_3 &= -(E_3, 0, \dots, 0, p_3 \sin \psi, p_3 \cos \psi), \\ p_4 &= -\frac{\sqrt{s_{45}}}{2}(1, 0, \dots, 0, \sin \theta_1 \sin \theta_2, \sin \theta_1 \cos \theta_2, \cos \theta_1), \\ p_5 &= -\frac{\sqrt{s_{45}}}{2}(1, 0, \dots, 0, -\sin \theta_1 \sin \theta_2, -\sin \theta_1 \cos \theta_2, \\ &\quad -\cos \theta_1). \end{aligned} \quad (10)$$

With this phase space parametrization, the helicity-dependent differential cross section at NLP is given by [93,97]

$$s_{12}^2 \frac{d^2 \sigma^{h_1 h_2 h_3 h_4 h_5}}{ds_{13} ds_{23}} \Big|_{\text{NLP}} = \mathcal{F}_{ab} \left(\frac{s_{45}}{\bar{\mu}^2} \right)^{-\epsilon} \mathcal{A}_{\text{NLP}}^2, \quad (11)$$

where

$$\mathcal{F}_{ab} = \frac{1}{2} K_{ab} G^2 \left(\frac{\alpha_s(\bar{\mu}^2)}{4\pi} \right)^2 \left(\frac{s_{13} s_{23} - m_H^2 s_{45}}{\bar{\mu}^2 s_{12}} \right)^{-\epsilon}, \quad (12)$$

and

$$\overline{\mathcal{A}_{\text{NLP}}^2} = \int_0^\pi d\theta_1 (\sin \theta_1)^{1-2\epsilon} \int_0^\pi d\theta_2 (\sin \theta_2)^{-2\epsilon} [\mathcal{A}]_{\text{NLP}}^2. \quad (13)$$

Here s_{45} serves as the threshold variable. Note that K_{ab} is a factor related to the color and spin average of the initial state particles, and G is the coupling constant associated with the massive particle. Since we are only interested in the leading logarithms at NLP, we can therefore treat $\mathcal{F}_{ab}$ as independent of the angular integrations. However, for next-to-leading logarithms at NLP, one would need to expand this factor around the threshold variable, though this is not necessary for the current purpose. The general form of the scattering cross section expanded around ϵ after performing the phase space integration is given by

$$\begin{aligned} s_{12}^2 \frac{d^2 \sigma^{h_1 h_2 h_3 h_4 h_5}}{ds_{13} ds_{23}} \Big|_{\text{NLP-LL}} &= \left\{ \mathcal{C}_{-1} \frac{1}{\epsilon} + \mathcal{C}_{45} \frac{1}{(s_{45})_+} + \mathcal{C}_{\text{LL}} \log \left(\frac{s_{45}}{\bar{\mu}^2} \right) \right\} (\mathcal{A}^{h_1 h_2 h_3 h_4})^2, \\ &\quad (14) \end{aligned}$$

where $\mathcal{C}$'s represent the coefficients of the corresponding factors that are multiplied by them, and the subscript LL denotes the leading logarithms.

Universality. Infrared singularities in the scattering cross section appear as divergences that manifest as poles in ϵ following dimensional regularization with $d = (4 - 2\epsilon)$. The singularity that appears in the differential cross section at NLP, as shown in Eq. (14), is canceled when the effects of mass factorization are taken into account, rendering the cross section finite. This finiteness ensures that the coefficient $\mathcal{C}_{-1}$ can be determined from the mass factorization contribution and is given by

$$\begin{aligned} \mathcal{C}_{-1} &= - \int_0^1 \frac{dx_1}{x_1} \Gamma_{a' \rightarrow ae}(x_1, \epsilon) \hat{s}_{12}^2 \frac{d^2 \sigma^{(0)}(\hat{s}_{12}, \hat{s}_{13}, s_{23})}{d\hat{s}_{13} ds_{23}} \frac{1}{(\mathcal{A})^2} \\ &\quad - \int_0^1 \frac{dx_2}{x_2} \Gamma_{b' \rightarrow be}(x_2, \epsilon) \hat{s}_{12}^2 \frac{d^2 \sigma^{(0)}(\hat{s}_{12}, s_{13}, \hat{s}_{23})}{ds_{13} d\hat{s}_{23}} \frac{1}{(\mathcal{A})^2}, \end{aligned} \quad (15)$$

where

$$\begin{aligned} (\mathcal{A})^2 &= (\mathcal{A}^{h_1 h_2 h_3 h_4})^2, \quad \hat{s}_{12} = x_1 x_2 s_{12}, \\ \hat{s}_{13} &= x_1 (s_{13} - m_{\mathcal{C}}^2) + m_{\mathcal{C}}^2, \\ \hat{s}_{23} &= x_2 (s_{23} - m_{\mathcal{C}}^2) + m_{\mathcal{C}}^2. \end{aligned} \quad (16)$$

σ_0 is the Born scattering cross section and Γ represents the helicity-dependent splitting kernel defined as

$$\Gamma_{i \rightarrow jk} = -\frac{\alpha_s}{2\pi} \frac{1}{\epsilon} \mathcal{P}_{i \rightarrow jk}, \quad (17)$$

with $\mathcal{P}$ being the helicity-dependent Altarelli-Parisi splitting functions. Note that $\mathcal{C}_{-1}$ can be determined, as outlined above, for the radiation of both next-to-soft gluons and soft quarks, irrespective of whether the production channel is diagonal (i.e., $q\bar{q}$ or gg initiated) or off diagonal (i.e., qg or $\bar{q}g$ initiated).

Since the Eikonal factor appears as an overall multiplicative term in the expression for $[\mathcal{A}]_{\text{NLP}}^2$, its contribution at NLP involves multiplication with a linear combination of s_{i5} with $i \in \{1, 2, 4\}$. This structure leads to six distinct types of terms, such as $\frac{1}{s_{i5}}$ and $\frac{s_{i5}}{s_{(i+1)5}s_{(i+2)5}}$, each accompanied by appropriate coefficients that are independent of s_{i5} . Upon performing the angular integration of these terms and multiplying by the appropriate phase space factor, it follows straightforwardly that

$$\mathcal{C}_{\text{LL}} = -\mathcal{C}_{-1}. \quad (18)$$

This represents one of the central findings of this article, exhibiting that the coefficients of the NLP leading logarithms can be derived from the computation of mass-factorization-related contributions. Remarkably, this relation holds across both diagonal and off-diagonal production channels. Although next-to-soft gluon and soft quark radiation differ in their physical origin and necessitate distinct treatments for extracting NLP logarithms, they nonetheless demonstrate the same universal structure in the threshold logarithms. This striking consistency strongly points toward the presence of an underlying universality governing these logarithmic contributions.

The NLP leading logarithms exhibit a universal analytical structure for both next-to-soft gluon emission and soft quark radiation, provided that the relevant helicity configurations are properly accounted for. This universality persists irrespective of whether the final-state massive colorless particle is of spin-0 or spin-1. In the case of soft (anti)quark emission, the NLP leading logarithms maintain a consistent analytical form when all contributing Born-level squared amplitudes are accurately identified—specifically, those constructed by “clubbing” the soft (anti)quark with the adjacent colored particle in color-ordered helicity amplitudes, along with a sum over all polarizations of the massive particle. Likewise, for next-to-soft gluon emission, the NLP leading logarithms exhibit universal analytical structures upon summation over the helicities of the emitted gluon and the polarizations of the massive particle.

In what follows, we consider two specific helicity configurations in the context of Higgs + jet production in the large top-mass limit—one involving next-to-soft gluon emission and the other involving soft quark radiation,

$$\begin{aligned} \mathcal{C}_{LL,1g2q^3H4\bar{q}5g(s)}^{++\Sigma-+} &= 4\pi\mathcal{F}_{qg} \left\{ C_A \left(\frac{4}{s_{23}} \right) \right. \\ &\quad \left. + 2C_F \left(\frac{1}{s_{23}} \right) \right\} \left| \mathcal{A}_{1g2q^3H4\bar{q}}^{++\Sigma-} \right|^2, \quad (19) \end{aligned}$$

$$\begin{aligned} \mathcal{C}_{LL,1g2q^3H4\bar{q}(s)5g}^{-+\Sigma-+} &= 2\pi\mathcal{F}_{qg} \left\{ C_A \left(\frac{2}{s_{23}} \right) \right. \\ &\quad \left. + 2C_F \left(\frac{1}{s_{23}} \right) \right\} \left| \mathcal{A}_{1\bar{q}2q^3H5g}^{-+\Sigma-} \right|^2. \quad (20) \end{aligned}$$

In each case, the soft particle is denoted by “(s)” in the subscript. The quantities $C_A = N$ and $C_F = \frac{N^2-1}{2N}$ represent the quadratic Casimir invariants associated with the adjoint and fundamental representations of the $SU(N)$ color group, respectively. Here Σ signifies that the Higgs boson, being a scalar particle, does not possess any polarization degrees of freedom. Note that $\mathcal{C}_{LL,1g2q^3H4q5g(s)}^{++\Sigma--} = 0$, as the maximally helicity violating amplitudes do not contribute to this process at NLP.

Based on the preceding discussion, it becomes evident that one can anticipate the structure of NLP leading logarithms for analogous processes, such as W + jet production. These are expected to follow the form as given below,

$$\begin{aligned} &\left[\mathcal{C}_{LL,1g2q^3W4q'5g(s)}^{++\Sigma-+} + \mathcal{C}_{LL,1g2q^3W4q'5g(s)}^{++\Sigma--} \right] \\ &= 4\pi\mathcal{F}_{qg} \left\{ C_A \left(\frac{4}{s_{23}} \right) + 2C_F \left(\frac{1}{s_{23}} \right) \right\} \left| \mathcal{A}_{1g2q^3W4q'}^{++\Sigma-} \right|^2, \quad (21) \end{aligned}$$

$$\mathcal{C}_{LL,1g2q^3W4q'(s)5g}^{-+\Sigma-+} = 2\pi\mathcal{F}_{qg} \left\{ 2C_F \left(\frac{1}{s_{23}} \right) \right\} \left| \mathcal{A}_{1q'2q^3W5g}^{-+\Sigma-} \right|^2. \quad (22)$$

The notation Σ in the superscript indicates a summation over the polarization states of the W boson is being carried out. Between q and q' , one denotes a quark and the other an antiquark, with the assignment depending on the electric charge of the W boson. It is noteworthy that no term proportional to C_A appears in Eq. (22), as there is no gg initiated channel contributing to W + jet production at the Born level. In Figs. 1 and 2, we present the NLP leading-logarithmic coefficients, with the corresponding $\mathcal{F}_{ab}$ factors and squared amplitudes removed (thus, $\mathcal{C}_{\text{LL}}$ is denoted as $\tilde{\mathcal{C}}_{\text{LL}}$ on the vertical axis), for helicity configurations analogous to those in Eqs. (21) and (22), respectively, but applied to a generic colorless particle $\mathcal{C}$. The mass of this particle is scaled to the Higgs boson mass m_H and varied over a wide range for different phase space points. The universal nature of the analytic forms of these coefficients enables the identification of the NLP leading-logarithmic values for the production of any colorless particles in association with a jet. We explicitly highlight the values for the W , Z , and Higgs bosons. By multiplying these values with the appropriate F_{ab} factors and squared amplitudes, the actual NLP leading-logarithmic values can easily be obtained. Note that, from Fig. 2, only the portion proportional to C_F can be estimated for the Higgs + jet production, as Eq. (22) does not contain any C_A term. A complete

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