

Quantum potential with no perturbative series and nonperturbative vacuum dominated by complex classical paths

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Spectra of standard one-dimensional potentials (double-well, sin-Gordon, etc.) are given by trans-series in coupling, including (badly divergent) perturbative series and nonperturbative terms $\sim \exp(-\text{const}/g^2)$ and $\log(1/g) \exp(-\text{const}/g^2)$. All of them are badly defined (e.g., perturbative series are badly divergent) but in sum supposed to be good. In this paper, we discuss an example of a potential with specially defined couplings making perturbative series completely absent. We calculate its nonperturbative vacuum energy and show that they are reproduced by the action of certain complex solutions to the holomorphic Newton equation.

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I. INTRODUCTION

Perhaps the most spectacular early example of how quantum mechanics is different from classical mechanics was the semiclassical description of tunneling [1]. It spectacularly described α decays and spontaneous fissions of heavy nuclei, covering the range of probabilities spanning about 20 orders of magnitude.

The instanton, a space-time soliton in gauge theories [2], plays a very important role in gauge dynamics. As was shown in early “instanton liquid model” [3] and confirmed in many subsequent works, instantons create spontaneous breaking of the chiral symmetry in QCD, being thus responsible for the effective quark mass (and thus the mass of nucleons and ourselves). Lately, numerical simulations on the lattice and phenomenological studies also point to the important role of correlated instanton-antiinstanton pairs, or molecules. For a recent review, see, e.g., Ref. [4].

Very extensive developments of semiclassical theory, based on Feynman path integrals in imaginary time, were slowly developed in the meantime. The *instantons* were discussed in many more settings. The most studied is the *double-well potential* (DWP) with a potential

$$V_{\text{DWP}} \sim (x^2 - f^2)^2 \quad (1)$$

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with two degenerate minima, at $x = \pm f$. Tunneling between them lifts degeneracy of the spectrum and establishes parity ($x \leftrightarrow -x$) as the true quantum number.

Further developments of semiclassical theory were formulating it in quantum field theory (QFT) terms. Using Feynman diagrams allows one in principle to calculate corrections to the instanton amplitude to any loop order. In particular, corrections up to three loops have been calculated [5]. In Fig. 1, we compare the one-, two-, and three-loop semiclassical expressions to actual *gaps* (splitting between lowest negative-to-positive parity states) $E_- - E_+$ computed numerically and given as a function of the

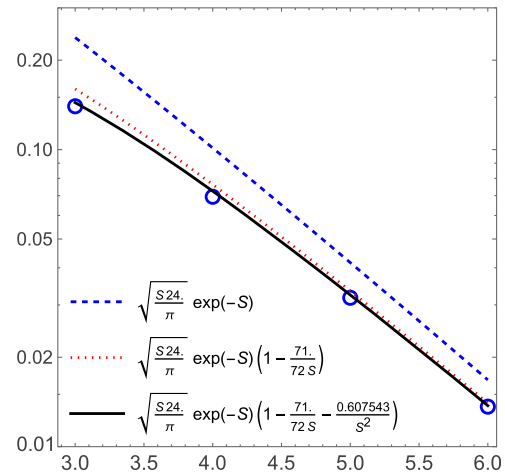


FIG. 1. The splitting of the negative-parity state from the positive-parity ground state $E_- - E_+$ as a function of instanton action S , shown as data points. Three curves correspond to one-, two-, and three-loop semiclassical approximations, with expressions given on the plot.

instanton action S . This plot demonstrates that, with fluctuations to few loops included, the semiclassical theory becomes rather accurate, even at not-so-large values of the action $S \sim 3\hbar$.

A wider perspective on semiclassics relates it to old questions of mathematics on representing solutions as series in parameters. Dependence of physical observables (such as the vacuum energy we will mostly discuss below) on coefficients of the action should in general be expressed as the so called *trans-series*. Those combine (badly divergent) perturbative series ($\sum a_n g^n$) with non-perturbative terms [$\sim \exp(-\text{const}/g^2)$ times another series $\sum b_n g^n$]; for a review, see, e.g., Ref. [6]. Nontrivial relations between a_n and b_n coefficients are known as *resurgence* relations. While those are fulfilled in known examples (e.g., by two- and three-loop corrections just mentioned), their generality, or even existence in the QFT context, remains a mystery.

Since the early 1980s, a number of boson-fermion examples were given in the context of *supersymmetric quantum mechanics* [7]. In those, supersymmetry holds perturbatively but gets broken nonperturbatively, by instantons. One example [8], with potential

$$V_{T0} = a^2 x^6 - 2ax^4 + (1 + 3a)x^2 - 1, \quad (2)$$

was studied extensively, showing that vacuum energy is indeed exponentially small $\sim a^{1/2} \exp[-\text{const}/a]$ at small a .

In the current paper, we focus on the other special one-dimensional (1D) quantum mechanical (QM) Hamiltonian [9]

$$H = (p^2 + x^2 - 1) + (a^2 x^4 + 2ax^3 - 2ax). \quad (3)$$

First of all, we will prove in the next section a total vanishing of perturbative series (all $a_n = 0$), done without any use of fermions or supersymmetry. Yet, the non-perturbative effects still apparently exist, and our further study would be to look for semiclassical paths responsible for the vacuum energy.

In doing so, we will follow Ref. [10], which introduced a notion of “complex bions,” solutions of the *holomorphic Newton’s equation* (HNE). Let us remind the reader that this work was set for a *tilted double-well* (TDW). The rescaled $x = \sqrt{g}r$ potential $\mathcal{W} = \frac{1}{g}W(\sqrt{g}r)$ leads to ordinary potential $V(r) = \frac{1}{2}[(\mathcal{W}'(r))^2 \pm p\mathcal{W}''(r)]$. A particular example considered had $W(x) = x^3/3 - x$. The parameter p was taken to be close to 1, the supersymmetric value.

Flipping the potential ($V \rightarrow -V$) in Euclidean time formulation, one observes that real paths starting from the global maximum cannot have a finite action. There exist, however, complex paths, solutions to the HNE with finite action. A solution of such a kind, or *complex bion*, was found in Ref. [10] by analytic continuation. Among other important observation made by Ref. [10], let us

mention that HNEs separated for real and imaginary parts do *not* correspond to motion in any two-dimensional potential.

(Another extreme path at small a can be a real “bounce” path, starting from the lower maximum and reflected from real turning point. Yet, it is of no interest for us now, as it is related to the false vacuum problem and is irrelevant to current work.)

A. Outline of the paper

In this work, we focus on a specific example of 1D QM problem with Hamiltonian (3). Note a bit nonstandard selection of units in the first Harmonic bracket, without usual $1/2$ and with some nonzero vacuum energy subtracted.

Three other terms, $\sim x, x^3, x^4$, depend on a single parameter a , to which we will refer as the coupling constant. We will only discuss the regime of relatively small a . Note further that its $O(a)$ term has different powers of the coordinate, $(2a)(x^3 - x)$, so a dependence of the action cannot be absorbed by the standard redefinition of the coordinate, as is the case in several examples discussed previously.

In Sec. II, we will start with arbitrary coefficients of these terms. Calculating the first perturbative correction by standard perturbative series, we will show that for this Hamiltonian the corrections $O(a^2)$ cancel each other. Pursuing the issue to higher orders, we evaluate generic Hamiltonian matrix and show, for this case, coefficient cancellations persists to higher orders as well. [We actually checked terms $O(a^4, a^6, a^8)$ and stopped, although it can be continued with larger and larger submatrices.]

In Sec. II B, we switch from Schrödinger equation to its Riccati form and show that coefficients of the perturbative series vanish to *all* orders [9].

The *nonzero* vacuum energy $E_0(a)$ we first simply calculate numerically in Sec. III; see Fig. 4. Indeed, its observed behavior is very different from that of generic potentials, for which the lowest perturbative series terms dominate at small a .

Then, following Ref. [10], we are looking for paths in the complex plane, classical solutions of the HNE. Unlike the authors of Ref. [10], we solve them numerically, thus deriving a *family* of the complex paths with various initial directions from their starting point, the global maximum (of inverted potential).

All such paths recoil from some complex turning point and come back, as an instanton-antiinstanton pair would do. Following Ref. [10], we also call them *complex bions*. All of them happen to possess the same action. Moreover, the imaginary part of the action is always π , so that $\exp(-S)$ remains real.

Finally, we demonstrate that the vacuum energies obtained from solutions of Schrödinger equation are in good correspondence to those actions.

II. PERTURBATIVE SERIES

A. Correction to vacuum energy of second order

At $a = 0$, the problem (3) it is just a harmonic oscillator, with spectrum given by even integers $E_n = 0, 2, 4, \dots$

Let us start using parameter a as a perturbative coupling. Explicit calculation of coefficient of standard perturbation theory is straightforward. The $a^2 x^4$ term appears already in the first order, and $2a(x^3 - x)$ appears in the second order. The results are

$$a2_{x^4} = \langle 0 | a^2 x^4 | 0 \rangle = (3/4)a^2 \quad (4)$$

$$a2_1 = \frac{|\langle 0 | 2a(x^3 - x) | 1 \rangle|^2}{E_0 - E_1} = -(1/4)a^2$$

$$a2_3 = \frac{|\langle 0 | 2a(x^3 - x) | 3 \rangle|^2}{E_0 - E_3} = -(1/2)a^2$$

$$\rightarrow a2 = \sum_i a2_i = 0. \quad (5)$$

Adding them together, one finds that the second-order perturbative coefficient $O(a_2)$ is indeed zero.

B. Using the Riccati form of equation, to all orders

Looking for higher-order coefficients, it is convenient to follow Turbiner and switch to the Riccati form of the Schrödinger equation

$$\begin{aligned} \psi &\sim \exp(-\phi(x)), & y(x) &= \phi'(x) \\ y(x)^2 - y'(x) &= V - E. \end{aligned} \quad (6)$$

Let us now look for a solution for the ground-state energy and $y(x)$ in form of perturbative series,

$$E_0 = \sum e_n a^n, \quad y(x) = \sum y_n(x) a^n. \quad (7)$$

Putting those into the Riccati equation above and separating powers of a , we get the set of equations

$$E_0 = 0, \quad y_0 = x \quad (8)$$

$$y'_1 - 2xy_1 = E_1 - 2x^3 + 2x \quad (9)$$

$$y'_2 - 2xy_2 = E_2 - x^4 - y_1^2, \quad (10)$$

and for $n > 2$,

$$y'_n - 2xy_n = E_n + \sum_{i=1}^{n-1} y_i y_{n-i} \quad (11)$$

Since in the zeroth order the potential is an oscillator, symmetric in $x \rightarrow -x$, it is clear that $O(a)$ terms in the

potential are both odd and thus their average value must be zero, so $E_1 = 0$. The equation gives then $y_1 = x^2$.

The second equation has in general a solution,

$$y_2 = e^{x^2} [c_2 + E_2/2\sqrt{\pi} \text{Erf}(x)].$$

However, as we have already shown above from ordinary perturbation theory, for the potential in question, $E_2 = 0$ due to cancellation of contributions, and the first term would not give a normalizable wave function. It then forces vanishing of the whole function as well, $y_2(x) = 0$.

The r.h.s. of the third equation contains in general a new term $y_1 y_2$, which in our case vanishes since $y_2(x) = 0$. Then, the equation for y_3 is the same as for y_2 , and therefore it leads to the same conclusion, $E_3 = 0$, $y_3 = 0$. This situation continues then to all orders, so one concludes that perturbation theory for all E_n , y_n terminates.

If so, then $y = x + ax^2$, and one comes to a solution of the Schrödinger equation,

$$\psi \sim \exp(-x^2/2 - x^3/3), \quad (12)$$

which is simple but, unfortunately, *unphysical* since it is *non-normalizable*. [In fact, starting from it, Turbiner invented the discussed potential (3)].

III. ACTUAL VACUUM ENERGY

Completing discussion of perturbative series, we return to study of the problem in full. The shape of the potential (3) is shown in Fig. 2. One can see that there are two regimes: at small enough a , there is a second minimum, which disappears above some critical value.

Since the derivative of our potential is a polynomial of power 3, in the complex plane, there are always three extrema. Analytic expressions for their location can be easily obtained, e.g., by Wolfram *Mathematica*, but we skip those as they are lengthy and not very relevant.

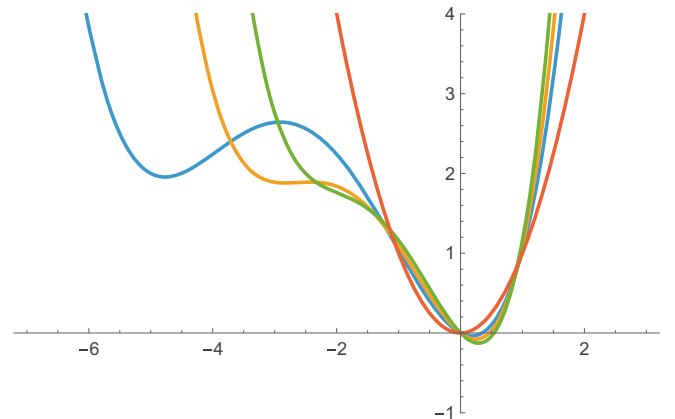


FIG. 2. Potentials $V(x)$ versus x , for $a = 0, 0.2, 0.3, 0.4$ (red, green, orange, and blue).

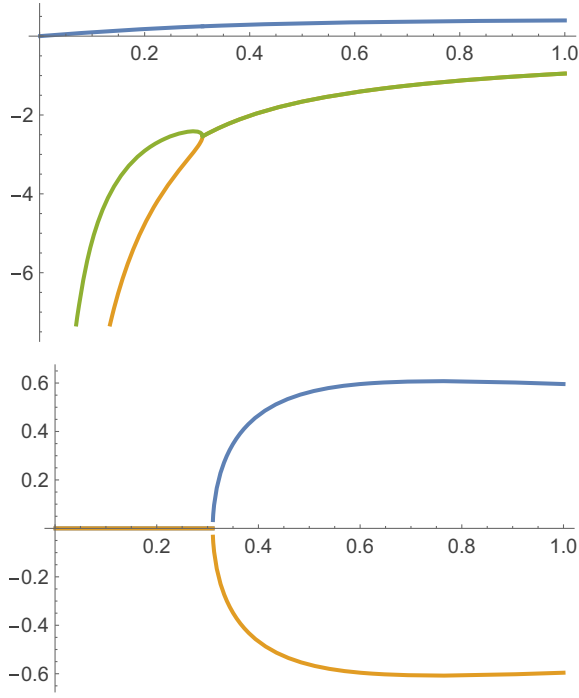


FIG. 3. The upper plot shows real parts of three extrema of the potential as a function of parameter a . The lower plot shows imaginary parts of two conjugated extrema, while the global minimum remains real.

Those one can clearly see in Fig. 3. For small $a < a_c$, all three extrema are real; then they collide; and above a_c , two are complex conjugates (see the lower plot).

There is no problem in finding the spectrum at any values of a numerically. It is indeed nonzero, and its dependence on a is shown in Fig. 4. Deviations of vacuum energy from zero are shown in logarithmic plot, with some part of it obtained with higher accuracy.

Such dependence on a does not occur for generic potentials which have perturbative corrections, e.g., of the lowest order $O(a^2)$. This plot clearly indicates that vacuum of this potential is completely nonperturbative, described by some version of the “instanton ensemble.”

IV. FAMILY OF COMPLEX BIONS

There is a well-known “bounce” real path starting from the lower maximum (of inverted potential $-V$). But since it does not touch the main global maximum, we also do not care about the two different regimes in a , with different arrangements of the two other extrema.

Relevant solutions to the HNE,

$$m \frac{d^2 z}{dt^2} = \frac{\partial V(z(t))}{\partial z},$$

should all start from the global maximum of $-V$ and move into the complex plane, where it may find a turning point and return back, closing the solution.

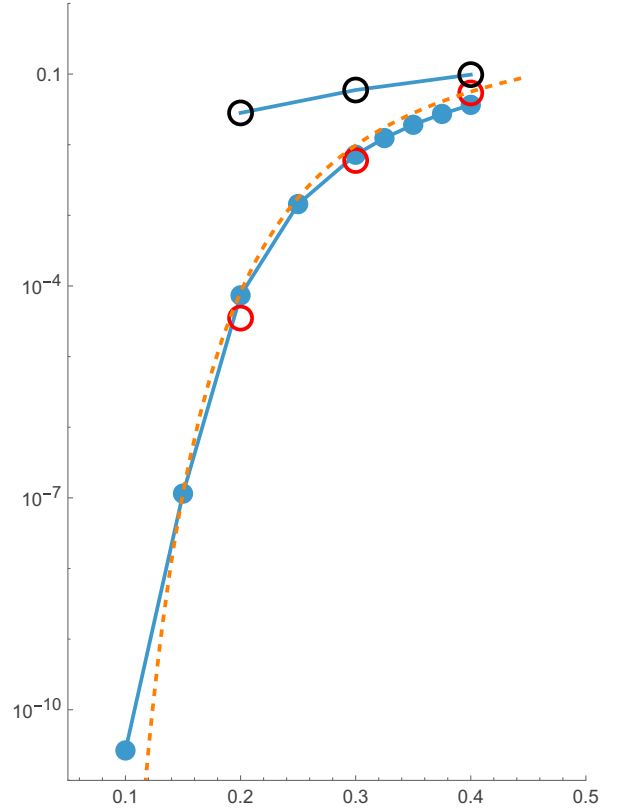


FIG. 4. The ground-state (vacuum) energy E_0 versus parameter a . Three black points above are for potential with only $a^2 x^4$ term, apparently dominated by the lowest-order perturbative contribution. The blue points are from the numerical solution of Schrödinger equation with the potential (3) under consideration: cancellation of perturbative series is apparent. The red dashed line is a crude fit of the form $\sim \sqrt{a} \exp[-0.33/a^2]$. The red circles show the contribution of the complex bions, according to (15).

Since we proceed numerically, we may start from the global maximum in any direction. This is done by small displacement of the initial point by $\delta(\cos(\phi), \sin(\phi))$, typically $\delta \sim 10^{-3}$. The angle ϕ is an arbitrary angle on which the complex bion solution depends. At $a = 0.2$, an example of it with $\phi = -\pi/2$ (starting along the imaginary axes) is shown in Fig. 5.

As a result, we find that holomorphic Newton’s equation has a family of complex bion solutions, parametrized by their initial angle ϕ . Not surprisingly, all of them have the same action. For our examples of bions,

$$\begin{aligned} S_{\text{kin}}(a=0.2) &= \int dt \frac{1}{4} (z'(t))^2 = 6.68618 + 1.57 \cdot i, \\ S_{\text{pot}}(a=0.2) &= \int dt V(z(t)) = 6.23949 + 1.57 \cdot i. \end{aligned} \quad (13)$$

The shape of these solutions is better shown in the parametric plot Fig. 5 (upper). The numerically calculated total action is thus

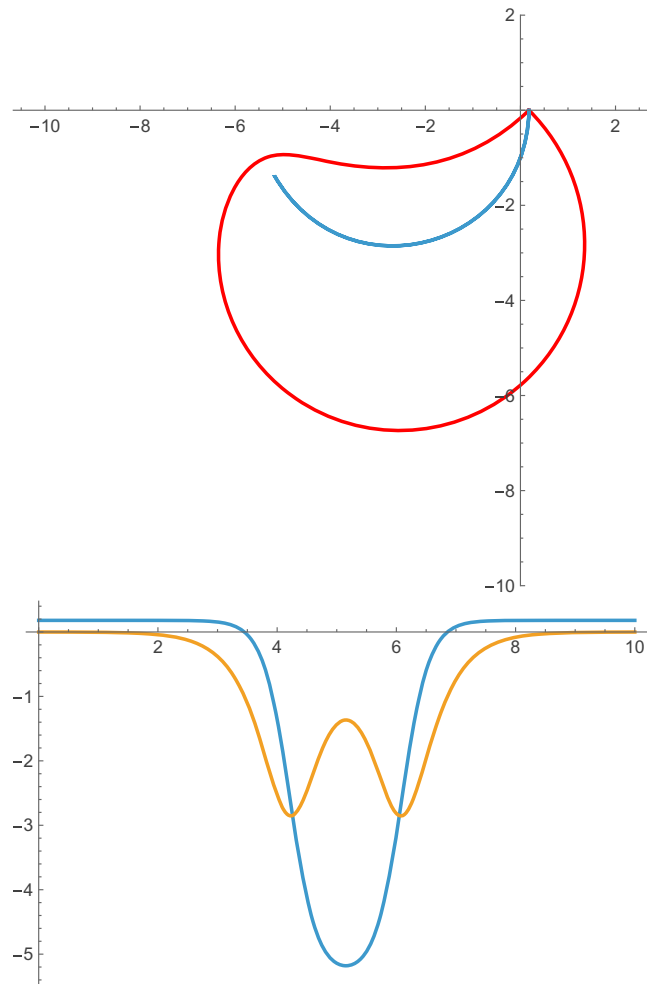


FIG. 5. Complex bions for $a = 0.2$. The upper parametric plot shows two solutions to the HNE in the complex X, Y plane. The blue curve (for angle $\phi = -\pi/2$) is passed twice. The red curve is for $\phi = -3\pi/4$ or $\phi = -\pi/4$, the same curve in the opposite direction. The lower plot shows $X(t)$ and $Y(t)$ as a function of “time” t for blue curve above: it confirms that the outgoing part of the path is the time-reflected incoming path.

$$S_{\text{tot}}(a = 0.2) = 12.9246 + i \cdot 3.14151. \quad (14)$$

It is the same for any shape of the path (within numerical accuracy) for any value of the initial angle ϕ . (Because of holomorphic nature of the equation, it is of course just another manifestation of Cauchy theorem, allowing deformations of the contour.)

Note that, while the action has a nonzero imaginary part, the amplitude $\exp(-S)$ remains real, since the imaginary

part happens to be exactly π , and $\exp(\pm i\pi) = -1$. Bions going into the upper half-plane are complex conjugate.

Originally found as a numerical curiosity, this observation has already been explained in Ref. [10]. This result can be traced to a limit in which a Cauchy contour is increased into a circle of large radius, where it becomes defined by the asymptotics of the potential, given in terms of the highest powers of the coordinate z in the potential.

After bion actions are found, we would like to compare their semiclassical contribution to vacuum energy with the numerical ones. The red circles in Fig. 4 correspond to

$$E_0 = 4 \cdot \sqrt{\text{Re}(S)} \exp(-\text{Re}(S)), \quad (15)$$

where $S(a)$ are bion actions and 4 is a fit parameter standing for the unknown functional one-loop determinant of the bion solution.

V. HAMILTONIAN AS A CUTOFF MATRIX, ACCURACY OF EIGENVALUES

Representation of Hamiltonians as finite-size matrices in some basis is a very popular way of solving spectroscopic quantum problems in atomic, nuclear, particle physics. Yet, systematic studies of accuracy of the results are in most cases missing.

In the QFT context, anything beyond some momentum cutoff is ignored by necessity. The relation between the cutoff scale and the value of the running couplings and masses is given by the renormalization group (RG), due to Gell-Mann and Law. One famous application of it to second-order phase transitions, by K. Wilson, was using ϵ expansion near space dimension $d = 4$ and derived the values of critical indices. A different form of it, the Hamiltonian RG (HRG), tries to compensate for the missing column and row of the Hamiltonian by changing some retained matrix elements. See also the paper by Glazek and Wilson [11].

Basic studies of the HRG for the quantum anharmonic oscillator were done in [12]. It was found there that the HRG approach to the fixed point happens via some spiral motion of peripheral matrix elements.

It is a straightforward exercise to calculate the Hamiltonian matrix of our Hamiltonian H_T in the harmonic basis. For example, here is 10×10 part of it. Since the highest power of the coordinate in the potential is x^4 , it limits the highest power of creation or annihilation operators of the harmonic basis, so it has a stripped form,

$\frac{3a^2}{4}$	$\frac{a}{\sqrt{2}}$	$\frac{3a^2}{\sqrt{2}}$	$\sqrt{3}a$	$\sqrt{\frac{3}{2}}a^2$	0	0	0	0	0
$\frac{a}{\sqrt{2}}$	$2 + \frac{15a^2}{4}$	$4a$	$5\sqrt{\frac{3}{2}}a^2$	$2\sqrt{3}a$	$\sqrt{\frac{15}{2}}a^2$	0	0	0	0
$\frac{3a^2}{\sqrt{2}}$	$4a$	$4 + \frac{39a^2}{4}$	$7\sqrt{\frac{3}{2}}a$	$7\sqrt{3}a^2$	$\sqrt{30}a$	$3\sqrt{\frac{5}{2}}a^2$	0	0	0
$\sqrt{3}a$	$5\sqrt{\frac{3}{2}}a^2$	$7\sqrt{\frac{3}{2}}a$	$6 + 75a^2/4$	$10\sqrt{2}a$	$9\sqrt{5}a^2$	$2\sqrt{15}a$	$\sqrt{\frac{105}{2}}a^2$	0	0
$\sqrt{\frac{3}{2}}a^2$	$2\sqrt{3}a$	$7\sqrt{3}a^2$	$10\sqrt{2}a$	$8 + 123a^2/4$	$13\sqrt{\frac{5}{2}}a$	$11\sqrt{\frac{15}{2}}a^2$	$\sqrt{105}a$	$\sqrt{105}a^2$	0
0	$\sqrt{\frac{15}{2}}a^2$	$\sqrt{30}a$	$9\sqrt{5}a^2$	$13\sqrt{\frac{5}{2}}a$	$10 + 183a^2/4$	$16\sqrt{3}a$	$13\sqrt{\frac{21}{2}}a^2$	$2\sqrt{42}a$	$3\sqrt{21}a^2$
0	0	$3\sqrt{\frac{5}{2}}a^2$	$2\sqrt{15}a$	$11\sqrt{\frac{15}{2}}a^2$	$16\sqrt{3}a$	$12 + 255a^2/4$	$19\sqrt{\frac{7}{2}}a$	$15\sqrt{14}a^2$	$6\sqrt{7}a$
0	0	0	$\sqrt{\frac{105}{2}}a^2$	$\sqrt{105}a$	$13\sqrt{\frac{21}{2}}a^2$	$19\sqrt{\frac{7}{2}}a$	$14 + 339a^2/4$	$44a$	$51\sqrt{2}a^2$
0	0	0	0	$\sqrt{105}a^2$	$2\sqrt{42}a$	$15\sqrt{14}a^2$	$44a$	$16 + 435a^2/4$	$75a/\sqrt{2}$
0	0	0	0	0	$3\sqrt{21}a^2$	$6\sqrt{7}a$	$51\sqrt{2}a^2$	$(75a)/\sqrt{2}$	$18 + 543a^2/4$

Note that diagonal matrix elements are finite even at $a \rightarrow 0$, except the first one. Therefore, its determinant is a polynomial proportional to certain powers of a because at $a \rightarrow 0$ the ground-state energy is zero.

Calculating the determinant analytically is rather straightforward, unlike the diagonalization, which can better be done at some fixed values of a . This power grows with the matrix size. For example, for a matrix of 4×4 , it is

$$\det(H_{44}) = 120a^4 + (1755a^6)/8 + (23625a^8)/256$$

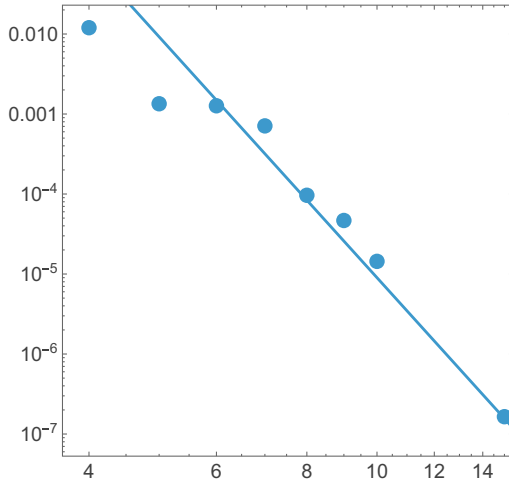


FIG. 6. Convergence of the lowest eigenvalue as a function of n , for $n \times n$ approximation. Points are the deviation $E_0(n, a) - E_0(\infty, a)$ of the ground-state energy from its asymptotic values, obtained numerically. This particular calculation is for $a = 0.4$, and others are similar. The line is power fit $\sim n^{-10}$, shown for comparison. Convergence is quite rapid; say, for $n = 15$, the accuracy improves by 5 orders of magnitude compared to the $n = 4$ case.

starting from a^4 and not a^2 as one could anticipate. For the 10×10 submatrix given above, the determinant starts with power a^8 . What it means is that corrections $\sim a^2, a^4, a^6$ to the ground-state energy have already canceled inside this matrix, while terms $\sim a^8, \dots$ are present because cancellations have not yet happened inside such a submatrix. Continuing this for larger and larger $n \times n$ matrices, one indeed observes subsequent disappearance of higher-order terms in the perturbative series, as anticipated above.

In Fig. 6, we show how the exact ground-state energies are approached when the size n of the set of included states (Hamiltonian in $n \times n$ form) is systematically increased. Converging to correct values is found to be quite rapid.

VI. CONCLUSIONS

We studied an example of a potential (3) suggested by [9]. By different means, we show that its perturbative series are vanishing to all orders. We show it directly, without usage of any fermions and supersymmetry.

The nonvanishing vacuum energy is calculated numerically for all couplings a . Its dependence on a is clearly nonperturbative. We give evidence that it is due to “instanton-antiinstanton correlated pairs,” in the form of the “complex bion” classical solutions to the HNE, found numerically.

We also studied how the results are approached when only the finite ($n \times n$ matrix) part of the Hamiltonian is used. The approach to correct values is found to be very rapid.

As an outlook, it would be interesting to see how our results correspond to different versions of “resurgence” relations and in particular whether using them one can find the series in higher-order fluctuations around such complex bions.

It is also tempting to ask if similar complex classical paths can be found for QFT problems, such as for the gauge four-dimensional theories.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the author.

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- [1] G. Gamow, *Z. Phys.* **51**, 204 (1928).
 - [2] A. Belavin, A. Polyakov, A. Schwarz, and Y. Tyupkin, *Phys. Lett.* **59B**, 85 (1975).
 - [3] E. V. Shuryak, *Nucl. Phys.* **B203**, 93 (1982).
 - [4] E. Shuryak and I. Zahed, [arXiv:2601.15085](#).
 - [5] M. A. Escobar-Ruiz, E. Shuryak, and A. V. Turbiner, *Phys. Rev. D* **92**, 025046 (2015); **92**, 089902(E) (2015).
 - [6] G. V. Dunne and M. Unsal, *Phys. Rev. D* **89**, 105009 (2014).
 - [7] E. Witten, *Nucl. Phys.* **B202**, 253 (1982).
 - [8] A. Turbiner, *Phys. Lett. B* **276**, 95 (1992); **291**, 519(E) (1992).
 - [9] A. Turbiner (private communication).
 - [10] A. Behtash, G. V. Dunne, T. Schäfer, T. Sulejmanpasic, and M. Ünsal, *Phys. Rev. Lett.* **116**, 011601 (2016).
 - [11] S. D. Glazek and K. G. Wilson, *Phys. Rev. D* **48**, 5863 (1993).
 - [12] M. Girgus and S. D. Glazek, [arXiv:2404.17446](#).