

Soft and semihard components of multiplicity distributions in the k_T -factorization approach

H. R. Martins-Fontes^{*} and F. S. Navarra[†]

Instituto de Física, Universidade de São Paulo, Rua do Matão 1371, Cidade Universitária, 05508-090, São Paulo, SP, Brazil



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Particle production in hadronic collisions can be studied in the low-momentum (soft) and high-momentum (hard) transfer regimes. While the latter can be well understood within perturbative QCD, the former contains nonperturbative effects which cannot be calculated from first principles. There is also an intermediate regime called semihard, in which the momentum transfer runs typically from 1 to 10 GeV. As the hadron-hadron collision energy increases, we expect to see a relative growth of the number of semihard events. It has been conjectured that this growth would be the cause of some changes observed in the multiplicity distributions measured in proton-proton collisions. In this paper we revisit the separation between soft and semihard events using the formalism of k_T factorization. The separation is implemented through the introduction of a scale that is the cutoff Λ in the transverse momentum of the produced gluon and allows us to compute the average number of particles produced in each regime. These numbers are used as input in the double negative binomial fit of data, from which we can extract correlations between the fraction of semihard events and the violation of Koba-Nielsen-Olsen scaling.

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I. INTRODUCTION

The multiplicity distribution (MD) of charged particles measured in high-energy proton-proton collisions is one of the observables that is not yet well understood in QCD. For a comprehensive review, see [1]. For the multiplicities of particles in jets, in the limit of very high energies and very high multiplicities, there are robust predictions of perturbative QCD (pQCD) [2]. For other types of events, there are significant nonperturbative effects and we cannot perform calculations from first principles. In the absence of a straightforward connection between theory and data, we have to resort to models. However, even this task is complex because of the nonperturbative nature of hadronization. The “top-down” models, such as those based on the color glass condensate (CGC) approach [3], or event generators, such as Pythia [4,5], give only an approximate description of the measured MD’s which exhibit a complex structure. Many works have taken a “bottom-up” approach, i.e., trying first to find empirical regularities in the data, obtaining successful parametrizations of these data, and

then searching for scaling laws and their violation. After this first step, one tries to connect the empirical observations with the theory. Along this line, the fits with the negative binomial distribution (NBD) were very successful for many years and especially for lower-energy data. But, at the $\sqrt{s} = 900$ GeV super proton synchrotron energy, the NBD started to fail in describing the measured multiplicity distributions, which exhibited a shoulder-shaped structure [1].

For LHC energies, a single NBD fit turned out to be problematic. In the early ALICE data [6], for the class of nonsingle diffractive (NSD) events, no strong deviation from a single NBD fit was observed at $\sqrt{s} = 0.9$ and 2.36 TeV for $|\eta| < 1$, but a hint of a substructure appeared at $|\eta| < 1.3$. However, in [7] for all event classes, already at $\sqrt{s} = 0.9$ TeV, the single NBD fits began to diverge from the data at the highest multiplicities for $|\eta| < 0.5$ and $|\eta| < 1.0$. Even more significant departures from the single NBD started at $\sqrt{s} = 2.76$ TeV.

A fruitful idea was to interpret the shoulder-like structure observed in the data as coming from a superposition of two negative binomial distributions (referred to as DNBD). The two NBDs could be associated to two different sources or two different types of processes. In the first case, as advocated, for example, in [8,9], the sources are well separated in rapidity. In the second case, the two NBDs would come from soft and semihard processes. This latter idea was advanced long ago in [10], further developed in [11], and more recently in [12]. A review on this subject can be found in [13]. For the TeV region, the expectation was

^{*}Contact author: henriquemartinsfontes@usp.br

[†]Contact author: navarra@if.usp.br

that, with increasing energies, the semihard component would become more important than the soft one. In parallel, a violation of the Koba-Nielsen-Olsen (KNO) scaling [14] was observed, suggesting a connection between the two findings. The latest LHC data [15,16] confirmed that for narrow pseudorapidity windows a single NBD was enough to reproduce the observed distributions and also that KNO scaling was valid. For larger windows, two NBDs were needed and KNO scaling was violated.

In [11,13], the difference between soft and semihard processes was only that the latter yield larger mean multiplicities. In the present work, using the k_T factorization formalism [17,18], we can make a better separation with the proper introduction of an energy scale. Moreover, the introduction of this formalism reduces the number of parameters. As will be seen, we will obtain fits with very good quality and we will extract from them, with less ambiguity, the behavior of the parameter α with the energy $\sqrt{s}$ and with the size of the pseudorapidity window. This parameter represents the fraction of soft events.

II. THE k_T FACTORIZATION APPROACH

At high-energy collisions between hadrons or heavy ions, a large number of gluons is produced due to the dense gluon population in the hadron wave function. Using the CGC formalism, particle production in high-energy hadronic collisions can be studied through the k_T factorization approach, where the inclusive production cross section is given by [18,19]

$$E \frac{d\sigma}{d^3p} = K \frac{4\pi N_c}{N_c^2 - 1} \alpha_s(Q^2) \frac{1}{p_\perp^2} \int^{p_\perp} dk_\perp^2 \varphi_1(x_1, k_\perp^2) \times \varphi_2(x_2, (p - k)_\perp^2), \quad (1)$$

where $x_{1,2} = \frac{p_\pm}{\sqrt{s}} \exp(\pm y)$, $\sqrt{s}$ is the center-of-mass energy, N_c is the number of colors and K is a normalization factor that describes the conversion of partons to hadrons. φ is the unintegrated gluon distribution (UGD) of a proton, and it can be related to the gluon density by

$$xG(x, \mu^2) = \int^{\mu^2} dk_\perp^2 \varphi(x, k_\perp^2). \quad (2)$$

The multiplicity per unit of rapidity can be computed by integrating (1) over $p_\perp$,

$$\frac{dN}{dy} = \frac{1}{S} \int_{p_{\perp, \min}}^{p_{\perp, \max}} d^2 p_\perp E \frac{d\sigma}{d^3 p}, \quad (3)$$

where S is a typical interaction area, and the minimum and maximum values of $p_\perp$ are determined by the experimental conditions. In (3), the main contribution is given by two regions of integration over $k_\perp$: $k_\perp \ll p_\perp$ and $|p_\perp - k_\perp| \ll p_\perp$. Hence, we can rewrite it as [20]

$$\begin{aligned} \frac{dN}{dy} &= \frac{K}{S} \frac{4\pi N_c}{N_c^2 - 1} \int_{p_{\perp, \min}}^{p_{\perp, \max}} \frac{dp_\perp^2}{p_\perp^2} \alpha_s(Q^2) \\ &\times \left[\varphi_1(x_1, p_\perp) \int^{p_\perp} dk_\perp^2 \varphi_2(x_2, k_\perp) + \varphi_2(x_2, p_\perp) \right. \\ &\times \left. \int^{p_\perp} dk_\perp^2 \varphi_1(x_1, k_\perp) \right] \\ &= \frac{K}{S} \frac{4\pi N_c}{N_c^2 - 1} \int_{p_{\perp, \min}}^{p_{\perp, \max}} \frac{dp_\perp^2}{p_\perp^4} \\ &\times \alpha_s(Q^2) xG_2(x_2, p_\perp^2) xG_1(x_1, p_\perp^2), \end{aligned} \quad (4)$$

where the equation in the first line was integrated by parts. The multiplicative constants K and S can be grouped into a single parameter, which is fixed by fitting the available pseudorapidity distributions.

The running coupling constant α_s follows the one-loop approximation and is assumed to freeze at 0.52:

$$\alpha_s(Q^2) = \min \left[\frac{12\pi}{27 \ln \left(\frac{Q^2}{\Lambda_{\text{QCD}}^2} \right)}, 0.52 \right], \quad (5)$$

with $\Lambda_{\text{QCD}} = 0.226$ GeV. The scale Q^2 at which the strong coupling $\alpha_s(Q^2)$ is evaluated in Eq. (4) is defined as

$$Q^2 = \max [p_\perp^2, \min (Q_{s,1}^2, Q_{s,2}^2)]. \quad (6)$$

In order to proceed with our calculation we need to choose the unintegrated gluon distribution. We will use two very well-known UGDs, namely the Golec-Biernat-Wüsthoff (GBW) [21,22] and the Kharzeev-Levin-Nardi (KLN) [20].

A. GBW model

We will use the GBW model [21,22] because it comes from a simple and well-established parametrization, which is known for its success in describing a wealth of experimental data on particle production, including both hard and soft processes. For the purposes of our work, the GBW model is especially appropriate because it reproduces very well the DIS data on F_2 at very low scales, which are consistent with the low transverse momenta of the majority of particles produced at the LHC. At these scales, the UGD derived from the GBW model is also consistent with much more sophisticated UGDs [23]. For a recent account of the successes of the GBW model, see [24] and references therein. The GBW UGD reads:

$$\varphi_{1,2}(x_{1,2}; k_\perp^2) = \frac{3\sigma_0}{4\pi^2 \alpha_s(Q_{s,1,2}^2)} \frac{k_\perp^2}{Q_{s,1,2}^2} \exp \left(-\frac{k_\perp^2}{Q_{s,1,2}^2} \right), \quad (7)$$

where the saturation scale Q_s^2 is defined as

$$Q_{s,1,2}^2 = Q_0^2 \left(x_0 \frac{\sqrt{s}}{Q_0} e^{\pm y} \right)^\lambda. \quad (8)$$

The parameters were fixed as in [19]: $Q_0 = 0.6$ GeV, $x_0 = 0.01$, $\bar{\lambda} = 0.205$, and $\sigma_0 = 23$ mb. The UGD (7) leads to the gluon distribution function expressed as

$$xG_{1,2}(x_{1,2}; p_\perp^2) = \frac{3\sigma_0}{4\pi^2\alpha_s(Q_{s_{1,2}}^2)} \left(Q_{s_{1,2}}^2 - \exp\left(-\frac{p_\perp^2}{Q_{s_{1,2}}^2}\right)(p_\perp^2 + Q_{s_{1,2}}^2) \right) (1 - x_{1,2})^4. \quad (9)$$

Here, as in [20], the factor $(1 - x)^4$ is introduced to account for large- x effects.

B. KLN model

The KLN model provides a simple form for xG , given by [20]

$$xG_{1,2}(x_{1,2}; p_\perp^2) = \begin{cases} \frac{S}{\alpha_s(Q_{s_{1,2}}^2)} p_\perp^2 (1 - x_{1,2})^4, & p_\perp < Q_{s_{1,2}}^2 \\ \frac{S}{\alpha_s(Q_{s_{1,2}}^2)} Q_{s_{1,2}}^2 (1 - x_{1,2})^4, & p_\perp > Q_{s_{1,2}}^2. \end{cases} \quad (10)$$

$$\begin{aligned} \frac{dN}{dy} &= \frac{K}{S} \frac{4\pi N_c}{N_c^2 - 1} \left[\int_{p_{\perp \min}}^{\Lambda} \frac{dp_\perp^2}{p_\perp^4} \alpha_s(Q^2) xG_2(x_2, p_\perp^2) xG_1(x_1, p_\perp^2) + \int_{\Lambda}^{p_{\perp \max}} \frac{dp_\perp^2}{p_\perp^4} \alpha_s(Q^2) xG_2(x_2, p_\perp^2) xG_1(x_1, p_\perp^2) \right] \\ &= \frac{dN_s}{dy} + \frac{dN_{sh}}{dy}, \end{aligned} \quad (11)$$

where N_s and N_{sh} are the soft and semihard components respectively. In the context of the CGC formalism, a similar separation criterion was introduced in [26]. To calculate the pseudorapidity distributions, one should rewrite expression (11) using the transformation

$$y(\eta) = \frac{1}{2} \log \frac{\sqrt{\cosh^2 \eta + \mu^2} + \sinh \eta}{\sqrt{\cosh^2 \eta + \mu^2} - \sinh \eta}, \quad (12)$$

where the scale μ is given as [25]

$$\mu(\sqrt{s}) = \frac{0.24}{0.13 + 0.32(\sqrt{s})^{0.115}} \quad (13)$$

and the Jacobian reads:

$$J(\eta) = \frac{\partial y}{\partial \eta} = \frac{\cosh \eta}{\sqrt{\cosh^2 \eta + \mu^2}}. \quad (14)$$

Changing variables and integrating (11) over the pseudorapidity we obtain the average multiplicity:

The saturation scale is parametrized in the same way as in the GBW model, and its parameters were fixed to the same values used in the GBW model [25].

C. Soft and semihard components

In expression Eq. (1), and also in Eq. (4), one gluon with $p_\perp$ is produced from the fusion of a gluon with $k_\perp$ and another gluon with $p_\perp - k_\perp$. Therefore the momentum $p_\perp$ is, to a good approximation, the minimum value of the momentum flowing in the three gluon vertex, i.e., the one to be used in the running coupling constant. For $p_\perp$ larger than a few GeV, the coupling $\alpha_s(Q^2)$ becomes sufficiently smaller than one and we are in the perturbative domain. We will thus follow the literature and call these events “semihard.” The events with lower values of $p_\perp$ will be called “soft.” Accordingly the total multiplicity per unit of rapidity is the sum of the contributions from the soft and semihard events. The separation is achieved by introducing a cutoff Λ in the integral over $p_\perp$, which defines the two contributions:

$$N = N_s + N_{sh}. \quad (15)$$

We emphasize that in the case of the GBW UGD the only free parameter is the constant K/S which is going to be determined by using Eq. (11) to fit the measured pseudorapidity distributions. When using the KLN UGD, the fitting parameter will be KS . It is also important to mention that Eq. (11) describes the gluon production and, after integration, the number of gluons produced. As done in previous works, we shall assume here that the number of produced partons is equal to the number of hadrons resulting from the hadronization of these partons (“parton-hadron duality”). The approximate equality between these numbers has been (successfully) tested recently in [27], where the authors used Pythia to calculate the hadron multiplicities within jets with and without using the LUND prescription for hadronization. They found a small difference between the number of partons and the number of hadrons.

In Fig. 1 we show the rapidity distributions measured by different collaborations of the LHC at different energies. The k_T factorization rapidity distribution obtained with the GBW and KLN UGDs are not very different from each other and both seem to be consistent with the data.

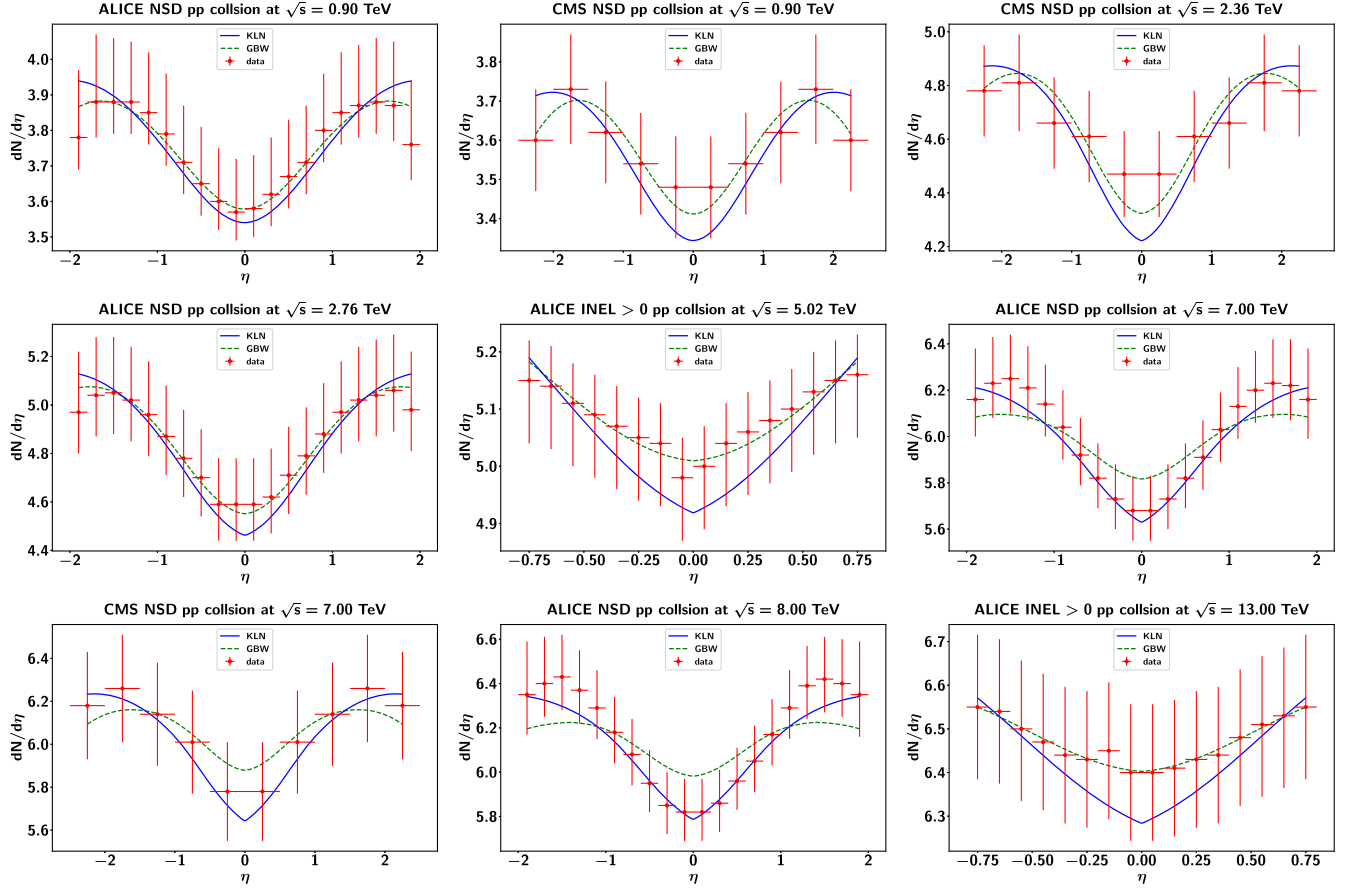


FIG. 1. Pseudorapidity distributions. The dashed lines are obtained from Eqs. (11) and (9). The solid lines are obtained from Eqs. (11) and (10). Data are from CMS [28,29] and ALICE [7,16]; the bars show the combined uncertainties.

Therefore we believe that for the purposes of this work, using Eq. (11) is sufficient.

III. DOUBLE NEGATIVE BINOMIAL DISTRIBUTION

In [30], Giovannini and Ugoccioni proposed a two-component model combining two NBDs associated with two event classes: “soft” and “semihard” processes. Recent analyses performed by the ALICE Collaboration showed that the double NBD fit yields a good description of the data on multiplicity distributions from 0.9 to 8 TeV [7,15,31] within several pseudorapidity windows. In this work, we will use their model. The DNBD is expressed as follows:

$$P(n) = \lambda[\alpha P(n, \langle n \rangle_s, k_s) + (1 - \alpha)P(n, \langle n \rangle_{sh}, k_{sh})], \quad (16)$$

where

$$P(n, \langle n \rangle, k) = \frac{\Gamma(k + n)}{\Gamma(k)\Gamma(n + 1)} \frac{\langle n \rangle^n k^k}{(\langle n \rangle + k)^{n+k}}. \quad (17)$$

In each NBD, $\langle n \rangle$ is the average multiplicity and the parameter k is related to the dispersion D ($D^2 = \langle n^2 \rangle - \langle n \rangle^2$) through

$$\frac{D_i^2}{\langle n \rangle_i^2} = \frac{1}{\langle n \rangle_i} + \frac{1}{k_i}, \quad (18)$$

where $i = \{s, sh\}$. The parameter α represents the fraction of soft events; consequently, $(1 - \alpha)$ denotes the fraction of semihard events; $\langle n \rangle_s$ and $\langle n \rangle_{sh}$ are the mean multiplicities of soft and semihard events respectively; k_s and k_{sh} are the negative binomial parameters of the soft and semihard components. The first few bins cannot be reproduced by any NBD and they are related to a different production mechanism, not included in our formalism. Therefore, these bins were removed from the fitting process. To account for this, a normalization factor λ was introduced [7,32]. The average multiplicity obtained from (16) can be expressed as

$$\langle n \rangle = \lambda[\alpha \langle n \rangle_s + (1 - \alpha) \langle n \rangle_{sh}]. \quad (19)$$

Comparing expression (19) with (15), and recalling that $N = \langle n \rangle$, we conclude that

$$\begin{aligned} \lambda \alpha \langle n \rangle_s &= N_s, \\ \lambda (1 - \alpha) \langle n \rangle_{sh} &= N_{sh}. \end{aligned} \quad (20)$$

Since N_s and N_{sh} are known, these two equations reduce the number of free parameters from six to four: α , λ , k_s , and k_{sh} .

IV. KNO SCALING

The Koba-Nielsen-Olesen (KNO) scaling [14] is a feature of MDs that asserts that, in the limit of $\langle n \rangle \rightarrow \infty$, the probability distributions $P(n)$ at different energies, once rescaled by $\langle n \rangle$, become copies of each other. In other words, if one plots $\langle n \rangle P(n)$ versus $n/\langle n \rangle$ for different energies, all curves collapse onto a universal function $\psi(n/\langle n \rangle)$:

$$\langle n \rangle P(n) = \psi\left(\frac{n}{\langle n \rangle}\right). \quad (21)$$

A direct consequence of KNO scaling is that the total scaled variance, in the scaling limit, increases linearly with $\langle n \rangle$ [33–35]:

$$\omega = \frac{D^2}{\langle n \rangle} = \kappa \langle n \rangle, \quad (22)$$

with $\kappa = \text{const} > 0$. In the case of the DNBD, using (19) we have

$$\begin{aligned} D^2 &= \lambda[\alpha \langle n^2 \rangle_s + (1 - \alpha) \langle n^2 \rangle_{sh}] - \langle n \rangle^2 = \lambda[\alpha(D_s^2 + \langle n \rangle_s^2) + (1 - \alpha)(D_{sh}^2 + \langle n \rangle_{sh}^2)] - \langle n \rangle^2 \\ &= \lambda \left[\alpha \left(\langle n \rangle_s + \frac{\langle n \rangle_s^2}{k_s} + \langle n \rangle_s^2 \right) + (1 - \alpha) \left(\langle n \rangle_{sh} + \frac{\langle n \rangle_{sh}^2}{k_{sh}} + \langle n \rangle_{sh}^2 \right) \right] - \langle n \rangle^2 \\ &= \langle n \rangle - \langle n \rangle^2 + \lambda \left[\alpha \langle n \rangle_s^2 \left(1 + \frac{1}{k_s} \right) + (1 - \alpha) \langle n \rangle_{sh}^2 \left(1 + \frac{1}{k_{sh}} \right) \right] \end{aligned} \quad (23)$$

where in the second line we made use of (18). Dividing the above equation by $\langle n \rangle$ we obtain the scaled variance:

$$\omega = \frac{D^2}{\langle n \rangle} = 1 - \langle n \rangle + \frac{\langle n \rangle}{\langle n \rangle^2} \lambda \left[\alpha \langle n \rangle_s^2 \left(1 + \frac{1}{k_s} \right) + (1 - \alpha) \langle n \rangle_{sh}^2 \left(1 + \frac{1}{k_{sh}} \right) \right] \quad (24)$$

and hence

$$\omega = 1 + \frac{1}{k_{\text{total}}} \langle n \rangle, \quad (25)$$

where

$$\frac{1}{k_{\text{total}}} = \frac{\lambda \alpha \langle n \rangle_s^2 \left(1 + \frac{1}{k_s} \right) + \lambda (1 - \alpha) \langle n \rangle_{sh}^2 \left(1 + \frac{1}{k_{sh}} \right)}{\langle n \rangle^2} - 1. \quad (26)$$

In the limit $\langle n \rangle \rightarrow \infty$, we finally obtain

$$\omega \approx \frac{1}{k_{\text{total}}} \langle n \rangle. \quad (27)$$

This is precisely (22), with $1/k_{\text{total}}$ being the positive constant. Hence the validity of KNO scaling requires that $1/k_{\text{total}}$ remains energy independent and in our analysis, we examine this quantity.

It is possible to improve the quantitative study of KNO scaling by analyzing the energy dependence of the C moments ($C_q = \langle n^q \rangle / \langle n \rangle^q$). In the limit $\langle n \rangle \rightarrow \infty$ they should become energy independent. In the case of single NBD models one can derive analytic expressions for the C_q moments as a function of $\langle n \rangle$. In [36], it was shown that

$$C_2 = \frac{1}{\langle n \rangle} + 1 + \frac{1}{k}.$$

In the limit of large mean multiplicity (and hence energy), C_2 remains constant if k is constant. In [36], it was shown that, when $|\eta| < 0.5$, this is approximately the case and C_2 is constant. For larger pseudorapidity windows, this is no longer true. The higher moments show a more complicated dependence on $\langle n \rangle$ and the data show larger error bars. We will not perform a study of the moments in this work. Nevertheless, we would like to point out that for the double NBD approach it is possible to establish a connection between the moments and k_{total} . For example, it is easy to show that in the scaling limit we have

$$C_2 = 1 + \frac{1}{k_{\text{total}}}.$$

Before concluding this section, we would like to mention that the study of KNO scaling belongs to the “bottom-up” approaches to multiplicity distributions mentioned in the Introduction and it is not the only scaling that has been investigated. Other empirical regularities have been found in the data, which might have consequences for the multiplicity distributions, such as geometric scaling [23] (and references therein) and entropy scaling [37].

TABLE I. Values of K/S and KS for different energies. The data used are taken from CMS [28,29] and ALICE [7,16].

Data	$\sqrt{s}$ (TeV)	K/S (GeV ²)	KS (GeV ⁻²)
		GBW	KLN
[28]	0.9	0.0968 ± 0.0011	0.3726 ± 0.0043
[7]	0.9	0.1015 ± 0.0011	0.3944 ± 0.0044
[28]	2.36	0.0993 ± 0.0011	0.3772 ± 0.0043
[7]	2.76	0.1001 ± 0.0012	0.3821 ± 0.0047
[16]	5.02	0.0929 ± 0.0005	0.3599 ± 0.0023
[29]	7.0	0.0975 ± 0.0012	0.3683 ± 0.0046
[7]	7.0	0.0965 ± 0.0007	0.3674 ± 0.0030
[7]	8.0	0.0952 ± 0.0007	0.3631 ± 0.0029
[16]	13.0	0.0888 ± 0.0005	0.3468 ± 0.0021

V. RESULTS AND DISCUSSION

The global normalization parameters K/S and KS are obtained by fitting Eq. (11) to the data at different energies. Their values are shown in Table I. It is reassuring to see that the normalization factors depend very weakly on the energy. As can be seen in Fig. 1, we obtain an overall good description of the pseudorapidity distribution data.

To obtain the soft and semihard average multiplicities from (11), we fix the separation scale to be $\Lambda = 1.4$ GeV. This value is approximately the same as that used in [38] and falls within the range of usual values used to define the separation between soft and semihard physics (see [38,39] for further references).

In [40] we have carefully studied the dependence of our results on the choice of Λ , varying it from 1 to 2 GeV. The conclusion is that the absolute values of the fitted parameters become different but their behavior with the energy and size of the pseudorapidity remains the same. Having determined the normalization factors and fixed the separation scale, we use Eq. (20) together with (16) to fit all LHC data on MDs to determine the parameters λ , α , k_s , and k_{sh} . For the GBW UGD, the values of these parameters can be found in Table II in the Appendix, together with the values obtained for the multiplicities and for k_{total} .

From Table II we can infer that the variables which are usually called $\langle n_1 \rangle$ and $\langle n_2 \rangle$ indeed correspond to $\langle n_s \rangle$ and $\langle n_{sh} \rangle$ respectively. Moreover, the previously found relation $\langle n_2 \rangle \simeq 3\langle n_1 \rangle$ becomes $\langle n_{sh} \rangle \simeq 3\langle n_s \rangle$, and remains valid, changing slightly with the center-of-mass energy and with the size of the rapidity window. This is consistent with the findings of the ALICE study published in [7] and in the analysis of CMS data performed in [11].

In Fig. 2, we show the comparison between the data and our model with the GBW UGD. We obtain fits of very good quality, which is reflected in the low values of χ^2 , among the smallest values reported in the literature for this class of model. The DNBD function provides a precise description of the entire set of multiplicity distributions measured at the LHC. This result is particularly remarkable given the reduction of free parameters from 6 to 4. For the KLN UGD we also obtain excellent fits. For the sake of conciseness, we do not show them here.

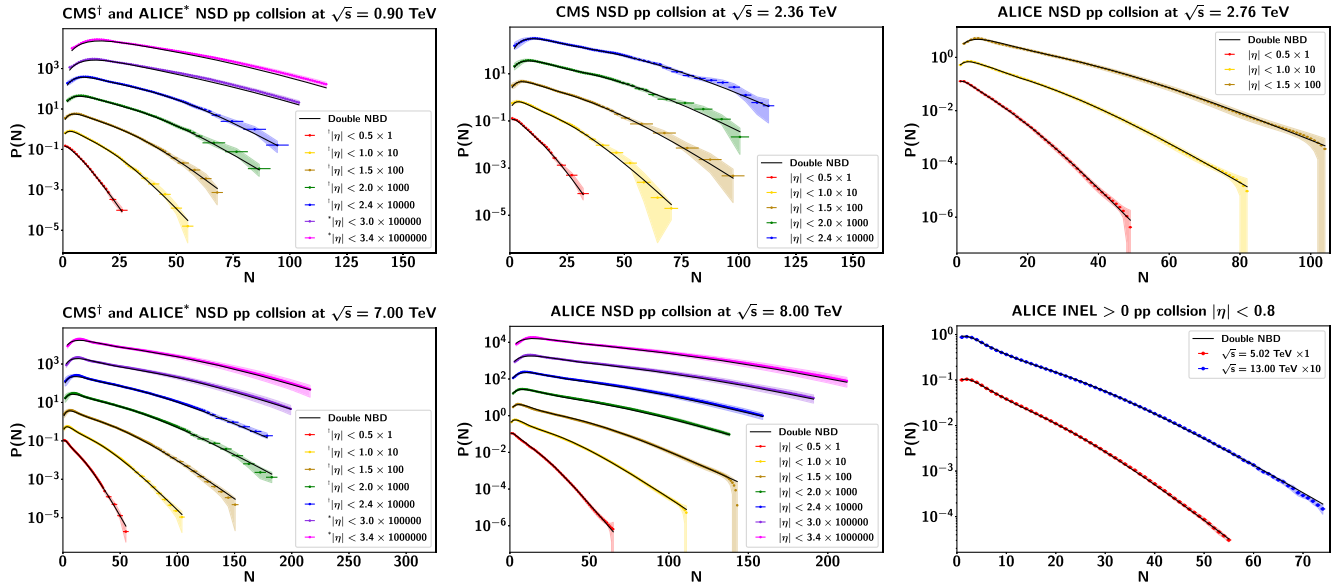


FIG. 2. Multiplicity distributions measured by different collaborations. In the bottom-right panel we show the MDs measured in inelastic events (INEL). All the other panels show data from nonsingle-diffractive (NSD) events. The fits are obtained with Eq. (16) using GBW UGD and the fit parameters are listed in Table II. Data are from CMS [41] and ALICE [7,15,31]. The colored bands show the combined uncertainties.

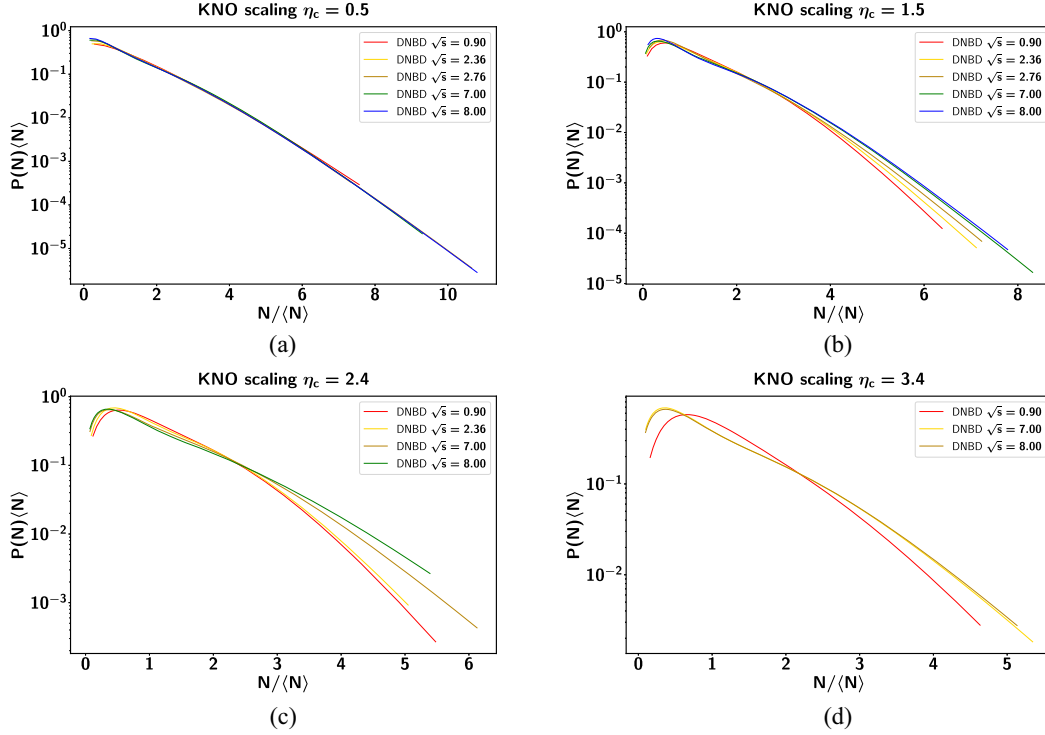


FIG. 3. KNO scaling of multiplicity distributions for various pseudorapidity windows ($-\eta_c < \eta < +\eta_c$). a) $|\eta_c| < 0.5$, b) $|\eta_c| < 1.5$, c) $|\eta_c| < 2.4$, d) $|\eta_c| < 3.4$.

In order to verify whether KNO scaling is observed in the LHC data, we could simply plot the data. However, this would produce some visually cumbersome figures, especially when including the errors. Instead, since our fit of the data is very good, we plot the multiplicity distributions computed with Eq. (16) and presented in terms of the KNO variable. For the GBW UGD, the resulting curves are shown in Fig. 3. It is very clear that for the smallest pseudorapidity window ($|\eta| < 0.5$) KNO scaling holds. For the others, the larger the window, the more KNO scaling is violated. We emphasize that using the KLN UGD we would obtain new versions of Figs. 2 and 3, which would be indistinguishable from them.

In [2] it was shown that in the very high energy limit pQCD predicts scaling. Having this in mind, we might think that the perturbative events are concentrated in the narrow, central rapidity window and are responsible for the scaling, whereas the nonperturbative events are increasingly important at larger rapidity windows. If this were the case, we would expect that the parameter α would fall strongly with the energy for $|\eta| < 0.5$ and stay constant or increase in the other windows. However, this is not the case. In Fig. 4 we show the evolution of the parameters α , k_s , k_{sh} , and k_{total} with the energy for different rapidity windows. They were obtained from fitting the data with Eq. (16) and the GBW UGD. In particular, in Fig. 4(a) we see the behavior of α with $\sqrt{s}$ for all windows multiplied by some numerical factor for the sake of clarity. We can see that the falling behavior is

the same for all windows and, contrary to previous expectations, there is no correlation between α and the violation of KNO scaling. In fact, if we did not multiply the values of α by the separating numerical factors, they would fall onto a single curve (within a few percent of uncertainty). Interestingly, in [7] the ALICE Collaboration performing a similar DNBD analysis of the same data found that α increases with the energy $\sqrt{s}$ in all rapidity windows (see Table 10 in that reference). This is an example of how the introduction of constraints can bring about not only quantitative but also qualitative changes in the behavior of the fit parameters.

As for the other parameters, we see a marked change in the behavior of k_s and k_{sh} with $\sqrt{s}$, going from constant k_s and k_{sh} with $\sqrt{s}$ for $|\eta| < 0.5$ to rapidly falling for wider windows, as can be seen in Figs. 4(b) and 4(c). Giovannini and Ugoccioni proposed in [10,42] three scenarios for the behavior of k_s and k_{sh} :

- (i) Both k_s and k_{sh} remain constant with energy, which means that KNO scaling is valid for both soft and semihard components.
- (ii) k_s is constant with energy and k_{sh} decreases linearly with increasing energy, implying that KNO scaling is strongly violated by the semihard component.
- (iii) k_s is constant with energy and k_{sh} starts decreasing with increasing energy, but asymptotically tends to a constant value, implying that KNO scaling violation is not very strong.

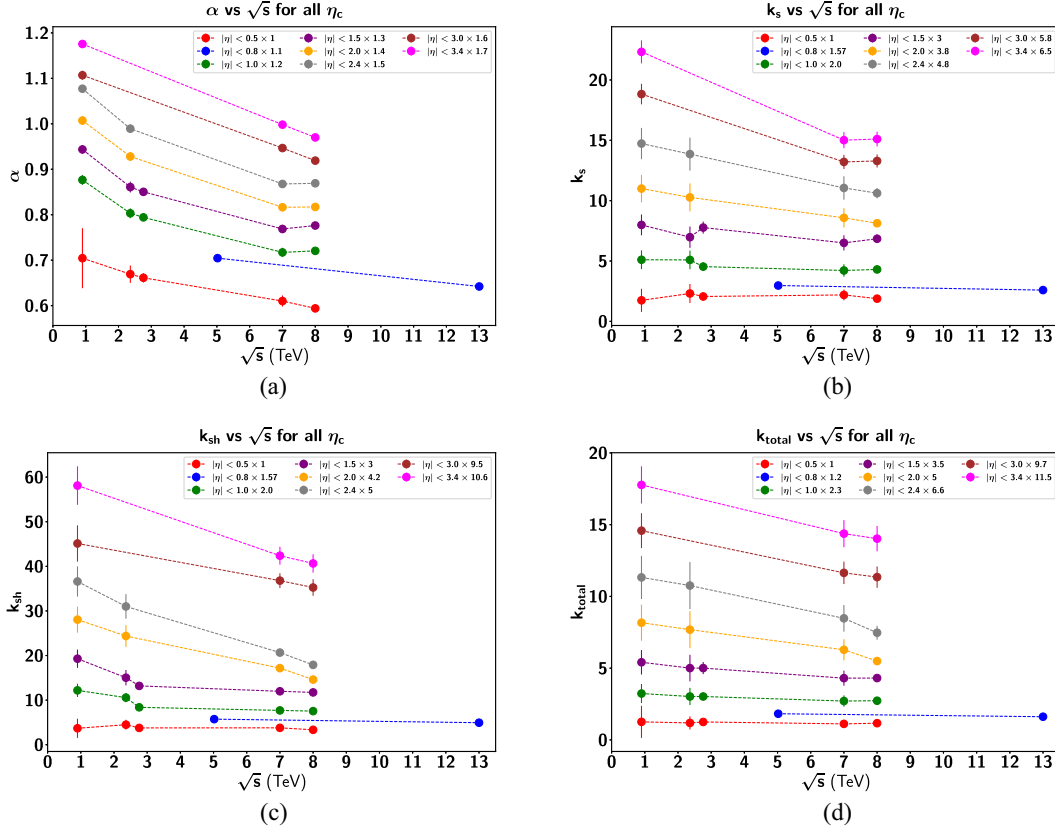


FIG. 4. GBW UGD. Behavior of the fit parameters with increasing energy for several pseudorapidity windows ($-\eta_c < \eta < +\eta_c$). a) α ; b) k_s ; c) k_{sh} , and d) k_{total} . The lines are just to guide the eyes. The data points in blue refer to INEL events. All the others refer to NSD events.

Our results presented in Fig. 4 suggest that none of these scenarios is supported by data. Moreover we emphasize the importance of the relation (27) and of studying the behavior of k_{total} . Indeed, since λ and α may depend on the energy, the simple constancy of k_s and k_{sh} does not ensure KNO scaling. The constancy of k_{total} does. In Fig. 4(d) we show k_{total} as a function of the energy for different pseudorapidity windows. It is constant only for $|\eta| < 0.5$ and energy dependent in the other pseudorapidity windows. The results and conclusions obtained with the KLN UGD, shown in Fig. 5, are quite similar, except for the behavior of k_s which might be constant with the energy.

We can compare our results with those reported in [7], where a similar DNBD fit was performed. The comparison shows that the order of magnitude of the parameters is the same. However, in contrast to [7], we find a clear trend which separates the narrower rapidity window, $|\eta| < 0.5$, from the wider ones. In the former, k_s and k_{sh} are constant with energy, whereas in the latter these parameters decrease with increasing energies. This feature, along with the observation that KNO scaling holds only in the narrower window, strongly suggests that there is a change of dynamics when we go from one region to the other.

In our formalism a change of dynamics can happen when we go from smaller to larger rapidities. Remembering that in the $2 \rightarrow 1$ kinematics the rapidity of the produced particle is $y = 1/2 \ln(x_1/x_2)$ we see that, when y is large x_2 is necessarily small. In contrast, when y is close to zero we have $x_1 \approx x_2$ and then both x_1 and x_2 can be large. Hence the change of dynamics means for us accessing the low x saturation region. We then might be tempted to say that “saturation breaks the KNO scaling.” However, before drawing this conclusion we must consider other aspects. One of them is the contribution of valence quarks, which was not included in our calculation and is known to be sizable at large rapidities. Other changes in dynamics, which could happen together with saturation are, for example, the emergence of double parton scattering [43] or changes in the number or properties of the clans [10,13].

To summarize:

- (1) Using the k_T factorization formalism and introducing a separation scale in the transverse momentum we were able to properly define soft and semihard multiplicities, giving a physical meaning to the previously called multiplicity components n_1 and n_2 , proposed in [30] and used in [7,15,31].
- (2) With this separation, we could perform a DNBD fit of the multiplicity distributions measured at the LHC

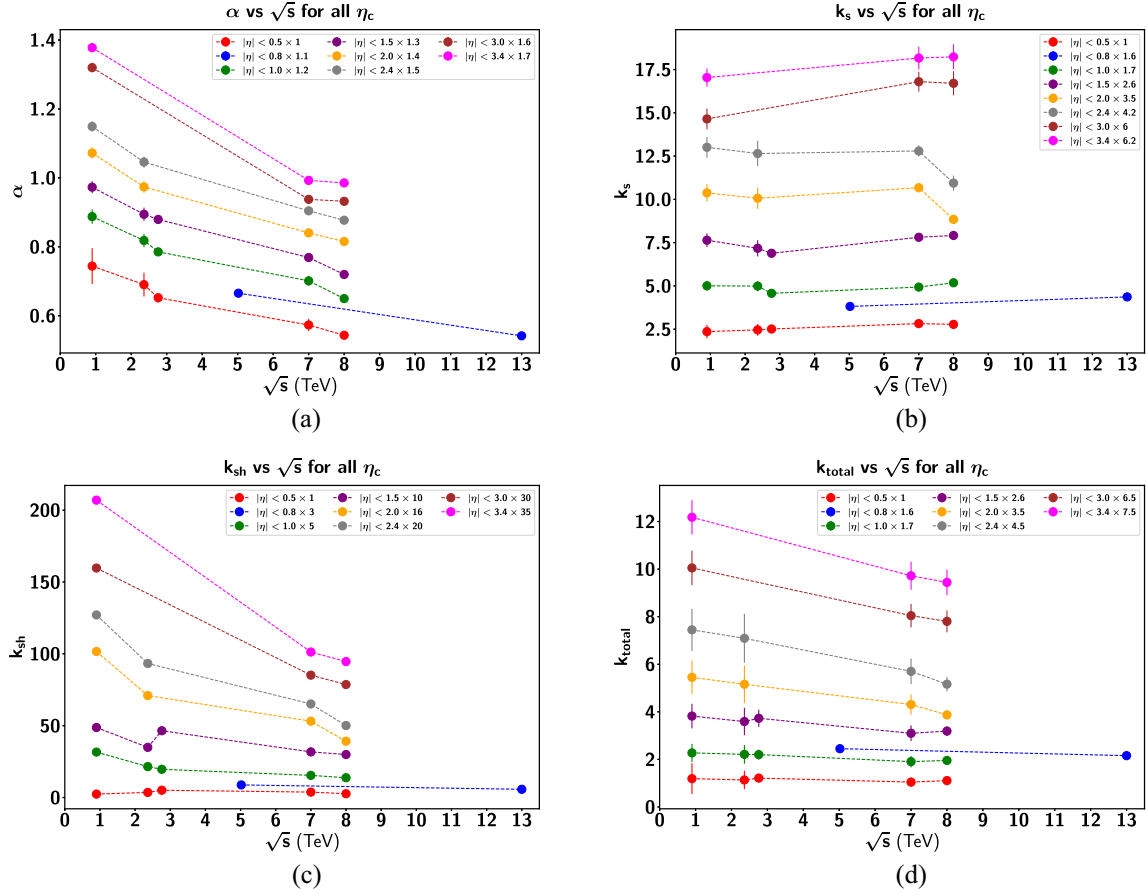


FIG. 5. KNL UGD. Behavior of α , k_s , k_{sh} , and k_{total} with increasing energy for various pseudorapidity windows ($-\eta_c < \eta < +\eta_c$). a) α , b) k_s , c) k_{sh} , and d) k_{total} . The lines are just to guide the eyes. The data points in blue refer to INEL events. All the others refer to NSD events. The error bars in (c) were omitted because they are too large, which would hinder the visualization of the mean points.

with fewer free parameters (4 instead of 6), obtaining very good fits.

- (3) The resulting behavior of the parameters α , k_s , and k_{sh} with the collision energy is consistent (but slightly different) with those found in previous analyses.
- (4) KNO scaling occurs for the total MD when k_{total} is independent of the $\sqrt{s}$ and this happens only for $|\eta| < 0.5$. Furthermore, within the same pseudorapidity window, KNO scaling is also satisfied for the soft and semihard components; that is, k_s and k_{sh} are also energy independent.
- (5) KNO violation is not related to changes in α and may have a dynamical origin due to changes in N_s , N_{sh} , k_s , and k_{sh} with pseudorapidity.
- (6) The results obtained with the GBW and KLN UGDs are similar.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [7,15,16,28,29,31,41].

APPENDIX: TABLES

In this Appendix we present the values of the fit parameters displayed in Table II.

TABLE II. Parameters obtained from fitting the data with the double NBD (16) for different pseudorapidity windows ($-\eta_c < \eta < +\eta_c$) and energies ($\sqrt{s}$). These results are for the GBW UGD. The data used for the fits are from CMS [41] and ALICE [7, 15, 31]. The break in trend in certain parameters, such as k_{sh} , for the energies 0.9, 7.0, and 8 TeV arises primarily because the data used for the fitting originate from different studies.

η_c	Data	$\sqrt{s}$ (TeV)	λ	α	$\langle n \rangle_s$	$\langle n \rangle_{sh}$	$\langle n \rangle$	k_s	k_{sh}	k_{total}	$\chi^2/d.o.f$
0.5	[41]	0.90	0.895 ± 0.054	0.704 ± 0.066	2.803 ± 0.234	6.319 ± 1.270	3.437 ± 0.040	1.743 ± 0.960	3.685 ± 2.154	1.257 ± 1.117	0.265
		2.36	0.875 ± 0.039	0.669 ± 0.019	3.150 ± 0.182	8.719 ± 0.689	4.370 ± 0.050	2.310 ± 0.786	4.489 ± 1.099	1.183 ± 0.457	0.587
	[7]	2.76	0.936 ± 0.011	0.661 ± 0.010	3.028 ± 0.065	8.599 ± 0.273	4.599 ± 0.057	2.061 ± 0.336	3.770 ± 0.279	1.249 ± 0.168	0.057
		7.00	0.920 ± 0.032	0.610 ± 0.012	3.376 ± 0.182	11.214 ± 0.711	5.915 ± 0.075	2.192 ± 0.444	3.788 ± 0.232	1.117 ± 0.235	0.102
	[7]	8.00	0.955 ± 0.009	0.594 ± 0.008	3.270 ± 0.055	10.733 ± 0.219	6.018 ± 0.049	1.882 ± 0.263	3.344 ± 0.152	1.164 ± 0.108	0.106
0.8	[15]	5.02	1.072 ± 0.006	0.640 ± 0.002	4.195 ± 0.041	13.634 ± 0.130	8.135 ± 0.052	1.890 ± 0.054	3.652 ± 0.054	1.519 ± 0.073	0.615
		13.00	1.083 ± 0.005	0.584 ± 0.001	4.429 ± 0.041	16.763 ± 0.136	10.353 ± 0.064	1.650 ± 0.037	3.144 ± 0.026	1.346 ± 0.043	0.910
1.0	[41]	0.90	0.891 ± 0.021	0.730 ± 0.009	5.514 ± 0.176	14.160 ± 0.680	6.987 ± 0.081	2.552 ± 0.388	6.096 ± 0.745	1.405 ± 0.289	0.254
		2.36	0.893 ± 0.023	0.669 ± 0.009	6.310 ± 0.227	17.494 ± 0.808	8.938 ± 0.103	2.546 ± 0.386	5.280 ± 0.515	1.315 ± 0.261	0.355
	[7]	2.76	0.936 ± 0.009	0.662 ± 0.007	6.176 ± 0.122	17.594 ± 0.442	9.393 ± 0.116	2.268 ± 0.178	4.195 ± 0.190	1.315 ± 0.111	0.040
		7.00	0.918 ± 0.022	0.598 ± 0.008	6.994 ± 0.267	21.978 ± 0.943	11.958 ± 0.151	2.108 ± 0.258	3.847 ± 0.161	1.178 ± 0.172	0.113
	[7]	8.00	0.947 ± 0.008	0.600 ± 0.005	6.608 ± 0.104	22.196 ± 0.392	12.157 ± 0.098	2.151 ± 0.129	3.767 ± 0.109	1.187 ± 0.074	0.080
1.5	[41]	0.90	0.910 ± 0.016	0.726 ± 0.007	8.270 ± 0.212	20.798 ± 0.795	10.650 ± 0.124	2.663 ± 0.287	6.430 ± 0.684	1.546 ± 0.243	0.143
		2.36	0.925 ± 0.020	0.662 ± 0.009	9.427 ± 0.299	25.370 ± 1.044	13.704 ± 0.158	2.325 ± 0.295	5.010 ± 0.578	1.429 ± 0.264	0.180
	[7]	2.76	0.941 ± 0.008	0.654 ± 0.006	9.504 ± 0.177	26.216 ± 0.599	14.386 ± 0.177	2.590 ± 0.167	4.397 ± 0.220	1.430 ± 0.121	0.100
		7.00	0.918 ± 0.018	0.591 ± 0.007	10.723 ± 0.347	32.745 ± 1.153	18.097 ± 0.228	2.168 ± 0.211	3.994 ± 0.139	1.229 ± 0.151	0.188
	[7]	8.00	0.947 ± 0.007	0.597 ± 0.004	10.061 ± 0.142	33.238 ± 0.497	18.372 ± 0.149	2.281 ± 0.119	3.910 ± 0.112	1.231 ± 0.073	0.144
2.0	[41]	0.90	0.909 ± 0.015	0.719 ± 0.006	11.233 ± 0.269	27.433 ± 0.939	14.344 ± 0.167	2.895 ± 0.300	6.679 ± 0.696	1.633 ± 0.251	0.167
		2.36	0.929 ± 0.018	0.663 ± 0.007	12.671 ± 0.359	34.319 ± 1.193	18.543 ± 0.214	2.703 ± 0.305	5.804 ± 0.576	1.537 ± 0.258	0.191
	[31]	7.00	0.908 ± 0.016	0.583 ± 0.006	14.731 ± 0.447	43.493 ± 1.400	24.252 ± 0.306	2.257 ± 0.211	4.089 ± 0.146	1.256 ± 0.147	0.310
		8.00	0.898 ± 0.005	0.584 ± 0.004	14.539 ± 0.175	45.398 ± 0.526	24.580 ± 0.199	2.139 ± 0.065	3.478 ± 0.116	1.099 ± 0.053	0.861
	[41]	0.90	0.913 ± 0.013	0.718 ± 0.005	13.464 ± 0.296	32.726 ± 1.001	17.245 ± 0.200	3.069 ± 0.270	7.321 ± 0.677	1.716 ± 0.228	0.180
2.4	[41]	2.36	0.934 ± 0.016	0.660 ± 0.006	15.252 ± 0.394	40.810 ± 1.273	22.381 ± 0.258	2.887 ± 0.284	6.201 ± 0.551	1.630 ± 0.248	0.153
		7.00	0.907 ± 0.015	0.578 ± 0.006	17.846 ± 0.504	51.731 ± 1.520	29.133 ± 0.367	2.303 ± 0.202	4.132 ± 0.154	1.284 ± 0.141	0.411
	[31]	8.00	0.897 ± 0.007	0.579 ± 0.005	17.579 ± 0.255	53.966 ± 0.818	29.495 ± 0.239	2.213 ± 0.089	3.581 ± 0.179	1.132 ± 0.073	0.337
		0.90	0.860 ± 0.006	0.692 ± 0.006	19.265 ± 0.303	41.306 ± 0.993	22.409 ± 0.253	3.247 ± 0.147	4.750 ± 0.428	1.503 ± 0.126	2.322
	[31]	7.00	0.912 ± 0.007	0.592 ± 0.004	21.332 ± 0.304	65.459 ± 0.960	35.886 ± 0.294	2.278 ± 0.101	3.873 ± 0.175	1.200 ± 0.080	0.281
3.0	[41]	8.00	0.896 ± 0.007	0.574 ± 0.005	22.050 ± 0.313	66.437 ± 0.954	36.684 ± 0.297	2.291 ± 0.093	3.711 ± 0.195	1.170 ± 0.077	0.311
		0.90	0.851 ± 0.006	0.691 ± 0.005	21.766 ± 0.321	46.526 ± 0.952	25.036 ± 0.283	3.435 ± 0.147	5.482 ± 0.407	1.545 ± 0.113	2.656
	[31]	7.00	0.916 ± 0.007	0.587 ± 0.004	24.058 ± 0.346	72.604 ± 1.059	40.405 ± 0.331	2.310 ± 0.102	3.998 ± 0.189	1.250 ± 0.082	0.216
		8.00	0.902 ± 0.007	0.571 ± 0.004	24.789 ± 0.344	73.676 ± 0.999	41.289 ± 0.334	2.324 ± 0.094	3.835 ± 0.195	1.220 ± 0.077	0.294

(Table continued)

TABLE II. (Continued)

$\sqrt{s}$ (TeV)	Data	η_c	λ	α	$\langle n \rangle_s$	$\langle n \rangle_{sh}$	$\langle n \rangle$	k_s	k_{sh}	k_{total}	$\chi^2/d.o.f$
0.90	[41]	0.5	0.895 ± 0.054	0.704 ± 0.066	2.803 ± 0.234	6.319 ± 1.270	3.437 ± 0.040	1.743 ± 0.960	3.685 ± 2.154	1.257 ± 1.117	0.265
		1.0	0.891 ± 0.021	0.730 ± 0.009	5.514 ± 0.176	14.160 ± 0.680	6.987 ± 0.081	2.552 ± 0.388	6.096 ± 0.745	1.405 ± 0.289	0.254
		1.5	0.910 ± 0.016	0.726 ± 0.007	8.270 ± 0.212	20.798 ± 0.795	10.650 ± 0.124	2.663 ± 0.287	6.430 ± 0.684	1.546 ± 0.243	0.143
		2.0	0.909 ± 0.015	0.719 ± 0.006	11.233 ± 0.269	27.433 ± 0.939	14.344 ± 0.167	2.895 ± 0.300	6.679 ± 0.696	1.633 ± 0.251	0.167
		2.4	0.913 ± 0.013	0.718 ± 0.005	13.464 ± 0.296	32.726 ± 1.001	17.245 ± 0.200	3.069 ± 0.270	7.321 ± 0.677	1.716 ± 0.228	0.180
	[31]	3.0	0.860 ± 0.006	0.692 ± 0.006	19.265 ± 0.303	41.306 ± 0.993	22.409 ± 0.253	3.247 ± 0.147	4.750 ± 0.428	1.503 ± 0.126	2.322
		3.4	0.851 ± 0.006	0.691 ± 0.005	21.766 ± 0.321	46.526 ± 0.952	25.036 ± 0.283	3.435 ± 0.147	5.482 ± 0.407	1.545 ± 0.113	2.656
2.36	[41]	0.5	0.875 ± 0.039	0.669 ± 0.019	3.150 ± 0.182	8.719 ± 0.689	4.370 ± 0.050	2.310 ± 0.786	4.489 ± 1.099	1.183 ± 0.457	0.587
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5.02	[15]	0.8	1.072 ± 0.006	0.640 ± 0.002	4.195 ± 0.041	13.634 ± 0.130	8.135 ± 0.052	1.890 ± 0.054	3.652 ± 0.054	1.519 ± 0.073	0.615
7.00	[41]	0.5	0.920 ± 0.032	0.610 ± 0.012	3.376 ± 0.182	11.214 ± 0.711	5.915 ± 0.075	2.192 ± 0.444	3.788 ± 0.232	1.117 ± 0.235	0.102
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