

Probing neutral triple gauge couplings via ZZ production at e^+e^- colliders with machine learning

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Neutral triple gauge couplings (nTGCs) first arise from the dimension-8 operators of the Standard Model effective field theory (SMEFT), rather than the dimension-4 SM Lagrangian and dimension-6 SMEFT operators, opening up a unique window for probing new physics at the dimension-8 level. In this work, we formulate the nTGC form factors of ZZV^* ($V = Z, \gamma$) that are compatible with the spontaneous breaking of the $SU(2) \otimes U(1)$ electroweak gauge symmetry and consistently match the dimension-8 nTGC operators in the broken phase. We study the sensitivities for probing both the ZZV^* form factors and the corresponding new physics scales through ZZ production (with visible/invisible fermionic Z decays) at high energy e^+e^- colliders including CEPC, FCC-ee, LCF, ILC, and CLIC. In particular, we identify the dimension-8 operator that contributes to the pure triple Z boson coupling ZZZ^* alone, but not the mixed $ZZ\gamma^*$ coupling. We further study the correlations between probes of the ZZZ^* and $ZZ\gamma^*$ couplings. Using machine learning, we show that angular distributions of the final-state fermions can play key roles in suppressing the SM backgrounds. The sensitivities can be further improved by using polarized $e^\pm$ beams, and we find that the optimal sensitivity bounds on the nTGC correlations are given by the mixed setting including both the unpolarized operation and the follow-up polarized operation. We demonstrate that machine learning is advantageous for handling the 4-body final states from ZZ decays and improves significantly the sensitivity reaches of probes of nTGCs in e^+e^- collisions. We find that nTGC new physics scales can be probed up to the multi-TeV scale at the proposed e^+e^- colliders.

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I. INTRODUCTION

Studies of neutral triple gauge couplings (nTGCs) open a unique window on new physics beyond the Standard Model (BSM), since they are absent both in the Standard Model (SM) and in the SM effective field theory (SMEFT) [1] at the dimension-6 level. However, nTGCs are generated by dimension-8 SMEFT operators, and offer a rare opportunity to probe directly BSM physics at the dimension-8 level,

without obscuration by dimension-4 SM contributions and dimension-6 SMEFT effects.

In a series of works [2–7], we have explored the potential sensitivities of probes of the nTGCs at e^+e^- and pp colliders through the reaction $f\bar{f} \rightarrow Z\gamma$, and further studied the UV completion of the nTGCs [8,9]. In the course of these studies, we have revisited the conventional nTGC form factor formalism [10,11], which respects only the residual $U(1)_{\text{em}}$ gauge symmetry of QED. In our formulation of nTGC form factors we have emphasized [2–4] the importance of incorporating the full electroweak gauge group $SU(2) \otimes U(1)$ of the SM with spontaneous symmetry breaking. In particular, we have shown that disregarding the full electroweak gauge group in the conventional nTGC formalism [10] gives unphysically large predictions for the nTGC sensitivity bounds that are stronger than the correct bounds by about two orders of magnitude in the cases of the LHC probes in initial CMS and ATLAS experimental

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analyses [12,13]. However, the ATLAS Collaboration [14] has recently performed a new LHC measurement of nTGCs by using the consistent nTGC form factor formulas [3,4] and confirmed our theoretical predictions for the prospective experimental sensitivities. The lesson is that, although the contributions of dimension-8 operators to collider processes generally have higher energy-power dependences than those of dimension-6 operators [15,16], it is important to ensure that high-energy behaviors of the dimension-8 contributions are consistent with the full electroweak gauge group $SU(2) \otimes U(1)$ with spontaneous symmetry breaking, because it can impose nontrivial energy cancellations for the physical scattering amplitudes that arise from BRST identities [2–4].

Previous studies have mainly focused on analyses of $Z\gamma$ production [2–7,17].¹ However, these cannot probe the pure Z boson nTGC ZZZ^* . For this purpose, we study in the present work a different reaction process, $e^+e^- \rightarrow ZZ$ with on shell final-state Z bosons, which can probe ZZZ^* and $ZZ\gamma^*$ nTGCs. The final-state Z bosons decay dominantly through the charged-fermion channels and invisible channels,² $Z \rightarrow q\bar{q}, \ell\bar{\ell}, \nu\bar{\nu}$, whose angular distributions can be used to suppress the SM backgrounds and enhance the sensitivities of probes of the nTGCs. We demonstrate in our analysis how the sensitivities of probes of nTGCs can be significantly enhanced by using machine learning to optimize the discriminatory power of the 4-body final states from ZZ decays. We show that using polarized electron and positron ($e^\pm$) beams can further increase the sensitivity reaches of probes of nTGCs. Moreover, we find that the optimal sensitivity bounds on the nTGC correlations are given by the mixed setting including both the unpolarized operation of e^-e^+ collisions and the follow-up polarized operation.

This paper is organized as follows. In Sec. II A we analyze how the nTGC couplings arise from dimension-8 SMEFT operators, and establish a consistent formulation of the nTGC form factors that incorporates the full electroweak gauge group $SU(2) \otimes U(1)$ with spontaneous symmetry breaking. Then, in Sec. II B we derive the unitarity

constraints on nTGCs that are significantly weaker than the sensitivity reaches for probes of the nTGCs at high-energy e^+e^- colliders, as we show later. In Sec. III we derive the SM and nTGC contributions to the scattering amplitudes and cross sections for ZZ production channels, and analyze the differences between their angular distributions. Section IV demonstrates how these may be used to probe ZZZ^* and $ZZ\gamma^*$ nTGCs at high-energy e^+e^- colliders. We analyze the visible fermionic decay channels $Z \rightarrow \ell\bar{\ell}, q\bar{q}$ in Sec. IV A and the invisible decay channels $Z \rightarrow \nu\bar{\nu}$ in Sec. IV B. We apply machine learning algorithms to enhance sensitivity reaches of probes of nTGCs in Sec. IV C. Then, we analyze the improved sensitivities with polarized electron/positron beams in Sec. IV D. We analyze correlations between the nTGC parameters in Sec. IV E. Comparisons with the previous analyses of probes of the nTGCs via the $Z\gamma$ production channel are presented in Sec. IV F. Finally, we conclude in Sec. V.

II. ZZV* COUPLINGS FROM OPERATORS AND FORM FACTORS

We present the formulation of the nTGC couplings of ZZV^* in terms of the gauge-invariant dimension-8 effective operators and the corresponding form factors in Sec. II A. Then, we analyze the unitarity constraints on the nTGCs in Sec. II B. We will show that these unitarity bounds are much weaker than our collider constraints, and thus do not affect our collider analyses in the next section.

A. nTGC operators and form factors for ZZV* couplings

In previous papers [6,7] we studied possible probes of nTGCs via on shell $Z\gamma$ production. In this section, we will analyze the nTGC contributions to the production process $e^-e^+ \rightarrow ZZ$ through the vertices ZZV^* (with $V = Z, \gamma$). We work in the general framework of the SMEFT, in which nTGCs are first generated by dimension-8 operators [11]. The general dimension-8 SMEFT Lagrangian can be written as

$$\begin{aligned} \Delta\mathcal{L}(\text{dim}-8) &= \sum_j \frac{c_j}{\tilde{\Lambda}^4} \mathcal{O}_j = \sum_j \frac{\text{sign}(c_j)}{\Lambda_j^4} \mathcal{O}_j \\ &= \sum_j \frac{1}{[\Lambda_j^4]} \mathcal{O}_j, \end{aligned} \quad (2.1)$$

where the dimensionless coefficients c_j may be $O(1)$, with signs $\text{sign}(c_j) = \pm$. We define the corresponding cutoff scale $\Lambda_j \equiv \tilde{\Lambda}/|c_j|^{1/4}$ and introduce the shorthand notation $[\Lambda_j^4] \equiv \text{sign}(c_j)\Lambda_j^4$.

A CP -conserving ZZV^* vertex can be parametrized as follows [11]:

¹In passing, we note that a recent preprint [18] studied ZZ production at e^+e^- colliders, but differs substantially from our work. It did not study the ZZZ^* coupling because it considered only the four operators $\mathcal{O}_{\tilde{B}W}, \mathcal{O}_{B\tilde{W}}, \mathcal{O}_{\tilde{W}W}, \mathcal{O}_{\tilde{B}B}$ that do not contribute to the ZZZ^* vertex, as shown in our Table I. Moreover, the operators $\mathcal{O}_{\tilde{W}W}$ and $\mathcal{O}_{\tilde{B}B}$ have zero contribution to the $ZZ\gamma^*$ vertex as well, whereas $\mathcal{O}_{B\tilde{W}}$ gives the same $ZZ\gamma^*$ contribution as $\mathcal{O}_{\tilde{B}W}$ (up to a trivial overall minus sign as shown in our Table I). Hence, Ref. [18] effectively studied only the $ZZ\gamma^*$ coupling generated by $\mathcal{O}_{\tilde{B}W}$, and could not study any correlations between different nTGCs as we do. Finally, it did not study the probe of any related nTGC form factors, which is a major subject of our work.

²We note that at e^+e^- colliders, in contrast to pp colliders, it is possible to make a kinematic selection of invisible decays of on shell Z bosons ($Z \rightarrow \nu\bar{\nu}$) in the mixed final states $ZZ \rightarrow \ell\bar{\ell}/q\bar{q} + \nu\bar{\nu}$.

$$ie\Gamma_{ZZV^*}^{\alpha\beta\mu}(q_1, q_2, q_3) = \frac{e(q_3^2 - M_V^2)}{M_Z^2} f_5^V \epsilon^{\mu\alpha\beta\rho} (q_1 - q_2)_\rho, \quad (2.2)$$

and a CP -conserving $Z\gamma V^*$ vertex can be parametrized as follows [4]:

$$\begin{aligned} ie\Gamma_{Z\gamma V^*}^{\alpha\beta\mu}(q_1, q_2, q_3) &= \frac{e(q_3^2 - M_V^2)}{M_Z^2} \left[\left(h_3^V + \frac{h_4^V}{2M_Z^2} q_3^2 \right) q_{2\nu} \epsilon^{\alpha\beta\mu\nu} \right. \\ &\quad \left. + \frac{h_4^V}{M_Z^2} q_2^\alpha q_{3\nu} q_{2\sigma} \epsilon^{\beta\mu\nu\sigma} \right]. \end{aligned} \quad (2.3)$$

The ZZV^* nTGC vertices can arise only from dimension-8 operators with two SM Higgs-doublet fields. As shown in Ref. [8], we note that there are seven independent CP -conserving nTGC operators of dimension-8 with two SM Higgs-doublet fields after taking into account the equivalence relations arising from integration by parts. We choose the following operator basis for our nTGC analysis:

$$\mathcal{O}_{\tilde{W}W} = iH^\dagger \tilde{W}_{\mu\nu} W^{\nu\rho} \{D_\rho, D^\mu\} H + \text{H.c.}, \quad (2.4a)$$

$$\mathcal{O}'_{\tilde{W}W} = iH^\dagger \tilde{W}_{\mu\nu} (D_\rho W^{\nu\rho}) D^\mu H + \text{H.c.}, \quad (2.4b)$$

$$\mathcal{O}_{\tilde{B}B} = iH^\dagger \tilde{B}_{\mu\nu} B^{\nu\rho} \{D_\rho, D^\mu\} H + \text{H.c.}, \quad (2.4c)$$

$$\mathcal{O}'_{\tilde{B}B} = iH^\dagger \tilde{B}_{\mu\nu} (D_\rho B^{\nu\rho}) D^\mu H + \text{H.c.}, \quad (2.4d)$$

$$\mathcal{O}_{\tilde{B}W} = iH^\dagger \tilde{B}_{\mu\nu} W^{\nu\rho} \{D_\rho, D^\mu\} H + \text{H.c.}, \quad (2.4e)$$

$$\mathcal{O}'_{\tilde{B}W} = iH^\dagger \tilde{B}_{\mu\nu} (D_\rho W^{\nu\rho}) D^\mu H + \text{H.c.}, \quad (2.4f)$$

$$\mathcal{O}_{\tilde{W}B} = iH^\dagger \tilde{W}_{\mu\nu} B^{\nu\rho} \{D_\rho, D^\mu\} H + \text{H.c.}, \quad (2.4g)$$

where H denotes the SM Higgs doublet and we define the dual field strengths $\tilde{B}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} B^{\alpha\beta}$ and $\tilde{W}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} W^{\alpha\beta}$, where $W_{\mu\nu} = W_{\mu\nu}^a \frac{\sigma^a}{2}$ and σ^a denotes the Pauli matrices. The above operators are all Hermitian and their Wilson coefficients c_j are assumed to be real, so that they conserve CP . As discussed in the text below Eq. (2.8) of Ref. [8], the operators of Eq. (2.4) are linear combinations of the operators $\mathcal{O}_{W^2\phi^2 D^2}^{(9)}$, $\mathcal{O}_{W^2\phi^2 D^2}^{(17)}$, $\mathcal{O}_{WB\phi^2 D^2}^{(14)}$, $\mathcal{O}_{WB\phi^2 D^2}^{(15)}$, $\mathcal{O}_{WB\phi^2 D^2}^{(18)}$, $\mathcal{O}_{B^2\phi^2 D^2}^{(10)}$, and $\mathcal{O}_{B^2\phi^2 D^2}^{(12)}$ that are listed in Ref. [19] (cf. its Table 2) for the dimension-8 SMEFT analysis, up to non-nTGC terms. These operators are not given in the basis of Refs. [20,21], but they can be derived from operators of the type $(D\phi)^2 F^2$ [20,21] via integration by parts.

The contributions of the operators (2.4) to ZZZ^* and $ZZ\gamma^*$ vertices are summarized in Table I, where $\langle H^0 \rangle = v/\sqrt{2}$ denotes the SM Higgs vacuum expectation value,

θ_W is the weak mixing angle, and we use the notations $(s_W, c_W) = (\sin \theta_W, \cos \theta_W)$.

We see that the operators $\mathcal{O}_{\tilde{B}W}$ and $\mathcal{O}_{\tilde{B}W}$ both contribute to the $ZZ\gamma^*$ coupling, but do not contribute to the ZZZ^* coupling. Moreover, their contributions to the $ZZ\gamma^*$ coupling differ only by an overall minus sign, so we need only consider the operator $\mathcal{O}_{\tilde{B}W}$ for our study of the $ZZ\gamma^*$ nTGC. In summary, we have four remaining relevant operators ($\mathcal{O}'_{\tilde{W}W}, \mathcal{O}'_{\tilde{B}B}, \mathcal{O}_{\tilde{B}W}, \mathcal{O}'_{\tilde{B}W}$), among which $\mathcal{O}_{\tilde{B}W}$ contributes to the $ZZ\gamma^*$ coupling alone and the other three operators contribute to both the ZZZ^* and $ZZ\gamma^*$ couplings.

We note in Table I that three dimension-8 operators ($\mathcal{O}'_{\tilde{W}W}, \mathcal{O}'_{\tilde{B}B}, \mathcal{O}'_{\tilde{B}W}$) contribute to the ZZZ^* vertex, as follows:

$$f_5^Z = v^2 M_Z^2 \left(\frac{1}{4 \tan \theta_W [\Lambda_{\tilde{W}W}^4]} + \frac{\tan \theta_W}{[\Lambda_{\tilde{B}B}^4]} + \frac{1}{2 [\Lambda_{\tilde{B}W}^4]} \right). \quad (2.5)$$

In order to study the ZZZ^* and $ZZ\gamma^*$ couplings separately, we can construct an operator that contributes to the ZZZ^* coupling alone,

$$\mathcal{O}_{3Z} \equiv \mathcal{O}'_{\tilde{W}W} + \frac{1}{4} \mathcal{O}'_{\tilde{B}B}, \quad (2.6)$$

which yields the following ZZZ^* coupling:

$$\frac{ev^2}{4s_W c_W} (p_3^2 - M_Z^2) (p_1^\rho - p_2^\rho) \epsilon_{\alpha\beta\mu\rho}. \quad (2.7)$$

Its contribution to the ZZZ^* form factor f_5^Z is as follows:

$$f_5^Z = \frac{v^2 M_Z^2}{4s_W c_W [\Lambda_{3Z}^4]}. \quad (2.8)$$

We also see in Table I that there are four dimension-8 operators ($\mathcal{O}'_{\tilde{W}W}, \mathcal{O}'_{\tilde{B}B}, \mathcal{O}_{\tilde{B}W}, \mathcal{O}'_{\tilde{B}W}$) that contribute to the $ZZ\gamma^*$ vertex, whose contributions to the $ZZ\gamma^*$ form factor are as follows:

$$f_5^\gamma = v^2 M_Z^2 \left(\frac{1}{4 [\Lambda_{\tilde{W}W}^4]} + \frac{1}{[\Lambda_{\tilde{B}B}^4]} + \frac{1}{2s_W c_W [\Lambda_{\tilde{B}W}^4]} + \frac{\tan \theta_W}{2 [\Lambda_{\tilde{B}W}^4]} \right). \quad (2.9)$$

Among the above four operators, the operator $\mathcal{O}_{\tilde{B}W}$ contributes to the form factor f_5^γ only, but not to f_5^Z . This is in contrast with the operator $\mathcal{O}_{3Z}$, which contributes to the form factor f_5^Z only, instead of f_5^γ . As discussed in Ref. [4], the operator $\mathcal{O}_{\tilde{B}W}$ also contributes to the $Z\gamma Z^*$ vertex, and the corresponding form factor h_3^Z is given as follows:

$$h_3^Z = \frac{v^2 M_Z^2}{2s_W c_W [\Lambda_{\tilde{B}W}^4]}. \quad (2.10)$$

TABLE I. Summary of contributions by the relevant CP -conserving dimension-8 operators to the nTGC vertices ZZZ^* and $ZZ\gamma^*$.

Operators	ZZZ^*	$ZZ\gamma^*$
$\mathcal{O}_{\tilde{W}W}$	0	0
$\mathcal{O}'_{\tilde{W}W}$	$\frac{ev^2}{4 \tan \theta_W} (p_3^2 - M_Z^2)(p_1^\rho - p_2^\rho) \epsilon_{\alpha\beta\mu\rho}$	$\frac{ev^2}{4} p_3^2 (p_1^\rho - p_2^\rho) \epsilon_{\alpha\beta\mu\rho}$
$\mathcal{O}_{\tilde{B}B}$	0	0
$\mathcal{O}'_{\tilde{B}B}$	$ev^2 \tan \theta_W (p_3^2 - M_Z^2)(p_1^\rho - p_2^\rho) \epsilon_{\alpha\beta\mu\rho}$	$-ev^2 p_3^2 (p_1^\rho - p_2^\rho) \epsilon_{\alpha\beta\mu\rho}$
$\mathcal{O}_{\tilde{B}W}$	0	$-\frac{ev^2}{2s_W c_W} p_3^2 (p_1^\rho - p_2^\rho) \epsilon_{\alpha\beta\mu\rho}$
$\mathcal{O}'_{\tilde{B}W}$	$\frac{ev^2}{2} (p_3^2 - M_Z^2)(p_1^\rho - p_2^\rho) \epsilon_{\alpha\beta\mu\rho}$	$\frac{ev^2}{2} \tan \theta_W p_3^2 (p_1^\rho - p_2^\rho) \epsilon_{\alpha\beta\mu\rho}$
$\mathcal{O}_{\tilde{W}B}$	0	$\frac{ev^2}{2s_W c_W} p_3^2 (p_1^\rho - p_2^\rho) \epsilon_{\alpha\beta\mu\rho}$

Inspecting Eqs. (2.9) and (2.10), we deduce the following relation:

$$f_5^\gamma = h_3^Z, \quad (\text{for } \mathcal{O}_{\tilde{B}W}). \quad (2.11)$$

As seen in Eq. (2.4), spontaneous electroweak symmetry breaking (EWSB) and the Higgs vacuum expectation value (VEV) are essential for generating ZZV^* couplings. We also note that these vertices could only generate on shell Z pairs with one Z polarized transversely and the other

Z polarized longitudinally, since the kinematic structure with the antisymmetric Levi-Civita tensor causes the production of Z pairs with the same polarizations to vanish. Due to the antisymmetric structure, the pure gauge operators $\mathcal{O}_{G\pm}$ that we introduced in Ref. [6] make vanishing contributions to the ZZV^* couplings.

Working in the electroweak broken phase of the SM in which $SU(2)_W \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$, we expand the operators (2.4) and derive the relevant neutral triple gauge vertices (nTGVs) as follows:

$$\mathcal{O}_{\tilde{W}W} \rightarrow -\frac{ev^2}{8s_W c_W} (s_W^2 \tilde{A}_{\mu\nu} A^{\nu\rho} + s_W c_W \tilde{A}_{\mu\nu} Z^{\nu\rho} + s_W c_W \tilde{Z}_{\mu\nu} A^{\nu\rho} + c_W^2 \tilde{Z}_{\mu\nu} Z^{\nu\rho}) (\partial^\mu Z_\rho + \partial_\rho Z^\mu), \quad (2.12a)$$

$$\mathcal{O}'_{\tilde{W}W} \rightarrow -\frac{ev^2}{8s_W c_W} Z^\mu (s_W^2 \tilde{A}_{\mu\nu} \partial_\rho A^{\nu\rho} + s_W c_W \tilde{A}_{\mu\nu} \partial_\rho Z^{\nu\rho} + s_W c_W \tilde{Z}_{\mu\nu} \partial_\rho A^{\nu\rho} + c_W^2 \tilde{Z}_{\mu\nu} \partial_\rho Z^{\nu\rho}), \quad (2.12b)$$

$$\mathcal{O}_{\tilde{B}B} \rightarrow \frac{ev^2}{2s_W c_W} (-c_W^2 \tilde{A}_{\mu\nu} A^{\nu\rho} + s_W c_W \tilde{A}_{\mu\nu} Z^{\nu\rho} + s_W c_W \tilde{Z}_{\mu\nu} A^{\nu\rho} - s_W^2 \tilde{Z}_{\mu\nu} Z^{\nu\rho}) (\partial^\mu Z_\rho + \partial_\rho Z^\mu), \quad (2.12c)$$

$$\mathcal{O}'_{\tilde{B}B} \rightarrow -\frac{ev^2}{2s_W c_W} Z^\mu (c_W^2 \tilde{A}_{\mu\nu} \partial_\rho A^{\nu\rho} - s_W c_W \tilde{A}_{\mu\nu} \partial_\rho Z^{\nu\rho} - s_W c_W \tilde{Z}_{\mu\nu} \partial_\rho A^{\nu\rho} + s_W^2 \tilde{Z}_{\mu\nu} \partial_\rho Z^{\nu\rho}), \quad (2.12d)$$

$$\mathcal{O}_{\tilde{B}W} \rightarrow \frac{ev^2}{4s_W c_W} (s_W c_W \tilde{A}_{\mu\nu} A^{\nu\rho} + c_W^2 \tilde{A}_{\mu\nu} Z^{\nu\rho} - s_W^2 \tilde{Z}_{\mu\nu} A^{\nu\rho} - s_W c_W \tilde{Z}_{\mu\nu} Z^{\nu\rho}) (\partial^\mu Z_\rho + \partial_\rho Z^\mu), \quad (2.12e)$$

$$\mathcal{O}'_{\tilde{B}W} \rightarrow \frac{ev^2}{4s_W c_W} Z^\mu (s_W c_W \tilde{A}_{\mu\nu} \partial_\rho A^{\nu\rho} + c_W^2 \tilde{A}_{\mu\nu} \partial_\rho Z^{\nu\rho} - s_W^2 \tilde{Z}_{\mu\nu} \partial_\rho A^{\nu\rho} - s_W c_W \tilde{Z}_{\mu\nu} \partial_\rho Z^{\nu\rho}), \quad (2.12f)$$

$$\mathcal{O}_{\tilde{W}B} \rightarrow -\frac{ev^2}{4s_W c_W} (-s_W c_W \tilde{A}_{\mu\nu} A^{\nu\rho} + s_W^2 \tilde{A}_{\mu\nu} Z^{\nu\rho} - c_W^2 \tilde{Z}_{\mu\nu} A^{\nu\rho} + s_W c_W \tilde{Z}_{\mu\nu} Z^{\nu\rho}) (\partial^\mu Z_\rho + \partial_\rho Z^\mu). \quad (2.12g)$$

Combining the above contributions to the nTGC vertices, we can derive the general Lorentz structures for the ZZV^* vertices with the form factors (f_5^Z, f_5^γ) in coordinate space that respect the residual gauge symmetry $U(1)_{\text{em}}$ of QED,

$$\mathcal{L} = \frac{e}{2M_Z^2} f_5^\gamma (c_W^2 \tilde{A}_{\mu\nu} Z^{\nu\rho} - s_W^2 \tilde{Z}_{\mu\nu} A^{\nu\rho}) (\partial^\mu Z_\rho + \partial_\rho Z^\mu) - \frac{e}{2M_Z^2} f_5^Z Z^\mu \tilde{Z}_{\mu\nu} \partial_\rho Z^{\nu\rho}. \quad (2.13)$$

TABLE II. Unitarity bounds on the new physics scales $\Lambda_{\bar{B}W}$ and Λ_{3Z} (in GeV) of the dimension-8 nTGC operators and on the nTGC form factors f_5^V and f_5^Z .

$\sqrt{s}$ (TeV)	0.25	0.5	1	3	5
$\Lambda_{\bar{B}W}$ (GeV)	48.8	82	138	314	461
Λ_{3Z} (GeV)	36.6	61.6	104	236	347
$ f_5^V $	106	13.2	1.65	0.061	0.013
$ f_5^Z $	166	20.7	2.59	0.096	0.021

B. Unitarity constraints on the CPC nTGCs

In this section we analyze the perturbative unitarity constraints on the CPC nTGCs from the process $e^-e^+ \rightarrow ZZ$, finding that these constraints are much weaker than the sensitivity reaches of the collider analysis presented in the following section.

We make a partial-wave expansion of the nTGC contributions to the scattering amplitude of $e^-e^+ \rightarrow ZZ$ as follows [22]:

$$a_J = \frac{e^{i(\nu'-\nu)\phi}}{32\pi} \int_{-1}^1 d(\cos\theta) d_{\nu'\nu}^J(\cos\theta) \mathcal{T}_{\text{nTGC}}^{s_f s_{\bar{f}}, \lambda_{Z1} \lambda_{Z2}}, \quad (2.14)$$

where the differences between initial-state and final-state helicities are given by $\nu = s_f - s_{\bar{f}}$ and $\nu' = \lambda_{Z1} - \lambda_{Z2}$, respectively. We note that it is sufficient to treat the initial-state electrons/positrons as massless, so we have $s_f = -s_{\bar{f}}$, leading to $\nu = \pm 1$. This implies that the $J = 1$ partial wave makes the leading contribution. The relevant Wigner d functions [23] are given by

$$d_{1,0}^1 = -\frac{1}{\sqrt{2}}, \quad d_{1,\pm 1}^1 = \frac{1}{2}(1 \pm \cos\theta), \quad (2.15)$$

and the relation $d_{m,m'}^J = d_{-m,-m'}^J$ holds in general.

For the process $e_L^- e_R^+ \rightarrow ZZ$, we compute the leading contribution to the scattering amplitude $\mathcal{T}_{\text{nTGC}}^{s_f s_{\bar{f}}, \lambda_{Z1} \lambda_{Z2}}$ as follows:

$$\mathcal{T}_{(8)}^{LR} = \pm \frac{c_L^V f_5^V e^2 s^{\frac{3}{2}}}{\sqrt{2} c_W s_W M_Z^3} \left(\sin^2 \frac{\theta}{2}, \cos^2 \frac{\theta}{2} \right), \quad (2.16)$$

and the corresponding leading p -wave amplitude a_1 is given by

$$|\Re(a_1)| = \left| \frac{c_L^V f_5^V e^2 s^{\frac{3}{2}}}{48 \sqrt{2} \pi c_W s_W M_Z^3} \right|. \quad (2.17)$$

The partial-wave unitarity condition is $|\Re(a_1)| < 1$, where we have included a factor of 2 as the final-state ZZ are identical particles [22]. Thus, we derive the following unitarity bound on the leading contributions to the nTGC form factor:

$$|f_5^V| < \left| \frac{48 \sqrt{2} \pi c_W s_W M_Z^3}{c_L^V e^2 s^{\frac{3}{2}}} \right|. \quad (2.18)$$

Analogously, for the process $e_R^- e_L^+ \rightarrow ZZ$, we derive the following unitarity bound:

$$|f_5^V| < \left| \frac{48 \sqrt{2} \pi c_W s_W M_Z^3}{c_R^V e^2 s^{\frac{3}{2}}} \right|, \quad (2.19)$$

which can be obtained from the unitarity bound (2.18) by the replacement $c_L^V \leftrightarrow c_R^V$. using these results we derive the following unitarity bounds on the nTGC form factors f_5^V and f_5^Z :

$$|f_5^V| < \frac{48 \sqrt{2} \pi M_Z^3}{e^2 s^{\frac{3}{2}}}, \quad |f_5^Z| < \frac{96 \sqrt{2} \pi c_W s_W M_Z^3}{e^2 (1 - 2s_W^2) s^{\frac{3}{2}}}. \quad (2.20)$$

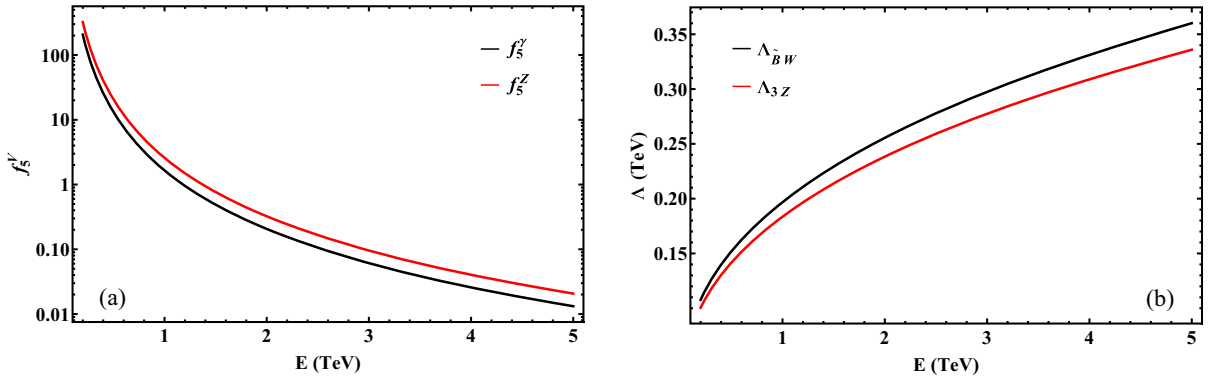


FIG. 1. Unitarity bounds on the nTGC form factors f_5^V and f_5^Z in plot (a) and on the new physics scales $\Lambda_{\bar{B}W}$ and Λ_{3Z} in plot (b). These bounds are derived from the p -wave amplitudes for the reaction $e^-e^+ \rightarrow ZZ$. The region below (above) each curve is allowed in plot (a) [in plot (b)].

Since the nTGC form factors f_5^γ and f_5^Z are connected to the new physics cutoff scales $\Lambda_{\tilde{B}W}$ and Λ_{3Z} of the corresponding dimension-8 nTGC operators as shown in Eqs. (2.9) and (2.8), we derive the following unitarity bounds on $\Lambda_{\tilde{B}W}$ and Λ_{3Z} :

$$\begin{aligned}\Lambda_{\tilde{B}W} &> \sqrt[4]{\frac{e^2 v^2 s^{3/2}}{96\sqrt{2}\pi s_W c_W M_Z}}, \\ \Lambda_{3Z} &> \sqrt[4]{\frac{e^2 v^2 s^{3/2}(1-2s_W^2)}{384\sqrt{2}\pi s_W^2 c_W^2 M_Z}}.\end{aligned}\quad (2.21)$$

We present the unitarity bounds (2.20) and (2.21) for the reaction $e^-e^+ \rightarrow ZZ$ in Table II, for the representative e^+e^- collider energies $\sqrt{s} = (0.25, 0.5, 1, 3, 5)$ TeV. We further present in Fig. 1 the unitarity bounds on the nTGC form factors and on the new physics scales of the nTGC operators as functions of the e^+e^- collision energy $\sqrt{s} = (0.25, 0.5, 1, 3, 5)$ TeV for the reaction $e^-e^+ \rightarrow ZZ$. The unitarity bounds on the nTGC form factors f_5^γ and f_5^Z are shown in plot (a), and in plot (b) we display the unitarity bounds on the new physics scales $\Lambda_{\tilde{B}W}$ and Λ_{3Z} . The region below (above) each curve is allowed in plot (a) [in plot (b)]. We find that these unitarity bounds on the nTGC form factor (f_5^γ, f_5^Z) and on the new physics scales ($\Lambda_{\tilde{B}W}, \Lambda_{3Z}$) are much weaker than our collider constraints, and hence do not affect our collider analyses in the next section.

III. SCATTERING AMPLITUDES, CROSS SECTIONS AND ANGULAR DISTRIBUTIONS

In this section we study the reaction $e^-e^+ \rightarrow ZZ$ including contributions from both the SM and the nTGCs. For this purpose, we compute helicity amplitudes and cross sections, and analyze the angular distributions of its final states.

As shown in Fig. 2, the SM contributions to the reaction $e^-e^+ \rightarrow ZZ$ are given by the t -channel and u -channel fermion-exchange diagrams, whereas the nTGC vertex ZZV^* contributes via s -channel diagrams mediated by a virtual Z or γ . We first derive the SM contributions to the scattering amplitudes of the reaction $e^-(p_1)e^+(p_2) \rightarrow Z(p_3)Z(p_4)$,

$$\begin{aligned}T_{\text{sm}} &= \bar{v}(p_2)\Gamma_Z^\mu c_\mu^*(p_4)\frac{(\not{p}_1 - \not{p}_3)}{t}\Gamma_Z^\nu c_\nu^*(p_3)u(p_1) \\ &+ \bar{v}(p_2)\Gamma_Z^\mu c_\mu^*(p_3)\frac{(\not{p}_1 - \not{p}_4)}{u}\Gamma_Z^\nu c_\nu^*(p_4)u(p_1),\end{aligned}\quad (3.1)$$

where we have defined the Mandelstam variables, $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, and $u = (p_1 - p_4)^2$. In the above the vertex function Γ_Z^μ is defined as follows:

$$\Gamma_Z^\mu = \frac{ie}{s_W c_W}\gamma^\mu(c_L^Z P_L + c_R^Z P_R),\quad (3.2)$$

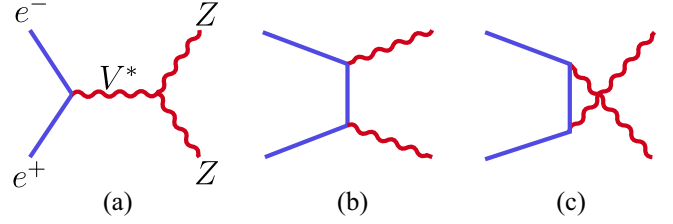


FIG. 2. Feynman diagrams that contribute to the reaction $e^-e^+ \rightarrow ZZ$. Plot (a) shows the signal process containing the nTGC vertices ZZV^* , and plots (b) and (c) show the leading SM background contributions.

where $P_{L,R} = \frac{1 \mp \gamma_5}{2}$ are projection operators, and $(c_L^Z, c_R^Z) = (-\frac{1}{2} + s_W^2, s_W^2)$ denote the (left, right)-handed gauge couplings of the electron to the Z boson, respectively.

Next we compute the new physics amplitudes contributed by the nTGCs for the reaction $e^-(p_1)e^+(p_2) \rightarrow Z(p_3)Z(p_4)$,

$$\begin{aligned}T_{(8)} &= \bar{v}(p_2)\Gamma_V^\mu u(p_1)\frac{-g_{\mu\nu}}{s - M_V^2}ie\Gamma_{ZZV^*}^{\alpha\beta\nu}c_\alpha^*(p_3)c_\beta^*(p_4) \\ &= \bar{v}(p_2)\Gamma_V^\mu u(p_1)\frac{-eg_{\mu\nu}}{M_Z^2} \\ &\quad \times f_5^V c^{\nu\alpha\beta\rho}(p_3 - p_4)_\rho c_\alpha^*(p_3)c_\beta^*(p_4),\end{aligned}\quad (3.3)$$

where we define the vertex functions Γ_V^μ ($V = \gamma, Z$) as follows:

$$\Gamma_V^\mu = \frac{ie}{s_W c_W}\gamma^\mu(c_L^V P_L + c_R^V P_R),\quad (3.4a)$$

$$c_L^\gamma = c_R^\gamma = -s_W c_W, \quad c_L^Z = -\frac{1}{2} + s_W^2, \quad c_R^Z = s_W^2,\quad (3.4b)$$

where c_L^V and c_R^V arise from the left-handed and right-handed gauge couplings of electrons to the gauge bosons $V = Z, \gamma$, respectively.

For the reaction $e^-(p_1)e^+(p_2) \rightarrow Z(p_3)Z(p_4)$, we parametrize the external momenta as follows:

$$\begin{aligned}p_1 &= \frac{\sqrt{s}}{2}(1, 0, 0, 1), & p_2 &= \frac{\sqrt{s}}{2}(1, 0, 0, -1), \\ p_3 &= \left(\frac{\sqrt{s}}{2}, q \sin \theta, 0, q \cos \theta\right), \\ p_4 &= \left(\frac{\sqrt{s}}{2}, -q \sin \theta, 0, -q \cos \theta\right),\end{aligned}\quad (3.5)$$

where $q = \sqrt{\frac{s}{4} - M_Z^2}$ is the magnitude of each Z boson's momentum and θ is the scattering angle between the directions of the incident $e^-(p_1)$ and the outgoing $Z(p_3)$.

In order to characterize the final state Z boson pair $Z(p_3, \lambda)Z(p_4, \lambda')$ with its nine helicity combinations, we define a 3×3 matrix amplitude $\mathcal{T}$ whose i th row and j th column element is defined as $\mathcal{T}(i, j) \equiv \mathcal{T}(\lambda + 2, \lambda' + 2)$. We derive the following SM contributions to the scattering amplitudes:

$$\mathcal{T}_{sm}^{LR} = \frac{2e^2 c_L^{Z2}}{s_W^2 c_W^2 (4M_Z^4 - 4\sin^2\theta M_Z^2 s + s^2 \sin^2\theta)} \times \begin{pmatrix} -sM_Z^2 \sin 2\theta & \sqrt{2}sM_Z (s\sin^2\theta - 4M_Z^2 \sin^2 \frac{\theta}{2}) & -2s(s - 2M_Z^2) \sin \theta \sin^2 \frac{\theta}{2} \\ \sqrt{2}sM_Z (s\sin^2\theta - 4M_Z^2 \cos^2 \frac{\theta}{2}) & 2sM_Z^2 \sin 2\theta & -\sqrt{2}sM_Z (s\sin^2\theta - 4M_Z^2 \sin^2 \frac{\theta}{2}) \\ 2s(s - 2M_Z^2) \sin \theta \cos^2 \frac{\theta}{2} & -\sqrt{2}sM_Z (s\sin^2\theta - 4M_Z^2 \cos^2 \frac{\theta}{2}) & -sM_Z^2 \sin 2\theta \end{pmatrix}, \quad (3.6)$$

for the process $e_L^- e_R^+ \rightarrow ZZ$, and

$$\mathcal{T}_{sm}^{RL} = \frac{2e^2 c_R^{Z2}}{s_W^2 c_W^2 (4M_Z^4 - 4s\sin^2\theta M_Z^2 + s^2 \sin^2\theta)} \times \begin{pmatrix} -sM_Z^2 \sin 2\theta & \sqrt{2}sM_Z (s\sin^2\theta - 4M_Z^2 \cos^2 \frac{\theta}{2}) & 2s(s - 2M_Z^2) \sin \theta \cos^2 \frac{\theta}{2} \\ \sqrt{2}sM_Z (s\sin^2\theta - 4M_Z^2 \sin^2 \frac{\theta}{2}) & 2sM_Z^2 \sin 2\theta & -\sqrt{2}sM_Z (s\sin^2\theta - 4M_Z^2 \cos^2 \frac{\theta}{2}) \\ -2s(s - 2M_Z^2) \sin \theta \sin^2 \frac{\theta}{2} & -\sqrt{2}sM_Z (s\sin^2\theta - 4M_Z^2 \sin^2 \frac{\theta}{2}) & -sM_Z^2 \sin 2\theta \end{pmatrix}, \quad (3.7)$$

for the process $e_R^- e_L^+ \rightarrow ZZ$.

As mentioned above, the ZZV^* coupling only contributes to the helicity amplitude for $e^- e^+ \rightarrow ZZ$ with one longitudinally-polarized Z boson and one transversely-polarized Z boson,

$$\mathcal{T}_{(8)} = \begin{pmatrix} 0 & O\left(\frac{E^3}{M_Z^3 \Lambda^4}\right) & 0 \\ O\left(\frac{E^3}{M_Z^3 \Lambda^4}\right) & 0 & O\left(\frac{E^3}{M_Z^3 \Lambda^4}\right) \\ 0 & O\left(\frac{E^3}{M_Z^3 \Lambda^4}\right) & 0 \end{pmatrix}, \quad (3.9b)$$

$$\mathcal{T}_{(8)}^{LR} = \frac{c_L^V f_5^V e^2 \sqrt{s}(s - 4M_Z^2)}{\sqrt{2}c_W s_W M_Z^3} \begin{pmatrix} 0 & \sin^2 \frac{\theta}{2} & 0 \\ \cos^2 \frac{\theta}{2} & 0 & -\sin^2 \frac{\theta}{2} \\ 0 & -\cos^2 \frac{\theta}{2} & 0 \end{pmatrix}, \quad (3.8a)$$

$$\mathcal{T}_{(8)}^{RL} = \frac{c_R^V f_5^V e^2 \sqrt{s}(s - 4M_Z^2)}{\sqrt{2}c_W s_W M_Z^3} \begin{pmatrix} 0 & -\cos^2 \frac{\theta}{2} & 0 \\ -\sin^2 \frac{\theta}{2} & 0 & \cos^2 \frac{\theta}{2} \\ 0 & \sin^2 \frac{\theta}{2} & 0 \end{pmatrix}. \quad (3.8b)$$

Making a high-energy expansion, we deduce the high-energy behaviors of the helicity amplitudes from the contributions of the SM and the nTGCs, respectively,

$$\mathcal{T}_{sm} = \begin{pmatrix} O\left(\frac{M_Z^2}{E^2}\right) & O\left(\frac{M_Z}{E}\right) & O(E^0) \\ O\left(\frac{M_Z}{E}\right) & O\left(\frac{M_Z^2}{E^2}\right) & O\left(\frac{M_Z}{E}\right) \\ O(E^0) & O\left(\frac{M_Z}{E}\right) & O\left(\frac{M_Z^2}{E^2}\right) \end{pmatrix}, \quad (3.9a)$$

where we denote the energy scale $E = \sqrt{s}$.

We see from Eq. (3.9) that the SM amplitude with the final states $Z_L Z_T$ would behave like $\mathcal{T}_{sm}(Z_L Z_T) = O\left(\frac{M_Z}{\sqrt{s}}\right)$ at high energies. This behavior can be understood via the electroweak equivalence theorem (ET) that connects the scattering amplitude of longitudinal gauge bosons to the scattering amplitude of the corresponding Goldstone bosons [24]. For the current scattering process we have the following ET relation:

$$\mathcal{T}[Z_L, Z_T] = \mathcal{T}[-i\pi^0, Z_T] + B[v, Z_T], \quad (3.10)$$

where π^0 denotes the neutral Goldstone boson that is converted to the longitudinal mode Z_L via the Higgs mechanism [25]. In Eq. (3.10), the B term is defined as $B = \mathcal{T}[v, Z_T]$, with $v \equiv v_\mu Z^\mu$ and $v^\mu = \epsilon_L^\mu - \epsilon_S^\mu = O(M_Z/E)$. Since the initial-state fermion masses are negligible, the SM contributions contain only the t - and u -channel exchanges. Thus we can deduce,

$$\mathcal{T}_{sm}[\pi^0, Z_T] \simeq 0, \quad \mathcal{T}_{sm}[Z_L, Z_T] \simeq B = O\left(\frac{M_Z}{\sqrt{s}}\right), \quad (3.11)$$

which explains the high-energy behavior of $\mathcal{T}_{sm}[Z_L Z_T]$. Moreover, Eq. (3.9) shows that $\mathcal{T}_{sm}[Z_L Z_L] = O(M_Z^2/E^2)$. This is because the ET gives

$$\mathcal{T}[Z_L, Z_L] = -\mathcal{T}[\pi^0, \pi^0] + B[LL], \quad (3.12)$$

where $\mathcal{T}[\pi^0, \pi^0] = 0$ holds because the initial-state fermions $e^\mp$ have very small masses, so that the Yukawa couplings between the Higgs boson (Goldstone boson) and

fermions can be ignored. Hence, for the ET identity (3.12), we deduce the following:

$$\begin{aligned} \mathcal{T}[Z_L, Z_L] &= B[LL] = \mathcal{T}[-i\pi^0, v] + \mathcal{T}[v, -i\pi^0] + \mathcal{T}[v, v] \\ &= \mathcal{T}[v, v] = O\left(\frac{M_Z^2}{E^2}\right). \end{aligned} \quad (3.13)$$

We can then compute the SM contributions to amplitude-squared as follows:

$$\begin{aligned} \overline{|\mathcal{T}_{sm}|^2}[ZZ] &= \frac{e^4 s(c_L^{Z4} + c_R^{Z4})[16M_Z^6(1 + \cos^2 \theta) - 4(3 - 7\cos^2 \theta + 4s\cos^4 \theta)M_Z^4]}{c_W^4 s_W^4 (4M_Z^4 - 4s\sin^2 \theta M_Z^2 + s^2 \sin^2 \theta)^2} \\ &\quad - \frac{e^4 s(c_L^{Z4} + c_R^{Z4})[8s^2 \sin^2 \theta \cos^2 \theta M_Z^2 + s^3(\cos^4 \theta - 1)]}{c_W^4 s_W^4 (4M_Z^4 - 4s\sin^2 \theta M_Z^2 + s^2 \sin^2 \theta)^2}, \end{aligned} \quad (3.14)$$

where we average over the spins of the initial-state fermions and sum over the helicity combinations of the final states. The squared scattering amplitudes for the SM contributions to the final states $Z_L Z_T$ and $Z_T Z_L$ take the following form:

$$\overline{|\mathcal{T}_{sm}|^2}[Z_L Z_T] = \frac{2e^4 s M_Z^2 (c_L^{Z4} + c_R^{Z4})[4(1 + \cos^2 \theta)M_Z^4 - 4s\sin^2 \theta M_Z^2 + s^2 \sin^4 \theta]}{c_W^4 s_W^4 (4M_Z^4 - 4s\sin^2 \theta M_Z^2 + s^2 \sin^2 \theta)^2}. \quad (3.15)$$

We have verified that our above formulas (3.14) and (3.15) agree with the literature [11]. We can further derive the SM contributions to the squared amplitudes for the final states $Z_L Z_L$ and $Z_T Z_T$,

$$\overline{|\mathcal{T}_{sm}|^2}[Z_L Z_L] = \frac{e^4 s^2 M_Z^4 (c_L^{Z4} + c_R^{Z4}) \sin^2 2\theta}{c_W^4 s_W^4 (4M_Z^4 - 4s\sin^2 \theta M_Z^2 + s^2 \sin^2 \theta)^2}, \quad (3.16a)$$

$$\overline{|\mathcal{T}_{sm}|^2}[Z_T Z_T] = \frac{e^4 s^2 (c_L^{Z4} + c_R^{Z4})[4(1 + 2\cos^2 \theta)M_Z^4 - 4s(1 + \cos^2 \theta)M_Z^2 + s^2(1 + \cos^2 \theta)] \sin^2 \theta}{8c_W^4 s_W^4 (4M_Z^4 - 4s\sin^2 \theta M_Z^2 + s^2 \sin^2 \theta)^2}. \quad (3.16b)$$

As we have discussed earlier, the new physics of ZZV^* nTGCs contribute only to the reaction $e^-(p_1)e^+(p_2) \rightarrow Z(p_3)Z(p_4)$ with the final states $Z_L Z_T$ and $Z_T Z_L$. Computing the interference between the contributions from the dimension-8 nTGC operator and from the SM, we find the following form:

$$2\text{Re}(\overline{\mathcal{T}_{sm} \mathcal{T}_{(8)}^*})[Z_L Z_T] = \frac{2se^4 f_5^V (c_L^V c_L^{Z2} - c_R^V c_R^{Z2})(4M_Z^2 - s)[2M_Z^2(1 + \cos^2 \theta) - s\sin^2 \theta]}{c_W^3 s_W^3 M_Z^2 (4M_Z^4 - 4s\sin^2 \theta M_Z^2 + s^2 \sin^2 \theta)}. \quad (3.17)$$

This interference term is of $O(\Lambda^{-4})$. We can further compute the squared amplitude of $O(\Lambda^{-8})$ from the dimension-8 nTGC contributions as follows:

$$\overline{|\mathcal{T}_{(8)}|^2}[Z_L Z_T] = \frac{e^4 s f_5^{V2} (c_L^{V2} + c_R^{V2})(s - 4M_Z^2)^2 (3 + \cos 2\theta)}{4s_W^2 c_W^2 M_Z^6}. \quad (3.18)$$

According to the energy power counting results (3.9), we can infer the leading high-energy behavior of each contribution to the squared amplitude $|\mathcal{T}|^2$,

$$|\mathcal{T}_{sm}|^2 \sim E^0, \quad \sum_{\lambda, \lambda'} \mathcal{T}_{sm}(\lambda \lambda') \mathcal{T}_{(8)}^*(\lambda \lambda') \sim E^2, \quad |\mathcal{T}_{(8)}|^2 \sim E^6. \quad (3.19)$$

However, we recall that the Z boson is unstable and decays inside the detector. The intermediate-state Z bosons with different polarizations interfere, and the leading high-energy dependence of $\mathcal{T}_{sm} \mathcal{T}_{(8)}^*$ is E^3 . We need to analyze the angular

distributions of final-state particles from Z boson decays to obtain information about the interference of different Z polarizations, which is discussed in the following section.

Using these results, we derive the total cross section of the reaction $e^-e^+ \rightarrow ZZ$ as follows:

$$\sigma = \frac{e^4(c_L^{Z4} + c_R^{Z4}) \left[-2s\beta(s - 2M_Z^2) + (4M_Z^4 + s^2) \ln \frac{s(1+\beta)-2M_Z^2}{s(1-\beta)-2M_Z^2} \right]}{16\pi c_W^4 s_W^4 (s - 2M_Z^2) s^2} + \frac{e^4 f_5^V (c_L^V c_L^{Z2} - c_R^V c_R^{Z2}) \left[s(s + 2M_Z^2)\beta + 4M_Z^2(s - M_Z^2) \operatorname{arccoth} \left(\frac{2M_Z^2 - s}{\beta s} \right) \right]}{16\pi c_W^3 s_W^3 M_Z^2 s^2} + \frac{e^4 f_5^V (c_L^{V2} + c_R^{V2}) \beta^5 s^2}{192\pi c_W^2 s_W^2 M_Z^6}, \quad (3.20)$$

where $\beta = p_Z/E_Z = \sqrt{1 - 4M_Z^2/s}$ and the symbol “ $\operatorname{arccoth}$ ” denotes the inverse function of the hyperbolic tangent function $\coth$. In the high-energy limit, we can deduce the following behaviors for each contribution to the total cross section:

$$\sigma = O(s^{-1}) + O\left(\frac{s^0 M_Z^2}{\Lambda^4}\right) + O\left(\frac{s^2 M_Z^2}{\Lambda^8}\right). \quad (3.21)$$

We see that the SM contribution scales as s^{-1} and decreases with energy, whereas the interference term scales as s^0/Λ^4 and thus is insensitive to the energy. Finally, the last contribution (arising from the squared amplitude) scales as s^2/Λ^8 and increases with energy rapidly.

For the current analysis, we define the following normalized angular distribution functions:

$$f_\theta^j = \frac{1}{\sigma_j} \frac{d\sigma_j}{d\theta}, \quad (3.22)$$

where σ_j (with $j = 0, 1, 2$) represents the SM contribution (σ_0), the $O(\Lambda^{-4})$ contribution (σ_1), and the $O(\Lambda^{-8})$ contribution (σ_2). Then, we compute the angular distribution functions as follows:

$$f_\theta^0 = \frac{s^2 \beta (2M_Z^2 - s) \{ 192M_Z^6 - 32sM_Z^4 - 8s^2 M_Z^2 + 5s^3 - s\beta^2 [4 \cos 2\theta (s + 2M_Z^2)^2 + s^2 \beta^2 \cos 4\theta] \} \sin \theta}{16 \left[2s\beta(s - 2M_Z^2) + (s^2 + 4M_Z^4) \ln \frac{s(1-\beta)-2M_Z^2}{s(1+\beta)-2M_Z^2} \right] (4M_Z^4 + s^2 \beta^2 \sin^2 \theta)^2}, \quad (3.23a)$$

$$f_\theta^1 = - \frac{s^3 \beta^3 [6M_Z^2 - s + (s + 2M_Z^2) \cos 2\theta] \sin \theta}{4 \left[s\beta(s + 2M_Z^2) + 4M_Z^2(s - M_Z^2) \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s} \right] (4M_Z^4 + s^2 \beta^2 \sin^2 \theta)}, \quad (3.23b)$$

$$f_\theta^2 = \frac{3}{16} (3 + \cos 2\theta) \sin \theta. \quad (3.23c)$$

We see in Fig. 4 that the SM distribution is concentrated around the forward and backward scattering regions with θ close to 0° or 180° . This feature is particularly clear for large collision energies $\sqrt{s} \gg M_Z$. It also shows that the interference distributions of $O(\Lambda^{-4})$ have negative sign in the small $\sin \theta$ region, namely, when the scattering angle θ obeys the condition $|\sin \theta| < 2M_Z/(s + 2M_Z^2)^{1/2}$.

Next, we further incorporate the fermionic decays of the final states ZZ . We first consider Z decay in its rest frame, $Z(q_1) \rightarrow f(k_1) \bar{f}(k_2)$, and express the momenta of the final-state fermions as follows:

$$k_1 = k(1, \sin \theta_a \cos \phi_a, \sin \theta_a \sin \phi_a, \cos \theta_a), \quad (3.24a)$$

$$k_2 = k(1, -\sin \theta_a \cos \phi_a, -\sin \theta_a \sin \phi_a, -\cos \theta_a), \quad (3.24b)$$

where $k = \frac{1}{2} M_Z$ holds since we treat the final-state fermions as effectively massless. Here the positive direction $\hat{z}_a$ in the Z rest frame is chosen to be the Z moving direction in the laboratory frame, and θ_a denotes the angle between the positive direction $\hat{z}_a$ and the fermion f direction of motion in the Z rest frame. We define ϕ_a as the angle between the e^-e^+ scattering plane and the Z -decay plane in the e^-e^+ center-of-mass frame. Similarly, we can express the kinematics of the other Z decay in terms of the angular variables θ_b and ϕ_b . This kinematic structure is illustrated in Fig. 3.

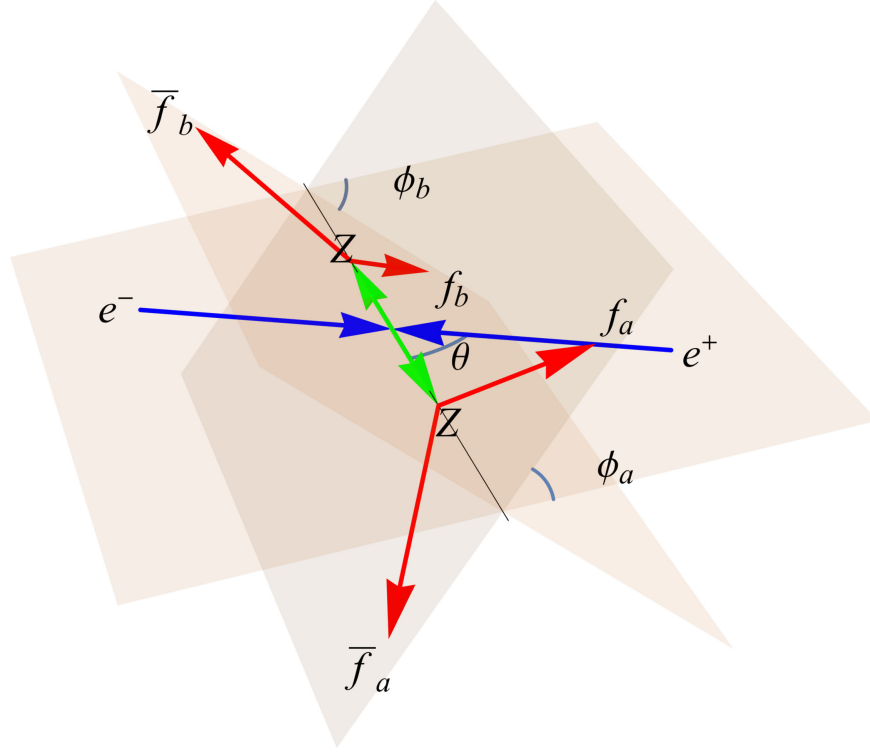


FIG. 3. Kinematic structure of the reaction $e^-e^+ \rightarrow ZZ$ followed by the fermionic decays $Z \rightarrow f\bar{f}$, in the e^-e^+ collision frame.

Then, we express the helicity amplitude for the reaction $e^-e^+ \rightarrow ZZ \rightarrow f_a\bar{f}_a f_b\bar{f}_b$ as follows:

$$\begin{aligned} \mathcal{T} = & \frac{e^2 M_Z^2 \mathcal{D}_Z(q_1) \mathcal{D}_Z(q_2)}{2s_W^2 c_W^2} \sum_{\lambda, \lambda' = -1}^1 e^{-i(\lambda\phi_a + \lambda'\phi_b)} \mathcal{T}_{ss'}[\lambda\lambda'] \\ & \times \{ (1 - \delta_\lambda^0) [f_R^a (1 - \lambda \cos \theta_a) - f_L^a (1 + \lambda \cos \theta_a)] \\ & + \sqrt{2} \delta_\lambda^0 \sin \theta_a (f_R^a + f_L^a) \} \\ & \times \{ (1 - \delta_{\lambda'}^0) [f_R^b (1 - \lambda' \cos \theta_b) - f_L^b (1 + \lambda' \cos \theta_b)] \\ & + \sqrt{2} \delta_{\lambda'}^0 \sin \theta_b (f_R^b + f_L^b) \}, \end{aligned} \quad (3.25)$$

where $\mathcal{D}_Z$ is the propagator of the Z boson, and $(f_L^{a,b}, f_R^{a,b}) = (f_L \delta_{\sigma, -\frac{1}{2}}, f_R \delta_{\sigma, \frac{1}{2}})$ denote the gauge couplings

of the fermions a and b with helicity σ . In the above, the gauge couplings $(f_L, f_R) = (T_3 - Qs_W^2, -Qs_W^2)$ denote the gauge couplings of the (left, right)-handed fermions. In Eq. (3.25), $\mathcal{T}_{ss'}[\lambda\lambda']$ represents the on shell helicity amplitude of the scattering process $e^-e^+ \rightarrow ZZ$,

$$\mathcal{T}_{ss'}[\lambda\lambda'] = \mathcal{T}_{sm}^{ss'}[\lambda\lambda'] + \mathcal{T}_{(8)}^{ss'}[\lambda\lambda'], \quad (3.26)$$

which include the contributions from both the SM and the dimension-8 nTGC operators.

Next, we derive the following normalized angular distributions $f_{\theta_a}^j = \frac{1}{\sigma_j} \frac{d\sigma_j}{d\theta_a}$ as functions of the angle θ_a :

$$\begin{aligned} f_{\theta_a}^0 = & \frac{3 \sin \theta_a (s - 2M_Z^2) [3s - 10M_Z^2 + (s + 2M_Z^2) \cos 2\theta_a]}{16\beta \left[s(s - 2M_Z^2)\beta + (s^2 + 4M_Z^4) \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s} \right]} \\ & + \frac{6 \sin \theta_a [(1 + \cos^2 \theta_a)(s^3 - 4s^2 M_Z^2 + 4s M_Z^4 + 8M_Z^6) - 32M_Z^6] \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s}}{16\beta^2 s \left[s(s - 2M_Z^2)\beta + (s^2 + 4M_Z^4) \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s} \right]}, \end{aligned} \quad (3.27a)$$

$$f_{\theta_a}^1 = f_{\theta_a}^2 = \frac{3}{32} \sin \theta_a (5 - \cos 2\theta_a), \quad (3.27b)$$

and the following normalized angular distributions $f_{\phi_a}^j = \frac{1}{\sigma_j} \frac{d\sigma_j}{d\phi_a}$ with respect to the angle ϕ_a :

$$f_{\phi_a}^0 = \frac{1}{2\pi} + \frac{3\pi\sqrt{s}\beta M_Z^3 f_-^2 (c_L^{Z^4} - c_R^{Z^4}) \cos \phi_a}{8f_+^2 (c_L^{Z^4} + c_R^{Z^4}) \left[s(s - 2M_Z^2)\beta + (s^2 + 4M_Z^4) \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s} \right]} - \frac{M_Z^2 (s - 2M_Z^2) \left[s\beta + (s - 2M_Z^2) \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s} \right] \cos 2\phi_a}{2\pi\beta^2 s \left[s(s - 2M_Z^2)\beta + (s^2 + 4M_Z^4) \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s} \right]}, \quad (3.28a)$$

$$f_{\phi_a}^1 = \frac{1}{2\pi} - \frac{3\pi\beta^3 f_-^2 s^3 (3s - 8M_Z^2) (c_L^V c_L^{Z^2} + c_R^V c_R^{Z^2}) \cos \phi_a}{256M_Z f_+^2 (c_L^V c_L^{Z^2} - c_R^V c_R^{Z^2}) \left[s(s + 2M_Z^2)\beta + 4M_Z^2 (s - M_Z^2) \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s} \right]} - \frac{\left[s(s - 2M_Z^2)\beta + 4M_Z^4 \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s} \right] \cos 2\phi_a}{8\pi \left[s(s + 2M_Z^2)\beta + 4M_Z^2 (s - M_Z^2) \operatorname{arccoth} \frac{2M_Z^2 - s}{\beta s} \right]}, \quad (3.28b)$$

$$f_{\phi_a}^2 = \frac{1}{2\pi} - \frac{\cos 2\phi_a}{16\pi}, \quad (3.28c)$$

where we have defined notations $f_{\pm}^2 = f_L^2 \pm f_R^2$, with $(f_L, f_R) = (T_3 - Qs_W^2, -Qs_W^2)$ denoting gauge couplings of the (left, right)-handed fermions in the final states. The other notations (c_L^V, c_R^V) denote the gauge couplings of the (left, right)-handed $e^\mp$ with the SM gauge boson $V(=Z, \gamma)$ as defined in Eq. (3.4b). We note that $f_{\theta_a}^1 = f_{\theta_a}^2$ in

Eq. (3.27b). In general, the angular distributions $f_{\theta_a}^i$ depend on the ZZ helicity combinations (TT, TL, LL). However, we see in Eq. (3.8) that $\mathcal{T}_{sm} \mathcal{T}_{(8)}^*$ and $|\mathcal{T}_{(8)}|^2$ both contain only the (TL) combination, hence the distributions $f_{\theta_a}^1$ and $f_{\theta_a}^2$ are the same.

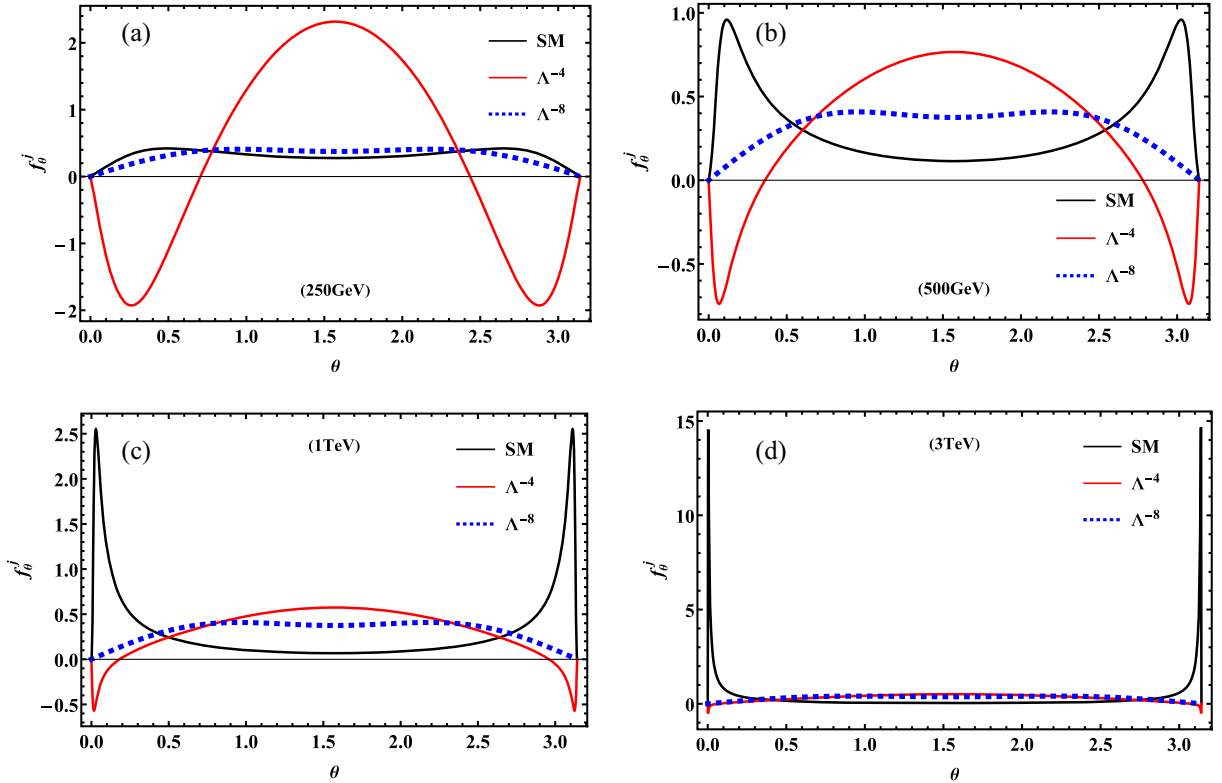


FIG. 4. Normalized angular distributions in the polar scattering angle θ for different collider energies, $\sqrt{s} = (0.25, 0.5, 1, 3)$ TeV, are shown in plots (a)–(d) respectively. In each plot, the black, red and blue curves denote the contributions from the SM, the $O(\Lambda^{-4})$ term, and the $O(\Lambda^{-8})$ term, respectively. The e^+e^- collision energy chosen for each plot is shown in the parentheses, such as (250 GeV), (500 GeV), and so on.

Using the above analytical formulas, we present numerical results for the normalized angular distribution functions of θ , θ_a , and ϕ_a in Figs. 4–6 respectively. In each figure, the four plots have input different collision energies, $\sqrt{s} = (0.25, 0.5, 1, 3)$ TeV, corresponding to the expected collision energies of the CEPC, FCC-ee, ILC, and its possible energy upgrades, and the design energy of CLIC. In Figs. 4 and 5 we use black, red and blue curves to denote the contributions from the SM, the $O(\Lambda^{-4})$ interference, and the $O(\Lambda^{-8})$ term, respectively. In Fig. 6, we use black, red, green, and blue curves to denote the contributions from the SM, the $O(\Lambda^{-4})$ interference with γ^* , the $O(\Lambda^{-4})$ interference with Z^* , and the $O(\Lambda^{-8})$ term, respectively. We find that the distributions with respect to θ_a and ϕ_a are similar to the corresponding distributions for the $Z\gamma$ final states given in [6,7]. The SM and pure dimension-8 distributions in ϕ_a are rather flat, whereas the $O(\Lambda^{-4})$ interference terms are dominated by cosine functions.

It is of interest to examine the high-energy behaviors of the angular distribution functions. In the case of the angular distribution respect to the scattering angle θ , the distribution from the SM, namely f_θ^0 , increases rapidly with collision energy in the small $\sin\theta$ region, whereas the distribution from the $O(\Lambda^{-4})$ interference term, f_θ^1 , decreases with

energy and the distribution from the pure $O(\Lambda^{-8})$ dimension-8 term, f_θ^2 , is rather flat. For all the functions $f_{\theta_a}^j$, the coefficients of all trigonometric functions approach constants in the high-energy limit $s \gg M_Z^2$. This is why Fig. 5 shows that the distributions in θ_a are not sensitive to the collision energy $\sqrt{s}$, as we vary the collision energy over the range $\sqrt{s} = (0.25, 0.5, 1, 3)$ TeV in the four plots. In the high-energy limit $s \gg M_Z^2$ we can derive the following angular distributions with respect to the angle ϕ_a :

$$f_{\phi_a}^0 = \frac{1}{2\pi}, \quad (3.29a)$$

$$f_{\phi_a}^1 = \frac{1}{2\pi} - \frac{9\pi(c_L^V c_L^{Z2} + c_R^V c_R^{Z2})(f_L^2 - f_R^2)\sqrt{s} \cos\phi_a}{256(c_L^V c_L^{Z2} - c_R^V c_R^{Z2})(f_L^2 + f_R^2)M_Z} - \frac{\cos 2\phi_a}{8\pi}, \quad (3.29b)$$

$$f_{\phi_a}^2 = \frac{1}{2\pi} - \frac{\cos 2\phi_a}{16\pi}. \quad (3.29c)$$

From the above formulas, we see that the angular function $f_{\phi_a}^0$ approaches the constant term $\frac{1}{2\pi}$ for $s \gg M_Z^2$. We see in Eq. (3.28c) or Eq. (3.29c) that the value of the angular

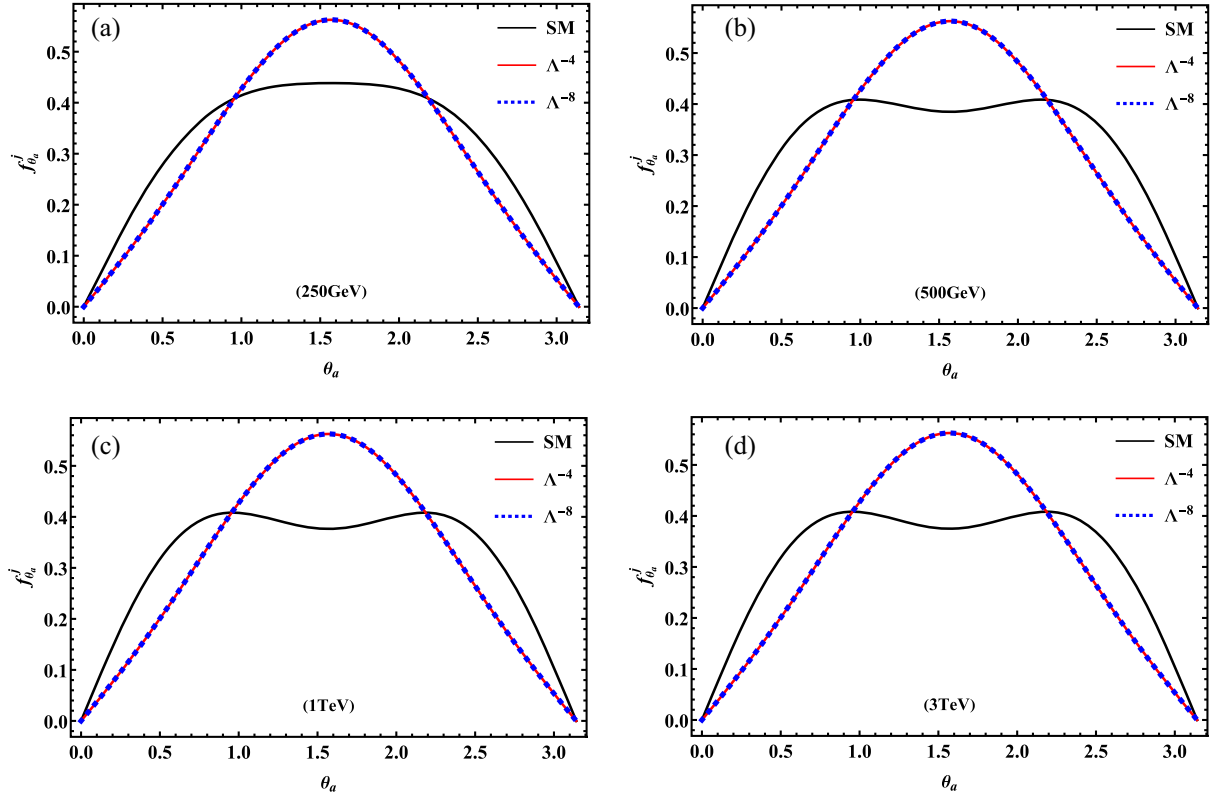


FIG. 5. Normalized angular distribution in the polar angle θ_a in the Z decay frame for different collider energies, $\sqrt{s} = (0.25, 0.5, 1, 3)$ TeV, as shown in plots (a)–(d), respectively. In each plot, the black, red and blue curves denote the contributions from the SM, the $O(\Lambda^{-4})$ interference, and the $O(\Lambda^{-8})$ term, respectively, where the red and blue curves exactly overlap. The e^+e^- collision energy chosen for each plot is shown in the parentheses, such as (250 GeV), (500 GeV), and so on.

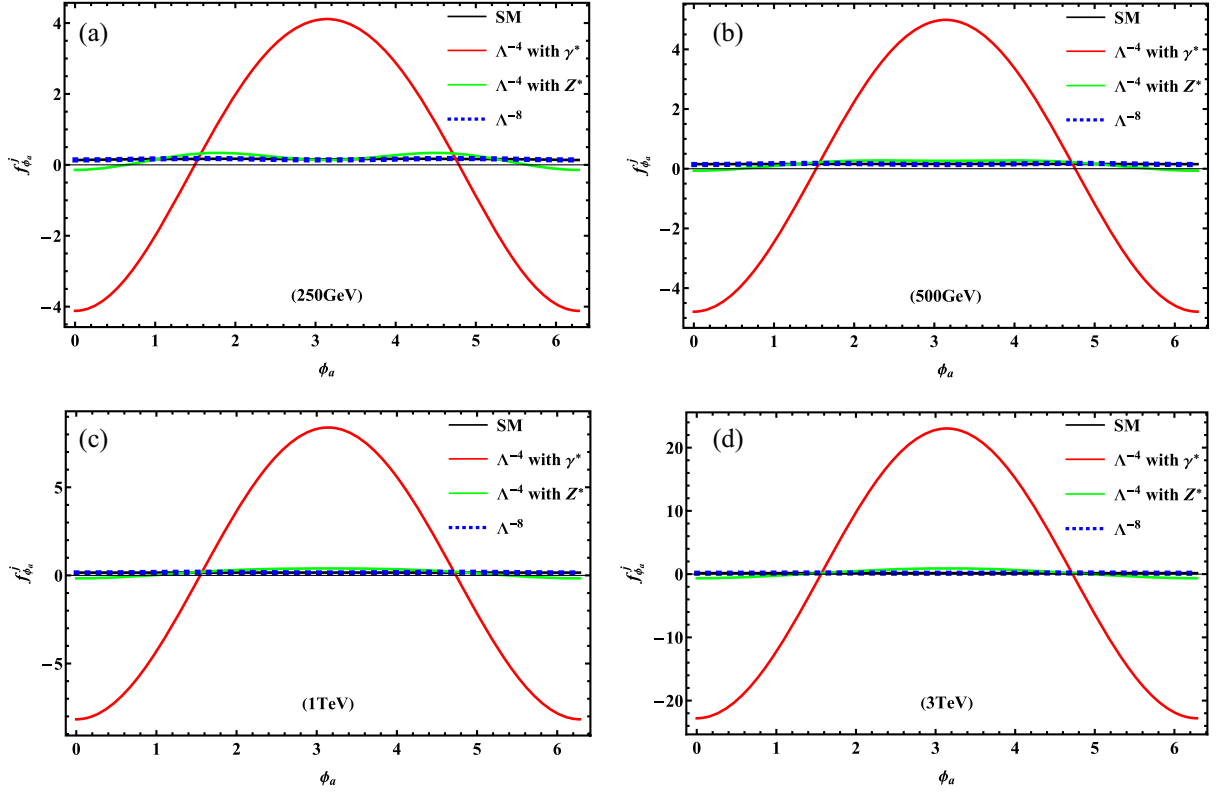


FIG. 6. Normalized angular distributions in the azimuthal angle ϕ_a of down-type quarks for different collider energies, $\sqrt{s} = (0.25, 0.5, 1, 3)$ TeV, are shown in plots (a)–(d), respectively. In each plot, the black, red, green and blue curves denote the contributions from the SM, the $O(\Lambda^{-4})$ interference with γ^* , the $O(\Lambda^{-4})$ interference with Z^* , and the $O(\Lambda^{-8})$ term, respectively, where the blue and black curves nearly overlap. The e^+e^- collision energy chosen for each plot is shown in the parentheses, such as (250 GeV), (500 GeV), and so on.

function $f_{\phi_a}^2$ fluctuates around $\frac{1}{2\pi}$ with a magnitude of only $\frac{1}{16\pi}$, which is why in Fig. 6 the angular functions $f_{\phi_a}^0$ and $f_{\phi_a}^2$ (shown as the black and blue curves) appear fairly flat and largely overlap with each other. In contrast, for the case of the angular function $f_{\phi_a}^1$, the coefficient of $\cos \phi_a$ is enhanced by an energy factor $\sqrt{s}/M_Z$, and can be much larger than the constant term.

IV. PROBING ZZV* COUPLINGS AT e^+e^- COLLIDERS

In this section we analyze systematically the capabilities of ZZ production in e^+e^- collisions followed by fermionic decays. We derive the nTGC signal sensitivity using the differential angular distributions of the cross section with respect to all angles $(\theta, \theta_a, \phi_a, \theta_b, \phi_b)$ for the case of Z bosons decaying to either leptons or quarks in Sec. IV A. Then, in Sec. IV B, we integrate over the angles θ_b and ϕ_b , which may be regarded as decay angles of Z bosons to neutrinos, thereby incorporating the neutrino decay channel of Z bosons. After this, we demonstrate in Secs. IV C and IV D that the signal sensitivity can be significantly enhanced through the implementation of machine learning

and the use of polarized $e^\pm$ beams. Finally, in Sec. IV E we compare our results with those obtained in previous studies that considered the $Z\gamma$ final state.

A. Analyzing ZZ production via leptonic/hadronic channels

For the present analysis, we define the observables,

$$\mathbb{O}_j \equiv \sigma_j \int_{\Omega} d\Omega |f_{\Omega}^j|, \quad (4.1)$$

where the angular distribution function is

$$f_{\Omega}^j = \frac{d^5\sigma_j}{\sigma_j d\theta d\theta_a d\phi_a d\theta_b d\phi_b}, \quad (4.2)$$

and $d\Omega = d\theta d\theta_a d\phi_a d\theta_b d\phi_b$. We note that Eq. (4.1) integrates the absolute value of the angular distribution function f_{Ω}^j over all the phase space, where $|f_{\Omega}^j| = \pm f_{\Omega}^j$ for $f_{\Omega}^j \gtrless 0$. In this way, the angular integration over $|f_{\Omega}^j|$ is enhanced without any cancellations between the $f_{\Omega}^j > 0$ regions and $f_{\Omega}^j < 0$ regions.

Then, we compute the signal significance,

$$\mathcal{Z}(f_a \bar{f}_a f_b \bar{f}_b) = \frac{|S_I|}{\Delta_{B_I}} = \frac{|\mathcal{O}_1(f_a \bar{f}_a f_b \bar{f}_b)|}{\sqrt{\sigma_0(f_a \bar{f}_a f_b \bar{f}_b)}} \times \sqrt{\mathcal{L} \times \epsilon}, \quad (4.3)$$

where $\mathcal{L}$ is the integrated luminosity and ϵ is the detection efficiency. When evaluating the signal significance by using the above formula (4.3), we include only the final-state leptons and quarks, since neutrinos cannot be detected directly. We note that reducible backgrounds contribute to the production process $e^+e^- \rightarrow 4f$. Most of these can be effectively eliminated by applying the Z on shell condition $|M_{ff} - M_Z| < 10$ GeV. Since the final-state fermions are nearly massless, there are certain soft and collinear divergences arising from these reducible backgrounds. The implementation of kinematic cuts on the separation angle $\Delta R(f\bar{f})$ and transverse momentum $P_T(f)$ can significantly suppress the contributions of these reducible backgrounds. Our MadGraph [26] simulations demonstrate that the reducible backgrounds can be reduced to below 5% of the total backgrounds for processes excluding e^-e^+ final states; and to $O(10\%)$ for processes containing e^-e^+ final states. The optimal cuts are process-dependent, varying with both the final-state configurations and collision energies [18]. Notably, the relative contribution of the reducible backgrounds become smaller as the e^+e^- collision energy rises. We further note that the SM backgrounds $\sigma_0(f_a \bar{f}_a f_b \bar{f}_b)$ contribute to the significance (4.3) through the square-root of σ_0 . Hence, for the present analysis, the effects of reducible backgrounds on the significance (4.3) are rather minor and mainly negligible. For this analysis we can focus on the signal extraction from irreducible backgrounds. A dedicated backgrounds simulation including both the irreducible and reducible backgrounds will be beneficial for future detector-level analyses.

The differential angular distribution f_θ^j with respect to the angle θ is presented in Fig. 4, and we observe that the SM background is predominantly concentrated within the forward and backward regions. Furthermore, as the scattering energy increases, the relative contributions of these SM backgrounds within the forward and backward regions

increase. Therefore, imposing a cut on the scattering angle θ (such that $\sin \theta > \sin \delta$) serves to suppress the SM backgrounds, and thus improves the significance of signals.

With the angular cut $\delta = (0, 0.36, 0.33, 0.34, 0.34)$ for $\sqrt{s} = (0.25, 0.5, 1, 3, 5)$ TeV, we derive the results in Table III for the sensitivities to the nTGC form factors (f_5^ℓ, f_5^Z) and the results in Table IV for the sensitivities to the new physics cutoff scales Λ_j of the dimension-8 nTGC operators. In the present analysis, for ease of comparison we choose an integrated luminosity $\mathcal{L} = 5 \text{ ab}^{-1}$ as the universal benchmark [27–34] for all collider energies. In addition, for the e^-e^+ collision energy of 250 GeV, we choose another benchmark of integrated luminosity $\mathcal{L} = 20 \text{ ab}^{-1}$ based on the CEPC TDR [28]. For any other choice of integrated luminosity, one can obtain the corresponding sensitivities by rescaling.

For reference, the FCC-ee Feasibility Study Report [29] quotes a baseline integrated luminosity 10.8 ab^{-1} for the 240 GeV collision energy and another 3.12 ab^{-1} for the (340–350) GeV and 365 GeV collision energies. The proposed project of LCF (Linear Collider Facility) [30] at CERN sets an envisioned integrated luminosity of 3 ab^{-1} for the 250 GeV collision energy and 8 ab^{-1} for the 550 GeV collision energy. The proposed CLIC project [31,32] at CERN plans to collect integrated luminosities of 4.3 ab^{-1} for the 380 GeV (and 350 GeV) collision energies and of 4.0 ab^{-1} for the 1.5 TeV collision energy. The proposed ILC project [34] is expected to deliver an integrated luminosities of 2 ab^{-1} at the 250 GeV collision energy and 4 ab^{-1} at the 500 GeV collision energy, with additional 200 fb^{-1} at the 350 GeV collision energy.

The sensitivity bounds on the new physics scale $\Lambda_{\bar{B}W}$ are consistently stronger than those on Λ_{3Z} , and this advantage grows with increasing scattering energy. Limits on the new physics scale Λ generally strengthen as the scattering energy increases, primarily because the differences between the differential angular distributions of the interference signal f_Ω^1 and the SM background f_Ω^0 become more pronounced at higher scattering energies.

In passing, we note that for the present analysis of ZZ production at the e^+e^- colliders, the dominant

TABLE III. Sensitivity reaches for the nTGC form factors f_5^ℓ and f_5^Z at the 2σ and 5σ levels, as derived by analyzing the reaction $e^-e^+ \rightarrow f_a \bar{f}_a f_b \bar{f}_b$ (with $f = \ell, q$), for a representative integrated luminosity $\mathcal{L} = 5 \text{ ab}^{-1}$ at selected collision energies and an ideal detection efficiency $\epsilon = 100\%$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab^{-1})	$ f_5^\ell _{2\sigma}$	$ f_5^\ell _{5\sigma}$	$ f_5^Z _{2\sigma}$	$ f_5^Z _{5\sigma}$
0.25	20	2.8×10^{-4}	7.0×10^{-4}	3.5×10^{-4}	8.9×10^{-4}
0.25	5	5.6×10^{-4}	1.4×10^{-3}	7.1×10^{-4}	1.8×10^{-3}
0.5	5	5.5×10^{-5}	1.4×10^{-4}	1.2×10^{-4}	2.9×10^{-4}
1	5	1.1×10^{-5}	2.9×10^{-5}	2.9×10^{-5}	7.3×10^{-5}
3	5	1.2×10^{-6}	3.0×10^{-6}	3.4×10^{-6}	8.4×10^{-6}
5	5	4.3×10^{-7}	1.1×10^{-6}	1.2×10^{-6}	3.1×10^{-6}

TABLE IV. Sensitivity reaches on the new physics scales Λ_{BW} and Λ_{3Z} (in TeV) via the reaction $e^-e^+ \rightarrow f_a \bar{f}_a f_b \bar{f}_b$, ($f = \ell, q$). The sensitivity reaches are shown at the 2σ and 5σ levels and for different collider energies. For illustration, we assume a representative integrated luminosity $\mathcal{L} = 5 \text{ ab}^{-1}$ and an ideal detection efficiency $\epsilon = 100\%$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab^{-1})	$\Lambda_{BW}^{2\sigma}$	$\Lambda_{BW}^{5\sigma}$	$\Lambda_{3Z}^{2\sigma}$	$\Lambda_{3Z}^{5\sigma}$
0.25	20	1.2	.96	.96	.76
0.25	5	1.0	.81	.81	.64
0.5	5	1.8	1.4	1.3	1.0
1	5	2.7	2.1	1.8	1.4
3	5	4.7	3.8	3.1	2.4
5	5	6.1	4.9	4.0	3.1

contribution comes from the interference term between the SM and the nTGC contributions, whereas the contribution of the quadratic nTGC term is negligibly small. For instance, we find that when the contribution of the interference term reaches a significance $\mathcal{Z}_{\text{int}} = 2$, the significance contributed by the quadratic term is only $\mathcal{Z}_{\text{qua}}(f_5^\ell) = (0.006, 0.009, 0.016, 0.047)$ and $\mathcal{Z}_{\text{qua}}(f_5^Z) = (0.003, 0.014, 0.036, 0.13)$ for $\sqrt{s} = (0.25, 0.5, 1, 3) \text{ TeV}$, showing that the contribution of the interference term always dominates the sensitivity.

B. Analyzing the invisible decay channel $Z \rightarrow \nu\bar{\nu}$

To compute the nTGC signal significance for the reaction $e^-e^+ \rightarrow ZZ \rightarrow f\bar{f}\nu\bar{\nu}$ (with $f = \ell, q$), we consider θ_b and ϕ_b to be the angles of the decay of one Z boson into neutrinos and integrate over them. Then, we compute the signal significance, with θ_a and ϕ_a being the angles of the other Z boson decays into leptons or quarks.

For this analysis, we define the following observable:

$$\mathcal{O}_j^n \equiv \sigma_j \int_{\Omega'} d\Omega' |f_{\Omega'}^j|, \quad (4.4)$$

where the superscript “n” of $\mathcal{O}_j^n$ denotes one of the final state Z bosons decaying into neutrinos, and $d\Omega' = d\theta d\theta_a d\phi_a$. In the above, the partially integrated angular distribution

function $f_{\Omega'}^j$ is given by

$$f_{\Omega'}^j = \int f_{\Omega}^j d\theta_b d\phi_b. \quad (4.5)$$

With these, we compute the nTGC signal significance,

$$\mathcal{Z}_n(f_a \bar{f}_a) = \frac{|S_I|}{\Delta_{B_I}} = \frac{|\mathcal{O}_1^n(f_a \bar{f}_a)|}{\sqrt{\sigma_0(f_a \bar{f}_a)}} \times \sqrt{\mathcal{L} \times \epsilon}. \quad (4.6)$$

We then derive the total signal significance for probing the nTGCs,

$$\mathcal{Z}_{\text{tot}} = \left[\sum_{(a,b) \in (\ell,q)} \mathcal{Z}^2(f_a \bar{f}_a f_b \bar{f}_b) + \sum_{a \in (\ell,q)} \mathcal{Z}_n^2(f_a \bar{f}_a) \right]^{\frac{1}{2}}, \quad (4.7)$$

by using the angular cut $\delta_n = (0.02, 0.30, 0.31, 0.31, 0.33)$ for $\mathcal{Z}_n$ at each e^+e^- collider energy $\sqrt{s} = (0.25, 0.5, 1, 3, 5) \text{ TeV}$. We present our findings for the sensitivity reaches on the nTGC form factors (f_5^ℓ, f_5^Z) in Tables V and VI, and on the nTGC new physics scales in Table VII.

It can be seen that the limits on $\Lambda^{\nu\bar{\nu}}$ are less stringent than those on $\Lambda^{f\bar{f}}$. This is due to the lower branching ratio of the Z boson decays to neutrinos as compared to its decays to charged fermions (including leptons and quarks),

$$\text{Br}[Z \rightarrow \nu\bar{\nu}] < \text{Br}[Z \rightarrow \ell\bar{\ell}, q\bar{q}], \quad (4.8)$$

and the absence of decay angle information. Consequently, incorporating the contribution from the neutrino decay channel of the Z boson yields only a modest improvement in the limit on the new physics scale Λ , as compared to the results obtained in Sec. IVA that include only Z boson decays to charged fermions.

C. Probability cuts from machine learning

In this section we explore the use of machine learning (ML) to improve the signal significance. ML has become a

TABLE V. Sensitivity reaches for the nTGC form factor f_5^ℓ at the $(2\sigma, 5\sigma)$ level, as derived by analyzing the reactions $e^-e^+ \rightarrow f_a \bar{f}_a f_b \bar{f}_b$, $e^-e^+ \rightarrow f_a \bar{f}_a \nu\bar{\nu}$ (with $f = \ell, q$) and their combination at selected collision energies, for a representative integrated luminosity $\mathcal{L} = 5 \text{ ab}^{-1}$ and an ideal detection efficiency $\epsilon = 100\%$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab^{-1})	$ f_5^\ell _{f\bar{f}}$	$ f_5^\ell _{\nu\bar{\nu}}$	$ f_5^\ell _{\text{tot}}$
0.25	20	$(2.8, 7.0) \times 10^{-4}$	$(.63, 1.6) \times 10^{-3}$	$(2.6, 6.4) \times 10^{-4}$
0.25	5	$(.56, 1.4) \times 10^{-3}$	$(1.3, 3.2) \times 10^{-3}$	$(.51, 1.3) \times 10^{-3}$
0.5	5	$(.55, 1.4) \times 10^{-4}$	$(1.2, 2.9) \times 10^{-4}$	$(.50, 1.2) \times 10^{-4}$
1	5	$(1.1, 2.9) \times 10^{-5}$	$(2.4, 6.0) \times 10^{-5}$	$(1.0, 2.6) \times 10^{-5}$
3	5	$(1.2, 3.0) \times 10^{-6}$	$(2.5, 6.3) \times 10^{-6}$	$(1.1, 2.7) \times 10^{-6}$
5	5	$(.43, 1.1) \times 10^{-6}$	$(.90, 2.3) \times 10^{-6}$	$(3.9, 9.7) \times 10^{-7}$

TABLE VI. Sensitivity reaches for the nTGC form factor f_5^Z at the $(2\sigma, 5\sigma)$ level, as derived by analyzing the reactions $e^-e^+ \rightarrow f_a \bar{f}_a f_b \bar{f}_b$, $e^-e^+ \rightarrow f_a \bar{f}_a \nu \bar{\nu}$ (with $f = \ell, q$) and their combination at selected collision energies, for a representative integrated luminosity $\mathcal{L} = 5 \text{ ab}^{-1}$ and an ideal detection efficiency $\epsilon = 100\%$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab^{-1})	$ f_5^Z _{f\bar{f}}$	$ f_5^Z _{\nu\bar{\nu}}$	$ f_5^Z _{\text{tot}}$
0.25	20	$(3.5, 8.9) \times 10^{-4}$	$(.57, 1.4) \times 10^{-3}$	$(3.0, 7.5) \times 10^{-4}$
0.25	5	$(.71, 1.8) \times 10^{-3}$	$(1.1, 2.9) \times 10^{-3}$	$(.60, 1.5) \times 10^{-3}$
0.5	5	$(1.2, 2.9) \times 10^{-4}$	$(2.3, 5.9) \times 10^{-4}$	$(1.0, 2.6) \times 10^{-4}$
1	5	$(2.9, 7.3) \times 10^{-5}$	$(.67, 1.7) \times 10^{-4}$	$(2.7, 6.7) \times 10^{-5}$
3	5	$(3.4, 8.4) \times 10^{-6}$	$(.86, 2.2) \times 10^{-5}$	$(3.1, 7.8) \times 10^{-6}$
5	5	$(1.2, 3.1) \times 10^{-6}$	$(3.2, 8.0) \times 10^{-6}$	$(1.1, 2.9) \times 10^{-6}$

TABLE VII. Sensitivity reaches on the new physics scales Λ_{BW} and Λ_{3Z} (in TeV) through the reactions $e^-e^+ \rightarrow f_a \bar{f}_a f_b \bar{f}_b$ and $e^-e^+ \rightarrow f_a \bar{f}_a \nu \bar{\nu}$ (with $f = \ell, q$) plus their combination. The bounds are shown at the $(2\sigma, 5\sigma)$ level and for different e^+e^- collider energies. For illustration, we choose a representative integrated luminosity $\mathcal{L} = 5 \text{ ab}^{-1}$ and an ideal detection efficiency $\epsilon = 100\%$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab^{-1})	$\Lambda_{BW}^{f\bar{f}}$	$\Lambda_{BW}^{\nu\bar{\nu}}$	$\Lambda_{BW}^{\text{tot}}$	$\Lambda_{3Z}^{f\bar{f}}$	$\Lambda_{3Z}^{\nu\bar{\nu}}$	$\Lambda_{3Z}^{\text{tot}}$
0.25	20	(1.2, .96)	(.99, .78)	(1.2, .98)	(.96, .76)	(.85, .68)	(1.0, .79)
0.25	5	(1.0, .81)	(.83, .66)	(1.0, .83)	(.81, .64)	(.72, .57)	(.84, .67)
0.5	5	(1.8, 1.4)	(1.5, 1.2)	(1.9, 1.5)	(1.3, 1.0)	(1.1, .85)	(1.3, 1.0)
1	5	(2.7, 2.1)	(2.2, 1.8)	(2.8, 2.2)	(1.8, 1.4)	(1.5, 1.2)	(1.8, 1.5)
3	5	(4.7, 3.8)	(3.9, 3.1)	(4.8, 3.9)	(3.1, 2.4)	(2.4, 1.9)	(3.1, 2.5)
5	5	(6.1, 4.9)	(5.1, 4.0)	(6.3, 5.0)	(4.0, 3.1)	(3.1, 2.5)	(4.0, 3.2)

transformative tool in collider physics, addressing challenges in data analysis, pattern recognition, and theoretical modeling [35]. ML algorithms (such as boosted decision trees, deep neural networks, nearest neighbor analysis, and support vector machines) are widely used to distinguish between rare particle collision signals and large background noise sources. These methods optimize event classification by learning complex correlations in high-dimensional datasets.

For this analysis, we employ the Mathematica's built-in function `Classify` to perform machine learning for distinguishing between the nTGC signals and the SM background events. The `Classify` function incorporates many machine learning algorithms and automatically identifies the optimal algorithm for the specific classification task to deliver the optimized performance. This implementation operates autonomously without requiring manual hyperparameter tuning.

The core methodology is to use large samples of (labeled) simulated scattering events (explicitly tagged as belonging to the signal or background categories), which we use to train a classification algorithm to discover automatically optimal criteria for distinguishing between the signal and backgrounds on the basis of measurements of physical quantities such as energies, momenta, angles, and their combinations within the events. After training, the classification algorithm functions as a classifier, which computes the probability that any experimental event belongs to the signal or background category.

We proceed by dividing the phase space of the final states into a positive region R_+ and a negative region R_- based on the signs of the angular distribution f_Ω^1 in Eq. (4.2) or f_Ω^1 in Eq. (4.5),

$$R_+ : f_\Omega^1 > 0, \quad \text{or} \quad f_{\Omega'}^1 > 0, \quad (4.9a)$$

$$R_- : f_\Omega^1 < 0, \quad \text{or} \quad f_{\Omega'}^1 < 0. \quad (4.9b)$$

Subsequently, we categorize the signal S and background B into positive dataset (S_+, B_+) and negative dataset (S_-, B_-) according to R_+ and R_- .

In this theoretical study, we generate events in Mathematica and then leverage its built-in ML function `Classify` with variables $(\theta, \theta_a, \theta_b, \phi_a, \phi_b)$ to distinguish the nTGC signals from SM backgrounds. For the positive portion of the signal and background dataset $(S_+$ and $B_+)$, we train a classifier $C_{\text{sig-bkg}}^+$ capable of distinguishing between signals and backgrounds, and providing the probability that each event belongs to either category. We apply the same approach to the negative dataset using a classifier $C_{\text{sig-bkg}}^-$. For instance, we illustrate in Figs. 7 and 8 the probability distributions for the collider energies $\sqrt{s} = 250 \text{ GeV}$ and $\sqrt{s} = 3 \text{ TeV}$, respectively. Comparing Fig. 7 with Fig. 8, we see that probability distributions significantly change as the collider energy increases from $\sqrt{s} = 250 \text{ GeV}$ to $\sqrt{s} = 3 \text{ TeV}$. Furthermore, it is shown

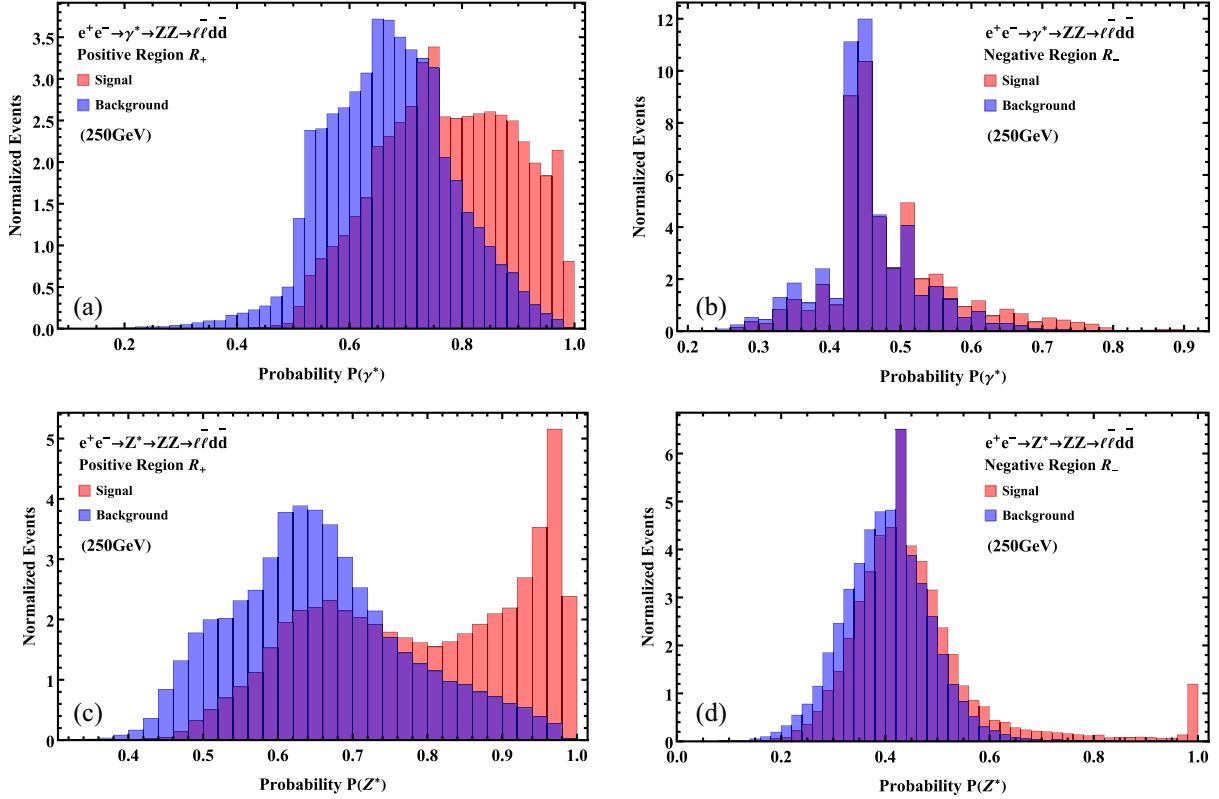


FIG. 7. Probability distributions of events being signals and backgrounds respectively, for the case of $e^+e^- \rightarrow V^* \rightarrow ZZ \rightarrow (\ell\bar{\ell})(d\bar{d})$ at a given collider energy $\sqrt{s} = 250$ GeV [marked as (250 GeV) in each plot]. Here we choose the particular hadronic decay channel $Z \rightarrow d\bar{d}$ [with d denoting the down-type quarks (d, s, b)] for illustration. Plots (a) and (b) present the probability distributions of events for the angular regions R_+ and R_- with the γ^* contributions, respectively; plots (c) and (d) show the probability distributions of events for the angular regions R_+ and R_- with the Z^* contributions, respectively.

that the higher collider energy can help to raise the probability of discriminating the signals from backgrounds.

In order to achieve higher sensitivity, we discriminate between signals and backgrounds by using the event probability distributions that are computed via ML. We divide events into bins of the probability distribution, whose widths are chosen as $\Delta P = 0.1$. We compute the significance $\mathcal{Z}_{\text{bin}}$ for each bin, and construct the following significance measure:

$$\mathcal{Z} = \sqrt{\sum \mathcal{Z}_{\text{bin}}^2}. \quad (4.10)$$

Finally, we compute the total signal significance $\mathcal{Z}_{\text{tot}}$ by combining the significances from both the positive region $\mathcal{Z}_+$ and negative region $\mathcal{Z}_-$:

$$\mathcal{Z}_{\text{tot}} = \sqrt{\mathcal{Z}_+^2 + \mathcal{Z}_-^2}. \quad (4.11)$$

Employing this approach, we have derived the sensitivity reaches on the nTGC form factors in Table VIII and on the nTGC new physics cutoff scales in Table IX.

We find that after applying machine learning to distinguish nTGC signals and SM backgrounds, the phenomenological constraints on the new physics cutoff scale

Λ are significantly enhanced as compared to the results prior to using machine learning. Specifically, in comparison with the results presented in Table V and Table VI of Sec. IV B, machine learning enhances the constraints on the form factor f_5^Z across different collider energies by 19% to 25%. The smallest enhancement of 19% is realized at $\sqrt{s} = 500$ GeV, whereas the largest enhancement of 25% occurs at $\sqrt{s} = 5$ TeV. Similarly, machine learning improves the constraints on the form factor f_5^Z by about (20–35)%. Here the smallest enhancement of 20% is found at the collider energy $\sqrt{s} = 1$ TeV. Conversely, the strongest enhancement of 35% is achieved at the collider energy $\sqrt{s} = 250$ GeV.

Machine learning also improves the sensitivity reaches for the new physics scale $\Lambda_{\tilde{B}W}$ across various collider energies by about (5.6–7.5)%. The smallest improvement of 5.6% is realized at $\sqrt{s} = 500$ GeV, whereas the largest improvement of 7.5% is achieved at $\sqrt{s} = 5$ TeV. Furthermore, machine learning improves the sensitivity reaches for Λ_{3Z} by about (5.7–11)% across different collider energies. Here the smallest improvement of 5.7% occurs at $\sqrt{s} = 1$ TeV, and the largest improvement of 11% is found at $\sqrt{s} = 250$ GeV.

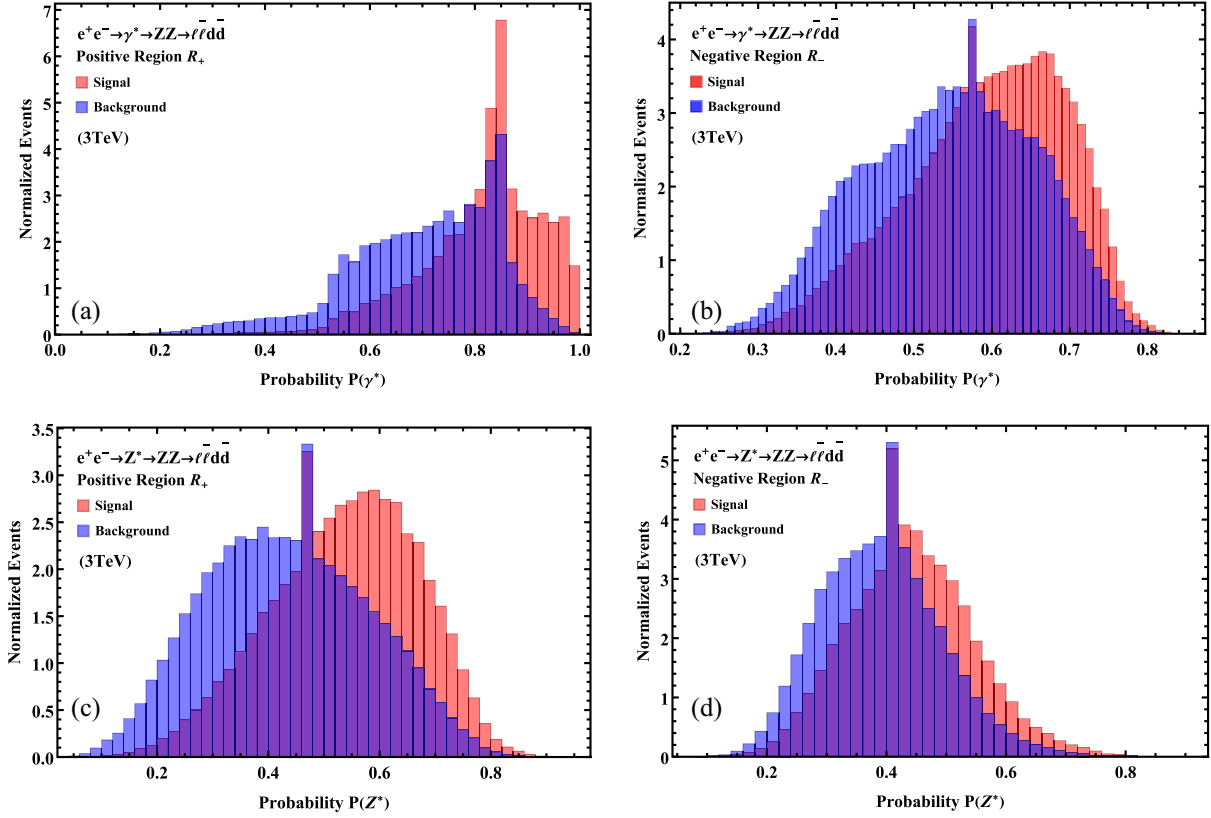


FIG. 8. Probability distributions of events being signals and backgrounds respectively, for the case of $e^+e^- \rightarrow V^* \rightarrow ZZ \rightarrow (\ell\bar{\ell})(d\bar{d})$ at a given collider energy $\sqrt{s} = 3$ TeV [marked as (3 TeV) in each plot]. Here we choose the particular hadronic decay channel $Z \rightarrow d\bar{d}$ [with d denoting the down-type quarks (d, s, b)] for illustration. Plots (a) and (b) present the probability distributions of events for the angular regions R_+ and R_- with the γ^* contributions, respectively; plots (c) and (d) show the probability distributions of events for the angular regions R_+ and R_- with the Z^* contributions, respectively.

TABLE VIII. Sensitivities to the nTGC form factors f_5^γ and f_5^Z at the 2σ and 5σ level after using ML, as derived by analyzing the combination of $e^-e^+ \rightarrow f_a\bar{f}_af_b\bar{f}_b$ and $e^-e^+ \rightarrow f_a\bar{f}_a\nu\bar{\nu}$ (with $f = \ell, q$), with an ideal detection efficiency $\epsilon = 100\%$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab^{-1})	$ f_5^\gamma _{2\sigma}$	$ f_5^\gamma _{5\sigma}$	$ f_5^Z _{2\sigma}$	$ f_5^Z _{5\sigma}$
0.25	20	2.0×10^{-4}	4.9×10^{-4}	2.0×10^{-4}	4.9×10^{-4}
0.25	5	3.9×10^{-4}	9.8×10^{-4}	3.9×10^{-4}	9.8×10^{-4}
0.5	5	4.0×10^{-5}	1.0×10^{-4}	8.2×10^{-5}	2.1×10^{-4}
1	5	8.3×10^{-6}	2.1×10^{-5}	2.1×10^{-5}	5.3×10^{-5}
3	5	8.4×10^{-7}	2.1×10^{-6}	2.5×10^{-6}	6.2×10^{-6}
5	5	2.9×10^{-7}	7.3×10^{-7}	8.6×10^{-7}	2.2×10^{-6}

TABLE IX. Sensitivity reaches for the new physics scales Λ_{BW} and Λ_{3Z} (in TeV) via the combination of $e^-e^+ \rightarrow f_a\bar{f}_af_b\bar{f}_b$ and $e^-e^+ \rightarrow f_a\bar{f}_a\nu\bar{\nu}$ (with $f = \ell, q$) after using ML. The ranges are shown at the 2σ and 5σ level and for different energies. For illustration, we choose a representative integrated luminosity $\mathcal{L} = 5 \text{ ab}^{-1}$ and an ideal detection efficiency $\epsilon = 100\%$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab^{-1})	$\Lambda_{BW}^{\text{tot}, 2\sigma}$	$\Lambda_{BW}^{\text{tot}, 5\sigma}$	$\Lambda_{3Z}^{\text{tot}, 2\sigma}$	$\Lambda_{3Z}^{\text{tot}, 5\sigma}$
0.25	20	1.3	1.1	1.1	.88
0.25	5	1.1	.88	.93	.74
0.5	5	2.0	1.6	1.4	1.1
1	5	2.9	2.3	1.9	1.5
3	5	5.2	4.1	3.3	2.6
5	5	6.7	5.4	4.3	3.4

TABLE X. Sensitivity reaches for the nTGC form factors f_5^γ and f_5^Z at the 2σ and 5σ level, as derived by analyzing the combination of $e^-e^+ \rightarrow f_a \bar{f}_a f_b \bar{f}_b$ and $e^-e^+ \rightarrow f_a \bar{f}_a \nu \bar{\nu}$ (with $f = \ell, q$), for (unpolarized, polarized) $e^\pm$ beams in each entry, with the $e^\pm$ beam polarizations $(P_L^e, P_R^e) = (0.9, 0.65)$ and an ideal detection efficiency $\epsilon = 100\%$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab $^{-1}$)	$ f_5^\gamma _{2\sigma}$ (unpol, pol)	$ f_5^\gamma _{5\sigma}$ (unpol, pol)	$ f_5^Z _{2\sigma}$ (unpol, pol)	$ f_5^Z _{5\sigma}$ (unpol, pol)
0.25	20	$(2.0, .96) \times 10^{-4}$	$(4.9, 2.4) \times 10^{-4}$	$(2.0, .90) \times 10^{-4}$	$(4.9, 2.3) \times 10^{-4}$
0.25	5	$(3.9, 1.9) \times 10^{-4}$	$(9.8, 4.8) \times 10^{-4}$	$(3.9, 1.8) \times 10^{-4}$	$(9.8, 4.5) \times 10^{-4}$
0.5	5	$(4.0, 3.2) \times 10^{-5}$	$(1.0, .81) \times 10^{-4}$	$(8.2, 5.2) \times 10^{-5}$	$(2.1, 1.3) \times 10^{-4}$
1	5	$(8.3, 7.0) \times 10^{-6}$	$(2.1, 1.7) \times 10^{-5}$	$(2.1, 1.2) \times 10^{-5}$	$(5.3, 3.0) \times 10^{-5}$
3	5	$(8.4, 7.6) \times 10^{-7}$	$(2.1, 1.9) \times 10^{-6}$	$(2.5, 1.3) \times 10^{-6}$	$(6.2, 3.2) \times 10^{-6}$
5	5	$(2.9, 2.6) \times 10^{-7}$	$(7.3, 6.4) \times 10^{-7}$	$(8.6, 4.2) \times 10^{-7}$	$(2.2, 1.1) \times 10^{-6}$

TABLE XI. Sensitivity reaches for the new physics cutoff scales Λ_{BW} and Λ_{3Z} (in TeV) through the combination of $e^-e^+ \rightarrow f_a \bar{f}_a f_b \bar{f}_b$ and $e^-e^+ \rightarrow f_a \bar{f}_a \nu \bar{\nu}$ (with $f = \ell, q$), with (unpolarized, polarized) $e^\pm$ beams in each entry. The bounds are presented at the 2σ and 5σ levels and for different collision energies. For illustration, we choose a representative integrated luminosity $\mathcal{L} = 5$ ab $^{-1}$, an ideal detection efficiency $\epsilon = 100\%$, and the $e^\pm$ beam polarizations $(P_L^e, P_R^e) = (0.9, 0.65)$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab $^{-1}$)	$\Lambda_{BW}^{\text{tot}, 2\sigma}$ (unpol, pol)	$\Lambda_{BW}^{\text{tot}, 5\sigma}$ (unpol, pol)	$\Lambda_{3Z}^{\text{tot}, 2\sigma}$ (unpol, pol)	$\Lambda_{3Z}^{\text{tot}, 5\sigma}$ (unpol, pol)
0.25	20	(1.3, 1.6)	(1.1, 1.3)	(1.1, 1.3)	(.88, 1.1)
0.25	5	(1.1, 1.3)	(.88, 1.1)	(.93, 1.1)	(.74, .90)
0.5	5	(2.0, 2.1)	(1.6, 1.6)	(1.4, 1.5)	(1.1, 1.2)
1	5	(2.9, 3.0)	(2.3, 2.4)	(1.9, 2.2)	(1.5, 1.8)
3	5	(5.2, 5.3)	(4.1, 4.2)	(3.3, 3.9)	(2.6, 3.1)
5	5	(6.7, 6.9)	(5.4, 5.5)	(4.3, 5.2)	(3.4, 4.1)

D. Improvements from polarized electron and positron beams

In this subsection, we study the role of polarized e^- and e^+ beams, assuming $(P_L^e, P_R^e) = (0.9, 0.65)$ for the fractions of (left, right)-handed (electrons, positrons) in the (e^-, e^+) beams [30,36], respectively.³ We present the sensitivity reaches on the nTGC form factors (f_5^γ, f_5^Z) in Table X and on the nTGC new physics scales Λ_j in Tables XI, where we compare the sensitivity reaches in the (unpolarized, polarized) cases in each entry.

Compared to the unpolarized results, polarized $e^\pm$ beams enhance the sensitivity reaches on the form factor f_5^γ across various e^-e^+ collider energies by (8.4–51)%. The smallest enhancement of 8.4% is realized at $\sqrt{s} = 3$ TeV, whereas the largest enhancement of 51% is achieved at $\sqrt{s} = 250$ GeV. Moreover, polarized $e^\pm$ beams improve the sensitivity bounds on the form factor f_5^Z by about (36–54)% across different collider energies.

³The degree of longitudinal beam polarization for electron e^- or positron e^+ is defined as $\hat{P} = P_R - P_L$ [36]. Because the sum of left-handed and right-handed fractions equals one ($P_L + P_R = 1$), the left-handed and right-handed fractions of e^- and e^+ can be derived as follows: $P_{L,R}^e = \frac{1}{2}(1 \mp \hat{P}^e)$ and $P_{L,R}^{\bar{e}} = \frac{1}{2}(1 \mp \hat{P}^{\bar{e}})$. For example, an unpolarized beam of e^- or e^+ has a vanishing degree of polarization $\hat{P} = 0$, whereas a polarized e^- beam with a fraction $P_L^e = 90\%$ has $\hat{P}^e = -0.8$ and a polarized e^+ beam with a fraction $P_R^e = 65\%$ has $\hat{P}^e = 0.3$.

Here the smallest enhancement of 36% is found at the collision energy $\sqrt{s} = 500$ GeV, and the largest enhancement of 54% occurs at the collision energy $\sqrt{s} = 250$ GeV.

We find that in most scenarios the beam polarizations can enhance Λ_{BW} by about 0.2 TeV as compared to the unpolarized case. However, the improvement of the sensitivity reaches for the new physics scale Λ_{3Z} facilitated by the polarized $e^\pm$ beams is significantly more pronounced. Specifically, relative to the unpolarized results, the polarized $e^\pm$ beams can improve the sensitivity reaches for Λ_{3Z} across different collision energies by about (12–21)%. The smallest improvement of 12% occurs at the collision energy $\sqrt{s} = 0.5$ TeV, whereas the largest improvement of 21% is achieved at the collision energy $\sqrt{s} = 0.25$ TeV.

Finally, we note that each e^+e^- collider will first operate with unpolarized $e^\pm$ beams and then operate with polarized $e^\pm$ beams. Thus, it is desired to consider an (ideally) mixed setting where one half of the total data is taken from the first-phase operation with unpolarized beams and the other half of the total data is from the second-phase operation with polarized beams. This mixed setting requires to combine the sensitivity bound using the unpolarized data with the sensitivity bound using the polarized data. We present the sensitivity reaches on the nTGC form factors (f_5^γ, f_5^Z) and the corresponding nTGC new physics scales $(\Lambda_{3Z}, \Lambda_{BW})$ in Table XII. It shows that the sensitivity bounds in the mixed setting always lie between the sensitivity bounds of the unpolarized and polarized cases.

TABLE XII. Same as Tables X–XI for the 2σ sensitivity bounds, but the sensitivity reaches for the mixed setting (including half of the data taken from the unpolarized operation and the other half of the data taken from the polarized operation) with beam polarizations $(P_L^e, P_R^e) = (0.9, 0.65)$.

$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab $^{-1}$)	$ f_5^\gamma _{2\sigma}$ (unpol, pol, mixed)	$ f_5^Z _{2\sigma}$ (unpol, pol, mixed)	$\Lambda_{BW}^{\text{tot}, 2\sigma}$ (unpol, pol, mixed)	$\Lambda_{3Z}^{\text{tot}, 2\sigma}$ (unpol, pol, mixed)
0.25	20	$(2.0, .96, 1.2) \times 10^{-4}$	$(2.0, .90, 1.2) \times 10^{-4}$	(1.3, 1.6, 1.5)	(1.1, 1.3, 1.3)
0.25	5	$(3.9, 1.9, 2.4) \times 10^{-4}$	$(3.9, 1.8, 2.3) \times 10^{-4}$	(1.1, 1.3, 1.3)	(.93, 1.1, 1.1)
0.5	5	$(4.0, 3.2, 3.6) \times 10^{-5}$	$(8.2, 5.2, 6.3) \times 10^{-5}$	(2.0, 2.1, 2.0)	(1.4, 1.5, 1.5)
1	5	$(8.3, 7.0, 7.5) \times 10^{-6}$	$(2.1, 1.2, 1.5) \times 10^{-5}$	(2.9, 3.0, 3.0)	(1.9, 2.2, 2.1)
3	5	$(8.4, 7.6, 8.0) \times 10^{-7}$	$(2.5, 1.3, 1.6) \times 10^{-6}$	(5.2, 5.3, 5.2)	(3.3, 3.9, 3.7)
5	5	$(2.9, 2.6, 2.7) \times 10^{-7}$	$(8.6, 4.2, 5.3) \times 10^{-7}$	(6.7, 6.9, 6.8)	(4.3, 5.2, 4.9)

As we will demonstrate in Sec. IV E, the mixed setting is important for optimizing the correlation bounds (contours) for each pair of nTGC parameters by avoiding certain poorly constrained parameter space in the case of the polarized beams.

E. Analyzing correlations of nTGC parameters by machine learning

In this subsection, we use machine learning to analyze correlations between two nTGC form factors or two dimension-8 nTGC operators. We find that using the machine learning method can significantly improve the sensitivities of probes of correlations between the nTGC parameters.

We divide the phase space of the final states into four regions based on the signs of the angular distribution f_Ω^1 in Eq. (4.2) or $f_{\Omega'}^1$ in Eq. (4.5),

$$R_{++}: (f_\Omega^1(\gamma) > 0, f_\Omega^1(Z) > 0), \quad \text{or}, \quad (f_{\Omega'}^1(\gamma) > 0, f_{\Omega'}^1(Z) > 0), \quad (4.12a)$$

$$R_{+-}: (f_\Omega^1(\gamma) > 0, f_\Omega^1(Z) < 0), \quad \text{or}, \quad (f_{\Omega'}^1(\gamma) > 0, f_{\Omega'}^1(Z) < 0), \quad (4.12b)$$

$$R_{-+}: (f_\Omega^1(\gamma) < 0, f_\Omega^1(Z) > 0), \quad \text{or}, \quad (f_{\Omega'}^1(\gamma) < 0, f_{\Omega'}^1(Z) > 0), \quad (4.12c)$$

$$R_{--}: (f_\Omega^1(\gamma) < 0, f_\Omega^1(Z) < 0), \quad \text{or}, \quad (f_{\Omega'}^1(\gamma) < 0, f_{\Omega'}^1(Z) < 0). \quad (4.12d)$$

Within these four regions, we can obtain the probability of each event belonging to the γ^* signal or Z^* signal category. These probabilities $P(\gamma^*)$ and $P(Z^*)$ are provided by classifiers trained specifically based on each dataset in the corresponding region.

In Fig. 9, we present the distribution of probabilities $P(\gamma^*)$ and $P(Z^*)$ of the simulated events belonging to the γ^* signal or Z^* signal category within the four regions of Eq. (4.12) for the case of $e^+e^- \rightarrow V^* \rightarrow ZZ \rightarrow (\ell\bar{\ell})(d\bar{d})$ at a collider energy $\sqrt{s} = 250$ GeV. For comparison, we present in Fig. 10 an analogous set of plots of $P(\gamma^*)$ and $P(Z^*)$ for a different e^+e^- collider energy $\sqrt{s} = 3$ TeV. For illustration, we choose in each plot a sample value of the nTGC cutoff scales $\Lambda_{3Z} = \Lambda_{BW} = 1$ TeV, which corresponds to the input of nTGC form factors $(f_5^\gamma, f_5^Z) = (6, 3) \times 10^{-4}$ according to Eqs. (2.8) and (2.9). As shown in Figs. 4 and 6 for the single-observable distributions, we see that the differences between the angular distributions (in θ and ϕ_a) of signals and backgrounds become much larger as the collision energy $\sqrt{s}$ increases. Hence, it is expected that using the machine learning algorithm we can separate the

signals (red and green points) from the SM backgrounds (blue points) much more efficiently at a higher collision energy (such as $\sqrt{s} = 3$ TeV) than at a lower collision energy (such as $\sqrt{s} = 250$ GeV). This enhanced discriminative power of signal-background separation can be seen by comparing the plots of Figs. 9 and 10.

We divide events into bins of the probability distribution, whose widths we take as $\Delta P(\gamma^*) = 0.1$ and $\Delta P(Z^*) = 0.1$. Then, we combine the contributions of two nTGC form factors into a global χ^2 function,

$$\chi^2 = \sum_{\text{region}} \sum_{\text{bin}} \frac{(\sigma_1^\gamma f_5^\gamma + \sigma_1^Z f_5^Z)^2}{\sigma_0} \times \mathcal{L} \times \epsilon. \quad (4.13)$$

Minimizing the χ^2 function, we derive correlated constraints on each pair of nTGC form factors and each pair of nTGC cutoff scales. We present these correlated constraints in Fig. 11 as 2σ contours, where the contours of plots (a) and (c) depict the correlations between the form factors (f_5^γ, f_5^Z) , whereas the contours of plots (b) and (d) demonstrate the correlations between the new physics scales $(\Lambda_{BW}', \Lambda_{3Z})$.

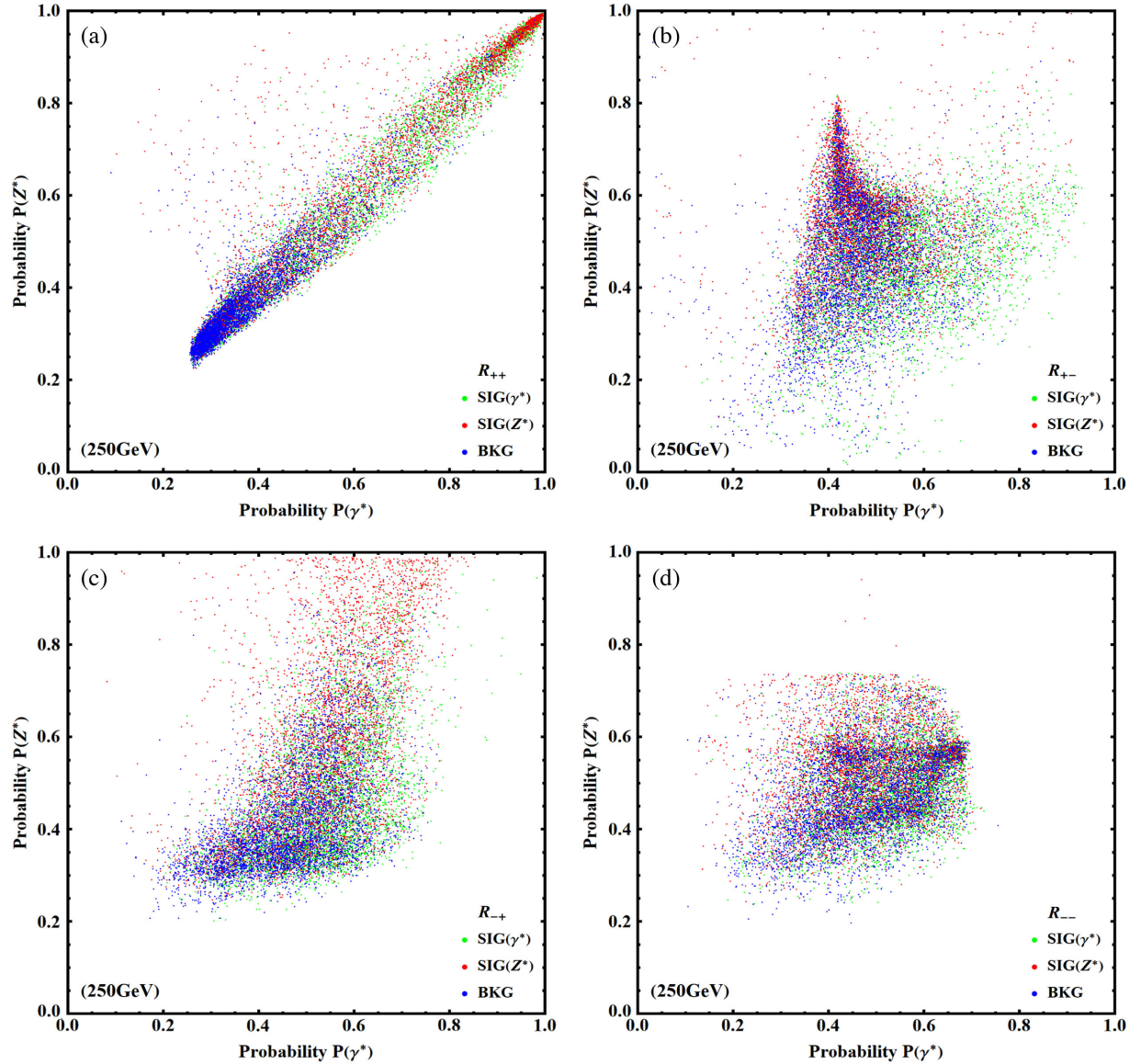


FIG. 9. Distribution of probabilities $P(\gamma^*)$ and $P(Z^*)$ of simulated events belonging to γ^* signal or Z^* signal category within the four regions of Eq. (4.12) for the case of $e^+e^- \rightarrow V^* \rightarrow ZZ \rightarrow (\ell\bar{\ell})(d\bar{d})$ at a collider energy $\sqrt{s} = 250$ GeV [marked as (250 GeV) in every plot], where d denotes the down-type quarks (d, s, b). The (green, red, blue) points in each plot represent the (γ^* , Z^* , SM) contributions, respectively. Plots (a)–(d) present the probability distributions of events for the angular regions R_{++} , R_{+-} , R_{-+} , and R_{--} , respectively.

In each plot of Fig. 11, we compare the correlation bounds for the case of unpolarized $e^\pm$ beams (shown as the blue contour) and the case of polarized $e^\pm$ beams (shown as the red contour). For the polarized case, we choose the $e^\pm$ beam polarizations $(P_L^e, P_R^e) = (0.9, 0.65)$. Figure 11 shows that using the beam polarizations can significantly tighten each correlation contour along the minor axis of the ellipse by about 50%, but makes the correlation bound much weaker along the major axis of the ellipse (which is orientated towards the northwestern direction in the parameter plane). Similar features hold for the correlation bounds in the parameter space of the cutoff scales $(\Lambda'_{BW}, \Lambda_{3Z})$. This can be understood as follows. The distributions of f_5^Z and

f_5^γ are actually the same for purely left-handed (or right-handed) electrons. So the fully polarized e_L^- (or e_R^+) beam would make f_5^Z and f_5^γ distributions indistinguishable and thus the correlation contour would collapse into a line with slope -1 . This explains why the polarized case makes the correlation in each plot poorly constrained along the major axis of the ellipse (orientated towards the northwestern direction in the parameter space of f_5^Z versus f_5^γ). Hence, we can modify the polarization setting and optimize the bound along this poorly constrained direction by combining the data-taking from both the unpolarized case and polarized case. To demonstrate this point, we present in each plot of Fig. 11 the correlation bound for a mixed

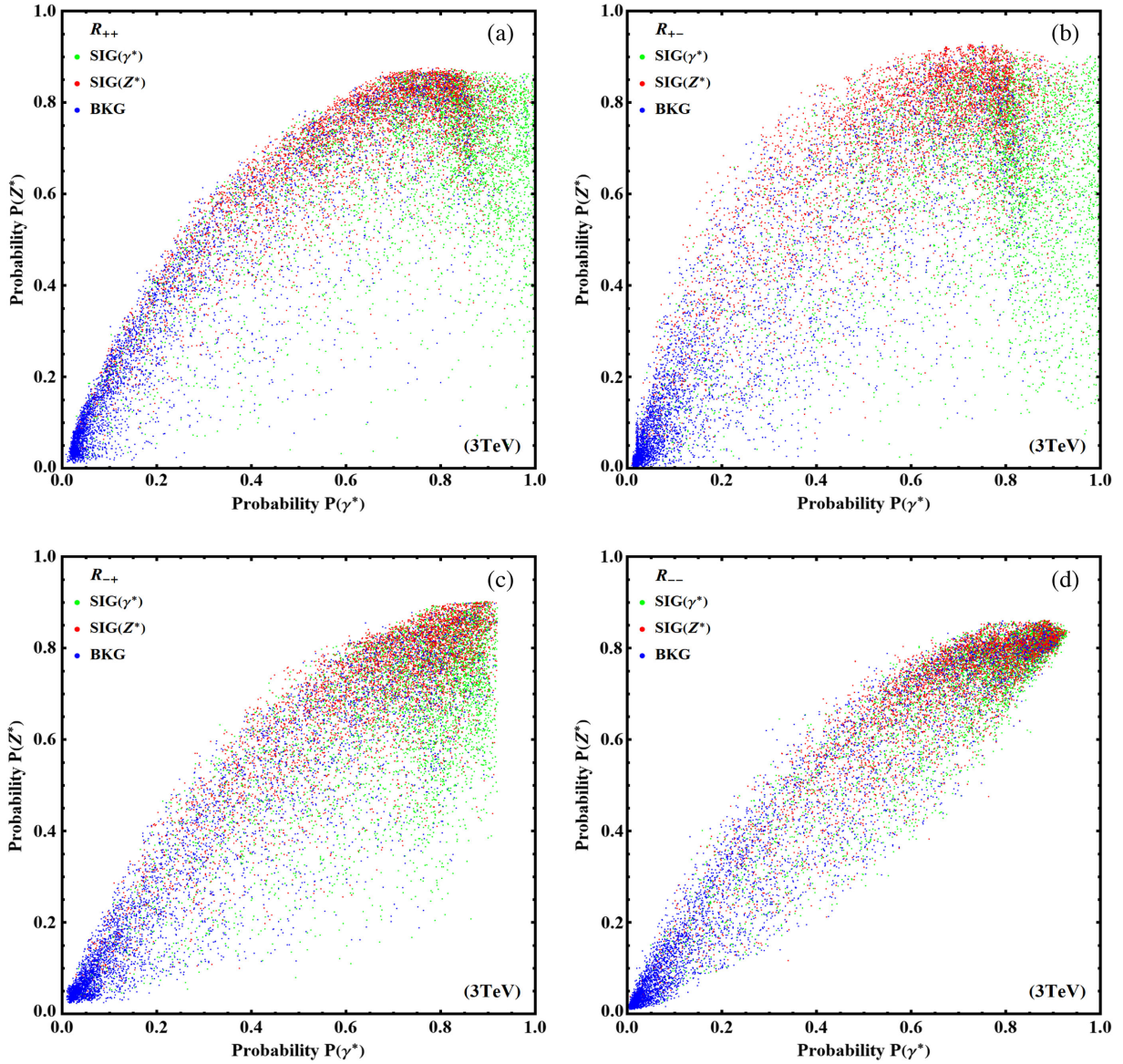


FIG. 10. Distribution of probabilities $P(\gamma^*)$ and $P(Z^*)$ of simulated events belonging to γ^* signal or Z^* signal category within the four regions of Eq. (4.12) for the case of $e^+e^- \rightarrow V^* \rightarrow ZZ \rightarrow (\ell\bar{\ell})(d\bar{d})$ at a collider energy $\sqrt{s} = 3$ TeV [marked as (3 TeV) in every plot], where d denotes the down-type quarks (d, s, b). The (green, red, blue) points in each plot represent the (γ^* , Z^* , SM) contributions, respectively. Plots (a)–(d) present the probability distributions of events for the angular regions R_{++} , R_{+-} , R_{-+} , and R_{--} , respectively.

setting in which half of the data comes from operation with unpolarized beams and the other half of the data comes from operation with polarized beams, as shown by the light-blue contour. We see that the correlation bound in the mixed setting is substantially tightened along the major axis of the ellipse and becomes comparable to that of the unpolarized case. On the other hand, the correlation bounds in the polarized and mixed cases are much stronger than that of the unpolarized case by about (40–50)% along the minor axis of the ellipse contours.

For Fig. 11, we further compare the correlation contours in the plots (a)–(b) for the collision energy $\sqrt{s} = 250$ GeV with the corresponding contours in the plots (c)–(d) for the

collision energy $\sqrt{s} = 3$ TeV. This shows substantial enhancements of the constraints by factors of $O(10^2)$ when the collider energy $\sqrt{s}$ increases from 250 GeV to 3 TeV. This feature can be understood by noting that the signal significance is determined by $S/\sqrt{B}$, where the signal S is dominated by the interference term (which scales with energy as $\sqrt{s}$) and the SM background B scales with energy as $1/s$ as shown by Eq. (3.28) for the differential cross sections. Hence, the signal significance scales with energy as s^1 and is increased by a factor of $O(10^2)$ when the collider energy rises by a factor of $(3 \text{ TeV}/250 \text{ GeV}) = O(10)$.

Finally, we compare the correlations bounds (2σ) on the nTGC form factors (f_5^V, f_5^Z) by using the conventional

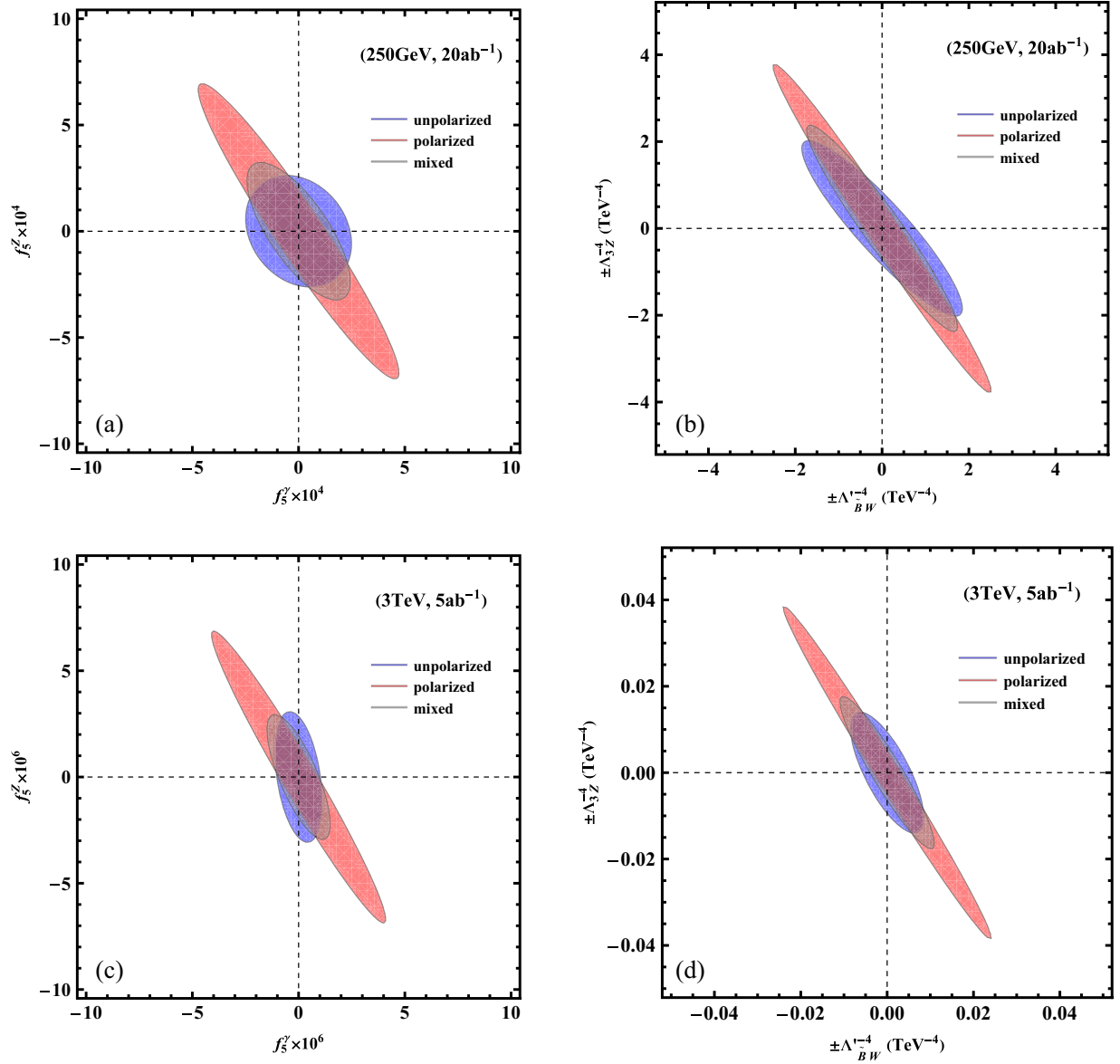


FIG. 11. Bounds on correlations of the nTGC form factors and on correlations of the nTGC new physics scales at the 2σ level. The plots (a)–(b) are for the collision energy $\sqrt{s} = 250$ GeV (with an integrated luminosity 20 ab^{-1}) and plots (c)–(d) are for the collision energy $\sqrt{s} = 3$ TeV (with an integrated luminosity 5 ab^{-1}), which are marked in the upper-right corner of each plot. The contours of plots (a) and (c) depict the correlations between the form factors (f_5^γ, f_5^Z) , whereas the contours of plots (b) and (d) present the correlations between the new physics scales $(\Lambda_{BW}^{'-4}, \Lambda_{3Z}^{'-4})$. In each plot, the correlation bound for the case of unpolarized $e^\mp$ beams is shown as the blue contour and that of the case for polarized $e^\mp$ beams is given by the pink contour, with the $e^\mp$ beam polarizations $(P_L^e, P_R^e) = (0.9, 0.65)$; whereas the light-blue contour presents the correlation bound for the mixed setting in which half of the data comes from unpolarized collisions and the other half of the data comes from polarized collisions.

manual cuts and by using the ML method as shown in Fig. 12. In plot (a), we present correlations bounds for the e^-e^+ collision energy 250 GeV and an integrated luminosity 20 ab^{-1} , but without using beam polarization. Plot (b) is the same as plot (a) except for the collision energy 3 TeV and an integrated luminosity 5 ab^{-1} . In both plots (a) and (b), the blue contours show the correlations bounds by using the conventional manual cuts, whereas the pink contours show the correlations bounds by using the

machine learning method [which are taken from the unpolarized contours of Figs. 11(a) and 11(c)]. From this comparison, we find that the machine learning can substantially improve the nTGC correlation bounds on (f_5^γ, f_5^Z) by about a factor of 2 to 3 relative to the conventional analysis by using manual cuts. This demonstrates that the machine learning method does provide a powerful tool for distinguishing the nTGC correlation signals from the SM backgrounds.

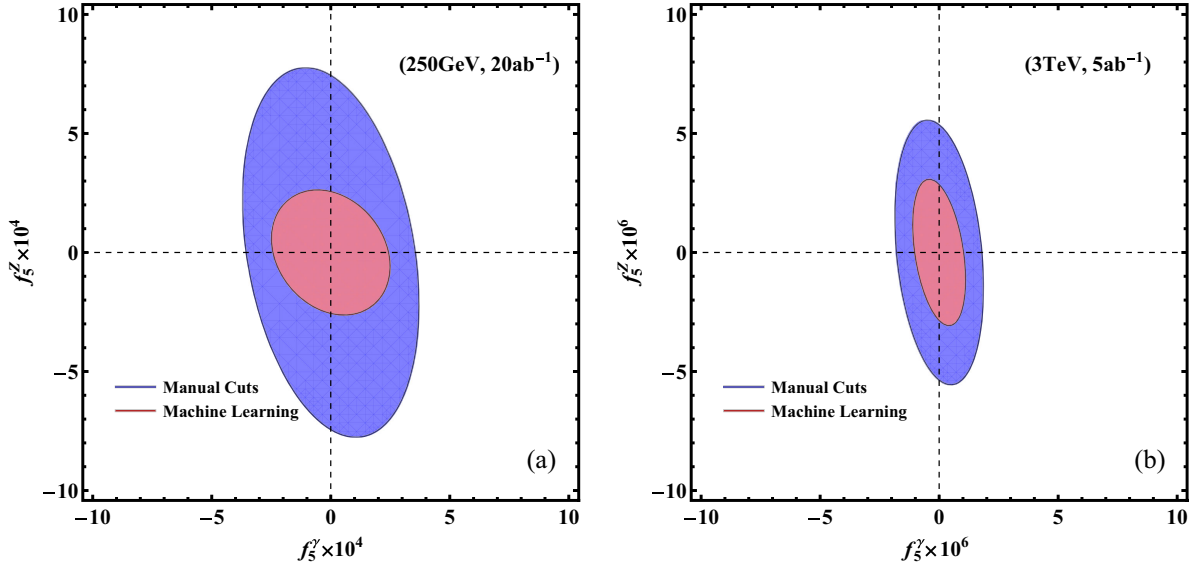


FIG. 12. Comparison of correlations bounds (2σ) on the nTGC form factors (f_5^Y, f_5^Z) by using the conventional manual cuts (blue contours) and by using the machine learning method (pink contours). Plot (a) shows correlations bounds for the e^-e^+ collision energy 250 GeV and an integrated luminosity 20 ab^{-1} , but without using beam polarization. Plot (b) is the same as plot (a) except for the e^-e^+ collision energy 3 TeV and an integrated luminosity 5 ab^{-1} .

F. Comparison with previous results

In this subsection we compare our present sensitivity analysis for probing nTGCs via ZZ final states with those bounds extracted from our previous study [4,6] via the $Z\gamma$ final states.

We extract the sensitivity reaches for the nTGC new physics scale $\Lambda_{\tilde{B}W}$ obtained via $Z\gamma$ production (based on Table 3 of Ref. [6]) and compare them with the reaches for $\Lambda_{\tilde{B}W}$ obtained via the ZZ production (based on the present analysis), which are summarized in Table XIII. The previous reaches were derived for the $Z\gamma$ production with $Z \rightarrow q\bar{q}$ [6] (shown in the second and third columns of this table), whereas the present reaches are derived for the ZZ production with $Z \rightarrow q\bar{q}, \ell\bar{\ell}, \nu\bar{\nu}$ (shown in the fourth and fifth columns of this table). These reaches are presented at 2σ and

5σ levels and for different collider energies. In each entry the sensitivity bounds correspond to (unpolarized, polarized) $e^\mp$ beams, with beam polarizations $(P_L^e, P_R^e) = (0.9, 0.65)$. For illustration, we choose a representative integrated luminosity $\mathcal{L} = 5 \text{ ab}^{-1}$ and an ideal detection efficiency $\epsilon = 100\%$. Moreover, for the 250 GeV collision energy of e^-e^+ , we choose another benchmark of integrated luminosity $\mathcal{L} = 20 \text{ ab}^{-1}$ based on the CEPC TDR [28], which is shown in the third row of Table 13.

In parallel, we can convert the sensitivity reaches for the nTGC new physics scale $\Lambda_{\tilde{B}W}$ obtained via the $Z\gamma$ production in Ref. [6] to the corresponding reaches for the nTGC form factor h_3^Z obtained via the $Z\gamma$ production [4]. Then, in Table XIV we compare these bounds with the sensitivity reaches for the form factor f_5^Y [contributed by the

TABLE XIII. Comparison of the previous sensitivity reaches for the nTGC new physics scale $\Lambda_{\tilde{B}W}$ (in TeV) obtained via the reaction $e^+e^- \rightarrow Z\gamma$ channel [6] with the present sensitivity reaches obtained via the reaction $e^+e^- \rightarrow ZZ$. These reaches are presented at 2σ and 5σ levels and for different collider energies. In each entry the sensitivity bounds correspond to (unpolarized, polarized) $e^\mp$ beams, with beam polarizations $(P_L^e, P_R^e) = (0.9, 0.65)$.

Energy and luminosity		Previous results [6]		Present results	
$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab^{-1})	$\Lambda_{\tilde{B}W}^{\text{tot}, 2\sigma}$ (unpol, pol)	$\Lambda_{\tilde{B}W}^{\text{tot}, 5\sigma}$ (unpol, pol)	$\Lambda_{\tilde{B}W}^{\text{tot}, 2\sigma}$ (unpol, pol)	$\Lambda_{\tilde{B}W}^{\text{tot}, 5\sigma}$ (unpol, pol)
0.25	20	(1.4, 1.5)	(1.2, 1.2)	(1.3, 1.6)	(1.1, 1.3)
0.25	5	(1.2, 1.3)	(.97, 1.0)	(1.1, 1.3)	(.88, 1.1)
0.5	5	(1.8, 1.9)	(1.4, 1.4)	(2.0, 2.1)	(1.6, 1.6)
1	5	(2.6, 2.6)	(2.0, 2.1)	(2.9, 3.0)	(2.3, 2.4)
3	5	(4.3, 4.5)	(3.5, 3.6)	(5.2, 5.3)	(4.1, 4.2)
5	5	(5.7, 5.9)	(4.5, 4.7)	(6.7, 6.9)	(5.4, 5.5)

TABLE XIV. Comparison of our previous sensitivity reaches for the nTGC form factors obtained via the $Z\gamma$ channel [4] with the present sensitivity reaches obtained via the ZZ channel. These reaches are presented at 2σ and 5σ levels and for different collider energies. In each entry the sensitivity bounds correspond to (unpolarized, polarized) $e^\pm$ beams, with beam polarizations $(P_L^e, P_R^e) = (0.9, 0.65)$.

Energy and luminosity		Previous results [4]		Present results	
$\sqrt{s}$ (TeV)	$\mathcal{L}$ (ab $^{-1}$)	$ h_3^Z _{2\sigma}$ (unpol, pol)	$ h_3^Z _{5\sigma}$ (unpol, pol)	$ f_5^Z _{2\sigma}$ (unpol, pol)	$ f_5^Z _{5\sigma}$ (unpol, pol)
0.25	20	$(1.4, 1.2) \times 10^{-4}$	$(3.4, 2.9) \times 10^{-4}$	$(2.0, 0.96) \times 10^{-4}$	$(4.9, 2.4) \times 10^{-4}$
0.25	5	$(2.7, 2.3) \times 10^{-4}$	$(6.8, 5.8) \times 10^{-4}$	$(3.9, 1.9) \times 10^{-4}$	$(9.8, 4.8) \times 10^{-4}$
0.5	5	$(6.2, 5.2) \times 10^{-5}$	$(1.6, 1.3) \times 10^{-4}$	$(4.0, 3.2) \times 10^{-5}$	$(1.0, .81) \times 10^{-4}$
1	5	$(1.5, 1.2) \times 10^{-5}$	$(3.8, 3.0) \times 10^{-5}$	$(8.3, 7.0) \times 10^{-6}$	$(2.1, 1.7) \times 10^{-5}$
3	5	$(1.7, 1.4) \times 10^{-6}$	$(4.3, 3.5) \times 10^{-6}$	$(8.4, 7.6) \times 10^{-7}$	$(2.1, 1.9) \times 10^{-6}$
5	5	$(5.8, 4.9) \times 10^{-7}$	$(1.5, 1.2) \times 10^{-6}$	$(2.9, 2.6) \times 10^{-7}$	$(7.3, 6.4) \times 10^{-7}$

same operator $\mathcal{O}_{\bar{B}W}$ via Eq. (2.9)] obtained through the ZZ production channel of the present study. The previous reaches were derived for the $Z\gamma$ channel with $Z \rightarrow q\bar{q}$ (shown in the second and third columns), whereas the present bounds are derived for the ZZ channel with $Z \rightarrow q\bar{q}, \ell\bar{\ell}, \nu\bar{\nu}$ (shown in the fourth and fifth columns). These reaches are presented at 2σ and 5σ levels and for different collider energies respectively. In each entry the reaches correspond to (unpolarized, polarized) $e^\pm$ beams, with beam polarizations $(P_L^e, P_R^e) = (0.9, 0.65)$. Same as in Table XIII, we choose a representative integrated luminosity $\mathcal{L} = 5 \text{ ab}^{-1}$ and an ideal detection efficiency $\epsilon = 100\%$. In addition, for the 250 GeV collision energy of e^-e^+ , we choose another benchmark of integrated luminosity $\mathcal{L} = 20 \text{ ab}^{-1}$ based on the CEPC TDR [28], as presented in the third row of Table 14.

As shown in Table XIII, it is evident that the sensitivity reaches for the nTGC new physics scales estimated in the present study are stronger than those reported in Ref. [6]. The current improvement is primarily attributable to the full use of the differential angular distribution f_Ω^1 incorporating all angular variables $(\theta, \theta_a, \phi_a, \theta_b, \phi_b)$, combined with machine learning classification algorithms that efficiently distinguish between the nTGC signals and SM backgrounds. For instance, the sensitivity reaches for the new physics scale $\Lambda_{\bar{B}W}$ estimated in the present study exhibit an enhancement up to 21% (18%) as compared to the previous bounds of Ref. [6] for the unpolarized (polarized) $e^\pm$ beams. The largest improvement of 21% (18%) is achieved for $\Lambda_{\bar{B}W}^{\text{tot}, 2\sigma}$ in the unpolarized (polarized) case at the collision energy $\sqrt{s} = 3 \text{ TeV}$.

As shown in Table XIV, the present sensitivity reaches on the nTGC form factors also surpass the previous reaches found in Ref. [4]. For instance, we find that the sensitivity reaches on the nTGC form factors exhibit enhancements of about (17–51)% as compared to the reaches found in Ref. [4]. The minimal improvement of 17% is realized for the sensitivity bounds on $|f_5^Z|$ in the polarized case at the collision energy $\sqrt{s} = 250 \text{ GeV}$, whereas the most significant enhancement of 51% is achieved for $|f_5^Z|$ in the

unpolarized case at the collision energy $\sqrt{s} = 3 \text{ TeV}$. We also see that using the polarized $e^\pm$ beams the sensitivity reaches on f_5^Z (h_3^Z) can be increased by about 51% (15%) at the collision energy $\sqrt{s} = 250 \text{ GeV}$, and by about 10% (18%) at the collision energy $\sqrt{s} = 3 \text{ TeV}$.

V. CONCLUSIONS

Neutral triple gauge couplings do not exist within the SM, nor do they receive contributions from any dimension-6 SMEFT operators. Hence, the nTGCs provide an important window for probing new physics beyond the SM that can be used to probe directly new physics encoded by dimension-8 SMEFT operators. The previous phenomenological studies [2–11] and experimental measurements [13,14] mainly focused on the $Z\gamma V^*$ nTGCs (with $V = Z, \gamma$) and did not probe the triple Z boson coupling ZZZ^* .

In this work, we formulated the nTGC form factors of ZZV^* ($V = Z, \gamma$) that are compatible with the spontaneous breaking of the $SU(2) \otimes U(1)$ electroweak gauge symmetry and consistently match the dimension-8 nTGC operators in the broken phase. Using machine learning, we have systematically studied the sensitivities of probing both the ZZV^* form factors and new physics scales through the reaction process $e^-e^+ \rightarrow ZZ \rightarrow f_a \bar{f}_a f_b \bar{f}_b$ (with $f_a = \ell, q$ and $f_b = \ell, q, \nu$) at high energy e^+e^- colliders, including CEPC, FCC-ee, LCF, ILC, and CLIC. We have analyzed the signal significance and derived sensitivity reaches for probing the nTGC form factors (f_5^Z, f_5^Z) and the corresponding nTGC new physics scales ($\Lambda_{3Z}, \Lambda_{\bar{B}W}$). We present a systematic summary of these sensitivity bounds in Figs. 13(a) and 13(b), allowing one nTGC parameter nonzero at a time.

In Sec. II we studied how the nTGC couplings of ZZV^* can arise from the relevant dimension-8 SMEFT operators as shown in Eq. (2.4) and Table I. In particular, we constructed the dimension-8 nTGC operator $\mathcal{O}_{3Z}$ in Eq. (2.6) that contributes to the ZZZ^* vertex only, but not to the $ZZ\gamma^*$ vertex, as shown in Table I and Eq. (2.8). This is in contrast with the other operator $\mathcal{O}_{\bar{B}W}$ that contributes to

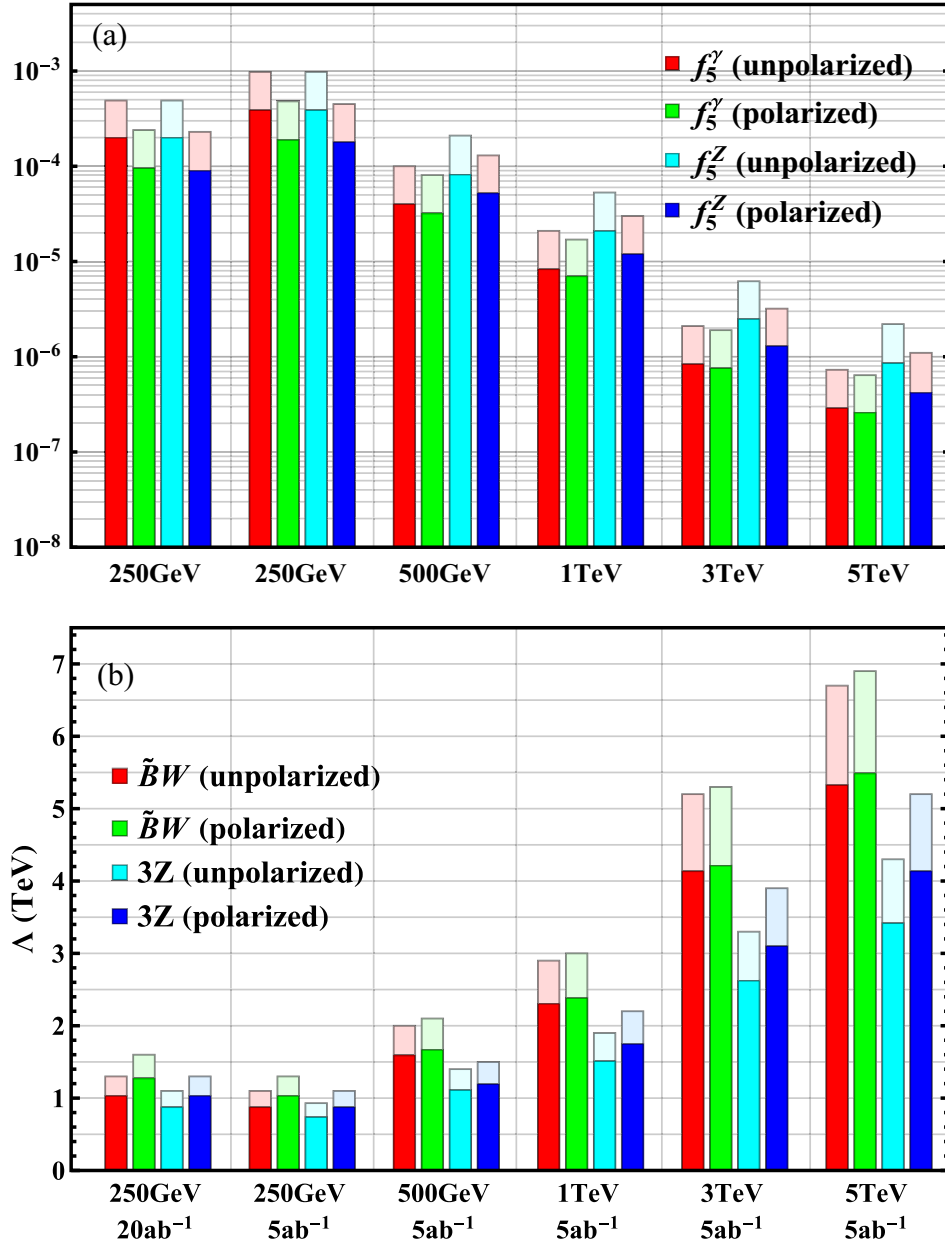


FIG. 13. Sensitivity reaches for the nTGC form factors in plot (a) and for the nTGC new physics scales in plot (b) for various e^-e^+ collision energies and integrated luminosities. In this figure, plot (a) presents the $(2\sigma, 5\sigma)$ sensitivity reaches in (heavy, light) colors, whereas plot (b) shows the $(2\sigma, 5\sigma)$ sensitivity reaches in (light, heavy) colors.

the $ZZ\gamma^*$ vertex alone, as shown in Table I and Eq. (2.9). We presented the consistent formulation of the nTGC form factors that are compatible with the full SM gauge symmetry $SU(2) \otimes U(1)$ with spontaneous electroweak symmetry breaking. The unitarity constraints on the nTGCs were derived in Table 2 of Sec. II B and are much weaker than the sensitivity bounds derived for the collider probes in Sec. IV.

In Sec. III, we derived the scattering amplitudes, cross sections, and angular distributions for the reaction process $e^-e^+ \rightarrow ZZ$. We analyzed systematically the contributions from both the SM and nTGCs, as given analytically in

Eqs. (3.23), (3.27), and (3.28). With these we demonstrated the differences in their differential angular distributions, as shown in Figs. 4–6. These are important for our phenomenological analysis of probing the nTGCs in the following Sec. IV.

In Sec. IV, we presented a systematic analysis of the ZZ production in different fermionic decay channels at various e^+e^- colliders. We computed the signal significance using the differential angular distributions of the cross section, considering both the visible leptonic and hadronic decay channels and the invisible decay channel ($Z \rightarrow \nu\bar{\nu}$). The estimated sensitivity reaches for probing the nTGC form

factors are shown in Tables III, V, VI and for probing the nTGC new physics scales are shown in Tables IV, VII. These sensitivity limits are presented for the leptonic and hadronic Z-decay channels in Sec. IVA and for the invisible Z-decay channels in Sec. IV B. We studied the significant improvements of the sensitivity reaches on the nTGC form factors and on the nTGC new physics scales (as well as their correlations) achieved through the use of machine learning algorithms in Secs. IV C and IV E. We further derived the enhanced sensitivity bounds on probing the nTGCs by using the polarized $e^\pm$ beams in Sec. IV D. A comprehensive summary of these nTGC sensitivity bounds for both the unpolarized and polarized $e^\mp$ beams is presented in Figs. 13(a) and (b). Moreover, we studied the correlation bounds for the nTGC form factors and for the nTGC new physics scales, which were presented in Figs. 11(a), (c) and 11(b), (d), respectively. We found that the optimal sensitivity bounds on the nTGC correlations are given by the mixed setting including the unpolarized operation and a follow-up polarized operation. We further demonstrated in Figs. 12(a) and 12(b) that the machine learning can substantially enhance the nTGC correlation bounds (pink contours) over those bounds derived by using the conventional manual cuts (blue contours). Finally, Sec. IV F compared the sensitivity reaches on probing the nTGCs as obtained in this work through ZZ production with the previous studies via $Z\gamma$ production [4,6], which are summarized in Tables XIII and XIV. We have demonstrated that the sensitivity bounds

on probing the nTGC form factors and the nTGC new physics scales as given in this work are stronger, primarily due to the full use of the differential angular distributions based on the machine learning algorithms.

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DATA AVAILABILITY

The data are not publicly available. But the data are available from the authors upon reasonable request.

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