

Production of hyperons, charmed baryons, and hadronic molecule candidates in neutrino-proton reaction

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We investigate the production of hyperons, charmed baryons, and potential hadronic molecular states in neutrino-proton ($\bar{\nu}_\mu p$) reaction, a process characterized by a particularly clean final state. Employing effective Lagrangians, chiral perturbation theory, and a hadronic molecular model, we perform theoretical calculations for several relevant channels, including those leading to the formation of the hadronic molecular candidate ($\bar{D}N$) and ($\bar{D}\Sigma$). Our results indicate that future neutrino facilities could serve as a complementary platform for exploring exotic baryonic states and provide valuable insights into the dynamics of strong interactions in the strange and charm sectors.

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I. INTRODUCTION

Over the past two decades, the search for pentaquark states has progressed from early, inconclusive hints to firmly established observations in the heavy-flavor sector [1–5]. In 2015, the LHCb Collaboration reported two hidden-charm pentaquarklike structures, $P_c(4380)^+$ and $P_c(4450)^+$, observed in the $J/\psi p$ invariant mass spectrum [6]. Subsequent analyses based on higher statistics revealed more refined structures of three narrow resonances $P_c(4312)^+$, $P_c(4440)^+$, and $P_c(4457)^+$ [7]. These hidden-charm states are located close to the $\Sigma_c \bar{D}^{(*)}$ thresholds, supporting earlier predictions [8–11] in the picture of hadronic molecules. More recently, evidence for a hidden-charm, strange pentaquark $P_{cs}(4459)$ in the $J/\psi \Lambda$ channel has also been reported [12].

On the other hand, neutrino-proton reactions offer a clean environment for probing hadron spectroscopy [13–15] and have been extensively investigated in fixed-target experiments over the past decades. With advances in accelerator and detector technologies, modern neutrino experiments can now achieve much higher statistics [16–19].

For example, the NOMAD experiment at CERN [20] conducted detailed measurements of inclusive strange-particle production, while the Fermilab MINER ν A experiment [21] has been designed for precision studies of exclusive strange-channel reactions. The MINER ν A program explicitly aims to perform “precision measurements of exclusive strange-particle production channels near threshold” (e.g. $\nu_\mu p \rightarrow \mu^- K^+ \Lambda$) and to determine hyperon production cross sections and polarizations, thereby enabling searches for pentaquarklike resonances. In summary, current and future neutrino experiments are expected to surpass the old bubble-chamber datasets by providing high-statistics samples of exclusive νp hadronic final states.

In this work, we investigate several representative processes involving hyperons or charmed baryons in the final state, such as $\mu^+ K \Lambda$, $\mu^+ \bar{D} \Lambda$, and $\mu^+ \bar{D} N$, and present their Dalitz plots along with the corresponding total cross sections. We employ effective Lagrangian methods and chiral perturbation theory to estimate their production rates.

Since the tree-level processes discussed above may involve resonant contributions from hadronic molecular states, we also investigate the neutrino-induced production of two molecular candidates, ($\bar{D}N$) [22–24] and ($\bar{D}\Sigma$) [24–26], denoted as $P_{\bar{c}}$ and $P_{\bar{c}s}$, respectively. In particular, we calculate the cross sections for the two-body processes $\bar{\nu}_\mu p \rightarrow \mu^+ P_{\bar{c}(s)}$, for comparison with the three-body channels.

We aim for the calculations of these representative processes to serve as a preparatory study for exploring

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their feasibility in future neutrino experiments, thereby providing valuable insights into hadron structure.

This paper is organized as follows. Section II presents the theoretical formalism, including the effective Lagrangians and form factors used for the νp scattering amplitudes. Section III demonstrates the numerical results for the form factors and computes the cross sections for selected exclusive channels, followed by a discussion of the results. Section IV provides a brief summary and concluding remarks.

II. FORMALISM

A. Feynman diagrams and lagrangians

At tree level, we consider three processes for comparison, as shown in Fig. 1. For the process $\bar{\nu}_\mu p \rightarrow \mu^+ K^0 \Lambda$, contributions from the contact term, s -, t -, and u -channel diagrams are all included. For the charm production processes $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 \Lambda$ and $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 n$, only the t -channel meson-exchange diagrams are considered, as the contact terms are omitted because their couplings, determined by the SU(4) chiral Lagrangian, are unreliable due to the severe breaking of SU(4) flavor symmetry by the large charm quark mass.

For the $(\bar{D}N)$ molecular state, which has been proposed as a hadronic molecule candidate [22,23], we consider the two isospin configurations $I = 0$ and $I = 1, I_3 = 0$. Their explicit isospin wave functions are

$$|(\bar{D}N), I = 0\rangle = \frac{1}{\sqrt{2}}(|D^- p\rangle - |\bar{D}^0 n\rangle) \quad (1)$$

$$|(\bar{D}N), I = 1, I_3 = 0\rangle = \frac{1}{\sqrt{2}}(|D^- p\rangle + |\bar{D}^0 n\rangle) \quad (2)$$

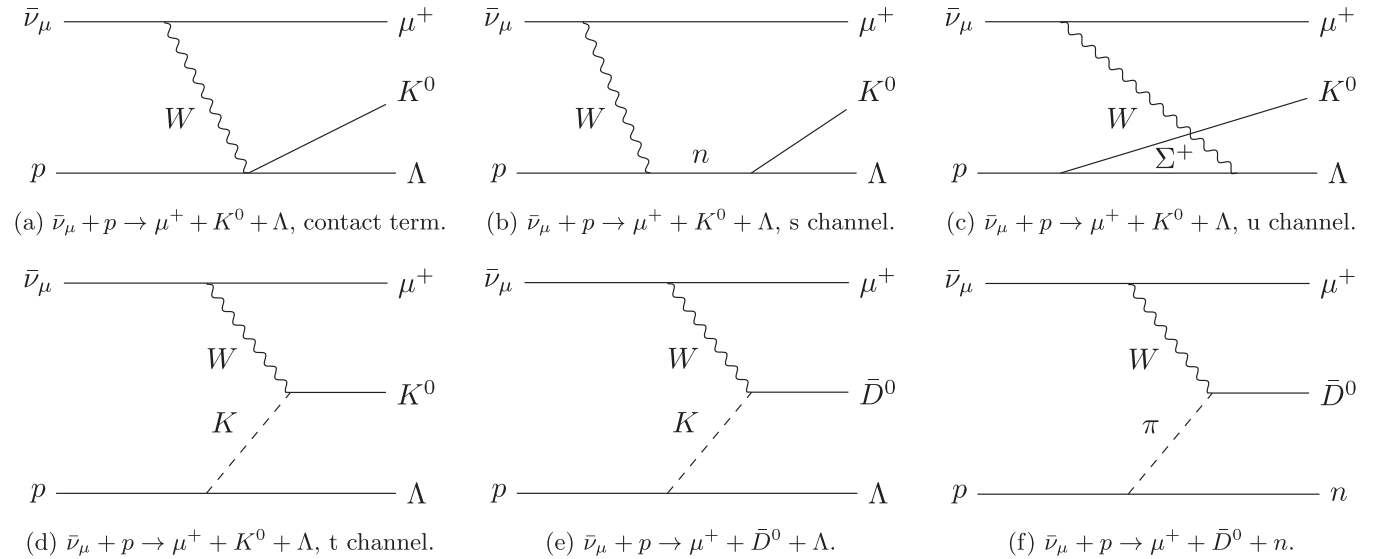


FIG. 1. Feynman diagrams for neutrino-proton scattering leading to three-body final states. Panels (a)–(d) correspond, respectively, to the contact term and the s -, u -, and t -channel contributions for hyperon production, while panels (e) and (f) show the t -channel diagrams for charm production.

For the $(\bar{D}\Sigma)$ molecular state, we restrict ourselves to the $I = 1/2$ configuration, which is the most relevant channel for forming a bound or near-threshold molecular state in existing theoretical studies [22,24]. The corresponding isospin wave function with $I_3 = \frac{1}{2}$ is given by

$$|(\bar{D}\Sigma), I = \frac{1}{2}, I_3 = \frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|D^- \Sigma^+\rangle - \sqrt{\frac{1}{3}}|\bar{D}^0 \Sigma^0\rangle, \quad (3)$$

We use effective Lagrangians and the electroweak Lagrangian to describe these processes. At the electroweak interaction vertex, the charged current part of the Lagrangian is given by

$$\mathcal{L}_c = -\frac{g}{\sqrt{2}} \left(\bar{\nu}_l \gamma^\mu \frac{1 - \gamma^5}{2} l \right) W_\mu^+ + \text{H.c.} \quad (4)$$

Here, ν_l, l and W_μ denote neutrino, lepton and W boson field, respectively. The SU(2) gauge coupling constant is $g = e/\sin\theta_W$, where θ_W is the weak mixing angle. Its relation to the Fermi constant G_F is given by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \quad (5)$$

where M_W is the mass of the W boson.

At the hadron level, we use chiral perturbation theory (ChPT) [27] to describe the interactions between hadrons and W bosons.

The pseudoscalar meson fields are collected in an SU(3) matrix $U = \exp(i\phi/F_0)$, where F_0 is the decay constant of the Goldstone bosons in the three-flavor chiral limit, and ϕ contains the pseudoscalar Goldstone boson fields

$$\phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta^0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta^0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta^0 \end{pmatrix}. \quad (6)$$

We also define $u = \sqrt{U}$. In ChPT, the W boson enters through the left-handed external current

$$l_\mu = -\frac{g}{\sqrt{2}}(W_\mu^+ T_+ + \text{H.c.}), \quad (7)$$

where T_+ is related to the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements V_{ij} , with $V_{ud} = 0.97367 \pm 0.00032$ and $V_{us} = 0.22431 \pm 0.00085$.

$$T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (8)$$

Since only the left-handed W -boson current is involved, the right-handed external current is set to $r_\mu = 0$.

The lowest-order chiral effective Lagrangian describing the coupling of the W boson to the pseudoscalar mesons is given by

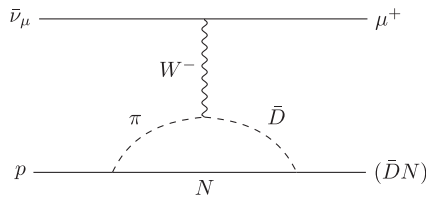
$$\mathcal{L}_{\phi W} = i\frac{F_0^2}{2}\text{Tr}(l_\mu \partial^\mu U^\dagger U) + \dots \quad (9)$$

where F_0 is taken to be $F_\pi = 92.4$ MeV in our numerical calculations, and we adopt the ratio $F_K/F_\pi \approx 1.2$ [28] when kaon vertices are involved.

The baryon octet with $J^P = \frac{1}{2}^+$ is represented by the matrix

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}. \quad (10)$$

The lowest-order chiral Lagrangian for the meson-baryon interaction reads



(a) $\bar{\nu}_\mu + p \rightarrow \mu^+ + (\bar{D}N)$.

$$\begin{aligned} \mathcal{L}_{\phi B} = & \text{Tr}(\bar{B}(i\not{D} - M_B)B) + \frac{D}{2}\text{Tr}(\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}) \\ & + \frac{F}{2}\text{Tr}(\bar{B}\gamma^\mu\gamma_5[u_\mu, B]), \end{aligned} \quad (11)$$

where M_B is the baryon mass matrix in the chiral limit, $D = 0.80$ and $F = 0.50$ are the axial-vector coupling constants at tree level [29]. The chiral connection Γ_μ , the chiral vielbein u_μ , and the covariant derivative $D_\mu B$ are defined as

$$\Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger], \quad (12)$$

$$u_\mu = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger], \quad (13)$$

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B]. \quad (14)$$

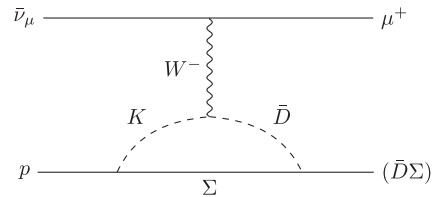
The explicit forms of the interaction vertices derived from $\mathcal{L}_{\phi B}$ are collected in Appendix (A4).

Although the Lagrangians for the KKW vertex can be obtained from ChPT, the form factors for the WDK and $WD\pi$ vertices used in this work are taken directly from lattice QCD calculations. These vertices are parametrized as

$$\begin{aligned} \langle P|V_\mu|D\rangle = & f_+(q^2)\left(p_{D\mu} + p_{P\mu} - \frac{m_D^2 - m_P^2}{q^2}q_\mu\right) \\ & + f_0(q^2)\frac{m_D^2 - m_P^2}{q^2}q_\mu, \end{aligned} \quad (15)$$

where P denotes a light pseudoscalar meson (π or K), and $f_+(q^2)$ and $f_0(q^2)$ are the vector and scalar form factors, respectively, describing the hadronic matrix element of the weak current between the D meson and P . More details can be found in Sec. II B.

One of the tree-level processes [Fig. 1(f)] involves a hadron pair in the final state, which could potentially form a hadronic molecule. To estimate the likelihood of such formation, we also consider the corresponding loop diagram as shown in Fig. 2(b). The effective Lagrangian for the hadronic molecule vertex is given by



(b) $\bar{\nu}_\mu + p \rightarrow \mu^+ + (\bar{D}\Sigma)$.

FIG. 2. $(\bar{D}N)$ and $(\bar{D}\Sigma)$ molecule production in antineutrino-proton scattering.

$$\begin{aligned} \mathcal{L}_{(\bar{D}N)}(x) &= ig_{\bar{D}N} \bar{P}_{\bar{c}}(x) \int d^4 y \Phi(y^2) N(x + w_{\bar{D}N} y) \\ &\times \bar{D}(x - w_{N\bar{D}} y) + \text{H.c.} \end{aligned} \quad (16)$$

where $P_{\bar{c}}$ denotes the $(\bar{D}N)$ molecular state with quantum numbers $J^P = 1/2^-$. The coupling constants are taken as $g_{\bar{D}N}^{I=0} = 1.68$ and $g_{\bar{D}N}^{I=1} = 2.62$, as determined in our previous work [30]. The coupling constant $g_{\bar{D}\Sigma} = 0.66$ is determined using the Weinberg compositeness criterion, corresponding to a binding energy of 0.2 MeV [24]. $\omega_{\bar{D}N}$ is a kinematic parameter that reflects the mass ratio between the two constituents of the molecule, defined as

$$\omega_{ij} = \frac{m_i}{m_i + m_j}, \quad (17)$$

where m_i and m_j are the masses of the constituent particles. $\Phi(x)$ is a correlation function that characterizes the distribution of the constituent momenta at the vertex. In momentum space, it is defined via a Fourier transform

$$\Phi(x^2) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot x} \tilde{\Phi}(-p^2) \quad (18)$$

We adopt a Gaussian form with a cutoff parameter Λ to model this vertex function

$$\tilde{\Phi}(p_E^2) \doteq \exp\left(-\frac{p_E^2}{\Lambda^2}\right) \quad (19)$$

where p_E denotes the Euclidean momentum.

We also introduce two form factors for the exchanged mesons to suppress their off-shell effects during the calculation

$$f_1(q^2) = \frac{\Lambda_1^4}{\Lambda_1^4 + (q^2 - m_{\text{ex}}^2)^2} \quad (20)$$

$$f_2(q^2) = \left(\frac{\Lambda_2^2 - m_{\text{ex}}^2}{\Lambda_2^2 - q^2}\right)^2 \quad (21)$$

where q is the four-momentum of the exchanged meson, and Λ_1, Λ_2 are phenomenological cutoff parameters.

The form factor f_1 is used for mesons exchanged in the triangle loop diagram, while f_2 is applied in the tree-level diagrams, as shown in Appendix.

B. Semileptonic form factors

Although we have incorporated some form factors, this treatment remains incomplete. The W -hadron vertex also requires a form factor to account for the internal structure of mesons. These form factors can be calculated using lattice QCD [28,31,32] or determined from fits to experimental data [33–35]. The vector and scalar form

factors $f_+(q^2)$ and $f_0(q^2)$ at the D -meson vertex can be parametrized as

$$\begin{aligned} \langle P | V_\mu | D \rangle &= f_+(q^2) \left(p_{D\mu} + p_{P\mu} - \frac{m_D^2 - m_P^2}{q^2} q_\mu \right) \\ &+ f_0(q^2) \frac{m_D^2 - m_P^2}{q^2} q_\mu, \end{aligned} \quad (22)$$

where P represents the final-state mesons π or K . The form factors in the q^2 -plane can be expressed using the $z(q^2, t_0)$ expansion, which exhibits rapid convergence

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}. \quad (23)$$

The semileptonic region is given by $m_\ell^2 \leq q^2 \leq t_-$, where $t_- = (M_D - M_P)^2$. A generic form factor contains poles and a branch cut $[t_+, \infty)$ along the real axis, where $t_+ = (M_D + M_P)^2$ is the pair-production threshold. With the choice $t_0 = 0$, the physical region $q^2 \in [0, q_{\text{max}}^2]$ maps to $z \in [0, -z_{\text{max}}]$.

A widely adopted parametrization is the BCL form [36]

$$f_0(z) = \frac{1}{1 - q^2(z)/M_{0+}^2} \sum_{n=0}^{M-1} b_n z^n, \quad (24)$$

$$f_+(z) = \frac{1}{1 - q^2(z)/M_{1-}^2} \sum_{n=0}^{N-1} a_n \left(z^n - \frac{n}{N} (-1)^{n-N} z^N \right) \quad (25)$$

The masses M_{J^P} in the denominators represent possible subthreshold poles. For $D \rightarrow \pi$ transitions, $M_{0+} = m_{D^{*0}}$ and $M_{1-} = m_{D_0^{*-}}$, while for $D \rightarrow K$ transitions, $M_{0+} = m_{D_s^0}$ and $M_{1-} = m_{D_s^{*-}}$. The coefficients a_n and b_n are the series expansion parameters that can be obtained from Ref. [32].

As for the KKW vertex, it is difficult to obtain the form factor from experiments since it does not have a semileptonic decay mode. One approach is to extract it from fits to $\tau^- K^- K_s \nu_\tau$ decay data [37,38], and there are phenomenological theoretical analyses for form factor calculations [39]. However, since we are primarily concerned with the order of magnitude of particle production rates, overly detailed form factor structures have minimal impact on the overall scale. Therefore, for simplicity and given our focus on order-of-magnitude estimates, we adopt the vector meson dominance (VMD) model to describe this vertex

$$F_V(q^2) = \frac{M_V^2}{M_V^2 - q^2} \quad (26)$$

In the numerical calculations, we set $M_V = 1$ GeV.

In loop calculations, to reduce computational complexity, we evaluate the loop integral contributions in advance and express them as form factors at the $W-p-(\bar{D}N)$ vertex.

However, this process can only be completed numerically, and directly using these results in subsequent steps would still be computationally expensive. To address this issue, we employ interpolation functions to handle these numerical results, with details provided in the following section.

The Lagrangian for the $W - p - (\bar{D}N)$ vertex can be written as

$$\mathcal{L}_{B'BV} = \bar{B}'_1 \left(g_{B'BV} \gamma_5 \gamma^\mu + \frac{f_{B'BV}}{m_1 - m_2} \gamma_5 \sigma^{\mu\nu} \partial_\nu \right) V_\mu B_2 + \text{H.c.} \quad (27)$$

where B' and B denote baryon fields with quantum numbers $J^P = \frac{1}{2}^-$ and $\frac{1}{2}^+$, respectively, m_1 and m_2 are their corresponding masses, and V denotes a vector field, which in this case is the W boson. The vector and tensor coupling constants $g_{B'BV}$ and $f_{B'BV}$ are obtained numerically from the loop integral and incorporate the loop-induced form factor, allowing the interaction to be treated effectively as a tree-level. Since we employ the multipole form factor of Eq. (20) at the exchanged-meson vertex and the molecular vertex form factor of Eq. (19) at the hadronic molecule vertex (see Appendix B for details), the coupling constants $g_{B'BV}$ and $f_{B'BV}$ are in practice form factors that depend on the cutoff parameters Λ and Λ_1 . Further details on their behavior are given in Sec. III A.

III. NUMERICAL RESULTS

A. Form factor behavior

For the DKW and $D\pi W$ vertices, lattice QCD calculations [32] only consider $q^2 > 0$ since they are compared with D semileptonic decay data. We extend the range to the $q^2 < 0$ region, and the results are shown below in Fig. 3.

The form factor at the hadronic molecule vertex is obtained through a loop integration that can only be evaluated numerically. The detailed derivation is presented in Appendix B.

As discussed in the previous section, this loop-induced form factor depends on two cutoff parameters, Λ and Λ_1 , whose correlation is illustrated in Fig. 4. In this plot, one parameter is fixed at 1 GeV while the other is varied from 0.5 to 1.5 GeV.

From the figure, we can see that the effect of the two cutoffs on the form factor is limited. Therefore, in the subsequent calculations, we set both Λ and Λ_1 to 1 GeV. The $(\bar{D}N)$ molecular states with different isospin values interact differently with their constituents, leading to variations in the corresponding coupling constants $g_{B'BV}$ and $f_{B'BV}$ defined in Eq. (27). We present these coupling constants to illustrate that they are of the same order of magnitude. As shown in Fig. 5, the two isospin states have

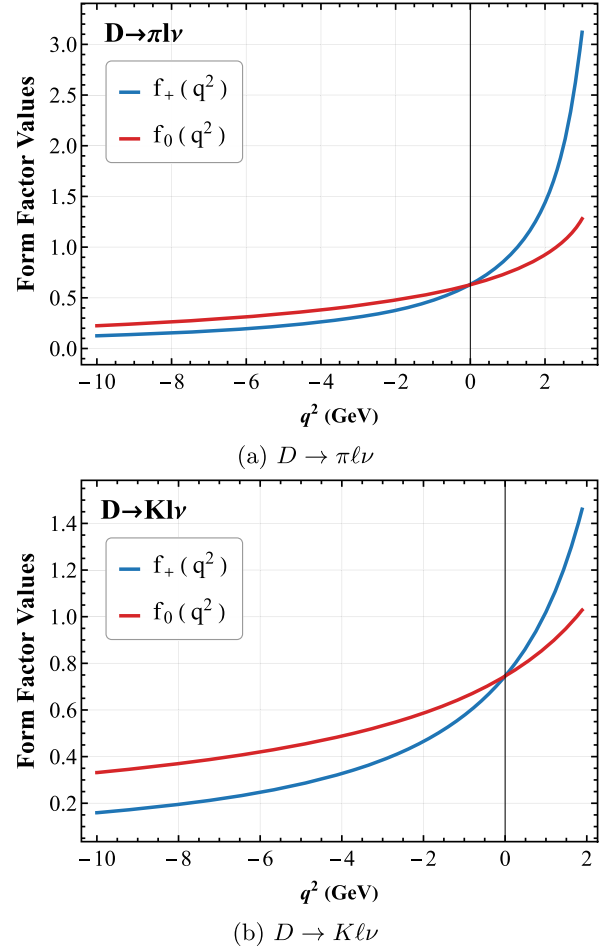


FIG. 3. Vector and scalar form factors in D meson semileptonic decays: (a) $D \rightarrow \pi \ell \nu$ and (b) $D \rightarrow K \ell \nu$.

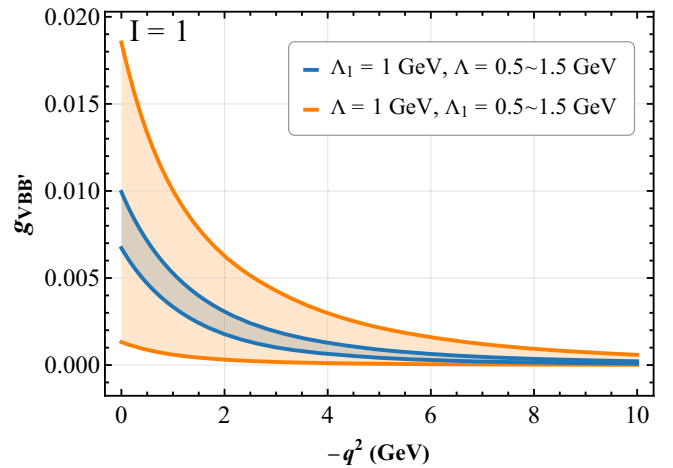


FIG. 4. Dependence of the coupling constant $g_{pP_c W}$ on the cutoff parameters Λ and Λ_1 , each varied from 0.5 to 1.5 GeV. Here, P_c denotes the $(\bar{D}N)$ state, and the isospin $I = 1$ configuration is chosen.

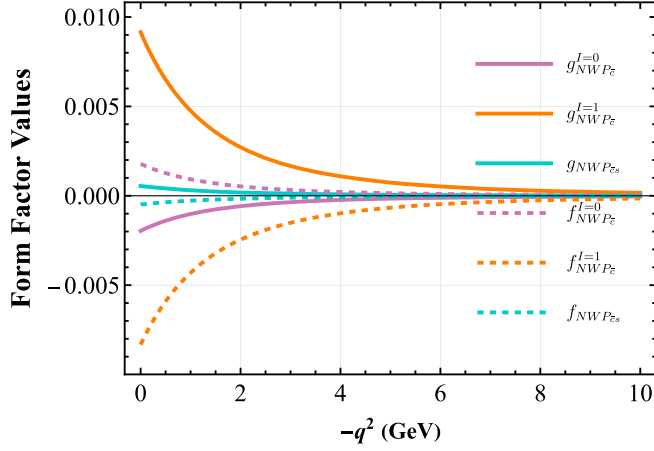


FIG. 5. Loop-induced form factors for the $P_{\bar{c}}N$ - W and $P_{\bar{c}s}N$ - W vertices, as defined in Eq. (27), shown as functions of $-q^2$, where q is the four-momentum of the W boson. Here $P_{\bar{c}}$ and $P_{\bar{c}s}$ denote the $(\bar{D}N)$ and $(\bar{D}\Sigma)$ hadronic molecular states, respectively.

opposite signs, and the coupling constants for $I = 0$ are slightly larger than those for $I = 1$. In contrast, the coupling constant of the $(\bar{D}\Sigma)$ molecular state is about one order of magnitude smaller than those of the $(\bar{D}N)$ states.

This suppression mainly originates from its smaller binding energy and the larger mass of the exchanged particle in the loop diagrams.

B. Effect of VMD form factor on results

In Fig. 6, we present the Dalitz plots for the process $\bar{\nu}_\mu p \rightarrow \mu^+ K^0 \Lambda$ at total energy $w = 5$ GeV, showing the individual contributions from the contact term, t -, s -, and u -channel diagrams with and without the VMD form factor. Since the effect of the VMD form factor near threshold is nearly identical to that at $w = 5$ GeV, we adopt this energy as a representative case for comparison among different channels. Although the contact term dominates the total cross section, we display all four channels to illustrate how the VMD form factor affects different amplitude structures. Overall, the inclusion of the VMD form factor suppresses the differential cross section by approximately one order of magnitude across all channels, while producing distinct structural modifications in each case. In the contact term [panels (a, e)], the VMD form factor relatively enhances the high- m_{23}^2 portion at low m_{12}^2 , shifting the peak structure toward that region. For the t -channel [panels (b, f)], the suppression predominantly affects the high- m_{23}^2 region, narrowing the bandlike enhancement structure toward the

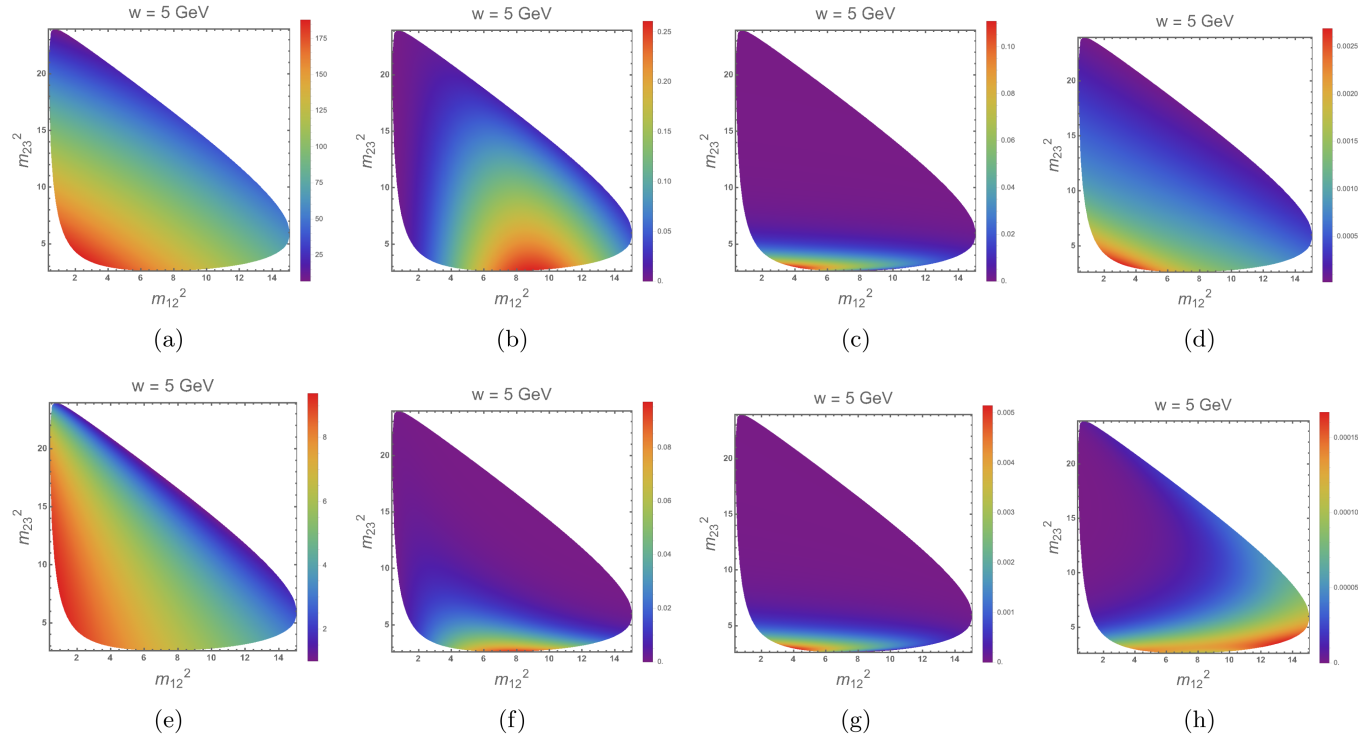


FIG. 6. Dalitz plots of the process $\bar{\nu}_\mu p \rightarrow \mu^+ K^0 \Lambda$ at total energy $w = 5$ GeV, illustrating the effect of the VMD form factor on individual contributions. In this process, m_1 , m_2 , and m_3 correspond to μ^+ , K^0 , and Λ , respectively. Panels (a, e) show the contact term, (b, f) the t -channel, (c, g) the s -channel, and (d, h) the u -channel contributions. The upper row [(a)–(d)] presents results without the VMD form factor, while the lower row [(e)–(h)] includes the VMD form factor. The invariant masses are defined as $m_{ij}^2 = (p_i + p_j)^2$, with the cutoff parameter fixed at $\Lambda_2 = 1$ GeV. The color scale represents the differential cross section $d\sigma/(dm_{12}^2 dm_{23}^2)$ in units of $10^{-41} \text{ cm}^2 \text{ GeV}^{-4}$.

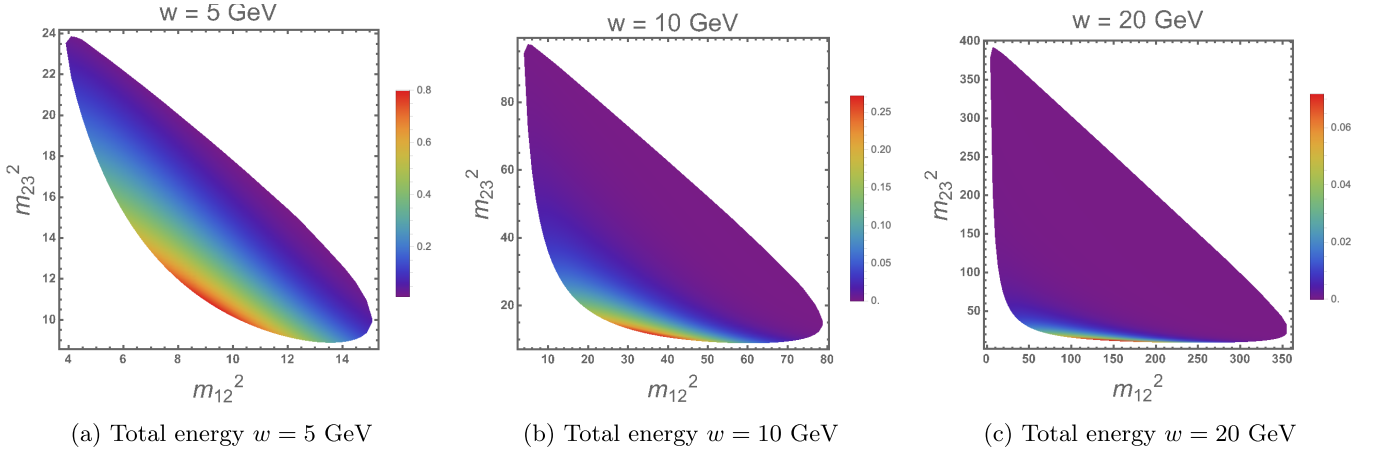


FIG. 7. Dalitz plots of the process $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 \Lambda$ at total energies $w = 5, 10$, and 20 GeV. In this process, m_1 , m_2 , and m_3 correspond to μ^+ , $\bar{D}^0$, and Λ , respectively. The invariant masses are defined as $m_{ij}^2 = (p_i + p_j)^2$, with the cutoff parameter fixed at $\Lambda_2 = 1$ GeV. The color scale represents the differential cross section $d\sigma/(dm_{12}^2 dm_{23}^2)$ in units of $10^{-42} \text{ cm}^2 \text{ GeV}^{-4}$.

low- m_{23}^2 boundary. The s -channel [panels (c, g)] exhibits a uniform suppression in magnitude with its overall kinematic structure largely preserved. For the u -channel [panels (d, h)], the VMD form factor suppresses the low- m_{12}^2 region while relatively enhancing the high- m_{12}^2 region, leading to a notable redistribution of the differential cross section across the Dalitz plot.

C. Neutrino-proton scattering

1. Dalitz plot and invariant mass spectrum

In this section, we present the Dalitz plots for the process $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 \Lambda$ at several representative energies to illustrate the evolution of the kinematic distributions with increasing total energy w . Since the other two processes exhibit similar behavior, their Dalitz plots are not shown here.

From Fig. 7, we observe that the events are concentrated in the low- m_{23}^2 region. This feature arises from the form factor at the DKW vertex. The horizontal position of the populated region is influenced by the form factor applied to the exchanged K or π meson, and therefore depends on the parameter Λ_1 , although the effect is relatively small. As the total energy increases, the dominant event region in the Dalitz plot appears visually narrower; however, the invariant mass spectrum shown in Fig. 8 reveals that its actual width in m_{23} becomes broader with increasing energy. The broadening rate slows down at higher energies, suggesting that the width will eventually saturate.

In Ref. [25], three narrow hadronic molecules of $\bar{D}\Sigma$ and $\bar{D}^*\Sigma$ are predicted to exist at energies around 3.05 and 3.19 GeV, respectively. They have large decay branching ratio to $\bar{D}\Lambda$, hence may be looked for here from the $\bar{D}\Lambda$ invariant mass spectrum of the reaction $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 \Lambda$.

2. Cross section

In this section, we present the total cross sections corresponding to the processes shown in Figs. 1 and 2. As discussed in the previous section, the results depend on the cutoff parameter introduced in the form factor for the exchanged particles. This parameter typically affects the overall magnitude of the final result. Empirically, the cutoff Λ_2 is usually about 0.4–1.0 GeV larger than the mass of the exchanged particle.

In Fig. 9, we show three representative curves for Λ'_2 ranging from 0.4 to 1.0 GeV in the process $\bar{\nu}_\mu + p \rightarrow \mu^+ + \bar{D}^0 + \Lambda$, where we define $\Lambda'_2 \equiv \Lambda_2 - m_\pi$ to make the effect more evident. As can be seen, the variation of Λ'_2 within this range changes the total cross section by roughly one order of magnitude, indicating that the cutoff has only a modest impact on the result.

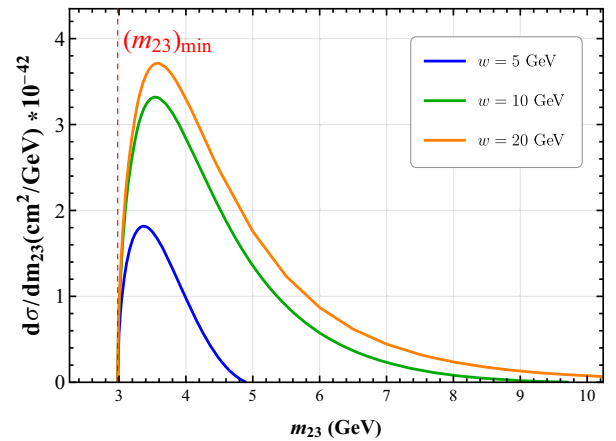


FIG. 8. Invariant mass spectra of $\bar{D}\Lambda$ corresponding to the Dalitz plots in Fig. 7, for total energies w of 5, 10, and 20 GeV, respectively.

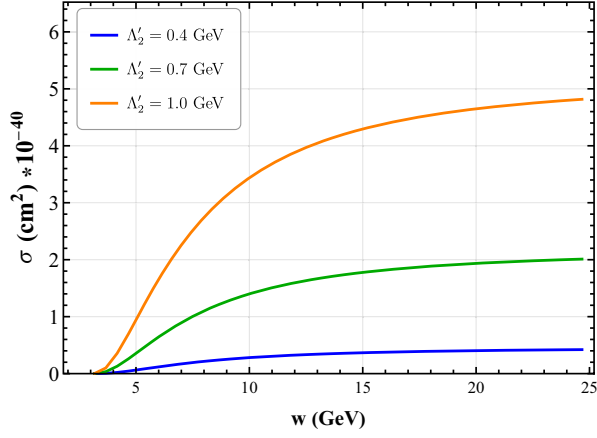


FIG. 9. Effect of the cutoff on the process $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 \Lambda$. The parameter $\Lambda'_2 = \Lambda_2 - m_\pi$ varies from 0.4 to 1.0 GeV.

In Fig. 10, we present the total cross sections for the processes $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 \Lambda$ and $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 n$, along with the individual channel contributions for $\bar{\nu}_\mu p \rightarrow \mu^+ K^0 \Lambda$ (shown in the inset), with the cutoff parameter fixed at $\Lambda_2 = 1$ GeV. The two $\bar{D}^0$ production processes share the same t -channel structure and, despite differing CKM matrix elements, yield cross sections of the same order of magnitude due to the significant difference in the masses of the exchanged mesons in the t -channel. For the process $\bar{\nu}_\mu p \rightarrow \mu^+ K^0 \Lambda$, the cross section is dominated by the contact term throughout the entire energy range. While the other channel contributions tend to saturate at higher energies, the contact term continues to grow within the energy range considered, driving the steady increase of the total cross section.

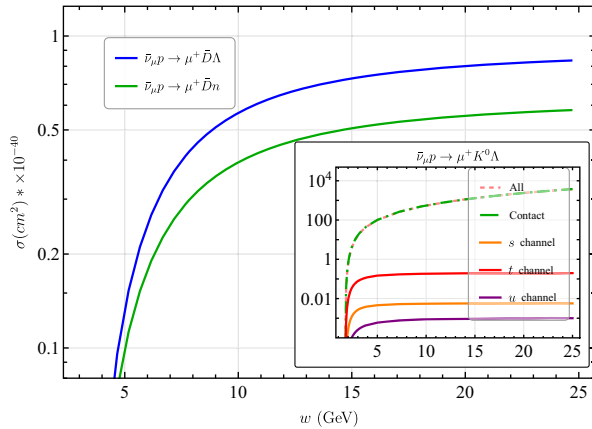


FIG. 10. Total cross sections of the processes $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 \Lambda$ and $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 n$ from threshold up to a total energy of $w = 25$ GeV, with the cutoff parameter fixed at $\Lambda_2 = 1$ GeV. The inset shows the total cross section of the process $\bar{\nu}_\mu p \rightarrow \mu^+ K^0 \Lambda$, along with the individual contributions from the contact term, s -, t -, and u -channel diagrams.

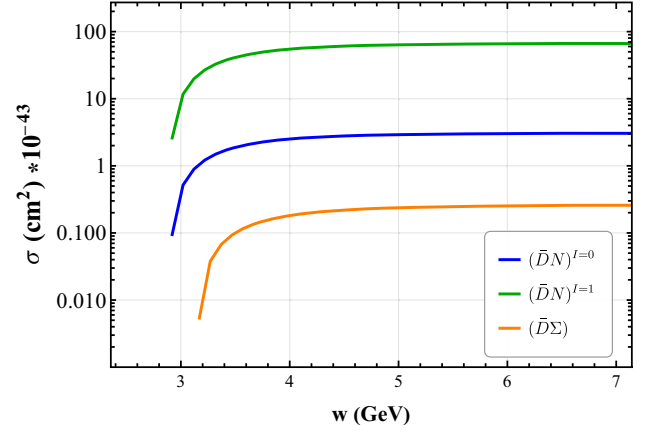


FIG. 11. Total cross section of the process $\sigma(\bar{\nu}_\mu p \rightarrow \mu^+ (\bar{D}N))$ and $\sigma(\bar{\nu}_\mu p \rightarrow \mu^+ (\bar{D}\Sigma))$, corresponding to Fig. 2, from threshold up to $w = 7$ GeV.

Due to the small coupling constants at the hadron vertices in the loop diagrams, the cross sections $\sigma(\bar{\nu}_\mu p \rightarrow \mu^+ (\bar{D}N))$ and $\sigma(\bar{\nu}_\mu p \rightarrow \mu^+ (\bar{D}\Sigma))$ are significantly smaller than the tree-level results, as shown in Fig. 11. The results for the two isospin channels, $I = 0$ and $I = 1$, are presented separately. In addition, the cross section for the $(\bar{D}\Sigma)$ channel is further suppressed by approximately one order of magnitude compared to that of the $(\bar{D}N)$ channel.

Finally, we present the differential cross section for the process $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 n$. Figure 12 shows the angular differential cross sections of the three final-state particles at a total center-of-mass energy of $w = 5$ GeV. The solid curves represent the results calculated in the center-of-mass frame, while the dashed curves correspond to those transformed to the laboratory frame. As the incident

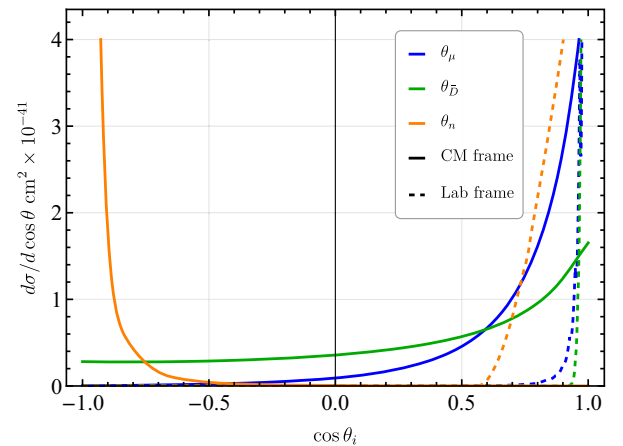


FIG. 12. Differential cross sections for $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 n$ at $\sqrt{s} = 5$ GeV in the center-of-mass (solid) and laboratory (dashed) frames. The variable θ_i denotes the scattering angle of the outgoing particle i (μ^+ , $\bar{D}^0$, or n) measured with respect to the incident $\bar{\nu}_\mu$ beam direction.

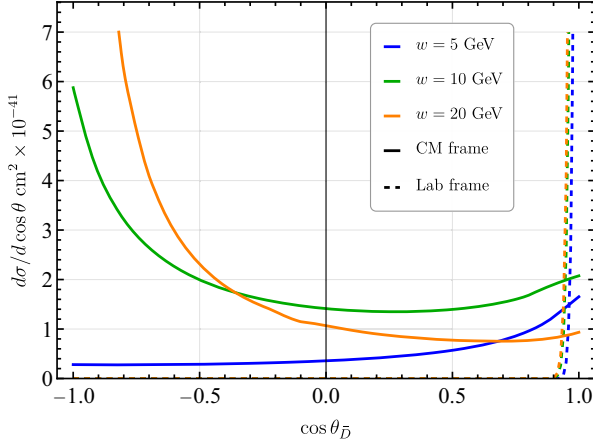


FIG. 13. Differential cross sections of the $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 n$ process as functions of the $\theta_{\bar{D}^0}$ scattering angle at different center-of-mass energies, illustrating the evolution of the angular distribution. Solid (dashed) lines correspond to the center-of-mass (laboratory) frame.

antineutrino is massless, the Lorentz transformation causes the baryon momenta in the laboratory frame to be strongly boosted in the forward direction. Consequently, the differential cross sections are concentrated in the region where $\cos\theta \approx 1$. As the energy increases, however, the $\bar{D}^0$ tends to be aligned with the incident proton direction, as illustrated in Fig. 13. This behavior reflects the kinematic constraints of the reaction and the increasing phase-space availability at higher energies. Such a trend indicates that forward production of heavy mesons becomes more dominant, which is consistent with expectations from hadronic models and may serve as a useful reference for future experimental searches.

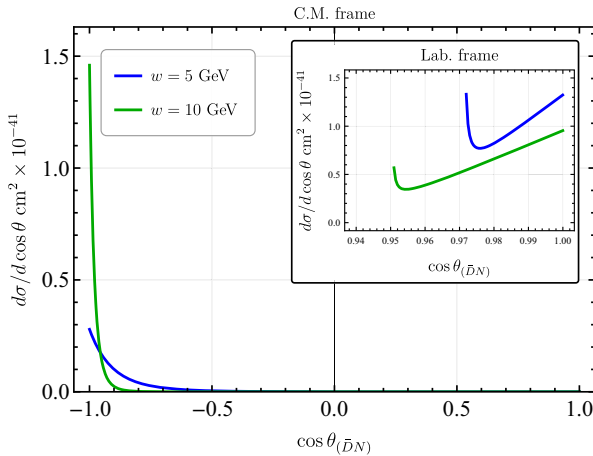


FIG. 14. Angular differential cross sections for the process $\bar{\nu}_\mu p \rightarrow \mu^+ (\bar{D}N)$ with isospin $I = 0$. Results are shown for total center-of-mass energies $w = 5$ GeV and $w = 10$ GeV. The main panel shows the results in the center-of-mass (C.M.) frame, while the inset illustrates the corresponding distributions in the laboratory (Lab.) frame.

In addition, we present the angular differential cross sections for the two-body final state process $\bar{\nu}_\mu p \rightarrow \mu^+ (\bar{D}N)$, where $(\bar{D}N)$ denotes a possible molecular state. Here, the case with isospin $I = 0$ is shown as a representative example, as shown in Fig. 14. It can be seen that, in the center-of-mass frame, the $(\bar{D}N)$ system is predominantly scattered in the backward direction. For the same reason as in the three-body case, due to the massless nature of the incident antineutrino, the Lorentz boost to the laboratory frame results in the $(\bar{D}N)$ momenta being strongly forward-focused, leading to a narrow angular distribution around $\cos\theta \approx 1$.

In particular, the kinematic features observed here provide valuable input for exploring the possible formation of bound states or pentaquark candidates, since the relative momentum distributions of the baryon-meson system play a key role in assessing the likelihood of molecular or multi-quark configurations in neutrino-proton scattering.

IV. SUMMARY

In this work, we have investigated exclusive hyperon, charmed baryon, and potential hadronic molecular state production in antineutrino-proton scattering within the framework of effective Lagrangians and chiral perturbation theory. Particular attention has been devoted to the $\bar{D}N$ final state, which could serve as a candidate component of hadronic molecular states. By constructing the relevant tree-level and loop amplitudes, incorporating phenomenological form factors, and employing lattice QCD-inspired parametrizations for semileptonic vertices, we have systematically calculated Dalitz plots, invariant mass spectra, and total cross sections for representative channels.

Our results indicate that processes such as $\bar{\nu}_\mu p \rightarrow \mu^+ K^0 \Lambda$, $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 \Lambda$, and $\bar{\nu}_\mu p \rightarrow \mu^+ \bar{D}^0 n$ yield cross sections of comparable magnitudes, with differences arising primarily from vertex form factors and CKM suppression effects. The molecular production channels $\bar{\nu}_\mu p \rightarrow \mu^+ (\bar{D}N)$, $\bar{\nu}_\mu p \rightarrow \mu^+ (\bar{D}\Sigma)$ are found to be significantly suppressed relative to the tree-level processes, yet still provides a potentially measurable signature at high-statistics neutrino facilities.

These findings highlight the role of neutrino-proton scattering as a complementary probe of hadronic dynamics in both the strange and charm sectors. In particular, future neutrino experiments with enhanced luminosity and detector precision could explore the formation of exotic baryonic states, offering new insights into the structure of hyperons, charmed baryons, and hadronic molecules.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [32].

APPENDIX A: LAGRANGIANS AND AMPLITUDES

Here we provide the explicit Lagrangians and amplitudes used in our calculation. The hadronic Lagrangians from Eqs. (9) and (10) are

$$\mathcal{L}_{WPBB} = -i \frac{g}{2\sqrt{2}} \frac{V_{xy}}{F_0} \bar{B} \gamma_\mu (C_V^{WPBB} - C_A^{WPBB} \gamma_5) B W^\mu P, \quad (\text{A1})$$

$$\mathcal{L}_{WPP} = -i \frac{g}{2\sqrt{2}} C_{WPP} V_{xy} W^\mu (\partial_\mu P_1 P_2 - P_1 \partial_\mu P_2), \quad (\text{A2})$$

$$\mathcal{L}_{BBW} = -\frac{g}{2\sqrt{2}} \bar{B} \gamma_\mu (C_V^{BBW} - C_A^{BBW} \gamma_5) B W^\mu, \quad (\text{A3})$$

where g is the $SU(2)_L$ gauge coupling constant, which also appears in the leptonic charged-current vertex. The strong baryon-baryon-pseudoscalar (BBP) Lagrangian reads

$$\begin{aligned} \mathcal{L}_{BBP} &= -\frac{D}{2F_0} \text{Tr}(\bar{B} \gamma_\mu \gamma_5 \{\partial^\mu P, B\}) - \frac{F}{2F_0} \text{Tr}(\bar{B} \gamma_\mu \gamma_5 [\partial^\mu P, B]) \\ &= -\frac{C^{BBP}}{2F_0} \bar{B} \gamma_\mu \gamma_5 \partial^\mu P B, \end{aligned} \quad (\text{A4})$$

where C^{BBP} denotes the appropriate linear combination of D and F for each baryon-meson channel. The numerical values of all coupling constants entering our calculation are collected in Table I.

We label the external momenta as follows (see also Fig. 1): k_1 and k_2 denote the momenta of the incoming

antineutrino and baryon, respectively, while p_1 , p_2 and p_3 denote those of the outgoing charged lepton, pseudoscalar meson and baryon, respectively. The tree-level amplitude in Fig. 1 takes the factorized form

$$i\mathcal{M} = -\frac{G_F}{\sqrt{2}} L^\mu H_\mu, \quad (\text{A5})$$

where the leptonic current is

$$L^\mu = \bar{v}(k_1) \gamma^\mu (1 - \gamma_5) v(p_1), \quad (\text{A6})$$

and H_μ is the hadronic current. The intermediate-state momenta for the s -, t -, and u -channel diagrams are

$$q_s = k_2 + q_1 = p_2 + p_3 \quad (\text{A7})$$

$$q_t = q_1 - p_2 = p_3 - k_2 \quad (\text{A8})$$

$$q_u = p_3 - q_1 = k_2 - p_2 \quad (\text{A9})$$

with $q_1 = k_1 - p_1$ the momentum transfer carried by the W boson. The explicit hadronic currents for each topology are

$$H_\mu^{\text{contact}} = \frac{V_{xy}}{F_0} \bar{u}(p_3) \gamma_\mu (C_V^{WPBB} - C_A^{WPBB} \gamma_5) u(k_2), \quad (\text{A10})$$

$$\begin{aligned} H_\mu^s &= \frac{C^{BBP}}{2F_0} \bar{u}(p_3) \not{p}_2 \gamma_5 \frac{\not{q}_s + M_s}{q_s^2 - M_s^2} \gamma_\mu \\ &\times (C_V^{BBW} - C_A^{BBW} \gamma_5) u(k_2) f_1(q_s^2), \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} H_\mu^t &= C_{WPP} V_{xy} (p_2 - q_t)_\mu \frac{1}{q_t^2 - m_t^2} \frac{C^{BBP}}{2F_0} \\ &\times \bar{u}(p_3) \not{q}_t \gamma_5 u(k_2) f_2(q_t^2), \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} H_\mu^u &= \frac{C^{BBP}}{2F_0} \bar{u}(p_3) \gamma_\mu (C_V^{BBW} - C_A^{BBW} \gamma_5) \\ &\times \frac{\not{q}_u + M_u}{q_u^2 - M_u^2} \not{p}_2 \gamma_5 u(k_2) f_1(q_u^2), \end{aligned} \quad (\text{A13})$$

where M_s (M_u) is the mass of the intermediate baryon in the s -channel (u -channel), m_t is the mass of the exchanged meson in the t -channel, and f_1 , f_2 are hadronic form factors introduced to account for the off-shell behavior of the intermediate states.

For the t -channel diagrams in which the WPP vertex is replaced by a transition form factor obtained from lattice QCD (the $D \rightarrow P$ transition), the hadronic current becomes

$$\begin{aligned} H^\mu &= V_{xy} \left[f_+(q_1^2) \left(p_2^\mu + q_2^\mu - \frac{m_{P_2}^2 - m_{Q_2}^2}{q_1^2} q_1^\mu \right) \right. \\ &\quad \left. + f_0(q_1^2) \frac{m_{P_2}^2 - m_{Q_2}^2}{q_1^2} q_1^\mu \right] \frac{1}{q_t^2 - m_{q_t}^2} f_2(q_t^2) \\ &\times \frac{C^{BBP}}{2F_0} \bar{u}(p_3) \not{q}_t \gamma_5 u(k_2), \end{aligned} \quad (\text{A14})$$

TABLE I. Coupling constants used in this work.

C_V^{KKW}	C_A^{KKW}	$C_V^{p\Lambda KW}$	$C_A^{p\Lambda KW}$
1	0	$-\frac{\sqrt{3}}{2}$	$-\frac{D}{2\sqrt{3}} - \frac{\sqrt{3}F}{2}$
C_V^{pnW}	C_A^{pnW}	$C_V^{\Sigma^+ \Lambda W}$	$C_A^{\Sigma^+ \Lambda W}$
$\frac{1}{2}$	$\frac{D+F}{2}$	0	$\frac{D}{\sqrt{6}}$
$C^{n\Lambda K}$	$C^{p\Sigma^+ K}$	$C^{p\Lambda K}$	$C^{p n \pi}$
$-\frac{D}{\sqrt{3}} - \sqrt{3}F$	$\sqrt{2}(D - F)$	$-\frac{D}{\sqrt{3}} - \sqrt{3}F$	$\sqrt{2}(D + F)$

where f_+ and f_0 are the vector and scalar form factors of the $D \rightarrow P$ transition, respectively.

APPENDIX B: LOOP-INDUCED FORM FACTOR

In the calculation of the diagram shown in Fig. 2(b), a triangle loop integration must be evaluated.

The resulting vertex can be expressed in the following effective form:

$$\begin{aligned} \mathcal{L}_{B'BV} &= \bar{B}' \left(g_{B'BV} \gamma_5 \gamma^\mu + \frac{f_{B'BV}}{m_1 - m_2} \gamma_5 \sigma^{\mu\nu} \partial_\nu \right) V_\mu B + \text{H.c.} \\ &= \bar{B}' \gamma_5 \Gamma^\mu V_\mu B + \text{H.c.}, \end{aligned} \quad (\text{B1})$$

where B' and B denote baryon fields with quantum numbers $J^P = \frac{1}{2}^-$ and $\frac{1}{2}^+$, respectively, m_1 and m_2 are their corresponding masses, and V denotes the W -boson field. The vertex function Γ^μ encodes the loop structure and serves as an effective form factor. It is given by

$$\begin{aligned} \Gamma^\mu &= \int d^4q \frac{A^\mu}{(q^2 - m^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)} \frac{\Lambda_1^4}{(q^2 - m^2)^2 + \Lambda_1^4} \\ &\times \exp\left(-\frac{(q_E - \omega_{21} p_E)^2}{\Lambda^2}\right), \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} A^\mu &= (\not{q}_2 + m_2) \cdot \not{q} \left[f_+(p_1^2)(q + q_1)^\mu \right. \\ &\quad \left. + (f_0(p_1^2) - f_+(p_1^2)) \frac{m_1^2 - m^2}{p_1^2} p_1^\mu \right], \end{aligned} \quad (\text{B3})$$

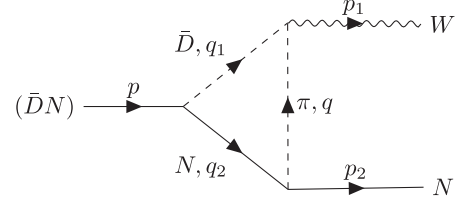


FIG. 15. Triangle loop diagram for the $(\bar{D}N)$ – W – N vertex with meson exchange.

The momentum labels and particle assignments are indicated in Fig. 15. Here $f_+(p_1^2)$ and $f_0(p_1^2)$ are the vector and scalar form factors of the $D \rightarrow P$ transition, and $\omega_{21} = m_2/(m_1 + m_2)$. The last two factors in the integrand of Γ^μ are regularization form factors introduced at the pion and hadronic molecule vertices, respectively; their explicit forms are given in Eqs. (19) and (20). With these two regularization form factors, the loop integral is rendered ultraviolet finite and can be evaluated numerically.

To simplify its implementation in subsequent calculations—such as those for the tree-level diagrams—we adopt an interpolation-based approximation. Specifically, we first evaluate the integral numerically to obtain a set of data points $\{q_i^2, \Gamma^\mu(q_i^2)\}$. To improve the uniformity of the data distribution, we transform $\Gamma^\mu(q_i^2)$ to $\log[\Gamma^\mu(q_i^2)]$, thereby reducing the relative variation among data points. Next, we construct an interpolation function $f(q^2)$ from the transformed dataset $\{q_i^2, \log[\Gamma^\mu(q_i^2)]\}$. We employed the Interpolation function in *Mathematica*, using the default InterpolationOrder. Finally, in subsequent calculations, we use $\exp[f(q^2)]$ as an efficient representation of the original form factor.

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