

Good qualities of the weak axion

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The presence of a topological susceptibility in the electroweak sector of the Standard Model motivates the existence of a *good quality* weak axion a_W , associated with the spontaneous breaking of $B + L$. Its anomalous couplings and tiny mass, generated from electroweak instantons, render a_W photophobic. We find that the strongest bound on the associated decay constant f_W stems from a loop-induced coupling to electrons, leading to $f_W \gtrsim 1000$ TeV from stellar cooling. Spontaneous breaking of the Abelian $B + L$ symmetry induces proton decay via higher-dimensional operators controlled by a new physics scale Λ . Existing Super-Kamiokande limits on these decay channels constrain the new physics scale to be $\Lambda \gtrsim 10^{12}$ GeV. The characteristic channel $p \rightarrow e^+ a_W$ and other possible operators mediating interactions with the Standard Model fields yield signals which are not detectable within the allowed parameter space. Future proton decay searches at the next generation of neutrino experiments offer the most promising avenues to test the good qualities of the weak axion paradigm.

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Introduction. The vacuum structure of non-Abelian Yang-Mills gauge theories is rich and features a nonvanishing topological susceptibility. For this reason, an $SU(N)$ gauge theory admits a topological θ -term,

$$\mathcal{L}_{\text{YM}} \supset \theta \epsilon^{\mu\nu\rho\sigma} \text{Tr} \mathcal{F}_{\mu\nu} \mathcal{F}_{\rho\sigma}, \quad (1)$$

where $\mathcal{F}_{\mu\nu}$ is the field strength tensor and the trace acts over the $N^2 - 1$ generators. In the presence of fermions, the θ -angle is tied to an anomalous global $U(1)$ symmetry [1], and its physical effects manifest through CP violation [2].

In QCD, this structure underlies the strong CP problem [3] and bounds on hadronic electric dipole moments [4] require an unnaturally small effective θ . The Peccei-Quinn (PQ) mechanism [5,6] dynamically relaxes θ via a spontaneously broken anomalous $U(1)_{PQ}$, predicting a light pseudoscalar, the axion. In this sense, $U(1)_{PQ}$ is a “good quality” symmetry: explicit breaking by renormalizable couplings is absent, and residual violations (e.g., from gravity [7]) must be sufficiently suppressed [8–10]. In contrast, the axial $U(1)_A$ is a “bad quality” symmetry as it is explicitly broken by quark masses; it is further spontaneously broken by the quark condensate leading to a would-be Nambu-Goldstone η' that, even with massless quarks, is heavy due to the axial anomaly. For massive quarks, θ becomes physical and its smallness requires tuning between the QCD vacuum angle and phases associated with the quark mass matrices. Generalizations to different matter representations and multisector extensions are reviewed in [11], where also the effective Lagrangian of [4], describing the interactions among baryons and pseudo-Nambu-Goldstone mesons, was corrected.

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An analogous topological structure exists in the electro-weak (EW) sector. The weak $SU(2)_L$ gauge group also admits a θ -term, but within the Standard Model (SM) it is unobservable [12–14] as it can be rotated away by linear combinations of baryon number (B) and lepton number (L) global transformations. More precisely, it is the $B + L$ combination that has an anomaly with $SU(2)_L$ affecting the weak vacuum of the SM. This symmetry remains a good quality one, as long as B and L violations remain small (typically arising from grand unification [15] and Majorana neutrino masses [16]).

Still, the presence of a nontrivial topological susceptibility suggests that a pseudoscalar degree of freedom can saturate the corresponding three-form dynamics [17]. In the language of Higgsed three-forms, an anomalous Abelian symmetry gauges a composite three-form that acquires a mass by “eating” a pseudoscalar, ensuring consistency of the infrared description [18,19]. This logic has motivated the identification of an EW-associated pseudoscalar, sometimes dubbed η_W [20].

At this point it is useful to distinguish two logically separate questions. First, in the “SM-only” case (with $B + L$ a good quality symmetry), one may ask how the $SU(2)_L$ three-form/topological susceptibility is saturated in the deep infrared. Second, one may consider a “beyond-the-SM” scenario in which $U(1)_{B+L}$ is spontaneously broken, thereby introducing a bona fide pseudo-Nambu-Goldstone boson whose mass is generated by weak instantons.

In our recent work [21] we showed that, in the SM with a good quality $B + L$, the η_W , associated with the $SU(2)_L$ three-form, is not an independent propagating singlet: once color confinement and SM gauge invariance are properly enforced, its low-energy manifestation maps onto the pseudoscalar hydrogen ground state rather than a new particle. This identification pertains to the SM-only infrared saturation mechanism discussed in [21]; we emphasize that the present work does not rely on this mapping for any of its phenomenological results. Henceforth, there is no need for a light new degree of freedom that receives a mass only through the anomaly, as argued in [20,22].

The purpose of the present Letter is therefore not to establish the existence of a new propagating η_W within the Standard Model alone. Rather, we investigate the phenomenology of a genuine weak axion arising when $U(1)_{B+L}$ is spontaneously broken beyond the SM. We remain interested in a generic scenario, described by a minimal effective field theory (EFT) that could naturally arise from many extensions of the SM, without committing to any specific model building direction (see, e.g., [23]).

Henceforth, in this Letter we investigate the good quality weak axion which we denote by a_W . We introduce it as a pseudo-Nambu-Goldstone boson of a spontaneously broken $U(1)_{B+L}$, as this is the only good quality global charge in the SM with an irreducible $SU(2)_L$ gauge anomaly. The weak axion (a_W), therefore, couples to EW gauge bosons

and fermions f as follows:

$$\mathcal{L} \supset \frac{a_W}{f_W} \left[c_W \frac{\alpha_2}{8\pi} W_{\mu\nu}^a \tilde{W}^{a\mu\nu} + c_B \frac{\alpha_Y}{8\pi} B_{\mu\nu} \tilde{B}^{\mu\nu} \right] + \sum_f c_f \frac{\partial_\mu a_W}{f_W} \bar{f} \gamma^\mu \gamma_5 f, \quad (2)$$

where $\tilde{W} = \frac{1}{2}\epsilon W$ and $\tilde{B} = \frac{1}{2}\epsilon B$, while f_W is the a_W decay constant which we assume is common to the gauge and fermion sectors. The anomaly coefficients c_W and c_B can be computed, leading to $c_W = -c_B = 1$ in our conventions. We note that this a_W has tree-level couplings to WW , ZZ and $Z\gamma$. However, as B and L do not have a gauge anomaly with QED, the a_W coupling to photons vanishes as $c_\gamma = c_W + c_B = 0$. Nevertheless, a coupling is generated at loop level [24], and it is proportional to the square of the a_W mass m_a^2 . If the a_W mass is smaller than all the masses of the SM, as expected in a good quality case where the a_W mass only stems from suppressed weak instantons, the coupling to photons is highly suppressed. Hence, the a_W is essentially “photophobic.” Couplings to fermions are more model dependent, and they may be absent at leading order but generated at loop level with a logarithmically enhanced coupling [24]. A photophobic axionlike particle (ALP) with such couplings has been studied in [25]. In the case of the a_W , there might also exist a coupling to SM fermions which stems from the $B + L$ charge of the source field: the minimal choice would be to couple it to an operator containing three quarks and one lepton, $qqql$ [20]. This coupling generates proton decay processes which we will study in the following section.

More specifically, we construct an effective model for the a_W and study the related phenomenological constraints. The motivation is twofold: (i) $U(1)_{B+L}$ is the unique good quality anomalous global symmetry of the SM, so its spontaneous breaking yields a well-defined minimal EFT with an unavoidable pseudoscalar; (ii) the resulting anomaly structure makes the axion essentially photophobic ($c_\gamma = 0$ at leading order) while generating a log-enhanced loop coupling to electrons, and the same construction generically permits baryon-violating portals testable by proton decay. Since we do not commit to a specific a_W the present analysis is general encompassing also the light η_W case investigated in [20,22,26].

The weak axion. The a_W can be introduced beyond the SM, as mentioned in [20], as a pseudo-Nambu-Goldstone boson arising from a new scalar field Φ_W that carries a global $B + L$ charge and acquires a vacuum expectation value (VEV),

$$\Phi_W = \langle \Phi_W \rangle e^{ia_W/(\sqrt{2}f_W)}, \quad (3)$$

where f_W is the axion decay constant. The two scales, the VEV and the decay constant, can be related. However, their

precise relation depends on the nature of the a_W : If Φ_W is an elementary weakly coupled scalar, then $\langle\Phi_W\rangle = f_W$. If a_W is generated by a composite sector, then $\langle\Phi_W\rangle > f_W$. We estimate that in such a scenario $\langle\Phi_W\rangle \sim g^* f_W$ with $g^* \sim 4\pi/\sqrt{N}$, N being the number of confining colors. The naive dimensional analysis and large- N counting stems from having the new fermions transforming according to the fundamental representation of an $SU(N)$ gauge group [27,28]. If the new fermions transform according to a generic representation R of the gauge group the scaling becomes $g^* \sim 4\pi/\sqrt{d(R)}$ [29].

The global $U(1)_{B+L}$ charge of Φ_W is communicated to the SM via couplings to the quarks and leptons. A minimal gauge and Lorentz invariant portal-like operator consistent with $U(1)_{B+L}$ is

$$\frac{\Phi_W}{\Lambda^3} qqql, \quad (4)$$

which fixes $Q_{B+L}(\Phi_W) = -2$. Here Λ denotes the ultra-violet scale associated with the dynamics generating the leading $B + L$ -violating portal; we keep Λ as a free UV parameter (not necessarily Planckian), since we do not commit to a specific UV completion. The effective field theory validity requires $\Lambda \gg \langle\Phi_W\rangle$, and our proton decay analysis should be read as an experimental lower bound on Λ for a given $(f_W, \langle\Phi_W\rangle)$. After spontaneous breaking of $B + L$, Eq. (4) yields

$$\frac{\langle\Phi_W\rangle}{\Lambda^3} qqql + i \frac{a_W}{\sqrt{2}f_W} \frac{\langle\Phi_W\rangle}{\Lambda^3} qqql + \dots, \quad (5)$$

where the first term matches the usual dimension-six $qqql$ operator responsible for proton decay. The second term sources a contact vertex for $p \rightarrow e^+ a_W$. In the following, we will investigate the phenomenology of these channels. We note that a lepton portal term, $(\Phi_W/\Lambda_l^2)(lH)(lH)$, is also allowed by the $B + L$ charges and enters one mass dimension lower than Eq. (4). It would contribute to the Weinberg neutrino mass operator [16] at the price of breaking $B - L$. Furthermore, as for any new scalar field, a renormalizable portal coupling to the SM Higgs can be added, $\Phi_W^\dagger \Phi_W H^\dagger H$, which is constrained by the Higgs coupling measurements.

Independent of its origins, the a_W mass is generated by nonperturbative EW instanton effects, leading to a highly suppressed value,

$$m_{a_W} \sim \kappa \frac{v_{EW}^2}{f_W} e^{-\pi/\alpha_2} \sim \kappa \frac{(1 \text{ TeV})}{f_W} \times 6 \times 10^{-29} \text{ eV}, \quad (6)$$

where κ is an $\mathcal{O}(1)$ coefficient. This nonperturbative suppression ensures the mass is far below the EW scale and much smaller than any SM mass (including neutrinos), hence validating a_W 's photophobic nature.

The weak axion couples to SM particles similar to the QCD axion, but without a direct coupling to gluons and photons. The bounds on f_W , therefore, follow those studied in [25] and are dominated by red giant cooling. We will first review such bounds and then discuss the complementary limits on the portal operators, such as that in Eq. (4).

Weak axion photophobia. For the case of the photophobic axionlike particle [25], the a_W has anomaly couplings $c_W = -c_B = 1$ and vanishing couplings to fermions at scales $\sim \langle\Phi_W\rangle$, where $U(1)_{B+L}$ is broken. Below this scale, radiative corrections generate couplings to all SM fermions [24]. For the light masses relevant for the a_W , the most important such coupling is to electrons,

$$\frac{c_e}{f_W} = -\frac{3\alpha^2}{16\pi^2 f_W} \left[\frac{3}{4\sin^4\theta_W} - \frac{Y_l^2 + Y_e^2}{\cos^4\theta_W} \right] \ln\left(\frac{\langle\Phi_W\rangle^2}{m_W^2}\right), \quad (7)$$

where $\alpha \sim 1/137$ is the fine-structure constant at the EW scale, and θ_W is the Weinberg angle, while $Y_l = -1/2$ and $Y_e = -1$ are the hypercharges of the SM lepton fields. This coupling is most strongly constrained by red giant cooling [30], which bounds the effective coupling of electrons to axionlike particles to be $y_e < 2.5 \times 10^{-13}$. In our model, it is defined as $y_e = (2m_e c_e)/f_W$. By using the expression in Eq. (7), we can extract a bound on f_W . In the elementary case ($\langle\Phi_W\rangle = f_W$), we find

$$f_W \gtrsim 1000 \text{ TeV} \quad (\text{red giants}). \quad (8)$$

This bound is larger than the one found in [26], which did not include the log enhancement. Due to the log dependence on the VEV, the bound is similar in the composite a_W case. As stressed above, the coupling to photons is extremely suppressed due to the tiny a_W mass, hence no further bounds arise. In view of the strong lower bound on f_W , it is phenomenologically contrived to identify the weak axion with an electroweak-scale object, as also noted in [26]. We further agree with [26] that by equating a_W with the light η_W [20,22] is contrived because current constraints already place f_W well above the electroweak scale. Consistently with [21], once confinement is included, η_W is not exotic but the CP -odd hydrogen eigenstate.

The a_W could also be directly produced at future colliders via the $Z\gamma$ coupling, leading to the decays of the Z boson into a single photon plus missing energy. At the Tera-Z run of the FCC-ee [31] or CEPC [32], some sensitivity could be achieved, as studied in [33,34]. However, taking the result from [34], we infer that the collider sensitivity can reach up to $f_W \sim 10 \text{ TeV}$, hence remaining within the region excluded by red giants.

Weak axion bounds from proton decay. In this section, we provide estimates of the new physics scales associated with the portals $(\Phi_W/\Lambda^3)qqql$ and $(\Phi_W/\Lambda_l^2)(lH)(lH)$.

We assume that a single operator dominates and neglect renormalization group (RG) running between the UV scale and the hadronic scale and effects from nuclear matrix elements. We estimate that RG and nuclear matrix-element effects can shift the inferred scales by factors of a few.

A distinctive feature of the weak axion, absent for the QCD axion and generic ALPs, is the portal in Eq. (4). Once Φ_W acquires a VEV, it induces the grand-unified-theory-like four-fermion operator

$$\frac{\langle \Phi_W \rangle}{\Lambda^3} qqql, \quad (9)$$

which mediates proton decay. The most stringent current bound comes from Super-Kamiokande's $p \rightarrow e^+ \pi^0$ search [35], $\tau_{p \rightarrow e^+ \pi^0} \geq 1.4 \times 10^{34}$ yr, implying [36]

$$\sqrt{\frac{\Lambda^3}{\langle \Phi_W \rangle}} \geq 2.4 \times 10^{16} \text{ GeV}. \quad (10)$$

Consistency of the EFT requires $f_W \simeq \langle \Phi_W \rangle \leq \Lambda$. Assuming $\langle \Phi_W \rangle \simeq f_W = 10^6$ GeV implies $\Lambda \gtrsim 8.3 \times 10^{12}$ GeV. We note that if $\langle \Phi_W \rangle = 4\pi f_W$, then Λ increases by a factor around 2 at most. Looking ahead, adopting the Hyper-Kamiokande 10-yr sensitivity $\tau_{p \rightarrow e^+ \pi^0} = 5 \times 10^{34}$ yr [37], we obtain the projection

$$\sqrt{\frac{\Lambda^3}{\langle \Phi_W \rangle}} \geq 3 \times 10^{16} \text{ GeV}, \quad (11)$$

which for the scenario $\langle \Phi_W \rangle = f_W = 10^6$ GeV implies $\Lambda \gtrsim 9.7 \times 10^{12}$ GeV.

A more direct signature arises from the first term in the expansion of Eq. (4) in the a_W field, which produces a direct coupling of a_W to proton and electron:

$$\frac{\langle \Phi_W \rangle}{\sqrt{2} f_W} \frac{a_W}{\Lambda^3} qqql \sim \frac{\langle \Phi_W \rangle}{\sqrt{2} f_W} \frac{\Lambda_{\text{QCD}}^3}{\Lambda^3} a_W \bar{\psi}_e \psi_p, \quad (12)$$

which induces the proton decay $p \rightarrow e^+ a_W$. The resulting Yukawa-type coupling gives

$$\Gamma_{p \rightarrow e^+ a_W} = \frac{m_p}{8\pi} \frac{\Lambda_{\text{QCD}}^6}{\Lambda^6} \frac{\langle \Phi_W \rangle^2}{f_W^2}. \quad (13)$$

Super-Kamiokande's search for a (quasi)massless axion yields $\tau_{p \rightarrow e^+ a_W} \geq 7.9 \times 10^{32}$ yr [38]. This implies

$$\Lambda \left(\frac{f_W}{\langle \Phi_W \rangle} \right)^{1/3} \geq 3.4 \times 10^{10} \text{ GeV}. \quad (14)$$

Assuming $\langle \Phi_W \rangle = f_W$, we find $\Lambda \gtrsim 3.4 \times 10^{10}$ GeV. Adopting a conservative, background-limited projection

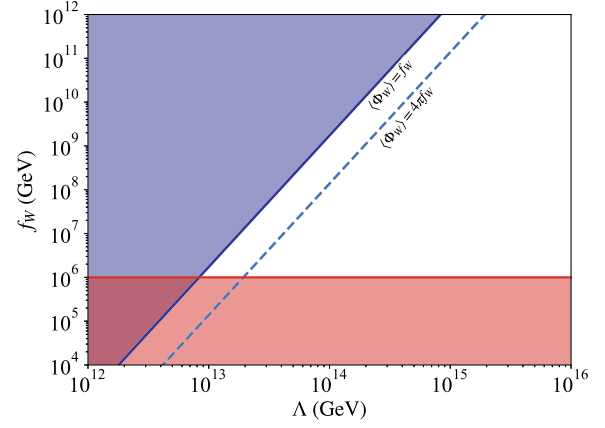


FIG. 1. Exclusion limits on the $f_W - \Lambda$ plane from red giant cooling (red) and proton decay (blue) from Super-Kamiokande. The solid lines correspond to the elementary case ($\langle \Phi_W \rangle = f_W$) and the dashed one to the composite one ($\langle \Phi_W \rangle = 4\pi f_W$).

for Hyper-Kamiokande with 10 years of running, assuming the sensitivity reached is $\tau_{p \rightarrow e^+ a_W} \simeq 2.1 \times 10^{33}$ yr, we find

$$\Lambda \left(\frac{f_W}{\langle \Phi_W \rangle} \right)^{1/3} \geq 4.0 \times 10^{10} \text{ GeV}. \quad (15)$$

We find that the constraint from $p \rightarrow e^+ \pi^0$ is strictly stronger than that from $p \rightarrow e^+ a_W$. Therefore, any parameter choice $(\Lambda, f_W, \langle \Phi_W \rangle)$ that satisfies $p \rightarrow e^+ \pi^0$ automatically satisfies $p \rightarrow e^+ a_W$. The allowed parameter space is illustrated in Fig. 1.

The $U(1)_{B+L}$ charge of Φ_W also allows one to add a lepton portal coupling

$$\frac{\Phi_W}{\Lambda_l^2} (lH)(lH), \quad (16)$$

which contributes to Majorana neutrino masses [the scale Λ_l may be different from the scale in Eq. (4) as the lepton portal breaks $B - L$]. As the absolute mass scale of neutrino masses is constrained by cosmological observations to be $m_\nu \lesssim 0.1$ eV [39], we find

$$\Lambda_\ell \gtrsim \sqrt{\frac{\langle \Phi_W \rangle v^2}{m_\nu}} \approx 1.7 \times 10^{10} \text{ GeV}, \quad (17)$$

where we assumed $\langle \Phi_W \rangle = 10^6$ GeV. We note that this bound is still weaker than the one from proton decay.

Finally, a quartic portal with the Higgs, $\lambda_h \Phi_W^\dagger \Phi_W H^\dagger H$, would affect Higgs couplings through mixing and threshold corrections. While Higgs coupling measurements constrain any induced mixing to be tiny, the large hierarchy $v_{\text{EW}}/\langle \Phi_W \rangle \lesssim 10^{-4}$ already drives collider-visible effects below the projected sensitivity [40]. However, a stronger constraint arises from the requirement of naturalness.

A nonzero portal coupling generates a tree-level contribution to the Higgs mass parameter proportional to the heavy scale $\langle\Phi_W\rangle$. To avoid fine-tuning,

$$\delta m_H^2 \sim \lambda_h \langle\Phi_W\rangle^2 \ll m_H^2 \Rightarrow \lambda_h \ll \frac{m_H^2}{\langle\Phi_W\rangle^2}, \quad (18)$$

which for $\langle\Phi_W\rangle \sim 10^6$ GeV yields $\lambda_h \lesssim 10^{-8}$ (up to order-one factors). Thus, avoiding fine-tuning pushes λ_h to values for which any Higgs-portal phenomenology is automatically unobservable in the parameter space considered here.

Conclusions and outlook. In this Letter we investigated a good quality weak axion, the a_W , realized as the pseudo-Nambu-Goldstone boson of a spontaneously broken $U(1)_{B+L}$ at the scale f_W . Through its $SU(2)_L$ anomaly, a_W dynamically relaxes the weak topological angle rendering the corresponding susceptibility unphysical. The anomaly coefficients, together with the tiny mass, make a_W photophobic, while loops generate a log-enhanced coupling to fermions.

The motivation for studying this minimal EFT is that $U(1)_{B+L}$ is the unique good quality anomalous global symmetry of the SM. If it is spontaneously broken beyond the SM, a weak axion is unavoidable and its anomaly structure implies a distinctive phenomenology: it is photophobic at leading order ($c_\gamma = 0$) but acquires a log-enhanced loop coupling to electrons, while the same construction generically permits baryon-violating portals that can be constrained by proton decay.

The constraints on the a_W parameter space are summarized in Fig. 1. The loop-induced coupling to electrons

allows stellar cooling bounds from red giants to constrain $f_W \gtrsim 10^6$ GeV. A distinctive feature of the $B + L$ construction is the portal $(\Phi_W/\Lambda^3)qqql$, which after symmetry breaking induces a dimension-six baryon-violating operator. Using Super-Kamiokande limits on $p \rightarrow e^+\pi^0$, we obtain $\Lambda \gtrsim 8.3 \times 10^{12}$ GeV for $f_W = 10^6$ GeV. The alternative channel $p \rightarrow e^+a_W$ yields a parametrically weaker constraint that is automatically satisfied whenever $p \rightarrow e^+\pi^0$ is satisfied. Looking ahead, Hyper-Kamiokande projections offer a mild improvement, pushing Λ into the 10^{13} GeV range for $f_W \sim 10^6$ GeV. We also examined other portals. The lepton portal $(\Phi_W/\Lambda_\ell^2)(lH)(lH)$ contributes to Majorana neutrino masses and the upper bound on neutrino masses from cosmology requires $\Lambda_\ell \gtrsim 10^{10}$ GeV for $f_W = 10^6$ GeV. A quartic Higgs portal $\lambda_h \Phi_W^\dagger \Phi_W H^\dagger H$ is likewise innocuous in the relevant region: collider-visible effects are suppressed by $v_{EW}/\langle\Phi_W\rangle$ and, independently, the naturalness requirement of the Higgs mass parameter drives λ_h to values $\lesssim 10^{-8}$.

In summary, the a_W is a good quality solution and is essentially photophobic, but its baryon-violating portal makes proton decay the most incisive probe. Further improvements in stellar cooling systematics and proton decay searches offer the most promising avenues to test the $B + L$ weak axion paradigm.

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