

Unitarization of $R + \alpha R^2$ gravity

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We make use of the improved K-matrix algorithm to obtain unitarized amplitudes in $R + \alpha R^2$ gravity (the so-called Starobinsky model, of cosmological relevance). The procedure is of some complexity because infrared divergences are present and need to be properly regulated. We focus on the behavior of a bona fide scalar resonance, known to exist in this model, and compare it to an apparent resonance detected in previous studies, thus confirming that the latter seems to be an artifact due to the introduction of the infrared regulator. We analyze the existence of other dynamical resonances and dwell on the amplitudes made unitary by this procedure.

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I. INTRODUCTION

It is widely understood that nonrenormalizable theories with a bad ultraviolet behavior may exhibit dynamical resonances that, when properly included, help mitigate their behavior at high energies. Unitarization of the partial wave amplitudes is probably the preferred method to unveil such resonances. In Refs. [1,2] the unitarization of Einstein-Hilbert gravity was carried out in the framework of the inverse amplitude method (IAM) and the one-loop-level graviton-graviton scattering in pure gravity was unitarized. Although the resulting amplitude was indeed unitary, we found no evidence of any genuine dynamical resonance, a fact that contradicted the conclusions of a previous study regarding a singularity found in the second Riemann sheet in the complex s plane (termed as “graviball” by the authors in Refs. [3–5]). We concluded that such singularity—that actually lies quite far away from the real s axis anyway—vanished as the infrared cutoff was removed. In this work we want to extend this study to an extension of the Einstein-Hilbert Lagrangian; namely, one that includes, in addition to the standard term, an R^2 piece. In fact, a gravity theory with a Lagrangian density of the form $R + \alpha R^2$ is known to possess a scalar degree of freedom

in addition to the familiar two helicities of the graviton. This scalar degree of freedom has been termed by some authors the “scalatron.” One of our purposes is to show how the unavoidable introduction of an infrared cutoff affects very differently both singularities (i.e., the graviball and the scalatron), reinforcing the conclusions in Refs. [1,2]. In addition we also want to explore possible physical consequences stemming from the unitarized partial waves and amplitudes, in particular, comparing with those of pure Einstein-Hilbert gravity. Although by no means a rigorous result, the emergence of a dynamical resonance would probably signal the existence of new underlying microscopic degrees of freedom. The approach followed here is consistent with assuming that gravity is an effective theory, with operators of increasing dimension that will be expanded around flat Minkowskian space. Specifically, we will search for resonances that are commensurate with the Planck scale. In other words, we do not consider the ultrahigh-energy regime where graviton scattering could possibly be made unitary by appealing to black hole formation and evaporation rather than relying on perturbative gravity.

II. THE GRAVITON

From the perspective of quantum field theory, the graviton that mediates the gravitational interaction is a fundamental particle analogous to the photon or the gauge bosons of the weak theory. It is a massless particle of spin $j = 2$ that has 2 degrees of freedom, corresponding to two possible physical helicity states of the graviton, $\lambda_{\pm} = \pm 2$. In order to establish conventions and properly define our starting point, we rederive the action for linearized gravity

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in a suitable gauge. Our starting point is Einstein-Hilbert action,

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R, \quad (1)$$

where $R = g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar and $R_{\mu\nu}$ is the Ricci tensor. Both quantities are defined from the Riemann tensor and the Christoffel symbols. In our conventions

$$\begin{aligned} R_{\nu}{}^{\mu}{}_{\sigma\rho} &= \partial_{\rho}\Gamma_{\nu\sigma}^{\mu} - \partial_{\sigma}\Gamma_{\nu\rho}^{\mu} + \Gamma_{\alpha\rho}^{\mu}\Gamma_{\nu\sigma}^{\alpha} - \Gamma_{\alpha\sigma}^{\mu}\Gamma_{\nu\rho}^{\alpha}, \\ \Gamma_{\mu\nu}^{\rho} &= \frac{1}{2}g^{\rho\sigma}(\partial_{\mu}g_{\sigma\nu} + \partial_{\nu}g_{\sigma\mu} - \partial_{\sigma}g_{\mu\nu}), \end{aligned} \quad (2)$$

contracting one of the indices in the first line of Eq. (2) such that $R_{\mu\nu} = R_{\mu}{}^{\sigma}{}_{\sigma\nu}$. We now expand Eq. (1) around Minkowski space-time,

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}, \\ g^{\mu\nu} &= \eta^{\mu\nu} - h^{\mu\nu} + h^{\mu\lambda}h_{\lambda}^{\nu} + \dots \end{aligned} \quad (3)$$

with $|h^{\mu\nu}| \ll 1$ and the interpretation of $h^{\mu\nu}$ as the graviton is immediate. The expansion of the metric in Eq. (3) induces, in turn, an expansion of the Ricci tensor in terms of $R_{\mu\nu}^{(n)}$, where $R_{\mu\nu}^{(n)}$ is of order h^n [see Eq. (2)], and the expanded Ricci scalar is

$$R = g^{\mu\nu} R_{\mu\nu} = (\eta^{\mu\nu} - h^{\mu\nu} + \dots)(R_{\mu\nu}^{(1)} + R_{\mu\nu}^{(2)} + \dots). \quad (4)$$

With this the following action for the Einstein gravity in its linearized version is

$$\begin{aligned} S_{\text{EH}} &= -\frac{1}{64\pi G} \int d^4x [\partial_{\mu}h_{\alpha\beta}\partial^{\mu}h^{\alpha\beta} - \partial^{\mu}h\partial_{\mu}h \\ &\quad + 2\partial_{\mu}h^{\mu\nu}\partial_{\nu}h - 2\partial_{\mu}h^{\mu\nu}\partial_{\rho}h_{\nu}^{\rho}], \end{aligned} \quad (5)$$

where we have defined $h \equiv h_{\mu}^{\mu}$, the trace of $h^{\mu\nu}$, and it has been used that $\sqrt{-g} \approx 1 + \frac{1}{2}h + \mathcal{O}(h^2)$. After choosing a reference frame where the expansion in Eq. (3) holds, a residual symmetry of diffeomorphism invariance remains

$$x^{\mu} \rightarrow x'^{\mu} = x^{\mu} + \xi^{\mu}(x) \quad (6)$$

that in the linearized version of gravity implies invariance under

$$h_{\mu\nu} \rightarrow h_{\mu\nu} - (\partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}). \quad (7)$$

The previous quadratic action can be written as

$$\begin{aligned} S_2 &= \frac{1}{2} \int d^4x [-\partial_{\rho}h_{\mu\nu}\partial^{\rho}h^{\mu\nu} + 2\partial_{\rho}h_{\mu\nu}\partial^{\nu}h^{\mu\rho} \\ &\quad - 2\partial_{\nu}h^{\mu\nu}\partial_{\mu}h + \partial^{\mu}h\partial_{\mu}h], \end{aligned} \quad (8)$$

after integration by parts and the field rescaling: $h_{\mu\nu} \rightarrow (32\pi G)^{-1/2}h_{\mu\nu}$. This rescaling means that now $h_{\mu\nu}$ should be thought of as a classical field with canonical mass dimensions. From now on the expansion of the metric on top of a Minkowski background will be

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \kappa h_{\mu\nu}, \\ g^{\mu\nu} &= \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda}h_{\lambda}^{\nu} + \dots, \end{aligned} \quad (9)$$

where

$$\kappa \equiv (32\pi G)^{1/2} \equiv \frac{(32\pi)^{1/2}}{M_{\text{pl}}}. \quad (10)$$

Of the 10 degrees of freedom in the symmetric tensor $h_{\mu\nu}$, which represents the graviton field, only 2 physical degrees of freedom remain after imposing the local symmetries. By applying the gauge invariance described in Eq. (7), one can adopt the Lorentz gauge¹

$$\partial^{\nu}\tilde{h}_{\mu\nu} = 0 \Leftrightarrow \partial^{\nu}h_{\mu\nu} = \frac{1}{2}\partial_{\mu}h, \quad (11)$$

where $\tilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$. These four equations reduce the degrees of freedom to 6 by choosing four independent functions ξ_{μ} satisfying $\square\xi_{\mu} = f_{\mu}(x)$ for any arbitrary $f_{\mu}(x)$ initial field configuration. In this very same gauge, the equations of motion for the graviton in the linearized theory reduce to

$$\square\tilde{h}_{\mu\nu} = 0, \quad (12)$$

which allows us to use four functions ξ_{μ} satisfying $\square\xi^{\mu} = 0$ to further reduce the number of independent degrees of freedom for the graviton to 2. These 2 physical degrees of freedom correspond to two independent helicity states, $\lambda_{\pm} = \pm 2$ that we simply characterize as $\pm$. Gauge fixing is required to find the propagator of the graviton field. The Lorentz gauge can be selected by introducing the following piece:

$$S_{gf} = - \int d^4x (\partial^{\nu}\tilde{h}_{\mu\nu})^2, \quad (13)$$

resulting in a propagator for the graviton $h_{\mu\nu}$ with momentum k ,

$$D_{\mu\nu\rho\sigma}(k) = (\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}) \left(\frac{-i}{k^2 - i\epsilon} \right). \quad (14)$$

After writing the gauge-fixing piece in Eq. (13) in terms of $h_{\mu\nu}$ and some integration by parts, one finds the action

¹Also called the Hilbert gauge, the harmonic gauge, or the De Donder gauge.

$$S = S_2 + S_{gf} = \int d^4x \left[-\frac{1}{2} \partial_\rho h_{\mu\nu} \partial^\rho h^{\mu\nu} + \frac{1}{4} \partial^\mu h \partial_\mu h \right]. \quad (15)$$

We will not consider the interaction with matter, as we are only interested in pure gravity. So much for the quadratic part of the action. Expansion around Minkowski space also introduces triple and quartic vertices (and, in fact, vertices including any number of gravitons, but only the former will

be required in the present study). Namely,

$$S = \int d^4x \left[-\frac{1}{2} \partial_\rho h_{\mu\nu} \partial^\rho h^{\mu\nu} + \frac{1}{4} \partial^\mu h \partial_\mu h + \kappa \bar{S}_3 + \kappa^2 \bar{S}_4 + \dots \right], \quad (16)$$

where

$$\begin{aligned} \bar{S}_3 = & \frac{1}{2} \left(\frac{1}{4} h_\mu^\mu \partial_\nu h_\alpha^\alpha \partial^\nu h_\beta^\beta - h^{\mu\nu} \partial_\mu h^{\alpha\beta} \partial_\nu h_{\alpha\beta} + 2 h^{\mu\nu} \partial_\mu h^{\alpha\beta} \partial_\alpha h_{\nu\beta} - \frac{1}{2} h^{\mu\nu} \partial_\alpha h_{\mu\nu} \partial^\alpha h_\beta^\beta \right), \\ \bar{S}_4 = & \frac{1}{4} \left(-\frac{5}{16} h_\mu^\mu h_\nu^\nu \partial_\alpha h_\beta^\beta \partial^\alpha h_\tau^\tau + \frac{1}{2} h_\mu^\mu h^{\nu\alpha} \partial_\nu h_{\beta\tau} \partial_\alpha h^{\beta\tau} - h_\mu^\mu h^{\nu\alpha} \partial_\nu h^{\beta\tau} \partial_\beta h_{\alpha\tau} \right. \\ & + h_\mu^\mu h^{\nu\alpha} \partial_\beta h_{\nu\tau} \partial^\beta h_\alpha^\tau - \frac{1}{8} h_{\mu\nu} h^{\mu\nu} \partial_\alpha h_\beta^\beta \partial^\alpha h_\tau^\tau + h^{\mu\nu} \partial_\mu h_{\nu\alpha} \partial^\beta h^{\alpha\tau} h_{\beta\tau} \\ & + \frac{1}{4} h^{\mu\nu} \partial_\nu h_\alpha^\alpha h_{\nu\beta} \partial^\beta h_\tau^\tau - 2 h^{\mu\nu} \partial_\mu h^{\alpha\beta} h_{\nu\alpha} \partial^\tau h_{\beta\tau} + h^{\mu\nu} \partial_\mu h_{\alpha\beta} h_{\nu\tau} \partial^\tau h^{\alpha\beta} \\ & \left. - 2 h^{\mu\nu} \partial_\mu h_{\alpha\beta} h^{\alpha\tau} \partial_\tau h_\nu^\beta + h^{\mu\nu} h_{\nu\alpha} \partial_\beta h_{\mu\tau} \partial^\beta h^{\alpha\tau} + 2 h^{\mu\nu} \partial_\nu h_{\alpha\beta} h^{\alpha\beta} \partial^\tau h_{\mu\tau} \right). \end{aligned} \quad (17)$$

Note that a mass term is conspicuously absent. It is actually forbidden by gauge invariance. The absence of a mass for the graviton renders many processes infrared divergent. This is a combined effect of the graviton being a gauge boson, which exchanges in all the three s , t , and u channels and being strictly massless. One may nevertheless be tempted to introduce a mass term to regulate the omnipresent infrared divergences. A popular choice could be the Pauli-Fierz mass [6]

$$\frac{m_g^2}{2} (h^{\mu\nu} h_{\mu\nu} - h^2). \quad (18)$$

This choice allows five possible polarizations to propagate, but at least prevents the propagation of the Boulware-Deser ghost (see Ref. [7]), typically present in massive spin two theories. However, it is known that even in this case the $m_g \rightarrow 0$ limit is not smooth. This is the celebrated Van Dam–Veltman–Zakharov discontinuity [8,9] which makes regulating the infrared with a mass problematic. Other methods have been used in dealing with the infrared singularities, including dimensional regularization and introducing a physical cutoff. In [10] it is shown that, when including bremsstrahlung diagrams, the graviton cross section is finite and it depends on a scale that could be considered a physical cutoff separating IR and UV regions. Some effort with varying degrees of success has been devoted to analyze whether these infrared singularities exponentiate; see, e.g., Ref. [11]. They are certainly ubiquitous and have to be dealt with one way or another. In what follows we will use a physical cutoff to deal with the IR singularities.

III. EFFECTIVE FIELD THEORIES FOR GRAVITONS

The fact that the parameter κ has dimensions of inverse energy, and that the action itself can be constructed as an expansion in this parameter, $\kappa \sim 1/M_{\text{pl}}$, is a clear indication that the quantization of the theory of gravity leads to a nonrenormalizable theory. The resulting theory from this expansion can be rendered finite order by order in κ , requiring at any order only a finite, though not necessarily small, new set of counterterms. From this perspective, we will be dealing anyway with an effective theory that describes the low-energy effects of a more fundamental theory governed by an unknown dynamics, and whose leading-order term is the Einstein-Hilbert action in Eq. (1) after the expansion of the metric. From the previous considerations it becomes apparent that gravity shares some features with other effective theories based on a derivative expansion, such as the nonlinear chiral Lagrangian describing the dynamics of massless Goldstone bosons in strong interactions, including both having massless quanta and an analogous power counting. A discussion about the possibility for the gravitons to be understood as Goldstones coming from a dynamical breaking of some UV theory can, e.g., be found in Refs. [12,13], but several other attempts exist. However, there is a notorious difference between gravity and the pion Lagrangian; namely, unlike the chiral effective theory that is built upon a global symmetry, the chiral symmetry, the Einstein-Hilbert theory has a local gauge symmetry that is responsible for the reduction of the number of degrees of freedom in the low-energy spectrum and the presence

of t and u exchange interactions. As in the case of the chiral Lagrangian, the next order in the expansion contributes to processes at $\mathcal{O}(p^4)$, and the set of operators forming $\mathcal{L}_4$ contains terms with four derivatives. In pure gravity—i.e., in the absence of matter and cosmological constant ($\Lambda = 0$)—at the next order in the chiral counting, we can only construct the following action:

$$S_{\text{NLO}} = \int d^4x \sqrt{-g} [\alpha_1 R^2 + \alpha_2 (R_{\mu\nu})^2 + \alpha_3 (R_{\mu\nu\alpha\beta})^2]. \quad (19)$$

In fact, one can reduce these three operators to two, since they are related by topology through the Gauss-Bonnet term, which is a total derivative (see, for instance, Ref. [7])

$$GB = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2. \quad (20)$$

At the one-loop level, Einstein-Hilbert theory is free from ultraviolet divergences when the equations of motion ($R_{\mu\nu} = 0$) are applied. This is as it should be because the lowest-order equation of motion forces the two remaining operators in $\mathcal{L}_4$ to vanish and one would be left without any counterterms to eliminate a possible divergence, in accordance with the familiar effective field theory counting. As a consequence, if one wants to go beyond Einstein-Hilbert gravity in the spirit of effective theories, the couplings α_i have to be “large,” meaning that the usual chiral counting is not applicable. This would also imply that the $\alpha_i \rightarrow 0$ limit is not necessarily smooth (just like the $m_g \rightarrow 0$ limit in the Pauli-Fierz mass term is known not to be smooth either [8,9]). Based on these considerations, we abandon at this point the chiral Lagrangian inspired counting and assume that the terms with two and four derivatives are equally important and on the same footing. This is, in fact, the logic behind the so-called $f(R)$ theories. See Refs. [14,15] for more information. A particularly interesting case from the perspective of inflation, and the one chosen for our study, is given by

$$f(R) = R + \alpha R^2, \quad (21)$$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} (R + \alpha R^2),$$

known as the Starobinsky model [16]. The equations of motion for R with $f(R) = R + \alpha R^2$ are

$$R + 6\alpha \square R = 0, \quad (22)$$

which, for nonvanishing values of α , immediately resembles the Klein-Gordon equation

$$(\square + m^2)R = 0, \quad (23)$$

describing a free scalar field with a mass given by

$$m^2 = \frac{1}{6\alpha}. \quad (24)$$

Indeed, by performing a conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = \exp(\beta\varphi), \quad \beta = \frac{1}{M_{\text{pl}}} \sqrt{\frac{16\pi}{3}}, \quad (25)$$

to rewrite the action in Eq. (21) in the Einstein frame (instead of the Jordan frame, see Ref. [17]), expressed in terms of quantities denoted with tildes, it is found that

$$R = \Omega^2 \tilde{R} - 6\Omega^{-1} \square \Omega. \quad (26)$$

In this frame, the action becomes

$$S = \int d^4x \sqrt{-\tilde{g}} \left(\frac{1}{2\kappa^2} \tilde{R} + \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - V(\varphi) \right),$$

$$V(\varphi) = \frac{3}{4\kappa^2} m^2 (\exp(-\beta\varphi) - 1)^2, \quad (27)$$

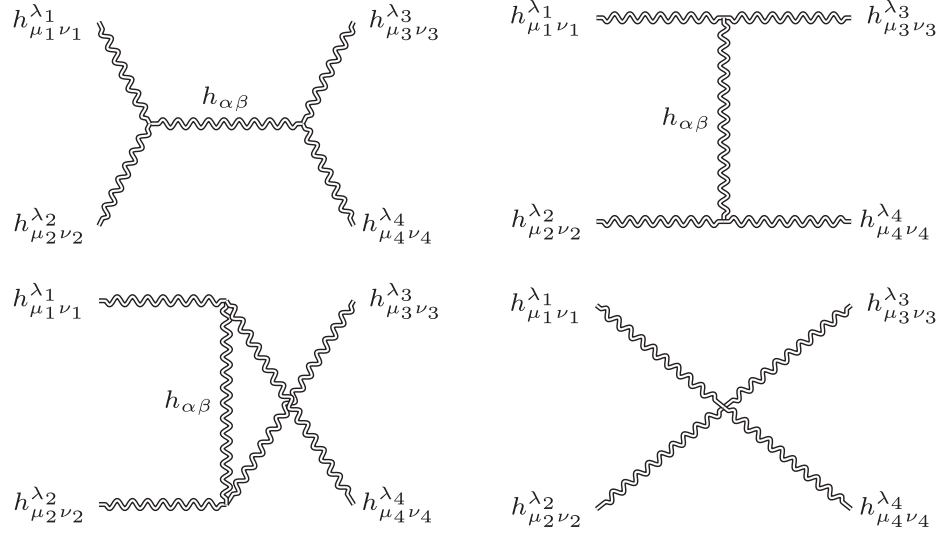
where m and β are defined in Eqs. (24) and (25), respectively. Expanding the potential $V(\varphi)$ for $\varphi/M_{\text{pl}} \ll 1$,

$$V(\varphi) \approx \frac{1}{2} m^2 \varphi^2 - 2\sqrt{\frac{\pi}{3}} \frac{m^2}{M_{\text{pl}}} \varphi^3 + \frac{14\pi}{9} \frac{m^2}{M_{\text{pl}}^2} \varphi^4 + \dots, \quad (28)$$

we find that the action at next to leading order in Eq. (21) rewrites as the leading-order Einstein-Hilbert term plus a massive, dynamical scalar field with self-interactions. This scalar field receives the name of scalaron. Recalling that $R \sim \partial\partial g \sim p^2$, R^2 corrections to Einstein-Hilbert action will be comparable for energies such that

$$p^2 \sim \alpha p^4 \Rightarrow s \sim \frac{1}{\alpha} = 6m^2. \quad (29)$$

It is worth mentioning that, when applying the Starobinsky model to cosmology, see Ref. [16], the scalaron mass of choice lies in the range $\sim 10^{-2} M_{\text{pl}}$ or less. Then the coefficient α , after pulling out a factor $\kappa/16\pi^2$, is definitely much larger than the natural order one would expect from chiral counting (see, e.g., [18]). This statement deserves some justification: once the theory is redefined in terms of the κ parameter, κ plays the role of v , the vacuum expectation value in the electroweak theory. Then, the natural expansion parameter of the effective theory should be $\kappa^2/16\pi^2$, implying that the coefficient of the p^4 terms is $\sim 1/16\pi^2 \simeq 10^{-3}$. In the $R + \alpha R^2$ model, the coefficient of the p^4 term is $\alpha/2\kappa^2$, which is a number of order 1 if the mass is $10^{-2} M_{\text{pl}}$, or even larger for lighter scalaron masses.

FIG. 1. Diagrams contributing to the elastic scattering of gravitons in the helicity states $\lambda_i = \pm 2$.

IV. THE SPECTRUM OF $R + \alpha R^2$

We saw in the previous section that getting qualitatively different results from the ones of the Einstein–Hilbert gravity requires somewhat large values for α . This brings us naturally into a nonperturbative regime. The scalaron has a squared mass inversely proportional to the coupling constant α in the next-to-leading-order effective theory of pure gravity, as given by Eq. (24). The more significant the corrections to Einstein–Hilbert action, the smaller the scalaron mass. Conversely, the smaller the coupling constant α associated with the higher-order term, the larger the scalaron mass, leading to greater decoupling from the pure gravity modes. The limit $\alpha \rightarrow 0$, where one might expect to recover as the mass tends to infinity ($m \rightarrow \infty$), can be pathological due to the structure of the potential. In its expanded form, as shown in Eq. (28), the potential includes interactions proportional to m —a clear manifestation of nondecoupling. This places us in an scenario where unitarization techniques can shed light on the high-energy behavior of the theory and perhaps reveal some resonances. We shall concentrate in the $J = 0$ channel hereafter. Previous works, such as those referenced in Refs. [1, 3–5, 19], have explored the presence of scalar resonances—and for higher values of J in the case of Ref. [1] in pure gravity and considering only the Einstein–Hilbert term. We want to perform a similar study in the framework of the Starobinsky model.

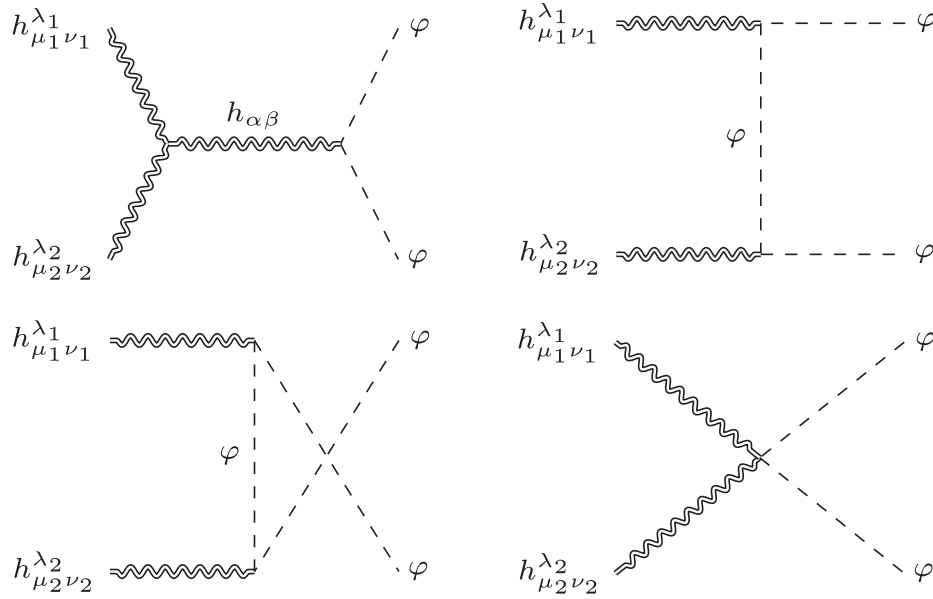
A. Relevant amplitudes

To study the scalar projection of $2 \rightarrow 2$ processes, three relevant amplitudes come into play: $h_{\mu_1 \nu_1}^{\lambda_1} h_{\mu_2 \nu_2}^{\lambda_2} \rightarrow h_{\mu_3 \nu_3}^{\lambda_3} h_{\mu_4 \nu_4}^{\lambda_4}$, $h_{\mu_1 \nu_1}^{\lambda_1} h_{\mu_2 \nu_2}^{\lambda_2} \rightarrow \varphi\varphi$, and $\varphi\varphi \rightarrow \varphi\varphi$. These are schematically represented as $\mathcal{A}^{hhhh}$, $\mathcal{A}^{hh\varphi\varphi}$, and $\mathcal{A}^{\varphi\varphi\varphi\varphi}$, respectively. In

general, we define the momenta and helicities—for the gravitons—of the incoming particles as $p_1, \lambda_1, p_2, \lambda_2$ and the outgoing particles as $p_3, \lambda_3, p_4, \lambda_4$. The helicity amplitudes are then expressed as

$$\begin{aligned} \mathcal{A}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^{hhhh} &= \langle p_3, \lambda_3; p_4, \lambda_4 | \mathcal{A}^{hhhh} | p_1, \lambda_1; p_2, \lambda_2 \rangle, \\ \mathcal{A}_{\lambda_1, \lambda_2}^{hh\varphi\varphi} &= \langle p_3; p_4 | \mathcal{A}^{hh\varphi\varphi} | p_1, \lambda_1; p_2, \lambda_2 \rangle, \\ \mathcal{A}^{\varphi\varphi\varphi\varphi} &= \langle p_3; p_4 | \mathcal{A}^{\varphi\varphi\varphi\varphi} | p_1; p_2 \rangle, \end{aligned} \quad (30)$$

where λ_i takes values ± 2 , denoted as $\pm$. The usual Mandelstam relations are defined as $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, and $u = (p_1 - p_4)^2$. Now, we present the explicit amplitudes for the three processes under study in this section. All of them are computed at tree level using the FEYNGRAV program (Ref. [20]). For each helicity combination, the diagrams involved in the elastic graviton scattering process include the s -, t -, and u -channel diagrams with a graviton exchange and a four-graviton contact diagram and are represented in Fig. 1. The explicit 16 decomposed helicity amplitudes $\mathcal{A}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^{hhhh}$ are gathered in Table I. Parity ($T_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} = T_{-\lambda_1, -\lambda_2, -\lambda_3, -\lambda_4}$) and time-reversal invariance ($T_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} = T_{\lambda_3, \lambda_4, \lambda_1, \lambda_2}$), corresponding to PT symmetry, are explicitly shown in Table I, whose entries are fully symmetric. The crossed channel, i.e., $h_{\mu_1 \nu_1}^{\lambda_1} h_{\mu_2 \nu_2}^{\lambda_2} \rightarrow \varphi\varphi$, has contributions from four diagrams that are depicted in Fig. 2. There is a s -channel graviton exchange, followed by two diagrams in the t and u channels with a scalar exchange. Finally, there is a contact vertex involving two gravitons and two scalarons. Four helicity combinations are possible, summarized as


 FIG. 2. Diagrams contributing to the crossed process $hh \rightarrow \phi\phi$ where the gravitons are in the helicity states $\lambda_i = \pm 2$.

$$\mathcal{A}^{hh\phi\phi} = -\frac{\kappa^2}{64s^2} \left(2s^3(h_1 h_2 + 1) + 2s(h_1 h_2 + 1)(t - u)^2 - \frac{s(s^2 + 2s(t + u) + (t - u)^2)^2}{(m^2 - t)(m^2 - u)} \right), \quad (31)$$

where $h_{1,2}$ represents the helicity with a normalization such that $h_{1,2} = \pm 1$. For elastic scattering of scalarons, seven types of diagrams are involved. Four of these, namely, the s , t , and u channels, and contact diagrams with scalar exchange, are identical to the equivalent Higgs processes described by anomalous triple and quartic self-interactions $\lambda_3 v h^3$ and $\lambda_4 h^4$, where v is the vacuum expectation value of the Higgs. These Higgs amplitudes are easily found in the literature [see, for instance, Eq. (17) of Ref. [21]] and the corresponding scalaron amplitudes are obtained straightforwardly through the following replacements:

$$\lambda_3 \rightarrow -2\sqrt{\frac{\pi}{3}} \frac{m^2}{M_{\text{pl}} v}, \quad \lambda_4 \rightarrow \frac{56\pi}{9} \frac{m^2}{M_{\text{pl}}^2} \quad (32)$$

for the triple and quartic couplings, respectively. The remaining three diagrams correspond to the s , t , and u

channels with graviton exchange. All of them together are shown in Fig. 3. The total amplitude for the elastic scattering of scalarons is

$$\mathcal{A}^{\phi\phi\phi\phi} = \frac{1}{12} \kappa^2 \left(18m^4 \left(\frac{1}{m^2 - s} + \frac{1}{m^2 - t} + \frac{1}{m^2 - u} \right) - 14m^2 - \frac{3}{8stu} (-s^2(6t^2 + 7tu + 6u^2) + s^3(t + u) + s(t + u)(t^2 - 8tu + u^2) + tu(t^2 - 6tu + u^2)) \right). \quad (33)$$

With these amplitudes, we can construct the associated partial waves using

$$t_J^{\lambda_1 \lambda_2 \lambda_3 \lambda_4}(s) = \frac{1}{32K\pi} \int_{-1}^1 d(\cos \theta) W_J^{\lambda, \lambda'}(\cos \theta) \mathcal{A}(s, \cos \theta), \quad (34)$$

where $\lambda = \lambda_1 - \lambda_2$ and $\lambda' = \lambda_3 - \lambda_4$. As mentioned earlier, we will focus on the $++++$ amplitudes, so $\lambda = \lambda' = 0$, and the Wigner functions W_J^{00} reduce to Legendre polynomials, which for the $J = 0$ projection is $P_0(\cos \theta) = 1$. For the elastic graviton-graviton case, the partial waves are not well defined even at tree level, as can be directly seen in Table I, since they diverge for $t \rightarrow 0$ ($\cos \theta \rightarrow \pm 1$), requiring an infrared regularization. Thus, the integral in Eq. (34) is redefined over $\cos \theta \in [-1 + \eta, 1 - \eta]$, with $0 < \eta < 1$, and the physical region where the partial wave is defined is $s > \mu^2$. These two free parameters are necessarily related because crossing symmetry requires

TABLE I. Table summarizing the 16 helicity combinations for elastic graviton scattering.

	$ +; +\rangle$	$ +; -\rangle$	$ -\; +\rangle$	$ -\; -\rangle$
$\langle +; + $	$\frac{\kappa^2 s^3}{4 tu}$	0	0	0
$\langle +; - $	0	$\frac{\kappa^2 u^3}{4 st}$	$\frac{\kappa^2 t^3}{4 su}$	0
$\langle -; + $	0	$\frac{\kappa^2 t^3}{4 su}$	$\frac{\kappa^2 u^3}{4 st}$	0
$\langle -; - $	0	0	0	$\frac{\kappa^2 s^3}{4 tu}$

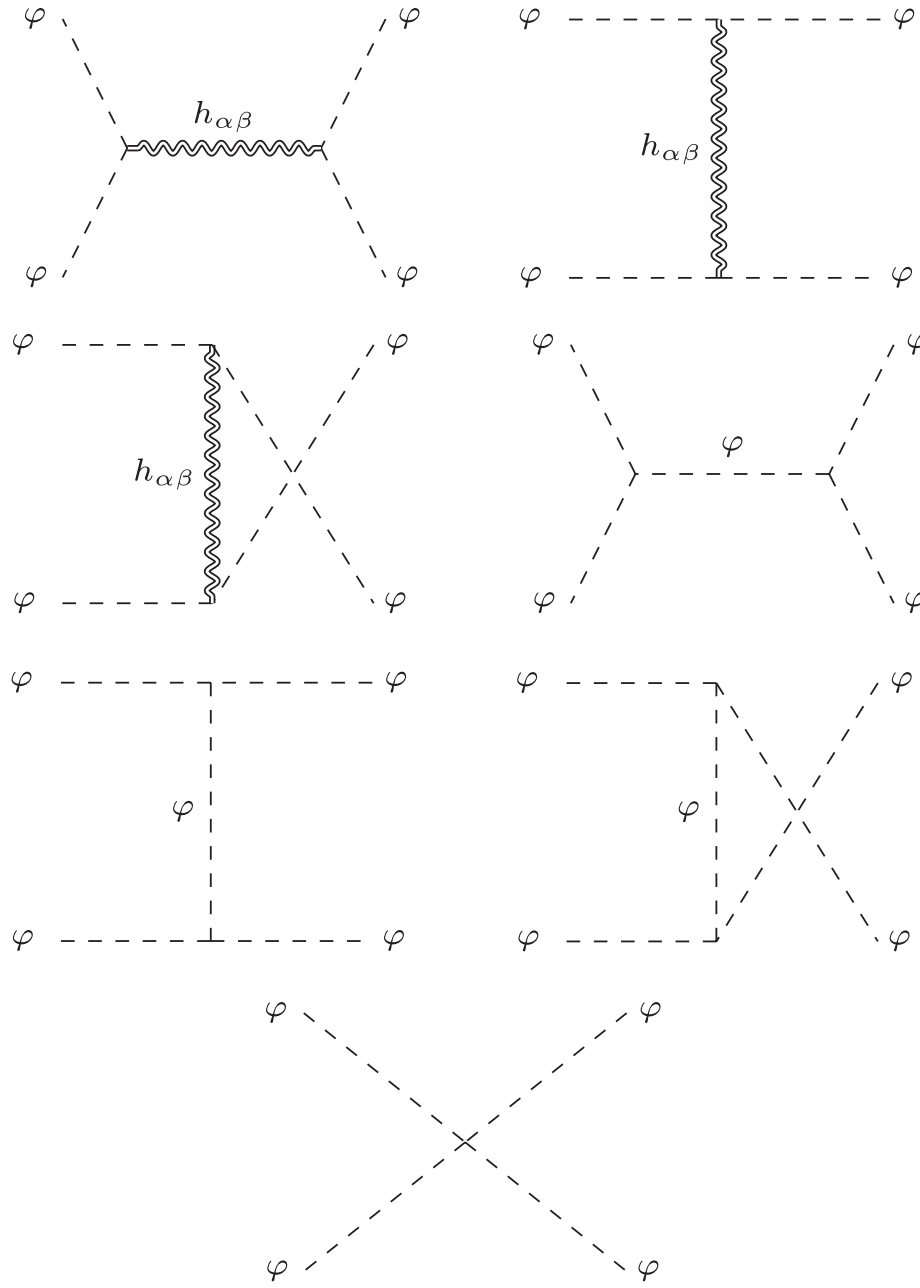


FIG. 3. Diagrams contributing to the elastic scattering of scalarons.

$t < -\mu^2$, as noted by the authors in Refs. [1,2], taking into account that for massless external states $t = -s/2(1 - \cos \theta)$ so

$$\int_{-1+\eta}^{1-\eta} d \cos \theta = \frac{2}{s} \int_{-s+\mu^2}^{-\mu^2} dt, \quad (35)$$

from where it follows that

$$\mu^2 = \eta \frac{s}{2}. \quad (36)$$

This procedure guarantees perturbative unitarity (see, e.g., Refs. [1,2]). The result of this integration for the three processes gives

$$t_0^{hhhh} = \frac{s}{2M_{\text{pl}}^2} \log \left(\frac{2}{\eta} \right), \quad (37)$$

$$t_0^{hh\varphi\varphi} = -\frac{s + 8m^2}{24M_{\text{pl}}^2} + \frac{m^4}{2\sigma M_{\text{pl}}^2 s} \log \left(\frac{1 + \sigma}{1 - \sigma} \right), \quad (38)$$

$$\begin{aligned}
 t_0^{\varphi\varphi\varphi\varphi} = & \frac{1}{24M_{\text{pl}}^2 s(s-m^2)(s-4m^2)} \left[s^4 \left(12 \log\left(\frac{2}{\eta}\right) - 11 \right) \right. \\
 & + m^2 s^3 \left(55 - 60 \log\left(\frac{2}{\eta}\right) \right) \\
 & - 4m^4 s^2 \left(19 - 18 \log\left(\frac{2}{\eta}\right) - 18 \log\left(\frac{s-3m^2}{m^2}\right) \right) \\
 & - 4m^6 s \left(-31 + 6 \log\left(\frac{2}{\eta}\right) \right. \\
 & \left. \left. - 18 \log\left(\frac{s-3m^2}{m^2}\right) \right) + 16m^8 \right], \quad (39)
 \end{aligned}$$

where σ is the two-body phase space $\sigma = \sqrt{1 - \frac{4m^2}{s}}$. All of them are gathered in a t_0 scalar wave matrix defined as

$$t_0 = \begin{pmatrix} t_0^{hhhh} & t_0^{hh\varphi\varphi} \\ t_0^{hh\varphi\varphi} & t_0^{\varphi\varphi\varphi\varphi} \end{pmatrix}, \quad (40)$$

which is suitable for the unitarization of the processes in the context of the coupled-channel formalism. According to this unitarization mechanism, using matrix notation, we refer to the elastic scattering of gravitons ($hhhh$), as the 11 channel [corresponding to the 11 entry Eq. (40)] and to the elastic scattering of scalarons ($\varphi\varphi\varphi\varphi$) as the 22 channel. This notation will be used for both unitarized and non-unitarized processes. It is interesting to note that in the $m = 0$ limit (i.e., formally taking $\alpha \rightarrow \infty$) the t_0^{hhhh} partial wave coincides with $t_0^{\varphi\varphi\varphi\varphi}$ and is fully dominated by the IR log. On the other hand, $t_0^{hh\varphi\varphi}$ is free of IR singularities.

V. UNITARIZATION AND RIEMANN SHEETS

The unitarization method chosen here is the K matrix in its improved version, the IK matrix (IK), and is described in, e.g., Ref. [22] (see also Ref. [23] for a recent review and Ref. [24] for applications to hadronic physics). This unitarization method admits amplitudes calculated solely at tree level, adding by hand a physical—also unitarity—cut for the relevant continuations. In contrast, the IAM requires the calculation of the amplitude at next-to-leading order for all three amplitudes. These amplitudes are not currently available and are exceedingly laborious to compute. To the best of our knowledge, no literature exists to date that includes these amplitudes with a self-interacting scalar contribution at the one-loop level. Only the contribution from pure gravity is available, see Refs. [1,2,25,26]. Therefore, the IK method turns out to be the most appropriate for a first approximation to the study of possible dynamical resonances in the $R + \alpha R^2$ theory. The presence of both massive particles, such as the scalaron, and massless particles, such as the graviton, in the asymptotic states of the processes for the unitarization of

the $J = 0$ channel leads to an analytical structure of the amplitudes with different cuts on the real axis. Through these cuts, the different partial waves can be analytically continued to access the different Riemann sheets. In general, with n coupled channels, 2^n Riemann sheets open up. In our case, we have two unitarity cuts: the first at the origin, described by the usual logarithm $L(s)$, and the second at $\text{Res} = 4m^2$, where m is the mass of the scalaron as defined in Eq. (24), described by the Chew-Mandelstam function $J(s)$. All the possible combinations of continuous crossings through these two physical cuts result in four Riemann sheets. The explicit expressions for $L(s)$ and $J(s)$ are

$$\begin{aligned}
 L(s) &= \log(-s/\Lambda^2), \\
 J(s) &= \sigma \log\left(\frac{\sigma+1}{\sigma-1}\right), \quad (41)
 \end{aligned}$$

and the corresponding continuation from the first to the second Riemann sheet (2RS),

$$\begin{aligned}
 L^{\text{II}}(s) &= \log|s| + i(\arg s - \pi), \\
 J^{\text{II}}(s) &= \sigma \log\left|\frac{\sigma+1}{1-\sigma}\right| + i\sigma\left(\arg\frac{\sigma+1}{1-\sigma} - \pi\right), \quad (42)
 \end{aligned}$$

where the superindex II indicates that the function is taken in the second Riemann sheet. These choices guarantee that, out of the possible 2^n sheets, one seeks for resonances in the so-called adjacent sheet—the one continuously connected to the physical region. In the present case with $n = 2$, four different Riemann sheets open up, but the sheet we refer to as the second Riemann sheet—where physical resonances are located—depends on Res . For $\text{Res} < 4m^2$, no continuation of $J(s)$ is required, since the second cut is not reachable, and the 2RS is defined by continuing only $L(s)$. Conversely, for $\text{Res} > 4m^2$, the adjacent sheet is achieved when both functions $L(s)$ and $J(s)$ are continued according to Eq. (42). In summary, the IK-matrix method in its coupled-channel version provides the following prescription:

$$t_0^{\text{IK}} = (1 + t_0^{(2)} \cdot G(s))^{-1} t_0^{(2)}, \quad (43)$$

where 1 represents the identity matrix and $t_0^{(2)}$ is the scalar wave matrix built solely with the leading-order Lagrangian—recall the tree-level calculation in this work—and is defined in Eq. (40). $G(s)$ is a matrix containing all the unitarity cuts for the different processes. In our case,

$$G(s) = \frac{1}{\pi} \begin{pmatrix} L(s) & 0 \\ 0 & g(s) \end{pmatrix}, \quad g(s) = \log\frac{m^2}{\Lambda^2} + J(s), \quad (44)$$

with $L(s)$ and $J(s)$ defined in Eq. (41). A straightforward calculation is needed to show that Eq. (43) with the

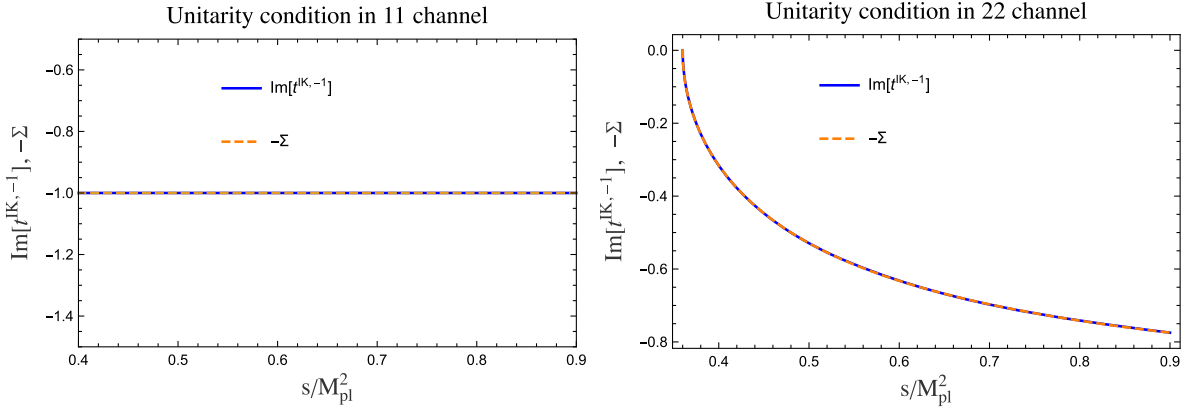


FIG. 4. Both sides of the full unitarity condition in Eq. (45) for the values $m = 0.3M_{\text{pl}}$ and $\mu = 10^{-10}M_{\text{pl}}$. Both lines coincide, meaning that the scalar waves fulfill unitarity as expected.

function $G(s)$ in Eq. (44) indeed fulfills all requirements from unitarity,

$$\text{Im} t_0^{\text{IK},-1} = (t_0^{(2)})^{-1} (1 + t_0^{(2)} \cdot G(s)) = -\Sigma, \quad \Sigma = \begin{pmatrix} 1 & 0 \\ 0 & \sigma \end{pmatrix} \quad (45)$$

since $\text{Im} g(s) = -\pi\sigma$ in the physical region. The matrix $G(s)$ contains the only functions that are to be continued to the second Riemann sheet for searching poles using Eq. (42), as tree-level amplitudes lack an imaginary part. Thus, together with the previous discussion, the 2RS is uniquely defined everywhere,

$$G^{\text{II}}(s < 4m^2) = \frac{1}{\pi} \begin{pmatrix} L^{\text{II}}(s) & 0 \\ 0 & g(s) \end{pmatrix},$$

$$G^{\text{II}}(s > 4m^2) = \frac{1}{\pi} \begin{pmatrix} L^{\text{II}}(s) & 0 \\ 0 & g^{\text{II}}(s) \end{pmatrix}, \quad (46)$$

where $g^{\text{II}} = \log \frac{m^2}{\Lambda^2} + J^{\text{II}}(s)$. The single-channel approximation—i.e., neglecting the mixing with scalaron contributions—can be obtained directly from the expression in Eq. (43) by replacing all matrices with their corresponding algebraic functions describing graviton-graviton scattering, yielding

$$t_0^{\text{IK-SC}} = \frac{t_0^{(2)}}{1 + t_0^{(2)} L(s)}. \quad (47)$$

Apart from the necessary checks to make contact with previous studies, in this work we adhere to the coupled-channel formalism in order to provide an accurate description of scalar resonances in $R + \alpha R^2$. To conclude this section, we demonstrate the unitarity of our partial waves after applying the IK method in Fig. 4, where we show both sides of Eq. (45) for the scalaron mass $m = 0.3M_{\text{pl}}$ and

$\mu = 10^{-10}M_{\text{pl}}$. The lhs is displayed as a blue solid line and the opposite-sign phase-space matrix in the rhs in dashed orange. Both lines coincide exactly revealing that full-unitarity is satisfied and partial waves are indeed well unitarized by the IK method.

VI. TRACKING THE POLES

Poles in the complex “ s ” plane are provided by the complex zeros of the determinant in the inverted term of Eq. (43). Since the determinant is common to all entries of the unitarized matrix of amplitudes, poles should, in principle, be visible in all of them, but the coupling of a given channel to a particular pole need not be the same, implying that they may be quite apparent in a given channel and difficult to see in another. All dimensional quantities and plots are shown in units of the Planck mass. Thus, we define

$$\bar{m} = \frac{m}{M_{\text{pl}}}, \quad \bar{\mu} = \frac{\mu}{M_{\text{pl}}}, \quad \bar{\Lambda} = \frac{\Lambda}{M_{\text{pl}}}, \quad \bar{s} = \frac{s}{M_{\text{pl}}^2}, \quad (48)$$

where m is the mass of the scalaron, μ is the IR cutoff regulating the bad behavior of the partial wave in the extremes, Λ is the UV scale that has to be included in the unitarization process, and s is the Mandelstam variable that is promoted to a complex quantity. We did not find significant variations when changing the ultraviolet cutoff $\bar{\Lambda}$. From this point onward, we fix $\bar{\Lambda} = 0.9$. Then our analysis is restricted to two parameters: the infrared cutoff $\bar{\mu}$ and the scalaron mass $\bar{m}$, which is related to the parameter α through Eq. (24). Numerically, one can reach extremely small values of the infrared cutoff, but this really makes no sense. Indeed, the appearance of infrared logarithms makes necessary to place a lower bound on the IR scale and the $\mu \rightarrow 0$ limit cannot be taken in a trivial manner. As we discussed before, the natural expansion parameter of the theory is $\kappa^2/16\pi^2$ and higher momentum

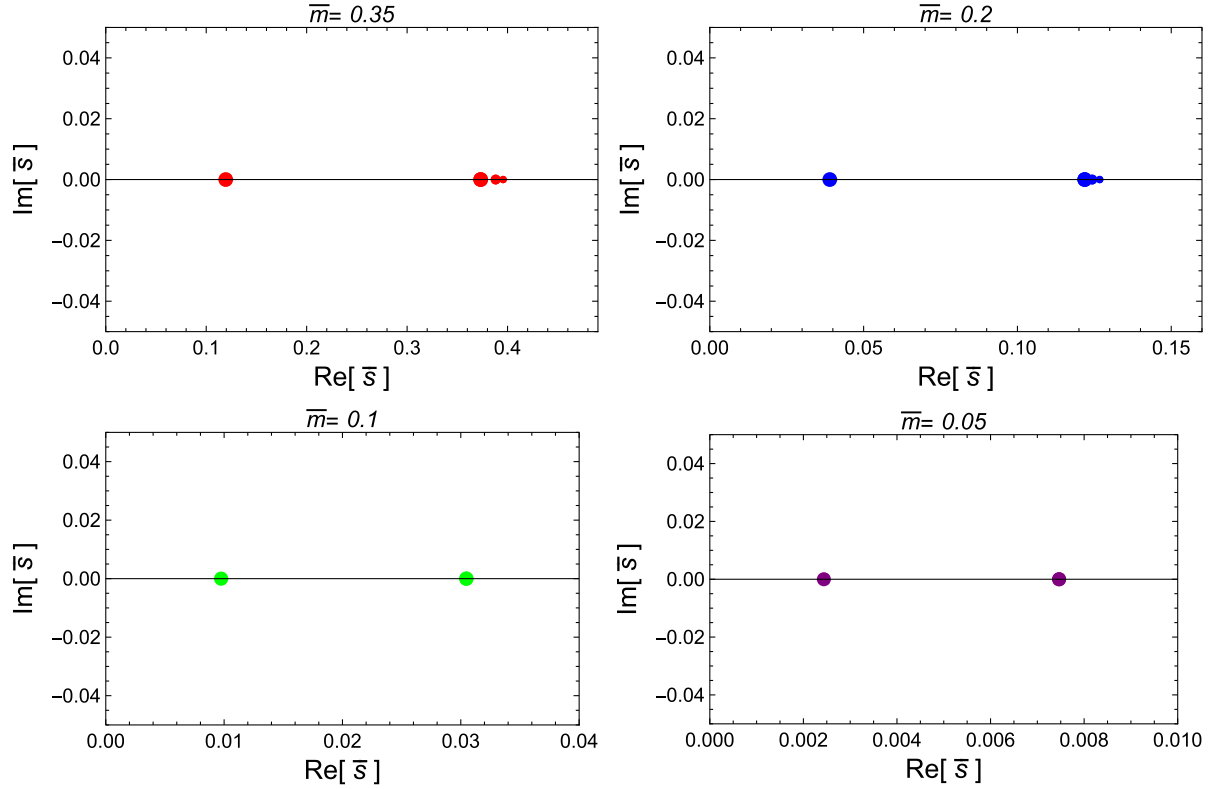


FIG. 5. Position of the scalaron and the $3m^2$ logarithmic branch point for various values of $\bar{m}$ and the infrared cutoff $\bar{\mu}$. In each panel, the values for the dimensionless IR regulator are $\bar{\mu} = 10^{-10}, 10^{-15}, 10^{-20}$ and are identified with different point sizes—bigger point sizes represent bigger values of $\bar{\mu}$. In the last two panels, the position of the logarithmic branch point is unchanged by the regulator.

corrections will appear in the form $\kappa^2 s / 16\pi^2$, but the iteration of IR divergences will presumably appear as $(\kappa^2 s / 16\pi^2) \log(s/\mu^2)$. In terms of the dimensionless variables previously introduced, this is equivalent to

$$\frac{4}{\pi} \bar{s} \log\left(\frac{\bar{s}}{\bar{\mu}^2}\right). \quad (49)$$

We need this effective expansion parameter to be smaller than 1, or at least not much bigger than 1 just to get qualitative results. If we place ourselves in the sub-Planckian region, simple considerations allow us to conclude that we cannot go to very small values of $\bar{\mu}$. Specifically, the results presented in this section should be considered tentative as we explore values of $\bar{\mu}$ down to 10^{-20} and focus mostly on the observed trend.

A. The scalaron

Our first task is to identify the pole due to the $J = 0$ scalaron and also the branch point that is expected to appear in the $J = 0$ partial wave due to the one-scalaron exchange in the t and u channels. We do not expect to find the poles exactly at the location of the Lagrangian because in some sense unitarization includes higher-order effects and they can be displaced, but in any case not too far away. A simple

estimation of the $1 \rightarrow 2$ one-loop diagrams involved shows that the expected width for the scalaron decaying into a pair of gravitons scales as $\sim m^7 / M_{\text{pl}}^6$, yielding a negligible imaginary part in the complex s plane for natural values of the scalaron mass. Indeed, after applying the unitarization method, we do not observe any displacement of the scalaron pole, neither along the real nor the imaginary directions in the complex s plane. Also, we naturally expect to visualize the poles just mentioned in the 22 channel; that is, the scattering of two scalarons yielding also two scalarons; indeed, this is the case. Figure 5 shows the position of the scalaron pole for a variety of Lagrangian values for m —recall that we use dimensionless quantities. The plot also includes the variation due to choosing a range of values for the infrared cutoff. The conclusion is obvious: there is almost no dependence on $\bar{\mu}$, especially for the pole of the scalaron at $\bar{m}^2$ due to s -channel exchange. In Fig. 6 we also depict the second (adjacent) Riemann sheet to visualize the two mentioned poles for a particular value of $\bar{m} = 0.3$.

B. The graviball

The conspicuous pole in the second Riemann sheet corresponding to two-graviton scattering, found in Refs. [3–5] and confirmed in Refs. [1,2], should also be

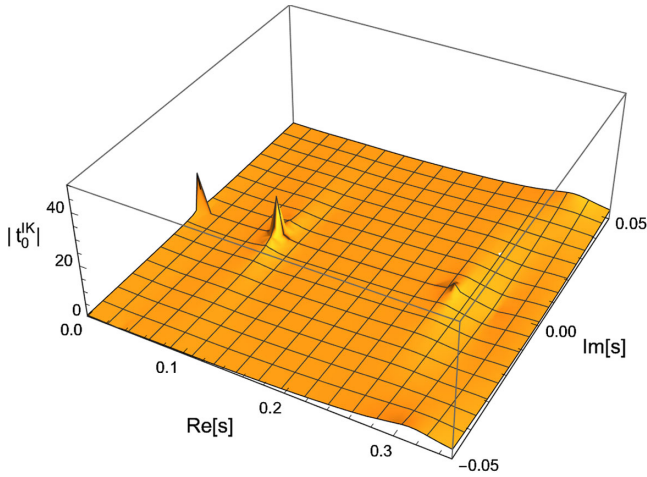


FIG. 6. The scalaron pole and the logarithmic singularity at $3\bar{m}^2$ in the second Riemann sheet for $\bar{m} = 0.3$. The value of the IR regulator is $\bar{\mu} = 10^{-5}$.

seen in the coupled-channel calculation, where it could be expected to mix somehow with the scalaron. However, we have already seen that the latter is fairly insensitive to the infrared cutoff. The graviball appears at very low energies in the $J = 0$ partial wave of graviton-graviton scattering in pure gravity. It has been interpreted as a putative two-graviton compound by gravitational forces, but its nature as a true dynamical resonance was challenged because it is very far from the real s axis and thus does not meet the usual criteria for genuine resonances and, at least in pure gravity, is highly dependent on the IR regulator. Nevertheless, some phenomenological speculations have already been proposed (see, for instance, Ref. [27]). The authors of Refs. [3–5,19] argue in favor of the presence of a resonance, whose position depends on a parameter a (their notation) that regulates infrared divergences appearing for the propagation of massless modes. This parameter redefines the energy scale $Q^2 = \pi G^{-1}/\log a$, which is characteristic of the elastic graviton process. The authors find the position of the graviball at $s = (0.22 - i0.63)(G \log a)^{-1}$. As already said, this could not be classified as a genuine resonance because it does not satisfy a natural relationship between its mass and width *à la* Wigner. Instead, the authors of Refs. [1,2] also identify this structure but describe it as an artifact due to the introduction of the parameter μ , which regulates partial waves in the infrared, analogous to a in the previous case. Using their unitarized analysis with the IAM at next-to-leading order, they determine that the position of this pole is consistent with $s = 0$ as $\mu \rightarrow 0$, distancing it from any physical significance as a dynamical state. Of course, this statement requires qualification because infrared divergences are definitely present in graviton scattering and the limit $\mu \rightarrow 0$ cannot be simply taken, as higher-order contributions would become more and more relevant. One has to stay with values of the infrared cutoff that do not conflict

with perturbation theory. The authors in Refs. [1,2] did two things: first, they found a value of their IR regulator that grossly coincided with the pole found by the authors in Refs. [3–5,19]; second, they reduced the value of the infrared cutoff and convinced themselves that the “pole” was moving fast toward the origin and therefore could be considered an artifact of the IR regulation procedure. It seems pertinent to recall that self-interactions of the graviton involve derivatives [see Eqs. (16) and (17)] implying that the theory is a free one in the infrared and the only degrees of freedom that can appear at $s = 0$ are the ones already existing in the Lagrangian. Consequently, the fact that a putative dynamical resonance is driven to $s = 0$ as the IR cutoff is removed signals that this resonance cannot possibly exist because the theory is a free one in that limit. Our results align here with this latter scenario. First, we verified that, indeed, using a single channel for unitarization (but now using the IK-matrix method)—that is, neglecting the mixing with the scalaron in the $J = 0$ channel—the main characteristics of the graviball found in Refs. [1,2] are reproduced. That is, as the infrared cutoff tends to zero, $\mu \rightarrow 0$, the pole moves closer and closer to the origin. Second, this behavior persists in the coupled-channel formalism when the coupling to a genuine scalar degree of freedom is considered. In Fig. 7, the position we obtain for the graviball (blue points) in the single-channel approximation is shown for values of $\bar{\mu} \in (0.001, 0.5)$ and two UV cutoffs, $\bar{\Lambda} = 0.8$ and $\bar{\Lambda} = 1.0$. Recall that quantities with a bar are in units of the Planck mass and are therefore dimensionless. We also show, for comparison, the position of the pole from Refs. [3–5]. Now we repeat the exercise in the coupled-channel formalism. As discussed before in the text, the functions with coupled channels depend not only on the IR regulator, but also on the mass of the scalaron. The results for the position of the graviball are shown in Fig. 8. The left panel displays the pole location for various values of the IR regulator and the scalaron mass, with the latter ranging from $\bar{m} = 0.35$ down to $\bar{m} = 0.05$. The IR regulator values are $10^{-8}, 10^{-10}, \dots, 10^{-20}$ and, for each series, the smaller the IR regulator, the closer the pole moves toward the origin. This behavior is shown explicitly in the right panel, which focuses on $\bar{m} = 0.05$ (the closest value in our scan to that of the Starobinsky model). Here, the graviball pole positions are plotted with point size encoding the value of the IR regulator—larger points correspond to larger regulators. In Fig. 8 we also show the scalaron pole, which is located at its expected position $\bar{s} = \bar{m}^2 = 0.0025$ no matter the choice of the IR regulator. This different behavior for the graviball and the scalaron indicates the difference between a proper resonance, as it is the case of the scalaron, and what we consider an artifact.

C. Other resonances

Quite interestingly, the study of the partial waves in the second Riemann sheet reveals further structures. This is

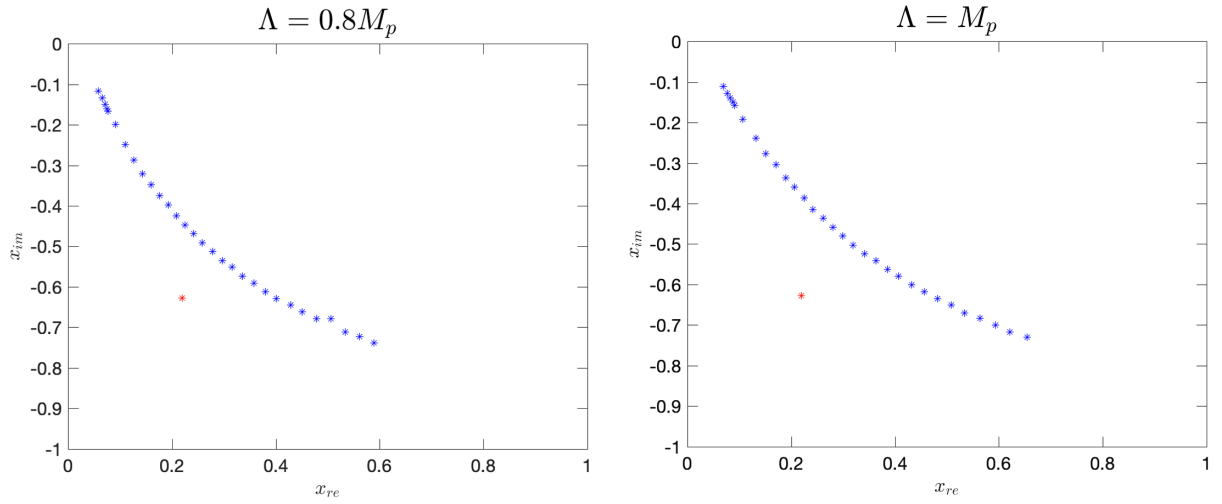


FIG. 7. Position of the graviball (blue points) for $\mu \in (0.001, 0.5)$ in pure gravity at leading order and two values of $\Lambda = 0.8M_{\text{pl}}$ (left) and $\Lambda = M_{\text{pl}}$ (right). The results show a trend toward the origin, ruling out the possibility of it being a physical resonance. The pole from Refs. [3–5] is also indicated with a red point.

best seen in the 22 partial wave after unitarization. In fact, without making use of the coupled-channel formalism, the 11 amplitude coincides with the one derived of the pure Einstein-Hilbert theory, where no proper dynamical singularity seems to be present (only the graviball, which progressively vanishes as the IR cutoff tends to zero). Therefore, additional singularities may appear in the 11 partial wave only after mixing with the 22 one and they tend to be fairly hard to see, indicating a small overlap of these resonances that are visible in the 22 channel with the

two-graviton initial state. Examining the 22 partial wave in the second Riemann sheet, we detect a singularity, clearly different from the ones due to the scalaron or to the scalaron exchange (that always lie on the real s axis) and the graviball. This new singularity moves closer to the real axis as $\bar{\mu}$ decreases and it moves closer to the imaginary s axis as $\bar{m}$ decreases; its real part is always located above the scalaron pole and the threshold at $s = 4\bar{m}^2$. Perhaps surprisingly, it seems to be associated with a mirror singularity in the first (physical) Riemann sheet. This is

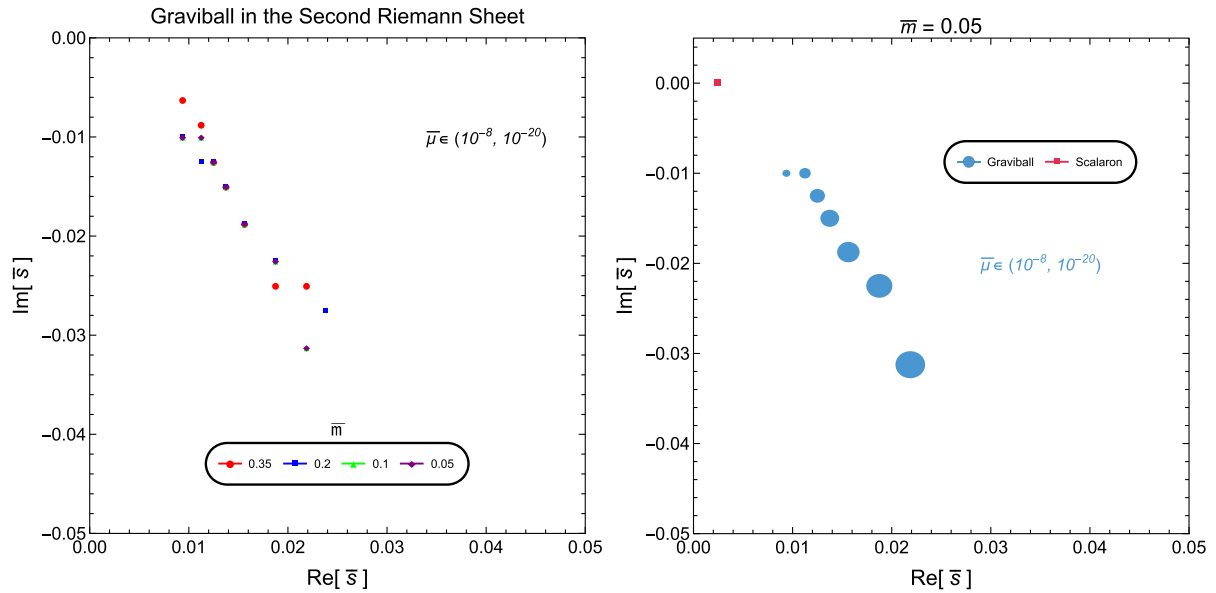


FIG. 8. A comparison between the displacement of the graviball pole under changes in the infrared regulator for different scalaron masses (left) and for $\bar{m} = 0.05$ (right). Left: the values of $\bar{\mu}$ vary from 10^{-8} to 10^{-20} , the latter producing graviball locations closer to the origin for all values of the scalaron mass. Right: the same for a specific value of $\bar{m}$ and we show in a red square the position of the scalaron too, independent of μ .

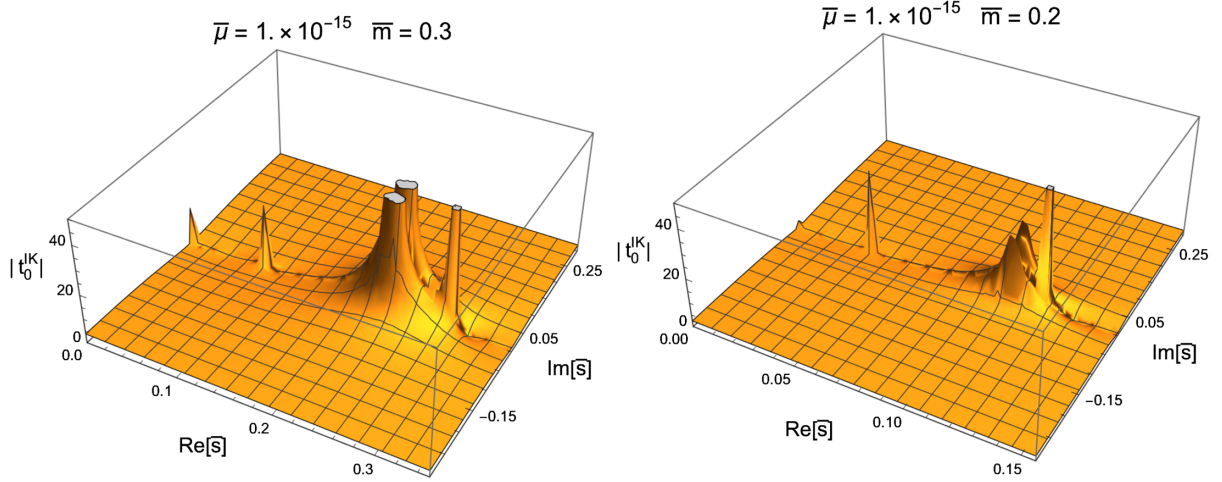


FIG. 9. Unitarized 22 partial wave for $\bar{\mu} = 10^{-15}$ and for different values of the dimensionless scalaron mass $\bar{m} = 0.3$ (left) and $\bar{m} = 0.2$ (right). The region of the complex plane above the real axis corresponds to the physical Riemann sheet (1RS) and the region below corresponds to the unphysical Riemann sheet (2RS), where the amplitude has been continued analytically.

clearly visualized in Fig. 9 for various values of the scalaron mass $\bar{m}$. However, there is actually nothing really odd here. If we consider the 22 partial wave by itself, that is, decoupling it from the 11 channel, it possesses a branch cut starting at $4m^2$. It turns out that the real part of the singularity being discussed now is always below threshold and, therefore, the singularities for negative and positive imaginary parts are, in fact, mirror resonances in the same Riemann sheet, something that is fine and relatively frequent. In the energy complex plane (as opposed to the s complex plane), the mirror image for positive values of the imaginary part of s actually lies quite far away from the physical region. Consequently, everything points in the direction that we are dealing with a genuine dynamical resonance. Also interestingly, and unlike the so-called graviball, it does not approach the origin when the infrared

cutoff is progressively removed, but it becomes narrower and closer to the positive real s axis. As said, it always stays for values of the real part of s below the branch point at $4\bar{m}^2$. This makes this resonance somewhat unusual, as one is more familiar with the situation where resonances are above threshold (in which case the mirror resonances would not be on the same physically relevant Riemann sheets). This state is heavier than the scalaron so it would be possible to interpret is as an excited state, needed to unitarize the amplitudes. In fact, given the way it approaches the positive real axis, everything suggests that the would-be resonance could be a bound state of two scalarons. In the limit when the IR cutoff is removed, it would be stable and it could only decay to two gravitons after the coupling to the 11 channel is reinstated. The evolution of this state for a given scalaron mass as a

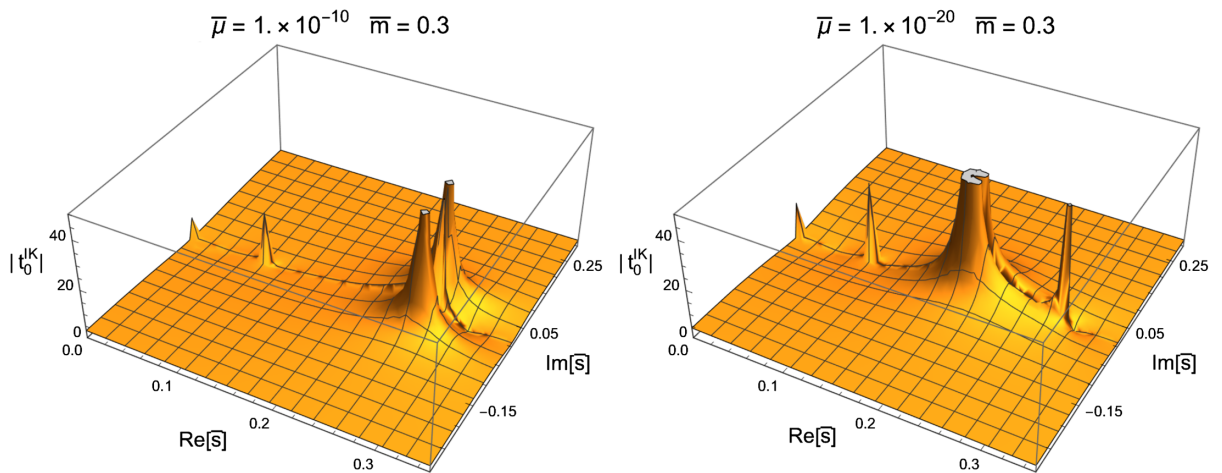


FIG. 10. Unitarized 22 partial wave for $\bar{m} = 0.3$ and for different values of the dimensionless IR regulator $\bar{\mu} = 10^{-10}$ (left) and $\bar{\mu} = 10^{-20}$ (right). The region of the complex plane above the real axis corresponds to the physical Riemann sheet (1RS) and the region below corresponds to the unphysical Riemann sheet (2RS), where the amplitude has been continued analytically.

function of the infrared cutoff is shown in Fig. 10. Of course, when the coupled-channel formalism is implemented, the analytic structure becomes more involved as the cuts from the various processes are superposed. However, for small values of the infrared cutoff, there is little coupling between the 11 and 22 channels because the channel 12 is much smaller than 11 or 22. In practice, the dynamical resonance just found stays there, more or less in the same place in the 22 channel, and it has no visible impact on the 11 one. It goes without saying that one should take the previous results with a grain of salt because, as we have repeatedly stated, the cutoff cannot be really removed and it cannot be made too small, lest the perturbative series explode; but the tendency is very clear.

VII. UNITARIZED AMPLITUDES

We insert a very small positive imaginary part in $\bar{s}$ in order to be on the physical Riemann sheet and visualize the partial waves for physical values of its argument. In the absence of dynamically generated scalar resonances close to the real axis, the only structures that we expect to find in the physical region are the poles inherent to the low-energy theory, i.e., before unitarization, and discontinuities at threshold. This is true for the unitarized partial wave due to the scalar matrix $t_0^{(2)}$ multiplying on the right in Eq. (43). This is precisely the situation seen in Fig. 11, which displays both the nonunitarized (blue) and unitarized (orange) scalar waves in the 22 channel for the values $\bar{m} = 0.3$ and $\bar{\mu} = 10^{-10}$. At $\bar{s} = \bar{m}^2 = 0.09$, the pole corresponding to the scalaron is visible in both cases. The logarithmic singularity at $\bar{s} = 3\bar{m}^2 = 0.27$ —arising from the scalaron exchange in the t and u channels—is also shown for both curves. The production threshold of the scalaron pair, located at $\bar{s} = 4\bar{m}^2 = 0.36$, is also noticeable in both curves: as a singularity in the tree-level scalar wave and as a glitch that prevents nonunitarity behavior in the physical region for the unitarized case. All these features appear at their corresponding locations for other values of the scalaron mass and the IR regulator. Similar features are observed for other values of the parameters. As we can see—and was expected—the unitarization process tames the high energy behavior of the 22 amplitude, making it tend to a constant value. The same behavior is observed in the 11 amplitude that corresponds to the process $hhhh$, i.e., the elastic scattering of two gravitons. Incidentally, its asymptotic value coincides with one of pure gravity (after unitarization). We want to draw a comparison between the leading-order and next-to-leading-order calculations in pure gravity. As explained earlier in the manuscript, performing the conformal transformation in Eq. (25) and adding the scalaron is equivalent to computing the next-to-leading-order term αR^2 , with α defined in Eq. (24). The task now is, therefore, to compare the unitarized calculation for $hhhh$ at tree level, Eq. (47), with this including coupled

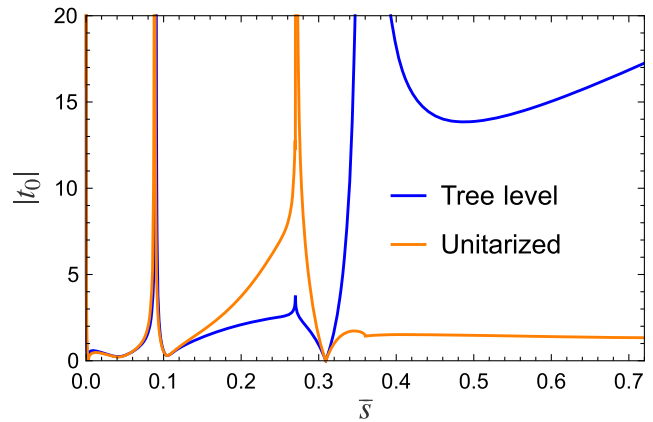


FIG. 11. Absolute value of the physical tree-level (blue) and unitarized (orange) partial wave in the 22 channel and in the $J = 0$ projection with the Mandelstam variable s in units of the Planck mass squared $\bar{s}$. The values for the dimensionless IR regulator and scalaron mass are $\bar{\mu} = 10^{-10}$ and $\bar{m} = 0.3$. The scalaron pole at $\bar{s} = 0.09$ and the logarithmic singularity at $\bar{s} = 0.27$ are evident for both partial waves. At $\bar{s} = 0.36$ the two scalaron channel opens.

channels, Eq. (43), for different values of the scalaron mass. The results are shown in Figs. 12 and 13 for both 11 and 22 channels, respectively. Interestingly, for the lower values of the scalaron mass, a decoupling of the two channels occurs: the crossed channel vanishes. Also, channels 11 and 22 coincide (this can be checked analytically too, even before unitarization) and their value tends to coincide with the one of pure R gravity. At first sight, this may appear contradictory since small values of the scalaron mass correspond to large values of the parameter α that controls the relevance of the R^2 term. In fact, in the limiting case only the R^2 term

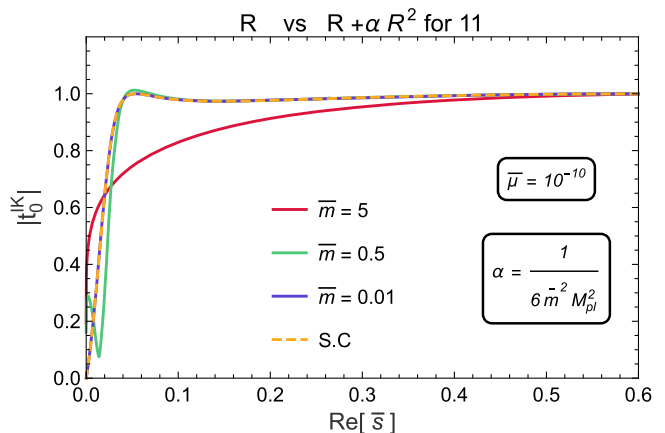


FIG. 12. Absolute value of the unitarized amplitude for (R) and $(R + \alpha R^2)$ for the 11 channel. The result for R coincides with the 11 amplitude in the single-channel approximation and it is shown as a dashed orange line, while the next-to-leading-order result, requiring the coupled-channel formalism, is shown as solid lines for different values of the scalaron mass.

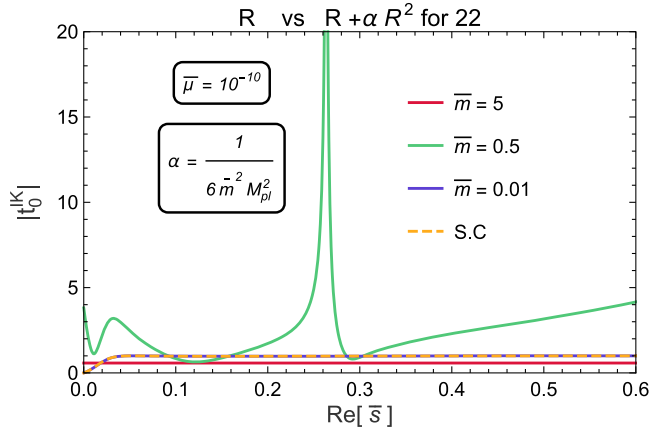


FIG. 13. Absolute value of the unitarized amplitude for the 22 channel (right). The leading-order result, obtained in the single-channel approximation, is shown as a dashed orange line, while the next-to-leading-order result, requiring the coupled-channel formalism, is shown as solid lines for different values of the scalaron mass.

would matter. Also, the equality between the two channels holds for large values of s , as, in practice, this coincides with the small m limit. All these results hold independent of the value of the IR cutoff provided that is very clearly separated from the physical region. For larger values of the scalaron mass, commensurate with the Planck mass, the $hhhh$ amplitude shows an enhancement at low values of s . As the mass of the scalaron grows, the sub-Planckian part of the amplitude becomes progressively featureless. Yet it tends to the same universal asymptotic value. Moving the scalaron mass to the super-Planckian region smoothens the amplitude further. However, it does not tend to the case where one sets $\alpha = 0$ directly in the Lagrangian. We interpret this as a manifestation of non-decoupling of the scalar mode that makes both limits noncommuting.

VIII. CONCLUSIONS

We have presented unitarized partial waves for the $R + \alpha R^2$ gravity theory, also known as the Starobinsky model, a particular class of $f(R)$ theories. Perturbatively, for generic values of the coupling α this model presents three states: two helicities for the graviton (which may combine to $J = 0$ in graviton-graviton scattering) and a genuine spin zero state, the scalaron. Projection of the partial waves in the scalar channel leads to a rather rich structure that has been studied in the framework of the (coupled channels) improved K-matrix unitarization mechanism. The analysis reveals the expected zero-width poles corresponding to the scalaron and the (logarithmic) pole due to the t and u scalaron exchange that are most visible in the $\varphi\varphi\varphi\varphi$ amplitude and have very small overlap with the two-graviton scattering $hhhh$. The latter

is as a whole not very affected by the coupled channel and shows characteristics somehow similar to those in pure Einstein gravity (i.e., $\alpha = 0$), but some differences have been seen. Incidentally, the limit $m \rightarrow \infty$ ($\alpha \rightarrow 0$) seems to be discontinuous because the scalaron shows signs of nondecoupling, something that can be understood from the couplings in the scalar potential. The limit $m \rightarrow 0$ ($\alpha \rightarrow \infty$) is interesting too and quite subtle. See Ref. [28] and references therein for a recent discussion about the pure αR^2 theory on a Minkowski background. One of purposes of this study was to identify the dependence of the various complex structures appearing in the second Riemann sheet on the infrared cutoff. The IR singularities are regulated by means of a physical cutoff in the angular distribution, a procedure that was verified to be consistent with perturbative unitarity. Indeed, as was somehow expected, a genuine physical resonance such as the scalaron shows almost no dependence on the infrared cutoff. On the contrary, both partial waves after unitarization display a pole, very far from the real axis, that is very strongly dependent on the IR cutoff and tends to move quickly to $s = 0$ as the cutoff tends to zero. We have analyzed the range of values for the IR cutoff that are, at least nominally, compatible with perturbation theory and adhered to those. The study has some other limitations: It is difficult to go to very low values of the mass for the scalaron because the structure in the second Riemann sheet become rather hard to see, but that limit tends clearly, at least at the level of the amplitudes, to pure Einstein-Hilbert gravity. The range of values of the Starobinsky model is about as low as one can get numerically. It is also delicate to go to large masses for the scalaron mass, something that can be understood in terms of nondecoupling. An additional pole is clearly seen in the $\varphi\varphi\varphi\varphi$ (or 22) amplitude below threshold. Actually an innocuous mirror resonance is seen on the physical Riemann sheet too. The dynamical resonance on the second Riemann sheet seems genuine and physical and should be considered as a valid prediction of the $R + \alpha R^2$ model after unitarization. In fact, all evidence points to the fact that it corresponds to a bound state of scalarons. Of course, further work with other unitarization methods would be very convenient in order to check the stability of the prediction. As for the amplitudes, they all tend to the same asymptotic value for scalaron masses in the sub-Planckian region. This limit coincides with the one of Einstein-Hilbert gravity, after unitarization, and shows a limiting constant behavior independent of s .

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DATA AVAILABILITY

No data were created or analyzed in this study.

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