

Potential of the reaction $e^+e^- \rightarrow p\bar{p}\pi^0$ for constructing higher ρ -meson spectroscopy above 2.4 GeV

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The spectrum of light unflavored vector mesons above 2.4 GeV has not yet been firmly established experimentally. In this work, we demonstrate that the process $e^+e^- \rightarrow p\bar{p}\pi$ serves as a particularly promising channel as a probe for higher isovector ρ excitations. In contrast to typical electron-positron annihilation processes into purely light-meson final states, the reaction $e^+e^- \rightarrow p\bar{p}\pi$ benefits from a relatively low continuum background, on the order of (50 ~ 100) pb around 2.4 GeV, which enhances the visibility of even modest resonance contributions. In addition, the comparatively high production threshold further suppresses potential contamination from lower ρ -meson excitations. Our analysis based on effective Lagrangian approach indicates that the observed line shape of the $e^+e^- \rightarrow p\bar{p}\pi$ cross section measured by the Beijing Spectrometer (BESIII) Experiment can be naturally interpreted by including the $\rho(5S)$ and $\rho(6S)$ contributions, namely the fourth and fifth radial excitations of $\rho(770)$. In particular, the pronounced rise of the cross section near 2.4 GeV provides strong evidence for the contribution of $\rho(5S)$ signal. Their branching ratios to the $p\bar{p}\pi$ final state are found to be below 4%, which do not contradict with usual expectations. In addition, we evaluate pion-induced production of these two ρ -meson states as a complementary probe, offering useful guidance for the relevant experiments such as the Common Muon and Proton Apparatus for Structure and Spectroscopy.

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I. INTRODUCTION

Light hadron spectroscopy provides a rich laboratory for exploring the strong interaction in the nonperturbative regime, where the color confinement dynamics are further complicated by active sea-quark effects inside hadrons and relativistic contributions become significant. Moreover, many candidates for exotic hadronic states such as glueballs or hybrids are expected to emerge in this sector [1,2]. Within

the light hadron family, unflavored vector mesons are of particular interest because their production can be directly accessed in electron–positron annihilation experiments such as BABAR [3], Belle [4] and Beijing Spectrometer (BES) [5,6], which have accumulated extensive and high-precision datasets over the past decades [7–9].

The spectroscopy of highly excited mesons usually offer a sensitive probe of the long-distance behaviors of color confinement, making systematic investigations of the ρ -, ω -, and ϕ -like excitations an important task for hadron physics. Research on highly excited states of light unflavored vector mesons has so far reached up to approximately 2.2 GeV. In the ϕ -meson family, the most well-known example is $\phi(2175)$ [10,11], initially proposed as a strange-sector partner of the charmoniumlike state $Y(4260)$ [12,13]. This state has attracted extensive theoretical discussions,

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and one popular interpretation is strangemonium excitation $\phi(3S)$ or $\phi(2D)$ [14–17]. A theoretical combined analysis of electron-positron annihilation to open-strange channels shows that the $\phi(2175)$ signal should simultaneously involve contributions from both the vector states $\phi(3S)$ and $\phi(2D)$, whose theoretical masses and decay behaviors based on an unquenched relativized potential model are taken as input of the fit analysis [18]. For the ω -meson family, the BESIII Collaboration recently had observed two enhancement structures near 2.2 GeV existing in $e^+e^- \rightarrow \omega\eta$ and $e^+e^- \rightarrow \omega\pi^0\pi^0$ [19,20], which have been suggested as the interference structures from highly excited states $\omega(4S)$ and $\omega(3D)$ [21]. In addition, the authors in Ref. [22] introduced the $4S-3D$ mixing effect in the highly excited ω spectroscopy to explain the ω -like structure $Y(2119)$ recently observed in $e^+e^- \rightarrow \rho\pi$ and $e^+e^- \rightarrow \rho(1450)\pi$ [23].

In the isovector ρ -meson family, there are three excited states $\rho(1900)$, $\rho(2000)$, and $\rho(2150)$ around 2 GeV collected in the Particle Data Group [10], whose dominant components were usually considered as ρ -meson excitations $\rho(3S)$, $\rho(2D)$ and $\rho(4S)$, respectively, in various theoretical approaches such as potential model [24–27], Regge trajectory [28,29], and other phenomenological models [30–32]. In recent years, the BESIII Collaboration performed the precise measurement of the cross sections for several isospin vector processes and also observed some ρ -like enhancement structures around 2 GeV [19,33,34]. According to a recent theoretical analysis for the six electron-positron annihilation processes with different light-meson final states, the line shapes of these enhancement structures can also be well reproduced by introducing the contribution from higher ρ meson excitations around 2.2 GeV [35].

Despite these advances, the higher light vector meson states above 2.2 GeV have remained elusive experimentally. The search for such higher excitations is undoubtedly crucial for extending our understanding of the spectroscopy of light vector mesons, and is essential for testing the validity of various theoretical models. In Ref. [36], an unquenched relativized potential model was employed to predict the next ρ -meson excitations, namely the $\rho(5S)$ and $\rho(6S)$, should lie around 2.4 GeV and 2.6 GeV, respectively, both with typical widths of about 100 MeV. Their isoscalar partners in the ω -meson family are also expected in this mass region, but none of these states have yet been discovered experimentally. Traditionally, searches for higher ρ excitations above 2.4 GeV should focus on typical light-meson final states, as observed for the ρ -meson states near 2 GeV, since such channels are generally considered to be their dominant decay modes. In contrast, baryonic final states, such as those involving a nucleon-antinucleon pair, are usually disfavored in this context, because the decay into these channels requires the creation of an additional light quark pairs from the vacuum, which suppresses their

branching ratios. As a result, these modes have often been overlooked in previous searches.

In this work, however, we point out that the process $e^+e^- \rightarrow p\bar{p}\pi$ provides a promising alternative channel for exploring the $\rho(5S)$ and $\rho(6S)$ states. Recently, BESIII reported precise measurements of the $e^+e^- \rightarrow p\bar{p}\pi$ cross section [37]. Although the available energy points are not sufficiently dense, we notice that a nontrivial rapidly enhanced point around 2.4 GeV, intriguingly close to the predicted mass of the $\rho(5S)$, seems difficult to be produced by the continuum term. Motivated by this, we perform a detailed analysis of the cross section using an effective Lagrangian approach, and find strong evidence suggesting the presence of both the $\rho(5S)$ and $\rho(6S)$ states. It is worth noting that the role of the $\omega(5S)$ and $\omega(6S)$ in this process basically can be ruled out. Moreover, two different analysis schemes consistently show that the branching fractions of $\rho(5S)$ and $\rho(6S)$ decays into the $p\bar{p}\pi$ final state do not exceed 4%, which is consistent with the expected suppression mechanism.

Although the $p\bar{p}\pi$ mode is not expected to be the dominant decay channel, it is important to emphasize that the visibility of a resonance signal in experiment does not solely depend on its coupling strength to a given final state. Instead, the relative size of the resonance contribution with respect to the continuum background plays a decisive role. If the continuum background is overwhelmingly large, even a signal with a sizable branching ratio may still be obscured. In the case of $e^+e^- \rightarrow p\bar{p}\pi$, a key advantage lies in its relatively small background contribution, at the level of about (50 ~ 100) pb around 2.4 GeV [37], which leaves room for the $\rho(5S)$ and $\rho(6S)$ signals to emerge. For comparison, in Fig. 1 we present the cross section data between 2.3 and 3.1 GeV for the e^+e^- annihilation channels into different light mesons together with the theoretical resonance contributions from the $\rho(5S)$ and $\rho(6S)$ states [35], including $e^+e^- \rightarrow \omega\pi^0$ [19,38], $e^+e^- \rightarrow \rho\eta$ [33], $e^+e^- \rightarrow a_2(1320)\pi^0$ [33], and $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ [39,40]. In all these channels, the continuum backgrounds are much larger, which may partly explain why the two higher ρ -meson excitations are still missing. Another notable advantage of the $e^+e^- \rightarrow p\bar{p}\pi$ reaction is its relatively high threshold, which strongly suppresses the contributions from lower ρ -meson excitations. From an experimental perspective, the proton in the final state is more stable compared with purely light-meson final states, offering further benefits in data reconstruction.

Finally, motivated by these findings, we have also calculated the pion-induced production of the $\rho(5S)$ and $\rho(6S)$ states and predicted the corresponding cross section, including both contact and t -channel amplitudes. These results may provide useful input for future searches for the $\rho(5S)$ and $\rho(6S)$ states at pion-proton collision facilities such as Common Muon and Proton Apparatus for Structure and Spectroscopy [41].

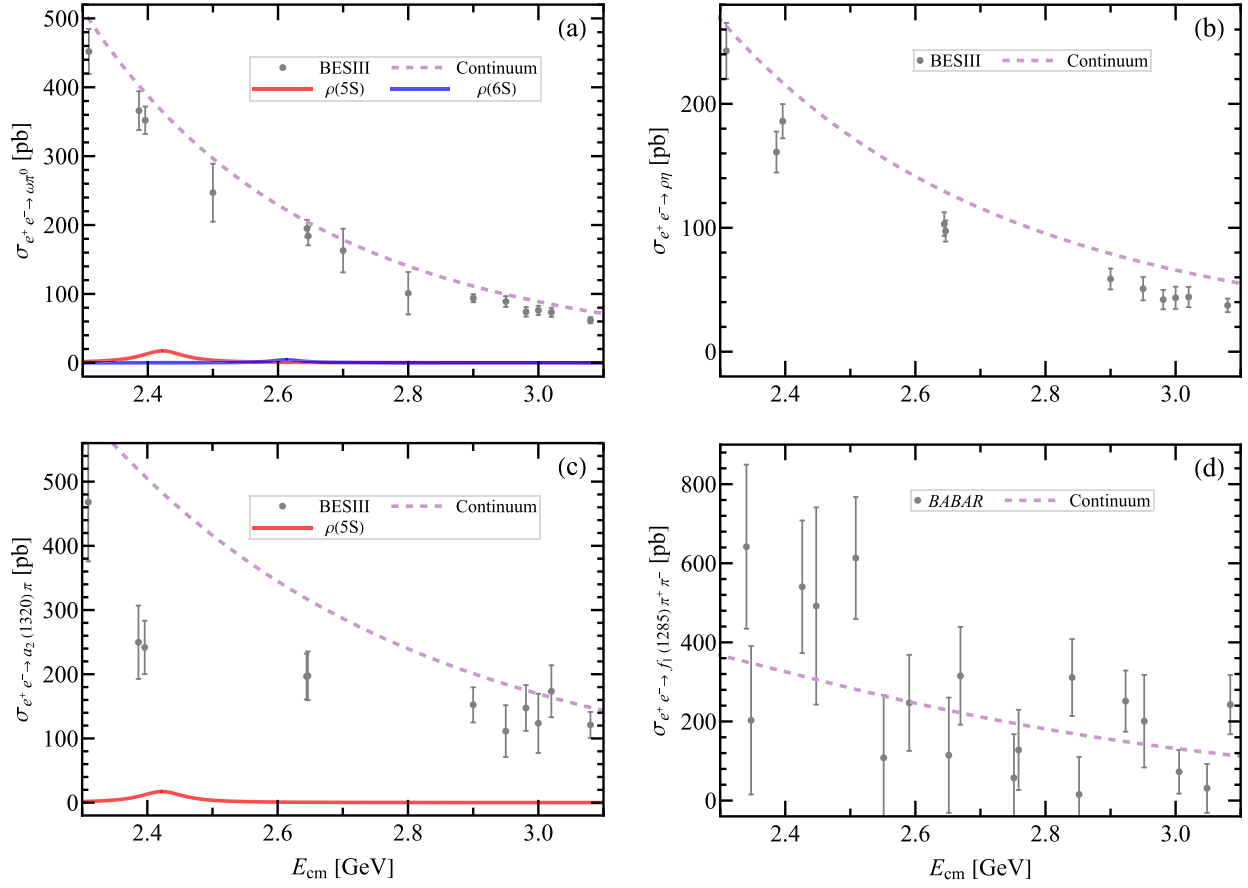


FIG. 1. Total cross sections of (a) $e^+e^- \rightarrow \omega\pi^0$ [19,38], (b) $e^+e^- \rightarrow \rho\eta$ [33], (c) $e^+e^- \rightarrow a_2(1320)\pi^0$ [33], and (d) $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ [39,40] above 2.3 GeV. The fitted continuum and theoretical resonance contributions of $\rho(5S)$ and $\rho(6S)$ are from Ref. [35]. Here, the reason for the absence of the resonance contribution in (b) and (d) is the tiny decay branching ratio of $\rho(5/6S)$ to the corresponding final states.

This paper is organized as follows: Sec. II presents a general analysis of t -channel dominance in baryonic and mesonic production mechanisms. In Sec. III, we investigate the contributions of the higher $\rho(5/6S)$ resonances to the $e^+e^- \rightarrow p\bar{p}\pi^0$ process, including both resonant and non-resonant amplitudes, and perform fits to the BESIII data. In Sec. IV, we extend our study to pion-induced reactions $\pi p \rightarrow \rho(5/6S)p$, where both contact and t -channel exchange mechanisms are considered, and we provide predictions for branching cross sections into different final states. Finally, Sec. V contains the summary.

II. DOMINANCE ANALYSES IN THE $\rho^* \rightarrow p\bar{p}\pi^0$ AND $\pi p \rightarrow \rho^*p$ PROCESSES

To clarify the production mechanisms in $e^+e^- \rightarrow p\bar{p}\pi^0$, we first analyze the relative importance of different topologies. For the resonant diagrams $e^+e^- \rightarrow \rho^*(p_1) \rightarrow (M \rightarrow p\bar{p})(p_3, p_4)\pi(p_2)$ and $e^+e^- \rightarrow \rho^*(p_1) \rightarrow (N^* \rightarrow p\pi)(p_3, p_2)\bar{p}(p_4)$, in which ρ^* stands for higher excited vector meson state, the corresponding amplitudes are denoted as

$$\mathcal{M}_1 \propto \frac{1}{m_{34}^2 - m_c^2}, \quad (1)$$

$$\mathcal{M}_2 \propto \frac{1}{m_{23}^2 - m_c^2} \quad (2)$$

with the kinematic constraint $m_3 + m_4 < m_{34} < m_1 - m_2$ and $m_2 + m_3 < m_{23} < m_1 - m_4$. Here, m_c is the mass of intermediate meson state M or nucleon state N^* . In the limit $m_\pi \simeq 0$ and $m_N \simeq 1$, the amplitudes approximately scale as

$$\mathcal{M}_1 \propto \frac{1}{[4m_N^2, m_{\rho^*}^2] - m_c^2} \propto \frac{1}{\mathcal{O}(4m_N^2) - m_c^2}, \quad (3)$$

$$\mathcal{M}_2 \propto \frac{1}{[m_N^2, (m_{\rho^*} - m_N)^2] - m_c^2} \propto \frac{1}{\mathcal{O}(m_N^2) - m_c^2}. \quad (4)$$

This comparison indicates that we should retain three kinds of dominant mechanisms for resonant diagrams of $e^+e^- \rightarrow p\bar{p}\pi^0$ in subsequent analysis: (1) the ρ^* couples to $M\pi$ with a mass of intermediate meson M around the $p\bar{p}$ threshold;

(2) the ρ^* couples to $N^*\bar{p}$ with an intermediate nucleon $N^* = p$; (3) the direct $p\bar{p}\pi$ coupling of the ρ^* resonance. We note that the discussions apply equally to the crossed channel $e^+e^- \rightarrow \rho^* \rightarrow \bar{N}^*p$.

For meson-induced reactions such as πp or Kp scattering to ρ^*p , the situation differs: both t -channel (light-meson exchange) and u -channel (nucleon exchange) diagrams contribute. In these cases, the generic amplitude takes the form

$$\mathcal{M}_1 \propto \frac{1}{p_{ex}^2 - m_c^2}, \quad (5)$$

where the exchanged momentum squared p_{ex}^2 equals to t or u . By setting $s = (m_N + m_{\rho^*} + \delta)^2$ with $m_\pi/m_K = \epsilon$, and expanding to leading order in $\mathcal{O}(\delta)$ and $\mathcal{O}(\epsilon)$, one obtains

$$t_{\min} = t_0 - \Delta t, \quad t_{\max} = t_0 + \Delta t, \quad (6)$$

$$u_{\min} = u_0 - \Delta u, \quad u_{\max} = u_0 + \Delta u, \quad (7)$$

where

$$\begin{aligned} t_0 &= -\frac{1}{(m_N + m_{\rho^*})^2} (m_N^2 m_{\rho^*}^2 + m_N m_{\rho^*}^3 \\ &\quad + \delta(m_{\rho^*}^3 + 4m_N^2 m_{\rho^*} + 5m_N m_{\rho^*}^2)), \\ \Delta t &= \frac{3m_N m_{\rho^*}^2 + m_{\rho^*}^3 + 2m_N^2 m_{\rho^*}}{(m_N + m_{\rho^*})^3} \\ &\quad \times \sqrt{\delta(2m_N^2 m_{\rho^*} + 2m_N m_{\rho^*}^2) + \delta^2(m_N^2 + 3m_N m_{\rho^*} + m_{\rho^*}^2)}, \\ u_0 &= -\frac{1}{(m_N + m_{\rho^*})^2} (-m_N^4 + 2m_N^2 m_{\rho^*}^2 + m_N m_{\rho^*}^3 \\ &\quad + \delta(m_{\rho^*}^3 + 4m_N^2 m_{\rho^*} + 5m_N m_{\rho^*}^2)), \\ \Delta u &= \frac{3m_N m_{\rho^*}^2 + m_{\rho^*}^3 + 2m_N^2 m_{\rho^*}}{(m_N + m_{\rho^*})^3} \\ &\quad \times \sqrt{\delta(2m_N^2 m_{\rho^*} + 2m_N m_{\rho^*}^2) + \delta^2(m_N^2 + 3m_N m_{\rho^*} + m_{\rho^*}^2)}. \end{aligned} \quad (8)$$

Since $\Delta u = \Delta t > 0$ and both t_0 and u_0 are negative, their magnitudes can be directly compared. It can be seen that t_0 or u_0 consists of a constant term and a term proportional to variable δ , relating to the c.m. energy $\sqrt{s}$, while the Δt term is entirely linear in δ . Near the production threshold, δ should be a small quantity, so the p_{ex}^2 always takes negative values. Consequently, one can conclude that the smaller the mass of the exchanged particle, the larger its contribution through the t - or u -channel near production threshold. To further compare the relative importance of the u - and t -channel contributions, we define the relative discrepancy as

$$\mathcal{A} = \frac{(u_0 - t_0)}{u_0}, \quad (9)$$

one finds

$$\mathcal{A}_{\max} = \mathcal{A}|_{\delta=0} = \frac{m_N(m_N - m_{\rho^*})}{m_N^2 - m_N m_{\rho^*} - m_{\rho^*}^2}. \quad (10)$$

Numerically, $\mathcal{A}_{\max} = 0.192$ for $\rho(5S)$ and $\mathcal{A}_{\max} = 0.187$ for $\rho(6S)$. These values indicate that the t - and u -channel contributions are comparable for exchange particles with equal mass. At lower energies, however, exchanges involving lighter mesons dominate in the t -channel, whereas the nucleon-exchange contribution in the u -channel is strongly suppressed. Consequently, in the following calculations we primarily focus on the pion exchange in the t -channel together with the direct contact coupling for the $\pi p \rightarrow \rho^* p$ reaction.

III. HIGHER EXCITED $\rho(5/6S)$ RESONANCE CONTRIBUTIONS IN $e^+e^- \rightarrow p\bar{p}\pi^0$

In e^+e^- collisions, the annihilation of an electron and a positron produces a virtual photon that subsequently couples to the final-state particles or other intermediate particles. Within the effective Lagrangian method, the process $e^+(k_2)e^-(k_1) \rightarrow p(p_2)\bar{p}(p_3)\pi^0(p_4)$ proceeds through both resonant and nonresonant channels.

In the energy region around 2.5 GeV, the modified Godfrey-Isgur model predicted the existence of unflavored excited light vector mesons belonging to the $\rho(5/6S)$ and $\omega(5/6S)$ [18,36]. This prediction is obtained by successfully reproducing the mass spectrum of the low-lying light mesons. Since the dielectron widths of the ω -meson excitations are usually found to be about 1 order of magnitude smaller than those of the corresponding ρ -meson partners [18,36], we therefore give priority to the resonance contributions from the $\rho(5S)$ and $\rho(6S)$ states in our analysis. The relevant resonance parameters, including the predicted masses, total widths, dilepton widths, and partial decay branching ratios, are collected in Table I, which provides the essential input for the present analysis. The resonant contributions include the decays of the intermediate $\rho(5/6S)$ resonance state. These decays include:

- (i) $\rho(5/6S) \rightarrow N\bar{N}^*$ (Fig. 2(a)) and $\rho(5/6S) \rightarrow \bar{N}N^*$ [Fig. 2(b)], where the two processes are distinguished by the momentum topologies of the intermediate nucleons or antinucleons, denoted as p_t and p_u .
- (ii) $\rho(5/6S) \rightarrow \pi^0 M$ [Fig. 2(c)], where M denotes a possible intermediate meson such as π , ρ , ω , or $a_2(1320)$, among others the direct three-body decay $\rho(5/6S) \rightarrow p\bar{p}\pi^0$ [Fig. 2(e)], which belongs to the contact term.

TABLE I. Masses, total widths, dilepton widths, and decay branching ratios of $\rho(5/6S)$ calculated in Refs. [18,36]. An ellipsis denotes that the branching ratios of $\rho(5/6S)$ are negligible.

	$\rho(5S)$	$\rho(6S)$
M_V	2422 MeV	2613 MeV
Γ_{total}	81.85 MeV	57.25 MeV
$\Gamma_{V \rightarrow e^+e^-}$	0.0377 keV	0.0204 keV
$\pi a_2(1700)$	13.61%	10.62%
$\rho\rho(1450)$	9.41%	9.13%
$\pi\pi(1300)$	8.40%	7.31%
$\pi\pi_2(1880)$	...	7.56%
$\omega a_0(1450)$	7.83%	4.07%
$\pi\omega(1420)$	7.56%	5.04%
$\pi\pi(1800)$	6.97%	7.33%
$\pi a_2(1320)$	4.11%	...
$\pi h_1(1170)$	3.28%	...
$\pi\pi$	2.33%	1.73%
$\pi\omega$	1.50%	0.57%

In addition, the nonresonant contribution corresponds to a direct coupling with virtual photon [Fig. 2(d)].

In principle, all five diagrams shown in Fig. 2 should be considered, and the intermediate $N^{(*)}$, $\bar{N}^*$ and M all corresponding to several possible candidate resonance in the relevant energy region. However, as discussed in Sec. II, the dominant nucleon and meson resonances are expected to have masses near the $p\bar{p}$ and πN thresholds, respectively. Based on model predictions on light meson decays into baryon-antibaryon pair [42,43], the branching fraction of $\rho(5/6S) \rightarrow p\bar{p}$ can be roughly estimated to be at the level of 10^{-3} . Based on this, we calculate the peak cross sections of the subprocesses in Figs. 2(a) and 2(b) as around 0.06 pb, which is about 3 orders of magnitude smaller than the experimental measurements [37]. From Table I, it can be seen that the $\pi^0 M$ decay ratios with an

intermediate meson M near $p\bar{p}$ threshold are too small for both $\rho(5S)$ and $\rho(6S)$ that their numerical results are not provided in Ref. [36]. For the channels shown in Fig. 2(c), taking $\rho(5S) \rightarrow (\rho(2150) \rightarrow p\bar{p})\pi^0$ as an example, the corresponding peak cross section is estimated to be about 8×10^{-3} pb if the branching ratio $\rho(5S) \rightarrow \pi\rho(2150)$ and $\rho(2150) \rightarrow p\bar{p}$ are taken to be at most 1% and 10^{-3} . Therefore, the contributions from the intermediate states in Figs. 2(a)–(c) are negligible compared to the experimental data, and in our calculation we mainly focus on the direct background term [Fig. 2(d)] and the dominant direct resonant contact term [Fig. 2(e)].

To account for the coupling between the photon and the vector mesons, we adopt the vector meson dominance mechanism. Within this framework, the $\gamma\rho(5/6S)$ interaction is described by the effective Lagrangian

$$\mathcal{L}_{\gamma V} = \frac{-eM_V^2}{f_V} V_\mu A^\mu, \quad (11)$$

where M_V and f_V denote the mass and decay constant of the vector meson, respectively, and V_μ and A_μ represent the vector meson and photon fields. The constant f_V is determined by the dilepton decay width $\Gamma_{V \rightarrow e^+e^-}$ through the relation

$$\frac{e}{f_V} = \left[\frac{3\Gamma_{V \rightarrow e^+e^-} M_V^2}{\alpha(M_V^2 - 4m_e^2)^{3/2}} \right]^{1/2}, \quad (12)$$

with the fine-structure constant $\alpha = 1/137$. In Refs. [18,36], the dilepton widths of light unflavored vector mesons were systematically calculated.

For the direct background contribution shown in Fig. 2(d), the virtual photon produced in e^+e^- annihilation couples directly to the $p\bar{p}\pi^0$ final state without involving intermediate resonances. The corresponding amplitude can be expressed as [44–46]

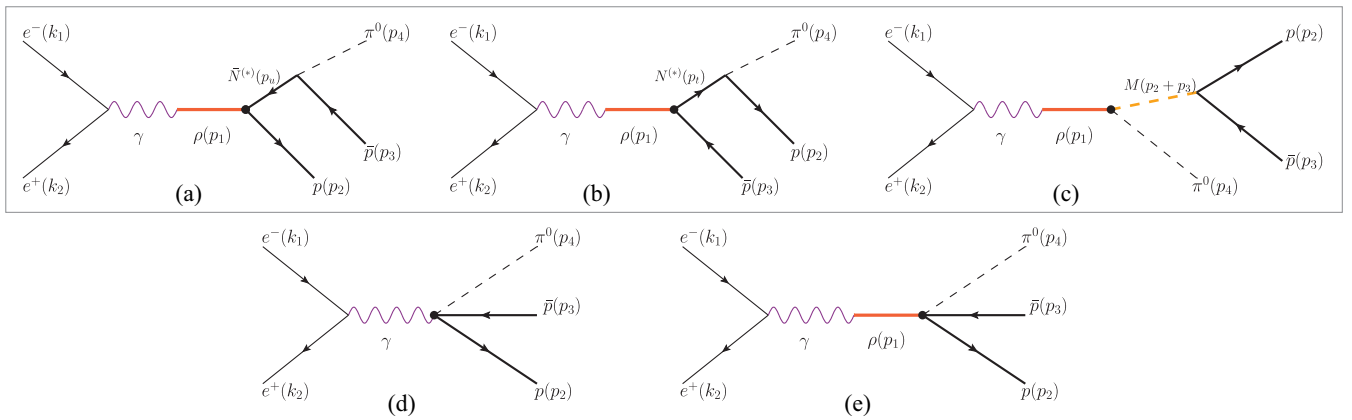


FIG. 2. Feynman diagrams for the process $e^+e^- \rightarrow p\bar{p}\pi^0$: (a)–(c) denote the resonant contributions from the cascade decays of $\rho(5/6S)$, which are found to be negligible in the present case. Diagrams (d) and (e) correspond to the direct background and the direct resonant contribution, respectively.

$$\mathcal{M}_{\text{NoR}} = g_{\text{NoR}} \bar{v}(k_2) e \gamma_\mu u(k_1) \frac{-g^{\mu\nu}}{\tilde{p}^2} \bar{u}(p_2) \gamma_\nu \gamma_5 v(p_3) \mathcal{F}_{\text{NoR}}(\tilde{p}^2), \quad (13)$$

where g_{NoR} is an effective coupling constant determined by fitting to experimental data. The background form factor is chosen in an optimized form, compared with that used in Refs. [44,45],

$$\mathcal{F}_{\text{NoR}}(s) = \exp \left[-a \left(\sqrt{s} - \sum_f m_f \right)^2 - b \left(\sqrt{s} - \sum_f m_f \right) \right], \quad (14)$$

where the m_f represents the mass of the final states. The parameters a and b are fitted to experimental data.

Similarly, the amplitudes for the resonant contributions are written as

$$\begin{aligned} \mathcal{M}_{\rho(5/6S)} &= g_{\rho(5/6S)} \frac{e M_{\rho(5/6S)}^2}{f_{\rho(5/6S)}} \bar{v}(k_2) e \gamma_\mu u(k_1) \frac{-g^{\mu\nu}}{\tilde{p}^2} \\ &\times \frac{-g_{\nu\alpha} + \tilde{p}_\nu \tilde{p}_\alpha / \tilde{p}^2}{\tilde{p}^2 - M_{\rho(5/6S)}^2 + i M_{\rho(5/6S)} \Gamma_{\rho(5/6S)}} \bar{u}(p_2) \gamma^\alpha \\ &\times \gamma_5 v(p_3), \end{aligned} \quad (15)$$

where $\tilde{p} = (k_1 + k_2)$.

In general, the interference effects from the contribution of different resonances should be taken into account. Thus, the total amplitude with relative phases can be written as

$$\mathcal{M}_{e^+e^- \rightarrow p \bar{p} \pi^0} = \mathcal{M}_{\text{NoR}} + e^{i\phi_1} \mathcal{M}_{\rho(5S)} + e^{i\phi_2} \mathcal{M}_{\rho(6S)}, \quad (16)$$

where the relative phases ϕ_1 and ϕ_2 are treated as free parameters fitted within the range $[0, 2\pi]$.

Then, the differential cross section for the process $e^+e^- \rightarrow \rho(5/6S) \rightarrow p \bar{p} \pi^0$ can be written as

$$d\sigma_{e^+e^- \rightarrow p \bar{p} \pi^0} = \frac{(2\pi)^4 |\overline{\mathcal{M}_{e^+e^- \rightarrow p \bar{p} \pi^0}}|^2}{4\sqrt{(k_1 \cdot k_2)^2}} d\Phi_3, \quad (17)$$

where the overline denotes averaging over the spins of the initial state and summing over the spins of the final state. The three-body phase-space element is given by

$$d\Phi_3 = \frac{1}{8(2\pi)^9 \sqrt{s}} |\vec{p}_3^*| |\vec{p}_2| d\Omega_3^* d\Omega_2 dM_{\bar{p}\pi}, \quad (18)$$

where $\vec{p}_3^*$ and Ω_3^* denote, respectively, the three-momentum and solid angle of the antiproton in the rest frame of the $\bar{p}\pi$ subsystem. After integrating over the three-body phase space, the total cross section $\sigma_{e^+e^- \rightarrow p \bar{p} \pi^0}$ is obtained.

Based on the above formalism, we fit the $e^+e^- \rightarrow p \bar{p} \pi^0$ cross section data with nine free parameters. Owing to the

TABLE II. Fitted parameters for the $e^+e^- \rightarrow p \bar{p} \pi^0$ process for two scenarios, Fit I and Fit II.

Parameters	Fit I	Fit II
g_{NoR}	8.4 ± 0.4	8.3 ± 0.32
a	-0.82 ± 0.14	-0.89 ± 0.14
b	3.71 ± 0.19	3.77 ± 0.17
$M_{\rho(5S)}$ [MeV]	2400(Fixed)	2422(Fixed)
$\Gamma_{\rho(5S)}$ [MeV]	150 ± 70	153 ± 22
$g_{\rho(5S)}$	21.9 ± 3.4	25 ± 10
ϕ_1 [rad]	3.54 ± 0.12	3.82 ± 0.09
$M_{\rho(6S)}$ [MeV]	2613(Fixed)	2613(Fixed)
$\Gamma_{\rho(6S)}$ [MeV]	100 ± 70	100 ± 40
$g_{\rho(6S)}$	10 ± 4	10.0 ± 1.7
ϕ_2 [rad]	3.0 ± 0.8	3.0 ± 0.5
$\chi^2/\text{d.o.f.}$	1.337	1.496

limited statistics and the sparse energy coverage near the $\rho(5S)$ region, two fitting scenarios are considered: Fit I, with the $\rho(5S)$ mass fixed at 2400 MeV, and Fit II, with the $\rho(5S)$ mass fixed at 2422 MeV. Using the 20 experimental data points reported in Ref. [37], we obtain minimum $\chi^2/\text{d.o.f.}$ values of 1.337 and 1.496 for Fits I and II, respectively. These correspond to p values of 0.196 and 0.125 for Fits I and II, respectively, indicating statistically good fits. The extracted resonance parameters, including total widths and coupling constants, are listed in Table II. Although the fitted total widths of $\rho(5/6S)$ are somewhat larger than the sum for two-body decay widths estimated in the quark pair production (QPC) model (Table I), they remain compatible within the present experimental uncertainties. From the fitted parameters, the branching fractions for $\rho(5/6S) \rightarrow p \bar{p} \pi^0$ are determined to be $\mathcal{B}[\rho(5S)] = 2.2\%$ (Fit I), $\mathcal{B}[\rho(5S)] = 3.3\%$ (Fit II), and $\mathcal{B}[\rho(6S)] = 2.4\%$, which do not contradict usual expectations [10]. It can also be seen that if the ρ -resonance contribution were replaced by the ω -resonance here, the resulting branching ratio for $\omega(5/6S) \rightarrow p \bar{p} \pi$ would be estimated at about 30%, which is obviously unphysical. This in turn supports the consistency of our initial treatment. For comparison, if we fix the masses and widths of the $\rho(5S)$ and $\rho(6S)$ states to the values in Table I and fit only the remaining seven parameters, we obtain $\chi^2 = 20.01$, which is worse than both Fits I and II and indicates that the additional parameters are statistically justified and overfitting is not severe.

The fitted results, incorporating both the direct background and the $\rho(5/6S)$ resonance contributions, are displayed in Fig. 3, together with the BESIII experimental Born cross section data [37]. As expected, the direct background term [Fig. 2(d)] provides the dominant contribution and gives a smooth energy dependence without any resonance structure. However, it alone cannot

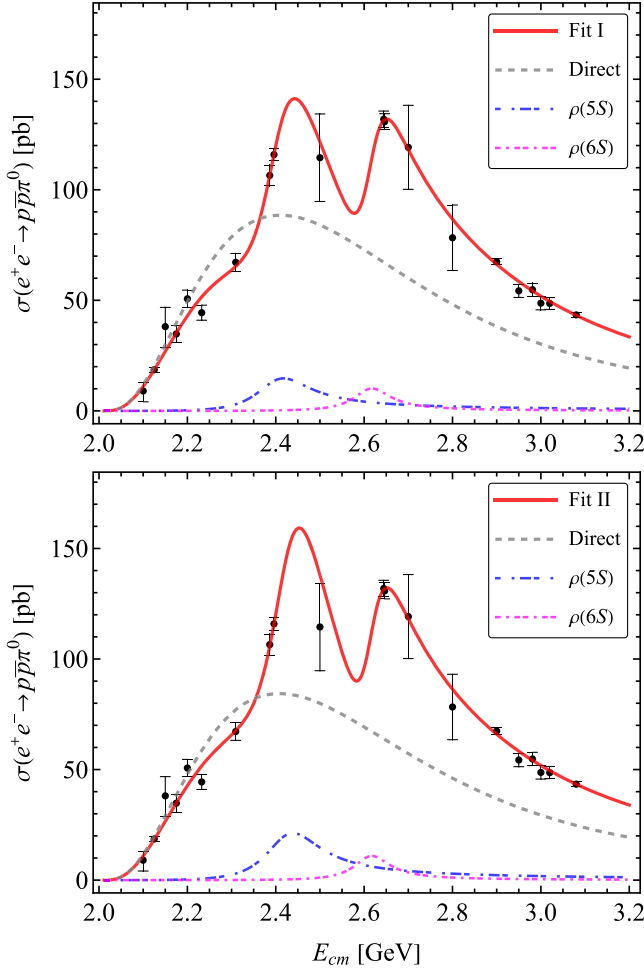


FIG. 3. Fitted Born cross sections for Fit I and Fit II scenarios of the $e^+e^- \rightarrow p\bar{p}\pi^0$ process [37] are shown by the red solid curves. The contributions from the direct background and the intermediate $\rho(5/6S)$ resonances are also displayed.

reproduce the experimental data, which strongly supports the necessity of including the $\rho(5/6S)$ resonance contributions. In particular, the pronounced rise near 2.4 GeV provides compelling evidence for the presence of the anticipated $\rho(5S)$ state.

Both Fit I and Fit II achieve good agreement with the measured cross sections. The main difference is that the $\rho(5S)$ peak in Fit II is slightly larger than that in Fit I. Because of the relatively larger $\rho(5S)$ contribution and smaller $\rho(6S)$ contribution in Fit II, the overall line shape is characterized by one prominent peak followed by another smaller one. Although the precise $\rho(5S)$ line shape is not well constrained due to the limited data quality, both fits require a clear Breit-Wigner structure, highlighting the role of an isovector resonance. Moreover, the total fitted curves show a slight shift of the resonance peaks toward higher energies, which arises naturally from interference effects.

The interference plays an indispensable role in shaping the cross sections. On the one hand, destructive interference

is observed around 2.3 GeV, where the total cross section even falls below the background term, and around 2.55 GeV, where a dip appears deeper than the standalone $\rho(6S)$ contribution. On the other hand, constructive interference occurs around 2.45 GeV and 2.65 GeV, where two pronounced peaks emerge and both are higher than the magnitudes of the individual resonances. These interference effects also slightly shift the effective peak positions away from the nominal $\rho(5/6S)$ masses.

Unfortunately, the current experimental data suffer from large uncertainties and sparse energy bins. More precise and finely binned measurements in the future will be crucial for improving the reliability of the analysis and confirming the existence of these two long-term unestablished isovector light vector meson excitations.

IV. PRODUCTION OF $\rho(5/6S)$ RESONANCES IN πp SCATTERING

Based on the fit procedure of the $e^+e^- \rightarrow p\bar{p}\pi^0$ reaction discussed in Sec. III, we can extract the effective contact couplings of the $\rho(5/6S)N\bar{N}\pi$ vertex, in which the results of Fit I are taken as input. With these couplings, we further investigate the production behaviors of $\rho(5/6S)$ in the $\pi p \rightarrow \rho(5/6S)p$ reaction, as illustrated in Fig. 4. In addition, following the discussion in Sec. II, we take into account the t -channel pion-exchange contribution, while neglecting the relatively suppressed u -channel nucleon-exchange diagram.

For the πNN interaction, we employ the well-established effective Lagrangian [47]

$$\mathcal{L}_{\pi NN} = \frac{g_{\pi NN}}{2M_N} \bar{N} \gamma^\mu \gamma_5 \partial_\mu \vec{\pi} \cdot \vec{\tau} N, \quad (19)$$

with the coupling constant fixed at $g_{\pi NN} = 13.26$. For the other relevant vertex in the t -channel, we adopt the effective Lagrangian [48]

$$\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi} [\vec{\pi} \times \partial_\mu \vec{\pi}] \cdot \vec{\rho}^\mu, \quad (20)$$

where $\vec{\pi}$ and $\vec{\rho}^\mu$ denote the pion- and ρ -meson fields, respectively. The coupling constants $g_{\rho(5/6S)\pi\pi}$ are extracted from the partial decay branching ratios listed in Table I together with the fitted total widths in Fit I given in Table II.

The differential cross sections for the process $\pi(k_1)p(p_1) \rightarrow \rho(5/6S)(k_2)p(p_2)$ in the c.m. frame are expressed as

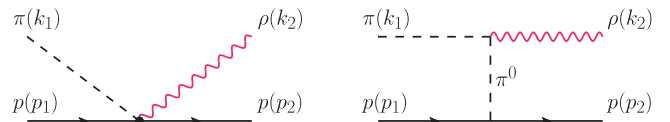


FIG. 4. Contact and t -channel Feynman diagrams of the process $\pi p \rightarrow \rho(5/6S)p$.

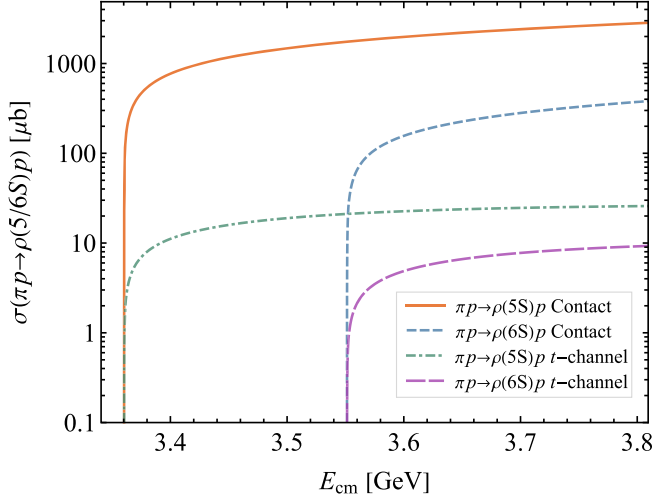


FIG. 5. Production cross sections of the $p\bar{p} \rightarrow \pi^0 \rho(5/6S)p$ reaction, showing the contributions from the contact term and the t -channel exchange.

$$\frac{d\sigma_{\pi p \rightarrow \rho(5/6S)p}}{d\cos\theta} = \frac{1}{32\pi s} \frac{|\vec{p}_2^{\text{cm}}|}{|\vec{p}_1^{\text{cm}}|} |\mathcal{M}_{\rho(5/6S)p}|^2, \quad (21)$$

then integrating over $\cos\theta$ yields the production cross sections.

In Fig. 5, we present the production cross sections of the $\pi p \rightarrow \rho(5/6S)p$ process, including both the contact term and the t -channel pion-exchange contributions. The production behaviors of $\rho(5S)$ and $\rho(6S)$ are qualitatively similar: the cross sections increase steeply just above the corresponding thresholds, reflecting the opening of phase space, and subsequently level off as the c.m. energy increases.

A clear hierarchy is observed between the two production mechanisms. For the contact term, the $\rho(5S)$ contribution exceeds that of the $\rho(6S)$ by approximately an order of magnitude, which can be attributed to the stronger effective coupling of the $\rho(5S)$ to the $p\bar{p}\pi^0$. In contrast, the t -channel pion-exchange contributions are significantly suppressed compared to the contact terms, being at least an order of magnitude smaller across the considered energy range. Interestingly, for the t -channel case, the peak cross section of $\rho(5S)$ is only about a factor of 2 larger than that of $\rho(6S)$, implying that in the considered energy region, the off-shell suppression of the pion propagator impacts both states in a similar way, such that the difference in effective couplings does not lead to as significant a hierarchy as observed in the contact contributions.

The percentage uncertainties of the predicted t -channel cross sections are directly proportional to the percentage uncertainties of the fitted decay widths, i.e., $\delta\sigma_t/\sigma_t = \delta\Gamma/\Gamma$. Numerically, the resulting t -channel cross sections $\sigma_t(5S)$ and $\sigma_t(6S)$ carry uncertainties of $\pm 46\%$ and $\pm 70\%$, respectively, relative to the central values plotted in Fig. 5. In addition, we estimate the percentage uncertainties

TABLE III. Branching cross sections for the reactions $\pi p \rightarrow (\rho(5/6S) \rightarrow M_1 M_2)p$, where the final states $M_1 M_2$ are listed below. Cross sections are given in units of μb . An ellipsis indicates a channel for which the $\rho(5/6S)$ branching fractions are negligible.

E_{cm} (GeV)	$\rho(5S)$				$\rho(6S)$		
	3.4	3.5	3.6	3.8	3.6	3.7	3.8
$\sigma(\pi p \rightarrow \pi\pi p)$	18	34	46	65	3	5	6
$\sigma(\pi p \rightarrow \pi\omega p)$	12	22	30	42	1	2	2
$\sigma(\pi p \rightarrow \rho\rho(1450)p)$	72	139	186	264	14	26	34
$\sigma(\pi p \rightarrow \pi\pi(1300)p)$	65	124	166	236	11	21	27
$\sigma(\pi p \rightarrow \pi\omega(1420)p)$	59	113	152	215	8	14	19
$\sigma(\pi p \rightarrow \pi a_2(1320)p)$	32	61	81	115	...	...	...
$\sigma(\pi p \rightarrow \omega a_0(1450)p)$	60	116	155	220	6	11	15
$\sigma(\pi p \rightarrow \pi h_1(1170)p)$	25	48	65	92	...	...	...

of the contact-channel cross sections σ_{Con} to be $\pm 31\%$ and $\pm 80\%$ for $\rho(5S)$ and $\rho(6S)$, respectively. Despite these sizable uncertainties, the results provide a useful estimate of the corresponding contributions.

Overall, these results demonstrate that the contact interaction dominates the $\pi p \rightarrow \rho(5/6S)p$ production, while the t -channel exchange provides only a minor correction. Nevertheless, the t -channel contribution may still play a role in shaping the detailed energy dependence of the cross sections and could become relevant in precision studies.

For reference in experimental analyses, Table III presents the three-body branching cross sections of the processes $\pi p \rightarrow (\rho(5/6S) \rightarrow M_1 M_2)p$ at several typical c.m. energies. The calculations are performed under the narrow-width approximation, which is well justified since the $\rho(5/6S)$ masses are much larger than their widths. The corresponding decay branching ratios of $\rho(5/6S)$ have been summarized in Table I. As shown in Table III, the branching cross sections increase with E_{cm} . Among the final states considered, $\rho\rho(1450)p$, $\pi\pi(1300)p$, $\pi\omega(1420)p$, and $\omega a_0(1450)p$ exhibit the largest measured cross sections. Nevertheless, the $\pi\pi p$ and $\pi\omega p$ final states are expected to be more favorable for experimental studies owing to their higher reconstruction efficiencies.

V. SUMMARY

The spectroscopy of unflavored light vector mesons above 2.4 GeV remains experimentally unknown, despite its importance for understanding strong interaction dynamics and testing quark-model predictions of higher radial excitations. In this work, we have identified the process $e^+e^- \rightarrow p\bar{p}\pi^0$ as a particularly potential channel for exploring the missing $\rho(5S)$ and $\rho(6S)$ states. Owing to its relatively small continuum background and higher production threshold, this reaction offers a cleaner environment compared to the electron-positron annihilation to typical

light-meson final states. Our fits to the BESIII data of $e^+e^- \rightarrow p\bar{p}\pi^0$ within an effective Lagrangian approach shows that the inclusion of $\rho(5S)$ and $\rho(6S)$ is essential to reproduce the observed line shape, with extracted branching fractions to $p\bar{p}\pi^0$ below 4%, consistent with usual expectations. The interference between the two ρ -meson resonances and the continuum background plays an important role, generating both constructive enhancements and destructive dips in the cross section distribution.

To provide complementary probes, we have further evaluated $\pi p \rightarrow \rho(5/6S)p$ production, including both contact and t -channel pion-exchange contributions. The results show that the contact term is dominant, while the t -channel provides only a minor correction. The predicted cross sections rise rapidly near threshold and reach the order of several tens to hundreds of μb at higher energies. Among various decay channels of $\rho(5/6S)$, $\rho\rho(1450)$, $\pi\pi(1300)$, $\omega a_0(1450)$, and $\pi\omega(1420)$ yield the largest branching cross sections, though the $\pi\pi p$ and $\pi\omega p$ modes remain the most favorable for experimental searches due to their reconstruction advantages.

Overall, our study indicates that the $e^+e^- \rightarrow p\bar{p}\pi^0$ and $\pi p \rightarrow \rho(5/6S)p$ processes provide promising avenues for establishing the long-missing $\rho(5S)$ and $\rho(6S)$ states. Improved precision measurements with finer energy scans at BESIII, Belle II or future facilities, together with dedicated pion-beam experiments, will be crucial for

confirming the existence of these excitations and deepening our understanding of the light vector meson spectroscopy above 2.4 GeV.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [19,33,35,37–40].

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